LECTURE III: NON LINEAR THEORY OF PARTICLE ACCELERATION

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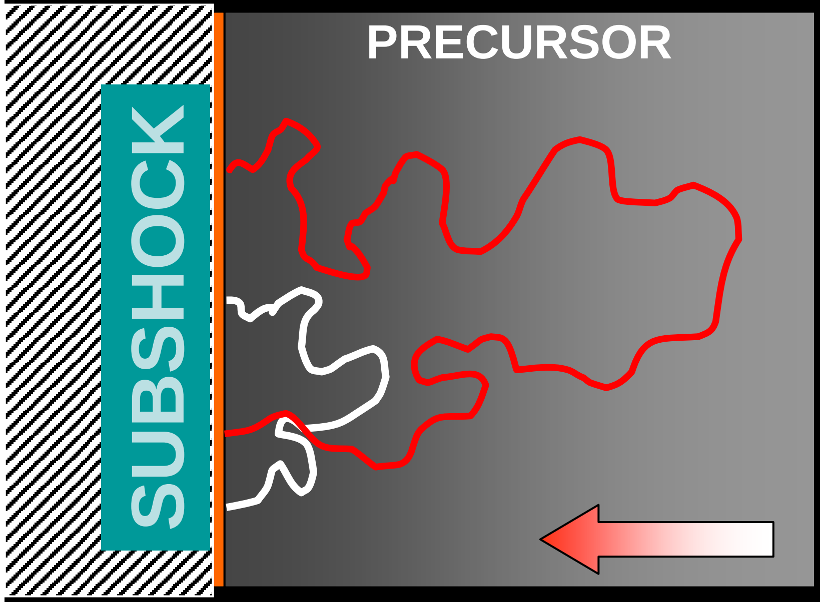
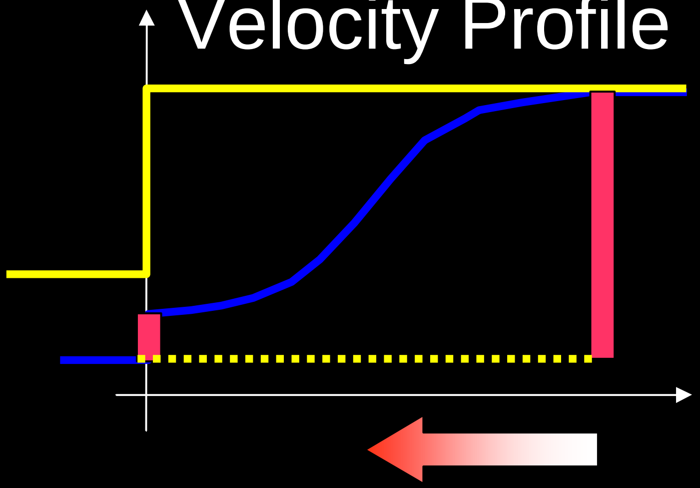
BEYOND THE TEST PARTICLE THEORY: NON LINEAR DSA

- AIM OF THE THEORY:
  - PREDICT THE EFFICIENCY OF ACCELERATION
  - ACCOUNT FOR THE DYNAMICAL REACTION OF THE ACCELERATED PARTICLES ON THE SHOCK
  - EXPLAIN WHY PARTICLES RETURN TO THE SHOCK
  - DETERMINATION OF THE SPECTRUM OF CR SEEN AT THE EARTH

Dynamical Reaction of Accelerated Particles





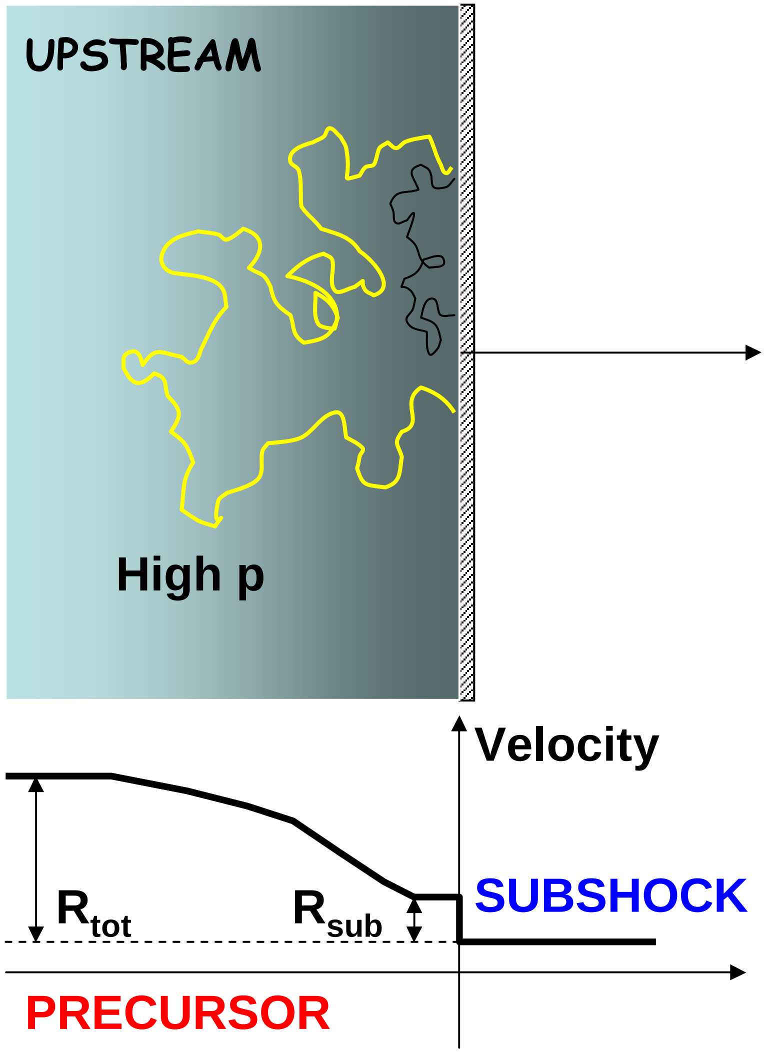
$$\rho_0 u_0^2 + P_{g,0} = \rho(x)\,u(x)^2 + P_g(x) + \boxed{P_{CR}(x)}$$

Conservation of Momentum

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

Transport equation for cosmic rays

WHAT SHOULD BE EXPECT?

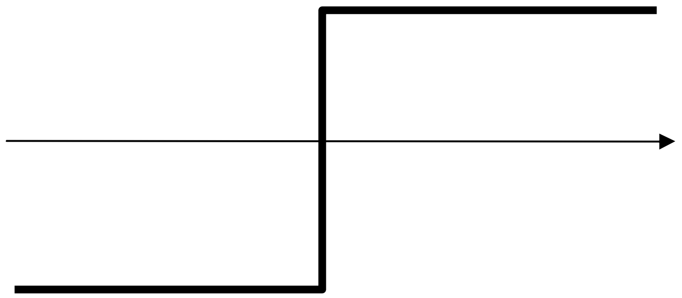


1. PARTICLES WITH HIGH P FEEL LARGER COMPRESSION FACTOR OF LOW P PARTICLES
2. THE TOTAL COMPRESSION BECOMES LARGER THAN 4
3. THE SPECTRUM SHOULD BE NO LONGER A POWER LAW!
4. AT HIGH P THE SLOPE SHOULD BE FLATTER THAN 2 !!!
5. THE GAS BEHIND THE SHOCK SHOULD BE COOLER THAN FOR AN ORDINARY SHOCK

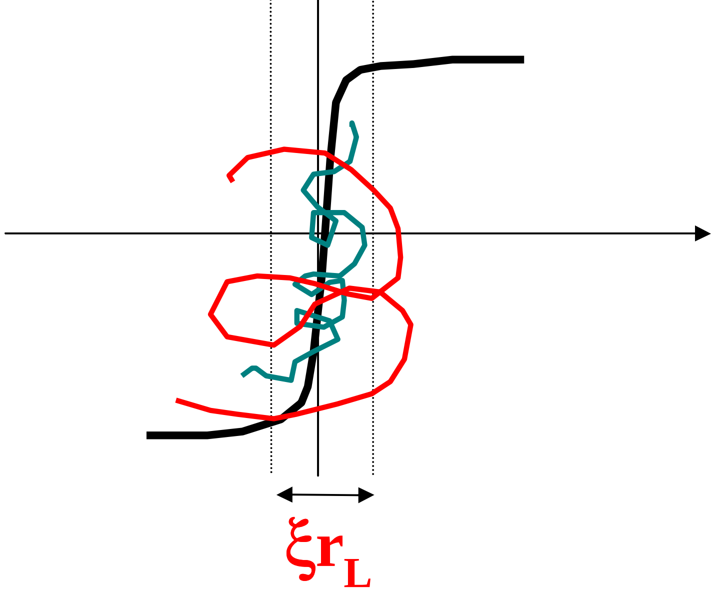
INJECTION

THIS IS THE LEAST UNDERSTOOD ASPECTS OF ALL !

Ideal Collisionless Shock

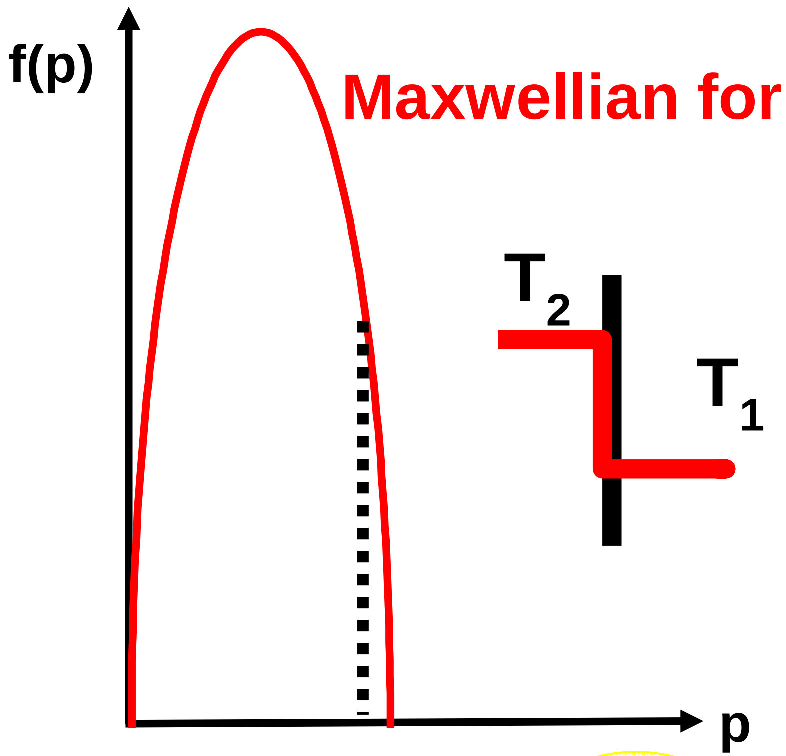


(More) Real Collisionless Shock



WHICH PARTICLES AND HOW MANY PARTICLES ENTER THE ACCELERATION CYCLE?

