Probing the structures of exotic and halo nuclei

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Two nucleon removal - what are useful regimes?

\[ \sigma_{\text{strip}} = \int db \left\langle \phi_0 \left| S_c \right|^2 (1 - \left| S_1 \right|^2)(1 - \left| S_2 \right|^2) \right| \phi_0 \right\rangle \]

Estimate assuming removal of a pair of uncorrelated nucleons -

\[ \phi_0 (A, r_1, r_2) = \Phi_c (A) \phi_{\ell_1} (r_1) \phi_{\ell_2} (r_2) \]

\[ \sigma_{\text{strip}} \Rightarrow \sigma_{\text{strip}} (\ell_1 \ell_2) \]

contribution from direct 2N removal \( \sigma_{-2N} \)

\[ \sigma_{-2N} = \frac{p(p-1)}{2} \sigma_{\text{strip}} (\ell_\alpha \ell_\alpha) + \frac{q(q-1)}{2} \sigma_{\text{strip}} (\ell_\beta \ell_\beta) \]

\[ + pq \sigma_{\text{strip}} (\ell_\alpha \ell_\beta) \]

D. Bazin et al., MSU preprint, submitted
Complications of 2 neutron removal reactions

B.A. Brown, P.G. Hansen and J.A. Tostevin, submitted

$^{22}\text{O}$ final states below n-threshold from the shell model (B.A. Brown)

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$I^\pi$</th>
<th>$\ell$</th>
<th>$C^2 S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0$^+$</td>
<td>0</td>
<td>0.797</td>
</tr>
<tr>
<td>3.38</td>
<td>2$^+$</td>
<td>2</td>
<td>2.130</td>
</tr>
<tr>
<td>4.62</td>
<td>0$^+$</td>
<td>0</td>
<td>0.115</td>
</tr>
<tr>
<td>4.83</td>
<td>3$^+$</td>
<td>2</td>
<td>3.079</td>
</tr>
<tr>
<td>5.32</td>
<td>1$^-$</td>
<td>1</td>
<td>0.851</td>
</tr>
<tr>
<td>5.93</td>
<td>0$^-$</td>
<td>1</td>
<td>0.332</td>
</tr>
<tr>
<td>6.50</td>
<td>2$^+$</td>
<td>2</td>
<td>0.242</td>
</tr>
</tbody>
</table>
Two neutron knockout from neutron rich nuclei
e.g. $^{23}$O → $^{21}$O (RIKEN measurement at 72A MeV on $^{12}$C)

$\sigma = 82(25)$ mb - is large!!

$\sigma_{\text{strip}}(02) = 0.9$ mb
$\sigma_{\text{strip}}(22) = 0.6$ mb

$\sigma_{-2n} = 6\sigma_{\text{strip}}(02) + 15\sigma_{\text{strip}}(52) = 14$ mb

but $\sigma_{sp}(p_{1/2}) = 12$ mb, $\sigma_{sp}(p_{3/2}) = 11$ mb

$\sigma_{-n}(p) = 2\sigma_{sp}(p_{1/2}) + 4\sigma_{sp}(p_{3/2}) = 68$ mb

leading to the $^{22}$O continuum - n evaporation

Shell model - 1 unit of p-strength leads to bound $^{22}$O

$\sigma_{-n}(p) = 57$ mb

Two proton knockout from neutron rich nuclei

D. Bazin et al., MSU preprint, submitted
Two proton knockout - a useful option?

D. Bazin et al., MSU preprint, submitted.

32Mg production rate R
could gain x100 rate
2p knockout (~1mb)

30Ne e.g. Coulex

D. Bazin et al., MSU preprint, submitted.

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Improving the eikonal approximation

\[ S_{\alpha\beta}(b) = \langle \phi_\beta | S_c(b_c) S_v(b_v) | \phi_\alpha \rangle \]

\[ u_\ell(r) \rightarrow (i/2) \{ H^-_\ell(kr) - S_\ell H^+_\ell(kr) \} \]

eikonal S(b) are poor for lower energy and light particle - small k

dashed - eikonal solid - exact

So, use instead the exact S_\ell, analytically continued to non-integer \ell, or b, in S_{\alpha\beta}

Beyond the eikonal approximation


Nucleon removal cross sections also corrected
Beyond the adiabatic approximation

The adiabatic approximation treats all break-up configurations, but with no explicit reference to \( \phi_k(r) \), by solution of:

\[
[T_R + U(r, R) - (E + \varepsilon_0)] \Psi_{K}^{AD}(r, R) = 0
\]

elastische part

\[
\Psi_{el}^{AD} = |\phi_0\rangle \langle \phi_0 | \Psi_{bu}^{AD}
\]

\[
H_p \Psi_{el}^{AD} = -\varepsilon_0 \Psi_{el}^{AD}
\]

is well approximated

breakup part is less well treated

\[
\langle \phi_k | \Psi_{bu}^{AD} \rangle \neq 0
\]

