
LECTURE II:
PARTICLE ACCELERATION
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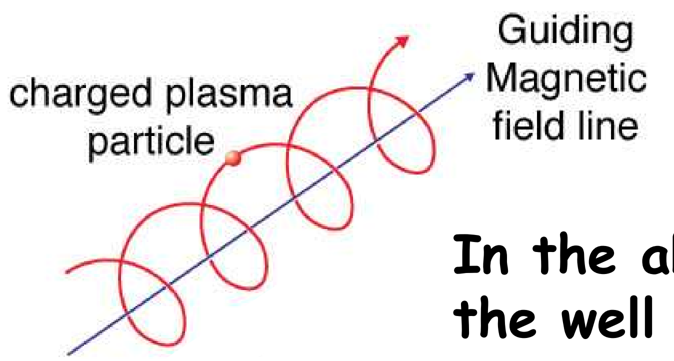
Charged Particles in a regular B-field



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

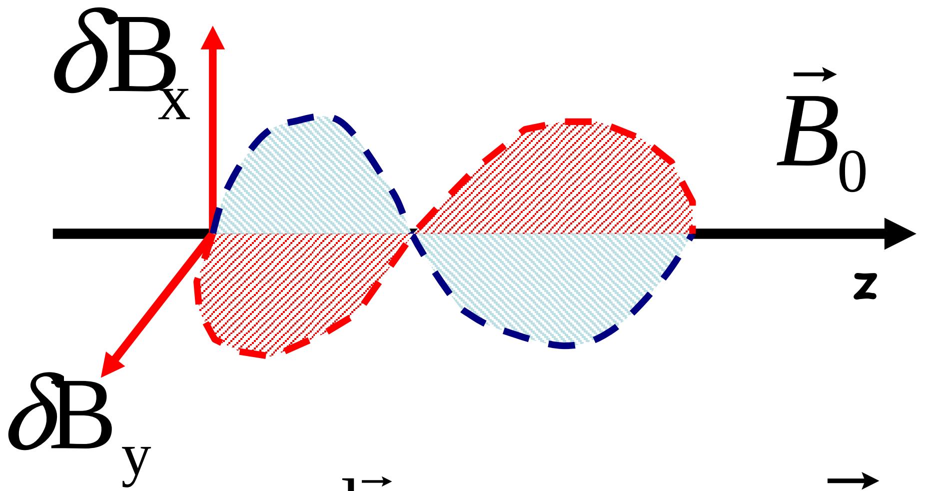
$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

LARMOR FREQUENCY

$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

Motion of a charged particle in a random magnetic field



$$\delta B << B_0$$

$$\delta\vec{B} \perp \vec{B}_0$$

$$\frac{d\vec{p}}{dt} = q\frac{\vec{v}}{c}\times(\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_z=P\mu$

SITTING IN THE REFERENCE FRAME OF THE THE WAVE, THERE IS NO ELECTRIC FIELD…AND IF THE WAVE IS SLOW COMPARED WITH THE PARTICLE (THIS IS GENERALLY THE CASE) THEN THE WAVE IS STATIONARY AND Z=vμt

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2}}{m\gamma c}\left[\cos(\Omega t)\, B_y - \sin(\Omega t)\, B_x\right]$$

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2} B_k}{m\gamma c}\left[\cos(\Omega t)\cos(kz+\psi) + \sin(\Omega t)\sin(kz+\psi)\right]$$

$$\boxed{\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2} B_k}{m\gamma c}\cos\left[(\Omega - kv\mu)t + \psi\right]}$$

RATE OF CHANGE OF THE PITCH ANGLE IN TIME

Diffusive motion

$$\boxed{\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2}B_k}{m\gamma c}\cos\left[(\Omega - kv\mu)t + \psi\right]}$$

ONE CAN TRIVIALLY SHOW THAT $\left\langle \frac{\mathrm{d}\mu}{\mathrm{d}t} \right\rangle = 0$

BUT:

$$\Delta\mu\Delta\mu = \frac{q^2(1-\mu^2)B_k^2}{m^2\gamma^2c^2}\int dt \int dt' \cos\left[(\Omega - kv\mu)t + \psi\right]\cos\left[(\Omega - kv\mu)t' + \psi\right]$$

$$\boxed{< \frac{\Delta\mu\Delta\mu}{\Delta t} >_\psi = \frac{q^2(1-\mu^2)\pi B_k^2}{m^2\gamma^2c^2}\frac{1}{v\mu}\delta(k - \frac{\Omega}{v\mu})}$$

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$< \frac{\Delta\mu\Delta\mu}{\Delta t} >= \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2}\frac{1}{v\mu}4\pi \int dk \frac{B_k^2}{4\pi}\delta(k - \frac{\Omega}{v\mu})$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\boxed{\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)k_{res}F(k_{res})}$$

$$\boxed{k_{res} = \frac{\Omega}{v\mu}}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $= G(k_{res})$

THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

PATHLENGTH FOR DIFFUSION ~ $v\tau$

$$\tau \approx \frac{1}{\Omega\, G(k_{res})} \Rightarrow \left\langle \frac{\Delta z \Delta z}{\Delta t} \right\rangle \approx v^2\, \tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.