INJECTION…cont'd



Maxwellian for the Downstream gas

$$\eta = \frac{4}{3\pi^{1/2}}\left(R_{sb} - 1\right)\xi^{3} e^{-\xi^{2}}$$

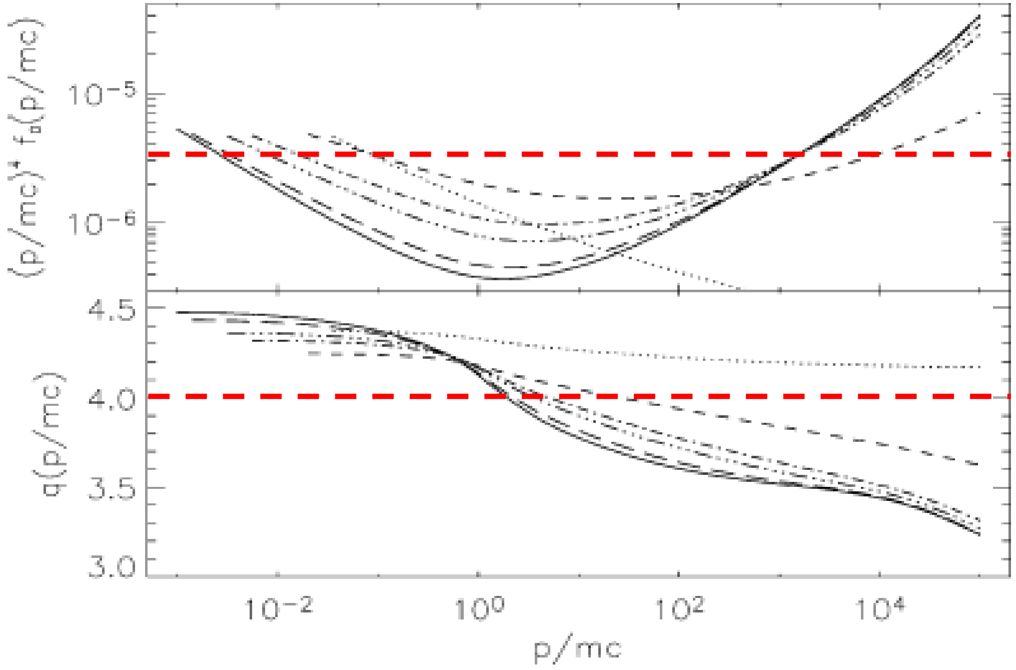
Relation between

R_{sub}, ξ and η

$P_{inj} = \xi\, p_{th,2}$

OUTPUT OF THE CALCULATIONS

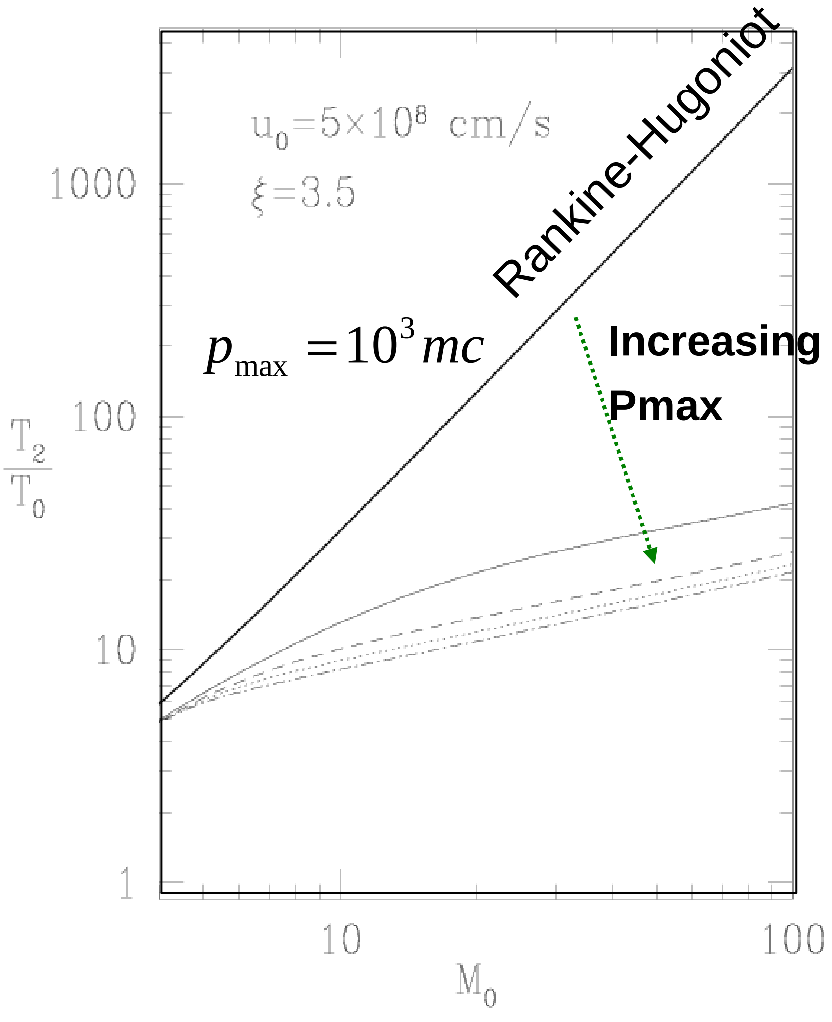
SPECTRA OF ACCELERATED PARTICLES

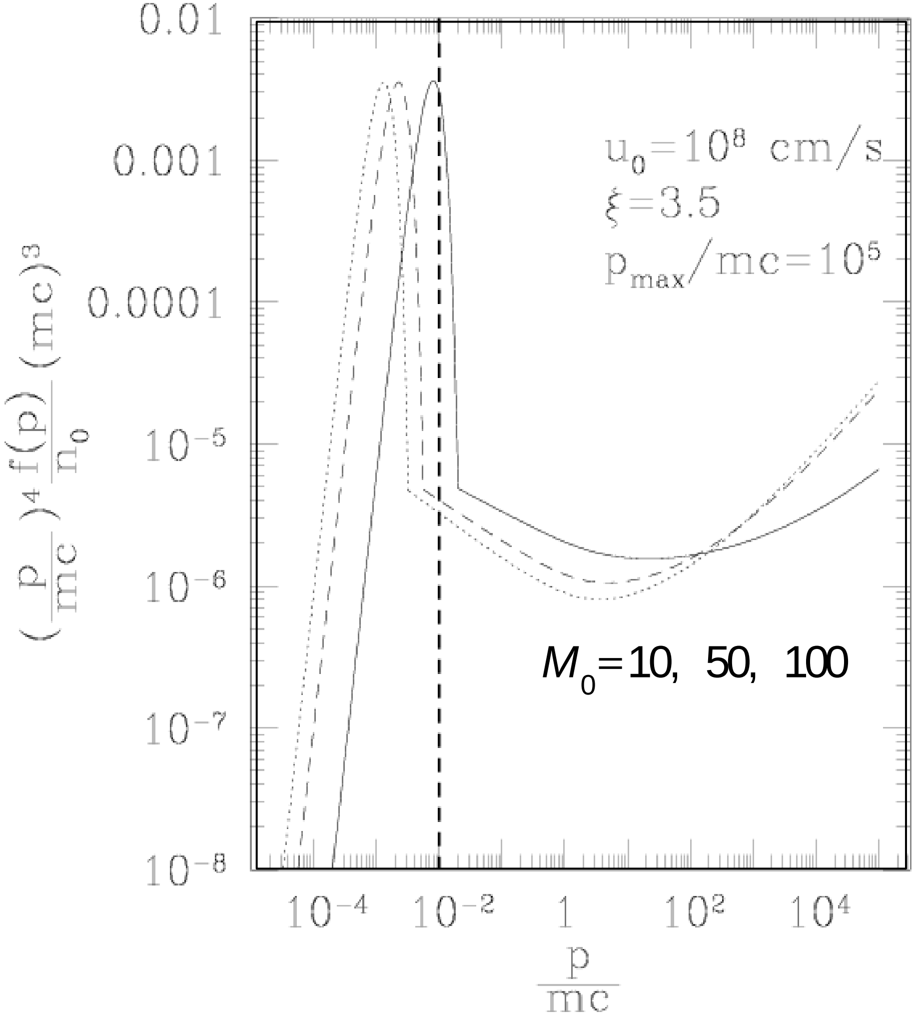


Amato&PB 2005

Mach number M_0	$R_{\rm sub}$	$R_{\rm tot}$	$\xi_c(0)$	$p_{\rm inj}$	η
4	3.19	3.57	0.1	0.035	3.4×10^{-4}
10	3.413	6.57	0.47	0.02	3.7×10^{-4}
50	3.27	23.18	0.85	0.005	3.5×10^{-4}
100	3.21	39.76	0.91	0.0032	3.4×10^{-4}
300	3.19	91.06	0.96	0.0014	3.4×10^{-4}
500	3.29	129.57	0.97	0.001	3.5×10^{-4}

SHOCK HEATING

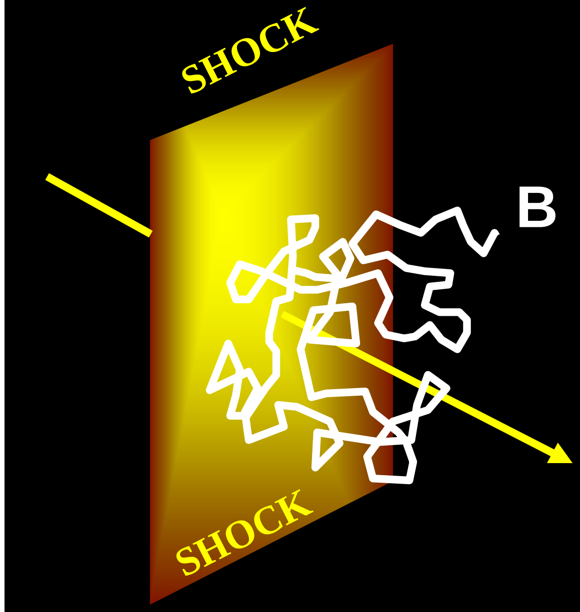




Cosmic Ray self-induced scattering: a primer

Small perturbations in a magnetized medium made of electrons and protons simply give ALFVEN WAVES

WHAT HAPPENS WHEN THERE IS A SHOCK AND IT IS ACCELERATING COSMIC RAYS?



$$\frac{dP_{CR}}{dt} = n_{CR}\, m\, \Gamma_{CR}\, (v_S - v_A)\Omega$$

$$\frac{dP_W}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{v_A}$$

$$\boxed{\gamma_W = \frac{n_{CR}}{n_{gas}} \Omega_{cyc} \left(\frac{v_S - v_A}{v_A}\right)}$$

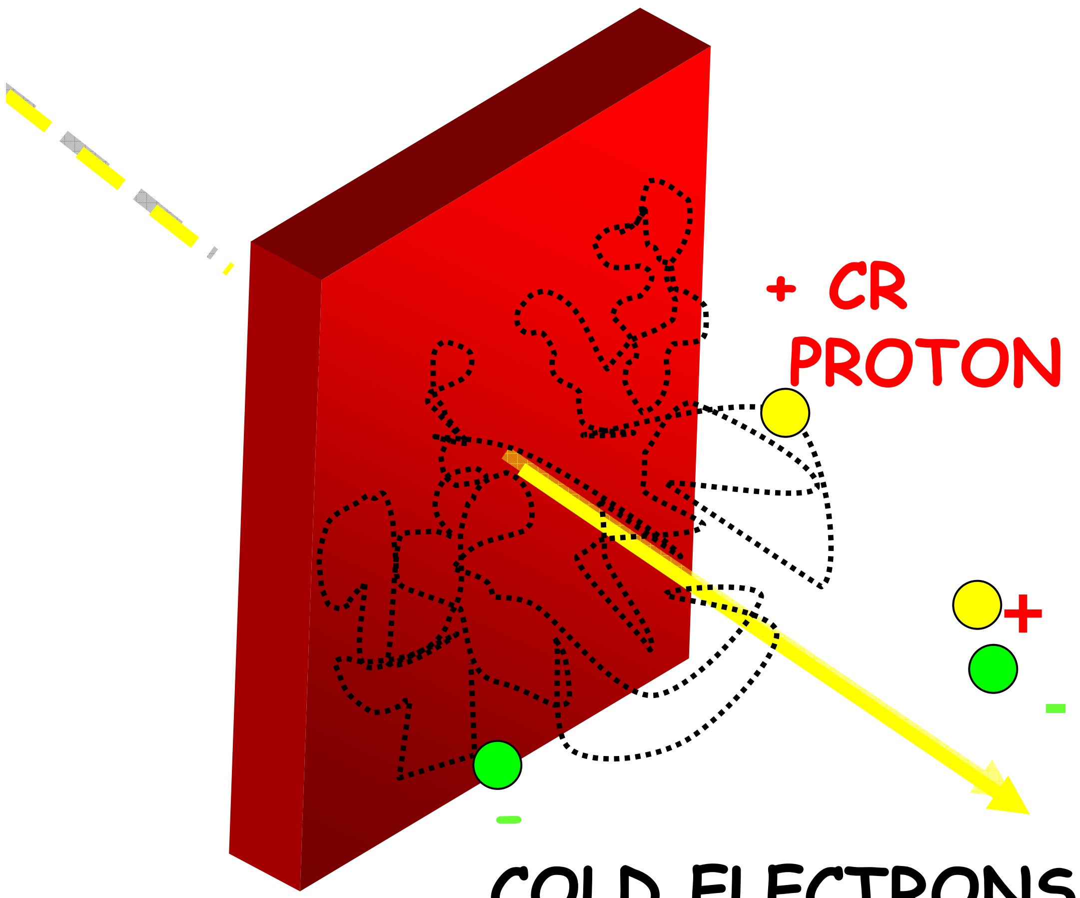
A HINT TO HOW THIS SHOULD BE DONE

WE HAVE ALREADY DISCUSSED A DERIVATION OF THE RESONANT MODES THAT LEAD TO WAVE PRODUCTION. HERE IS THE MORE GENERAL AND FORMALLY CORRECT APPROACH:

ONE SHOULD PERTURB THE FOLLOWING EQUATIONS:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{c} \left(\vec{v} \times \vec{B} \right)_\beta \frac{\partial f_\alpha}{\partial p_\beta} = 0$$

+ Maxwell Equations



COLD ELECTRONS
COMPENSATING
THE CR CHARGE

$$\begin{aligned}
f_i(p,\mu) &= \frac{n_i}{2\pi p^2}\delta(p-m_i v_s)\delta(\mu-1) \\
f_e(p,\mu) &= \frac{n_e}{2\pi p^2}\delta(p-m_e v_s)\delta(\mu-1) \\
f_e^{cold}(p) &= \frac{N_{CR}}{4\pi p^2}\delta(p) \\
f_{CR}(p) &= \frac{N_{CR}}{4\pi}g(p).
\end{aligned}$$

THE DISPERSION RELATION

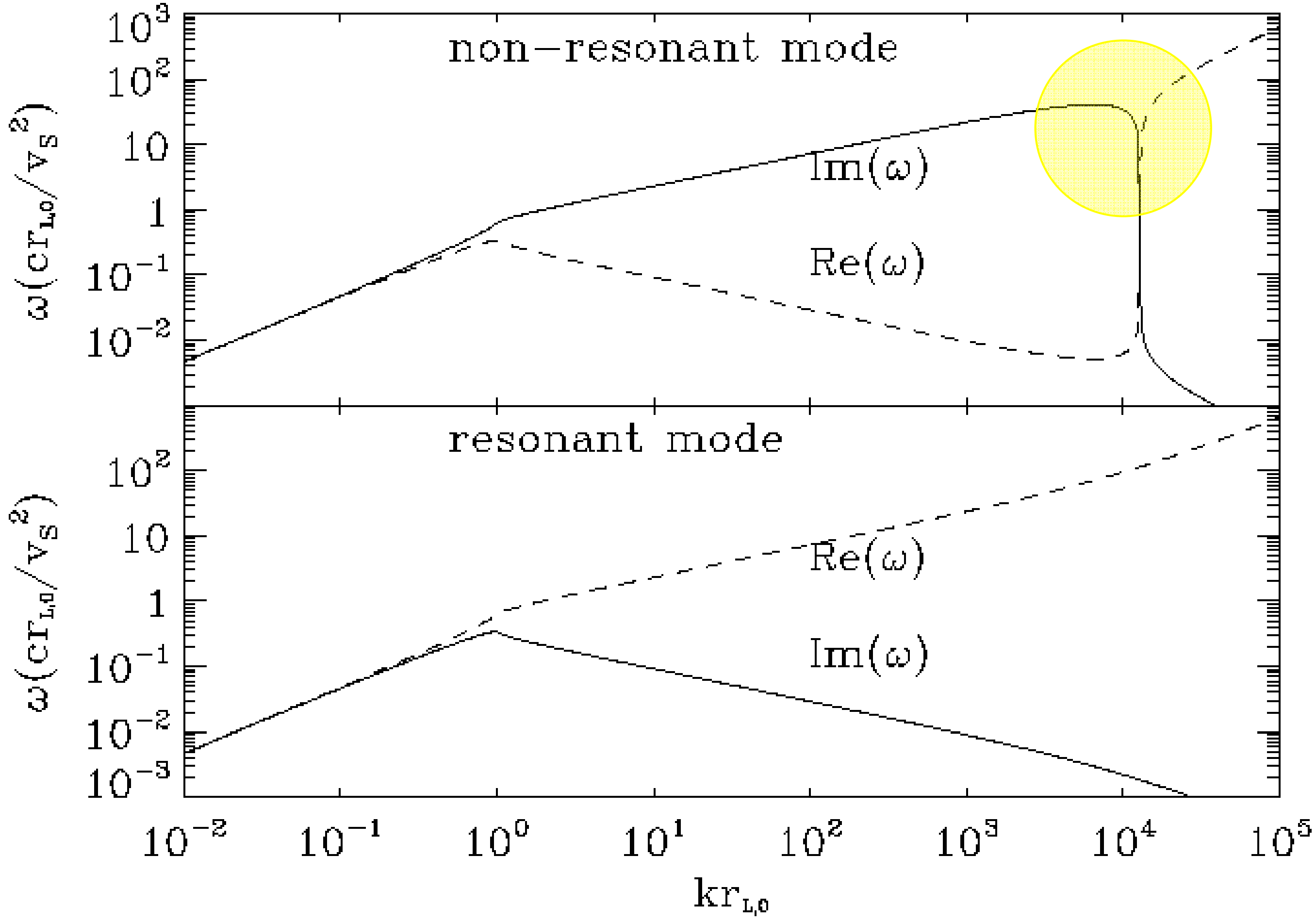
$$v_A^2 k^2 = \tilde{\omega}^2 \pm \frac{N_{CR}}{n_i}(\tilde{\omega} - k v_s)\Omega_i^* \left[1 \pm I_1^{\pm}(k) \mp i I_2(k)\right]$$

$$I_1^{\pm}(k) = \frac{p_{min}(k)}{4}\int_{p_0}^{p_{max}} dp \frac{dg}{dp}\left[(p^2 - p_{min}(k)^2)\ln\left|\frac{1 \pm p/p_{min}}{1 \mp p/p_{min}}\right| \pm 2 p_{min} p\right]$$

$$I_2(k) = \frac{\pi}{4} p_{min}(k) \int_{Max[p_0, p_{min}(k)]}^{p_{max}} dp \frac{dg}{dp}(p^2 - p_{min}(k)^2).$$

