QCD thermodynamics with continuum extrapolated Wilson fermions

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in collaboration with

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Motivation

Let’s do continuum QCD thermodynamics with a fermion formulation which is known to be correct!

- Rooting procedure not fully understood $\rightarrow$ Steve Sharp Lattice 2006
- Domain wall fermions $\rightarrow$ expensive
- Overlap fermions $\rightarrow$ even more so (but: Stefan Krieg today 3:50)
- Wilson fermions $\rightarrow$ theoretically sound, correct and straightforward continuum limit (even in practice)
Motivation 2

Let’s check the staggered thermodynamics results!

- Even though ugly, probably correct

- Successful check for some quantities lends support for unchecked other quantities

- Check only meaningful in continuum limit for fully renormalized finite quantities

- For the check only, physical quark masses not essential (heavier quarks also universal)
Outline

- Staggered strategy vs Wilson strategy for thermodynamics
- Renormalization of $\bar{\psi}\psi$, $\chi_s$ and $L$
- Action and simulation parameters
- Results and comparison with staggered results in continuum limit
<table>
<thead>
<tr>
<th>Staggered</th>
<th>Wilson</th>
</tr>
</thead>
<tbody>
<tr>
<td>change $\beta$ ($N_t$ fix)</td>
<td>change $T = 1/aN_t$</td>
</tr>
<tr>
<td>continuous</td>
<td>steps in $T$</td>
</tr>
<tr>
<td>low temperature</td>
<td>large discr. errors</td>
</tr>
<tr>
<td>many times</td>
<td>tuning masses (LCP)</td>
</tr>
<tr>
<td>$N_t \to \infty$</td>
<td>continuum limit</td>
</tr>
<tr>
<td>yes</td>
<td>chiral symmetry</td>
</tr>
<tr>
<td>simple</td>
<td>renormalization</td>
</tr>
</tbody>
</table>
Our quantities

- quark number susceptibility: $\chi_s/T^2$, finite in continuum limit (as in staggered)

- chiral condensate: $\bar{\psi}\psi$, needs renormalization (more tricky than staggered)

- Polyakov loop: $L$, needs renormalization
Renormalization of chiral condensate

Additive

\[ \Delta_{\bar{\psi}\psi}(T) = \langle \bar{\psi}_0\psi_0 \rangle(T) - \langle \bar{\psi}_0\psi_0 \rangle(T = 0) \quad \text{Cancellation } O(a^{-3}) \]

\[ \Delta_{PP}(T) = \int d^4x \langle P_0(x)P_0(0) \rangle(T) - \int d^4x \langle P_0(x)P_0(0) \rangle(T = 0) \]

\(P_0(x):\) bare pseudo-scalar condensate, cancellation \(O(a^{-2})\)

From axial Ward identity: \(2m_{PCAC}Z_A \Delta_{PP}(T) = \Delta_{\bar{\psi}\psi}(T) + O(a)\)
Renormalization of chiral condensate

Multiplicative

\[ m_R \langle \overline{\psi} \psi \rangle_R(T) = m_{PCAC} Z_A \Delta_{\overline{\psi} \psi}(T) \]

Last year we didn’t have \( Z_A \) so used Ward identity and

\[ m_R \langle \overline{\psi} \psi \rangle_R(T) = \frac{\Delta_{\overline{\psi} \psi}^2(T)}{2 \Delta_{PP}(T)} \]

Now we have \( Z_A \) (and also \( m_{PCAC} \)) so can use

\[ m_R \langle \overline{\psi} \psi \rangle_R(T) = 2N_f m_{PCAC}^2 Z_A^2 \Delta_{PP}(T) \]

Best scaling among 3 choices!
Renormalization of chiral condensate, measurement of $Z_A$

$Z_A$ is finite and $Z_A \rightarrow 1$ in the continuum limit

- $Z_A$ is defined in the chiral limit

- $N_f = 3$ simulations at four quark masses, $m_s/3 < m_q < m_s$

- fixed volume $V \sim (2 \text{ fm})^4$

- RI-MOM: compute $Z_V$ then $Z_A = Z_V \Gamma_V(p)/\Gamma_A(p)$

- dependence on $p$ very small (systematic error)