PARTICLE SCATTERING

- EACH TIME THAT A RESONANCE OCCURS THE PARTICLE CHANGES PITCH ANGLE BY $\Delta\theta \sim \delta B/B$ WITH A RANDOM SIGN

- THE RESONANCE OCCURS ONLY FOR RIGHT HAND POLARIZED WAVES IF THE PARTICLES MOVES TO THE RIGHT (AND VICEVERSA)

- THE RESONANCE CONDITION TELLS US THAT 1) IF $k \ll 1/r_L$ PARTICLES SURF ADIABATICALLY AND 2) IF $k \gg 1/r_L$ PARTICLES HARDLY FEEL THE WAVES

WAVES? Who asked them?

WAVES MAY BE GENERATED BY DIFFERENT SOURCES (SN EXPLOSIONS FOR INSTANCE) BUT THERE IS A MORE INTERESTING AND PHYSICALLY IMPORTANT PHENOMENON:

ASSUME PARTICLES ARE DRIFTING WITH VELOCITY $V_D > V_A$

THE EFFECT OF SCATTERING IS TO ISOTROPIZE CR:

$$N_{CR} m v_D \longrightarrow N_{CR} m v_A$$

Initial momentum → **final momentum**

$$\frac{dP_{CR}}{dt} = \frac{n_{CR}\, m\, \Gamma_{CR}\, (v_D - v_A)}{\tau}$$

$$\frac{dP_W}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{v_A}$$

$$\longrightarrow \boxed{\gamma_W = \frac{n_{CR}}{n_{gas}} \Omega_{cyc} \left(\frac{v_D - v_A}{v_A} \right)}$$

LATER A MORE RIGOROUS AND GENERAL TREATMENT

Some Galactic Numbers

FOR $N_{CR}=10^{-10}$ CM^{-3}, $N_{gas}=10^{-1}$ cm^{-3} AND $B_0=1\mu G$, AND ASSUMING $V_D=2V_A$, ONE FINDS:

$V_A = 7\ 10^5$ cm/s

$\Omega_{cyc} = 10^{-2}$ s^{-1}

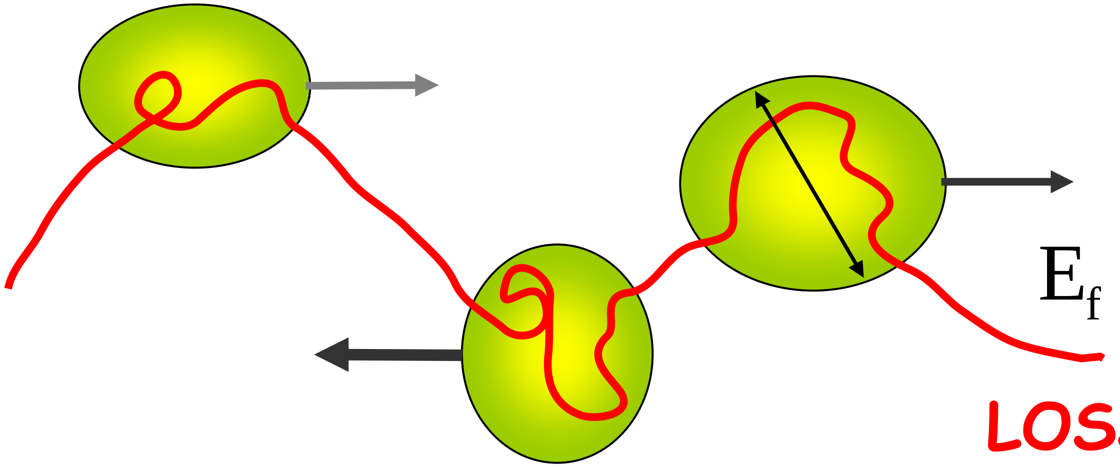
$$\boxed{\gamma_W = \frac{n_{CR}}{n_{gas}} \Omega_{cyc} \left(\frac{v_D - v_A}{v_A} \right) = 10^{-3}\ \mathrm{yr}^{-1}}$$