AT FIRST SIGHT THE EXCITED WAVES ARE BASICALLY MODIFIED ALFVEN WAVES WITH A SMALL FORCING PROPORTIONAL TO THE COSMIC RAY DENSITY

REAL AND IMAGINARY PARTS OF THE FREQUENCY



GROWTH OF THE WAVES

THE WAVES ARE GENERATED UPSTREAM AND EVENTUALLY ADVECTED WITH THE FLUID. THE EQUATION FOR THE NORMALIZED WAVE ENERGY IS THEREFORE

$$\frac{\partial \mathcal{F}_w(k,x)}{\partial x} = u(x)\frac{\partial \mathcal{P}_w(k,x)}{\partial x} + \sigma(k,x)\mathcal{P}_w(k,x) - \Gamma(k,x)\mathcal{P}_w(k,x)$$

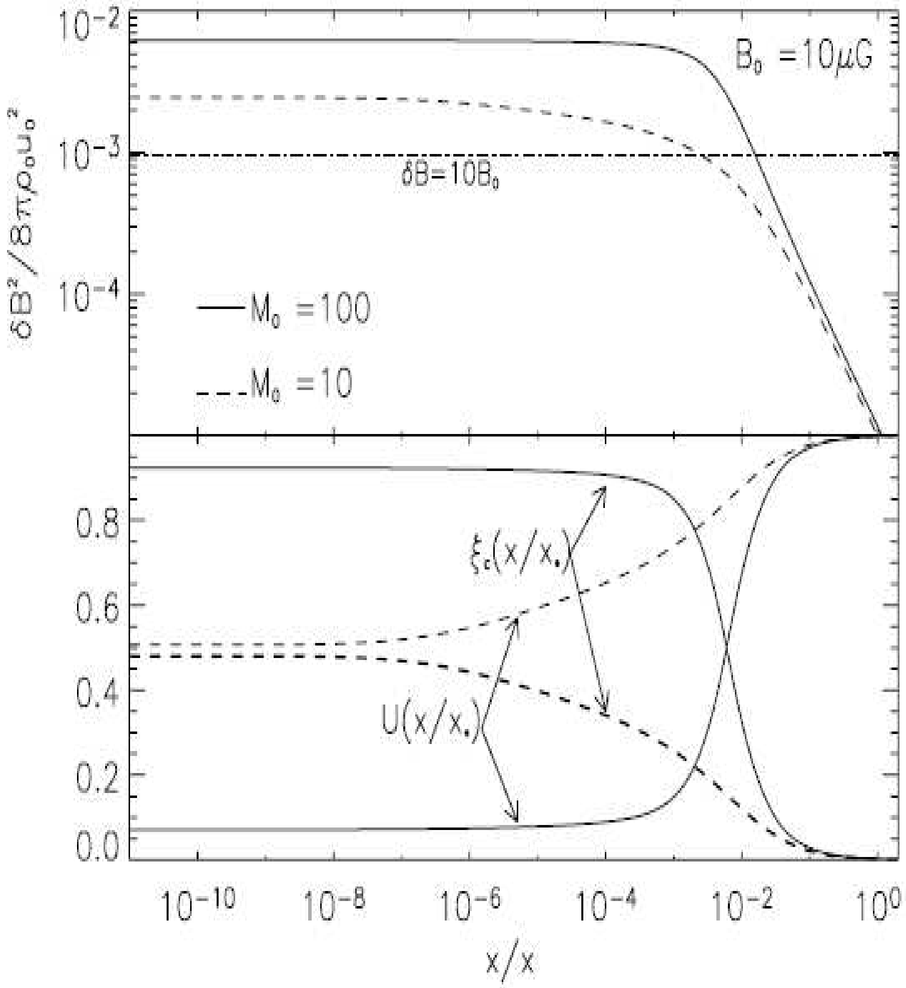
ADVECTION AMPLIFICATION DAMPING

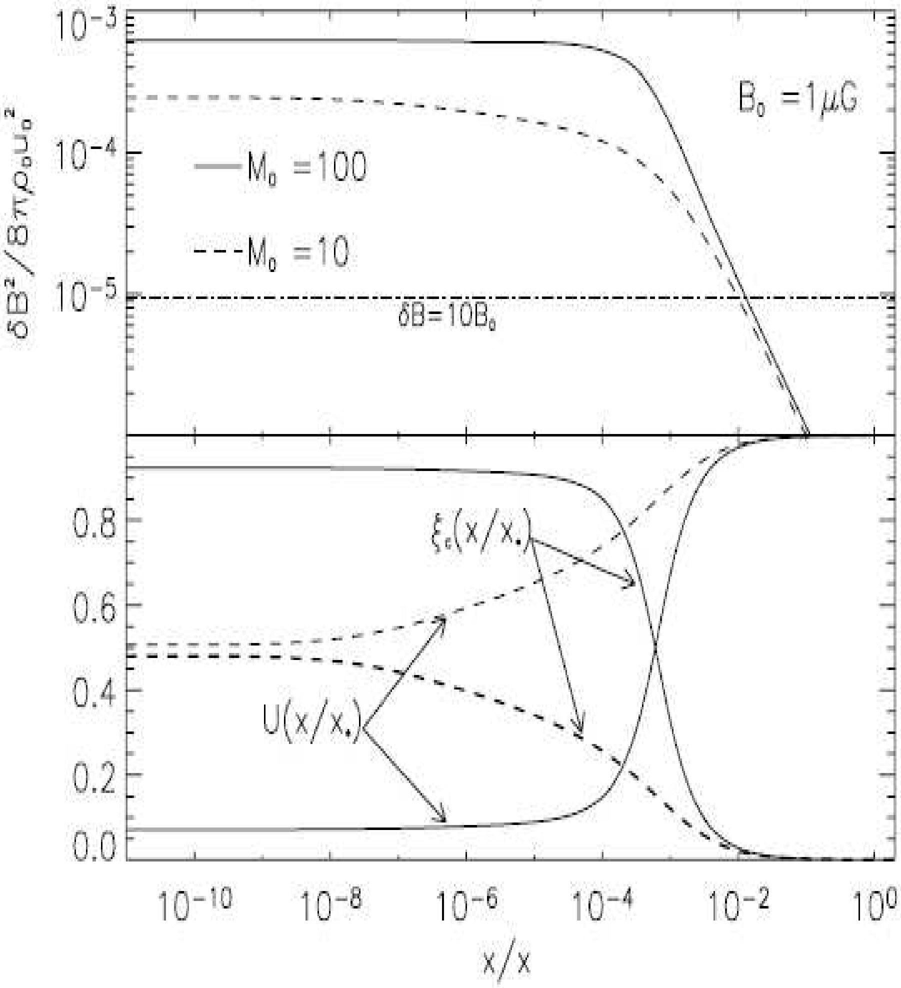
$$\boxed{\sigma = \frac{P_C(x)-P_C(x-dx)}{dx/v_A} = \frac{4\pi}{3} v_A c \left[p^4 \frac{\partial f}{\partial x} \right]_{p_{res}(k)} \frac{1}{U_M}}$$

INTEGRATING IN K:

$$\frac{dF_w(x)}{\partial x} = u(x)\frac{dp_w(x)}{\partial x} + v_A \frac{dp_c(x)}{dx} \qquad F_w(x) \simeq 3u(x)p_w(x).$$

TYPICALLY: $\delta B=[10^{-2}\ \rho u^2]^{1/2}$ = 30-50 μG UPSTREAM
And further compressed to about 100 μG downstream





Amato&PB 2006

SOME POINTS TO MAKE

- THE IMPLICATIONS FOR THE ORIGIN OF COSMIC RAYS ARE EVIDENT: LARGER MAX MOMENTA BECOME POSSIBLE (see below)
- THE PREDICTION OF LARGE TURBULENT FIELDS AT THE SHOCK HAS VERY IMPORTANT OBSERVATIONAL CONSEQUENCES ON RADIATION FROM THE ACCELERATORS (see below)
- BUT A LOT OF CARE SHOULD BE USED TOO: REMEMBER THAT WE OBTAINED A LARGE FIELD BY ASSUMING A PERTURBATIVE APPROACH…WHAT DO REALITY AND SIMULATIONS TELL US?

IMPLICATIONS FOR THE ORIGIN OF GALACTIC COSMIC RAYS

WE ARE NOW IN THE UNCHARTED WATERS OF $\delta B/B>>1$. WE DO NOT KNOW HOW TO CALCULATE D(E) IN THESE CONDITIONS. BUT REMEMBER WHAT WE OBTAINED:

$$D(E) \approx \mathrm{v}^2\,\tau = \frac{\mathrm{v}^2}{\Omega\,\mathrm{G}(\mathrm{k}_{\mathrm{res}})} \approx r_L(E)c\,\frac{1}{G(k(E))}$$

IF G=1 FOR ALL K's then

$$D(E) \approx r_L(E)c$$

BOHM DIFFUSION

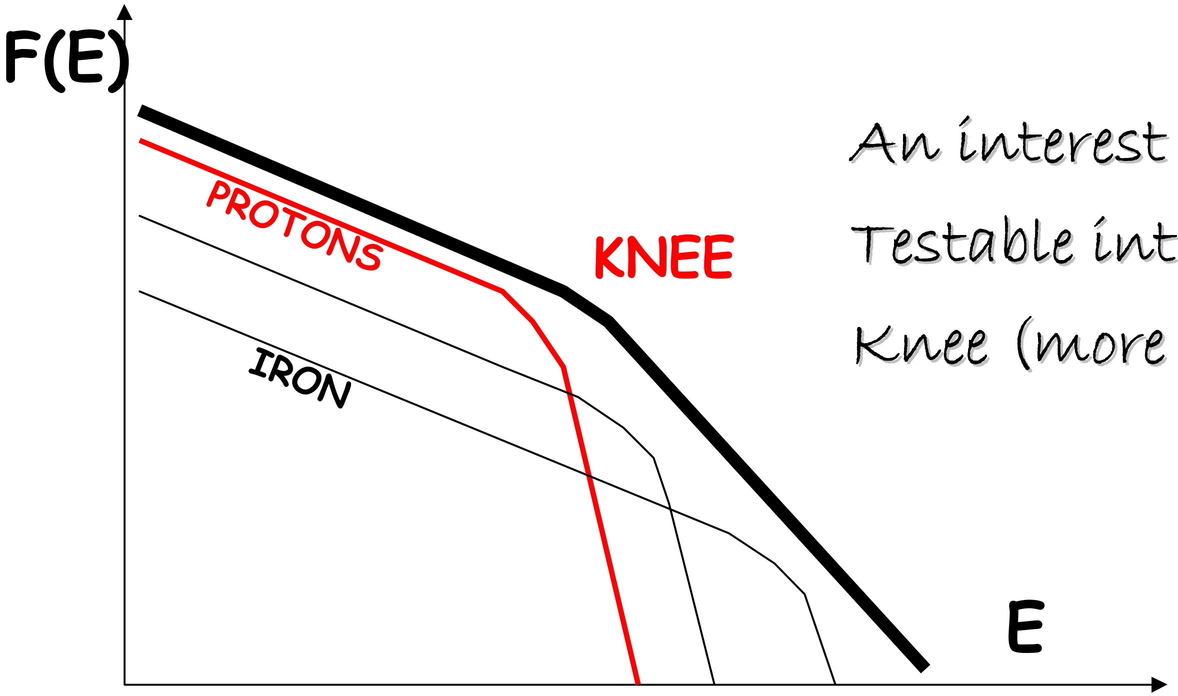
$$\sigma = \frac{P_C(x)-P_C(x-dx)}{dx/v_A} = \frac{4\pi}{3} v_A c \left[p^4 \frac{\partial f}{\partial x}\right]_{p_{res}(k)} \frac{1}{U_M}$$

INDEPENDENT OF K IF $f\sim p^{-4}$

IF WE ASSUME BOHM DIFFUSION IN THE SELF-GENERATED MAGNETIC FIELD:

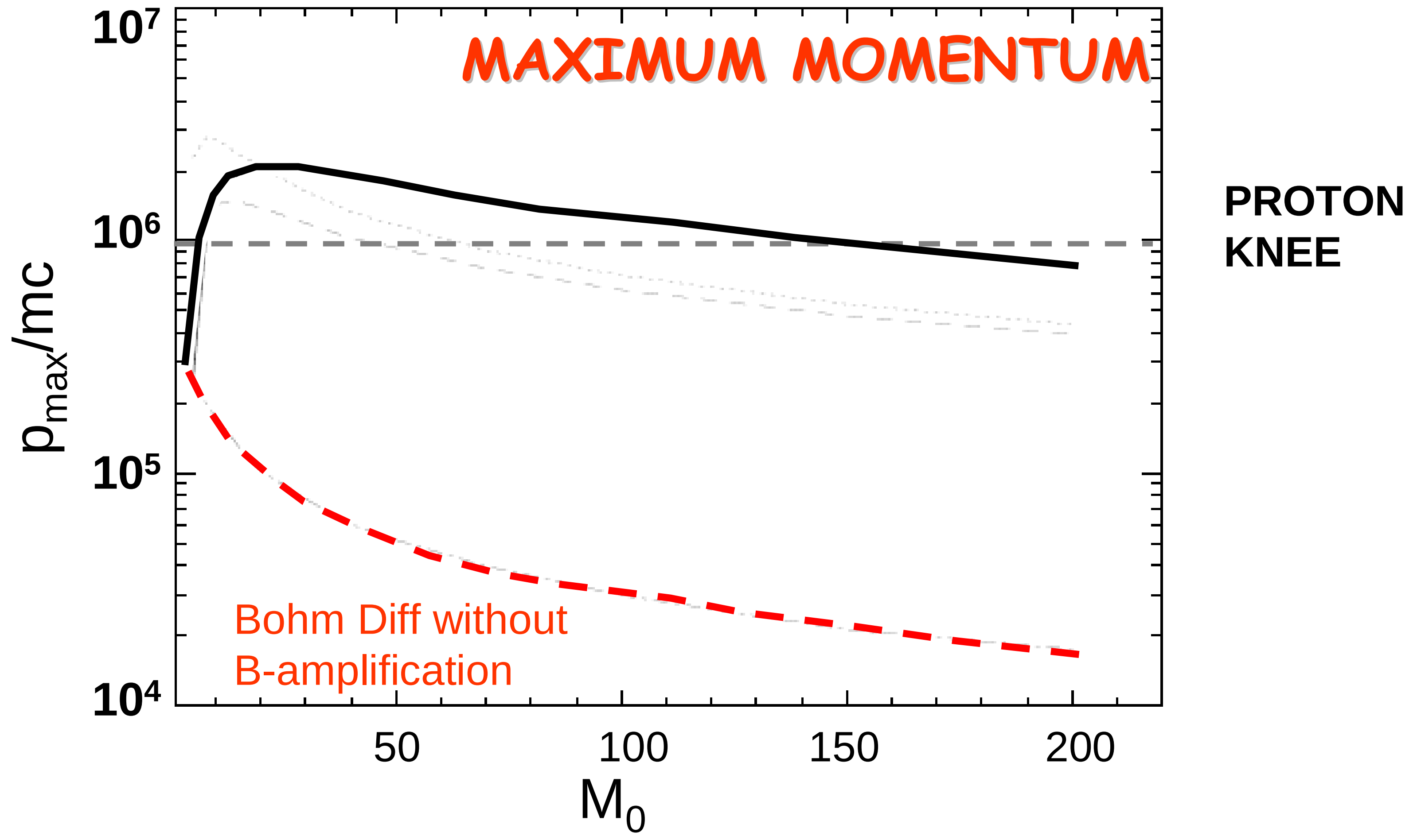
$$\boxed{E_{MAX} \approx 3\times10^{15}\, Z\, u_{3000}^{2}\ \tau_{1000}\ B_{100}\ \ \mathrm{eV}}$$

