ANISOTROPY STUDY TECHNIQUES

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LECTURE 1: INTRODUCTORY MATERIAL

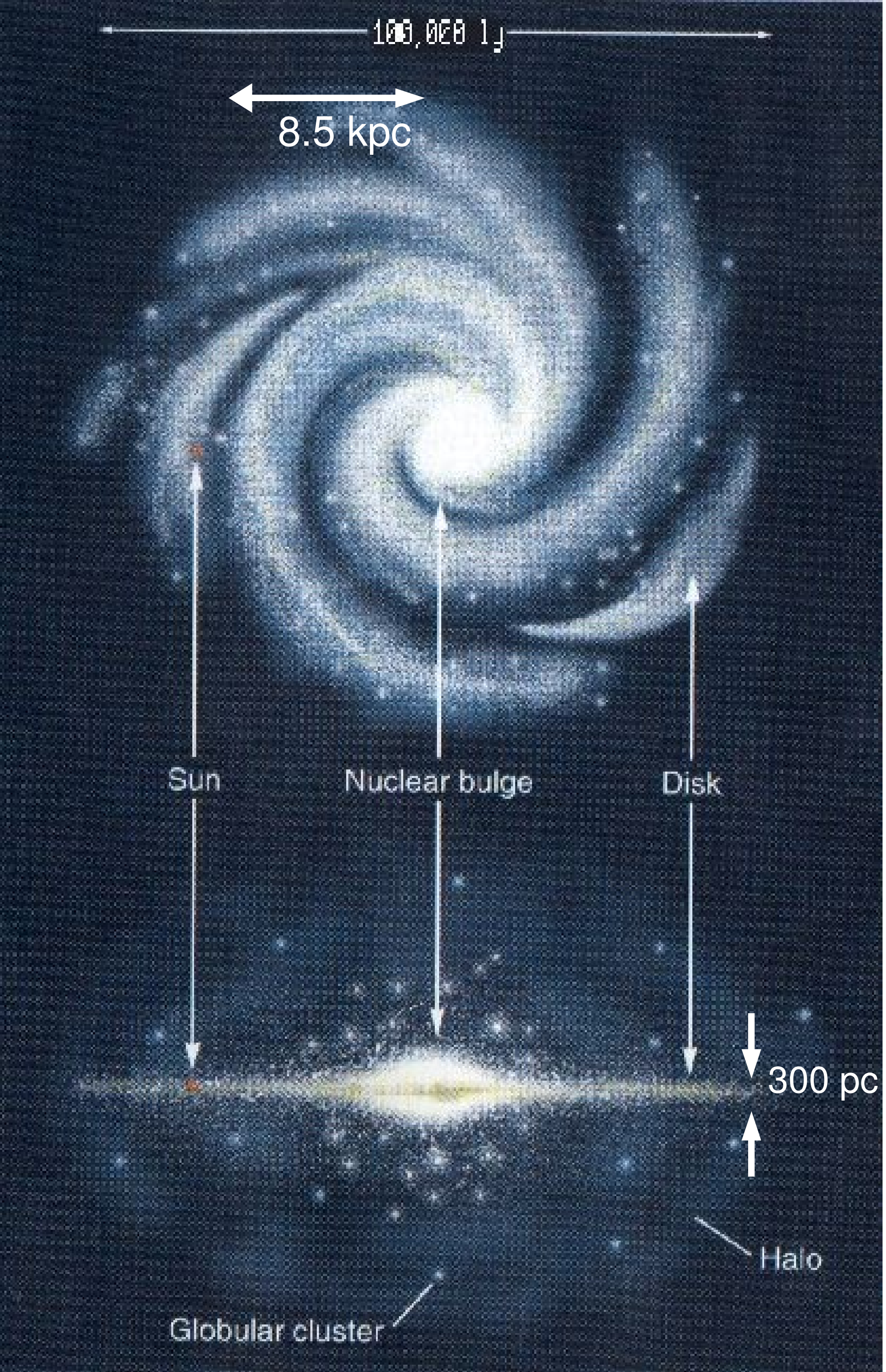
- The matter distribution in our neighbourhood
- Galactic & extragalactic magnetic fields
- Magnetic field effects on CR propagation: deflections, flux amplification, multiple images
- GZK horizon
- Inhomogeneous experimental exposure

Main aims:

anisotropy studies provides a handle to study several open questions

- **which are the sources of cosmic rays?**
- **how do they propagate?**
- **how are the galactic and extragalactic magnetic fields?**
- **which is the CR composition?**

THE COSMIC RAY FLUX IS VERY CLOSE TO ISOTROPIC
CAREFUL STUDIES ARE NEEDED TO DETECT ANISOTROPIES



THE GALAXY

Disk: contains most of the visible stars and atomic gas (90% H and 10% He, $n \sim 1/cm^3$) forming a spiral arm pattern. It has a regular magnetic field, $B_\odot = 3\ \mu G$.

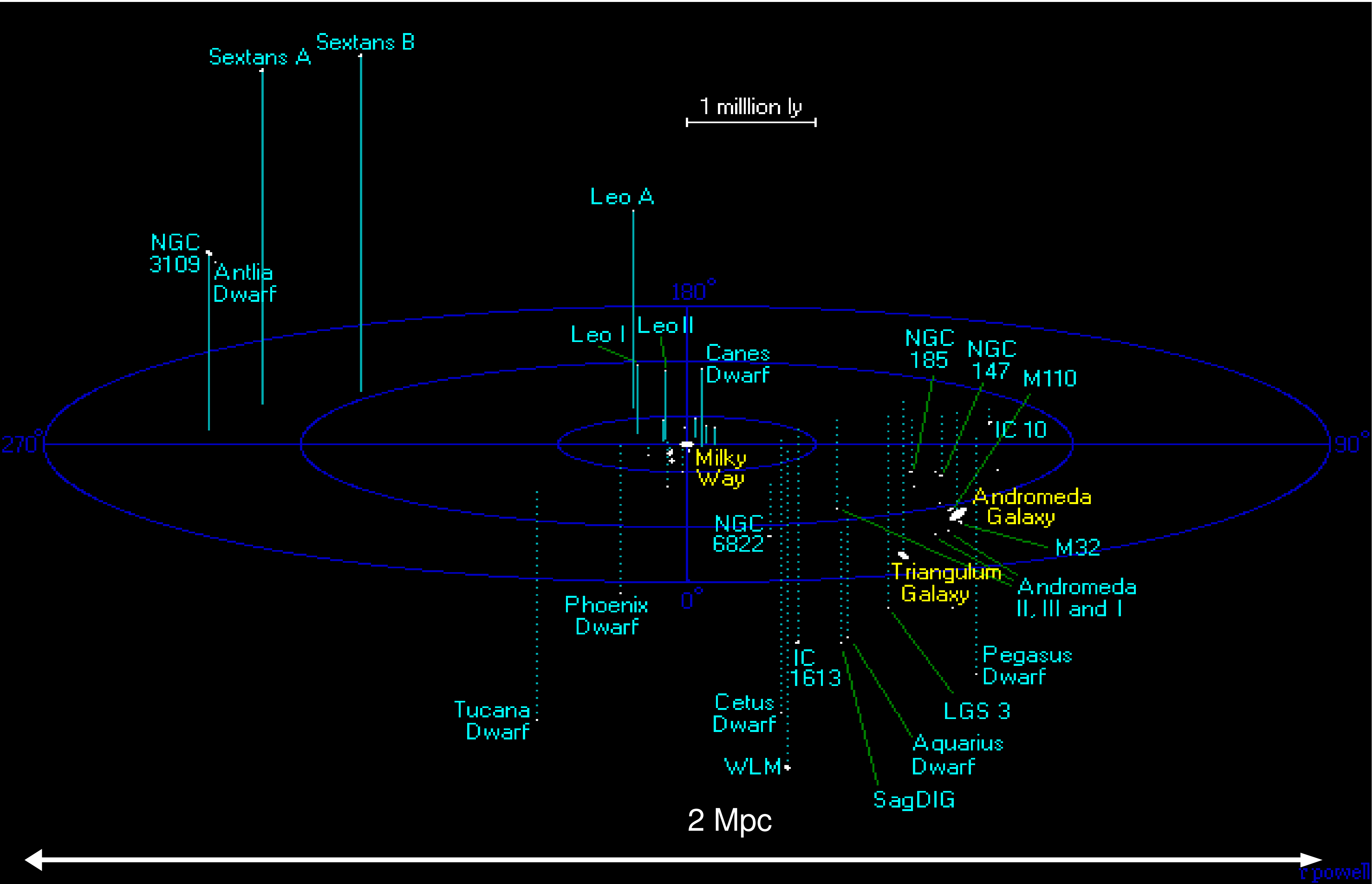
Spheroid (or stellar halo): older stellar population and gas ($n \sim 0.01/cm^3$). Extends up to ~15 kpc. Central bulge. Turbulent magnetic field than can exceed the regular one.

Dark matter halo: evidence from rotation curves of a much larger halo.

Massive black hole in the center $M \sim 10^6\ M_\odot$

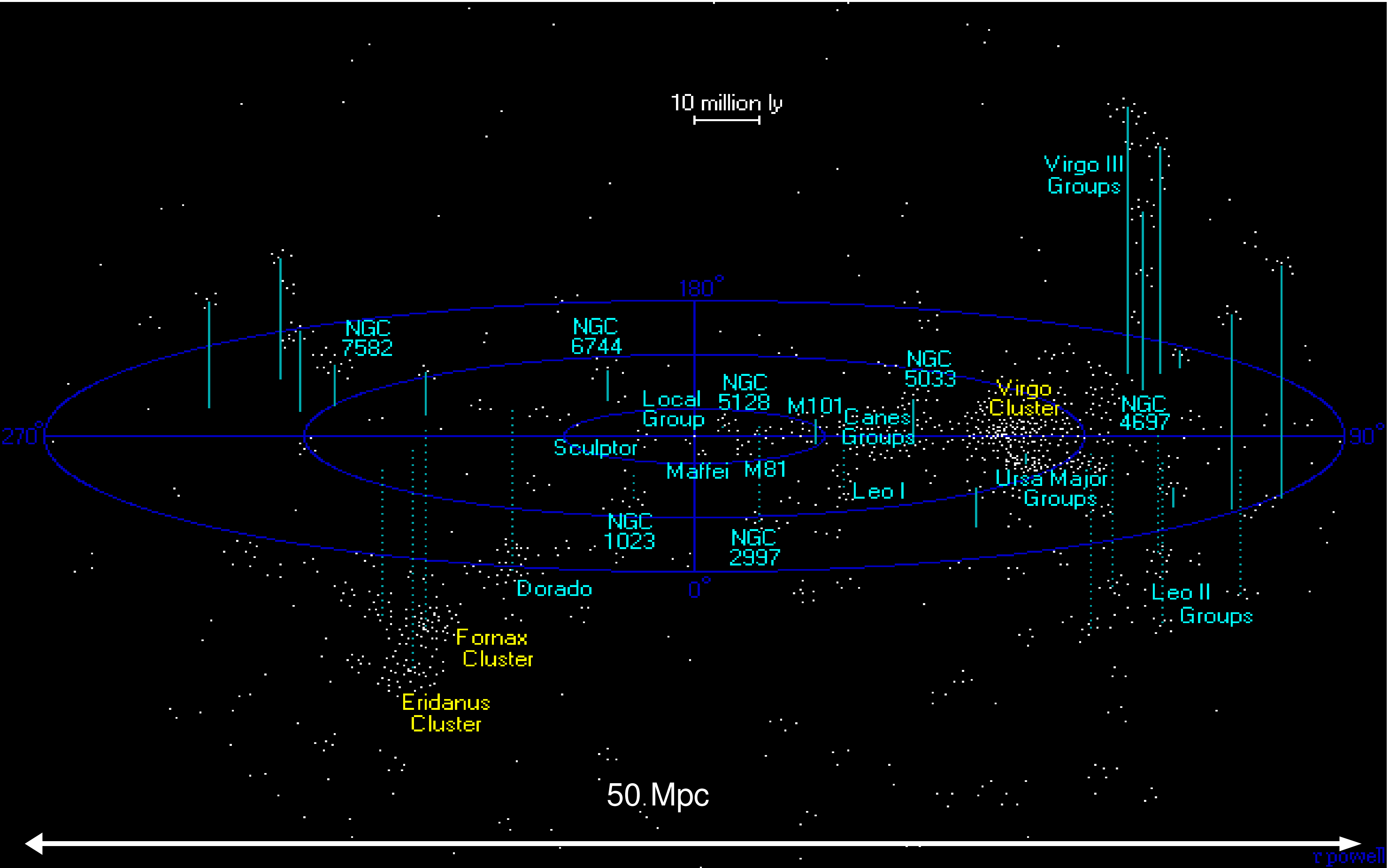
1 pc = 3 ly

THE LOCAL GROUP: Milky Way and Andromeda are the most prominent members of this small cluster of about 30 galaxies

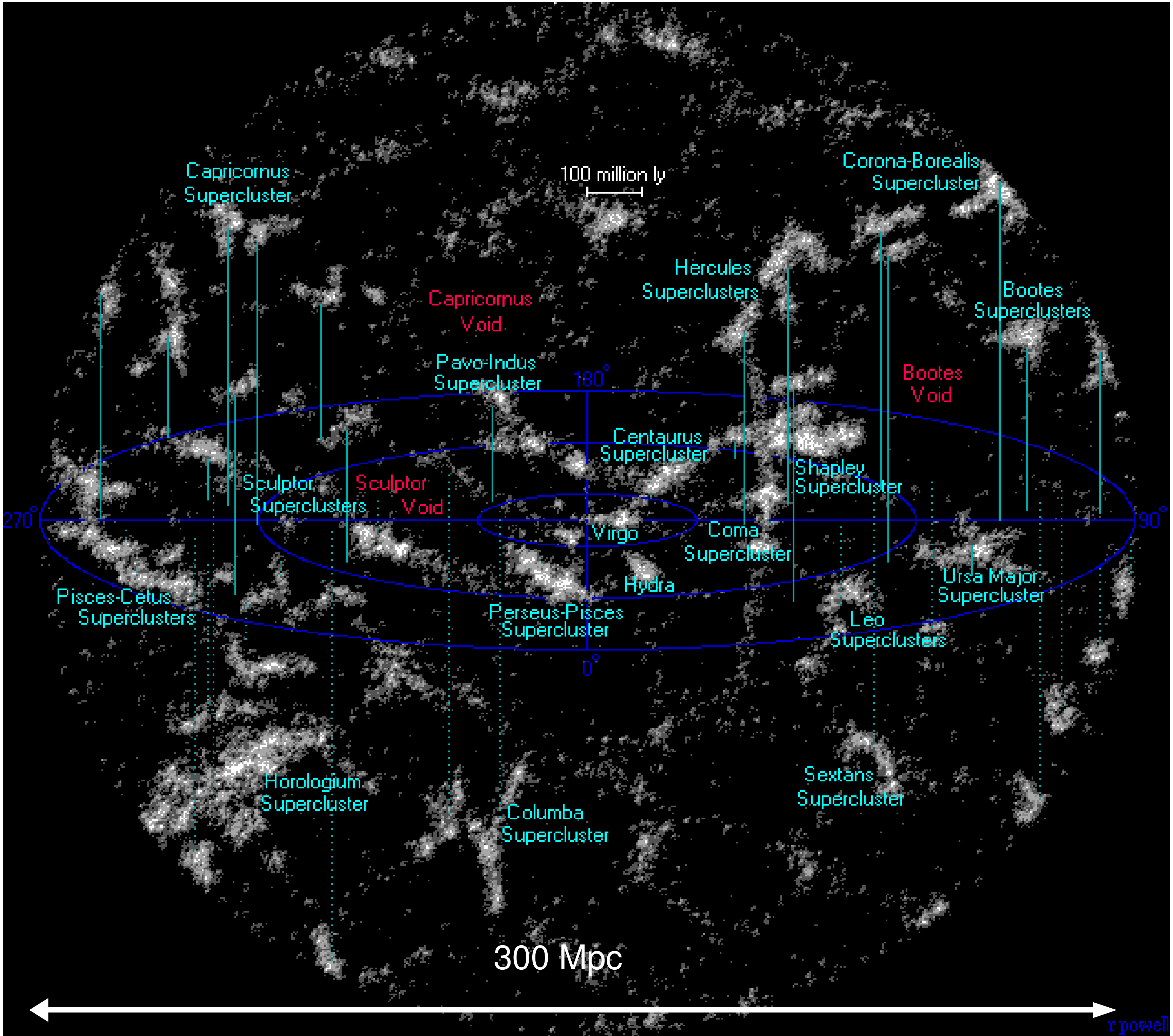


THE VIRGO SUPERCLUSTER:

Our closest cluster of galaxies Virgo (~18 Mpc), a large cluster (more than 2000 galaxies) including the prominent radio galaxy M87



THE NEARBY SUPERCLUSTERS



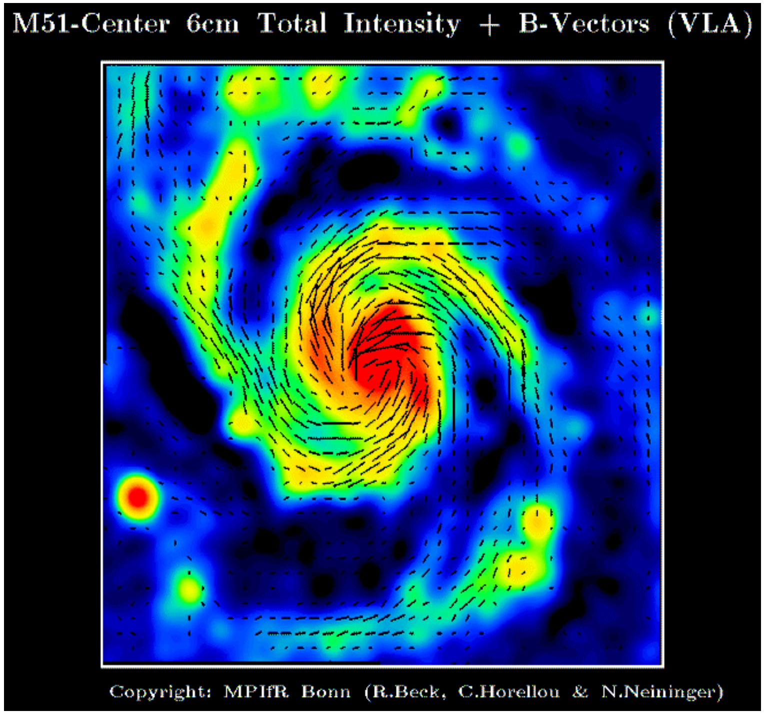
Galactic magnetic field

It has a regular + a turbulent component

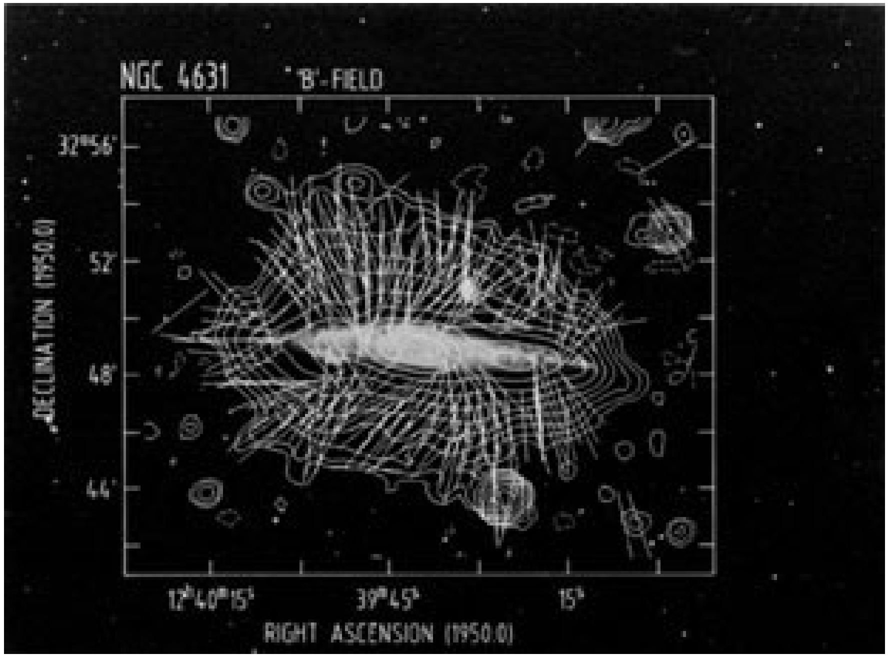
It is not very well known

From observations of other spiral galaxies it is known that:

M51



NGC4631



Hummel et al. '88

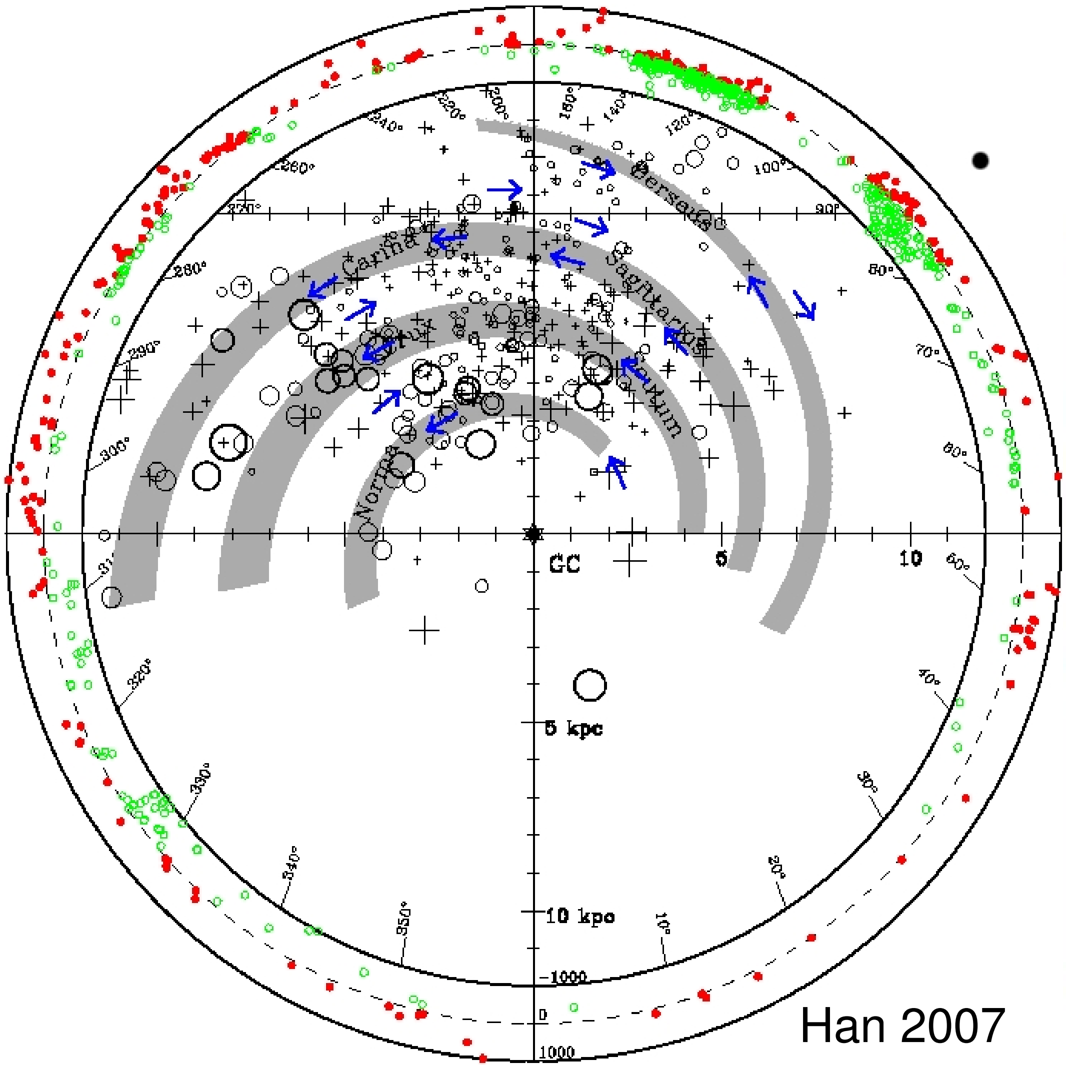
Regular B field follows spiral arms as observed from polarized radio emission in face-on galaxy

Magnetic halo from radio polarization measurement in edge-on galaxy (z_h~ few kpc)

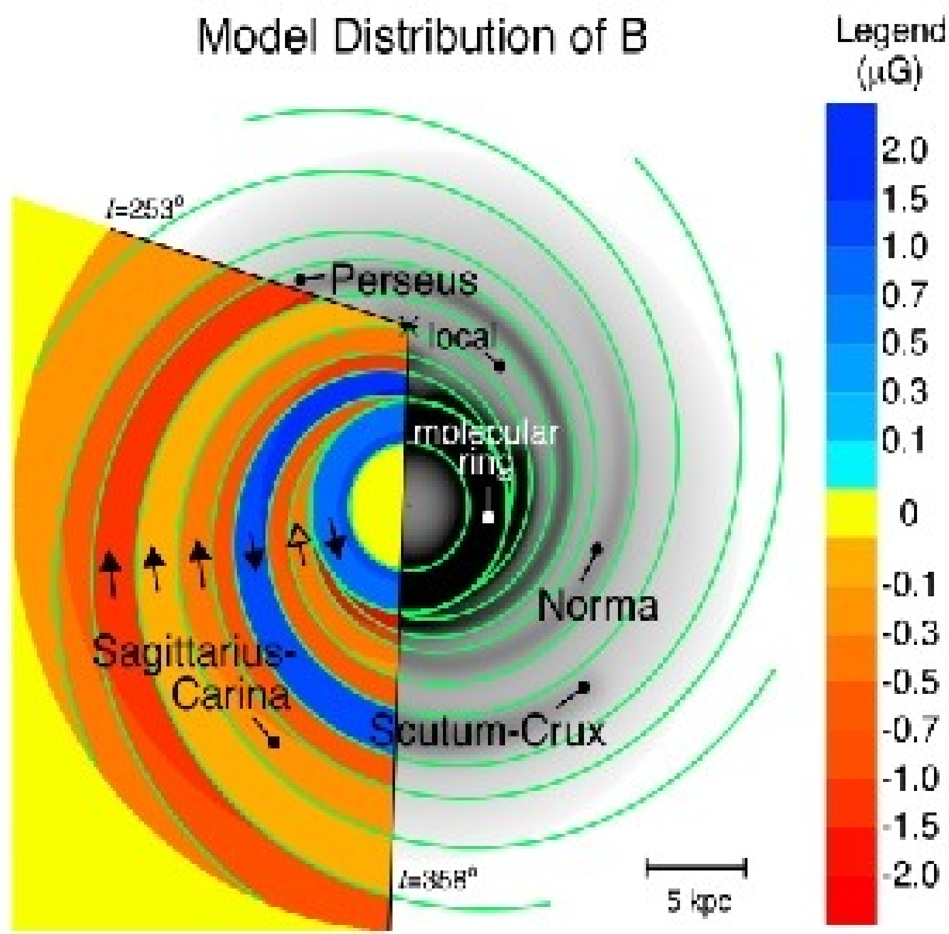
In our own Galaxy: Faraday rotation measures of pulsars and extragalactic radio sources

$$RM: \theta \propto \lambda^2 \int dx\, B_{par} N_e \qquad DM: \tau \propto \lambda^2 \int dx\, N_e$$

$$\boxed{\langle B_{par} \rangle = \frac{RM}{DM}}$$



Han 2007

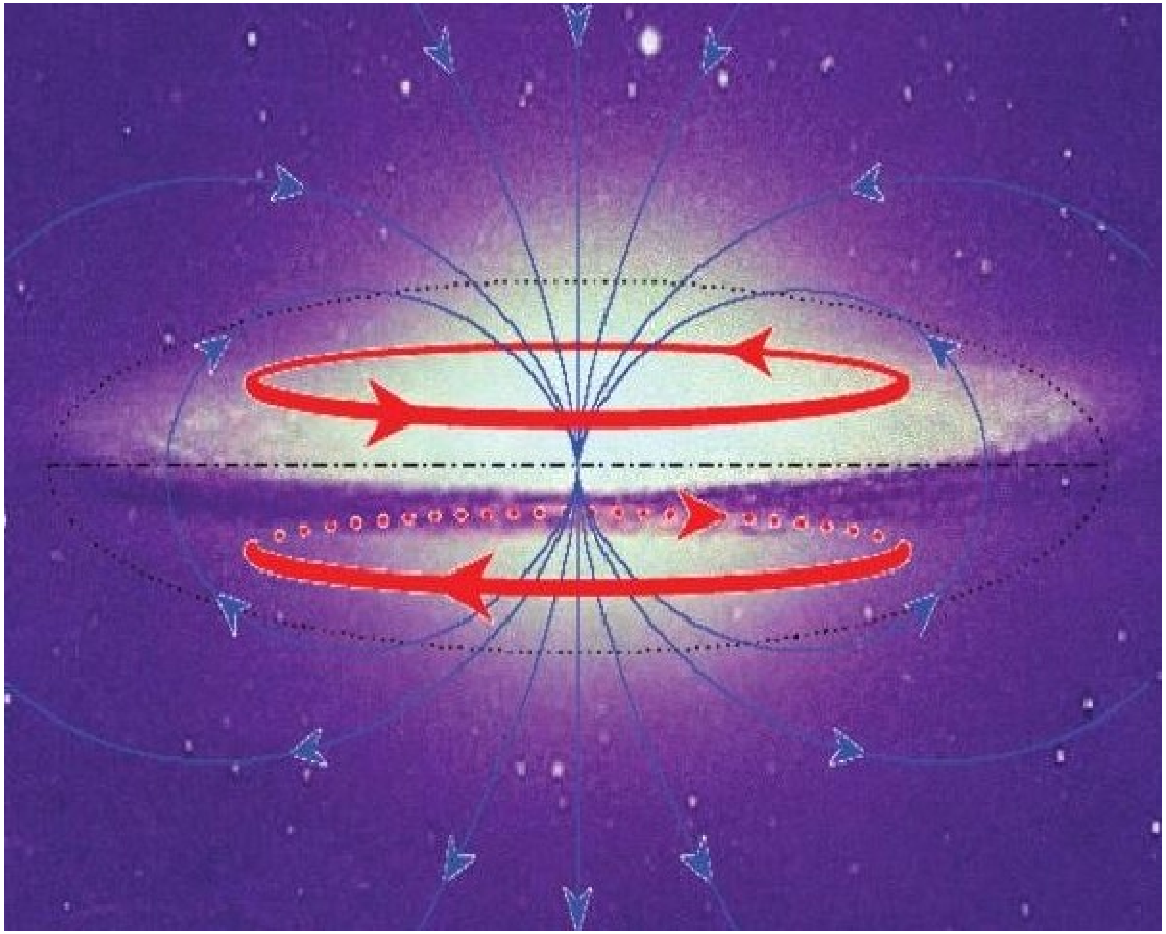


Brown 2007

Magnetic field in the disk follows the spiral pattern → $B_\odot \sim 2 – 3\ \mu G$

Not well known: reversal from arm to arm, or between arm and interarm region?

Halo component: symmetric or antisymmetric? Extension?



Turbulent component: coherence length L_c ~ 100 pc
and Kolmogorov spectrum $dE/dk \sim k^{-5/3}$ (like ISM density fluctuations),
$\langle B_{rms} \rangle$ = 2 - 4 μG

Propagation of CR in galactic and extragalactic magnetic fields

Deflection of charged particles is inversely proportional to their energy

$$\vec{F} = m\,\gamma\,\dot{\vec{v}} = \frac{q}{c}\,\vec{v}\times\vec{B} \qquad (\vec{v} = c\,\hat{u}\,, \quad E = m\,\gamma\,c^2)$$

$$\hat{u} = \hat{u}_0 + \frac{Ze}{E}\int dl\,\hat{u}\times\vec{B} \qquad \text{(c =1)}$$

Larmor radius:

$$r_L \simeq \frac{E/Z}{10^{18} eV}\,\frac{\mu G}{B}\,kpc$$

In regular field, particles have helical trajectories, with $V_\perp = V\cos\theta$ (θ being the pitch angle).

For E/Z < 10^{18} eV (1 EeV) CR confined in the galactic MF

For E/Z > 10^{18} eV unconfined

But for E/Z < 10^{17} eV they scatter off magnetic field irregularities with scale $l \sim r_L$, they make a random walk and diffuse.

Liouville theorem → An isotropic flux of CR remains isotropic after propagating through a magnetic field

LT: Phase space distribution $f(\mathbf{r},\mathbf{p})$ is constant along CR trajectories

The intensity $\quad I = dN/(dA\ dt\ d\Omega\ dE)$

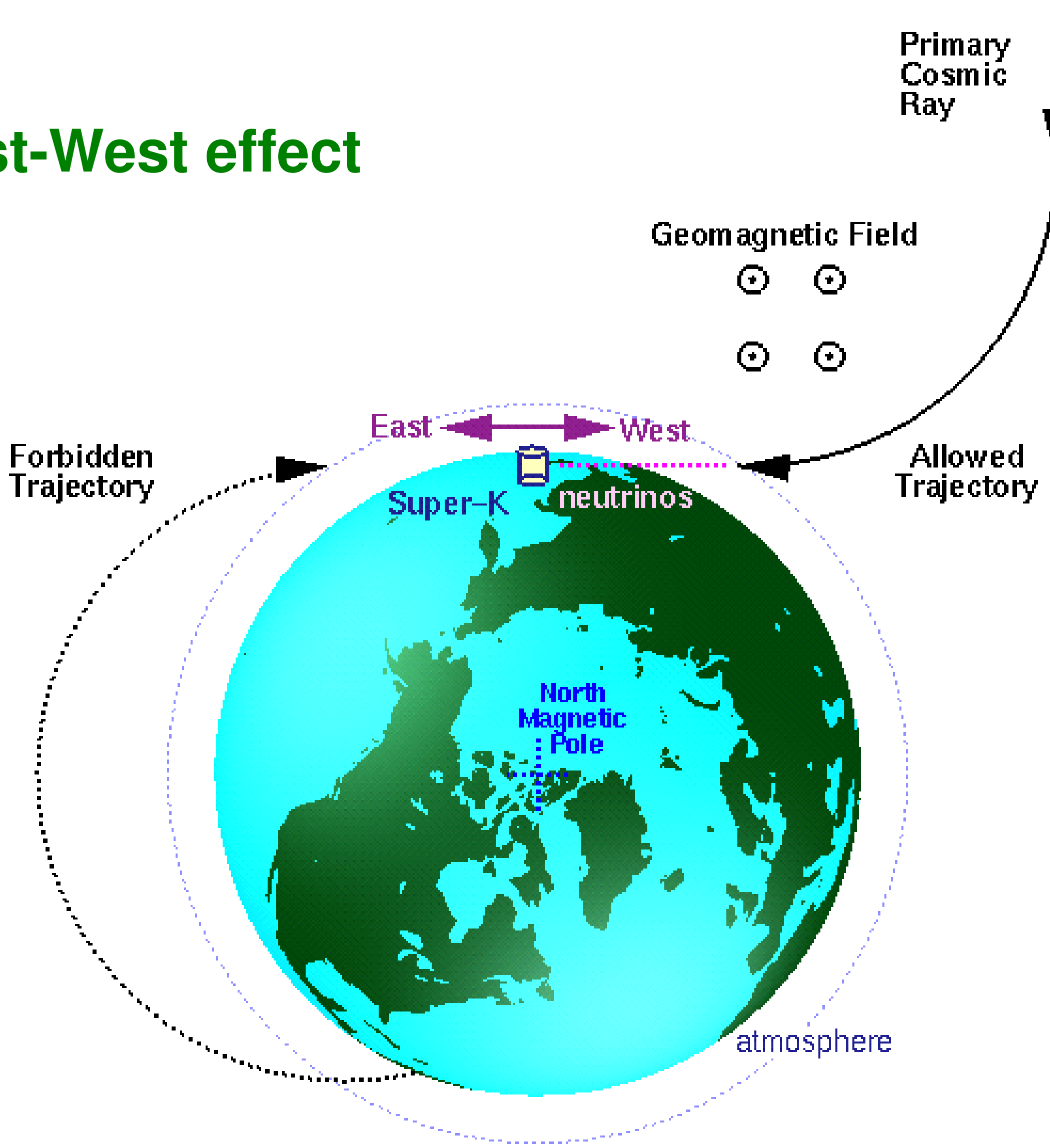
and $\quad dN = f(\mathbf{r},\mathbf{p})\ d^3r d^3p \qquad d^3r = dA\ vdt\ ,\ d^3p = p^2\ dp\ d\Omega$

Then $\quad I = f(\mathbf{r},\mathbf{p})\ v\ p^2\ dp/dE = f(\mathbf{r},\mathbf{p})\ p^2$

Since p is constant along the trajectory → I is constant

An isotropic CR flux remains isotropic unless there is a 'shadowing effect': directions from which particles cannot reach the detector coming from infinity.
For example at low energies this happens because of the Earth `shadow': trajectories of antiparticles leaving from the detector hit the Earth due to the deflections in the geomagnetic field

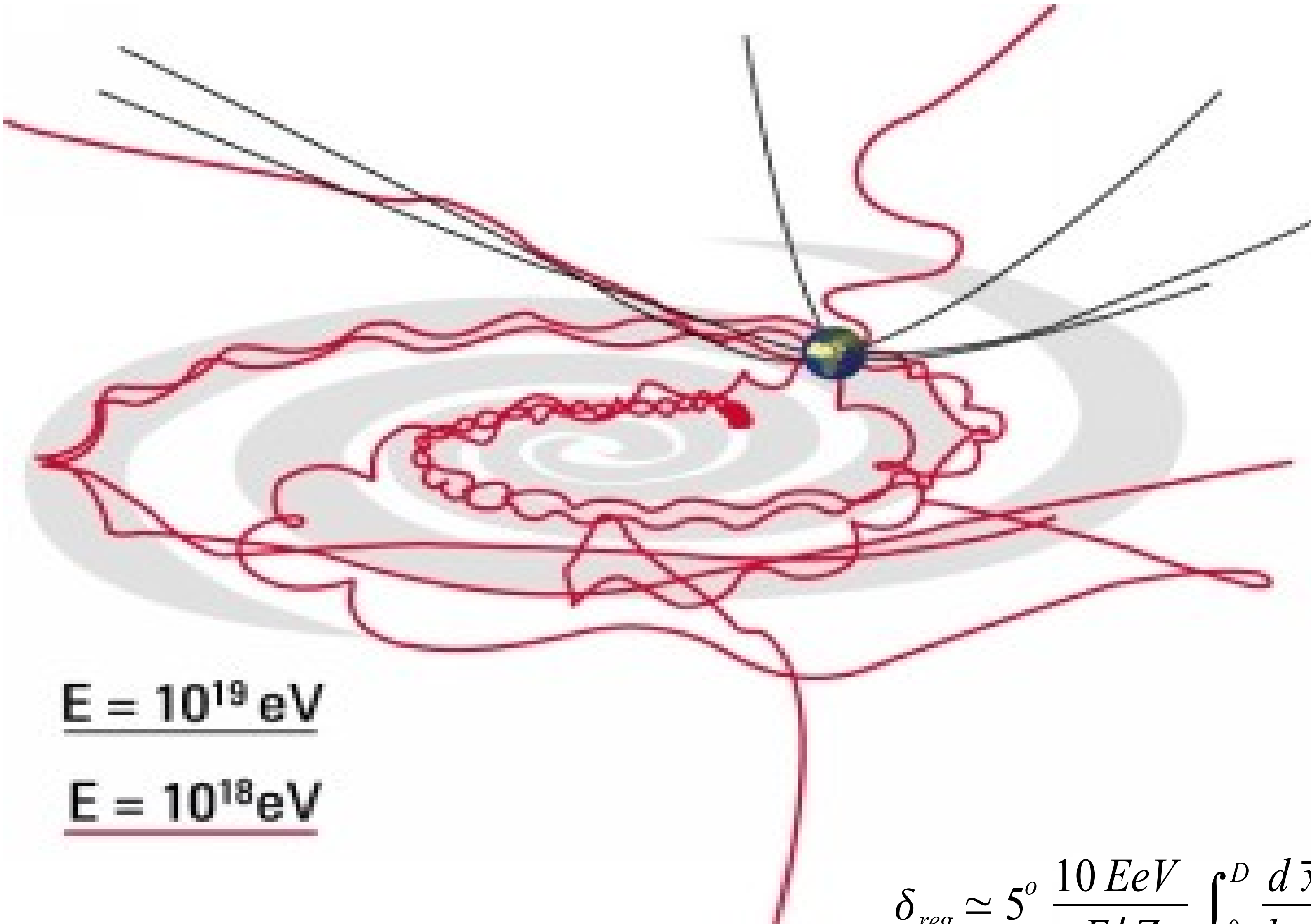
East-West effect



Protons with energy smaller than few GeV are not able to reach the Earth from the east. The sign of this E-W asymmetry was used to infer that CR primaries are positively charged.