WAVES MAY **GROW VERY FAST**, ON TIME SCALES MUCH SHORTER THAN THE MEASURED DIFFUSION TIME SCALES…

THE GROWTH ENDS WHEN $V_D = V_A$

PARTICLE ACCELERATION

A quick look at 2nd order Fermi Acceleration (Fermi, 1949)



$$E' = \gamma E_i (1 - \beta\mu)$$

$$E_f = \gamma^2 E_i (1 - \beta\mu)(1 + \beta\mu')$$

LOSSES AND GAINS ARE PRESENT BUT DO NOT COMPENSATE EXACTLY

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu'} = 2[\gamma^2 (1 - \beta\mu) - 1]$$

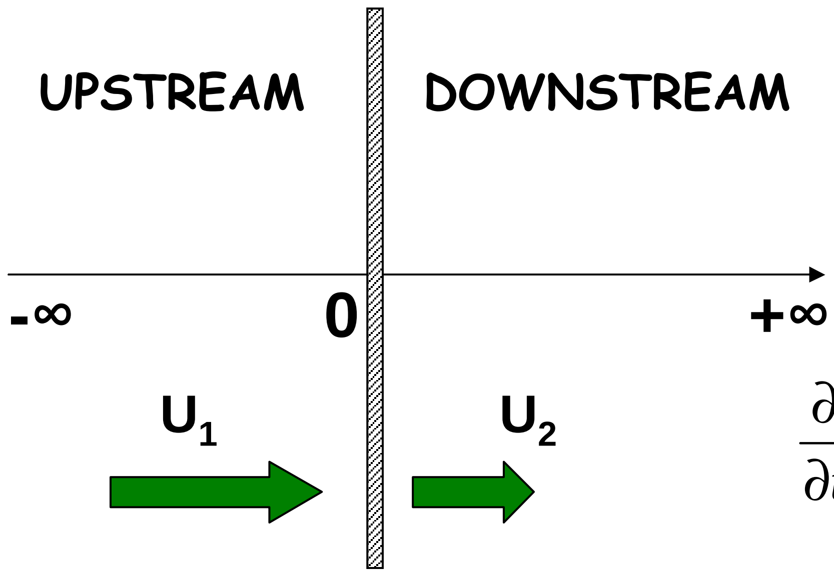
$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu} = \int_{-1}^{1} d\mu \frac{1}{2}(1 - \beta\mu)\, 2\left(\gamma^2 (1 - \beta\mu) - 1\right) \propto \beta^2$$

PROBABILITY OF ENCOUNTER

A PRIMER ON SHOCK WAVES

For **σ~10^{-25}** cm^2 and density n~1 cm^{-3} the typical interaction length is ~3 Mpc >> than the typical size of astrophysical objects and even Larger than the Galaxy!!!

COLLISIONLESS SHOCKS



$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}[\rho u]$$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x}\left[\rho u^2 + P_{gas}\right]$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_{gas}}{\gamma - 1}\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma P_{gas} u}{\gamma - 1}\right]$$

STATIONARY SHOCKS

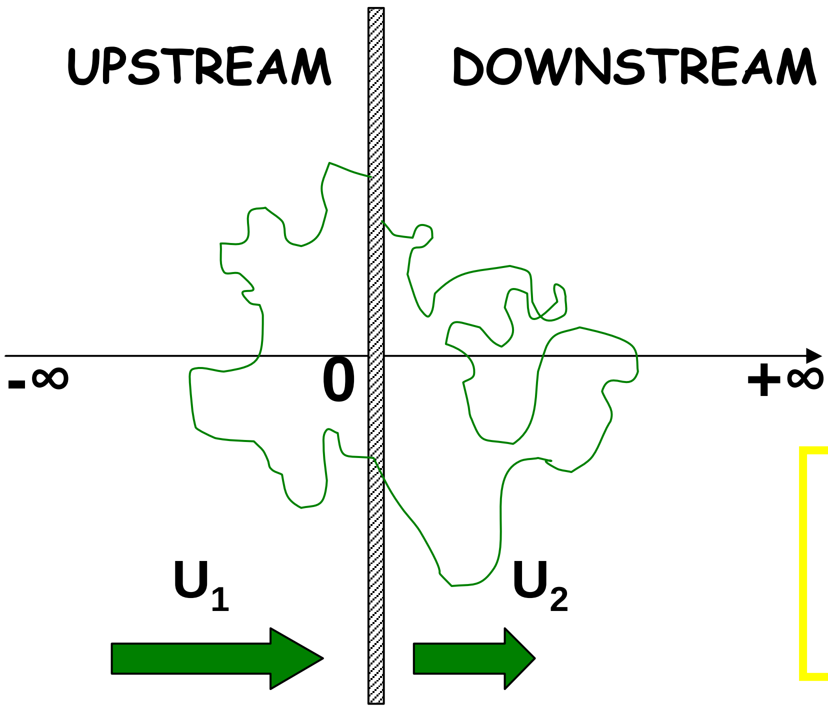
$$\frac{\rho_2}{\rho_1} = \frac{4M^2}{M^2+3} \xrightarrow[M\to\infty]{} 4$$