- extrapolate to $m_q \rightarrow 0$ (very smooth)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>3.30</th>
<th>3.57</th>
<th>3.70</th>
<th>3.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_A$</td>
<td>0.892(7)</td>
<td>0.951(2)</td>
<td>0.966(2)</td>
<td>0.976(5)</td>
</tr>
</tbody>
</table>
Renormalization of Polyakov loop

Additive divergence in free energy

Get rid of it by the scheme \( L_R(\mathcal{T}_0) = L_*, \) for some \( \mathcal{T}_0 > T_c \)

For Wilson: \( L_{R}(T) = \left( \frac{L_*}{L_0(\mathcal{T}_0)} \right)^{\mathcal{T}_0/T} L_0(T) \) at each \( \beta \)

For staggered a bit more tricky: first usual renormalization via static potential, then finite scheme change to above scheme
Action and simulation parameters

2 + 1 flavor, tree level Symanzik gauge action, 6 steps stout and tree level clover improved fermion action

\[ m_\pi/m_\Omega = 0.326(4), \quad m_K/m_\Omega = 0.366(4) \] for both staggered and Wilson

Quark mass ratios \( (2m_K^2 - m_\pi^2)/m_\pi^2 = 1.530(7) \) are tuned very precisely, \( m_s \) physical

\[ m_\Omega = 1672 \text{ MeV sets scale} \rightarrow m_\pi \approx 545 \text{ MeV}, \quad m_K \approx 612 \text{ MeV} \]

Large volumes \( m_\pi L \gtrsim 8 \)

4 lattice spacings:

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>3.30</th>
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<th>3.70</th>
<th>3.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \ [\text{fm}] )</td>
<td>0.139(1)</td>
<td>0.093(1)</td>
<td>0.070(1)</td>
<td>0.057(1)</td>
</tr>
<tr>
<td>( V )</td>
<td>( 32^3 \times 6 - 16 )</td>
<td>( 32^3 \times 8 - 16 )</td>
<td>( 48^3 \times 8 - 28 )</td>
<td>( 64^3 \times 12 - 28 )</td>
</tr>
</tbody>
</table>
Continuum limit

Wilson simulation: discrete $N_t \rightarrow$ discrete $T$

Cubic spline interpolation in $T$

Continuum extrapolation of cubic spline coefficients using $O(a^2)$ and $O(a\alpha)$

Always have at least 3 lattice spacings
Results, $\chi_s/T^2$ Wilson

\[ \beta = 3.30 \]

Trick for disconnected part used from Ejiri et al. arXiv:0909.2121
Results, $\chi_s/T^2$ Wilson

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Results, $\chi_s/T^2$ Wilson

Trick for disconnected part used from Ejiri et al. arXiv:0909.2121
Results, $\chi_s/T^2$ staggered

![Graph showing $\chi_s/T^2$ vs. $T/m_\Omega$ for $N_t = 6$.]
Results, $\chi_s/T^2$ staggered
Results, $\chi_s/T^2$ staggered

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi_s/T^2 \]
Results, $\chi_s/T^2$ staggered
Results, $\chi_s/T^2$ staggered
Results, $\chi_s/T^2$ Wilson vs. staggered

The graph shows the comparison between the staggered continuum and Wilson continuum for $\chi_s/T^2$ as a function of $T/m_\Omega$. The staggered continuum is represented by blue markers, while the Wilson continuum is shown in red. The x-axis represents $T/m_\Omega$ ranging from 0.08 to 0.16, and the y-axis represents $\chi_s/T^2$ ranging from 0 to 1.
Results, $\bar{\psi}\psi$ Wilson

\[ m_{R\bar{\psi}\psi_R(T)} = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0)) \]
Results, $\bar{\psi}\psi$ Wilson

\[ m_{R\bar{\psi}\psi_R}(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0)) \]
Results, $\bar{\psi}\psi$ Wilson

\[ m_{R\bar{\psi}\psi R}(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0)) \]
Results, $\bar{\psi}\psi$ Wilson

\[ m_{R\bar{\psi}\psi_R}(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0)) \]
Results, $\bar{\psi}\psi$ Wilson