Towards the poles the threshold is smaller → CR intensity increase with latitude at low energies ('latitude effect')

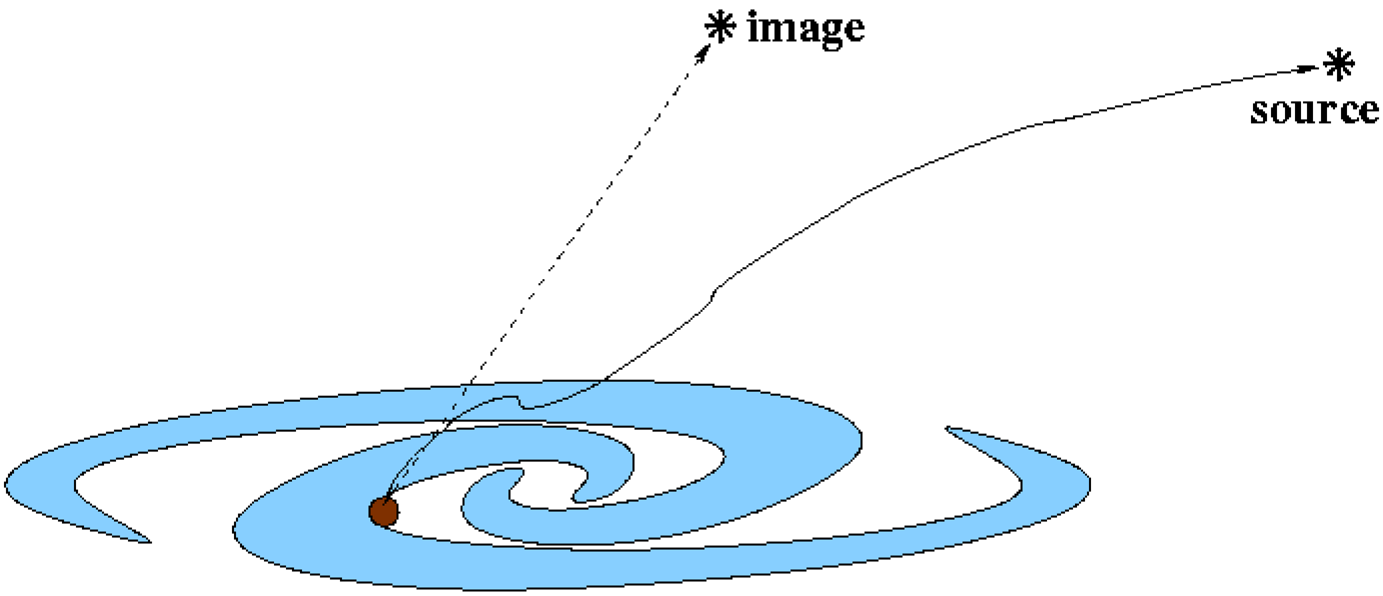
DEFLECTION OF CHARGED PERTICLES IN GALACTIC MAGNETIC FIELD



$$\delta_{reg} \simeq 5^{o}\,\frac{10\,EeV}{E/Z}\int_0^D \frac{d\,\vec{x}}{kpc}\times\frac{\vec{B}}{\mu\,G}$$

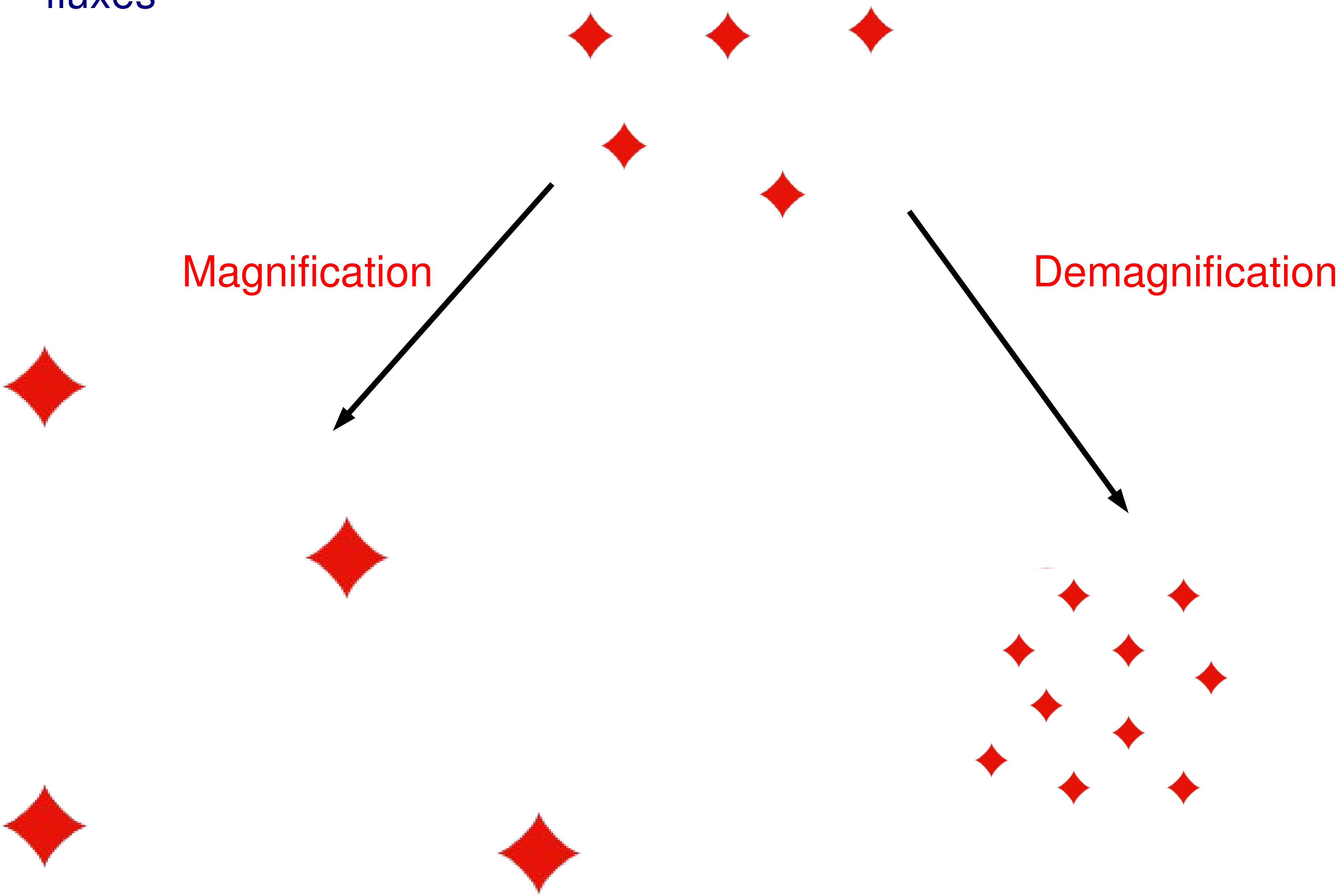
only for E/Z >> 10^{19} eV deflections become less than a few degrees and CR astronomy could become feasible

If Galactic B field (and composition) were known, one could correct the arrival direction to search for the source



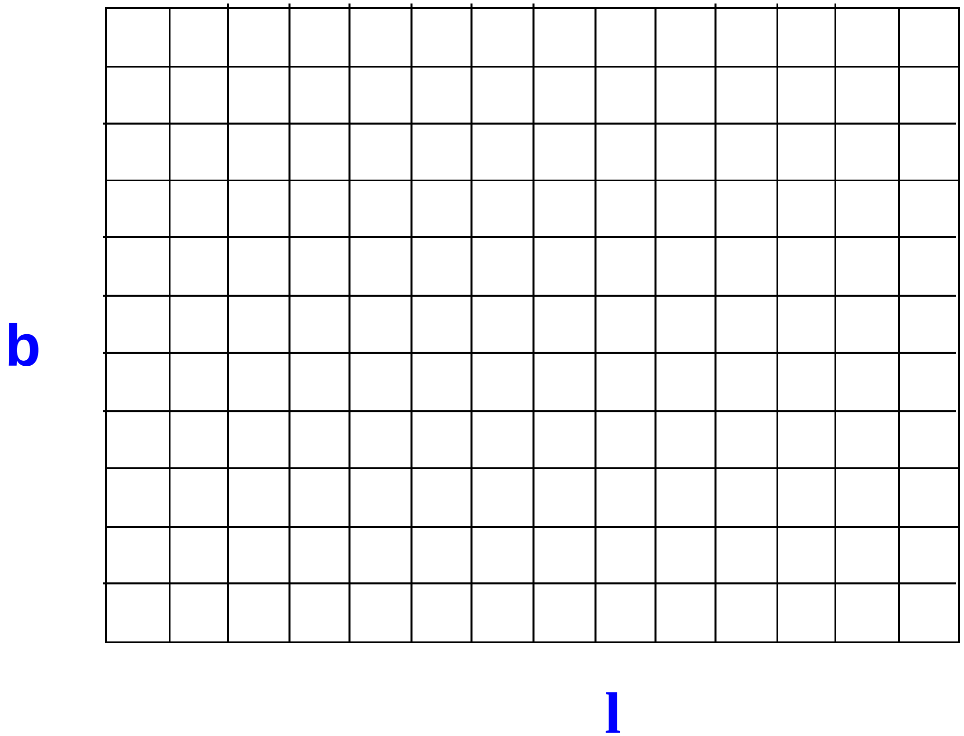
Need to 'backtrack antiprotons' (antinuclei)

Liouville theorem says that the magnetics fields cannot produce anisotropies in an isotropic flux, but they do affect anisotropic fluxes

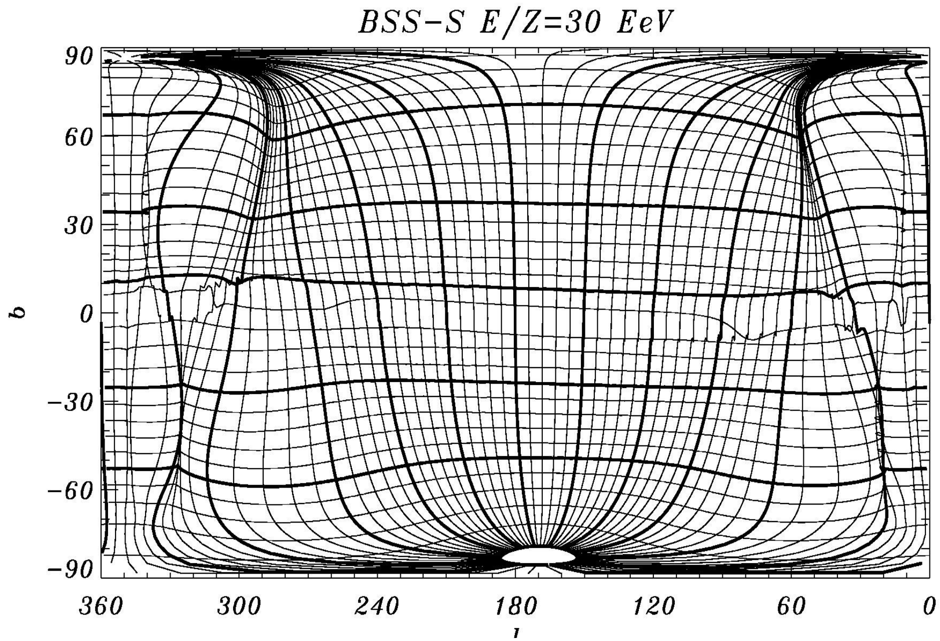


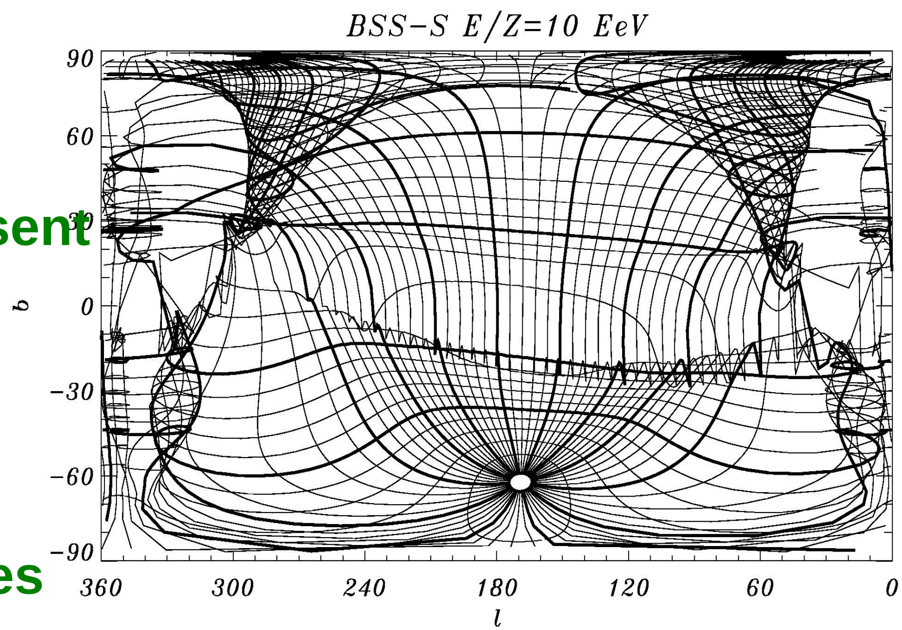
Multiple images can appear

Sky as seen on Earth



Sky projected into the halo





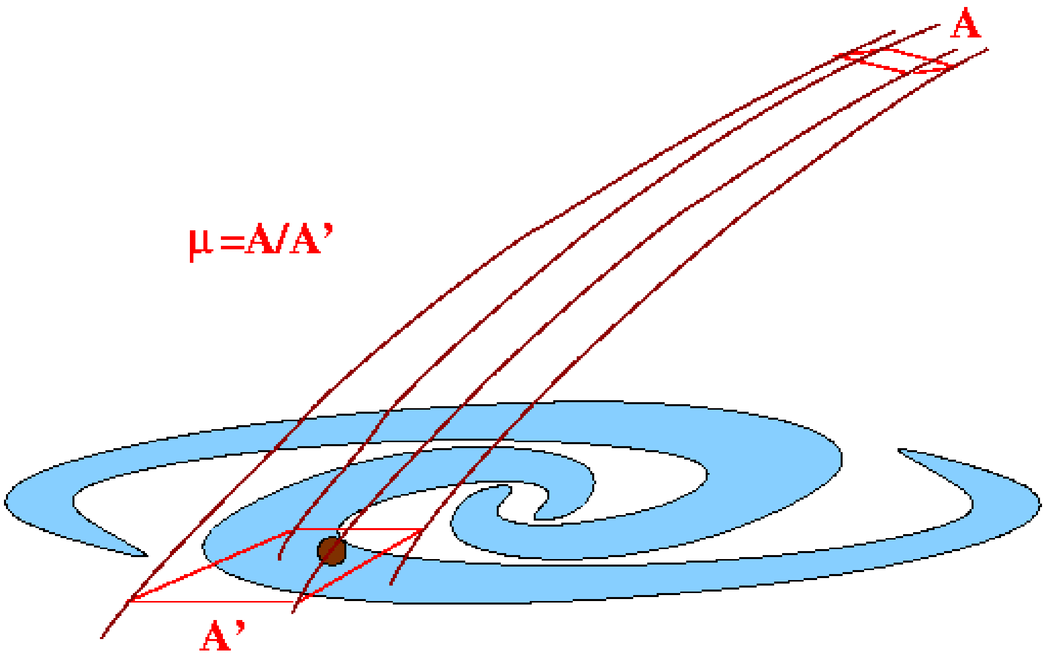
For every fold, two new images are present

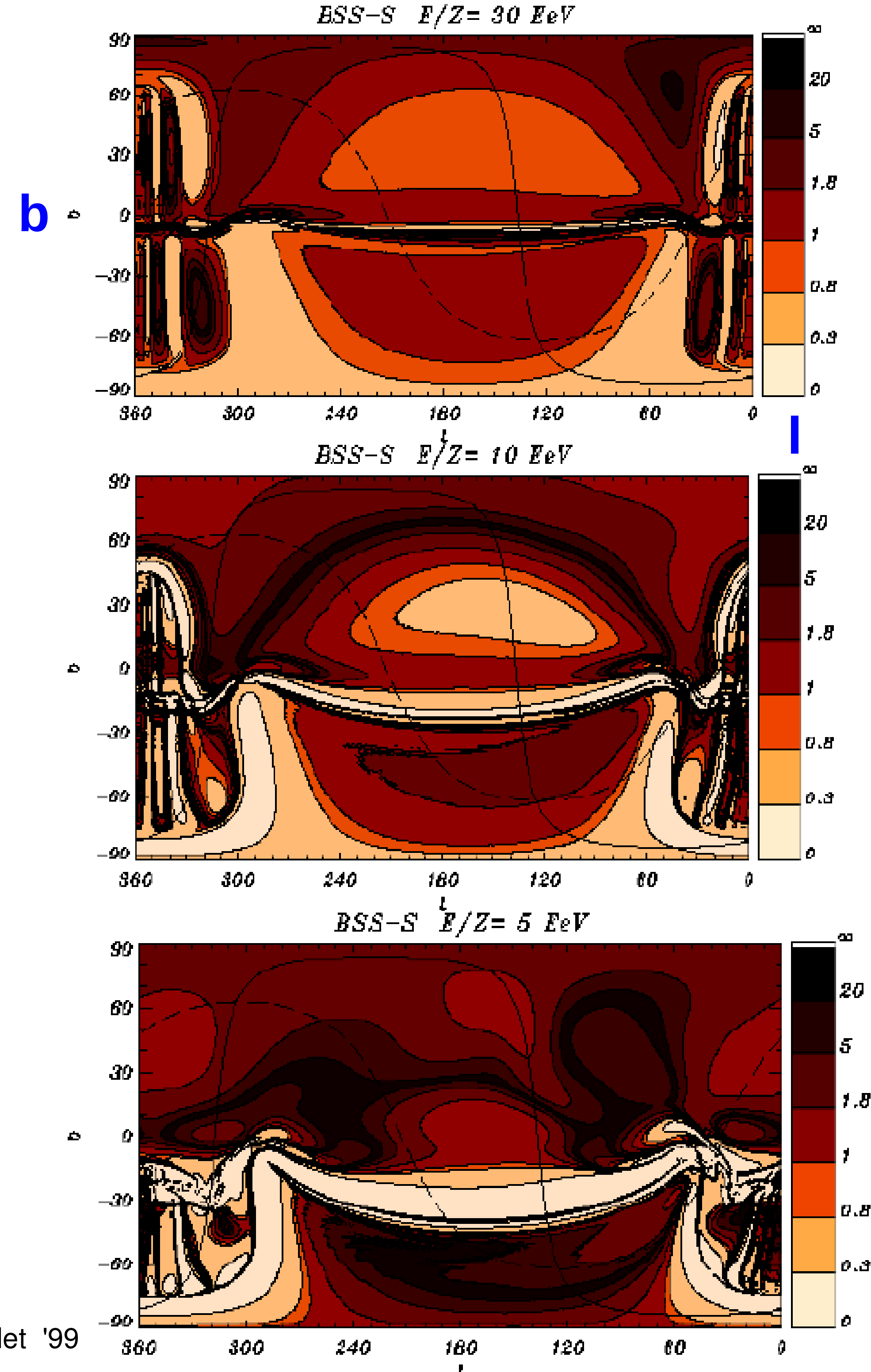
sky stretching → demagnification

sky compression → magnification

at folds (caustics) magnification diverges

CRs are not only deflected, but also magnified





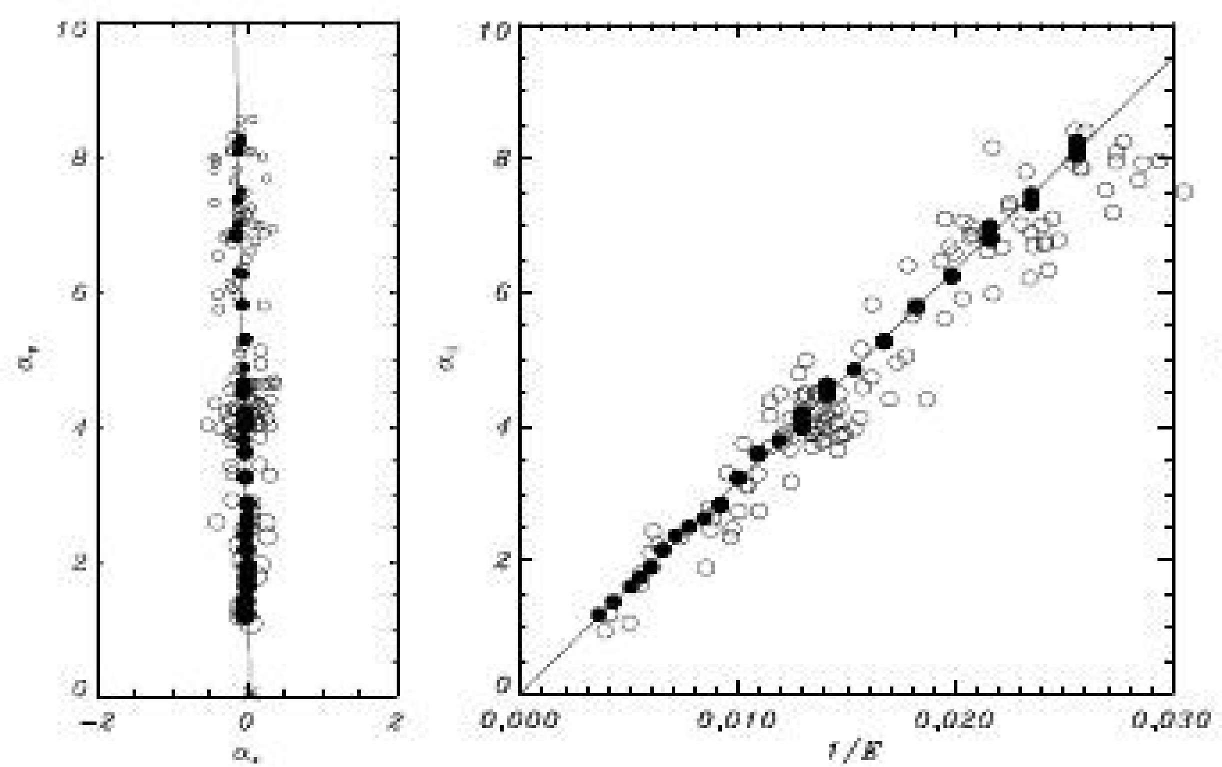
The magnification diverges at the critical lines (which are the images of the caustics on the source plane)

D Harari, S M, E. Roulet '99

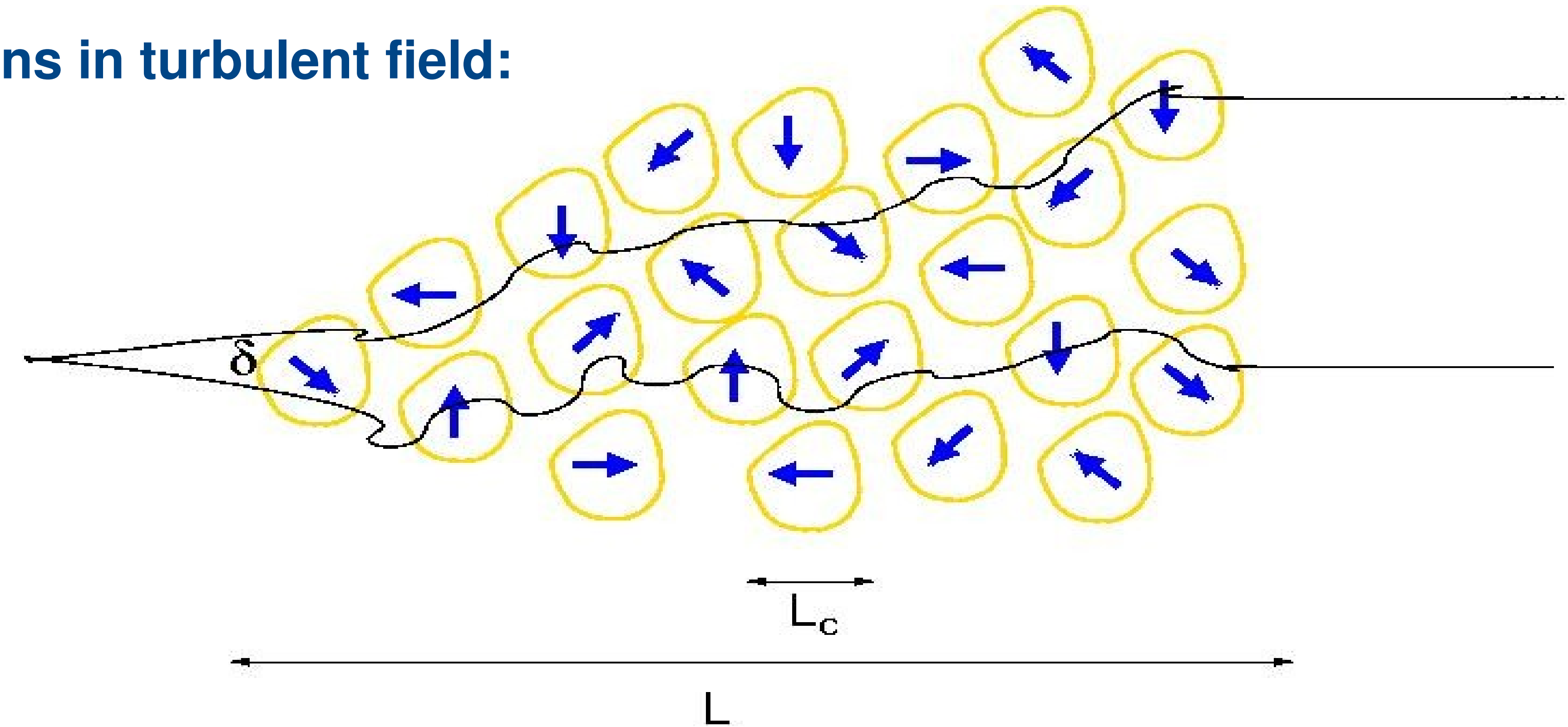
Magnetic field reconstruction

If several CR with different energies coming from one sources are detected it would be possible to measure the integrated perpendicular component of the magnetic field along the CR trajectory

$$\delta \simeq 8.1^{\circ} \frac{40\,\mathrm{EeV}}{E/Z} \boxed{\left| \int_0^L \frac{\mathrm{d}\mathbf{s}}{3\,\mathrm{kpc}} \times \frac{\mathbf{B}}{2\,\mu\mathrm{G}} \right|}$$



Deflections in turbulent field:



Random walk $\rightarrow \delta_{rms}$ = sqrt(n of deflections) x typical deflection in one domain

$$\delta_{rms} \simeq \sqrt{\frac{L}{L_c}}\frac{ZeB_{rms}L_c}{E} \simeq 1^o \frac{10^{19}\,eV}{E/Z}\frac{B_{rms}}{\mu G}\sqrt{\frac{L}{kpc}}\sqrt{\frac{L_c}{50\,pc}}$$

L: distance traversed in B field L_c: coherence length

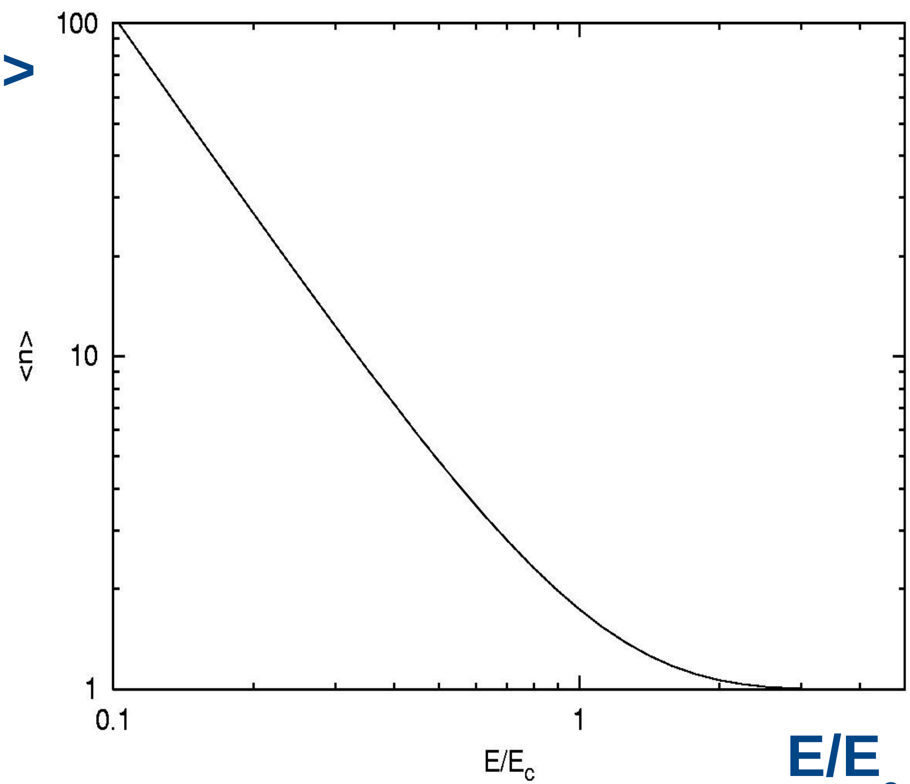
For $B_{rms} \sim B_{reg}$, deflections are smaller than those produced by the regular field, but can dominate the magnetic lensing effects

Multiple images appear below a critical energy E_c, such that typical transverse displacements among different paths become of order the correlation length of the B field ($\delta_{rms} \sim L_c/L$)

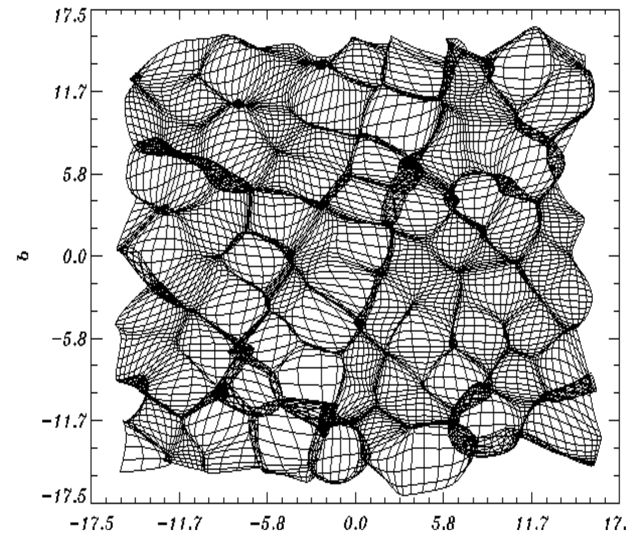
typically $E_c \sim 4\times10^{19}$ eV Z ($B_{rms}/5\mu G$) ($L/2$ kpc)$^{3/2}$ ($L_c/50$ pc)$^{-1/2}$

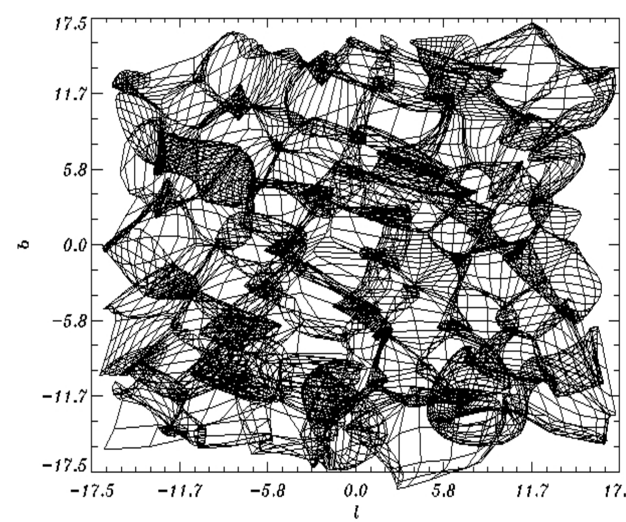
for $E \ll E_c$, the number of images grows exponentially