$$\frac{p_2}{p_1} = \frac{5}{4}M^2 - \frac{1}{4} \xrightarrow[M\to\infty]{} p_2 = \frac{6\rho_1 u_1^2}{8}$$

$$\frac{T_2}{T_1} = \frac{\left(\frac{10}{3}M^2 - \frac{2}{3}\right)\left(\frac{2}{3}M^2 + 2\right)}{\left(\frac{8}{3}M\right)^2} \xrightarrow[M\to\infty]{} T_2 = \frac{3}{16}mu_1^2$$

SHOCK WAVES ARE MAINLY HEATING MACHINES!

BOUNCING BETWEEN APPROACHING MIRRORS



$V=U_1-U_2>0$ **Relative velocity**

INITIAL ENERGY DOWNS: E

$$E_d = E(1-V\mu) \qquad -1<\mu<0$$

$$E_u = E(1-V\mu)(1+V\mu') \qquad 0<\mu'<1$$

TOTAL FLUX

$$J=\int_0^1 d\Omega \frac{N}{4\pi} v\mu = \frac{Nv}{4} \Rightarrow P(\mu)d\mu = \frac{AN v\mu}{\frac{Nv}{4}} d\mu = 2\mu d\mu$$

$$\left\langle \frac{\Delta E}{E} \right\rangle = \int_0^1 d\mu \, 2\mu \int_{-1}^0 d\mu' \, 2\mu' \left[(1 - V\mu)(1 + V\mu') - 1 \right] = \boxed{\frac{4}{3}(U_1 - U_2)}$$

FIRST ORDER

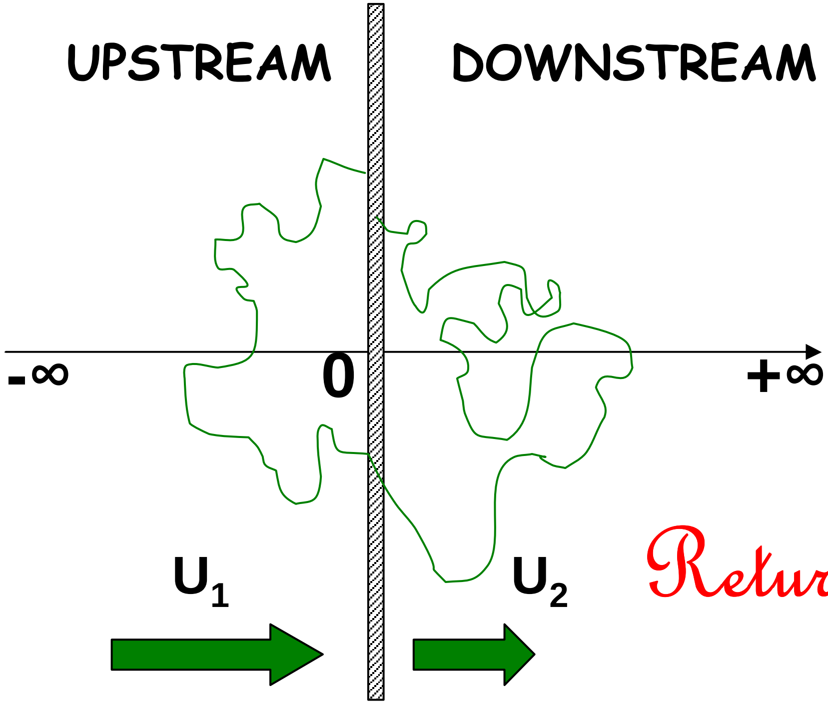
A FEW IMPORTANT POINTS:

I. There are no configurations that lead to losses

II. The mean energy gain is now first order in V

III. The energy gain is basically independent of any detail on how particles scatter back and forth!

RETURN PROBABILITIES AND SPECTRUM OF ACCELERATED PARTICLES



$$\varphi_{in} = \int_{-u_2}^{1} d\mu\, f_0(u_2+\mu) = \frac{1}{2}(1+u_2)^2$$

$$\varphi_{out} = \int_{-1}^{-u_2} d\mu\, f_0(u_2+\mu) = \frac{1}{2}(1-u_2)^2$$

Return Probability from Downstream

$$P_d = \frac{\varphi_{out}}{\varphi_{in}} = \frac{(1-u_2)^2}{(1+u_2)^2} \approx 1-4u_2$$

HIGH PROBABILITY OF RETURN FROM DOWNSTREAM BUT TENDS TO ZERO FOR HIGH U_2

ENERGY GAIN: $$E_{k+1} = \left(1 + \frac{4}{3}V\right)E_k$$

$$\mathbf{E_0 \rightarrow E_1 \rightarrow E_2 \rightarrow --- \rightarrow E_K = [1+(4/3)V]^K\ E0}$$

$$\Downarrow$$

$$\ln\left(\frac{E_K}{E_0}\right) = K \ln\left(1 + \frac{4}{3}(U_1 - U_2)\right)$$

$$\mathbf{N_0 \rightarrow N_1 = N_0 {*} P_{ret} \rightarrow --- \rightarrow N_K = N_0 {*} P_{ret}{}^K}$$

$$\ln\left(\frac{N_K}{N_0}\right) = K \ln(1 - 4U_2)$$

Putting these two expressions together we get:

$$K = \frac{\ln\left[\frac{N_K}{N_0}\right]}{\ln[1-4U_2]} = \frac{\ln\left[\frac{E_K}{E_0}\right]}{\ln\left[1+\frac{4}{3}(U_1-U_2)\right]}$$

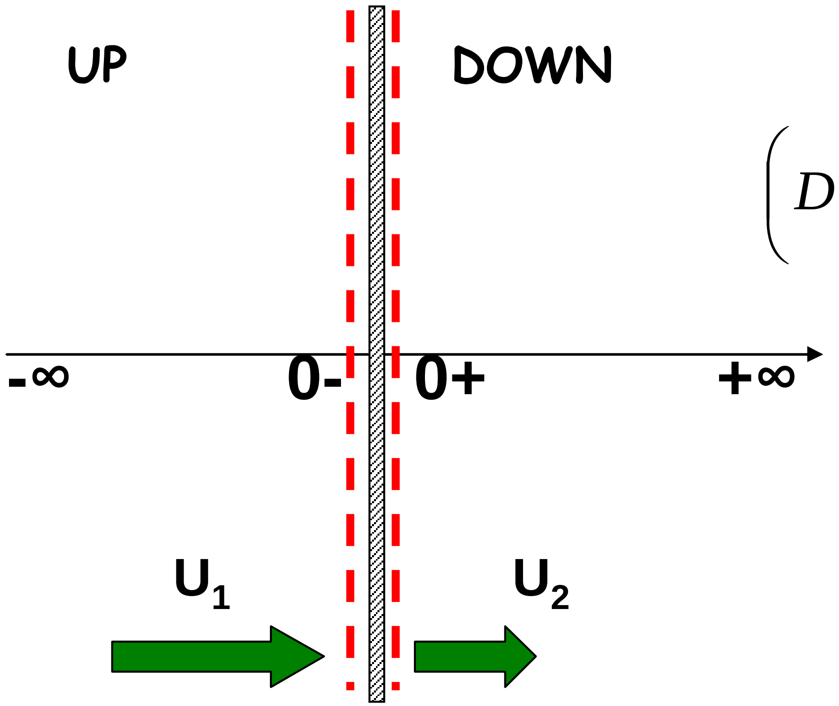
Therefore:

$$N(>E_K) = N_0\left(\frac{E_K}{E_0}\right)^{-\gamma} \qquad \gamma = \frac{3}{r-1} \qquad r = \frac{U_1}{U_2}$$

THE SLOPE OF THE DIFFERENTIAL SPECTRUM WILL BE **γ+1=(r+2)/(r-1)** → **2** FOR r→4 (STRONG SHOCK)

THE TRANSPORT EQUATION APPROACH

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$



Integrating around the shock:

$$\left(D\frac{\partial f}{\partial x}\right)_2 - \left(D\frac{\partial f}{\partial x}\right)_1 + \frac{1}{3}(u_2 - u_1)p\frac{df_0(p)}{dp} + Q_0(p) = 0$$