$m_{R\bar{\psi}_R\psi_R}(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0))$
Results, $\bar{\psi}\psi$ staggered

\[ m_{R\bar{\psi}\psi R} / m_\pi \]

\[ T/m_\Omega \]

\[ M_{\text{MeV}} \]

\[ N_t = 6 \]
Results, $\bar{\psi}\psi$ staggered

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Plot of $m_R\bar{\psi}\psi R / m_\pi$ vs $T/m_\Omega$ for $N_t = 6$ (solid green line) and $N_t = 8$ (dashed magenta line).}
\end{figure}
Results, $\bar{\psi}\psi$ staggered

- $N_t = 6$
- $N_t = 8$
- $N_t = 10$
Results, $\bar{\psi}\psi$ staggered
Results, $\bar{\psi}\psi$ staggered

\begin{figure}
\centering
\hspace{-1cm}
\includegraphics[width=\textwidth]{figure}
\caption{Plot showing $m_{R\bar{\psi}\psi R}/m_{\pi}$ vs $T/m_{\Omega}$ for different $N_t$.}
\end{figure}

The figure shows the continuum and the results for $N_t = 6, 8, 10, 12$.
Results, $\bar{\psi}\psi$ Wilson vs staggered

![Graph showing results of $\bar{\psi}\psi$ Wilson vs staggered continuum. The graph plots $m_{R\bar{\psi}\psi R}/m_\pi$ against $T/m_\Omega$ with MeV on the y-axis and $T/m_\Omega$ on the x-axis. The staggered continuum is represented by blue diamonds, and the Wilson continuum by red crosses. The data points show a decrease in the ratio as the temperature increases.]
Results, $L$ Wilson

\[ \beta = 3.30 \]
Results, $L$ Wilson

\[ L_R \text{ vs. } T/m_\Omega \]

- $\beta = 3.30$
- $\beta = 3.57$
Results, $L$ Wilson

\begin{align*}
\beta &= 3.30 \\
\beta &= 3.57 \\
\beta &= 3.70
\end{align*}

MeV

\begin{align*}
L_R &
\end{align*}

T/m$_\Omega$

150 175 200 225 250 275
Results, \( L \) Wilson

\[
\begin{array}{c}
\beta = 3.30  \quad \text{green line} \\
\beta = 3.57  \quad \text{pink dashed line} \\
\beta = 3.70  \quad \text{blue dotted line} \\
\beta = 3.85  \quad \text{black dotted line}
\end{array}
\]
Results, L Wilson

![Graph showing continuum and different beta values]

- Continuum
- $\beta = 3.30$
- $\beta = 3.57$
- $\beta = 3.70$
- $\beta = 3.85$
Results, $L$ staggered

$N_t = 6$

$T/m_\Omega$ vs. $MeV$

Plot shows a trend with $L_R$ vs. $T/m_\Omega$ for different values of $MeV$. Notable values are shown in the image.
Results, $L$ staggered

$N_t = 6$

$N_t = 8$
Results, $L$ staggered

\begin{align*}
\text{MeV} & \\
\text{T/m Ω} & \\
N_t = 6 & \\
N_t = 8 & \\
N_t = 10 & \\
\end{align*}
Results, $L$ staggered

\[ \begin{array}{cccccc}
N_t &=& 6 & & & \\
N_t &=& 8 & & & \\
N_t &=& 10 & & & \\
N_t &=& 12 & & & \\
\end{array} \]
Results, $L$ staggered
Results, \( L \) Wilson vs staggered

\[ \begin{array}{cccccc}
0 & 0.08 & 0.1 & 0.12 & 0.14 & 0.16 \\
150 & 175 & 200 & 225 & 250 & 275
\end{array} \]

\[ \begin{array}{cccccc}
L & R & T/m & Ω & MeV & staggered continuum & Wilson continuum
\end{array} \]
Summary and outlook

- Continuum thermodynamics with Wilson fermions is feasible

- Agreement between continuum staggered and continuum Wilson results

- Lighter pions are currently ongoing (look feasible)
Backup slides
Taste breaking in staggered simulations

\[ m_\pi \text{ [MeV]} \]

\begin{align*}
N_t = 12 & \quad \text{transition} \\
N_t = 8 & \quad \text{transition}
\end{align*}

\[ \gamma_i, \gamma_j, \gamma_i\gamma_5, \gamma_5 \]

\[ a \text{ [fm]} \]