WHIGH IS TANTALIZINGLY CLOSE TO THE KNEE REGION, AND EVEN HIGHER FOR HEAVIER (HIGHER Z) NUCLEI



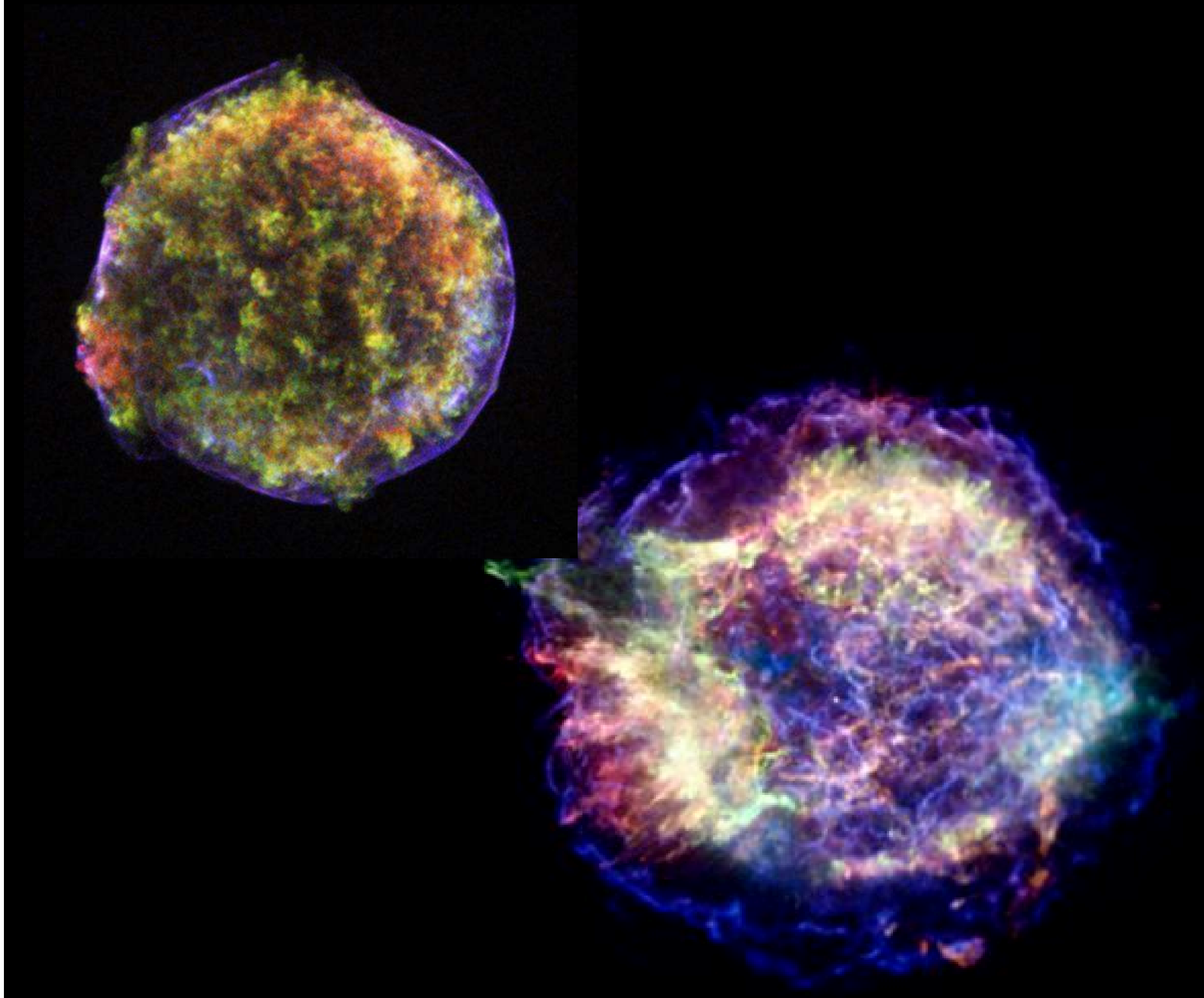
An interest new possible and Testable interpretation of the Knee (more later!)

Implications of B-field amplification



PB, Amato & Caprioli, 2007

HOW DOES THIS GENERAL THEORY CONFRONT NATURE?



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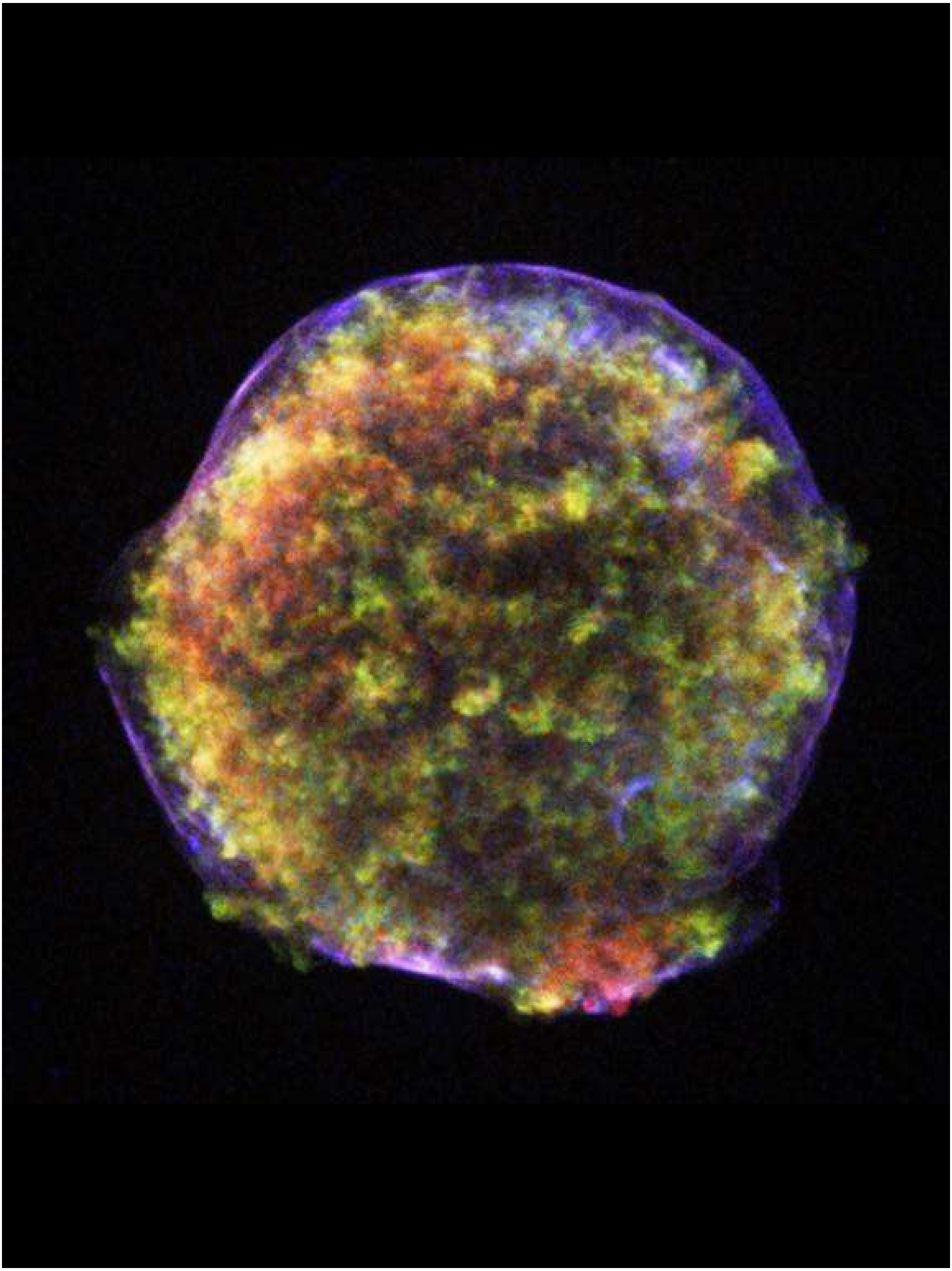
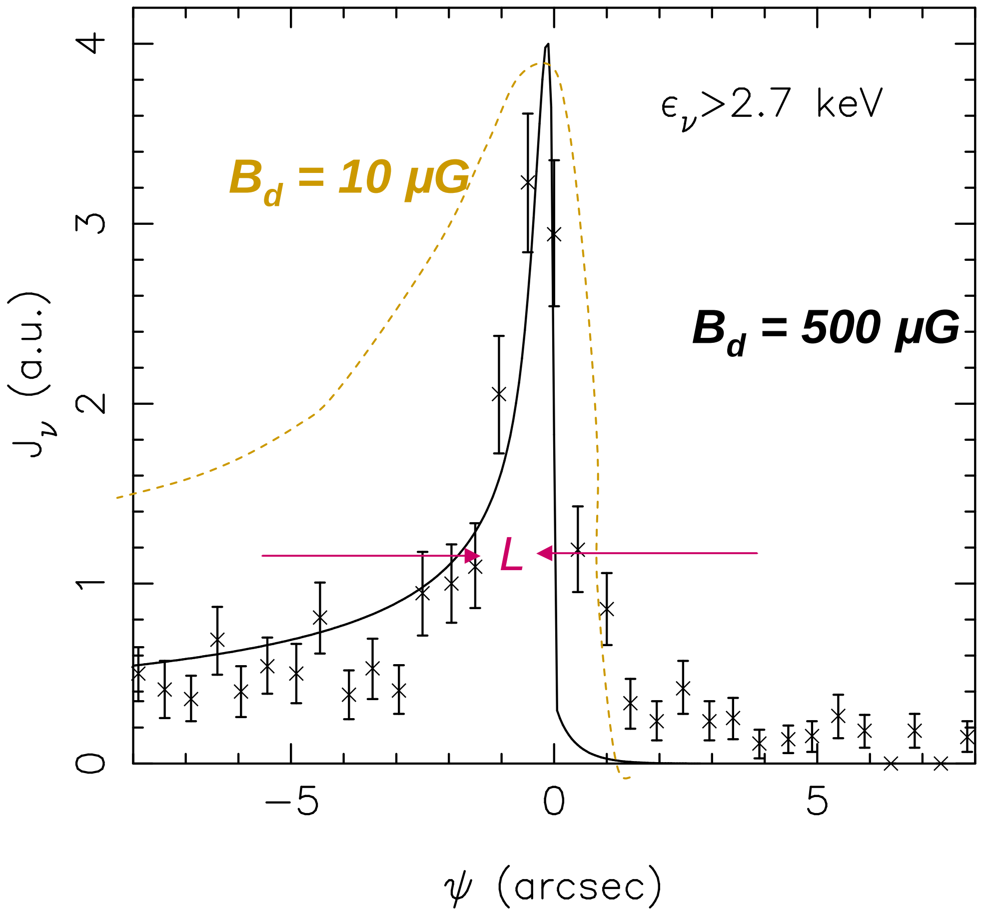
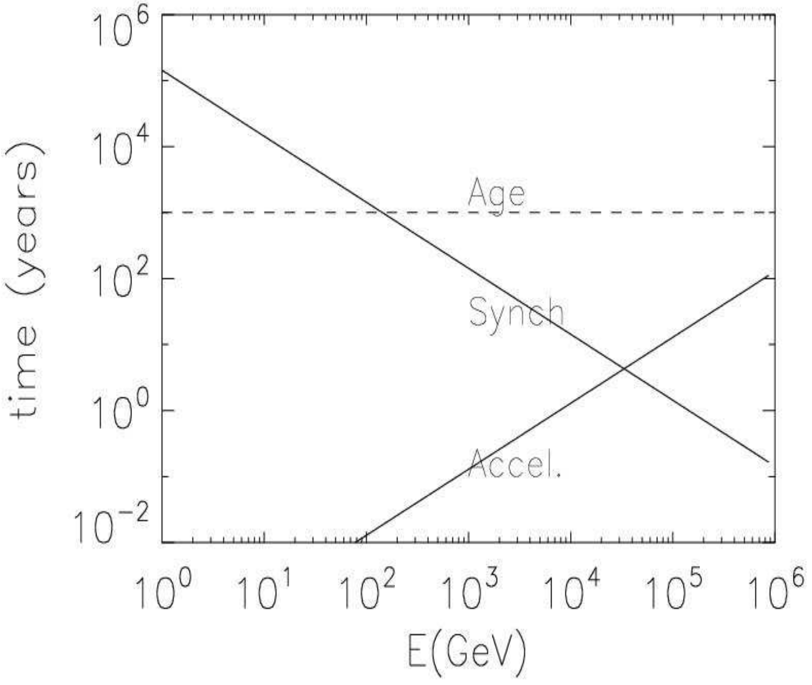






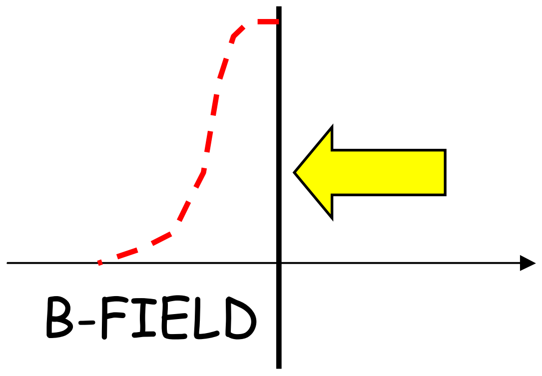
IN THE DOWNSTREAM PLASMA THE THICKNESS OF THE RIM IS:

$$\Delta L \approx \mathrm{Min}\left\{u_2\tau_{loss}, \sqrt{4D(E)\tau_{loss}(E)}\right\}$$

AND FOR BOHM DIFFUSION THE EMISSION ALWAYS ENDS UP IN THE X-RAY BAND

AN ALTERNATIVE EXPLANATION: THE IMPORTANCE OF DAMPING

THE BRIGHT RIMS COULD BE PRODUCED IF THE MAGNETIC FIELD DOWNSTREAM WERE DAMPED



IT IS NOT CLEAR WHAT IS THE CORRECT INTERPRETATION, BUT THERE IS A SIMPLE DIAGNOSTIC:

IF THE FIELD IS DAMPED DOWNSTREAM THE FILAMENTS SHOULD APPEAR IN THE RADIO AS WELL.