Integrating from upstr. infinity to 0-:

$$\left(D\frac{\partial f}{\partial x}\right)_1 = u_1 f_0$$

and requiring homogeneity downstream:

$$p\frac{df_0}{dp} = \frac{3}{u_2 - u_1}(u_1 f_0 - Q_0)$$

THE TRANSPORT EQUATION APPROACH

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p) = \frac{3u_1}{u_1 - u_2} \frac{N_{inj}}{4\pi p_{inj}^2} \left(\frac{p}{p_{inj}} \right)^{\frac{-3u_1}{u_1 - u_2}}$$

NOTE THAT THIS IS IN P SPACE NAMELY

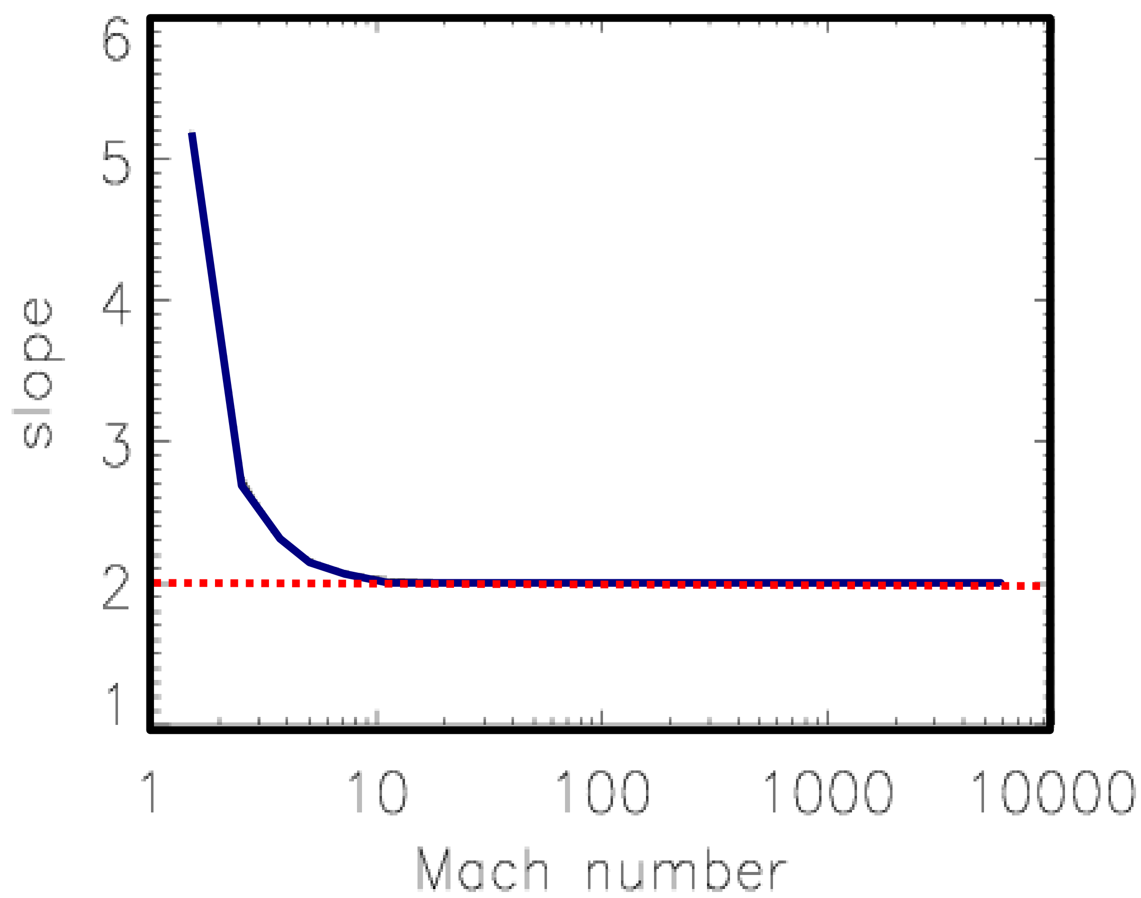
$N(p)dp = 4\pi\ p^2\ f(p)dp$

Therefore the slope is

$3r/(r-1)$

1. THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW EXTENDING TO INFINITE MOMENTA
2. THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES
3. INJECTION IS TREATED AS A FREE PARAMETER WHICH DETERMINES THE NORMALIZATION