<n>



E/E_c

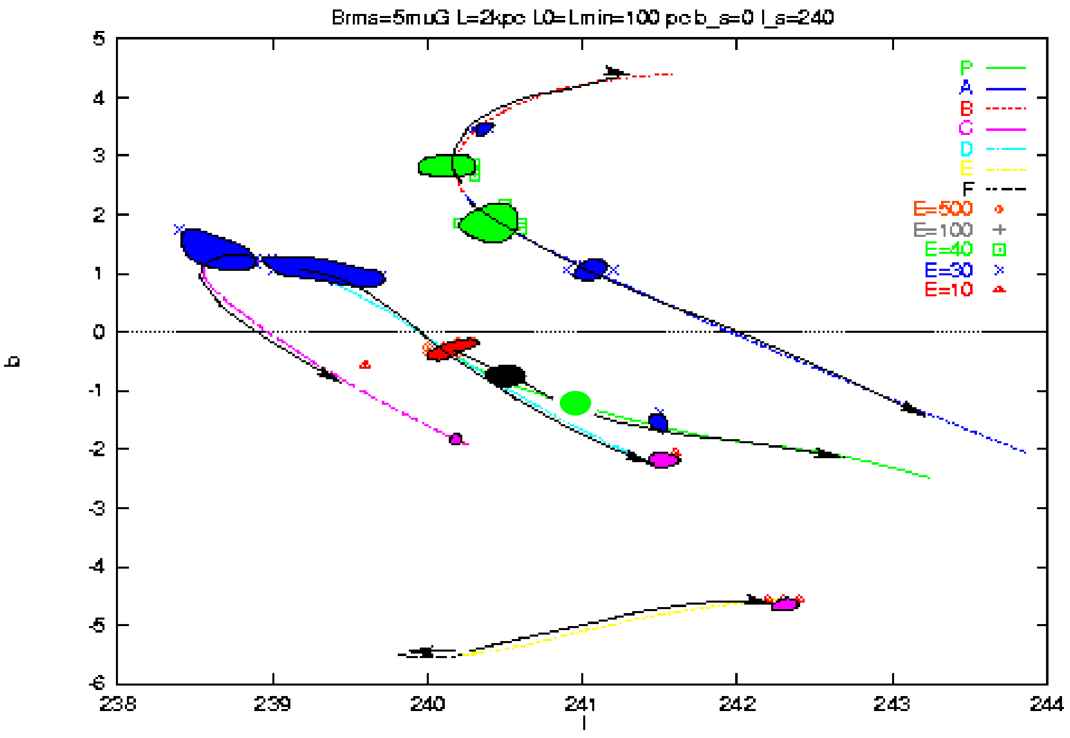




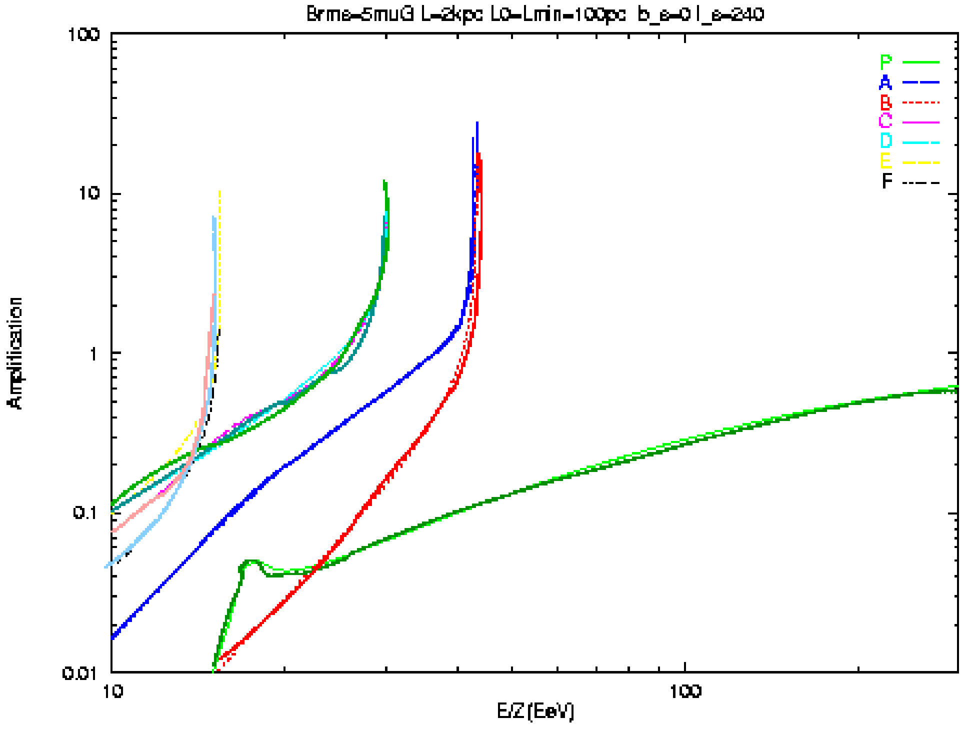
D Harari, S M, E. Roulet , F. Sanchez '02

example:

images

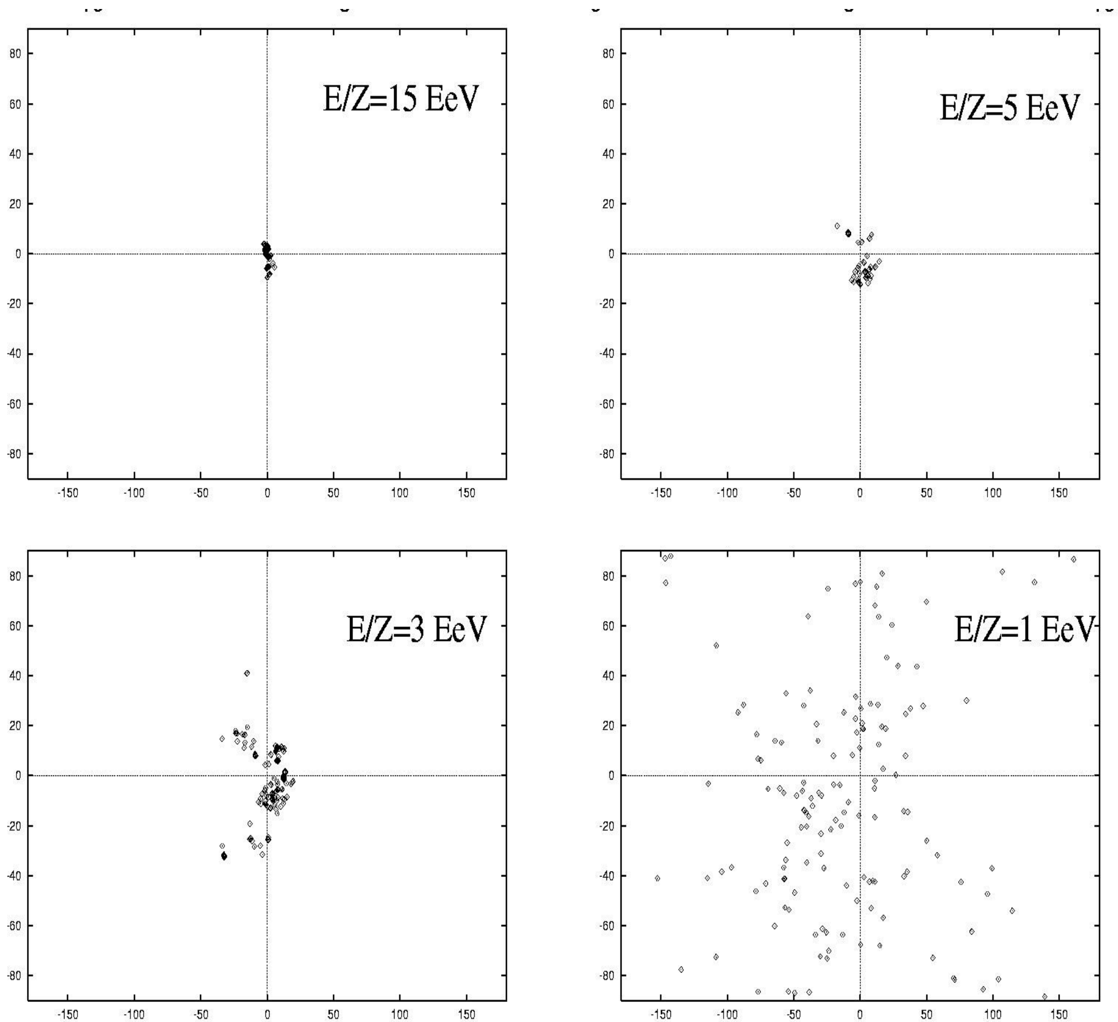


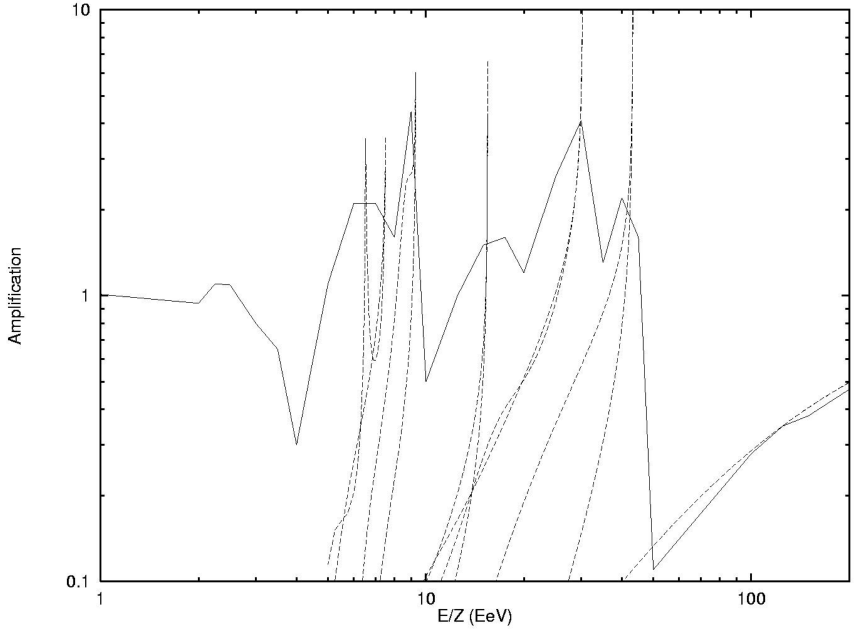
magnification



5×10^{20}, 10^{20}, 4×10^{19}, 3×10^{19}, 10^{19}

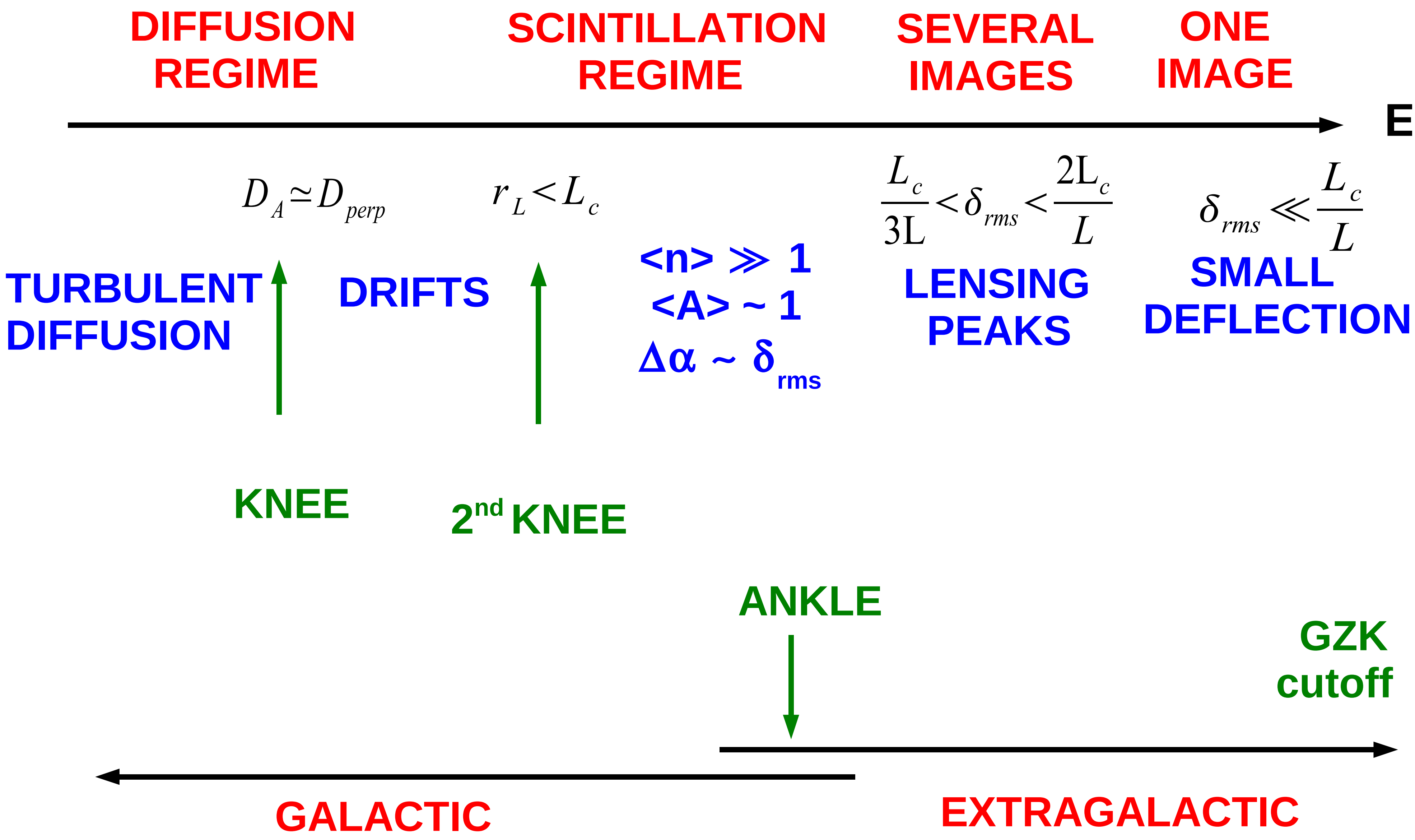
The scintillation regime





A regime is reached with a large number of images, spread over $\Delta\alpha \sim \delta_{rms}$ and with <A> ~ 1 (like twinkling stars)

TURBULENT MAGNETIC FIELD EFFECTS



EXTRAGALACTIC MAGNETIC FIELDS

Magnetic fields are also present outside galaxies, but the observational constraints are still very poor.

The amplitude in central region of clusters may reach ~ μG

The distribution may follow the filamentary pattern of the large scale matter distribution

In most of the space $\langle B_{rms} \rangle = 10^{-8} - 10^{-9}$ G

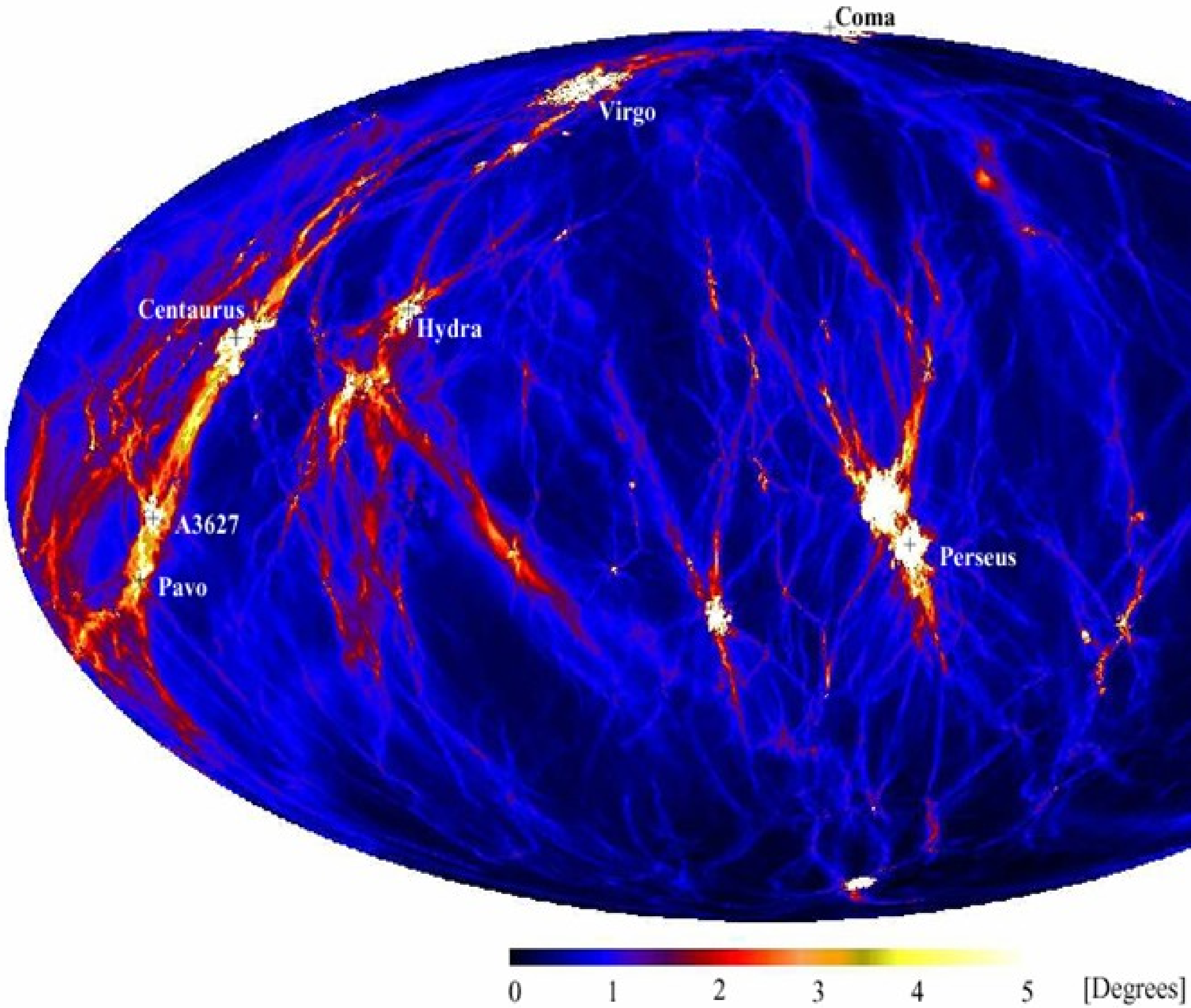
Coherence length L_c ~ Mpc

$$\delta_{ran} \simeq 10^{o} \frac{10\, EeV}{E/Z} \frac{B}{10^{-9}\, G} \sqrt{\frac{L_c}{1\, Mpc}} \sqrt{\frac{D}{10\, Mpc}}$$

Magneto-hydrodynamic simulations: not consensus on the amplitude of CR deflections

Deflection in extragalactic MF - Protons - E > 4 x 10^{19}eV (D< 107 Mpc)

0°

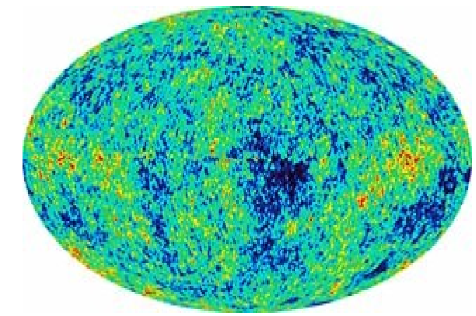


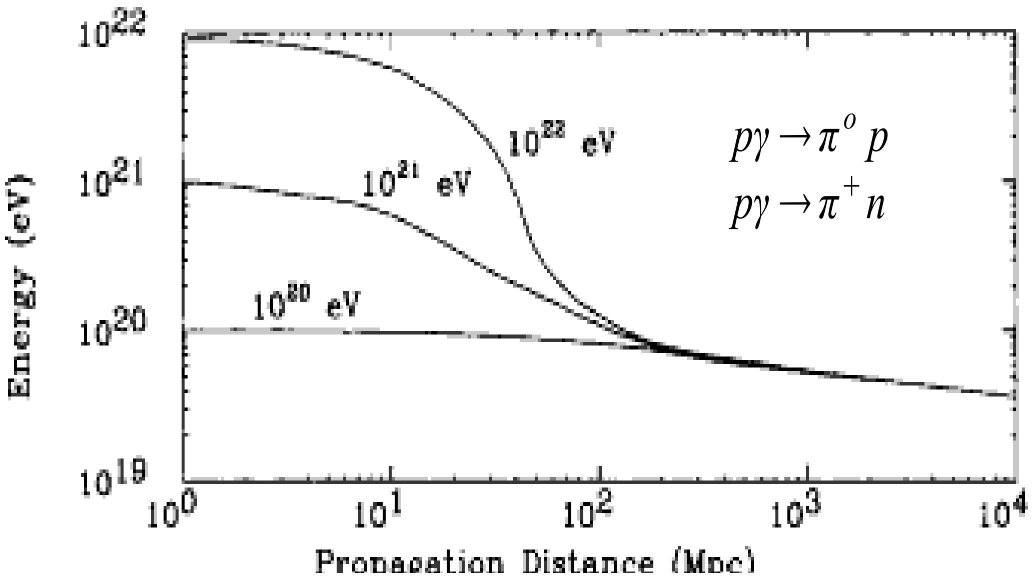
360°

Dolag, astro-ph/0310902

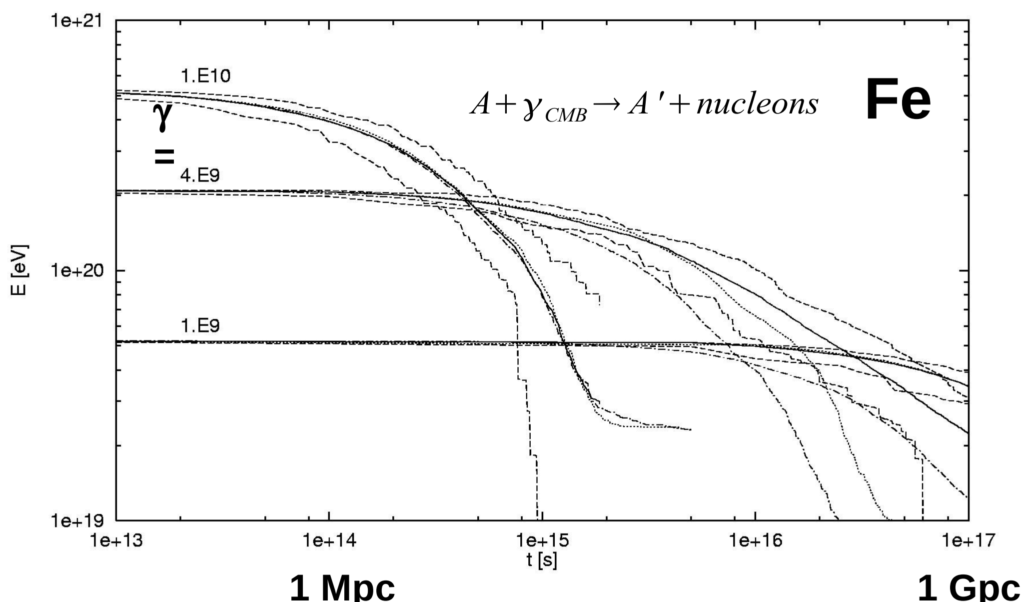
The Greisen-Zatsepin-Kuzmin effect (1967)

AT THE HIGHEST ENERGIES, PROTONS LOOSE ENERGY BY INTERACTIONS WITH THE CMB BACKGROUND





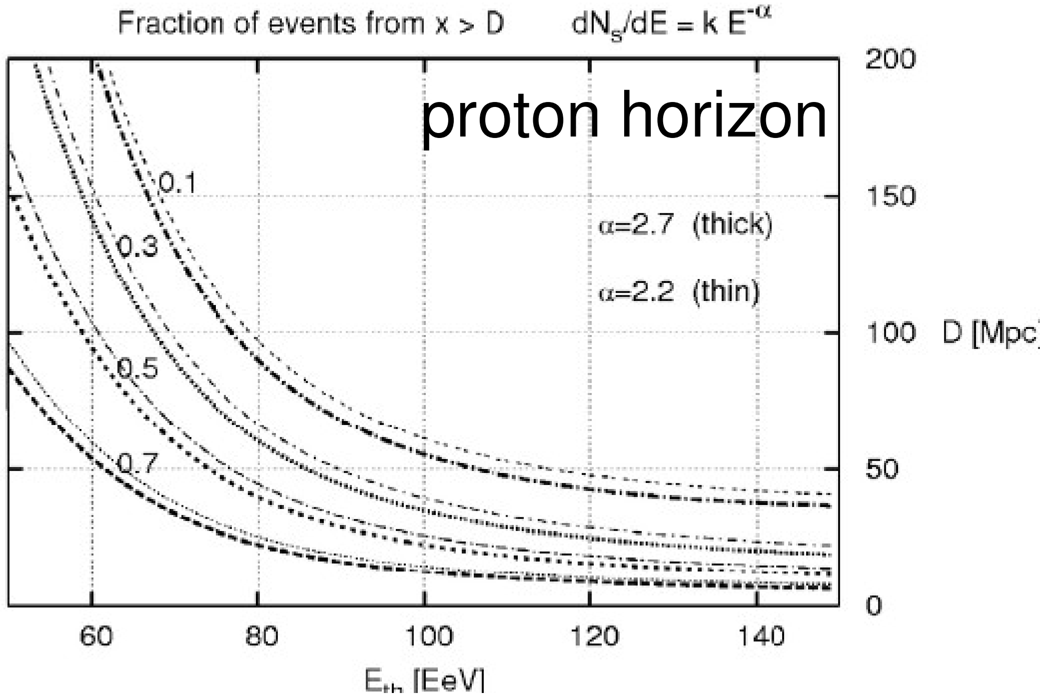
PROTONS WITH E > 6×10^{19} eV CAN NOT ARRIVE FROM D > 200 Mpc



For Fe nuclei:
after ~ 200 Mpc the leading fragment has E < 6×10^{19} eV

ligther nuclei get disintegrated on shorter distances

GZK HORIZON



if CRs are protons:

for E > 6×10^{19} eV

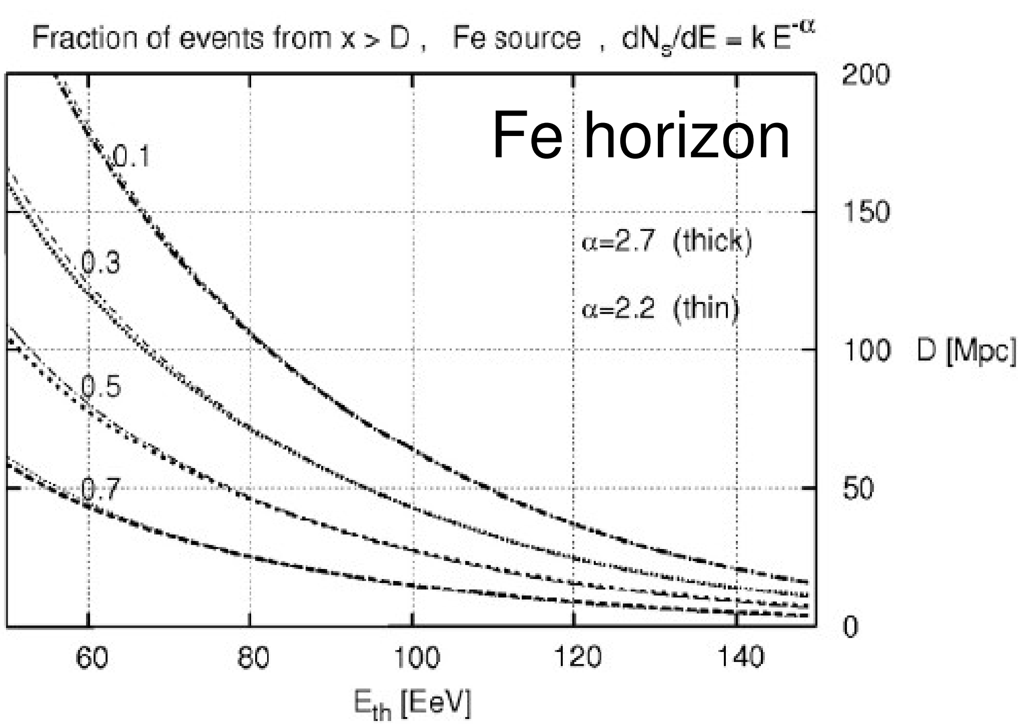
90% from D < 200 Mpc

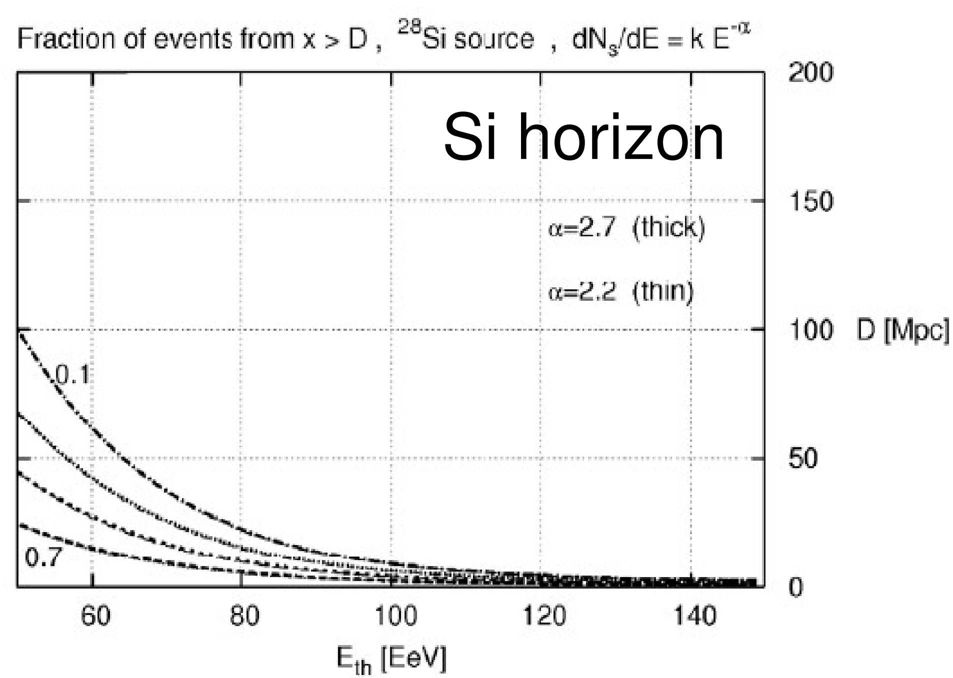
50% from D < 100 Mpc

for E > 8×10^{19} eV

90% from D < 100 Mpc

50% from D < 40 Mpc





D Harari, S M, E. Roulet JCAP 2006

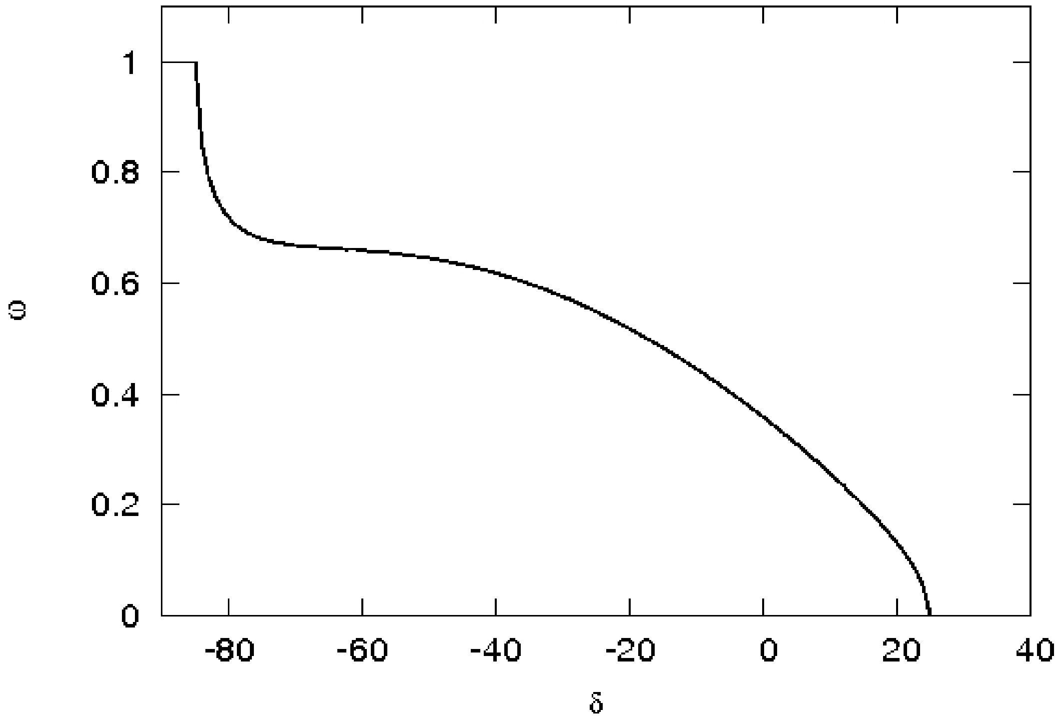
EXPOSURE

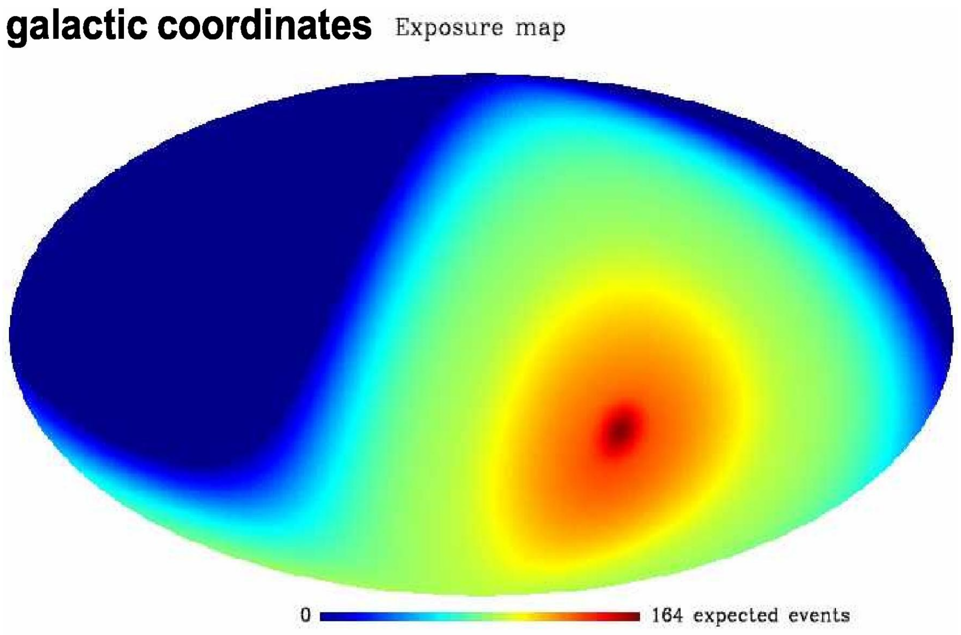
For any CR anisotropy analysis it is very important to have a good estimate of the expectation in the isotropic case.
The exposure measures the time integrated effective collecting area in units of km^2 yr. For each direction of the sky $\omega(\delta,\alpha)$ gives the relative exposure
For a detector in continuous operation it is uniform in RA $\rightarrow$ $\omega(\delta)$

If the detector is fully efficient for particles arriving with zenith angle $\theta < \theta_{m,}$ the exposure has only $\cos\theta$ modulation due to the change in the effective area.
Zenith of a detector at latitude $\delta_0 \rightarrow \boldsymbol{\xi} = (\cos\delta_0 \cos\alpha_\xi, \cos\delta_0 \sin\alpha_\xi, \sin\delta_0)$
Source at position $\mathbf{s} = (\cos\delta_s \cos\alpha_s, \cos\delta_s \sin\alpha_s, \sin\delta_s)$ is seen at a zenith
$\cos\theta = \boldsymbol{\xi}.\mathbf{s} = \cos\delta_0 \cos\delta_s (\cos(\alpha_s - \alpha_\xi)) + \sin\delta_0 \sin\delta_s$
Exposure to a direction **s** prop to integral of $\cos\theta(t)$ for t when $\theta(t) < \theta_m$

$$\omega(\delta) \propto \cos\delta \cos\delta_0 \sin\alpha_m + \alpha_m \sin\delta \sin\delta_0$$

$$\alpha_m = \begin{cases} 0 & if\ z > 1 \\ \pi & if\ z < -1 \\ \arccos(z) & otherwise \end{cases} \qquad z \equiv \frac{\cos\theta_m - \sin\delta_0 \sin\delta}{\cos\delta_0 \cos\delta}$$





Example: detector at lat= -35 (Auger)

Perfect exposure holds at the highest energies where every shower triggers the detector.
At lower energies the more inclined showers are more attenuated than the vertical ones and are less effective to trigger the detector → exposure not proportional to cosθ
Strategies:
- Semi-analytical method: obtain the zenith angle distribution from the data itself and proceed as before with the integration
- Shuffling technique: Many faked events are simulated using the time and zenith distribution of real events from which the isotropic expectation is obtained.