AT THE PRESENT TIME THERE DOES NOT SEEM TO BE EVIDENCE OF FILAMENTS IN THE RADIO BAND

DYNAMICAL REACTION OF THE B-FIELD

BEFORE MOVING ON TO OTHER PHENOMENOLOGICAL TESTS OF THE THEORY, WE NEED TO ASSESS ONE LAST POINT, THE DYNAMICAL REACTION OF THE LARGE FIELD ON THE SHOCK

RECALL THAT WE HAVE SHOWED THAT

$$\frac{P_W(x)}{\rho_0 u_0^2} = \alpha(x) = \frac{1 - U(x)^2}{4 M_A(x) U(x)} << 1$$

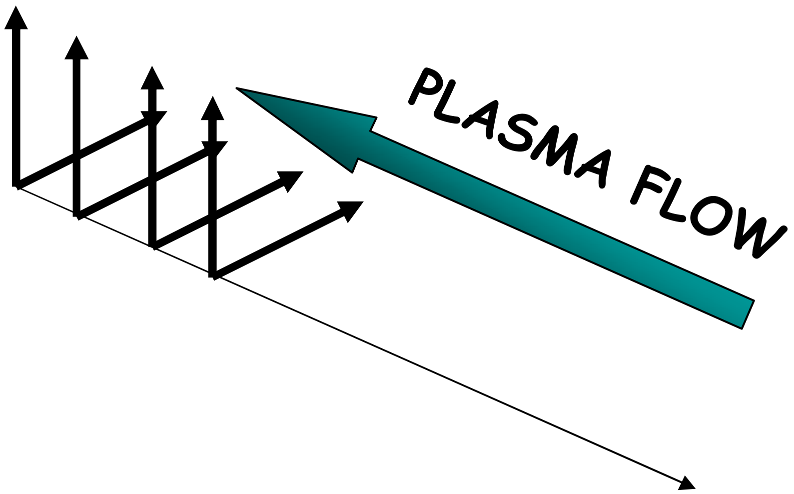
BUT THE PRESSURE TERM IS:

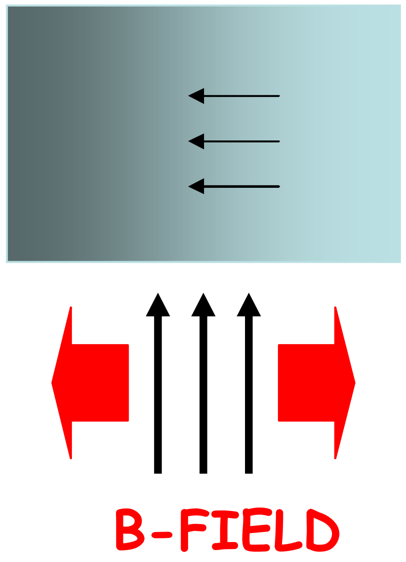
$$\frac{P_{gas}(x)}{\rho_0 u_0^2} = \frac{U(x)^{-\gamma}}{\gamma M_0^2}$$

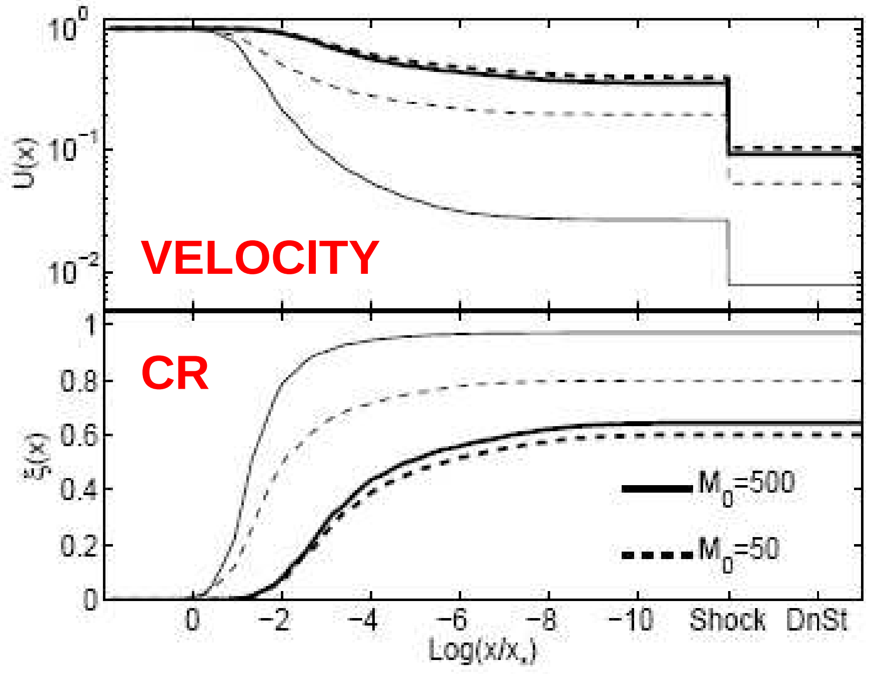
DYNAMICAL REACTION OF THE B-FIELD

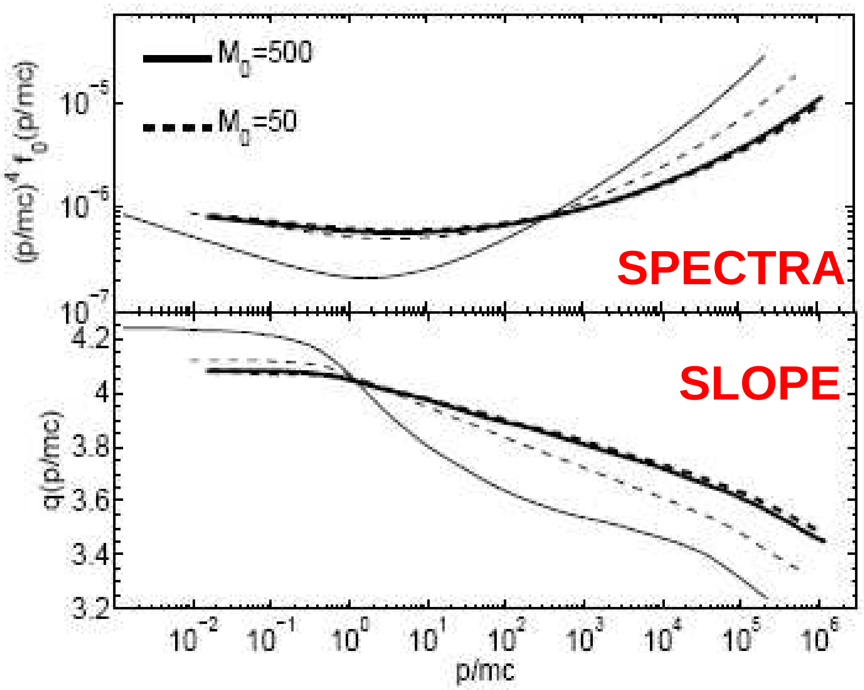
IT FOLLOWS THAT THE MAGNETIC TERM, THOUGH SMALL, BECOMES COMPARABLE WITH THE PRESSURE TERM WHEN:

$$M_{A,0} \approx M_0^2 \qquad v \approx 3800\, T_8\, n^{1/2}\, B_\mu^{-1} \quad \text{km/s}$$





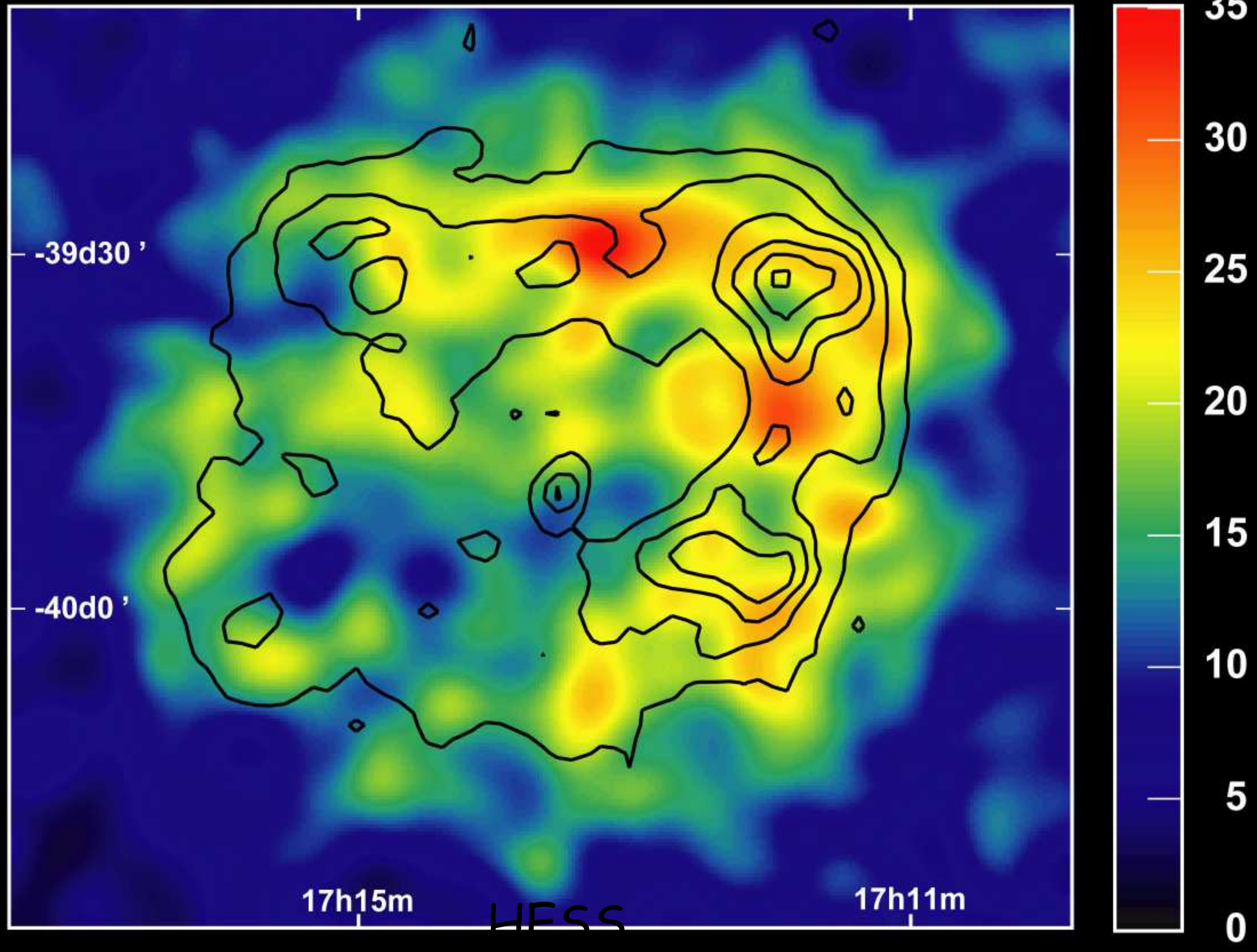




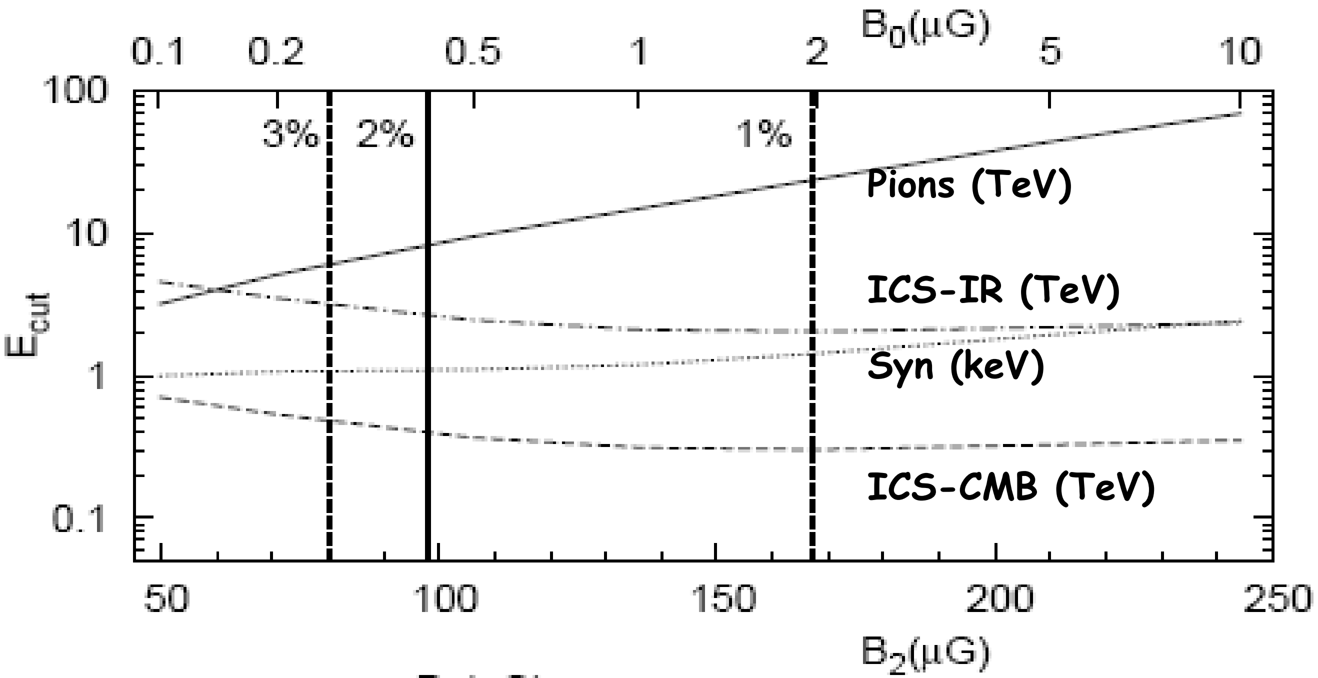
T_0(K)	Λ_B	ξ_1	$p_{max}(10^6 GeV)$	R_{sub}	R_{tot}	S_{sub}	S_{tot}	$B_2(\mu G)$	$T_2(10^6$ K)
10^4	No	0.97	0.22	3.38	125.1	3.43	128.6	679.4	0.67
10^4	Yes	0.64	1.19	3.75	10.6	3.76	10.7	525.0	87.7
10^6	No	0.80	0.53	3.67	18.6	3.69	18.7	236.7	33.1
10^6	Yes	0.60	1.17	3.76	9.52	3.77	9.57	475.6	114.6