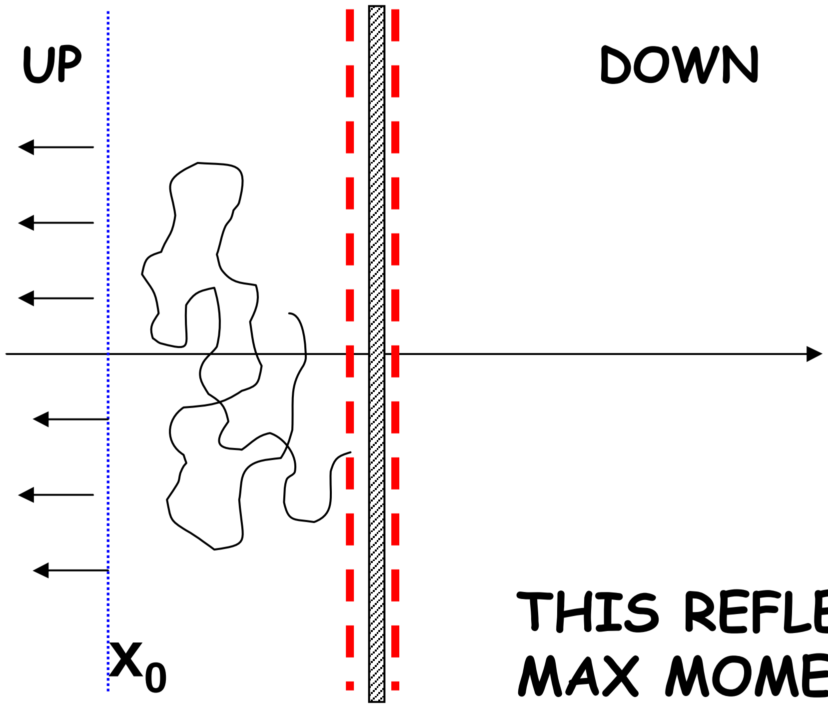
TEST PARTICLE SPECTRUM



SOME IMPORTANT COMMENTS

- THE STATIONARY PROBLEM DOES NOT ALLOW TO HAVE A MAX MOMENTUM!
- THE NORMALIZATION IS ARBITRARY THEREFORE THERE IS NO CONTROL ON THE AMOUNT OF ENERGY IN CR
- AND YET IT HAS BEEN OBTAINED IN THE TEST PARTICLE APPROXIMATION
- THE SOLUTION DOES NOT DEPEND ON WHAT IS THE MECHANISM THAT CAUSES PARTICLES TO BOUNCE BACK AND FORTH
- FOR STRONG SHOCKS THE SPECTRUM IS UNIVERSAL AND CLOSE TO E^{-2}
- IT HAS BEEN IMPLICITELY ASSUMED THAT WHATEVER SCATTERS THE PARTICLES IS AT REST (OR SLOW) IN THE FLUID FRAME