LECTURE 2: ANISOTROPY STUDY TECHNIQUES & RESULTS

- Anisotropy signals at different scales
- Large scale anisotropies: Rayleigh analysis, Compton-Getting effect, sidereal and solar frequencies, observational results, 2-D analysis
- Small and intermediate scale anisotropies: autocorrelation function

ANISOTROPIES : different signals are expected at different energies

Highest energies: small magnetic deflections and small GZK horizon $\rightarrow$ events coming from 'nearby' sources
- small scale clustering of events from the same source
- correlation of events with a population of source candidates
- intermediate scale clustering reflecting the clustering of local sources

Lowering the energy: deflections increase and GZK horizon increase
- CR flux expected to be more isotropic
- Some intermediate scale clustering and correlation with source distribution

Lower energies:
- Large scale anisotropies from diffusion and drifts

For neutral component: point-like signal & correlation with source

Large scale anisotropy from motion of the detector

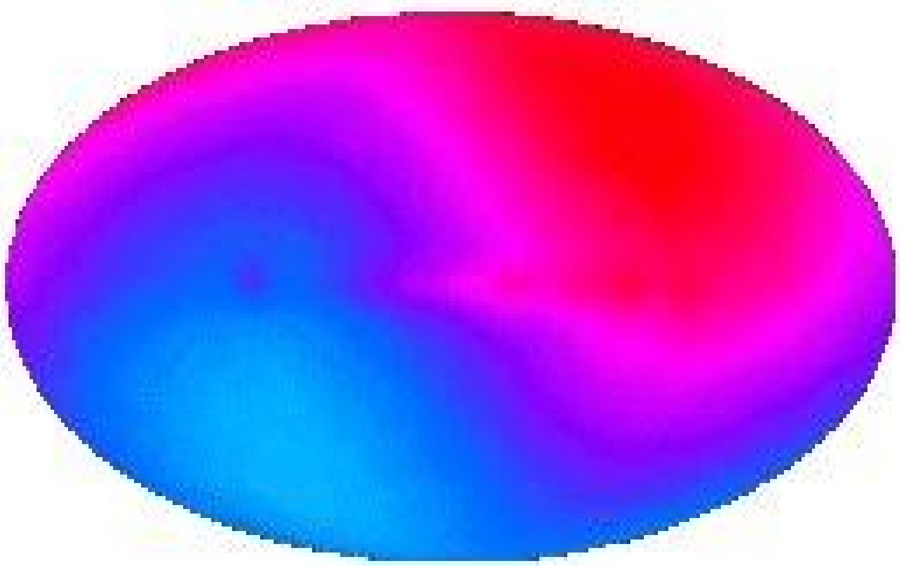
LARGE SCALE ANISOTROPIES

Intensity **I**: number of particles per unit solid angle that pass per unit time through a unit of area perpendicular to the direction of observation **û**

Differential (spectral) intensity **I(E)** → I(E) dE is the intensity of particles with energy in the interval from E to E+dE

Dipole in the direction ĵ:

$$I(\hat{\mathbf{u}}) = I_0 + I_1\, \hat{\mathbf{j}} \cdot \hat{\mathbf{u}}$$



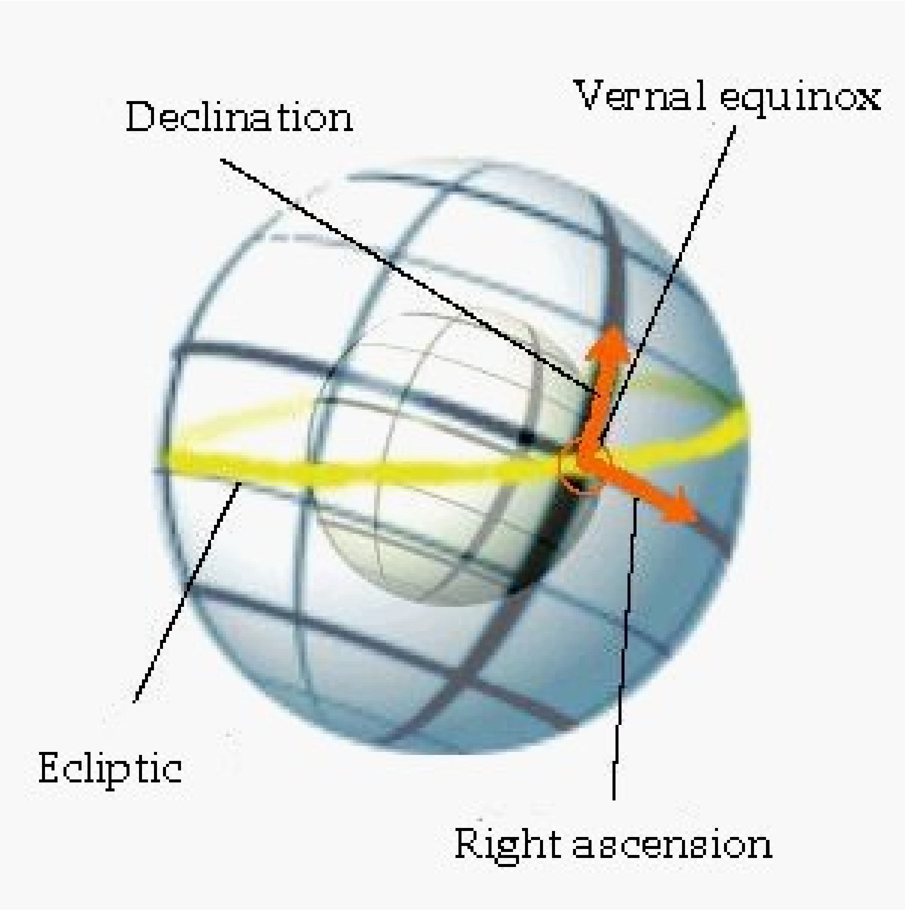
Amplitude

$$\Delta = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_1}{I_0}$$

LARGE SCALE ANISOTROPY MEASUREMENTS: 1- D ANALYSIS

Study only the RA harmonic dependence (of full dataset or a fixed declination band)

-Some experiments cannot reliably determine the δ dependence of the exposure



Celestial coordinates in equatorial system

Right ascension: α
(like longitude)

Declination: δ
(like latitude)

Rayleigh analysis: for n events with right ascension α_i, the k-order harmonic

$$a_k = \frac{2}{n}\sum_{i=1}^{n}\cos(k\,\alpha_i)$$

$$b_k = \frac{2}{n}\sum_{i=1}^{n}\sin(k\,\alpha_i)$$

amplitude $r_k = \sqrt{a_k^2 + b_k^2}$

phase $\phi_k = \arctan\left(\frac{b_k}{a_k}\right)$

$$I(\alpha) = I_0\,(1 + r_1\cos(\alpha - \phi_1) + r_2\cos(2(\alpha - \phi_2) + \ldots)$$

Significance: probability that an amplitude larger or equal than the observed r_k arises from an isotropic dataset

$P(\geqslant r_k) = \exp(- n\, r_k^2 /4)$

Central Limit Theorem: the sum of n random variables $x_1,\ldots,x_n$ distributed with any p.d.f. has a p.d.f. approaching a Gaussian for large n

$y = \sum x_i \rightarrow$ Gaussian $(n \gg 1)$, $\langle y \rangle = \sum \langle x_i \rangle$, $\sigma_y^2 = \sum \sigma_i^2$

$$a_k = \frac{2}{n} \sum_{i=1}^{n} \cos(k\,\alpha_i)$$

Isotropic dist.$\rightarrow$ α_i uniform in $[0,2\pi]$ $\rightarrow$ a_k and b_k Gaussians with $\langle a_k \rangle = \langle b_k \rangle = 0$ and $\sigma_{ak}^2 = (2/n)^2 \sum_i \underbrace{\langle \cos^2(k\,\alpha_i) \rangle}_{1/2} > = 2/n = \sigma_{bk}^2$

$$r_k = \sqrt{a_k^2 + b_k^2}$$

For N Gaussian var. $\{y_i\}$ $\rightarrow$ $z = \sum_i (y_i - \langle y_i \rangle)^2/\sigma_i^2$ is χ^2 distributed with N d.o.f.

$z = (n/2)(a_k^2 + b_k^2) = (n/2)\, r_k^2$ is $\chi^2(2)$ $\rightarrow$ $P(z) = ½ \exp(-z/2)$

$P(r_k)\, dr_k = ½ \exp(-\, n\, r_k^2/4)\, n\, r_k\, dr_k$

$P(\geq r_k) = \int_{r_k}^{\infty} P(r_k)\, dr_k = \exp(-\, n\, r_k^2/4)$

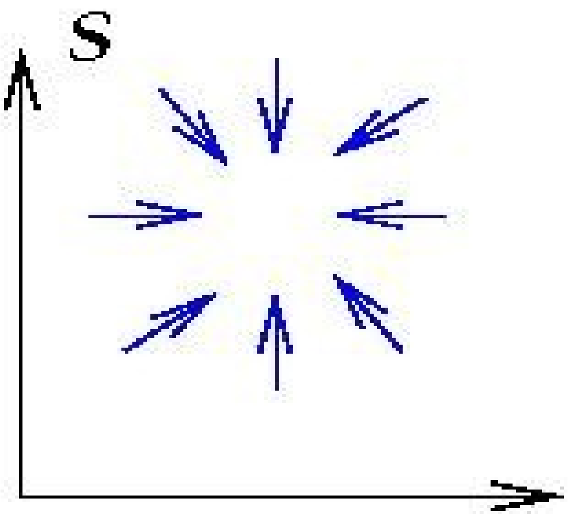
Rayleigh method: r_k, ϕ_k , $P(\geq r_k)$

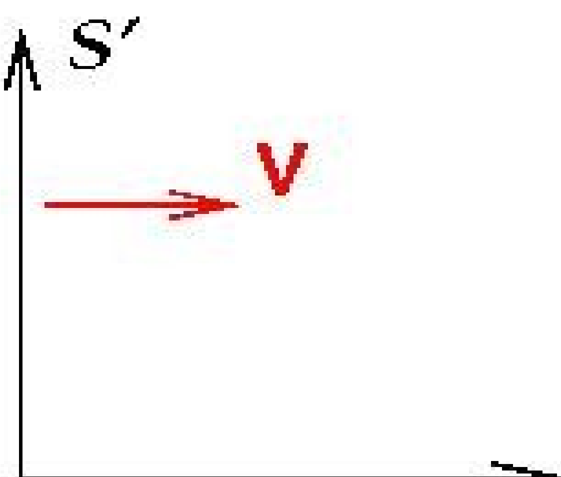
Only recontruct the projection to the equatorial plane of the real dipole

For full sky uniform exposure: $r_1 = (\pi/4)\, \Delta \cos(\delta_{Dip})$ and ϕ_1: RA of the dipole

Compton Getting effect

If the CR flux is isotropic in a reference system S and the observer is moving with respect to that coordinate system with a velocity **V**, he will measure an anisotropic flux → DIPOLE





f(**p**,**r**) → distribution func. of part. in S, f ' → in S'

Lorentz invariance → f(**p**,**r**) = f '(**p'**,**r'**)

$$\vec{p}\,' = \gamma_V \left(\vec{p} - \frac{p}{u} \vec{V} \right) \qquad (V \ll c \,, \gamma_V \sim 1)$$

u: velocity of the relativistic particles

$$f\,'(\vec{p}\,') = f(\vec{p}\,') - \frac{\partial f}{\partial \vec{p}\,'} \cdot \vec{V} \, \frac{p}{u} = f \left(1 - \frac{\vec{V} \cdot \vec{p}}{u\, p} \frac{\partial \ln f}{\partial \ln p} \right)$$

I (t,E,**r**,**n**) = p^2 f(t,**r**,**p**) → ln I = 2 ln p – ln f

$$\frac{\partial \ln f}{\partial \ln p} = \frac{\partial \ln I}{\partial \ln p} - 2 \simeq \frac{\partial \ln E}{\partial \ln p} \frac{\partial \ln I}{\partial \ln E} - 2 = \frac{E^2 - m^2 c^4}{E^2} (-\gamma) - 2 \simeq -(\gamma + 2)$$

for particles with spectrum I = $E^{-\gamma}$

I' (E') = I (1 + (V/u) (γ+2) cos θ)

(ex: V = 100 km/s → Δ = 1.6 10^{-3})

CG from orbital motion of the Earth around the Sun

Rotation velocity V = 29.8 km/s

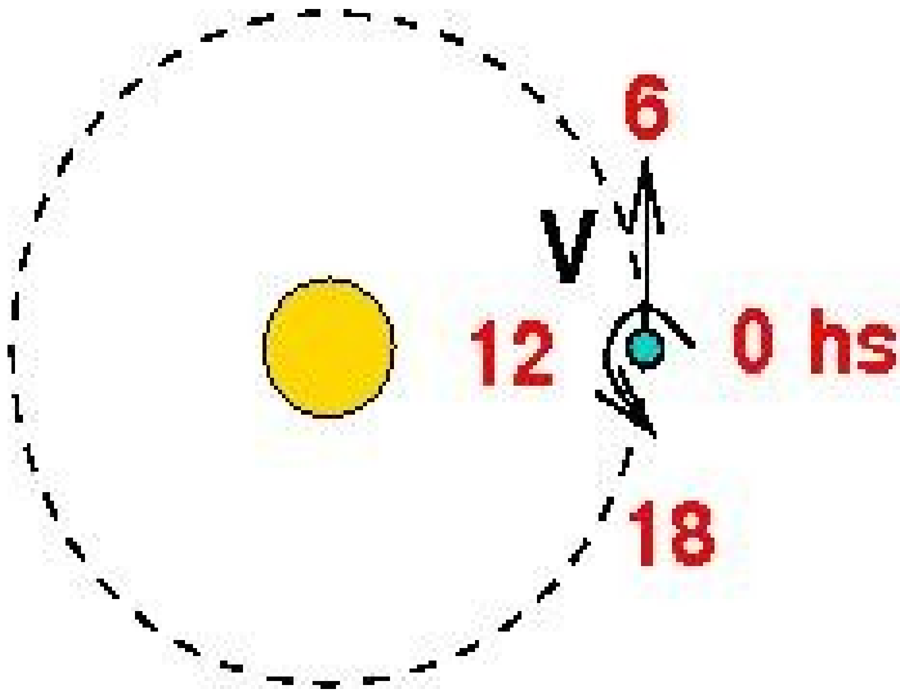
A vertically looking detector sees
a modulation of the intensity
with the solar time

$I(t) = I_0\,(1 + r\cos((t - t_0)\,2\pi/24\text{hs}))$

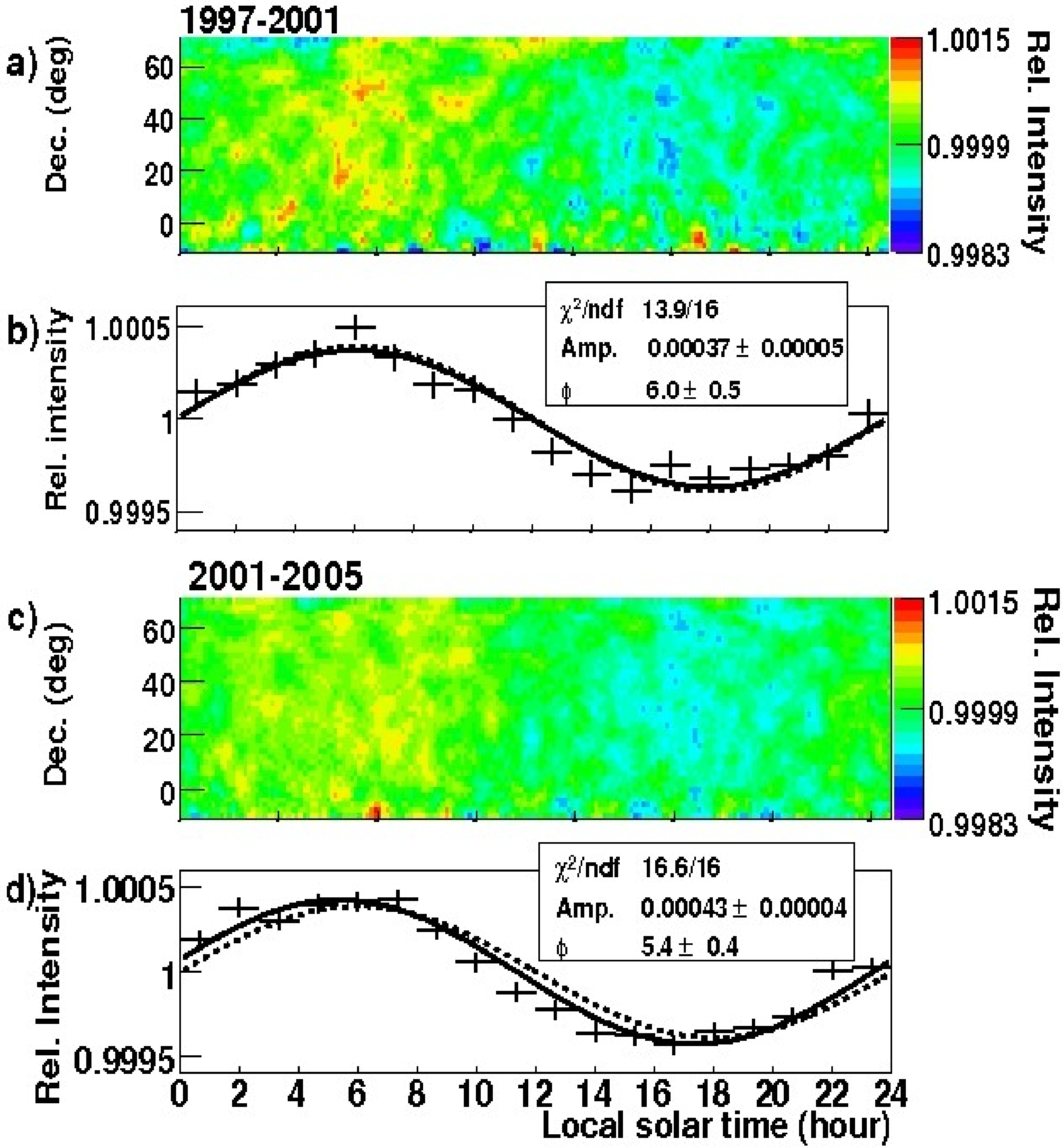
maximum at t_0 = 6 hs

Amplitude depends on the detector
latitude and can be as large as

$\Delta = (V/c)\,(\gamma + 2) \sim 5 \times 10^{-4}$



Solar time CR intensity map and Rayleigh amplitude for Tibet AS (E~10 TeV)



Amenomori '06

Best fit → solid line
CG exp → dashed line

Good agreement for the solar frequency, where the expectations are well known: this gives confidence that the measurements are reliable for the sidereal frequency analysis, where expectations are uncertain.

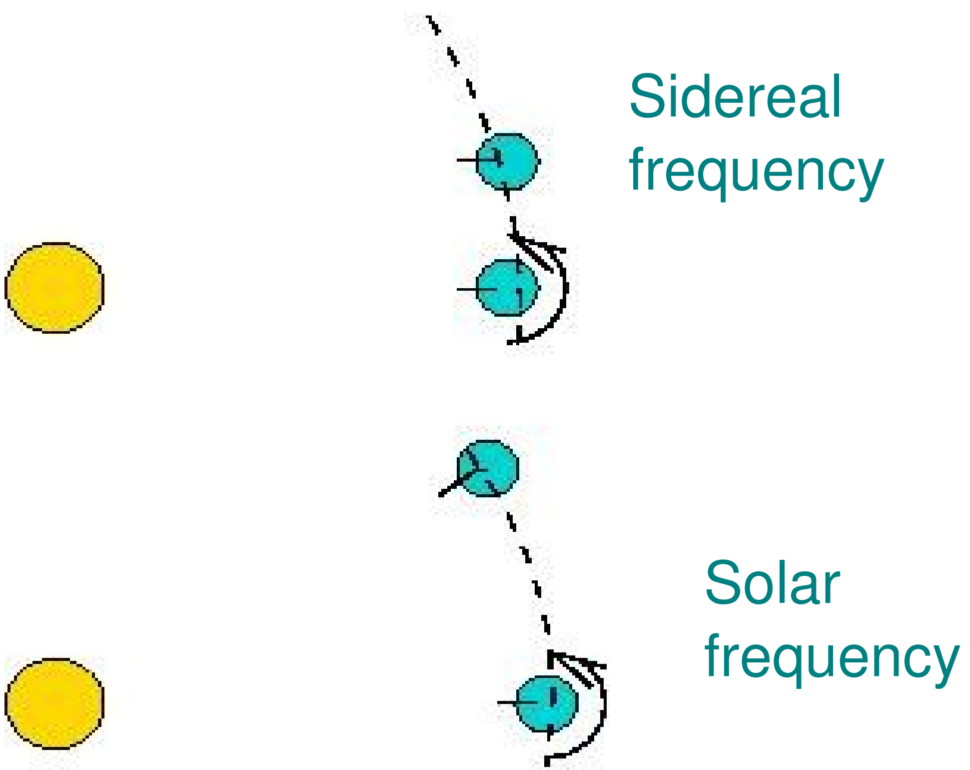
Also used to estimate the spectral index
$\gamma = 3.03 \pm 0.55$ (6-40 TeV)

Amenomori '07

CG from the motion of the Solar system

Motion of the solar system with respect to the rest frame of CR →
modulation in the sidereal time frequency (RA).

The solar day is bit longer than the sidereal day
(1 year = 365.24 solar days = 366.24 sidereal days)



The sidereal time (or RA) modulation has the most interesting information

Results of sidereal harmonic analysis

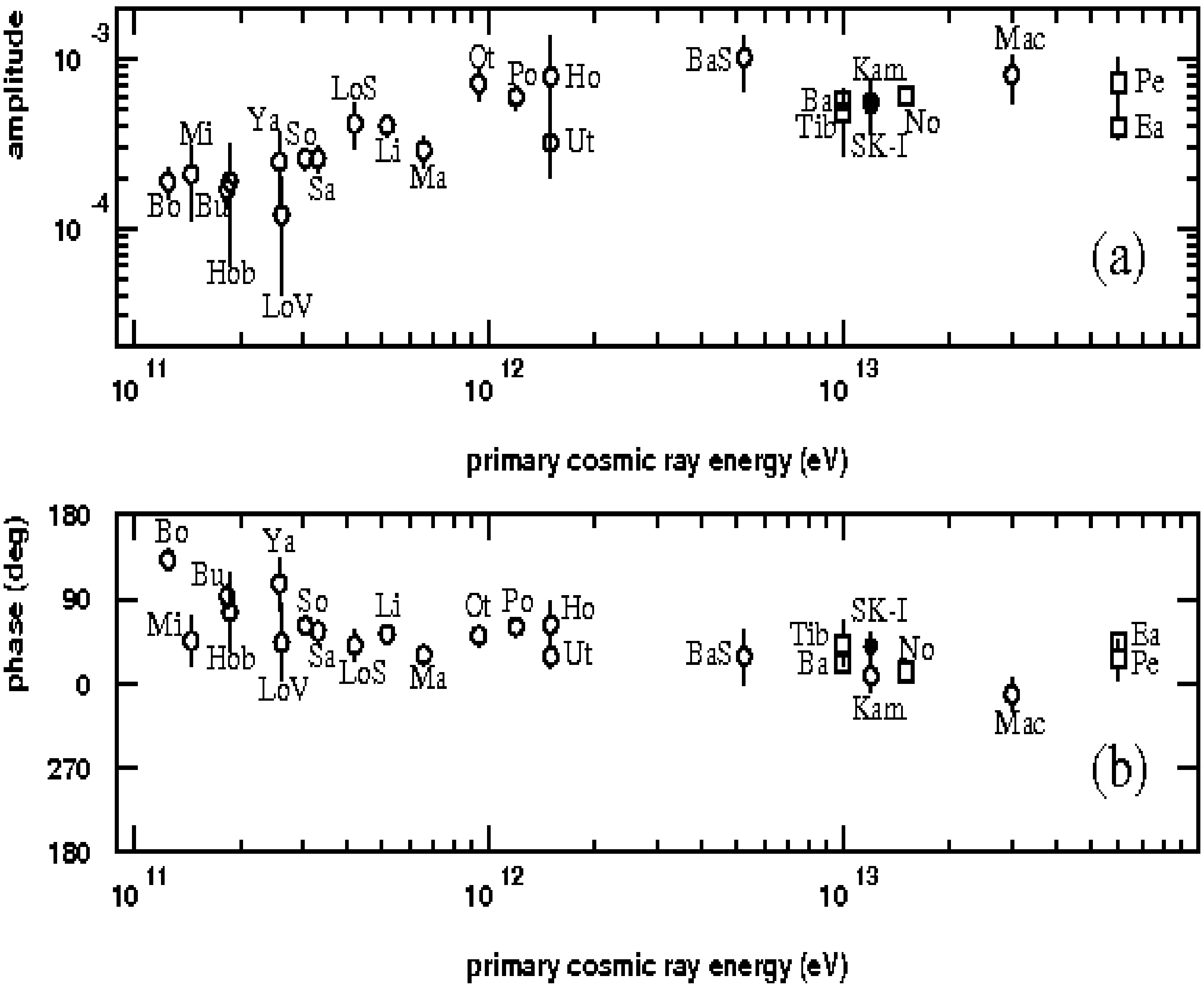
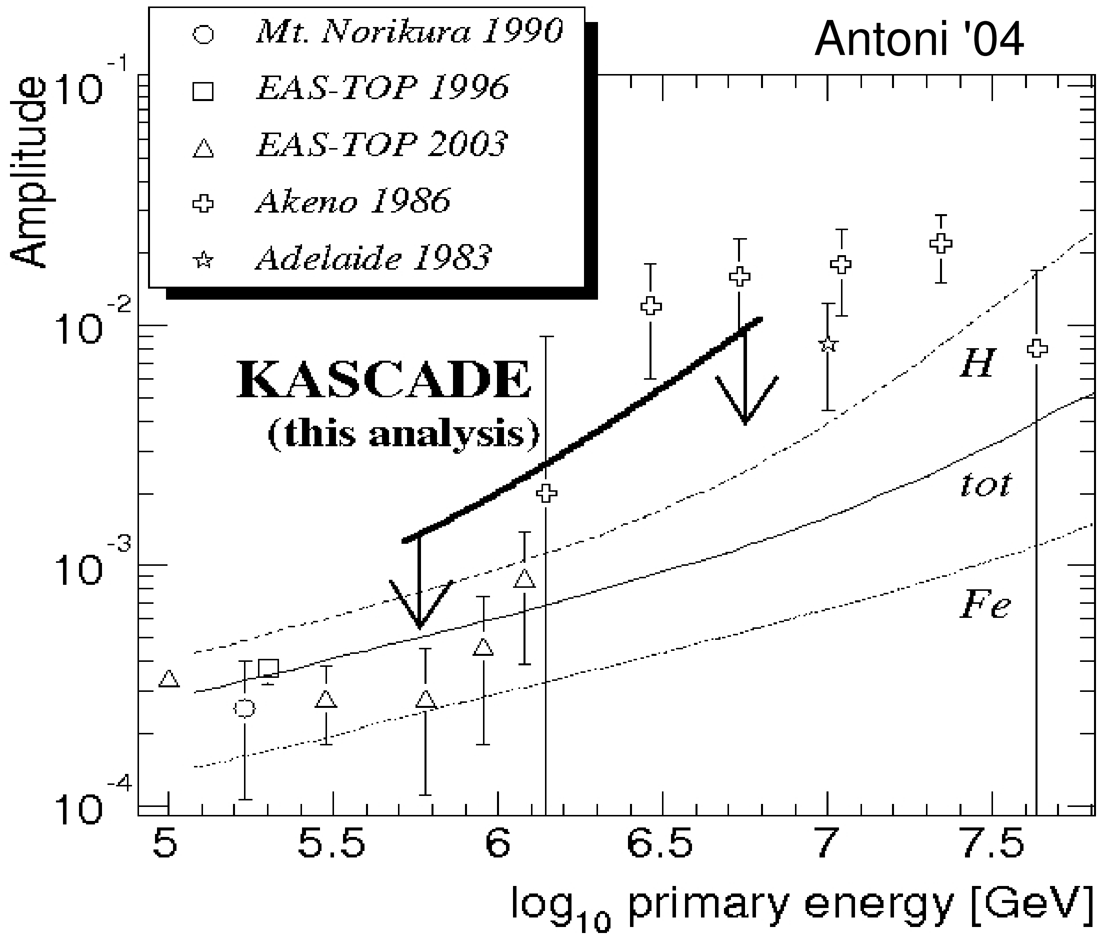


FIG. 3: Amplitude and phase of the first harmonic fit to zenith-type plots from various cosmic ray experiments. The energy in the horizontal axis is either the median or the log-mean energy. Circles: muon detectors. Squares: extensive air shower array. Filled circle: SK-I. Data references: Bo = Bolivia (vertical) [12], Mi = Misato (vertical) [13], Bu = Budapest [13], Hob = Hobart (vertical) [13], Ya = Yakutsk [13], LoV = London (vertical) [13], So = Socorro (vertical) [12], Sa = Sakashita (vertical) [14], LoS = London (south) [15], Li = Liapootah (vertical) [16], Ma = Matsushiro (vertical) [17], Ot = Ottawa (south): [18], Po = Poatina (vertical) [19], Ho = Hong Kong [20], Ut = Utah [21], BaS = Baksan (south) [22], SK-I (this report), Kam = Kamiokande [10], Mac = MACRO [11], Tib = Tibet (vertical) [23], Ba = Baksan air shower [24], No = Mt. Norikura [3], Ea = EAS-TOP [25], Pe = Peak Musala [26].

Amplitude
Δ = few x 10^{-4}

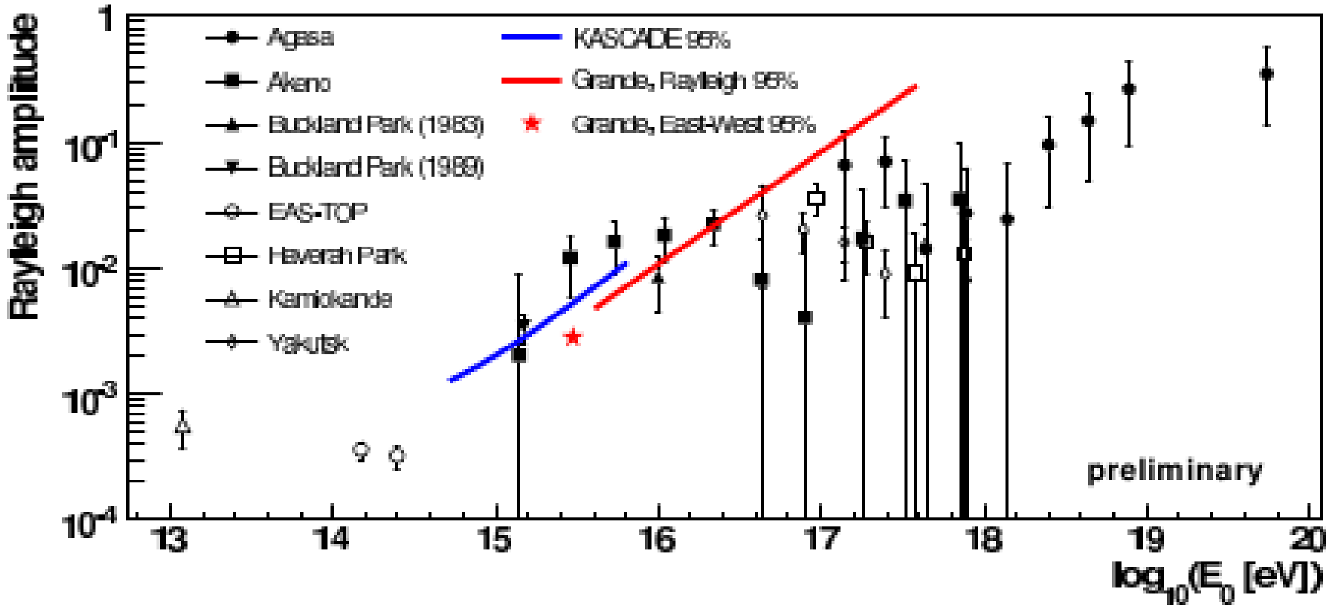
Velocity of solar system around the Galaxy is
V $\simeq$ 220 km/s
$\rightarrow$ Δ = few x 10^{-3}

CR plasma corrotate with the local stars



Higher energy results

Theoretical expectations: transport of CR in a magnetized plasma is governed by anisotropic diffusion, drift, convection
→ anisotropies are expected with increasing amplitude at larger energies, as particles start to escape from the Galaxy more easily



Over '07

AGASA: 4% amplitude for the dipole at E > 10^{18} eV

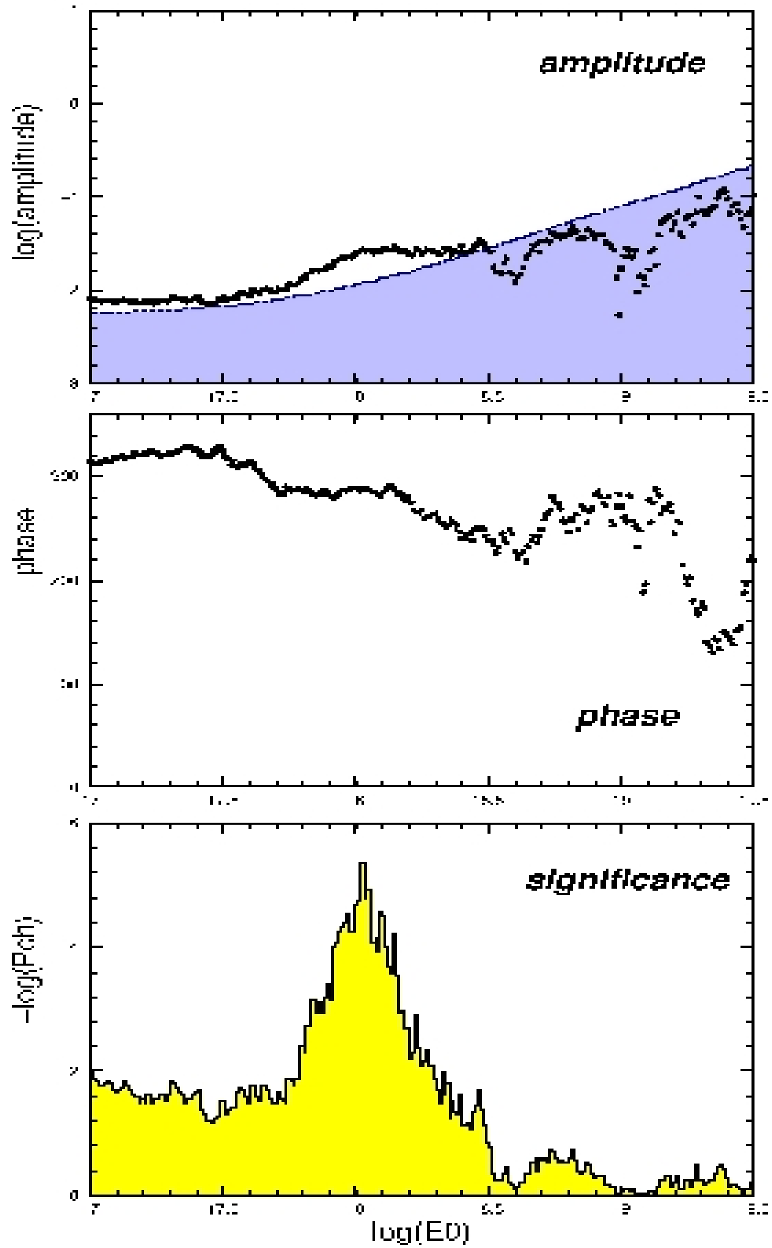


Fig. 1. The first harmonic analysis in the right ascension direction. **Top:** The amplitude of first harmonics is shown as a function o threshold energy. The shaded region represents statistical fluctua tion with 90 % C.L.. **Middle:** The direction of maximum ampli tude in the right ascension. **Bottom:** The statistical significance o the first harmonic is shown as a function of threshold energy. Th chance probability at 10^{18}eV threshold energy is 10^{-5}.

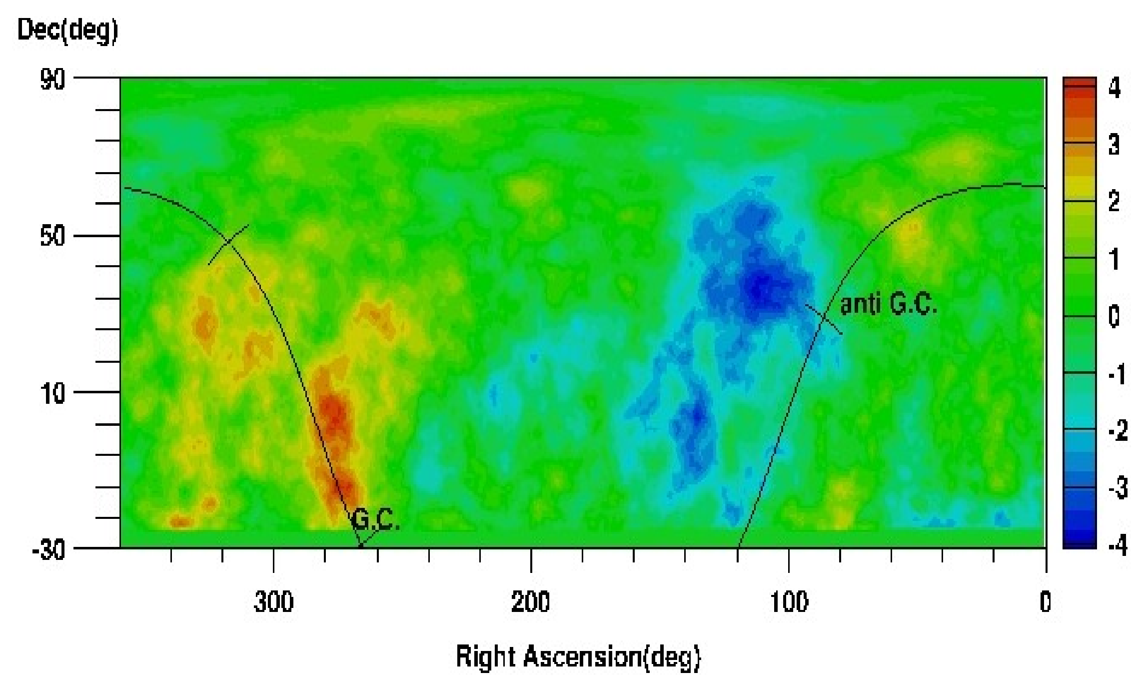
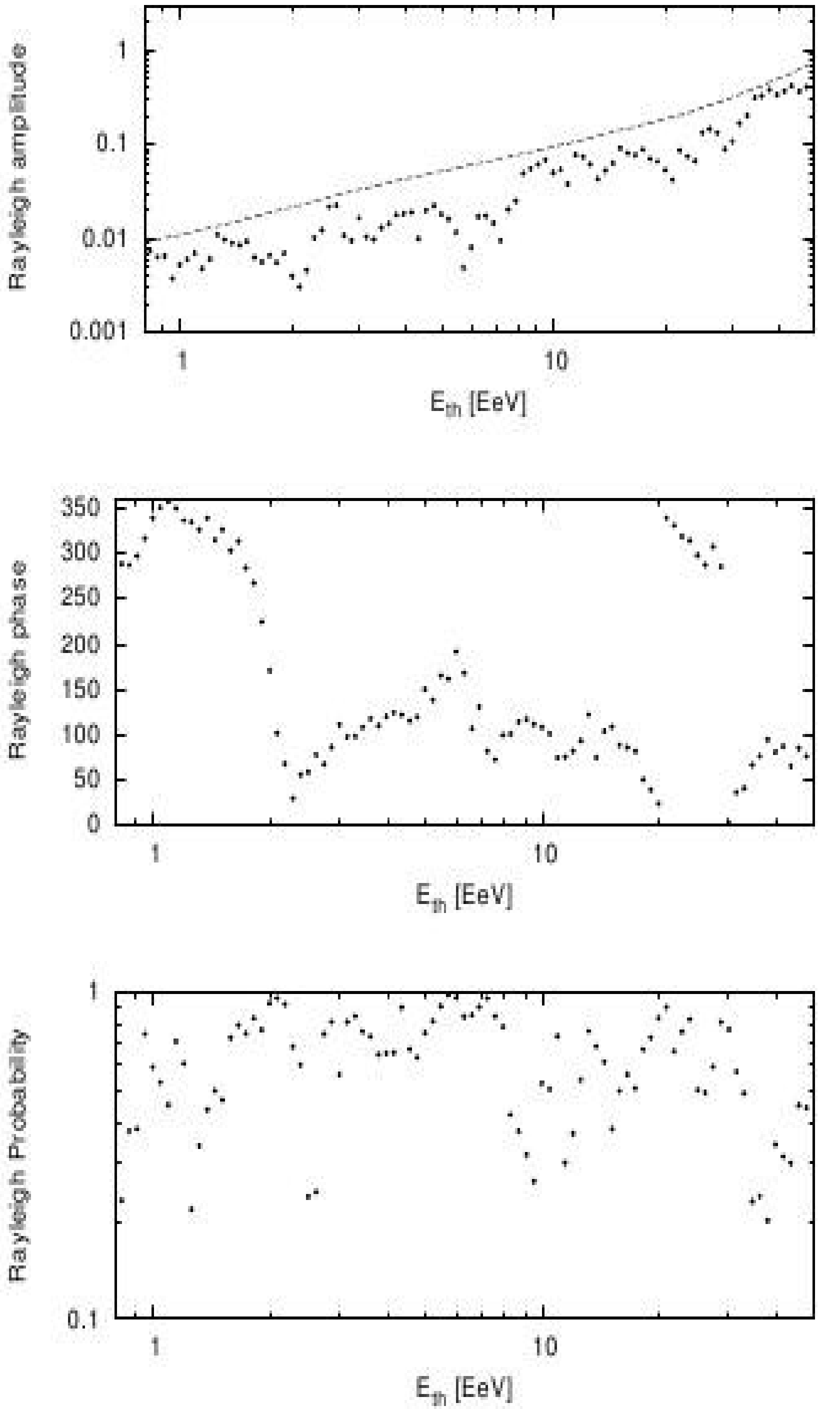
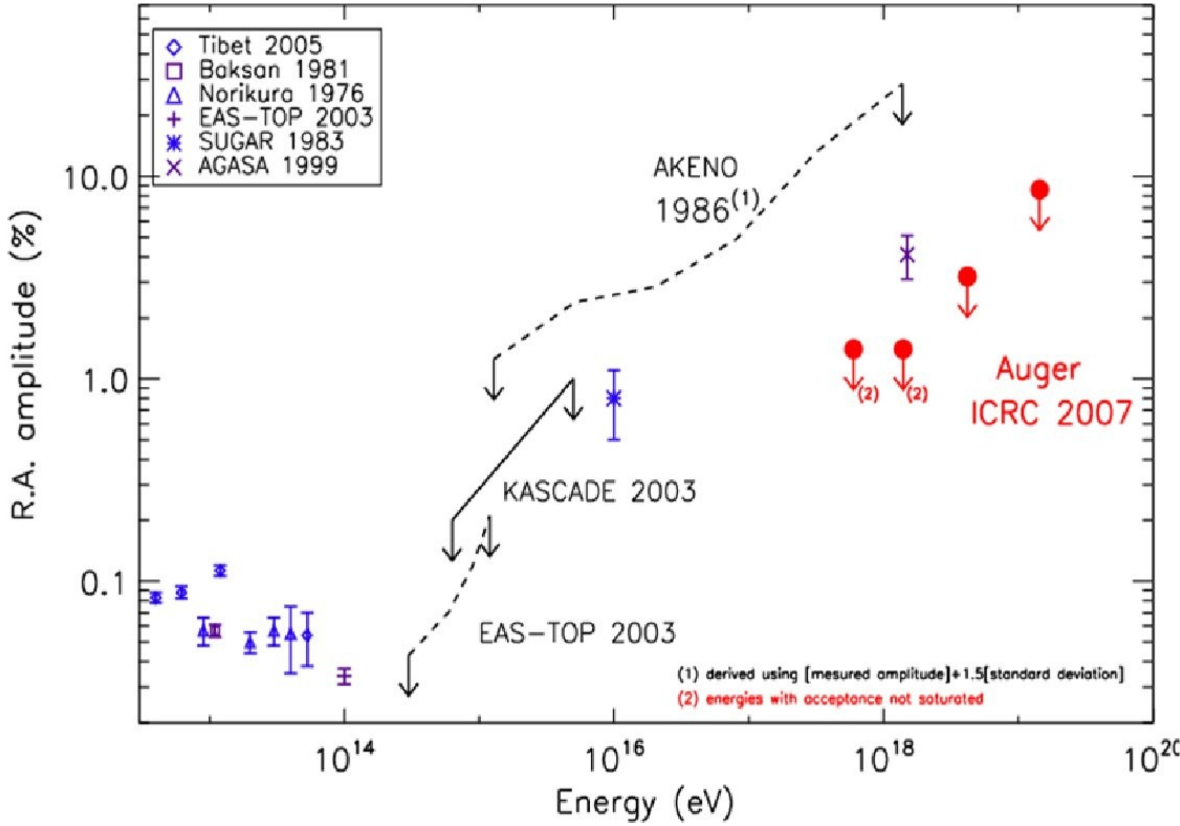


Fig. 2. The significance of event density in equatorial coordinates. The statistical significance of deviation is evaluated for each 1° grid with the aperture of 20 degrees radius. The excess and deficit can be seen with 4 σ statistical significance near the Galactic center and anti-galactic center, respectively.



