Properties of G2 gauge theories

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Overview

• Why G2?
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• G2 Yang-Mills theory
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  - Phase diagram [Danzer, Gattringer, Maas JHEP'09]
  - Topological properties [Ilgenfritz, Maas unpublished]
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  • Exploring the full phase diagram [Maas, von Smekal, Wellegehausen, Wipf '12: 1203.5653]
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  • Details: Next talk by Björn Wellegehausen

• Summary
QCD as a gauge theory

- QCD is a gauge theory
QCD as a gauge theory

• QCD is a gauge theory

\[ L = -\frac{1}{4} F_{\mu \nu}^a F^{\mu \nu}_a \]

\[ F_{\mu \nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a \]

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- Numbers \( f^{abc} \) determined by the gauge group
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- QCD: SU(3)
- Here: G2
Why G2?

- Conceptual
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- Conceptual – Quenched QCD
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  ● Full phase diagram accessible
    ● Test of methods and models
    ● Qualitative insights
G2 facts sheet

- G2 is an exceptional Lie group
  - Rank 2 (like SU(3))
  - Subgroup of SO(7)
  - Can be formulate as a product of SU(3) and the 6-sphere
  - All representations are equivalent to real representations
  - Fundamental representation 7 dimensional
  - Adjoint representation 14 dimensional
G2 quenched QCD

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[Wellegehausen et al. PRD 11, Liptak et al. PRD 08, Greensite et al. PRD 06]
G2 quenched QCD

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- Intermediate distance static quark-antiquark potential similar to SU(3) Yang-Mills theory
  - Used to set the scale
- Qualitatively similar glueball spectrum
  - Similar gluon-gluon potential [Wellegehausen et al. PRD 11]
What about confinement?
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- No center symmetry: What about the phase diagram?
Phase diagram of G2 quenched QCD
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    [Pepe et al. NPA 07, Greensite PRD 07, Cossu et al. JHEP 07]
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  - Complicated by a bulk transition
    [Cossu et al. JHEP 07]

- Remains with quarks

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Polyakov loop

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- Similar to full QCD – what about chiral symmetry?
Chiral symmetry in G2

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- Non-anomalous chiral symmetry breaking for 1 flavor possible

[Holland et al. NPA 03, Pepe et al. NPA 07]
Quenched G2 QCD

- Chiral symmetry 'broken' in quenched G2 QCD
Quenched G2 QCD

Polyakov loop transition

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**Quenched G2 QCD**

- Chiral symmetry 'broken' in quenched G2 QCD
- 'Restoration' at the phase transition

[Danzer, Gattringer, Maas, JHEP09]
Quenched G2 QCD

- Chiral symmetry 'broken' in quenched G2 QCD
- 'Restoration' at the phase transition
  - Like in QCD
  - Unlike adjoint QCD [Bilgici, Ilgenfritz, Gattringer, Maas JHEP 09]
Topological properties

• Coincidence in SU(N) gauge theory possibly connected to topological properties
  • Center vortices, monopoles, calorons,…
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  • What is the reason in G2?
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  - Direct product of two SU(3) instantons
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- Change of topological properties at the phase transition?
Picture of G2 topological lumps

- Identified by cooling
Picture of G2 topological lumps

- Identified by cooling – single time slice
  - Action density
Picture of G2 topological lumps

- Identified by cooling – single time slice
  - Action density
  - Topological charge density
Topological susceptibility

- Fewer topological lumps the higher the temperature
Topological susceptibility

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- Topology reflects phase transition
Finite density

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  - Important for compact stellar objects
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- **Unquenched** 1 flavor calculation
Phase diagram
Phase diagram

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Introduction – G2 Yang-Mills theory – G2 QCD – Summary

Start of lattice artifacts
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• Conceptual insights
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  • Quenched G2 QCD is almost the same as quenched QCD, up to the static potential
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● Practical insights: Phase diagram
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- Practical insights: Phase diagram
  - Rough shape of the phase diagram of a gauge theory is similar to the expected one
  - More details: Next talk