Caprioli, PB, Amato & Vietri 2008

THE CASE OF RX-J1713



Streaming Instability ON



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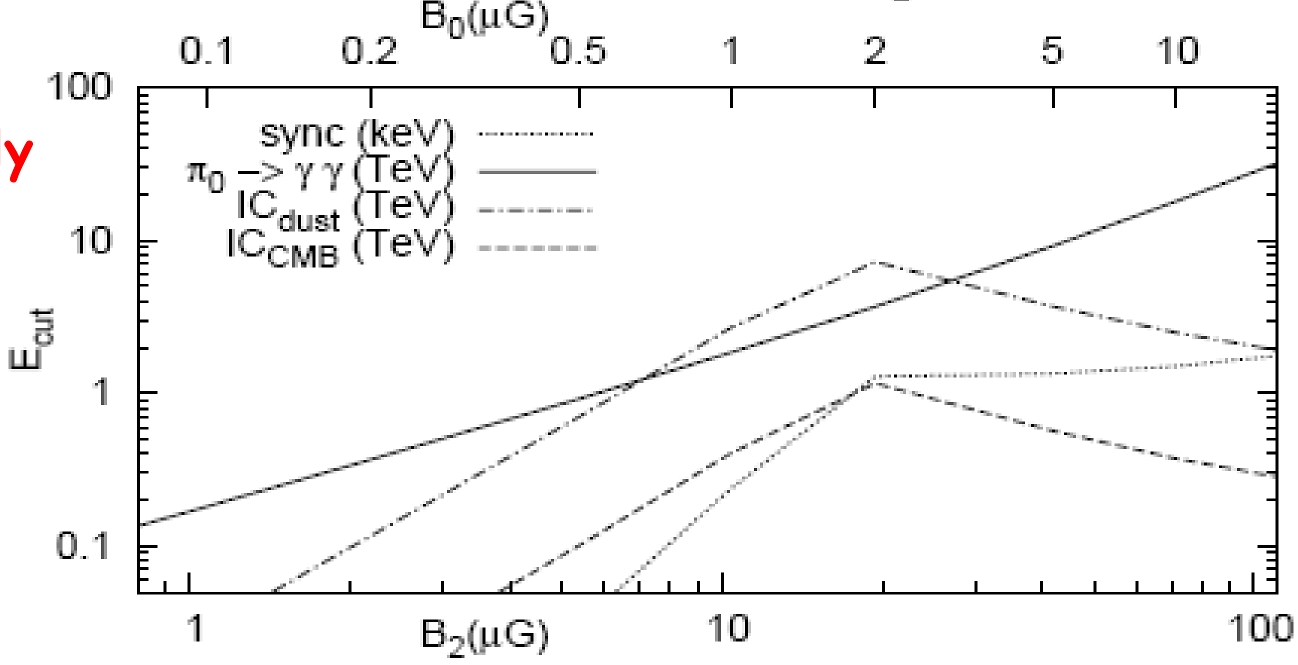
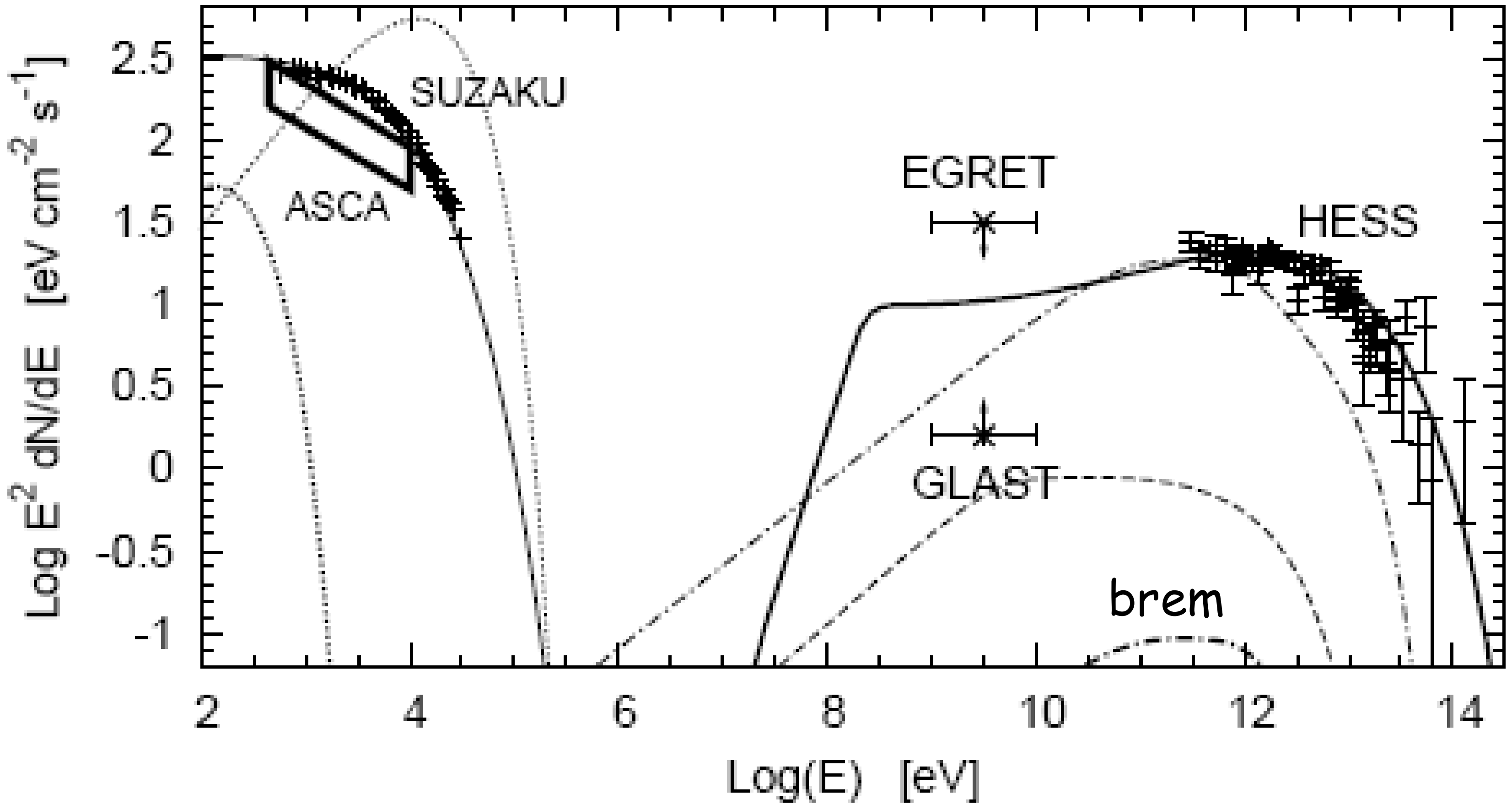


Morlino, PB & Amato 2008

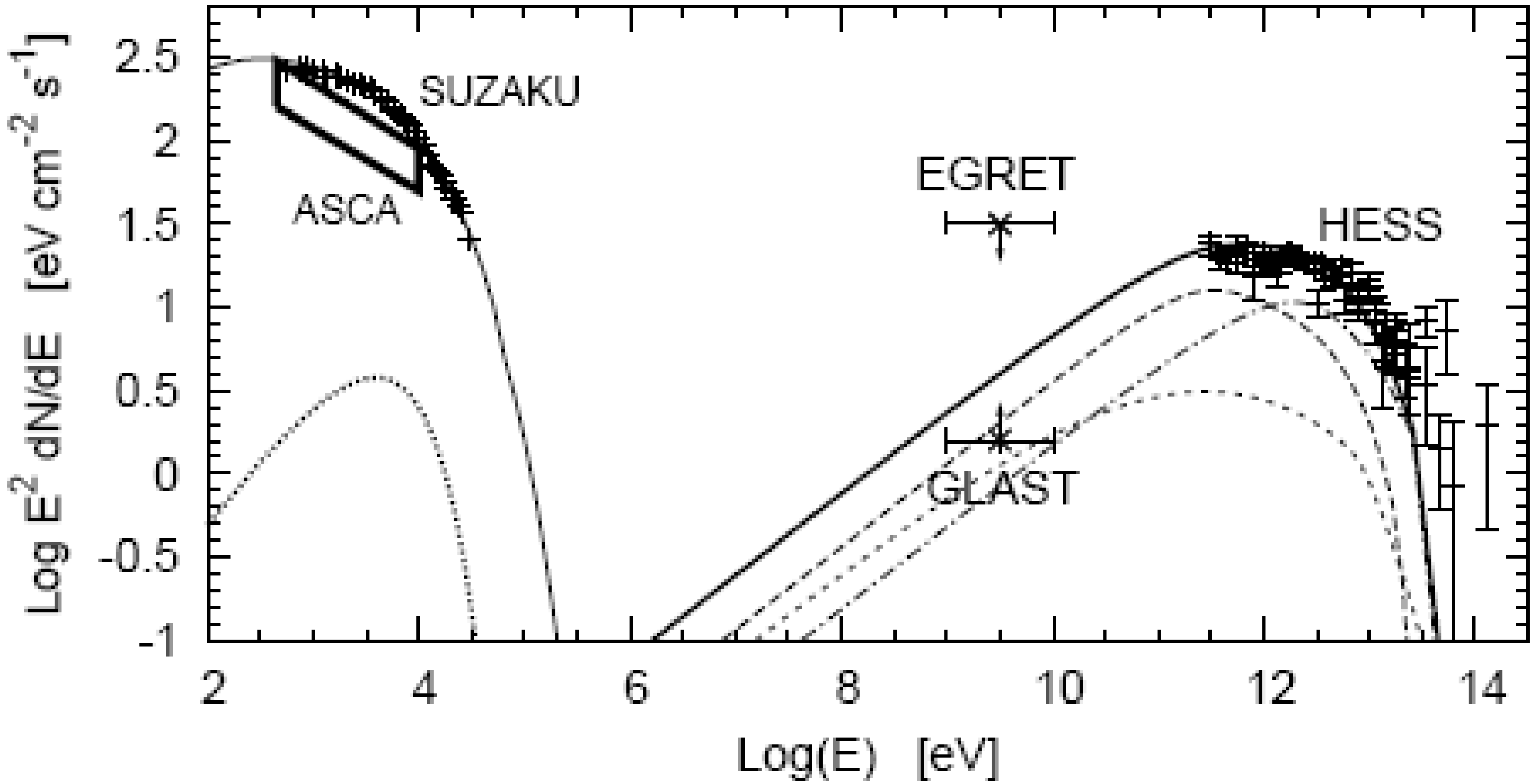
RXJ1713 **Morlino, PB & Amato 2008**

HADRONIC FIT – LARGE B FIELDS



PROBLEMS: 1. LARGE THERMAL X-RAYS (BUT…)
2. VERY LOW RATIO OF ELECTRONS AND PROTONS

LEPTONIC FIT – LOW B FIELDS



PROBLEMS: 1. VERY HIGH PHOTON DENSITY FOR ICS

2. LOW B FIELDS (IGNORES X-RAY OBSERVATIONS)
3. BAD FIT TO HIGHEST-E HESS DATA POINTS

WHICH ACCELERATED PARTICLES BECOME COSMIC RAYS?

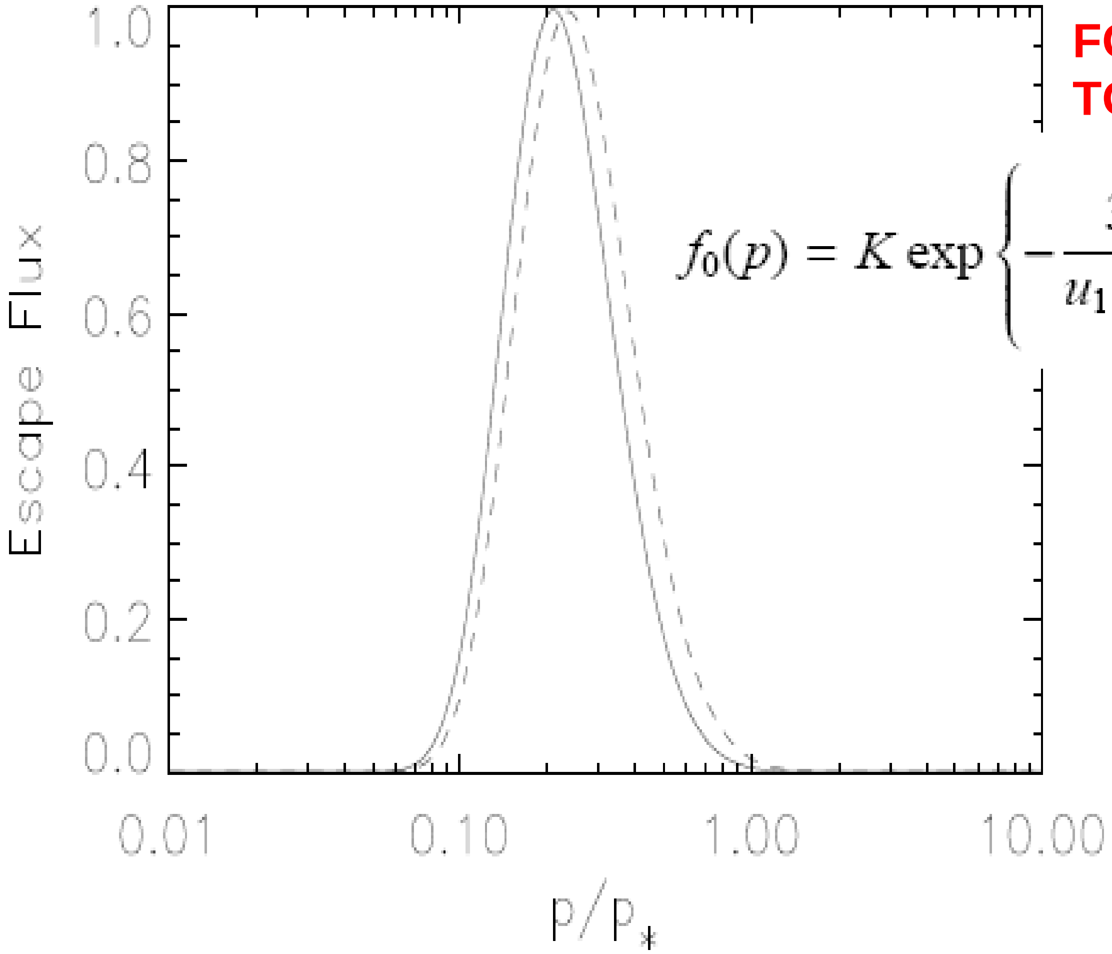
WE HAVE SEEN THAT COSMIC RAYS ACCELERATED IN SNR HAVE CONCAVE SPECTRA

WE HAVE ALSO SEEN THAT THE RETURN PROBABILITY FROM UPSTREAM IS UNITY! ALL PARTICLES COME BACK, WITH THE POSSIBLE EXCEPTION OF THOSE AT P_{MAX}

WE HAVE SEEN THAT THE SPECTRUM OF THE ESCAPING PARTICLES AT UPSTREAM INFINITY IS CLOSE TO A DELTA FUNCTION

SO…WHICH ONES ARE THE COSMIC RAYS?