Quasi-adiabatic continuum of \( H_p \) is assumed degenerate with a new energy \( \varepsilon \) which is a better representation of the states excited.
Quasi-adiabatic type approximations

Using the non-adiabatic few-body model equation

\[
[T_R + U(r, R) + H_p - E] \Psi_{K}^{(+)}(r, R) = 0
\]

\[
\Psi_{el} + \Psi_{bu}
\]

\[
|\phi_k(r)\rangle \quad \varepsilon
\]

\[
\varepsilon(R) \approx \langle \Psi_{bu}^{AD} | H_p | \Psi_{bu}^{AD} \rangle_r / \langle \Psi_{bu}^{AD} | \Psi_{bu}^{AD} \rangle_r
\]

inhomogeneous equation with source term

Important corrections in transfer reactions which are sensitive to near- and far-side interference effects

and then iterate if necessary

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Non-adiabatic - but trajectory based

Time-dependent (finite difference) solution of the valence particle motion - assuming the heavy core, or c.m., follows a trajectory: [See: Bertsch and Esbensen, Baur and Typel, Suzuki, Melezhik and Baye]

\[ V_{VT}(r + R_{cT}(t)), \quad R_{cT}(t) = b + vt \]

Solved on an \((r,t)\) grid and care is needed.

\[ \phi_0(r) \rightarrow \psi_f(r, T_0) \]

Not exact - but non-adiabatic
Dynamics of \(V_{cT}\) is not included and no energy transfer/sharing between core and internal motion. For heavy targets - Coulomb path
The time-dependent approach - observables

\[ i\hbar \frac{\partial \psi}{\partial t} = (H_p + V_{vT})\psi(r, t) \]

as \( t \to -\infty \), \( \psi(r, t) \to \phi_0(r) \)

\( t \to +\infty \), \( \psi(r, t) \to \psi_f(r, T_0) \)

At an impact parameter \( b \) then (for a neutron valence particle):

- **neutron removal probability**
  \[ P_{\text{r-n}}(b) = 1 - |\langle \phi_0 | \psi_f \rangle|^2 \]

- **neutron stripping probability**
  \[ P_{\text{str}}(b) = 1 - \langle \psi_f | \psi_f \rangle \]

- **diffractive break-up probability**
  \[ P_{\text{diff}}(b) = \langle \psi_f | \psi_f \rangle - |\langle \phi_0 | \psi_f \rangle|^2 \]

- **with cross sections**
  \[ \sigma_\alpha = 2\pi \int_{b_{\text{min}}}^{\infty} db \ b \ P_\alpha(b) \]

absorptive effects of target have to be put in ‘by hand’ - restricting impact parameters \( b \) to values \( b > b_{\text{min}} \approx R_T + R_c \)

Only absorption/loss of flux in the equation is due to \( V_{vT} \) and so
Beyond the adiabatic limit - the CDCC

Coupled channels solution of break-up by discretisation of the continuum

\[ \hat{\phi}_{i\ell m}(\mathbf{r}) = u_{i\ell}(\mathbf{r}) Y_{\ell m}(\mathbf{r}) \]

\[ u_{i\ell}(\mathbf{r}) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} dk \ f_i(k) \phi_\ell(k, \mathbf{r}) \]

\[ \Psi_K^{(+)}(\mathbf{r}, \mathbf{R}) = \sum_i \hat{\chi}_i(\mathbf{R}) \hat{\phi}_i(\mathbf{r}) \]

M. Kamimura et al, Prog Theor Phys (Suppl) 89 (1986), 1

\[ \langle \hat{\phi}_{i\ell} | H_p | \hat{\phi}_{i\ell} \rangle \]

\[ k_i \]

\[ k_{i-1} \]

Projectile excitation

5/2+ -0.480

1/2+ -1.218

15C

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Properties of CDCC bin (basis) states

\[ u_{i\ell}(r) = \sqrt{\frac{2}{\pi N_i}} \int \Delta k_i f_i(k) \phi_\ell(k, r) \]

bin states \[ \hat{\phi}_{i\ell m}(r) \]

\[ \hat{\mathcal{E}}_i, \ell \]