Auger: amplitude of the dipole
< 1.4% at 10^{18} eV and
< 10% at 10^{19} eV



HIGHEST ENERGIES: extragalactic cosmic rays may also show Compton-Getting anisotropy.

The sun moves at v = 368 km/s towards the great attractor $\rightarrow$

$\Delta = (2+2.7)\ v \approx 0.006$ (much smaller than current upper limits)

LARGE SCALE ANISOTROPY MEASUREMENTS: 2 - D ANALYSIS

Reconstruct the amplitude and direction of the dipole
Observed flux = real flux x exposure

$$I(\hat{\mathbf{u}}) = \tilde{I}(\hat{\mathbf{u}}) \times \omega(\hat{\mathbf{u}})$$

If $\omega \neq 0$ everywhere (full sky observatory)

$$\tilde{I}(\hat{\mathbf{u}}) = I(\hat{\mathbf{u}})/\omega(\hat{\mathbf{u}}) = I_0(1 + \Delta\, \hat{\mathbf{j}} \cdot \hat{\mathbf{u}})$$

$$I_0 = \frac{1}{4\pi} \int_S d\Omega \frac{I(\hat{u})}{\omega(\hat{u})} \qquad \Delta\, \hat{j} = \frac{3}{4\pi I_0} \int_S d\Omega\, \hat{u}\, \frac{I(\hat{u})}{\omega(\hat{u})}$$

For an experiment detecting N events with directions $\hat{\mathbf{u}}_i$

$$I_0 = \frac{1}{4\pi} \sum_i \frac{1}{\omega(\hat{u}_i)} \qquad \Delta\, \hat{j} = \frac{3}{4\pi I_0} \sum_i \frac{\hat{u}_i}{\omega(\hat{u}_i)}$$

The accuracy depends on the number of points (better for large N) and on the amplitude of the dipole (better for large Δ).
$S \approx 0.065\, \Delta \sqrt{N} \rightarrow$ for CG $\Delta \approx 0.005$ and a 5σ detection requires 2 million CR

Sommers '00

- If there is no full coverage → the method cannot be used directly
  Some modifications have been proposed, but they have limitations
- Also upper harmonics can be reconstructed for full sky coverage

$$I(\hat{u}) = \sum_{i=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{u}) \qquad a_{lm} = \int_S I(\hat{u})\, Y_{lm}(\hat{u})\, d\Omega$$

For N events with arrival directions $\{\hat{\mathbf{u}}_i\}$ and exposure ω

$$a_{lm} = \frac{1}{C} \sum_{i=1}^{N} \frac{1}{\omega(\hat{u}_i)} Y_{lm}(\hat{u}_i) \qquad C = \sum \frac{1}{\omega(\hat{u}_i)}$$

l=1 dipole
l=2 quadrupole: may arise when sources are correlated with a plane

For 2-D analysis it is important to know well the exposure

2-D maps for which ω(δ) is not known are normalized in each declination band → loose any modulation in the N - S direction, in particular they miss the z component of the dipole

SMALL AND INTERMEDIATE SCALE CLUSTERING

Clustering at small scales can be the clue to detect repeating sources $\rightarrow$ the amount of clustering (or the fraction of repeaters) gives a measure of the number of sources that give rise to CR above a given energy threshold $\rightarrow$ local density of sources

Clustering at intermediate scales contains information on the pattern of the distribution of local sources.

AUTOCORRELATION FUNCTION

Measures the excess(deficit) in the number of pairs with respect to an isotropic distribution as a function of the angle

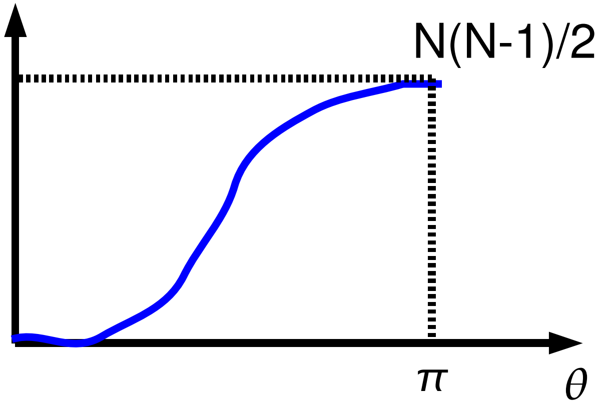
For an isotropic distribution of N points on the full sphere

$$dn_P = \frac{N(N-1)}{2} \sin(\theta)\, d\theta$$

number of pairs in [θ,θ+dθ]

$$n_P(\theta) = \frac{N(N-1)}{2}(1-\cos(\theta))$$

number of pairs with angle < θ



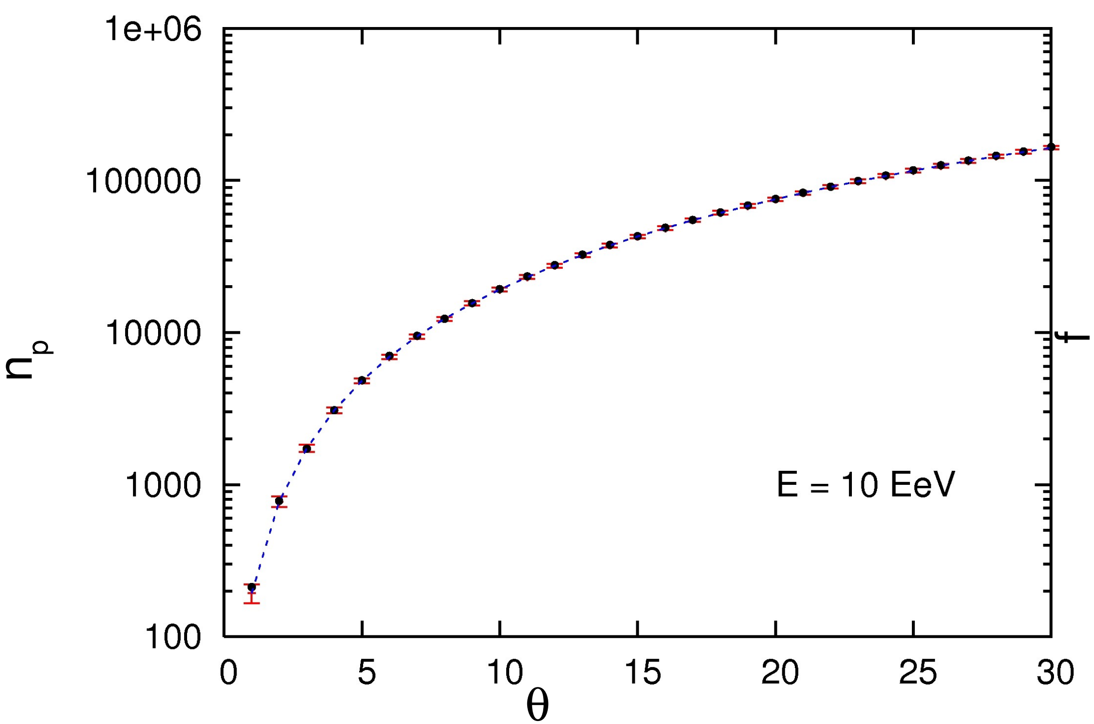
For partial/non-uniform sky coverage the expected number of pairs from an isotropic flux has to be computed simulating isotropic event realizations following the exposure and counting the pairs as a function of the angle

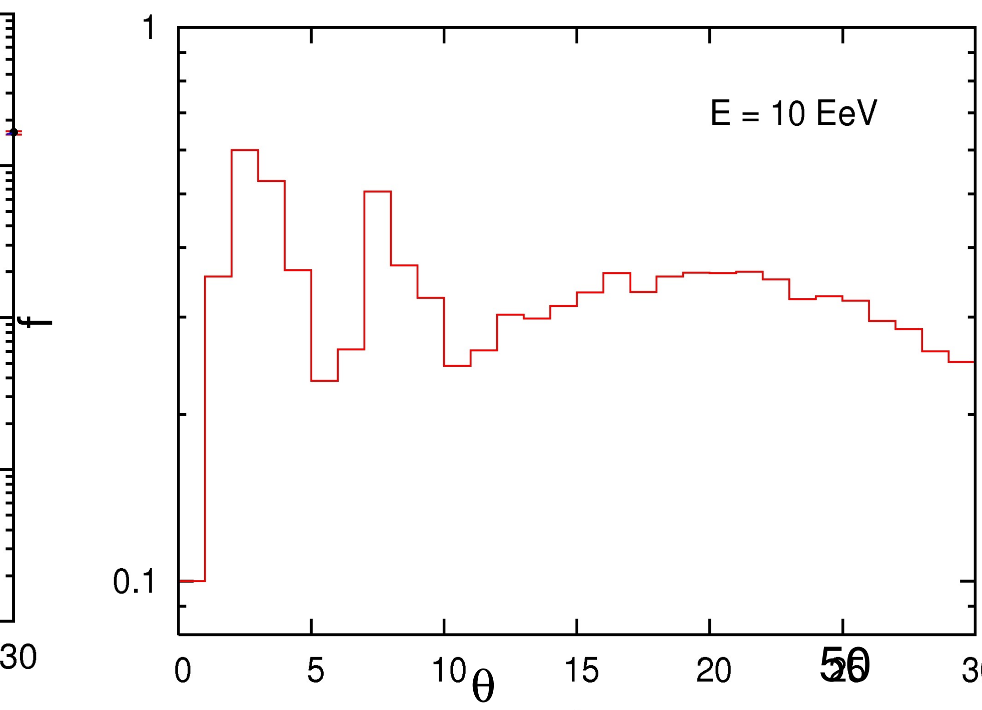
CUMULATIVE CORRELATION FUNCTION

- Count the # of pairs n_p with separation < θ: $$n_p(\theta)=\sum_{i=2}^{N}\sum_{j=1}^{i-1} H(\theta-\theta_{ij})$$

- Using Monte Carlo simulations with an isotropic distribution: obtain the expected $\langle n_p \rangle$
- If an excess is observed at some angular scale: how to estimate the significance? Run many simulations with the same # of events N
Probability of observing n_p or more pairs with separation < θ = fraction of simulations with larger $n_p(\theta)$ than the data

Auger '07





Caveat: if one looks at many angular scales, this is not a good estimation of the significance
This probability needs to be penalized for choosing a posteriori the angle where we found the largest departure from isotropy

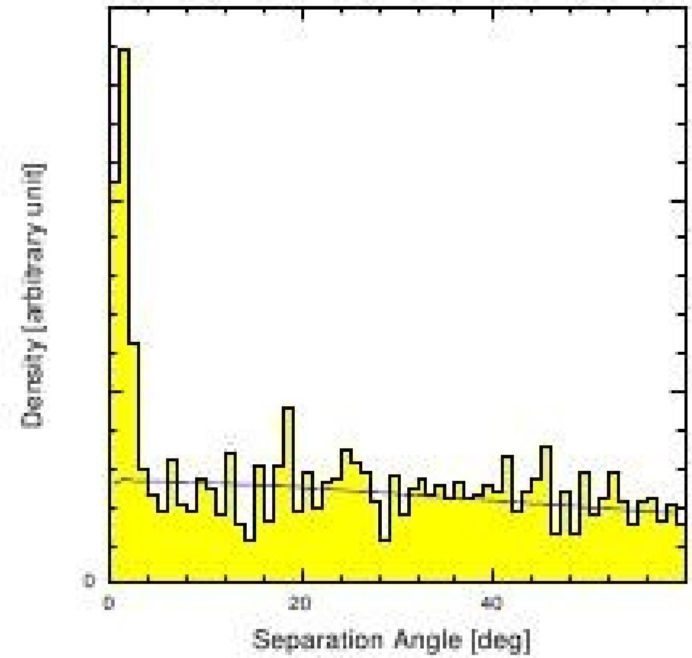
Possible solution: do the same for a large set of simulations → for each one choose the angle with maximum departure from isotropy and compute f_{sim} (fraction of simulations with $n_p(\theta)$ larger than the value obtained in that simulation)
Chance Probability P = fraction of simulations with $f_{sim} < f_{data}$

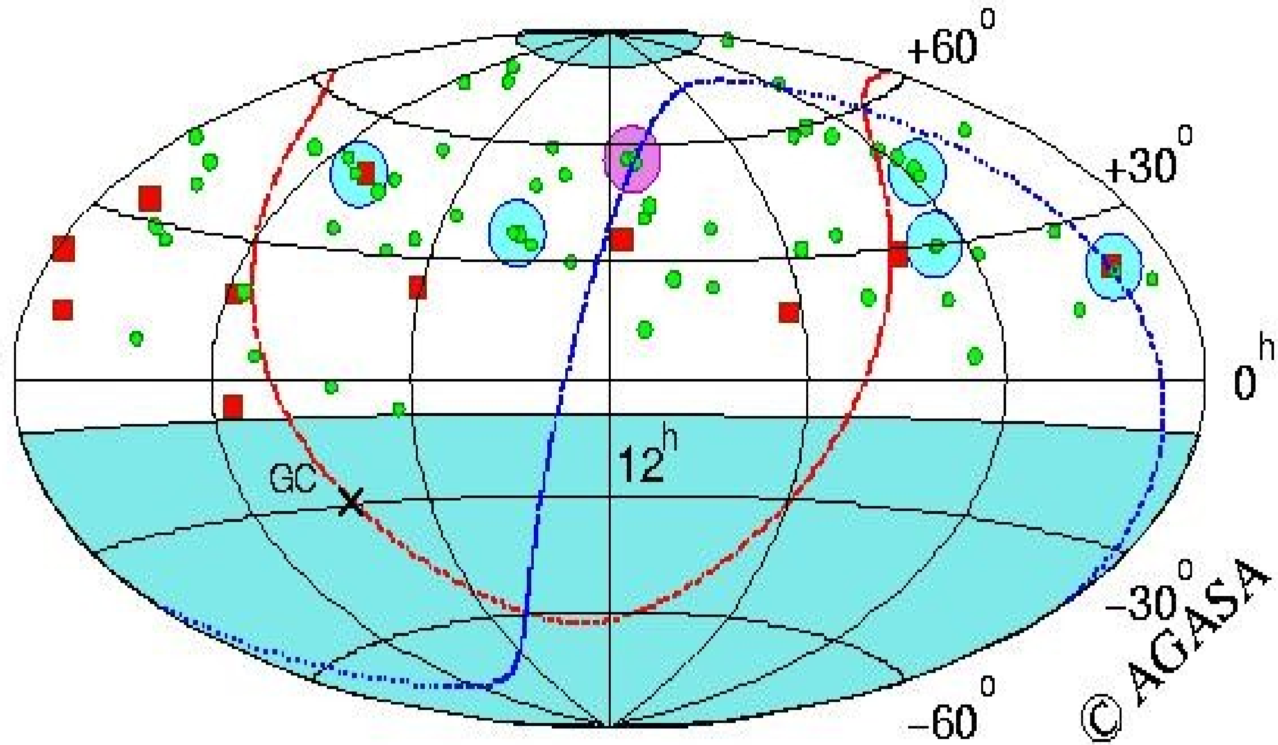
The same problem arises if the energy threshold (or range) is not fixed a priori and we look for excesses of pairs in different E ranges
Same solution: scan the data in angle and energy and choose the values θ and E with smallest f_{data} (larger number of pairs compared to isotropic sims).
Perform the same scan in a large set of simulations → f_{sim}
Chance probability that an excess like the one observed arises by chance from an isotropic flux → P = fraction of simulations with $f_{sim} < f_{data}$

CLAIMS OF CLUSTERING:

AGASA:

5 doublets and 1 triplet
with separation $\theta < 2.5^o$
out of 59 events
with E > 40EeV





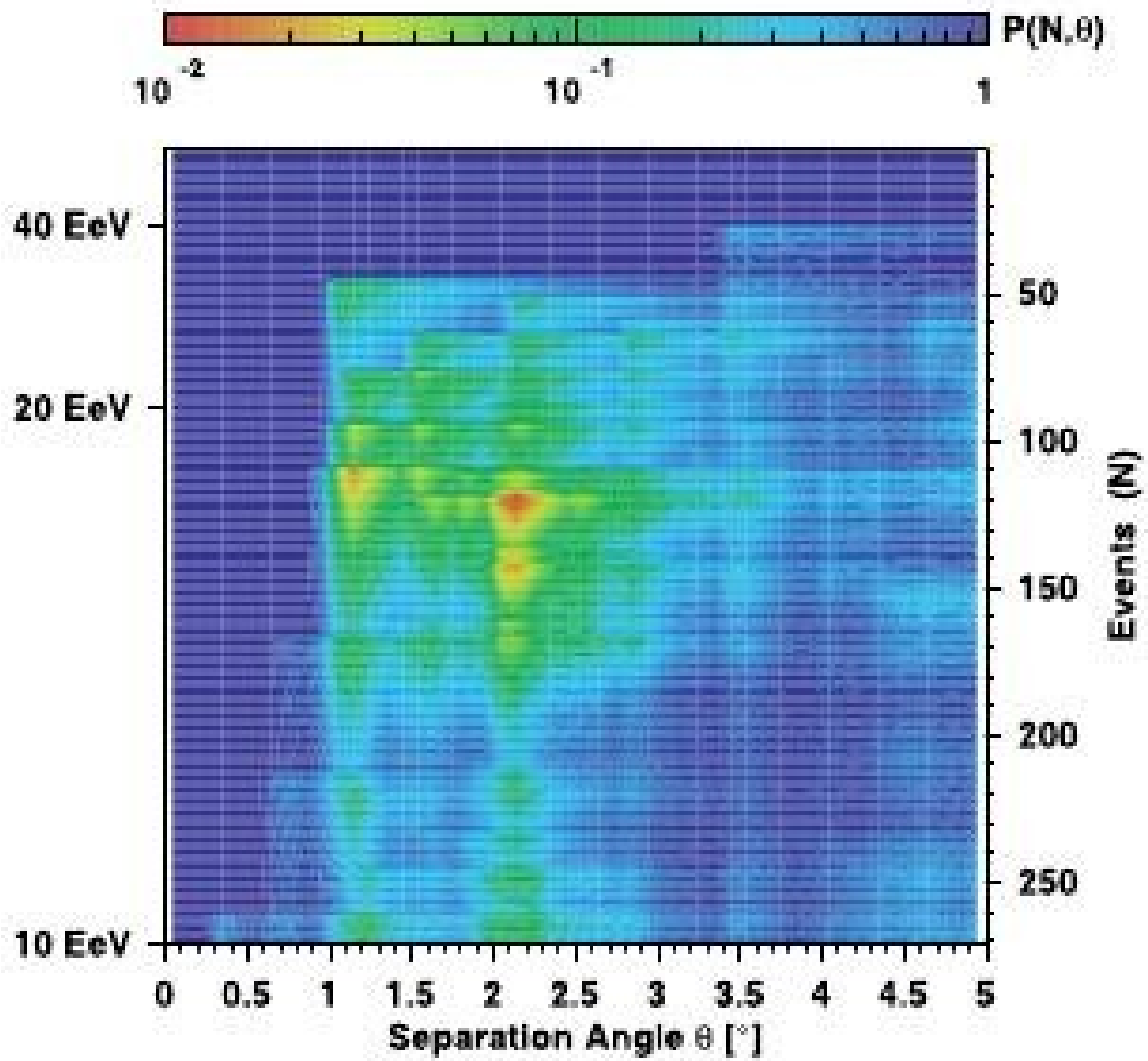
Takeda et al 1999,
Teshima et al. 2003

Chance probability was estimated P $\sim 10^{-4}$

Controversy arose because the angular scale and the energy threshold had not been fixed a priori

Taking into account a penalization for scanning in angle and energy
P $\sim 3 \times 10^{-3}$

HiRes: NO evidence of clustering for any energy E >10 EeV



f_{min} = 0.019

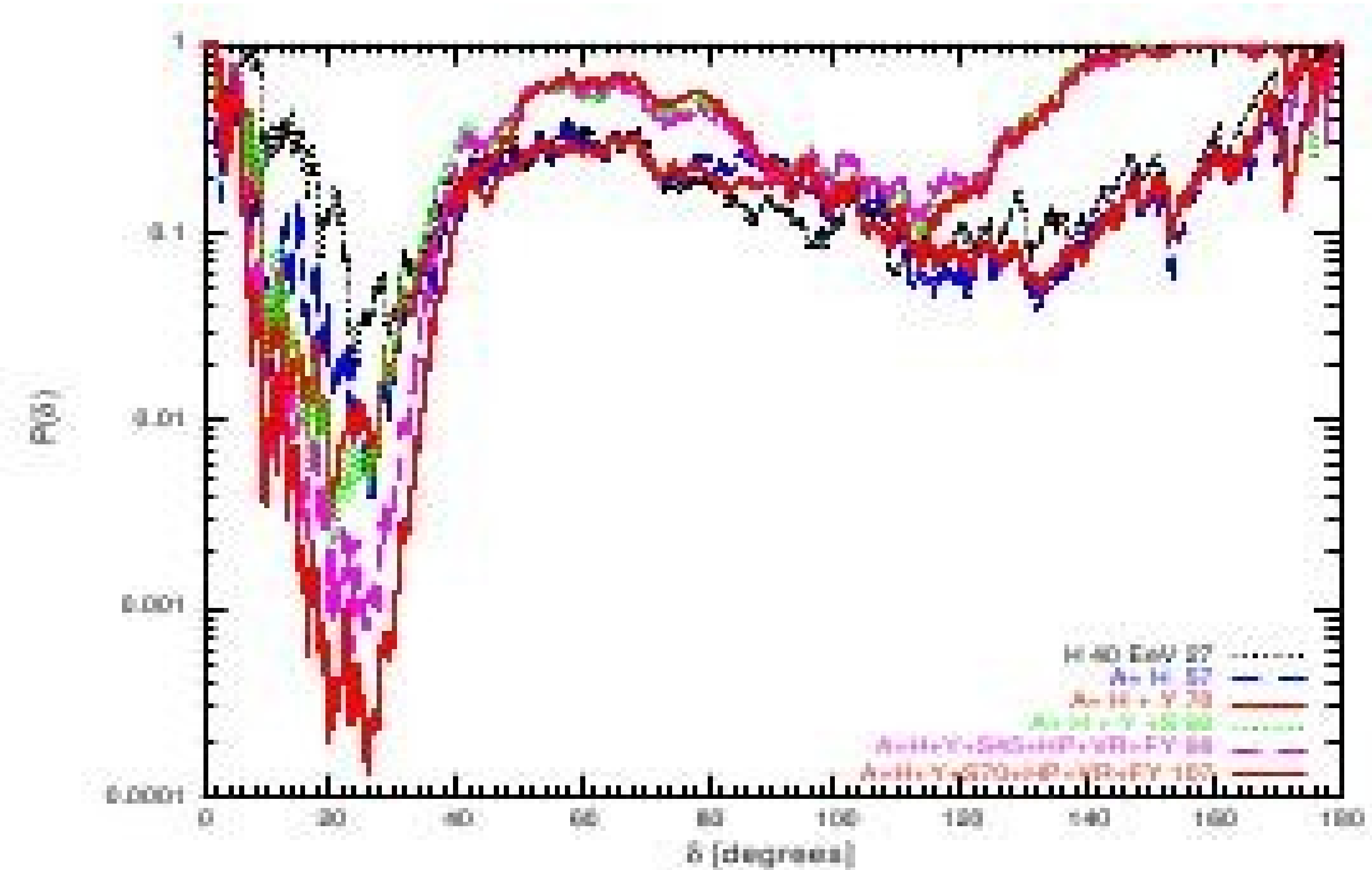
Chance probability

P_{ch} = 0.52

Abbasi et al. 2004

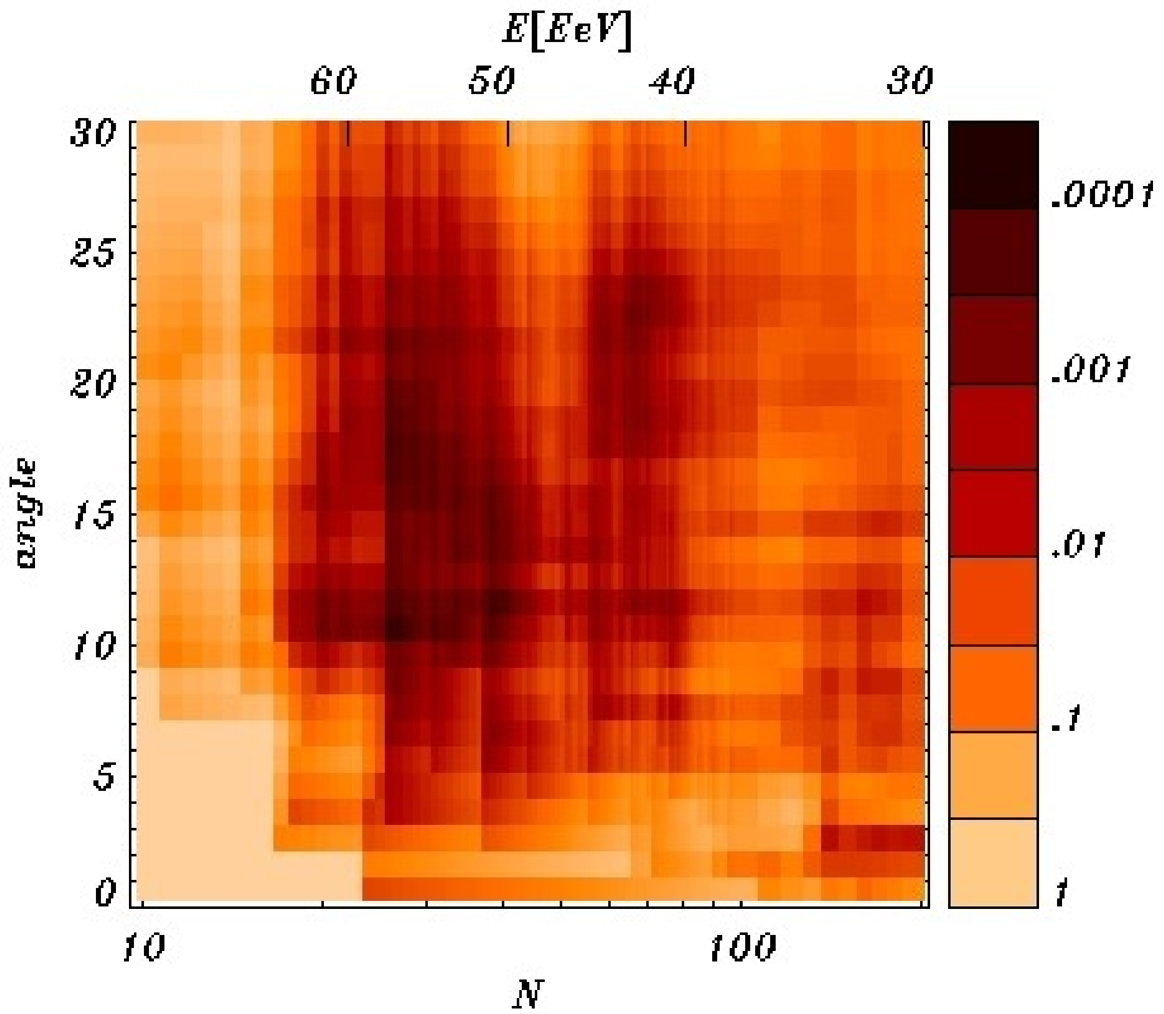
AGASA + HiRes stereo + Yakutsk + SUGAR

correlation at intermediate scales (25º) for E > 40 EeV (Kachelriess & Semikoz '06)



medium scale clustering → inhomogeneous distribution of the sources

AUTOCORRELATION SCAN: AUGER RESULTS

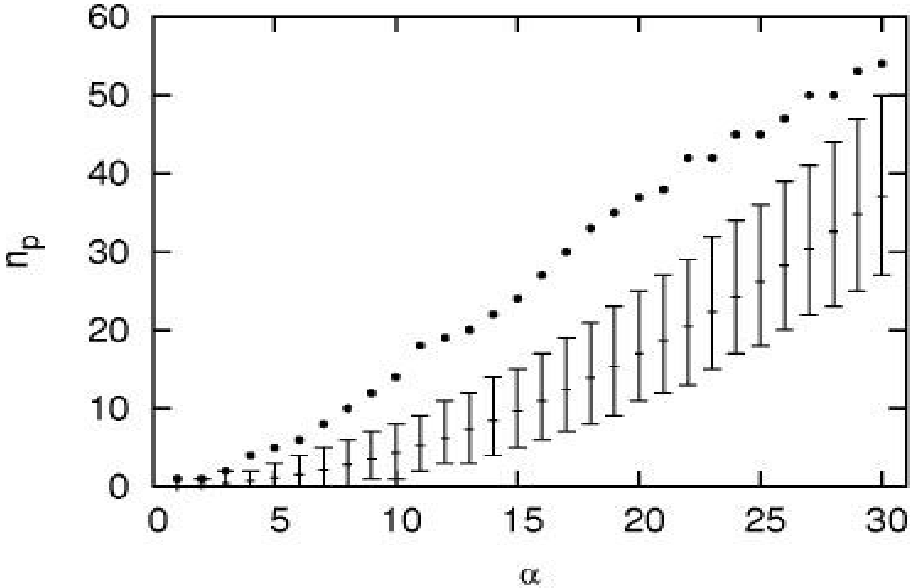


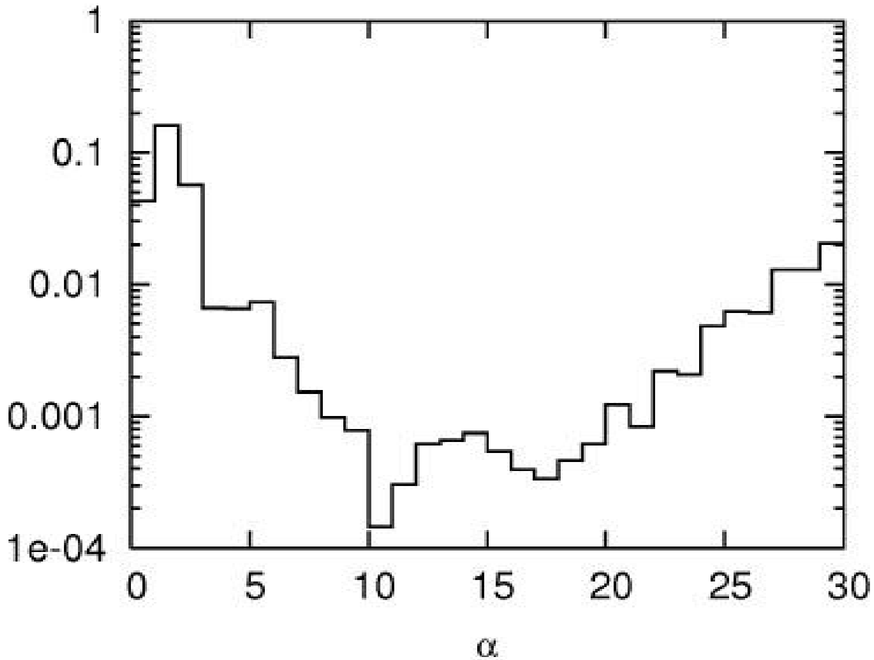
Data from 1/1/2004 to 31/8/2007

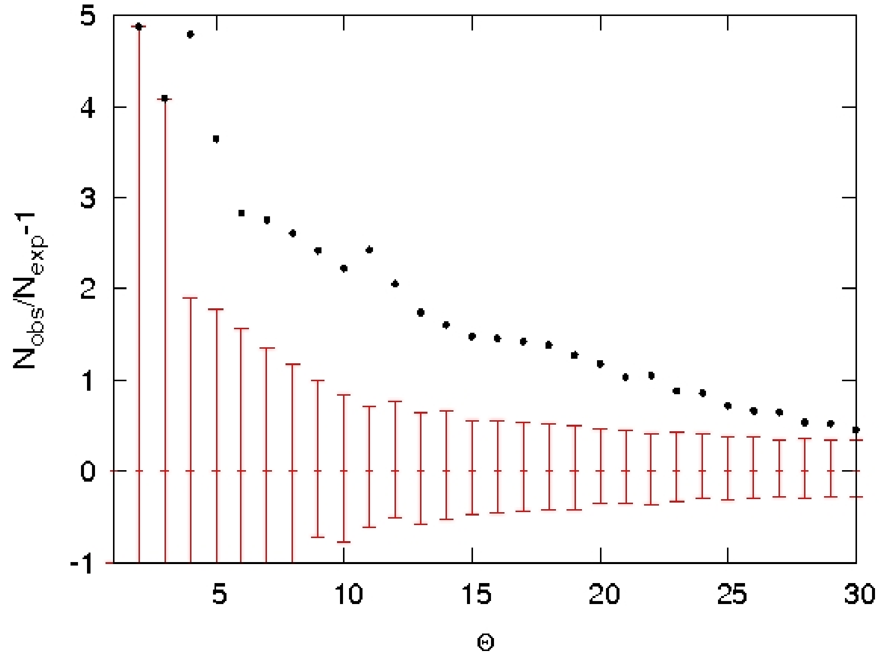
Auger '07

CORRELATION EXCESS AT INTERMEDIATE ANGLES (9 – 22 deg) AND LARGE ENERGIES (>50 EeV)

MINIMUM: θ = 11°, E > 57 EeV (N = 27), obs/exp = 18/5.2, $P_{min} \simeq 10^{-4}$



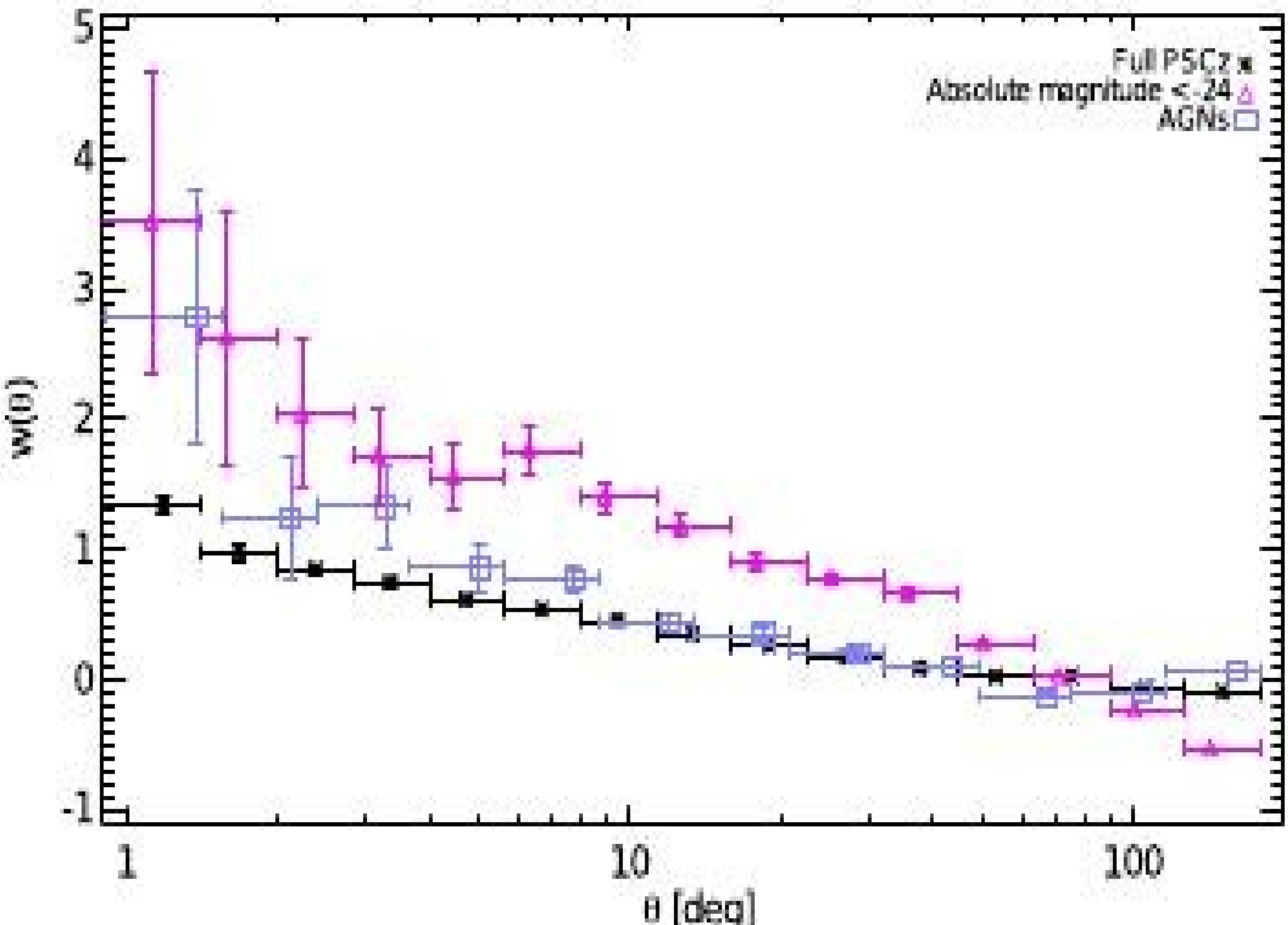




CHANCE PROBABILITY: P $\simeq$ 1.6 %

The autocorrelation function can give information on the spatial density of sources → identify source population and on the distribution of the sources

Galaxies, bright galaxies, AGNs, some subset of them?