ESCAPE FLUX IN TEST PARTICLE THEORY



FOR D(E) PROPORTIONAL
TO E (BOHM DIFFUSION):

$$f_0(p) = K \exp\left\{-\frac{3u_1}{u_1 - u_2}\int_{p_{inj}}^{p}\frac{dp'}{p'}\frac{1}{1-\exp\left(-\frac{p_*}{p'}\right)}\right\}$$

$$p_{MAX} = \frac{r-1}{3r}\,p_*$$

DIFFERENT PHASES OF A SNR

THERE IS AN INITIAL PERIOD DURING WHICH THE SHELL OF THE SN EXPANDS FREELY (FREE EXPANSION PHASE -BALLISTIC MOTION):

MASS OF THE EJECTA: M_{ej}

TOTAL KINETIC ENERGY: E_{51}

FREE EXPANSION VELOCITY:

$$V_s = \sqrt{\frac{2E_{ej}}{M_{ej}}} = 10^9\ E_{51}^{1/2}\ M_{ej,\Theta}^{-1/2}\ \text{cm/s}$$

BUT THE SHOCK SWEEPS THE MATERIAL IN FRONT OF IT AND AT SOME POINT IT ACCUMULATES ENOUGH MATERIAL TO SLOW DOWN THE EXPANDING SHELL:

SEDOV PHASE: $T_{Sedov} = 300\, E_{51}^{-1/2}\, n^{-1/3}\, M_{\Theta}^{5/6}$ years

$$R_{sh}(t) = 2.7 \times 10^{19}\mathrm{cm}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{2/5}$$

$$V_{sh}(t) = 4.7 \times 10^{8}\mathrm{cm/s}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{-3/5}$$

The sound speed in the ISM is about 10^6 cm/s

↓

Mach number ≈ 100 - 1000

STRONG SHOCK

MAX ENERGY DURING SEDOV

THE MAXIMUM ENERGY OF ACCELERATED PARTICLES INCREASES DURING THE FRE EXPANSION PHASE AND REACHES A MAXIMUM AT THE BEGINNING OF THE SEDOV PHASE.

IN THE SEDOV PHASE:

$$\delta B(t) = 65\, n_1^{1/4}\, B_{0,\mu G}^{1/2} \left(\frac{E_{51}}{n_1}\right)^{1/10} t_{kyr}^{-3/10}\, \xi_c(t)^{1/2}\, \mu G$$

$$E_{max}(t) = 2.5 \times 10^6 \left(\frac{E_{51}}{n_1}\right)^{1/2} n_1^{1/4} B_{0,\mu G}^{1/2}\, \xi_c(t)^{1/2}\, t_{kyr}^{-1/2}\, GeV$$

OVERLAP OF ESCAPE FLUXES: A SIMPLE ESTIMATE

$$E_{MAX}(t) \propto \xi_c(t)\, t^{-1/2}$$

$$R_{sh}(t) = 2.7 \times 10^{19}\mathrm{cm}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{2/5}$$

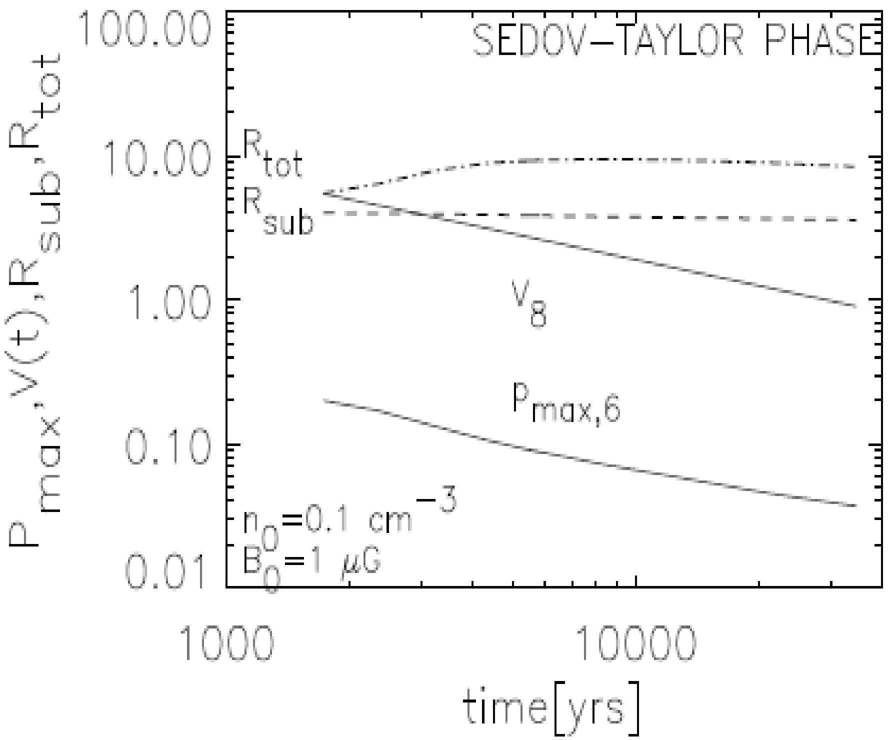
$$V_{sh}(t) = 4.7 \times 10^{8}\mathrm{cm/s}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{-3/5}$$

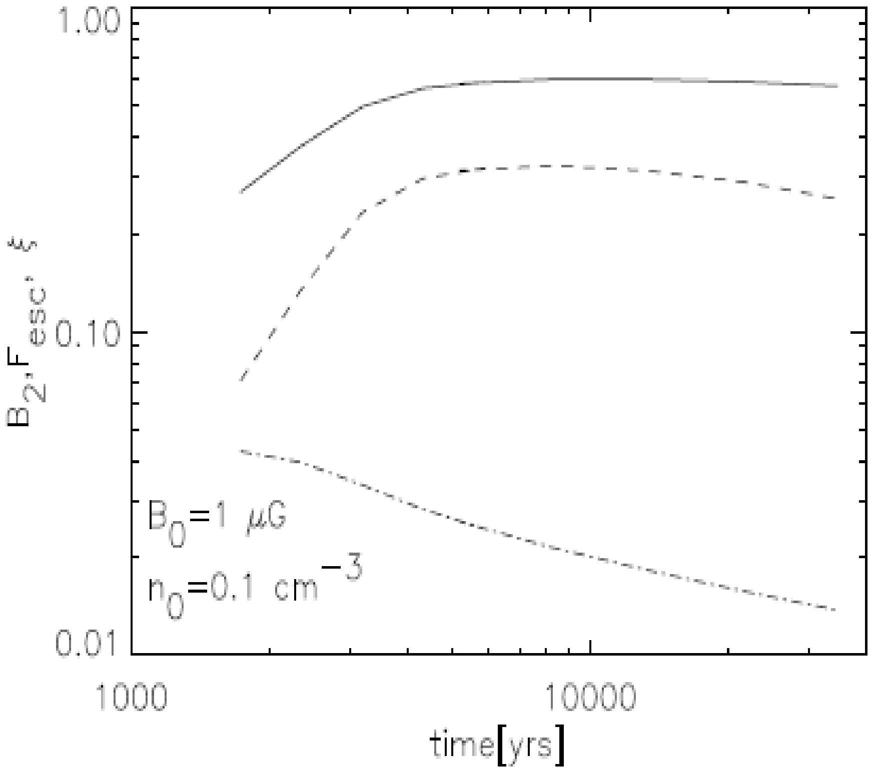
$$\boxed{\mathrm{EQ(E)dE} \approx \mathrm{F_{esc}(t)}\ \frac{1}{2}\rho \mathrm{V_s^3}\ 4\pi\pi_{\mathrm{sh}}^{2}\ \frac{\mathrm{dE_{max}}}{\mathrm{dt}}\mathrm{dE} \propto\ \mathrm{t^{1/2}\,dE} \propto\ \mathrm{E^{-1}\,dE}}$$

BE VERY CAREFUL…THIS IS JUST A WAY TO SHOW HOW YOU GET ROUGHLY A POWER LAW BUT SUMMING NON-POWER LAWS.

MORE DETAILED CALC'S SHOW DEPARTURES FROM THIS SIMPLE TREND

ESCAPE FLUX IN NON LINEAR REGIME





Caprioli, PB & Amato (2008)

A LOT OF PHYSICS IN THESE PLOTS !!!

RECALL THAT OUR IGNORANCE OF HOW THINGS EVOLVE WHEN THE MAGNETIC FIELD BECOMES AMPLIFIED TO NON LINEAR LEVELS IS HUGE:

☐ WE DO NOT KNOW HOW B GROWS WHEN $\Delta B/B>1$

☐ WE DO NOT KNOW IF IT GROWS ANISOTROPICALLY

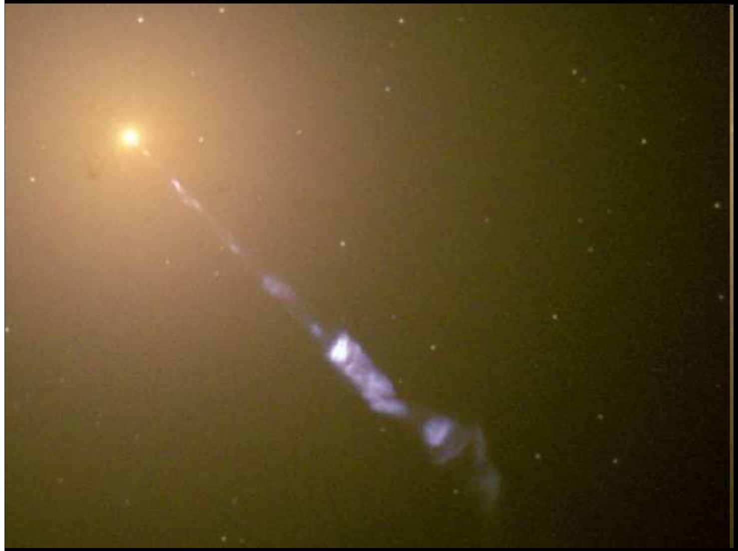
☐ WE DO NOT KNOW HOW TO EXTRACT A DIFFUSION COEFFICIENT FROM IT (PARTICLE-WAVE INTERACTION)

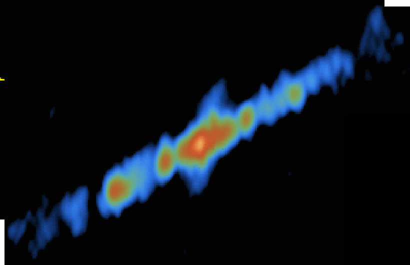
☐ EVEN THE TRANSPORT EQUATION ITSELF COULD BE CHANGED IN THE REGIME OF STRONG TURBULENCE

THIS IGNORANCE REFLECTS IN MANY OF OUR FINDINGS WHICH ARE STRONGLY AFFECTED BY ASSUMING BOHM DIFFUSION…

THEREFORE THERE IS A LOT FOR YOU TO DISCOVER!!!

SOME BASIC ASPECTS OF PARTICLE ACCELERATION AT RELATIVISTIC SHOCKS





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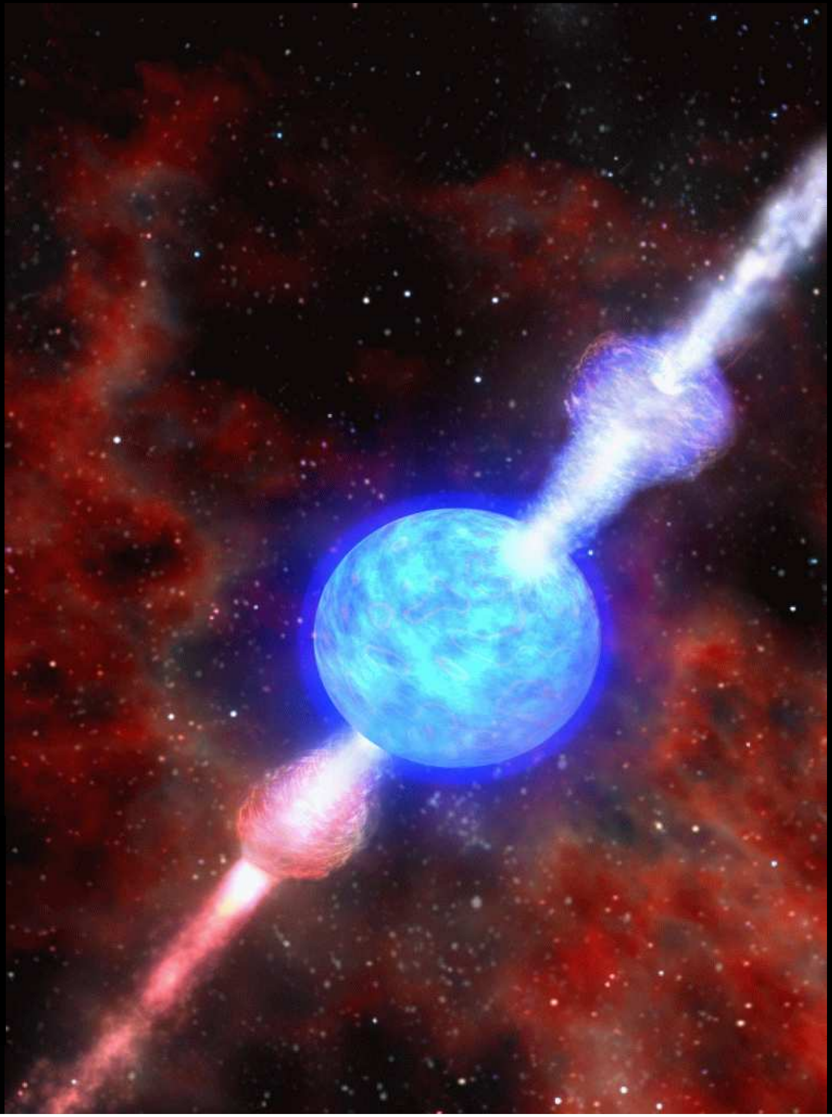


BASICS OF ACCELERATION AT RELATIVISTIC SHOCKS

$$\gamma_1 \beta_1 n_1 = \gamma_2 \beta_2 n_2$$

$$\gamma_1^2 \beta_1 (\varepsilon_1 + p_1) = \gamma_2^2 \beta_2 (\varepsilon_2 + p_2)$$

$$\gamma_1^2 \beta_1^2 (\varepsilon_1 + p_1) + p_1 = \gamma_2^2 \beta_2^2 (\varepsilon_2 + p_2) + p_2$$

