Recent results in large-$N$ lattice gauge theories

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Introductory reviews on large-$N$ QCD:

- Y. Makeenko, hep-th/0001047
- M. Teper, 0912.3339
- B. Lucini and M. P., in preparation

At this conference, parallel talks relevant for this topic are presented by M. García Pérez, A. González-Arroyo, M. Hanada, M. Honda, D. Kadoh, L. Keegan, M. Koreń, J.-W. Lee, R. Lohmayer, F. Negro, M. Okawa and P. Orland.
1. Introduction

2. A selection of physical results

3. Concluding remarks
Consider a generalization of QCD with $\text{SU}(N \to \infty)$ gauge group

- Take $g \to 0$, with $\lambda = g^2 N$ fixed, to have a perturbatively smooth limit
- Keep track of the number of independent color indices in Feynman diagrams through double-line notation for propagators
- Dominance of planar diagrams without dynamical quark loops
- Terms proportional to different powers of $1/N$ can be arranged in a topological series

$$A = \sum_{h,b=0}^{\infty} N^{2-2h-b} \sum_{n=0}^{\infty} c(h,b),n \lambda^n$$

Analogous to a loop expansion in Riemann surfaces for string theory, upon replacing $1/N \to g_s$

- This also holds according to the conjectured holographic correspondence: In the large-$N$ limit, loop effects on the string side become negligible (see also plenary talk by Hanada)
Consider a generalization of QCD with SU\((N \to \infty)\) gauge group

- Take \(g \to 0\), with \(\lambda = g^2 N\) fixed, to have a perturbatively smooth limit
- Keep track of the number of independent color indices in Feynman diagrams through *double-line notation* for propagators
  - Quark: fundamental rep. \(\Rightarrow\) single line
  - Gluon: adjoint rep. \(\Rightarrow\) double line

- Dominance of *planar* diagrams without dynamical quark loops
- Terms proportional to different powers of \(1/N\) can be arranged in a *topological series*

\[
\mathcal{A} = \sum_{h,b=0}^{\infty} N^{2 - 2h - b} \sum_{n=0}^{\infty} c_{(h,b),n} \lambda^n
\]

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Consider a generalization of QCD with $SU(N \to \infty)$ gauge group

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- Keep track of the number of independent color indices in Feynman diagrams through *double-line notation* for propagators
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QCD in the ’t Hooft limit

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Assuming that the large-$N$ limit of QCD is confining:

- The spectrum consists of infinitely many stable glueballs and mesons, with masses $\mathcal{O}(1)$ and interactions suppressed by powers of $1/\sqrt{N}$: large-$N$ QCD is a theory of weakly coupled hadrons
- Exotica (e.g. tetraquarks, molecules, et c.) are absent
- The OZI rule is exact
- Loop effects in the chiral Lagrangian are suppressed by $1/N$
- The axial anomaly is suppressed by $1/N$, and the square of the $\eta'$ mass is $\mathcal{O}(1/N)$
- Baryons can be interpreted as solitons of the theory, with masses $\mathcal{O}(N)$
- Quantitative predictions for baryon-meson couplings, baryon masses, magnetic moments, et c. from consistency conditions based on unitarity
- Implications for the QCD phase diagram—quarkyonic matter (McLerran and Pisarski, 2007)?
- Further implications for high-energy QCD (evolution equations, hadronic cross-sections, parton distributions and structure functions, large-$N$ Standard Model, ... )
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Factorization, volume reduction and large-$N$ equivalences

"You can hide a lot in a large-$N$ matrix"
—Stephen Shenker

- Large-$N$ counting rules imply that vev's of products of gauge-invariant operators are dominated by disconnected contributions $\Rightarrow$ Factorization of vev's of physical operators
  $\langle O_1 O_2 \rangle = \langle O_1 \rangle \langle O_2 \rangle + \mathcal{O}(1/N)$

The analogy with a classical limit can be made explicit by constructing appropriate coherent states (Yaffe, 1982)

- Factorization leads to volume independence
- Volume reduction can be interpreted as a large-$N$ "orbifold" equivalence: Projection using a discrete subgroup of the global symmetries of the theory (Kovtun, Ünsal and Yaffe)
- Orbifold equivalences at large $N$ also relate theories with different field content—e.g., orientifold planar equivalence (Armoni, Shifman and Veneziano—see also numerical studies by Lucini et al.)
- Finally, orbifold projections are also relevant for lattice supersymmetry (Catterall, Kaplan and Ünsal; see also Tsuchiya et al., Nishimura et al.)
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- Factorization leads to Eguchi-Kawai volume independence: the lattice theory can be formulated in an arbitrary small volume \textit{provided center symmetry is unbroken}
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- But center symmetry *does* get broken in a small volume in the continuum limit; fixes:
  - Quenched EK (Bhanot, Heller and Neuberger)—but see (Bringoltz and Sharpe)
  - Twisted EK
  - Add dynamical adjoint fermions
  - Double-trace deformations
  - Partial reduction
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  - Twisted EK (González-Arroyo and Okawa; Teper and Vairinhos, Azeyanagi et al., Bietenholz et al., García Pérez et al.)

![Graph showing the relationship between $\Lambda_{MS}/\sqrt{\sigma}$ and $1/N^2$ with data points for SU(N) and TEK N=841, along with the formula $0.515(3) + 0.34(1)/N^2$.]