Galaxies
Bright galaxies
AGNs

LECTURE 3: FURTHER TECHNIQUES & RESULTS

- Point-like or extended excesses in the sky
- The Galactic center region
- Searches for correlation with objects: crosscorrelation function, maximum likelihood, binomial probability scan, loglikelihood per event, two dimensional Kolmogorov Smirnov
- The UHECR anisotropy and possible sources

SEARCH FOR POINT-LIKE OR EXTENDED EXCESS OF EVENTS AROUND A GIVEN DIRECTION IN THE SKY

- Measure the observed number of events in a window (top-hat, Gaussian,...) around the given direction. For a point-like excess (as could arise for neutral primaries) use the angular resolution size $\rightarrow N_{ON}$

- Estimate the background: use the detector measurements in other regions of the sky $\rightarrow$ $N_B = \alpha\ N_{OFF}$ with $\alpha = t_{ON}/t_{OFF}$ $(= \omega_{ON}/\omega_{OFF})$

- Estimate the signal $\rightarrow$ $N_S = N_{ON} - N_B = N_{ON} - \alpha\ N_{OFF}$

- Estimate the significance

Variance of the signal (N_{ON} and N_{OFF} independent measurements)

$\sigma^2(N_S) = \sigma^2(N_{ON}) + \alpha^2\, \sigma^2(N_{OFF})$

Different possibilities to evaluate the variances:

1) 2 Poisson processes

$$\hat{\sigma}_1(N_S) = \sqrt{N_{ON} + \alpha^2 N_{OFF}} \qquad S_1 = \frac{N_{ON} - \alpha N_{OFF}}{\sqrt{N_{ON} + \alpha^2 N_{OFF}}}$$

2) to estimate the significance of an excess we want to evaluate the probability that it arises only from the background

$P(N_{ON}) =$ Poisson $(\langle N_{ON}\rangle = \sigma^2(N_{ON}) = \langle N_B\rangle)$

$P(N_{OFF}) =$ Poisson $(\langle N_{OFF}\rangle = \sigma^2(N_{OFF}) = \langle N_B\rangle/\alpha)$

$$\sigma^2(N_S) = \sigma^2(N_{ON}) + \alpha^2\sigma^2(N_{OFF}) = \langle N_B\rangle(1+\alpha)$$

$$\langle N_B\rangle = \frac{N_{ON}+N_{OFF}}{t_{ON}+t_{OFF}}t_{ON} = \frac{\alpha}{1+\alpha}(N_{ON}+N_{OFF})$$

$$\hat{\sigma}_2(N_S) = \sqrt{\alpha(N_{ON}+N_{OFF})}$$

$$\boxed{S_2 = \frac{N_{ON}-\alpha N_{OFF}}{\sqrt{\alpha(N_{ON}+N_{OFF})}}}$$

An observed excess N_S is said to be an 'S standard deviation detection'

If $\langle N_S\rangle = 0$, S approximates a Gaussian variable with mean=0 and variance=1

Gaussian(S) $\rightarrow$ confidence level of the observational result.

Tests with numerical simulations indicates that S_2 is a better estimator than S_1

S_1 underestimates (overestimates) significances for $\alpha <$ (>) 1

3) Likelihood ratio method: Li-Ma

$$\lambda = \frac{L(data|null\,hyp)}{L(data|alternative\,hyp)} = \frac{P(data|\langle N_S\rangle = 0)}{P(data|\langle N_S\rangle = N_{ON} - \alpha N_{OFF})}$$

If the null hyp is true and N_{ON}, N_{OFF} (>10) are large then sqrt(-2 ln λ) is Gaussian distributed with σ^2=1

$$P(data|\langle N_S\rangle=0) = P\left(N_{ON}, N_{OFF}|\langle N_S\rangle=0, \langle N_B\rangle = \frac{\alpha}{1+\alpha}(N_{ON}+N_{OFF})\right)$$

$$= Poisson(N_{ON}, \langle N_{ON}\rangle = \langle N_B\rangle)\, Poisson(N_{OFF}, \langle N_{OFF}\rangle = \langle N_B\rangle/\alpha)$$

$$P(data|\langle N_S\rangle = N_{ON} - \alpha N_{OFF}, \langle N_B\rangle = \alpha N_{OFF}) = Poisson(N_{ON}, \langle N_{ON}\rangle = N_{ON})\, Poisson(N_{OFF}, \langle N_{OFF}\rangle = N_{OFF})$$

$$Poisson(n,\nu) = \nu^n \frac{e^{-\nu}}{n!}$$

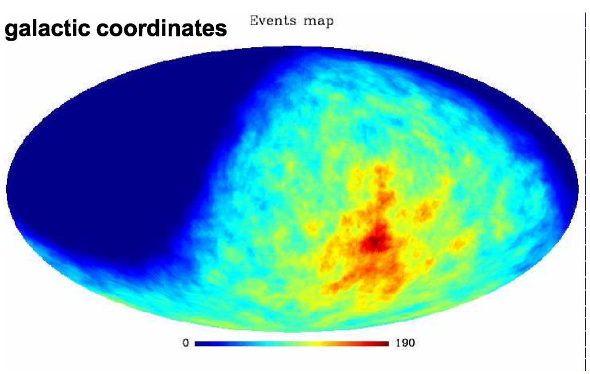
$$\lambda = \left[\frac{\alpha}{1+\alpha}\left(\frac{N_{ON}+N_{OFF}}{N_{ON}}\right)\right]^{N_{ON}} \left[\frac{1}{1+\alpha}\left(\frac{N_{ON}+N_{OFF}}{N_{OFF}}\right)\right]^{N_{OFF}}$$

$$\boxed{S_3 = \sqrt{-2\ln(\lambda)}}$$

Li-Ma significance follows a Gaussian better than S_1 and S_2

SEARCH FOR POINT-LIKE OR EXTENDED EXCESSES OF EVENTS FROM ANY DIRECTION IN THE SKY

- Smoothed map of the observed events: to look for a point-like excess in any direction use a window matching the angular resolution of the experiment (relevant for neutral particles), for charged particles larger angles are more relevant.
- Isotropic expectation from the same smoothing of the exposure map



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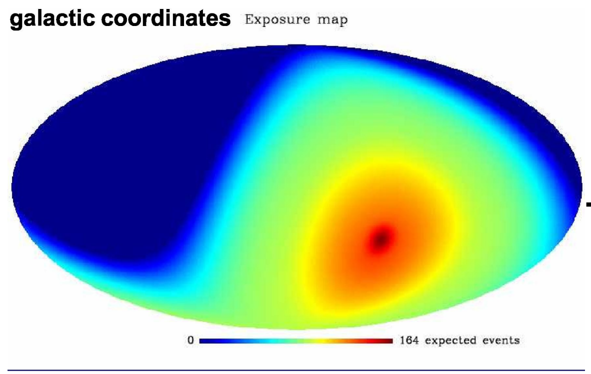

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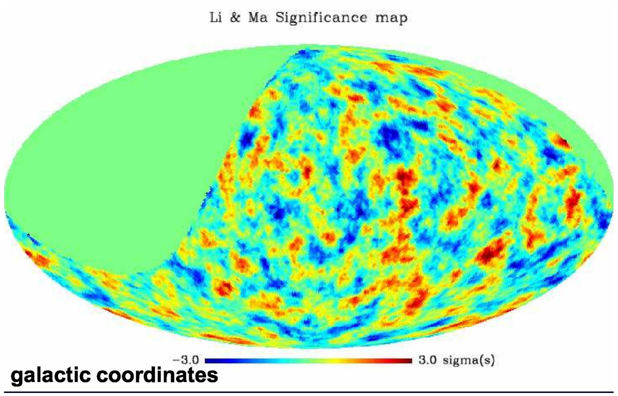


Observed events in an angular window

Expected events from an isotropic flux

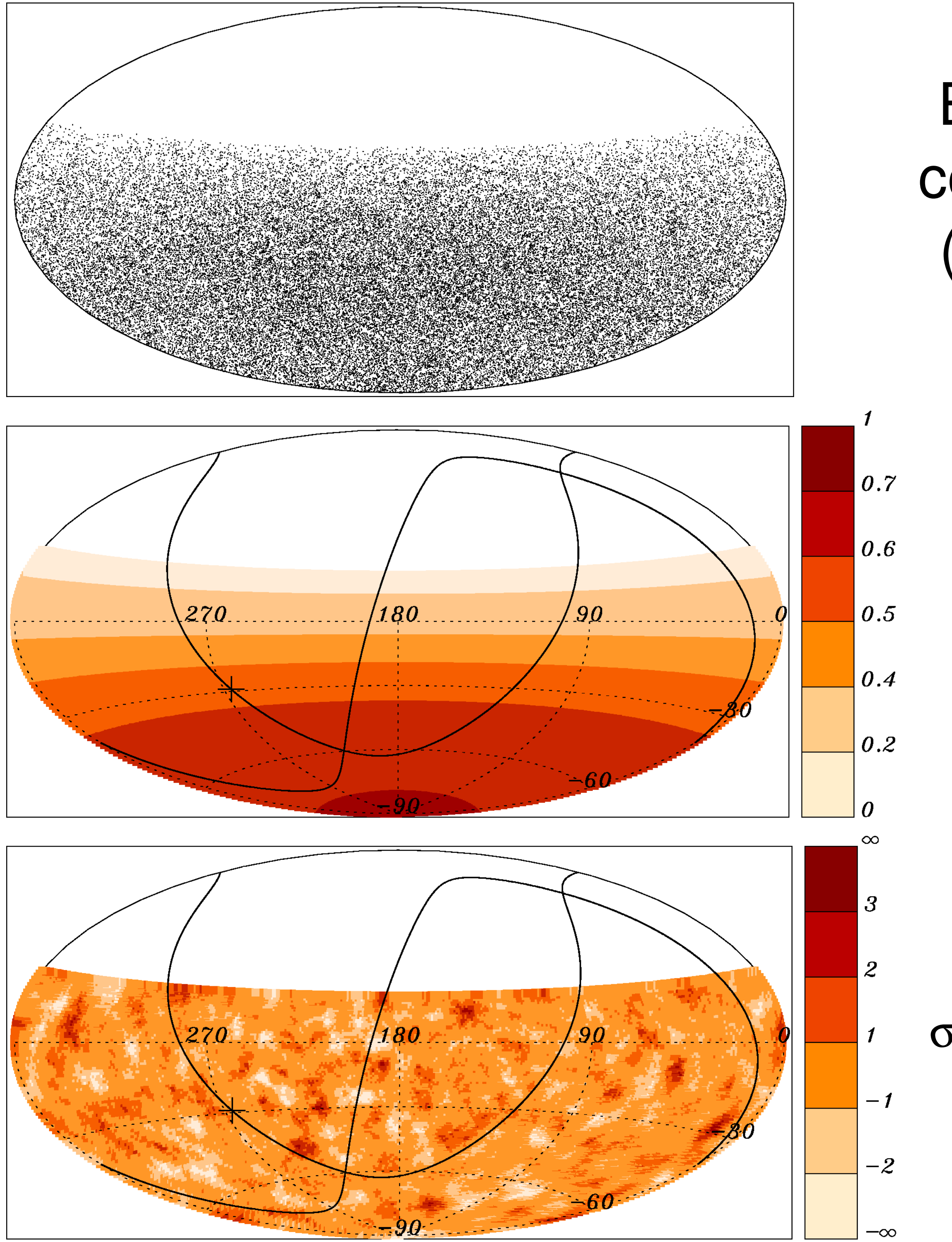
Significances of excesses and deficits

Equatorial coordinates (RA,DEC)

Events map

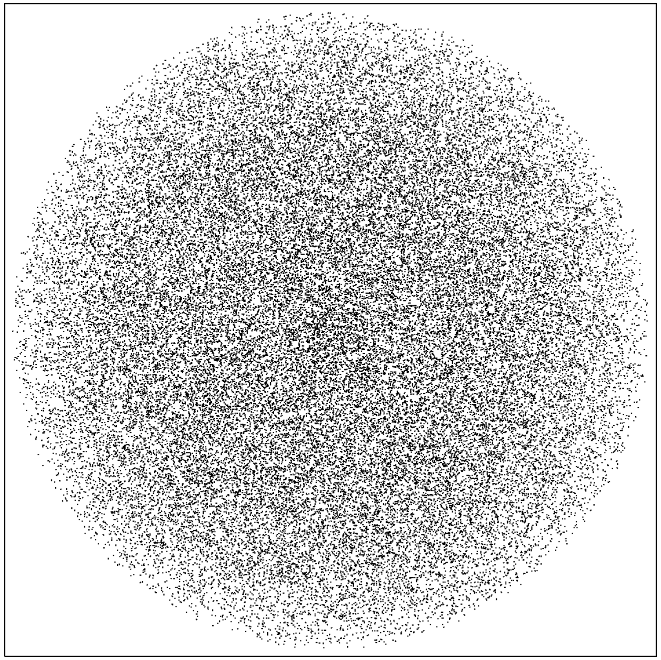
Exposure map
(5^0 radius windows)

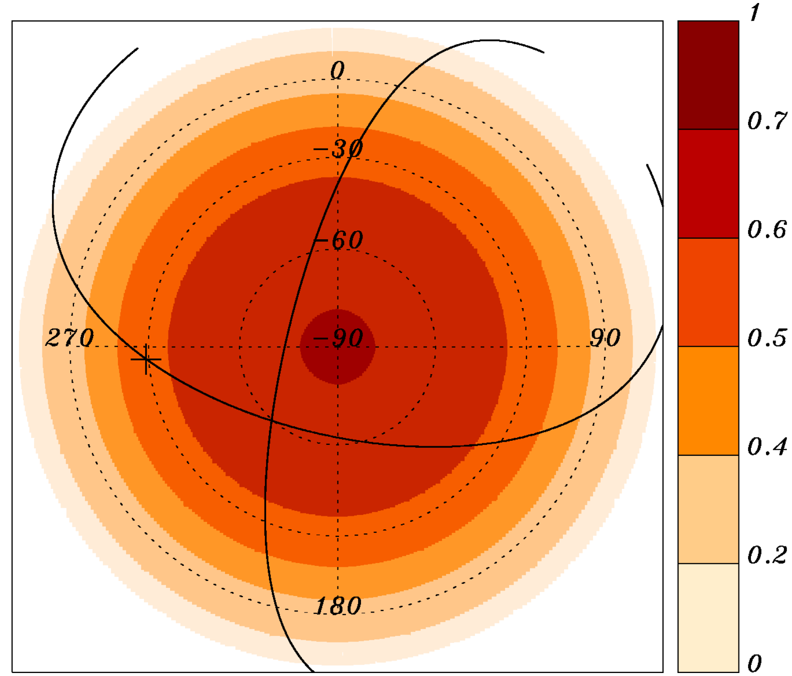
Excesses/deficits significance map (Li-Ma)



SOUTHERN PERSPECTIVE

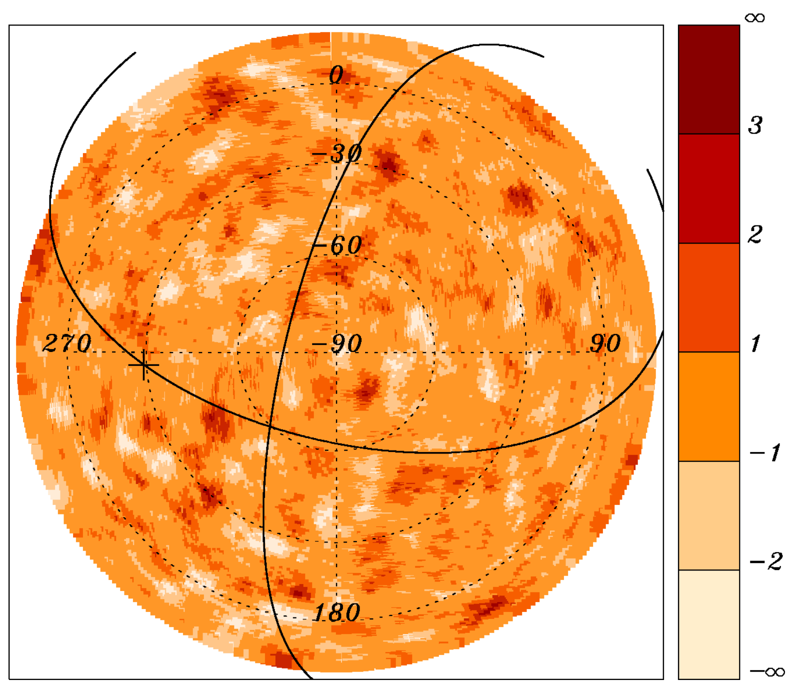
Events map





Exposure map

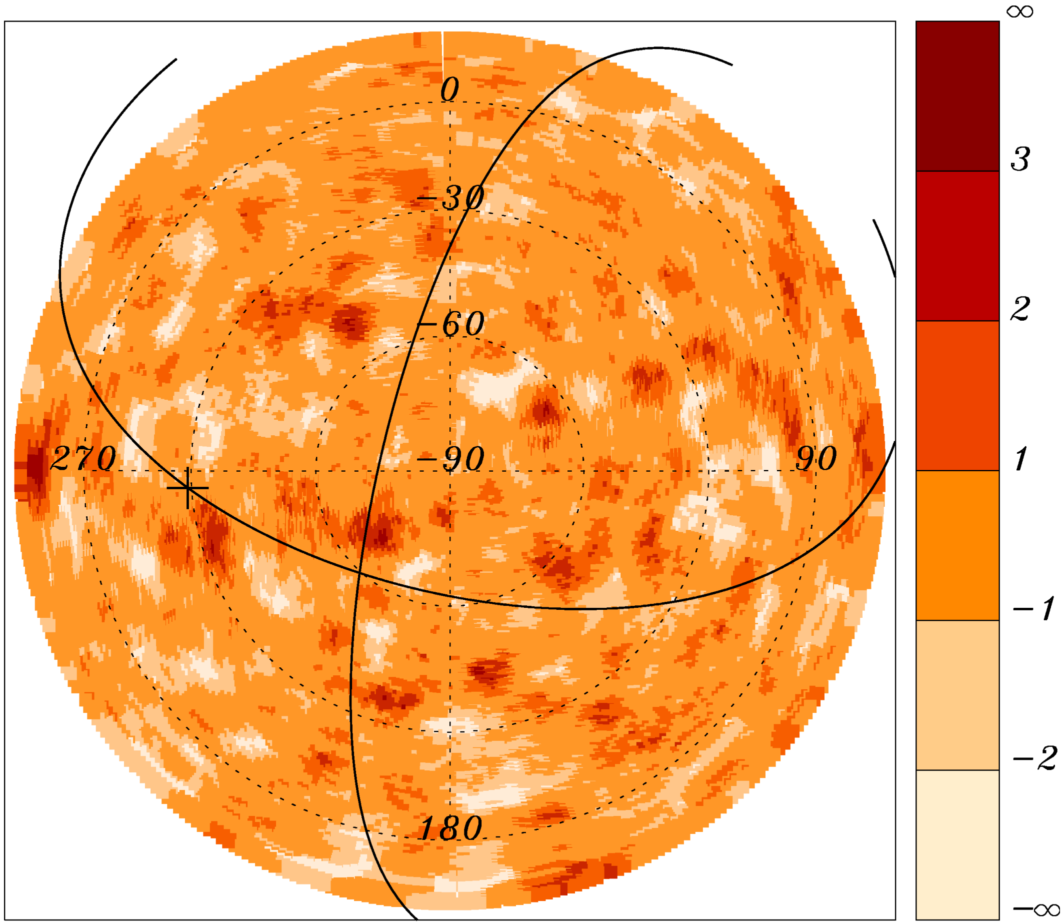
Excesses/deficits significance map (Li-Ma)

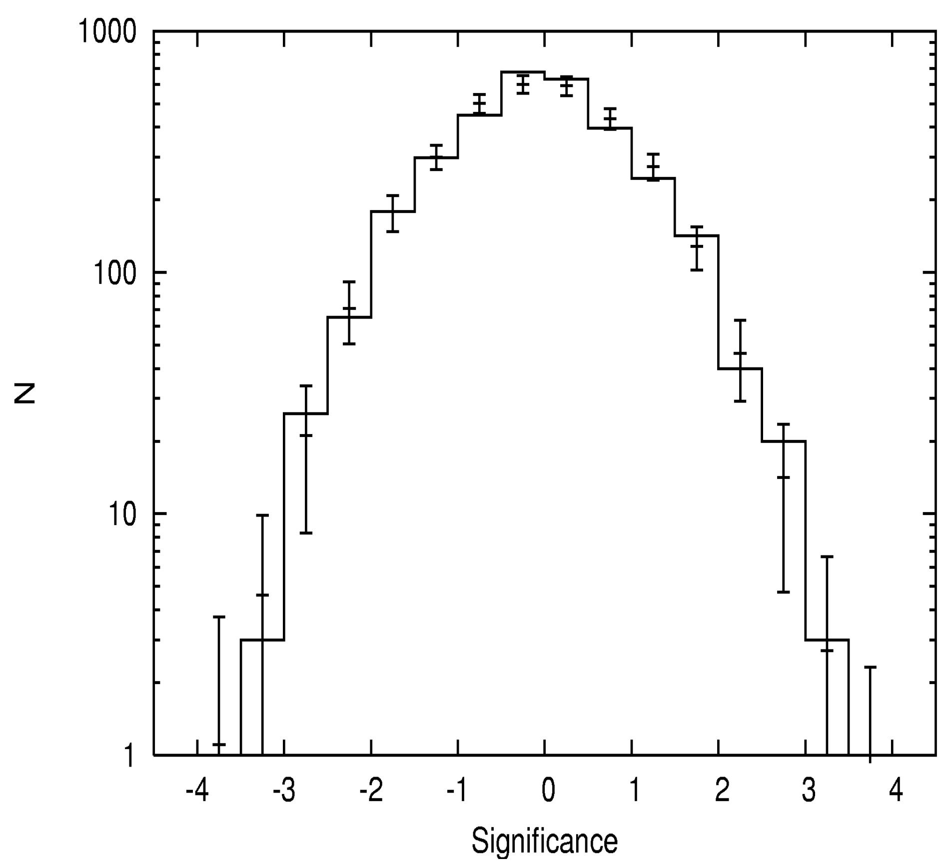


AUGER ANISOTROPY SEARCHES (august 2006)

5° radius top-hat windows
E > 5 EeV (2445 good-quality events)

Excesses/deficits compatible with isotropic expectations

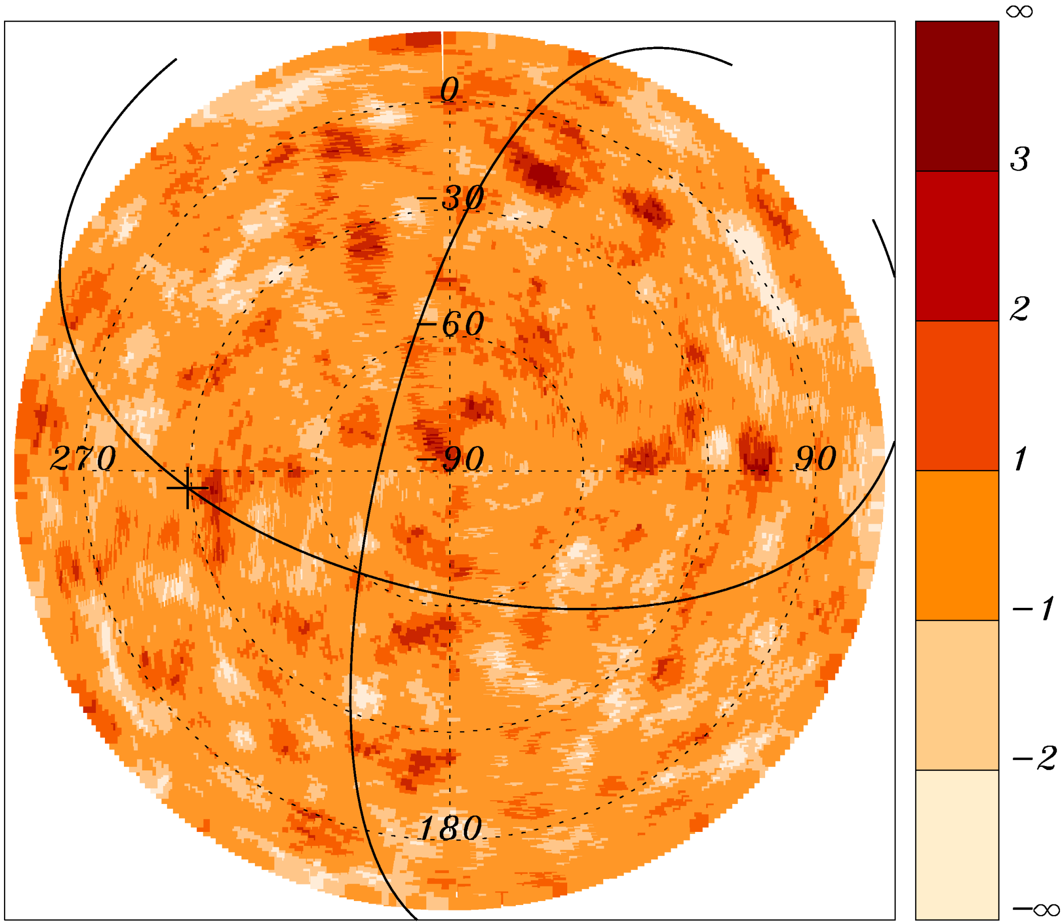


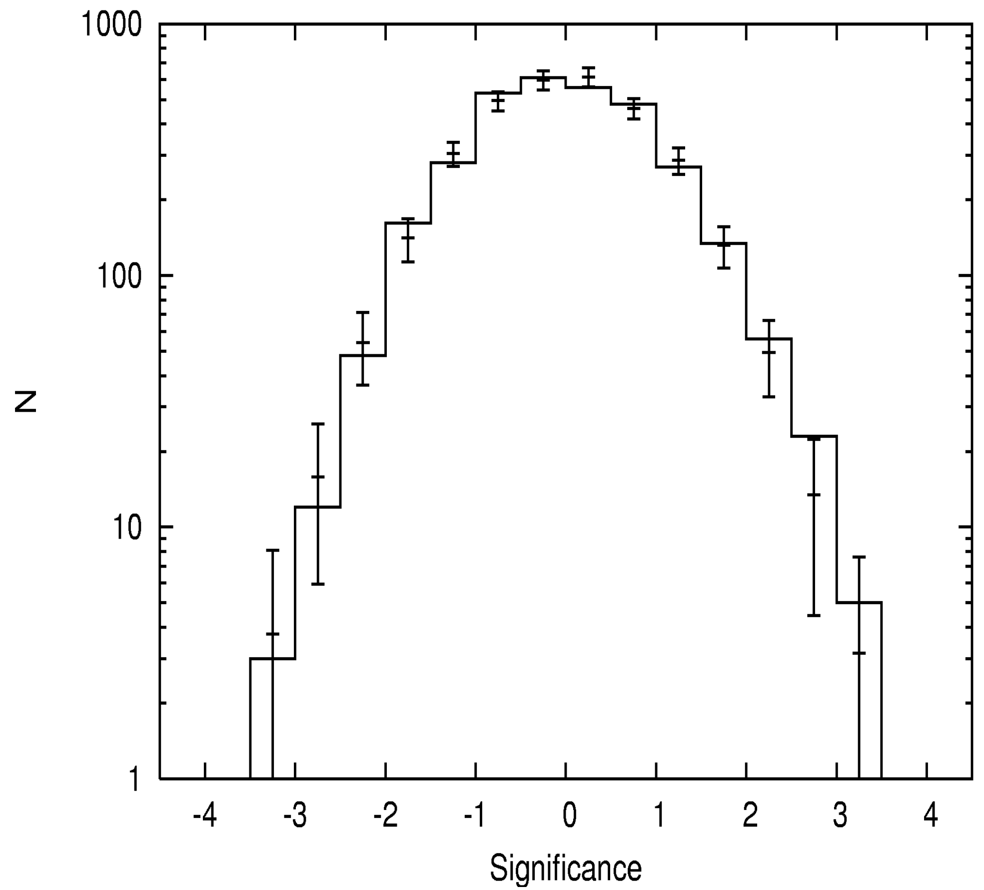


AUGER ANISOTROPY SEARCHES (august 2006)

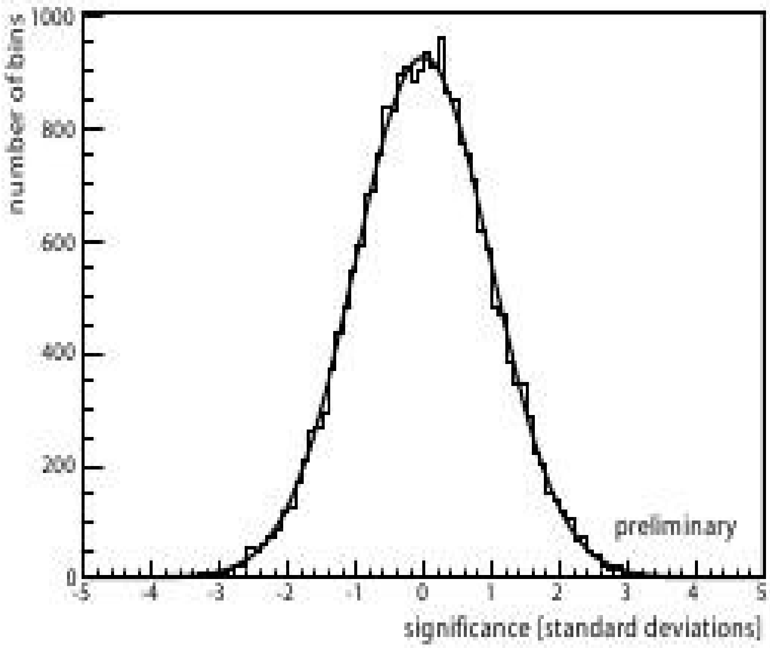
5° radius top-hat windows
1EeV < E < 5 EeV (54931 good-quality events)

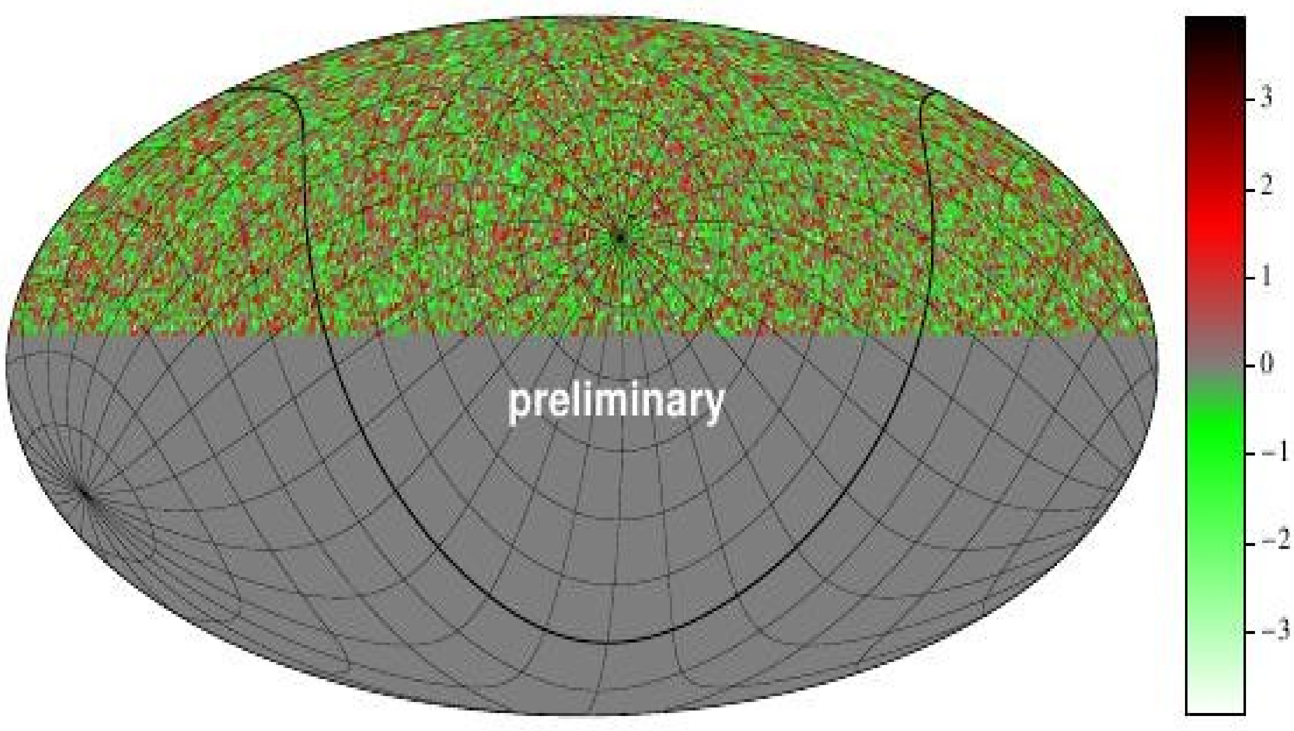
Excesses/deficits compatible with isotropic expectations