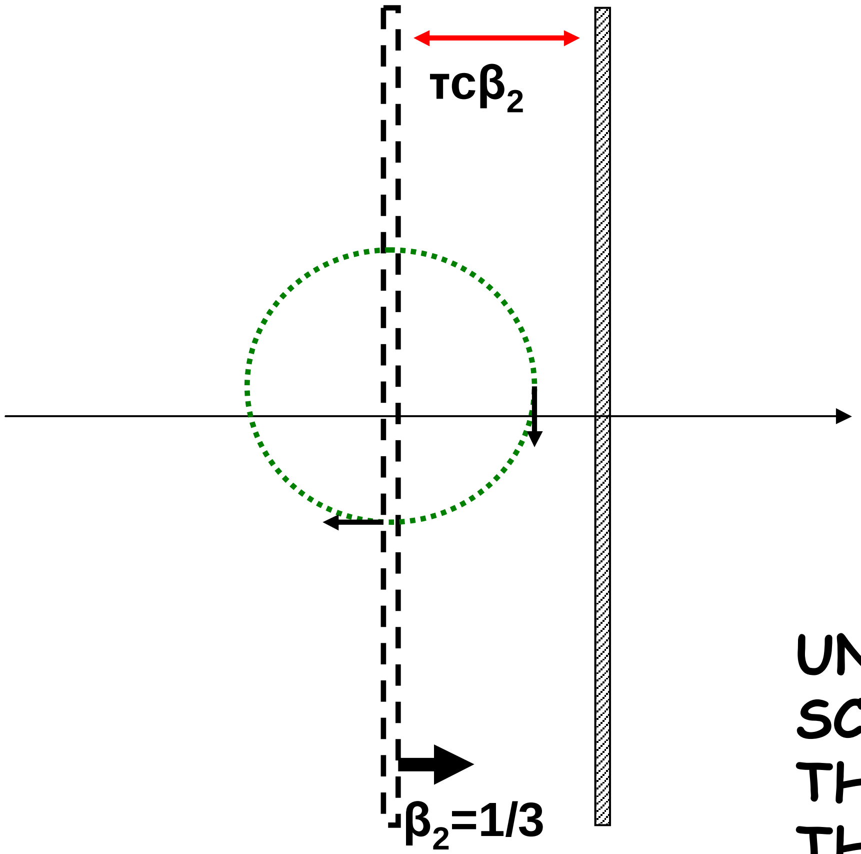
IN THE ASSUMPTION THAT:

$$\boxed{\frac{B_1^2}{4\pi} << (\varepsilon_1 + p_1)} \quad \boxed{\gamma_1 >> 1} \quad \boxed{p_1 = 0}$$

No equipartition   ultrarelativistic   pressureless

WE FIND THAT:

$$p_2 = \frac{1}{3}\varepsilon_2 \qquad \beta_2 = \frac{1}{3}$$



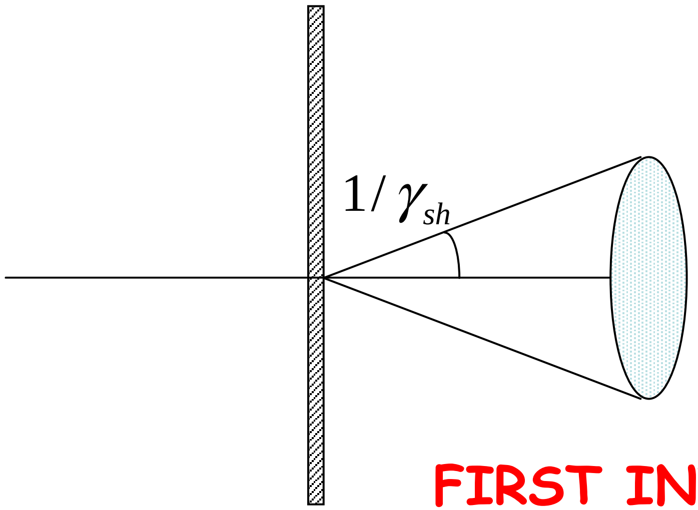
$$\tau \approx \frac{3}{4}\frac{2\pi\, r_L}{c}$$

$$\Delta x = \frac{1}{3}c\,\tau = \frac{\pi}{2} r_L > r_L$$

UNLESS THERE IS STRONG SCATTERING DOWSNTREAM THE PARTICLES ARE TRAPPED THERE

THE RETURN PROBABILITY FROM DOWNSTREAM IS EXPECTED TO BE SMALLER THAN FOR NEWTONIAN SHOCKS: STEEPER SPECTRA

ANISOTROPY



$$\delta\mu = \left[-1 + \frac{1+3\beta_{\mathrm{rel}}}{3+\beta_{\mathrm{rel}}}\right] \approx \frac{1}{4}\frac{1}{\gamma_{\mathrm{rel}}^2}$$

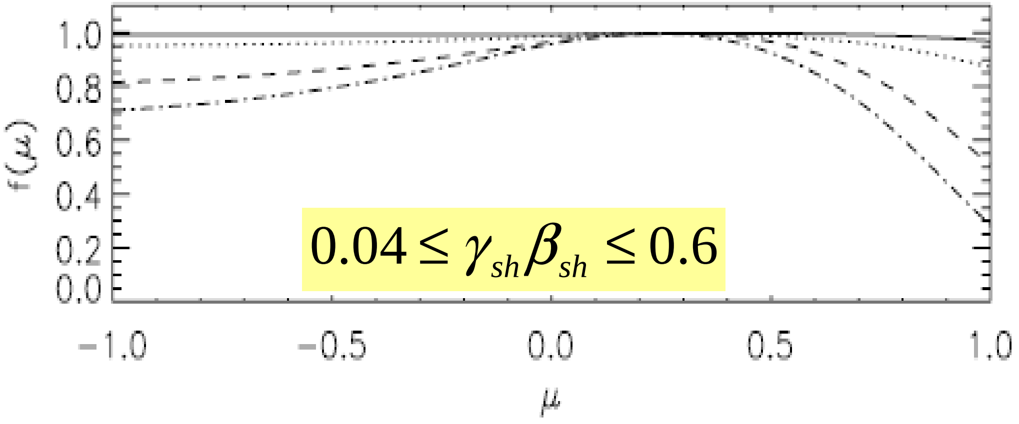
FIRST INTERACTION:

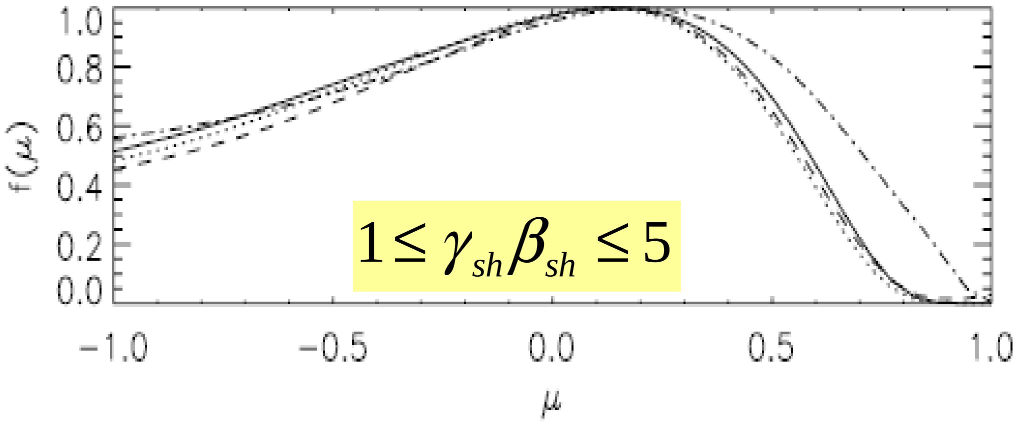
$$E_i \Rightarrow E_d = \gamma_{rel} E_i (1+\beta_{rel}) \Rightarrow E_f = \gamma_{rel}^2 E_i (1+\beta_{rel})^2 \approx 4\gamma_{rel}^2 E_i$$

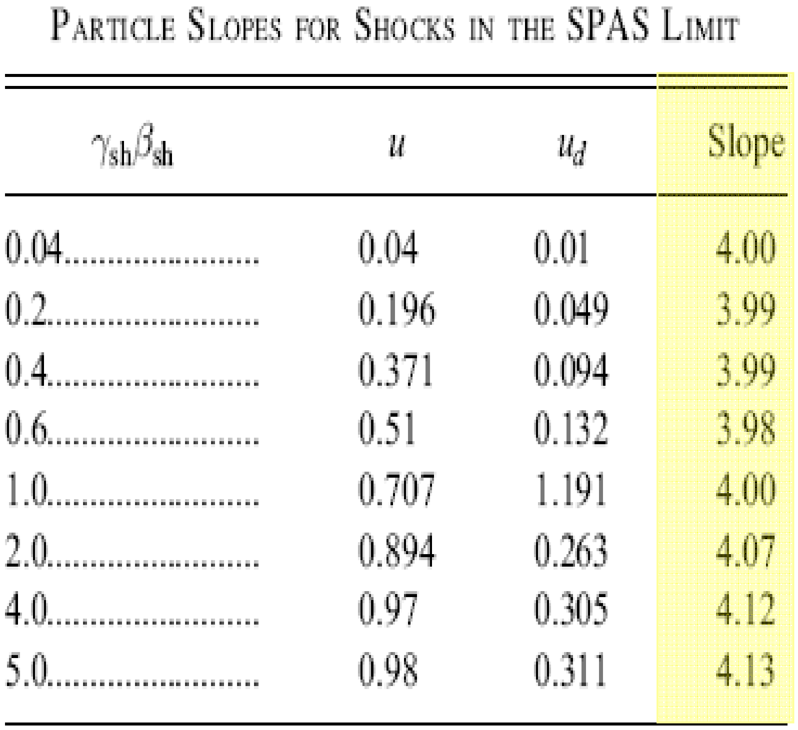
FURTHER INTERACTIONS:

$$E_f \approx 2\ E_i$$

EFFECTS OF ANISOTROPY







PARTICLE SLOPES FOR SHOCKS IN THE SPAS LIMIT

$\gamma_{sh}\beta_{sh}$	u	u_d	Slope
0.04	0.04	0.01	4.00
0.2	0.196	0.049	3.99
0.4	0.371	0.094	3.99
0.6	0.51	0.132	3.98
1.0	0.707	1.191	4.00
2.0	0.894	0.263	4.07
4.0	0.97	0.305	4.12
5.0	0.98	0.311	4.13

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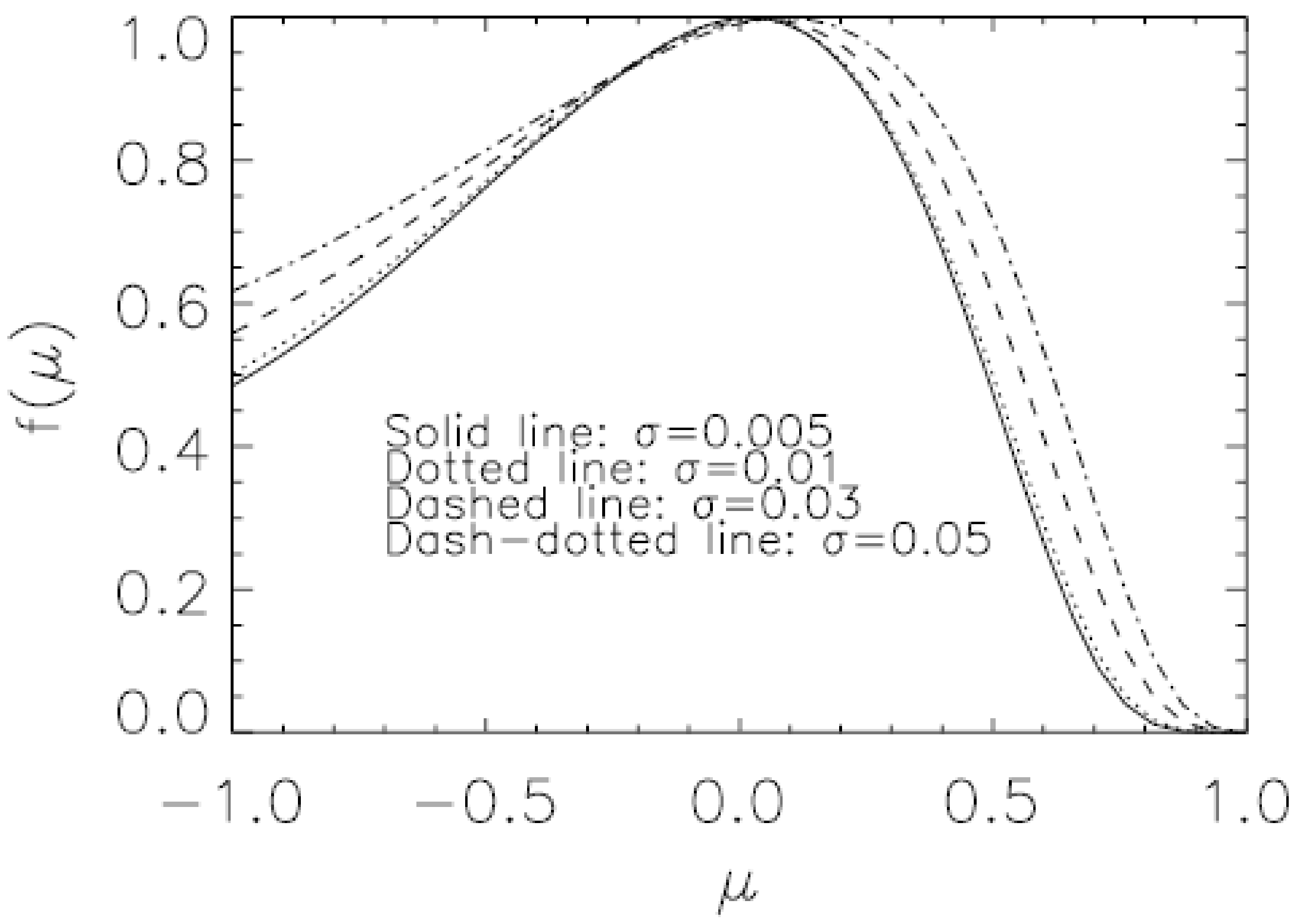
EFFECTS OF THE SPAS



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SOME REMARKS

- THE SPECTRUM OF ACCELERATED PARTICLES IN THE RELATIVISTIC CASE IS STILL A POWER LAW
- THE SLOPE OF THIS POWER LOW IN THE ULTRAREL CASE IS ABOUT 2.3
- HOWEVER THE SLOPE CAN BECOME APPRECIABLY HARDER (FLATTER SPECTRA) FOR LARGE ANGLE SCATTERING
- OR APPRECIABLY SOFTER (STEEP SPECTRA) DUE TO …BASICALLY ANYTHING ELSE YOU DO (FOR INSTANCE COMPRESSION OF TURBULENCE AT THE SHOCK)
- SHOCK ACCELERATION AT RELATIVISTIC SHOCKS DEPENDS ALSO ON THE EQUATION OF STATE OF THE DOWNSTREAM PLASMA (PROTONS, PAIRS, B-FIELDS ALL CHANGE THE RESULTS DRAMATICALLY)
- THERE IS NO NON LINEAR THEORY OF PARTICLE ACCELERATION AT RELATIVISTIC SHOCKS