Uncertainty principle: so these must be chosen carefully

\[ \Delta k_i \]

normalised and orthogonal

\[ \langle \phi_i | U(r, R) | \phi_j \rangle_r \]

Couplings between bin states (channels) are

Uncertainty principle: so these must be chosen carefully
Coupled channels model space is needed

Example of a coupled channel (CDCC) model space for $^{15}$C break-up on a $^9$Be target at $E=54A$ MeV

J.A. Tostevin et al, PRC 66 (2002) 024607
Residue parallel momentum distributions

Calculations of $^{10}\text{Be}$ residue $p_{\parallel}$ momentum distributions following neutron knockout from a $^{11}\text{Be}$ beam at 60A MeV/, with no coincident photon - $^{10}\text{Be}$ in its ground state.

There is asymmetry in data?

T. Aumann et al. PRL 84 (2000) 35
Momentum distributions from the CDCC

$^{14}$C

$^{15}$C + $^9$Be
54A MeV

CDCC and eikonal calculations agree in most forward directions, but CDCC develops an asymmetry for deflected residues.

J.A. Tostevin et al, PRC 66 (2002) 024607

$^9$Be ($^{15}$C, $^{14}$C(gs)) X
Non-adiabatic and non-eikonal effects for $^{15}\text{C}$

$^{15}\text{C} + ^9\text{Be}$

Coupled channels (CDCC)

$^{14}\text{C}$

1$^-$ 6.09 MeV $^{1}\text{p}_{3/2}$

0$^+$ 0

1$^-$ 6.09 MeV $^{1}\text{p}_{3/2}$

$^{14}\text{C}$ (15C, 14C(π)) X

J.A. Tostevin et al, PRC 66 (2002) 024607

$^9\text{Be}$ ($^{15}\text{C}, ^{14}\text{C}(\pi)$) X

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Core fragment differential cross sections

\[ \frac{d\sigma}{d\Omega} \] [mb/(sr GeV/c)]

\[ {^{15}}C + {^{9}}Be, 54 \text{ MeV/nucleon} \]

\[ p_{\parallel} \left( {^{14}}C(\text{gs}) \right) \] (GeV/c)

\[ 0-1^0, 1-2^0, 2-3^0, 3-4^0 \]

these yields almost entirely due to diffractive dissociation

\[ {^{9}}Be \left( {^{15}}C, {^{14}}C(\text{gs}) \right) X \]

J.A. Tostevin et al, PRC 66 (2002) 024607
Coupled channels and Coulomb break-up

Do CDCC calculations converge in the case of Coulomb couplings?

$\Delta k = k_i - k_{i-1}$ must be small

$\hat{\phi}_i(r)$ and associated couplings $\langle \hat{\phi}_i \mid U(r, R) \mid \hat{\phi}_j \rangle$ of very long range

$^{19}$C + Pb → $^{18}$C+n+X
E= 67A MeV Coulomb dominated

Convergence is not proven!

...... the foundation and general validity of the continuum-discretized-coupled-channel (CDCC) method (Sakuragi et al 1986) is under criticism. Clearly, it is just a model and does not provide a general solution of the three-body problem. The question remains whether it might be a general approximation that can converge in some sense to a three-body scattering theory. It has been revealed that ‘CDCC is valid for special three-body models, constructed with absorptive phenomenological interactions’ (Austern and Kawai 1988). In the particular situation of long-range Coulomb forces, absorptive interaction plays a small role and the applicability of the CDCC method is in serious doubt. Results of the calculations depend on the choice of the model-space and the way of discretization. The convergence is by no means convincingly demonstrated.

8B - a weakly bound proton nucleus

\[ \text{Projectile excitation} \]

\[ E_{\text{max}} \]

\[ 3/2^-, -0.137 \]

\[ ^8\text{B} \]

\[ \theta \]

\[ ^7\text{Be} \]

Convergence with \( E_{\text{max}} \)
CDCC can reproduce data at low energy

$^{8}\text{B} + ^{57}\text{Ni} \rightarrow ^{7}\text{Be} + X, 25.8\text{ MeV}$

(Notre-Dame)

Double differential cross sections for breakup

\[ \frac{d^2\sigma}{dE_c d\Omega_c} \]

\[ ^8\text{B} + ^{57}\text{Ni} \rightarrow ^7\text{Be} + X \]