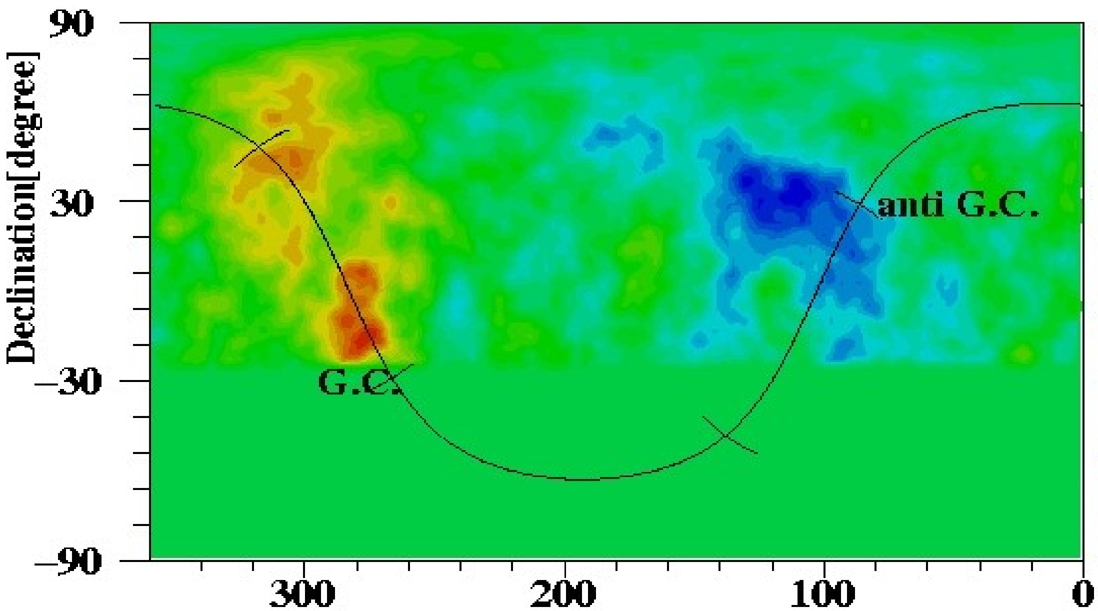
KASCADE-GRANDE search for point-like sources



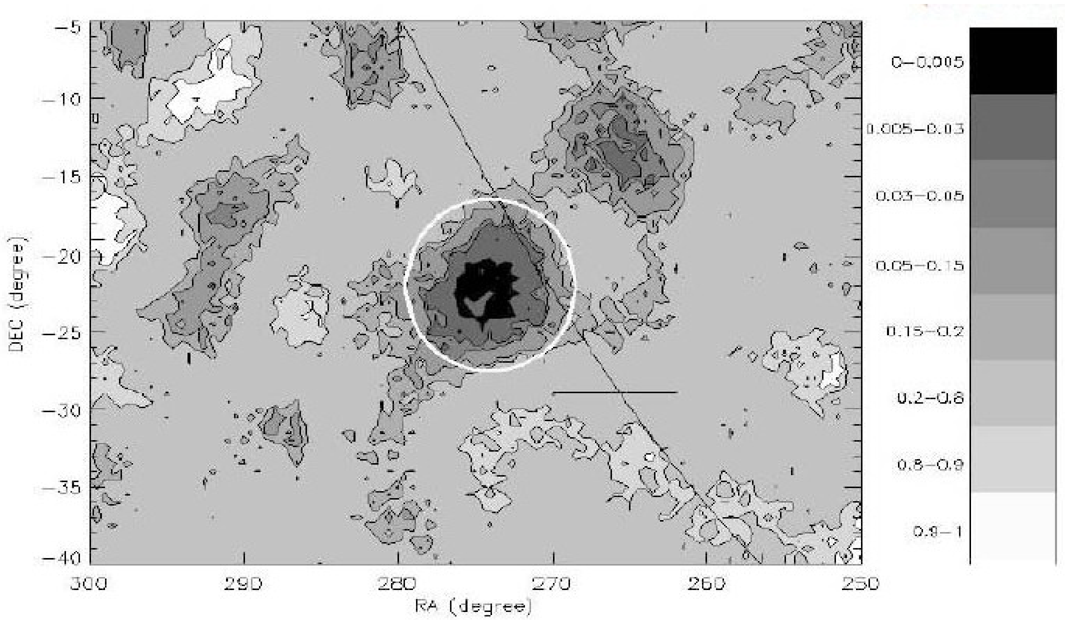


Distribution of significances compatible with isotropic distribution

Anisotropies – Galactic center

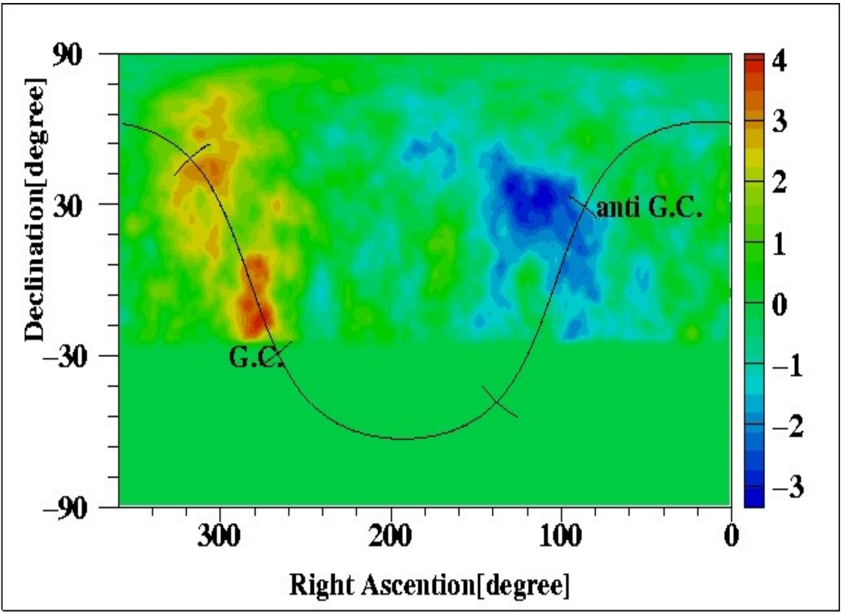


AGASA (Japan):
Excess 4.5 σ (1999)
Circle of 20º radius
Energy 1 - 2.5 EeV



SUGAR (Australia):
Excess 2.9 σ (2001)
Circle of 5.5º radius
Energy 0.8 - 3.2 EeV

AGASA ANISOTROPIES ON 20º SCALES (10^{18} – 10$^{18.4}$ eV)



$$\frac{obs}{\exp} = \frac{506}{413.6} \quad (+4.5\sigma)$$

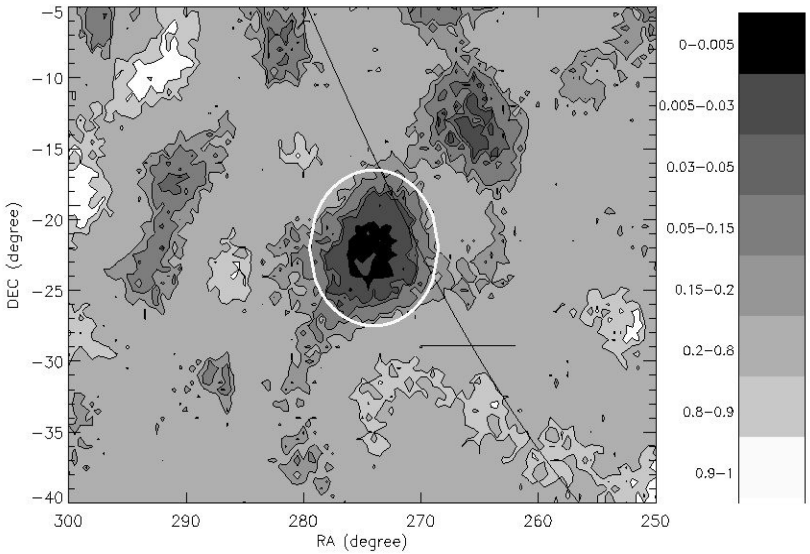
at $(\delta, \alpha) = (-15, 280)$

(22% excess)

AUGER got $\frac{obs}{\exp} = \frac{2116}{2159.6} \quad (-1\sigma)$

Astroparticle Physics 27 (2007) 244–253

SUGAR galactic center search (5.5º for 10$^{17.9}$ – 10$^{18.5}$ eV)



$$\frac{obs}{\exp} = \frac{21.8}{11.8} \quad (+2.9\sigma)$$

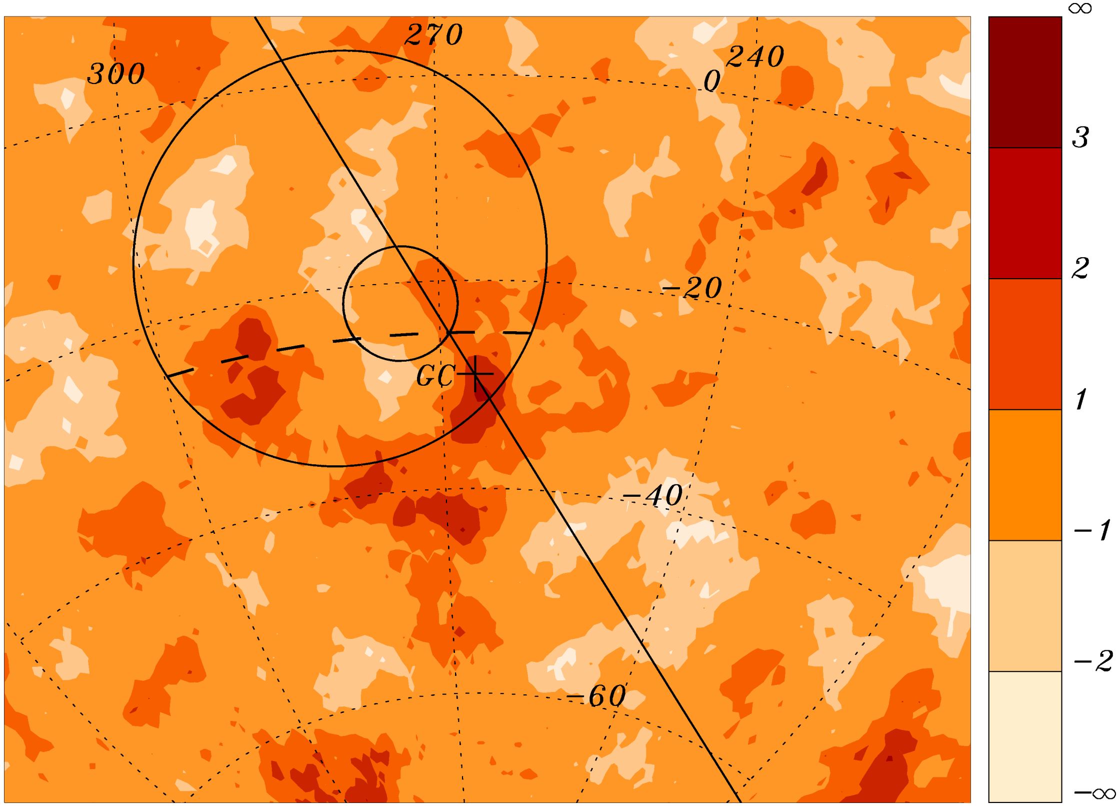
at $(\delta, \alpha) = (-22, 274)$

(85% excess)

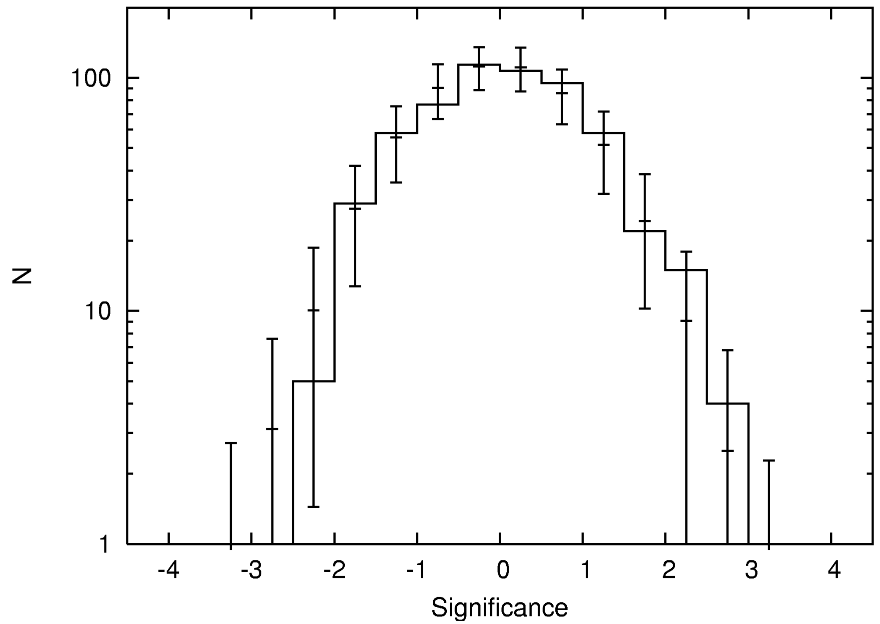
AUGER got $\frac{obs}{\exp} = \frac{286}{289.7} \quad (-0.3\sigma)$

THE GALACTIC CENTRE REGION

$10^{17.9}$ eV < E < $10^{18.5}$ eV



Sky map and distribution of significances of overdensities near the GC on 5° radius windows



E_{min}[eV]	E_{max}[eV]	n_{obs} / n_{exp}
$10^{17.9}$	$10^{18.3}$	3179 / 3153.5 = 1.01 ± 0.02(*stat*) ± 0.01(*syst*)
10^{18}	$10^{18.4}$	2116 / 2159.5 = 0.98 ± 0.02(*stat*) ± 0.01(*syst*)
$10^{18.1}$	$10^{18.5}$	1375 / 1394.5 = 0.99 ± 0.03(*stat*) ± 0.01(*syst*)

Events in 20° radius AGASA region

SEARCHES FOR CORRELATIONS WITH OBJECTS

Given a population of candidate sources there are different proposed tests to search a correlation with CR arrival directions

CROSSCORRELATION FUNCTION:

Looks for an excess of pairs of CR separated less than a given angle from any candidate source in the set with respect to the expectations from an isotropic CR distribution.

Similar procedure as in the autocorrelation analysis:

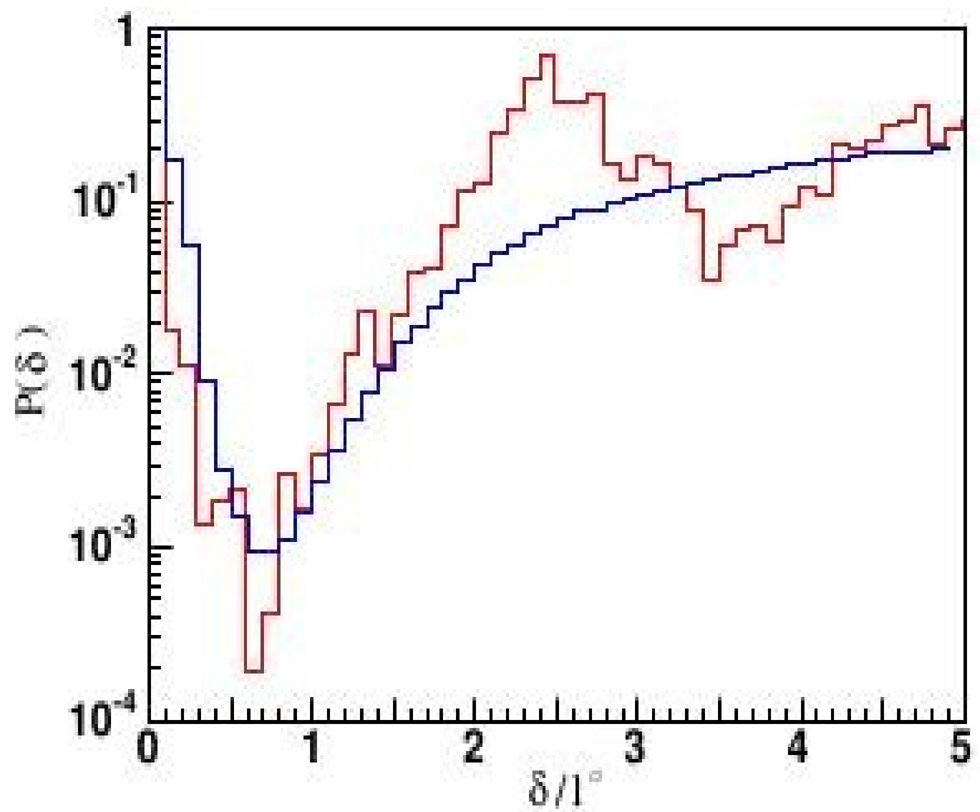
- Count the number of pairs CR-objects as a function of the angle in the data.
- Repeat the procedure for a large number of isotropic simulated datasets.
- To estimate the significance of any excess compute the fraction of the simulations with larger number of pairs than those present in the data

Example: correlation of CR with E > 10 EeV and BL Lacs with m < 18 at the resolution angular scale (first found in HiRes data by Gorbunov et al. 2004)

156 BL Lacs
271 events

$n_p(0.8^o) = 11$ (data)
$\langle n_p(0.8^o)\rangle_{iso} = 3$
$f\,(0.8^o) = 4 \times 10^{-4}$

Penalization for search at different angles and with different sets of objects not included



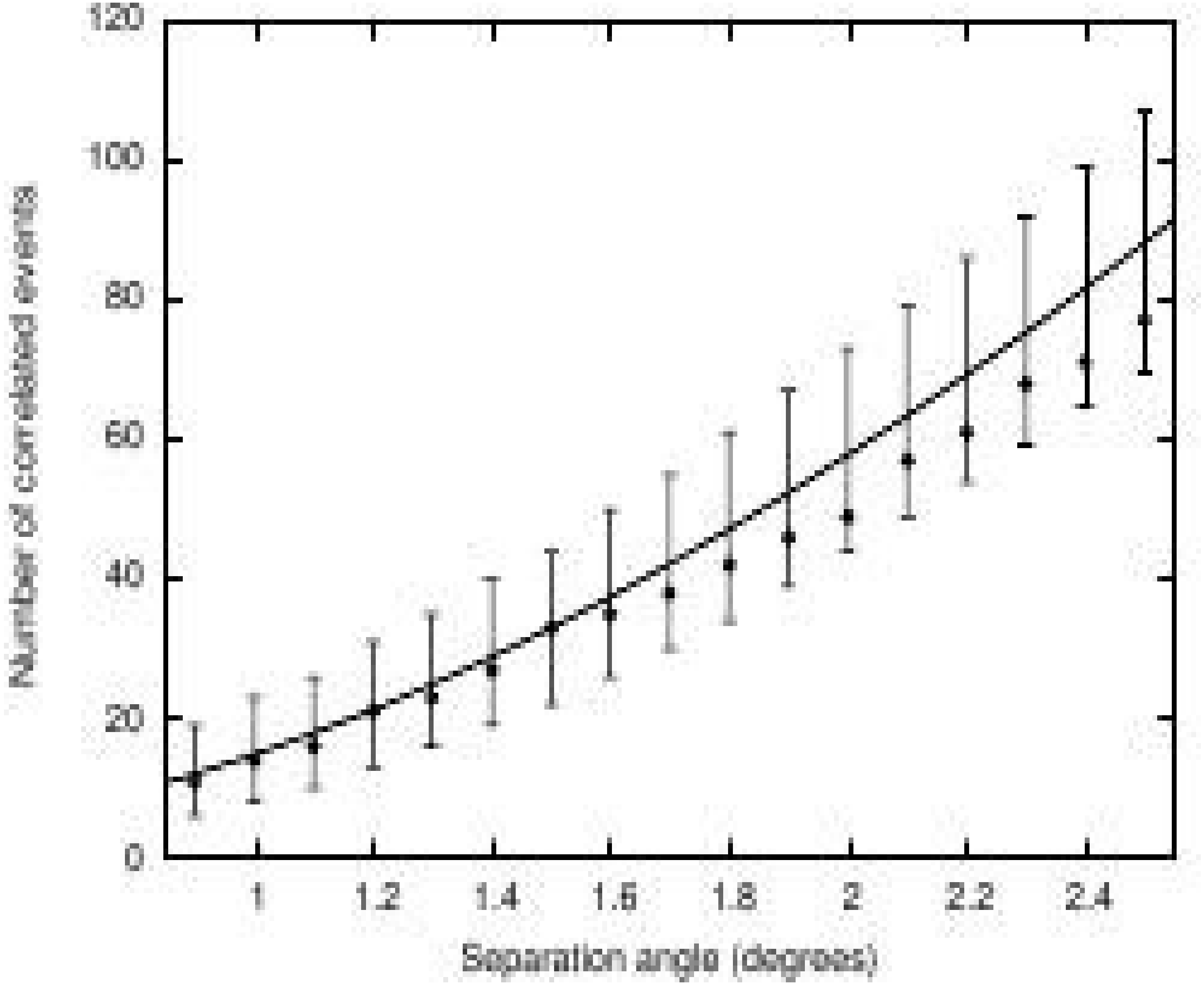
data
9 from BL Lac + 262 isotropic

Test of the signal with Auger data

1736 events with E > 10 EeV

No evidence of excess in Auger data

Harari et al. ICRC07



MAXIMUM LIKELIHOOD RATIO METHOD

Idea:

- The set of N measured events consist in n_s source events and N - n_s background events.
- A background event arrives with a probability distribution proportional to the exposure $\omega(\hat{\mathbf{u}})$
- A source event comes from the source direction $\hat{\mathbf{s}}$ with a distribution $Q(\hat{\mathbf{u}},\hat{\mathbf{s}})$ given by the angular resolution of the experiment (for charged particles a larger spread can be introduced to account for magnetic deflections).

For M sources, we add the contribution of all the sources weighted by the exposure and eventually by a relative source intensity. Simplest option: equally apparent bright sources

$Q(\hat{\mathbf{u}}) = \sum_j \omega(\hat{\mathbf{s}}_j) Q(\hat{\mathbf{u}},\hat{\mathbf{s}}_j) / \sum_j \omega(\hat{\mathbf{s}}_j)$

- The probability distribution for any event is

$P(\hat{\mathbf{u}}) = (n_s/N)\, Q(\hat{\mathbf{u}}) + (N-n_s)/N \;\; \omega(\hat{\mathbf{u}})$

- The likelihood for the set of N events is $$L(n_s) = \prod_{i=1}^{N} P(\hat{u}_i)$$
- Search the value of n_s that maximizes the ratio $R(n_s) = L(n_s)/L(0)$, with $L(0)$ the likelihood of the null hypothesis ($n_s = 0$)
- Significance: $\chi^2 = 2 \ln R$ (for $n_s \geq 0$) and $\chi = -\sqrt{(2 \ln R)}$

check computing R for isotropic simulations and counting the fraction $R^{sim} \geq R^{dat}$

Example: test of correlation of CR with E > 10 EeV and BL Lacs with m < 18 at the resolution angular scale in HiRes data (Abbasi et al. 2005)

For event i use $Q(\hat{\mathbf{u}}_i,\hat{\mathbf{s}}_j)$ a Gaussian centred at $\hat{\mathbf{s}}_j$ with a dispersion equal to the angular resolution of that event

ln R is maximized for $n_s = 8.0$ corresponding to ln R = 6.08
Fraction of simulations with higher ln R, $f = 2 \times 10^{-4}$

- Similar significance to the crosscorrelation analysis
- Can be adapted to give different weight to each candidate source, for example depending on the distance or known brightness in some band. It is also possible to consider different angular scales depending on the angular resolution of the event, or e.g. expected magnetic deflections in different directions.

BINOMIAL PROBABILITY SCAN

For a given candidate source population, e. g. AGNs, galaxy clusters, radio galaxies, there are different unknown variables that will influence the correlation with events

- angular scale: magnetic deflections are not known
- maximum distance to the objects: UHE events from distant sources will have their energy diminished by interactions with CMB (GZK effect)
- energy threshold: only high energy events are expected to be correlated with local sources

Idea: scan in all of them. For a given candidate source population

1) Compute: $p(\psi, D_{max}) \rightarrow$ Probability that a CR from an isotropic flux arrives with angular separation smaller than ψ from a candidate source at a distance smaller than D_{max}

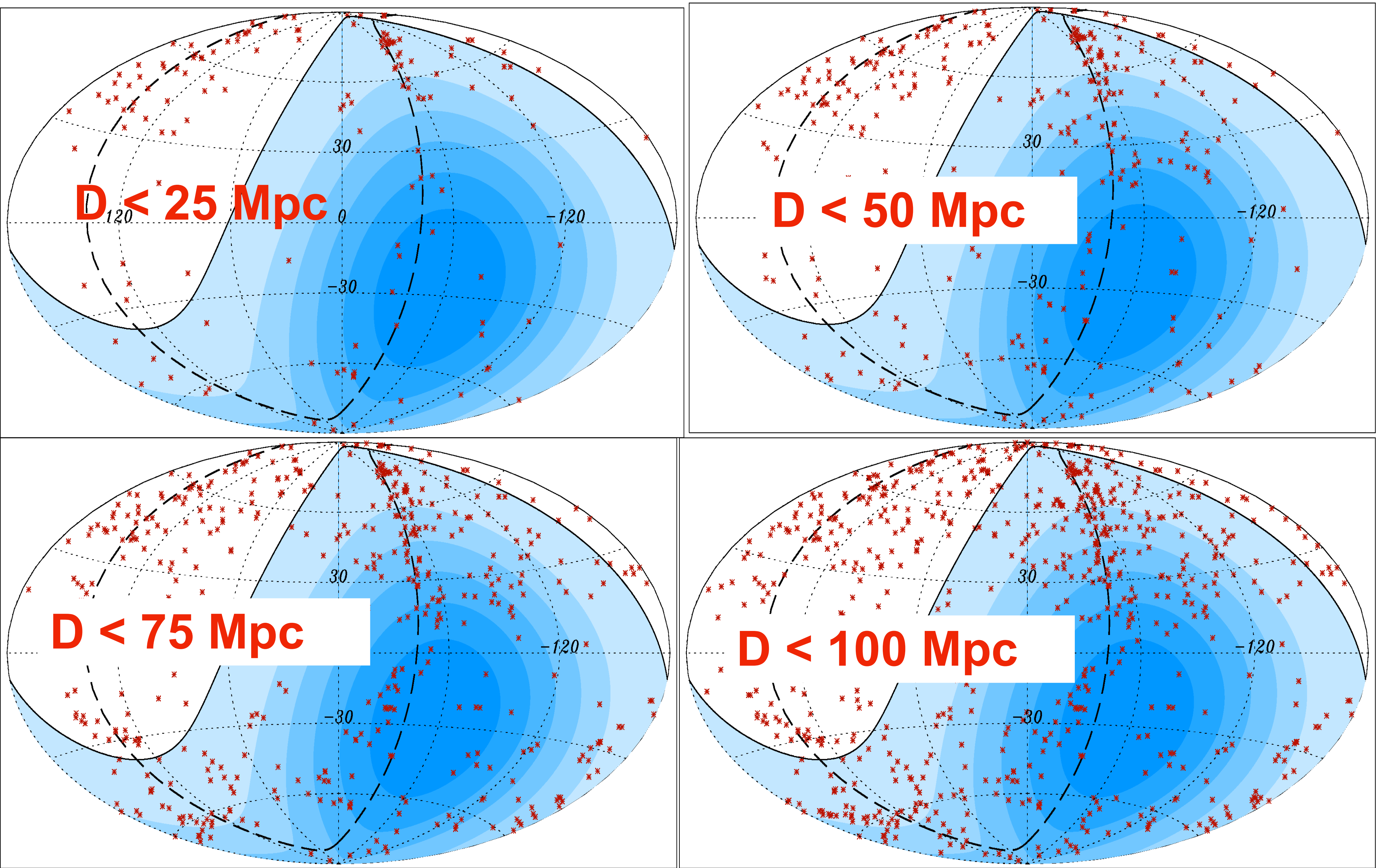
2) For each ψ, D_{max} and E_{min} obtain the number k of correlated events, and look for the set of values having the minimum probability

$$P = \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$\rightarrow P_{min}$

3) Significance: perform isotropic simulations and under the same scan in ψ, D_{max} and E_{min} obtain the fraction f having a P_{min} smaller than the data

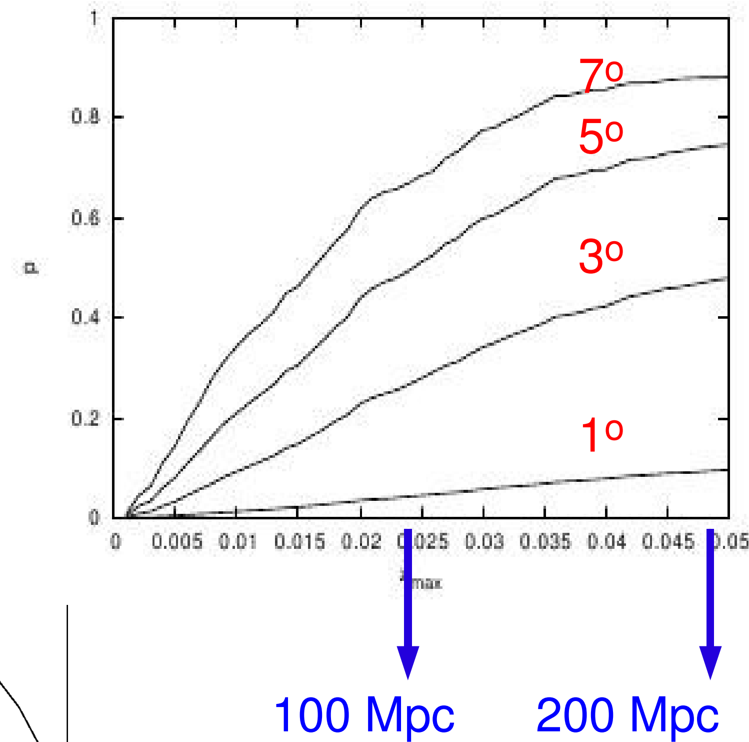
AUGER SEARCH FOR CORRELATIONS WITH NEARBY ACTIVE GALAXIES (AGN) Véron-Cetty & Véron catalog (2006)

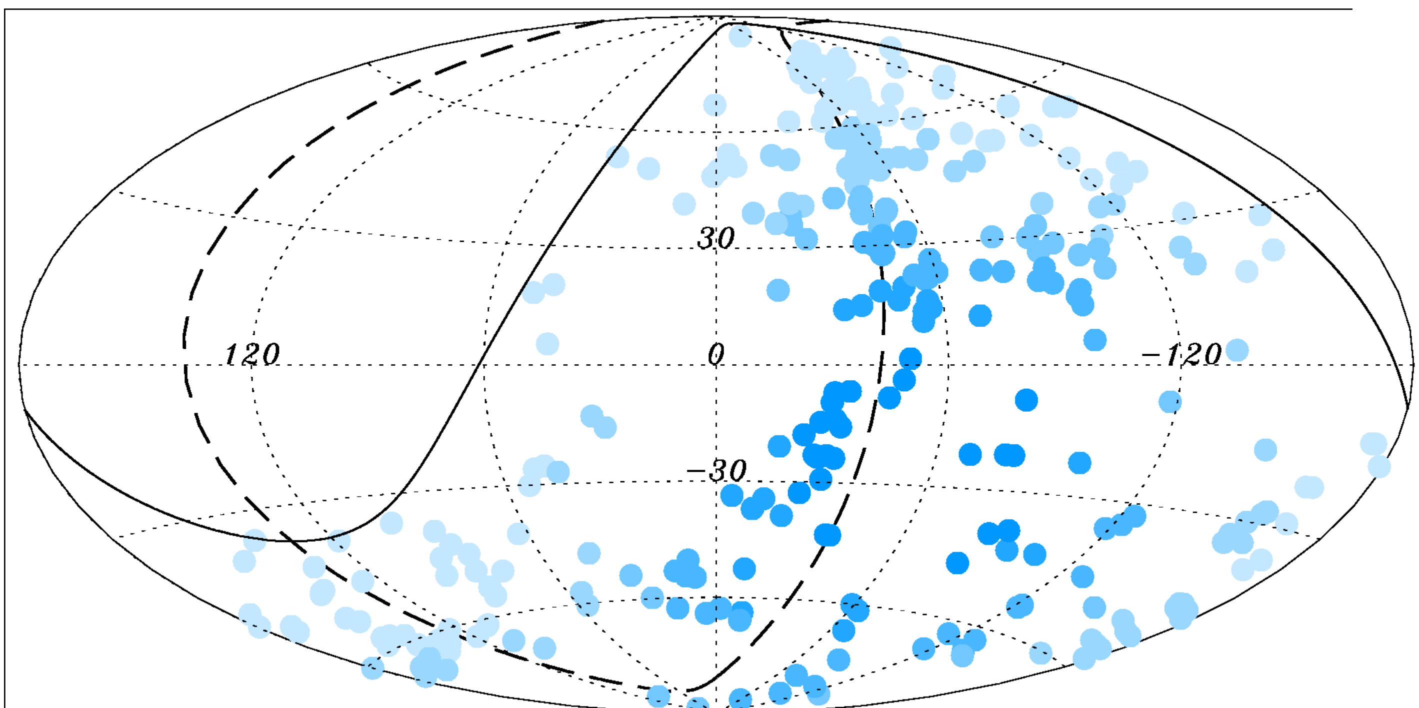


1. Evaluate p(ψ , D_{max})

Probability that a CR from an isotropic flux arrives with angular separation smaller than ψ from an AGN at a distance smaller than D_{max}

p = Fraction of the area, weighted by the relative exposure, covered by circular windows of radius ψ





e.g. p = 0.21 for D_{max} = 75 Mpc and ψ = 3.1°

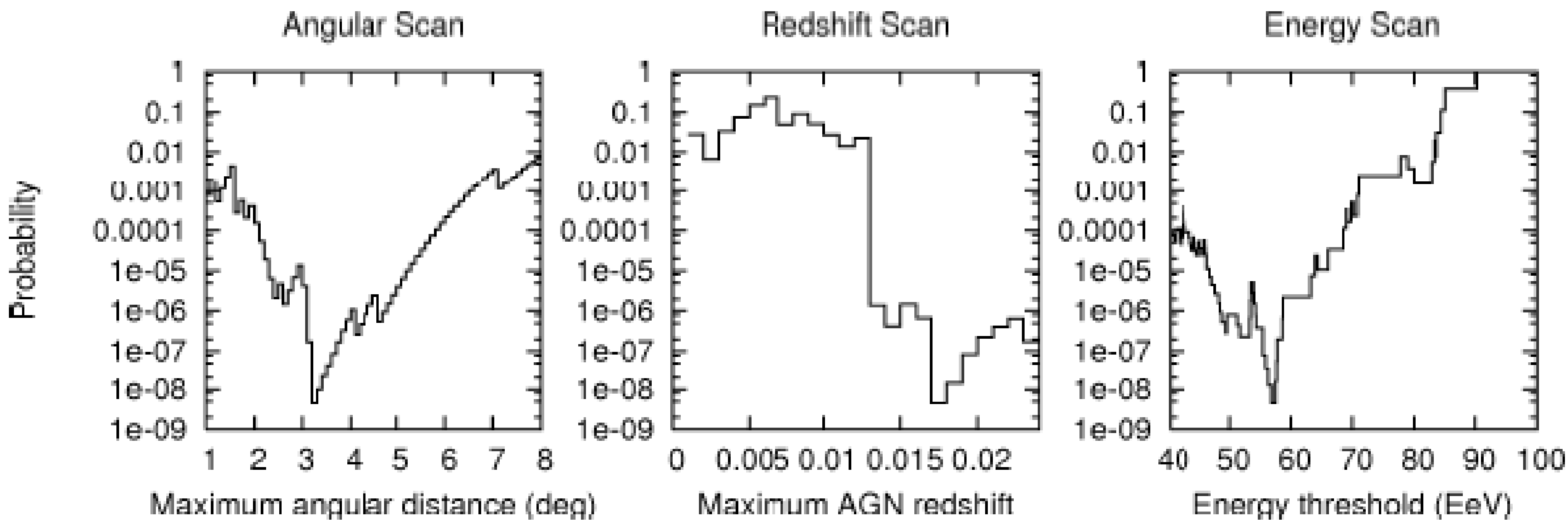
2. Scan over ψ, D_{max}, E_{min}

(using 81 events up to 31 august 2007 with $E > 4\text{x}10^{19}$ eV)

P = probability to have k or more correlations in a set of n isotropic events, with individual chance probability $p(\psi, D_{max})$:

$$P = \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

Minimum of P: $E_{min} \sim 6\text{x}10^{19}$ eV (n = 27) , $\psi \sim 3^o$, D ~ 75 Mpc



3. fraction of isotropic simulations of 81 events which have a smaller P_{min} under the same scan

f ~ 10^{-5}

HOW TO MINIMIZE THE POSSIBILITY THAT A REPORTED SIGNAL BE A FLUCTUATION (AS ALREADY HAPPENED IN THE PAST)?