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$$S_{YM} \longrightarrow S_{YM} + \frac{1}{N^3} \sum_{\vec{x}} \sum_{n=1}^{[N/2]} a_n |\text{tr}(L^n(\vec{x}))|^2$$

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  - Partial reduction (Kiskis, Narayanan and Neuberger)

\[ m(k) = 0.099(16)k + 0.34(12) - 0.55(19)/k \]

\[ b=0.348, \ L=6, \ N=47 \]
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Outline

1. Introduction
2. A selection of physical results
3. Concluding remarks
Recent results

**SU(N)** is a confining theory in the large-$N$ limit (see, e.g., Meyer and Teper hep-lat/0411039)

- Confining flux tubes behave like Nambu-Goto strings
- Glueball masses have a smooth dependence on $N$
- Well-behaved scale-dependence of the coupling
- The deconfinement temperature has a smooth dependence on $N$
- The equation of state appears to have only a *trivial* dependence on $N$
- Topological susceptibility and $\theta$-dependence
- Quenched mesonic spectrum
- Quenched baryonic spectrum
Results in 4D

- SU(N) is a confining theory in the large-\(N\) limit
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\[ E = \sqrt{\sigma} \]

\[ l = \sqrt{\sigma} \]

Torelon spectrum in SU(5) (Athenodorou et al., 1007.4720)

See also: Lucini and Teper, hep-lat/0107007; Lohmayer and Neuberger, 1206.4015; Mykkänen, in progress

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YM spectrum (Lucini, Rago and Rinaldi, 1007.3879)
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Bare coupling in pure YM (Allton et al., 0803.1092)
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Numerical results

- One-loop $\beta$-function
- Two-loop $\beta$-function

SU(4) YM, SF scheme (Lucini and Moraitis, 0805.2913)

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Mass anomalous dimension in QCD$_N$ with $n_f = 2$ 2S fermions
(DeGrand, Shamir and Svetitsky, 1202.2675)
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See also: Lucini, Teper and Wenger, hep-lat/0307017 and hep-lat/0502003; Piemonte et al., in progress
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(Datta and Gupta, 1006.0938)

See also: Bringoltz and Teper, hep-lat/0506034, M.P., 0907.3719; Mykkänen et al., 1202.2762

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(Del Debbio et al., 2002)

See also: Lucini and Teper, hep-lat/0110004, hep-lat/0401028, Cundy et al., hep-lat/0203030, Panagopoulos et al., 1109.6815
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• Quenched mesonic spectrum

(Bali et al., in progress)

See also: Del Debbio et al., 0712.3036, Bali and Bursa, 0806.2278, Hietanen et al., 0901.3752
- SU($N$) is a confining theory in the large-$N$ limit
- Confining flux tubes behave like Nambu-Goto strings
- Glueball masses have a smooth dependence on $N$
- Well-behaved scale-dependence of the coupling
- The deconfinement temperature has a smooth dependence on $N$
- The equation of state appears to have only a *trivial* dependence on $N$
- Topological susceptibility and $\theta$-dependence
- Quenched mesonic spectrum
- Quenched baryonic spectrum

(DeGrand, 1205.0235)
Recent results

**Results in 3D**

Much like in 4D:

- **SU(N)** is a confining theory in the large-\(N\) limit (Teper, hep-lat/9804008)
- Confining flux tubes behave as Nambu-Goto strings (Athenodorou et al., 1103.5854; Caselle et al., 1102.0723; Mykkänen, in progress)
- Glueball masses have a smooth dependence on \(N\) (Johnson and Teper, hep-ph/0012287; Meyer, hep-lat/0508002)
- The equation of state depends only trivially on \(N\) (Caselle et al., 1105.0359 and 1111.0580)
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\[
\frac{M}{\sqrt{\sigma}} = \frac{1}{N^2}
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Several exact results are known; in particular:

- The continuum spectrum of large-$N$ QCD in 2D was computed by ’t Hooft in 1974
- In 1979, Gross and Witten found a third-order transition in the lattice theory
- The spectral density of Wilson loops was studied by Durhuus and Olesen in 1981

In general, a 2D world can be a useful laboratory for QCD toy models (see, e.g., works by Narayanan, Neuberger and Vicari; Orland et al., . . . )

Recently, the eigenvalue density of Wilson loops in 2D has been studied by Lohmayer, Neuberger and Wettig; similar studies have also been done in 4D (Lohmayer and Neuberger)

Various groups (e.g. Bringoltz; Galvez, Hietanen and Narayanan, et c.) have addressed the problem of 2D large-$N$ theories at finite chemical potential
Outline

1. Introduction
2. A selection of physical results
3. Concluding remarks
Lattice studies of gauge theories in the large-$N$ limit are theoretically very appealing, numerically tractable, and interesting for a very broad community.
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From my personal point of view, particularly promising research directions for further numerical studies at large $N$ include:

- Simulations with dynamical fermions, in various representations
- Finite temperature/finite density; comparisons with perturbative computations, with holography, or with effective models
- Topological properties (see, e.g., Lucini et al., hep-lat/0401028, hep-lat/0502003; Panagopoulos and Vicari, 0803.1593, 1109.6815; D’Elia and Negro, 1205.0238)
- Large-$N$ equivalences and volume reduction