25.8 MeV

J. Tostevin et al.,
Phys Rev C 63
(2001) 024617

J. Kolata et al.,
Phys Rev C 63
(2001) 024616
Recoil limit of the adiabatic few-body model

$$V_{cT}(r_{cT}) \approx 0$$
$$V_{VT}(r_{VT}) \text{ dominates}$$

Removal of $v$ is by core recoil or shake-off mechanism

closed-form solution in adiabatic approximation

$$\Psi_K^{Ad}(r, R) = \exp(i\alpha K \cdot r)\phi_0(r)\chi_K^{(+)}(R_{cT}), \quad \alpha = \frac{m_v}{(m_c + m_v)}$$

R.C. Johnson et al., PRL 79 (1997) 2771

and provides limit against which model calculations can be tested - e.g. CDCC
Application to elastic scattering of composites

\[ T_{el}(K', K) = \langle K' \phi_0 | V_{CT} | \Psi_K^{Ad}(r, R) \rangle = \langle \alpha Q, \phi_0 | \phi_0 \rangle \langle K' | V_{CT} | \chi_{K}^{(+)} \rangle, \]

\[ \frac{d\sigma}{d\Omega}_{el} = |F_{00}(\alpha Q)|^2 \frac{d\sigma}{d\Omega}_{point} \]

\( Q = K' - K \)

structure formfactor \( F_{00}(\alpha Q) \)
x point projectile scattering \( V_{CT} \)

core scattering

\[ ^{10}\text{Be} + ^{12}\text{C}, \ 59.4 \text{ MeV/A} \]

GANIL

composite scattering

\[ ^{11}\text{Be} + ^{12}\text{C}, \ GANIL \text{ data} \]

49.3 A MeV

R.C. Johnson et al., PRL 79 (1997) 2771
Inelastic scattering, similarly

\[
T_{\text{inel}}(K', K) = \langle K'\phi_1 | V_{cT} | \Psi_K^{\text{Ad}}(r, R) \rangle = \langle \alpha Q, \phi_1 | \phi_0 \times K' | V_{cT} | \chi_K^{(+)} \rangle
\]

![Graph showing elastic and inelastic scattering cross sections](image)

\[
\frac{d\sigma}{d\Omega} = |F_{10}(\alpha Q)|^2 \left| \frac{d\sigma}{d\Omega} \right|_{\text{point}}
\]

No measurement as of yet

\[\text{Elastic scattering cross section of point projectile by potential felt by the core c} \]

\[\text{Inelastic scattering cross section of point projectile by potential felt by the core c} \]

\[\text{prediction} \]
Coulomb break-up of the deuteron


\[
T_{bu}(k_v, k_c, K) = \langle k_v \chi_{k_c}^{(-)}(R_c) \mid V_{cv} \mid \Psi_K^{Ad} (r, R) \rangle
\]

\[
= \langle P_v \mid V_{cv} \mid \phi_0 \rangle \langle Q_v \chi_{k_c}^{(-)} \mid \chi_K^{(+)} \rangle
\]
Exact 3-body amplitude in the adiabatic limit

\[ \Psi^{\text{Ad}}_K (\mathbf{r}, \mathbf{R}) = \exp(i\alpha \mathbf{K} \cdot \mathbf{r}) \phi_0 (\mathbf{r}) \chi^{(+)}_K (\mathbf{R}_{cT}), \quad \alpha = \frac{m_v}{m_c + m_v} \]

\[ T_{el} (K', K) = \langle K' | V_{cT} | \Psi^{\text{Ad}}_K (\mathbf{r}, \mathbf{R}) \rangle = \langle \alpha Q | \phi_0 \langle K' | V_{cT} | \chi^{(+)}_K \rangle \]

includes effects of long range Coulomb couplings without partial wave decomposition or truncation

\[ \int_{-1}^{1} dx \, P_L (x) [f_{el}(\theta) - f_C(\theta)] \]

\[ S_L \]

\[ f_{el}(\theta) = F(\alpha Q) \, f_{pt}(\theta) \]

Subtract point Coulomb amplitude and invert to give \( S_L \) to compare with that calculated using CDCC, in the limit that \( H_p \rightarrow \varepsilon_0 \)
Coupled channels for Coulomb break-up?

Coupled channels (CDCC) calculations with all channel energies equal to that of the elastic channel.
Messages to take away …..

Weak beams of rare weakly bound nuclei pose challenges to reaction theories – continuum of states, non-perturbative

Approximate schemes are being developed which allow sp spectroscopy on beams with of order 1pps – show Shell Model ideas are working away from stability

Apparently simple problems (the Coulomb interaction and its induced break-up) remain to be fully resolved.

Insight is being gained in the light nucleus domain and extended rapidly to heavier systems as new facilities are planned and commissioned (NSCL, RIKEN, GSI, RIA ..)
thanks for your attention and hospitality ..... 
- and also for the cricket!