Exploratory scan:

Data analysed from 1/1/2004 up to 26/5/2006

First hints of correlations obtained through this scan

12/15 correlations (about 3 expected)

D_{max} = 75 Mpc ψ = 3.1º E_{min} = 5.6 x10^{19} eV   (p = 0.21)

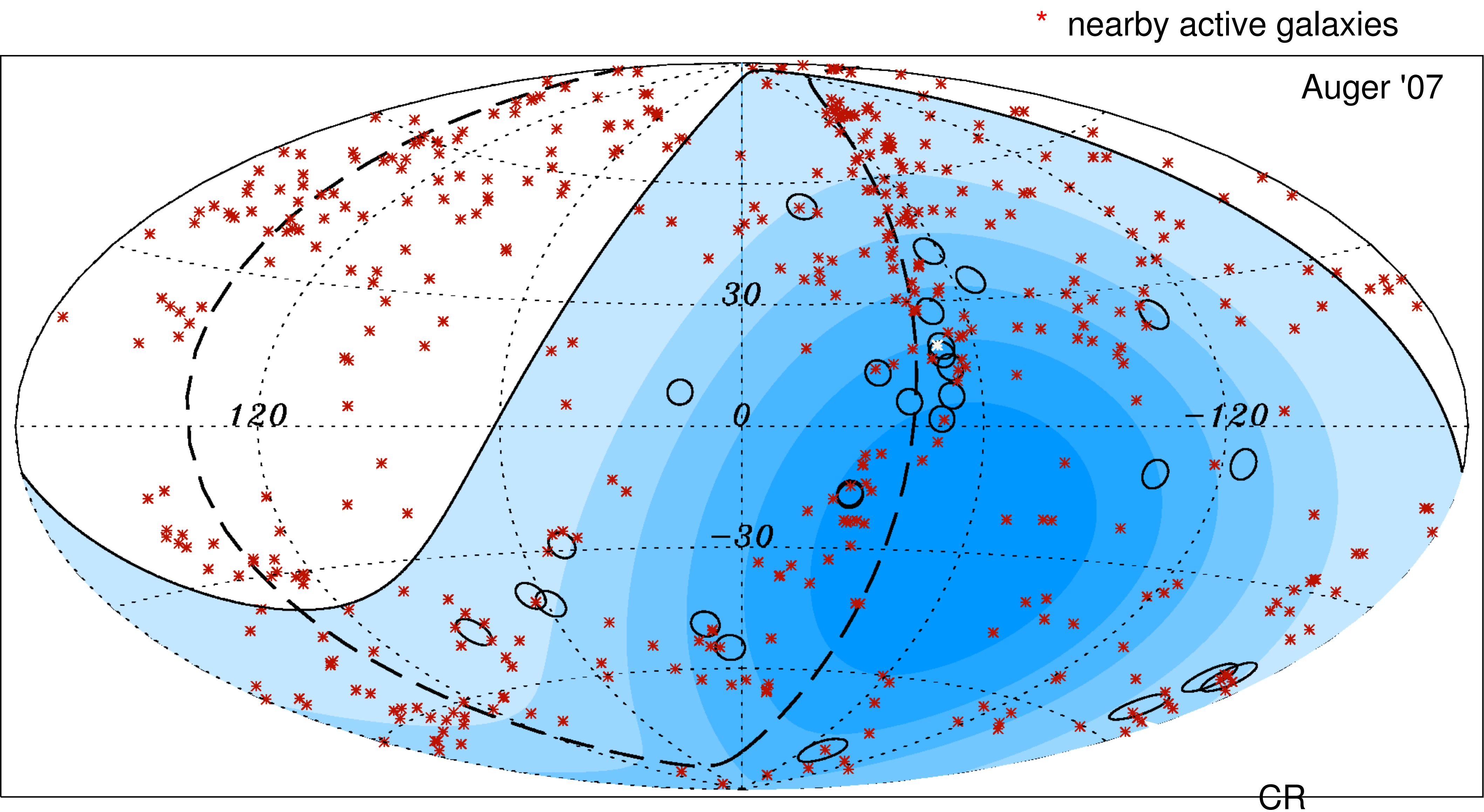
$f \sim 10^{-3}$

Test using an independent data set with parameters fixed a priori

Data analysed from 27/5/2006 up to 31/8/2007

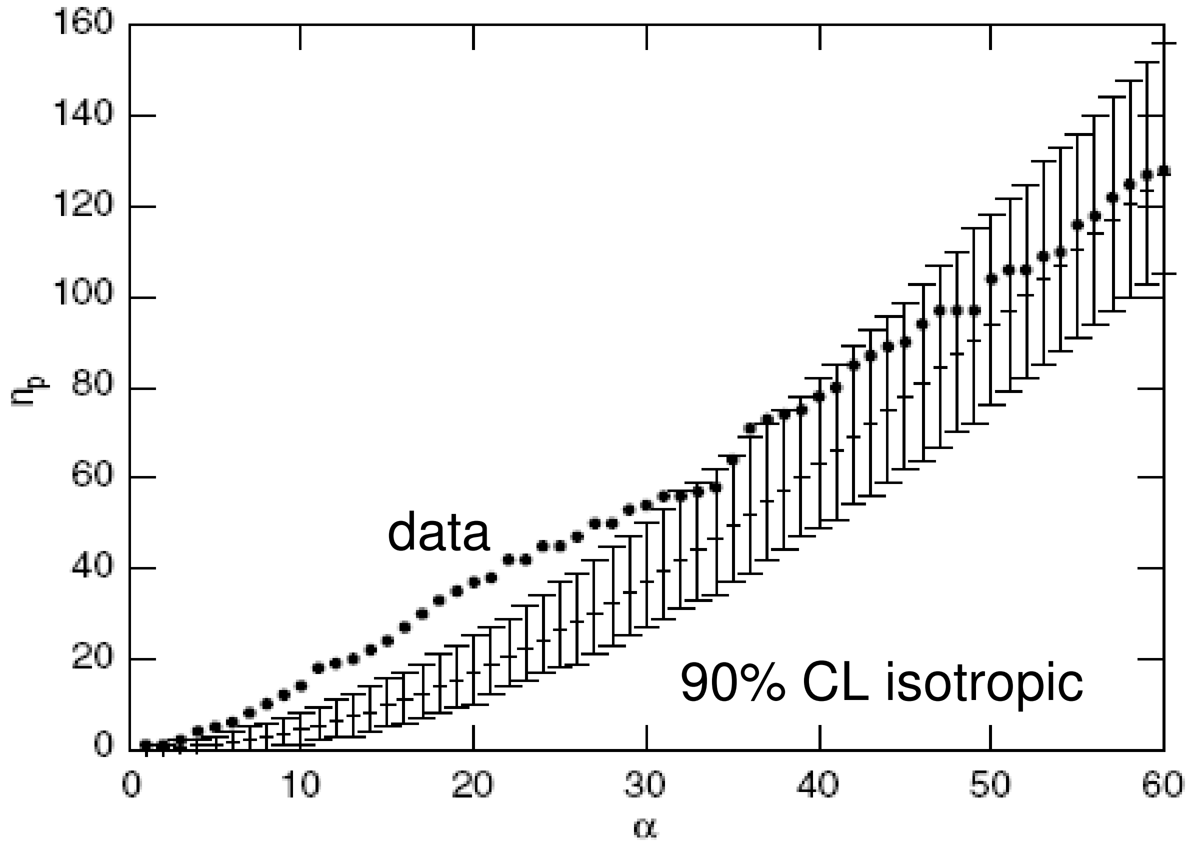
8/13 correlations

in this independent set with parameters specified apriori (thus no penalization for scanning is needed) the probability that flux be isotropic is < 1%



with the data up to 31 august 2007, from the 27 CRs with highest energies, 20 are at less than ~3 degrees from an active galaxy at less than ~ 75 Mpc , while 6 were expected (from the 7 which are not, 5 have $|b_G| < 12$ deg, where catalog is largely incomplete)

Autocorrelation function for the same dataset



Number of pairs vs. angular separation
for 27 events E > 57 EeV

10^{-3} isotropic simulations have comparable departures under similar angular scan

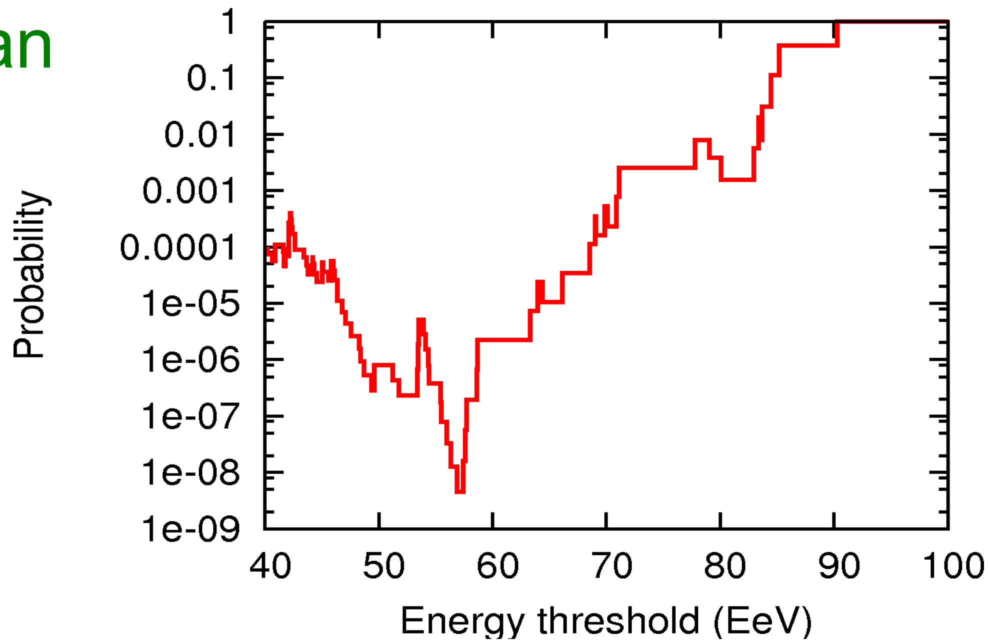
ENERGY THRESHOLD ~ 60 EeV : consistent with GZK effect

Energy scan

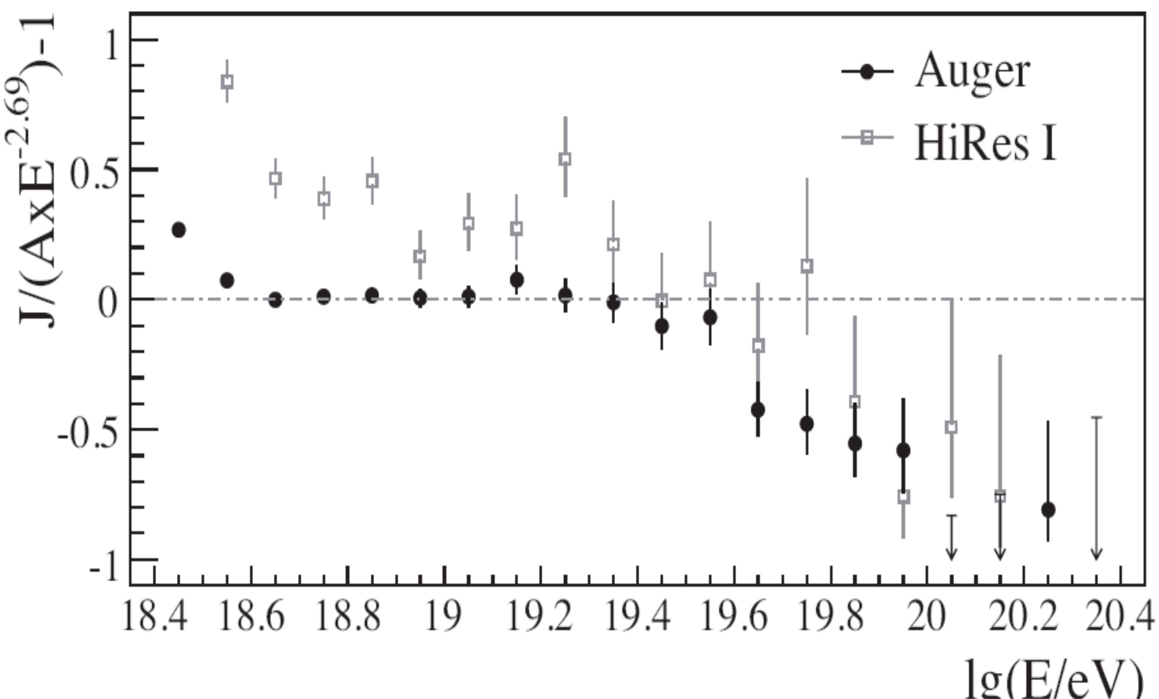
$z_{max} = 0.017$

$\psi = 3.2^o$

p=21%



Sharp transition to isotropic expectations at lower energies



Energy threshold for correlation coincides with ~ 50% flux suppression compared to extrapolation from lower energies as measured by Auger

ANISOTROPY: confirmed at 99% CL with a priori test on independent data set.
Correlation with AGN positions for $E > 6 \times 10^{19}$ eV and $D < 100$ Mpc compatible with origin in extragalactic sources within GZK horizon.

Scenarios with galactic origin ruled out:
Young neutron stars, pulsars, black holes, halo dark matter decay, …

Angular scale of few degrees suggests predominantly light composition at highest energies (unless magnetic fields are very weak)

TOWARD SOURCE IDENTIFICATION:
Correlation does not imply AGN are the sources.

LSS distribution of matter similar as that of local AGN.
GRBs, starburst galaxies, quasar remnants, massive clusters,…

Plausible that only a subclass of AGN be sources (or tracers).
Striking alignment along supergalactic plane, particularly around Cen A.

LOG LIKELIHOOD PER EVENT

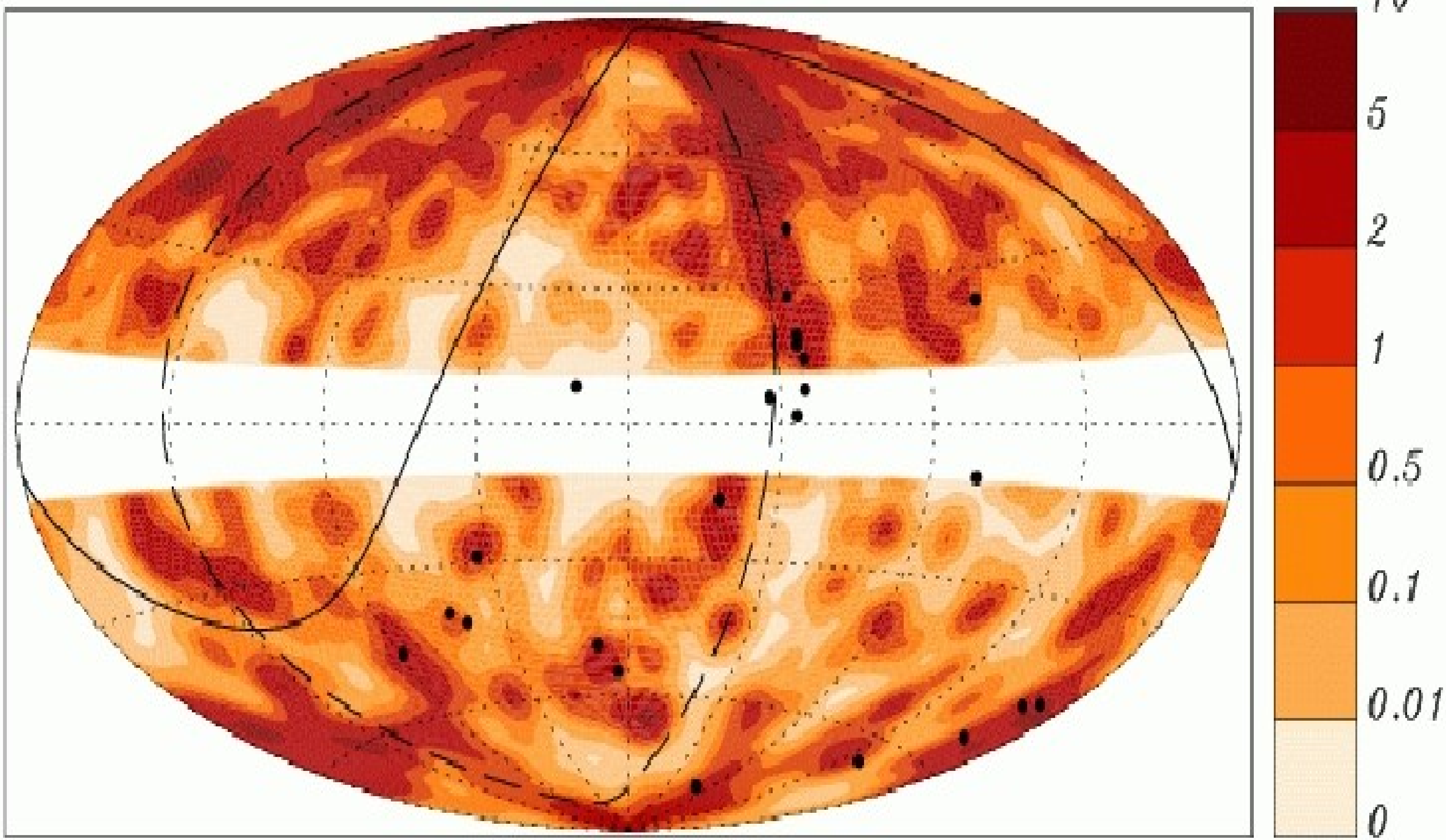
The previous method cannot be applied when the candidate source population is large, as the fraction of the sky covered p becomes very large.
A more useful method in this case is to build a probability map for the expected arrival directions of events above a given threshold

Map construction: Take a Gaussian of given σ around the direction of each object in the catalog and weight them by a factor

$$w(z, E_{th}) = \frac{1}{4\pi d_L^2(z)\phi(z)} \int_{E_i(z,E_{th})}^{\infty} E^{-s} \mathrm{d}E.$$

d_L: distance to the object
ϕ (z): selection function of the catalog
integral term: fraction of the flux from a source at redshift z that reaches the Earth with an energy larger than the threshold E_{th}. E_i is the initial energy that the particle needs to have at the source to arrive at Earth with E_{th}, s is the source spectral index.

Example: AGNs in the Veron-Cetty & Veron catalogue



Log Likelihood per event

- The likelihood associated to a given set of N observed events is

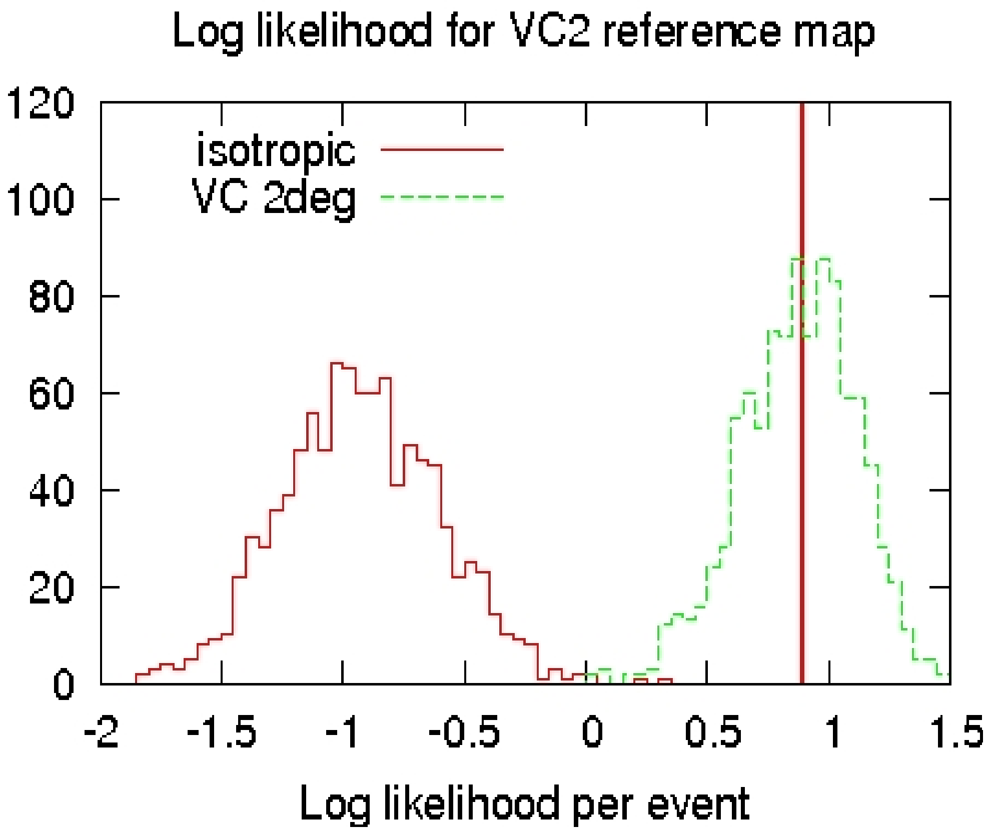
$$L = \prod_{i=1}^{N} P(\hat{u}_i)$$

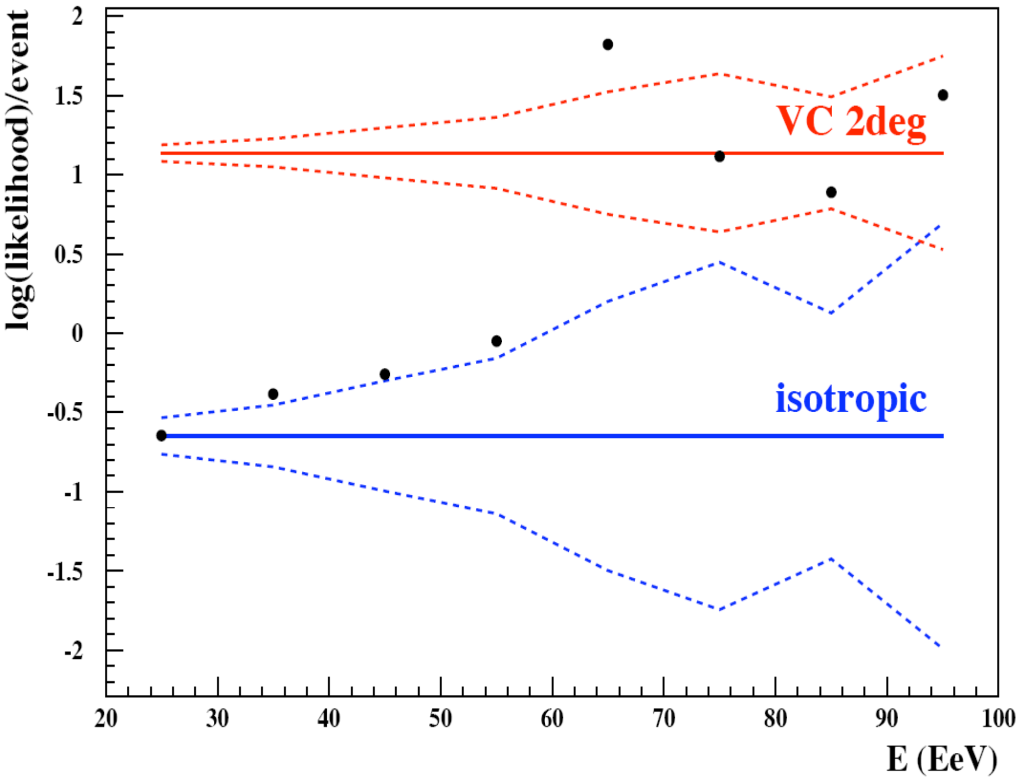
with P proportional to the map density in each event direction.
- Better to consider the log likelihood per event

$$LL = \frac{1}{N} \sum_{i=1}^{N} \ln(P(\hat{u}_i))$$

- Measure LL for a model reference map for the data
- Simulate events distributed according to some alternative hypothesis: isotropic, following AGNs,... and compute LL for the reference model map → histogram the LLs for each hypothesis: the mean of the distribution is independent of the number N of observed events, but the width becomes smaller as N grows

When the histograms corresponding to the different hypothesis are separated → the test is good to discriminate among them

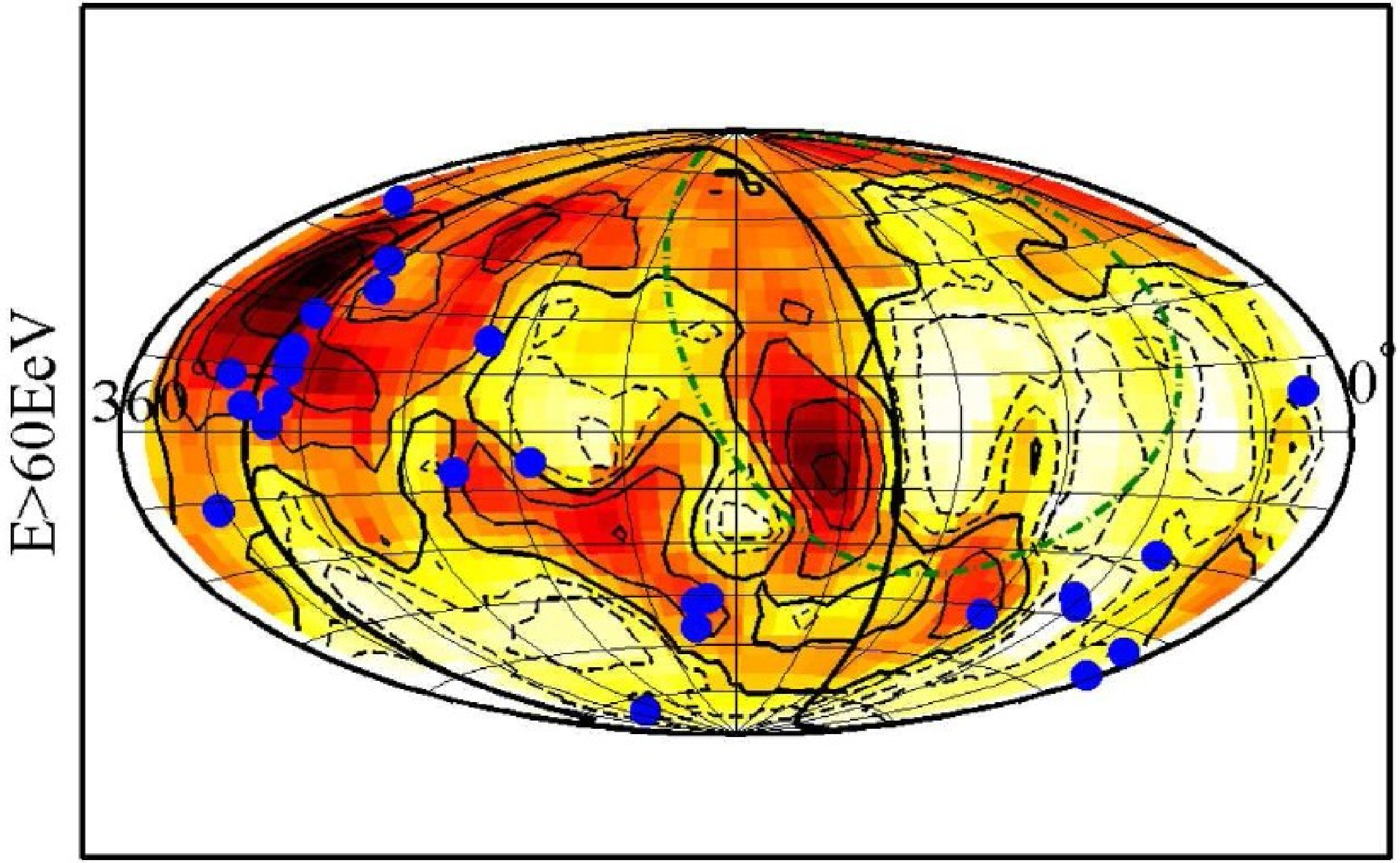




SHARP TRANSITION TO ISOTROPY BELOW 60 EeV

IRAS PSCz galaxy catalog as tracer of LSS

Map of expected UHECR intensity **(Kashti and Waxman - 2008)**



Using a χ^2 statistics:

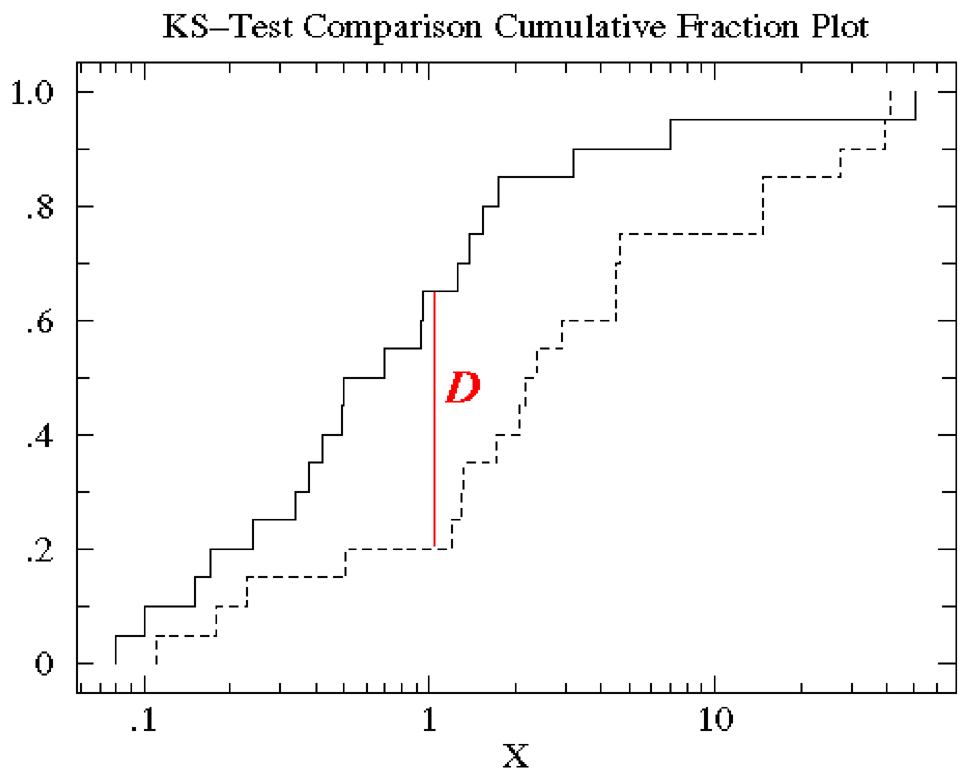
Auger data inconsistent with isotropy at 98% CL

Consistent with distribution that traces LSS

TWO DIMENSIONAL KOLMOGOROV SMIRNOV TEST

Another statistics proposed to test the probability that a given dataset be a realization of a model is a generalization to two variables of the well know KS test for one variable (Fasano&Franceschini `87, Press et al.)

In the 1 variable test we look for the maximum difference of the normalized cumulative distributions of the dataset with respect to that expected for the model → D

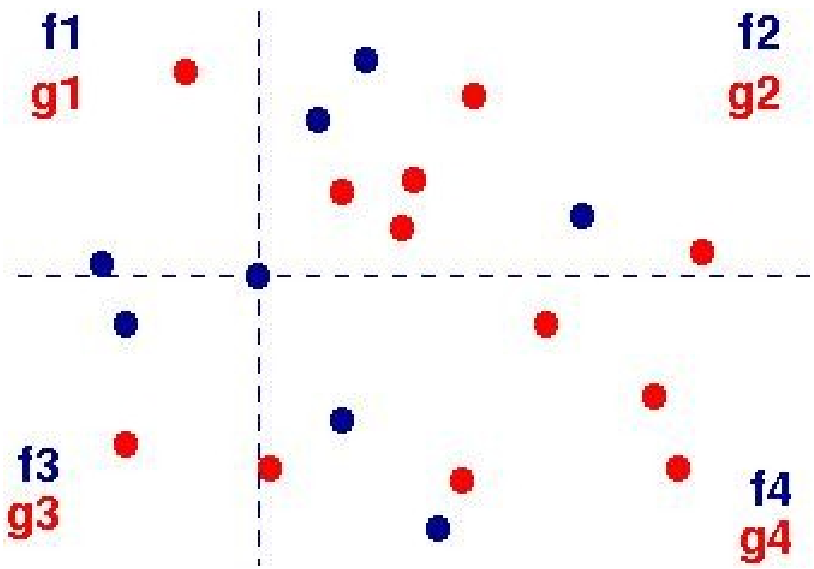


We then estimate the probability that a larger or equal value of D arises by chance

The 1-D KS test can be used for example to probe the RA distribution of a dataset

Generalization to probe a 2-D distribution:

ex: determine if a given dataset is distributed as objects in a given catalog



For each CR determine the fractions of CR: f1,f2,f3,f4 and of catalog objects: g1,g2,g3,g4 associated to the 4 quadrants. Compute the absolute value of the differences

dk = |fk – gk| (k=1,...,4)

and select the maximum value

D ≡ max(dk(j))

To estimate the similarity of the distributions repeat the procedure for a large number of simulated isotropic sets and count the fraction P that have a larger value of D

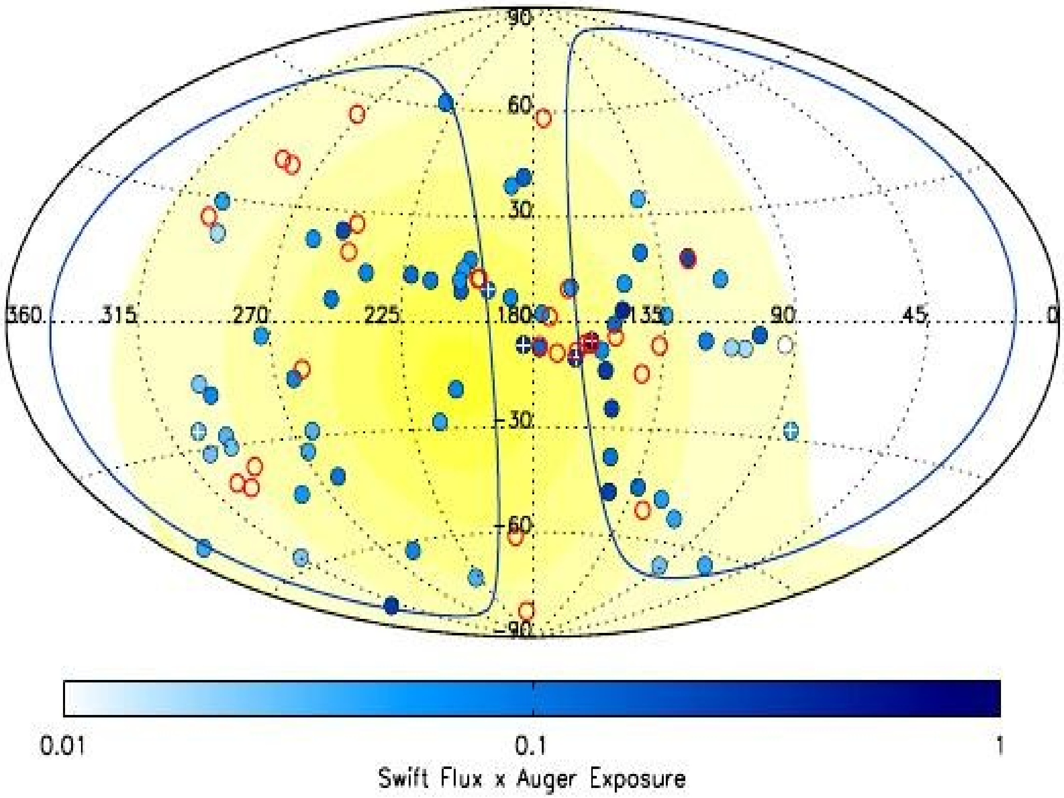
low (high) P → low (high) probability that the 2 distributions arise from the same source population

Variant: Define the quadrants around the events (→ D1) and around the catalog objects (→ D2) and define D = (D1+D2)/2

EXAMPLES

George et al. '08

Ghisellini et al '08



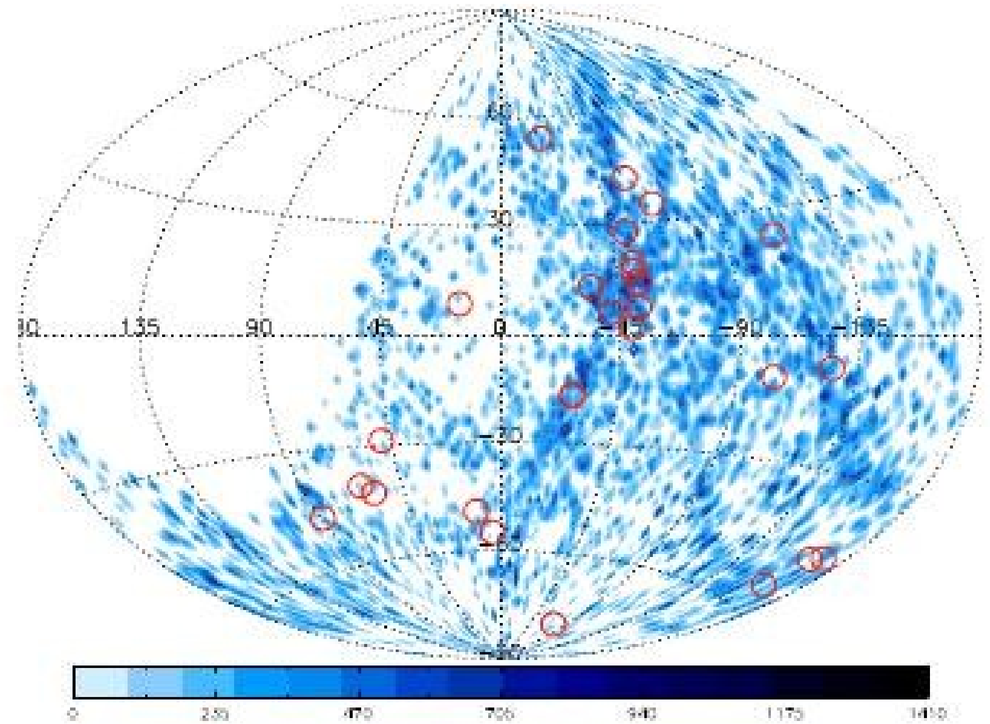


Figure 2. HIPASS galaxy H I flux, in galactic coordinates. Blue levels corresponds to the integrated flux (in bins of 2° × 2° and units of Jy km s^{-1}) of H I emission, multiplied by the relative AUGER exposure. Red circles are the locations of the 27 AUGER UHECRs above 57 EeV.

Hard X-ray selected AGN (Swift BAT)
cutting galactic plane |b| < 15º
138 AGN, 19 UHECR

P = 0.98   AGN weighted by Swift flux
x Auger exposure
(P = 0.5 if no weight is considered)

Galaxies in the HI Parkes All Sky survey

P = 0.72   galaxies weighted by HI flux
x Auger exposure
P = 0.99   ¼ most massive galaxies

FINAL REMARKS

A variety of different methods are needed to study anisotropies at different eenrgies and angular scales

CR astronomy is finally starting and we are getting the first clues on the UHECR origin: they are correlated with nearby extragalactic matter

Still many open questions: sources, composition, magnetic fields?

New data waited to clarify these issues ...

MANY THANKS