A FREE ESCAPE BOUNDARY CONDITION



THE ESCAPE OF PARTICLES AT X=X0 CAN BE SIMULATED BY TAKING

$$f(x_0, p) = 0$$

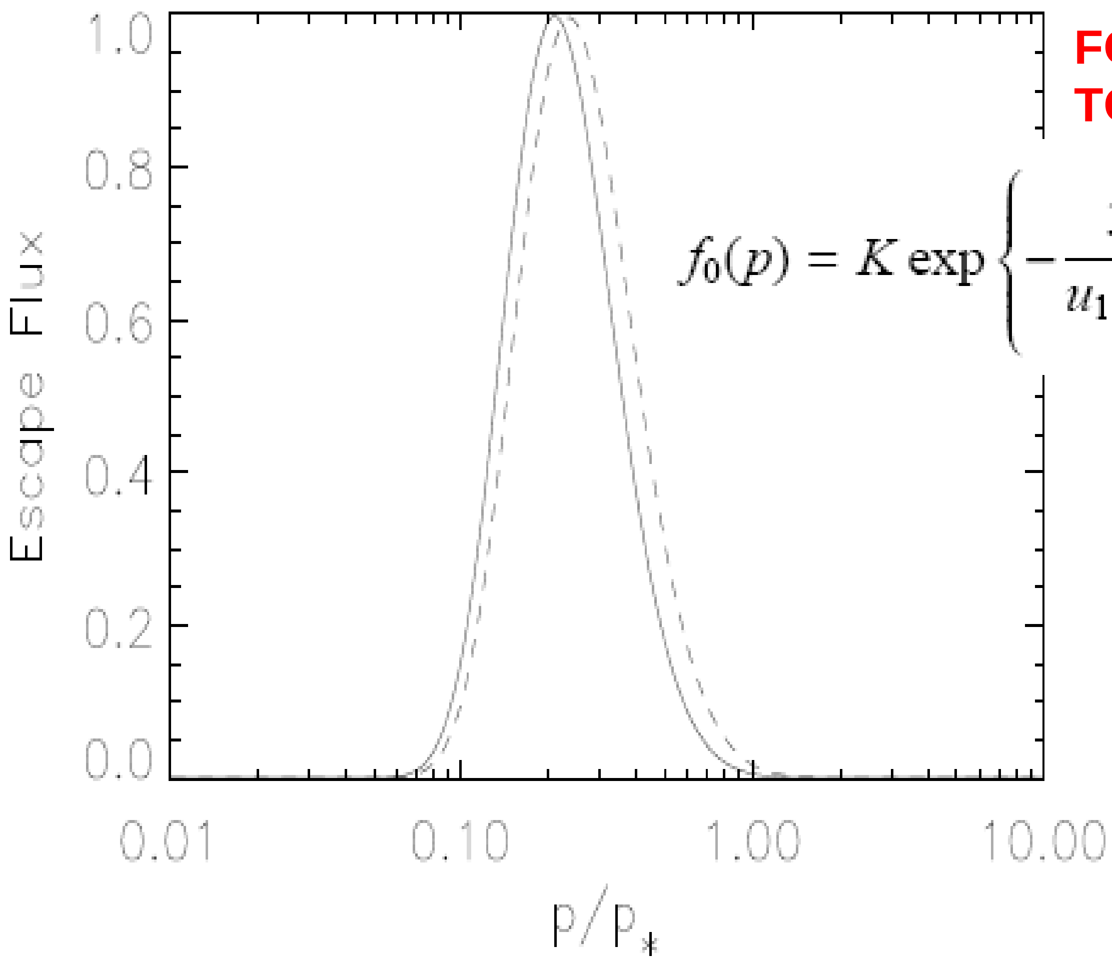
$$f_0(p) = K \exp\left\{-\frac{3u_1}{u_1 - u_2}\int_{p_{inj}}^{p}\frac{dp'}{p'}\frac{1}{1-\exp\left(\frac{x_0 u_1}{D(p')}\right)}\right\}$$

THIS REFLECTS IN AN EXP CUTOFF AT SOME MAX MOMENTUM

$$\phi(x_0, p) = u_1 f(x_0, p) - D(p)\frac{\partial f(x_0)}{\partial x} = -\frac{u_1 f_0(p)}{1-\exp\left(\frac{u_1 x_0}{D(p)}\right)}\exp\left(\frac{u_1 x_0}{D(p)}\right) < 0$$

ESCAPE FLUX TOWARDS UPSTREAM INFINITY!!!

ESCAPE FLUX IN TEST PARTICLE THEORY



FOR D(E) PROPORTIONAL TO E (BOHM DIFFUSION):

$$f_0(p) = K \exp\left\{-\frac{3u_1}{u_1-u_2}\int_{p_{inj}}^{p}\frac{dp'}{p'}\frac{1}{1-\exp\left(-\frac{p_*}{p'}\right)}\right\}$$

$$p_{MAX} = \frac{r-1}{3r}p_*$$

SOME FOOD FOR THOUGHT

- WHAT DETERMINES THE MAX MOMENTUM IN REALITY?

- IF THE RETURN PROBABILITY FROM UPSTREAM IS UNITY, WHAT ARE COSMIC RAYS MADE OF?

- ARE WE SURE THAT THE 10-20% EFFICIENCY WE NEED FOR SNR TO BE THE SOURCES OF GALACTIC CR ARE STILL COMPATIBLE WITH THE TEST PARTICLE REGIME?

MAXIMUM MOMENTUM OF ACCELERATED PARTICLES

THE ACCELERATION TIME IS GIVEN BY:

$$\tau_{acc} = \frac{3}{U_1 - U_2}\left[\frac{D_1(E)}{U_1} + \frac{D_2(E)}{U_2}\right]$$

AND SHOULD BE COMPARED WITH THE AGE OF THE ACCELERATOR, FOR INSTANCE A SUPERNOVA REMNANT

AS AN ESTIMATE:

$$\tau_{acc} \approx \tau_{age} \Longrightarrow \mathbf{E_{max}}$$

IF THE SHOCK IS PROPAGATING IN THE ISM ONE WOULD BE TEMPTED TO ASSUME $D(E)=D_{gal}(E)$

$$D_{gal}(E) = A\left(\frac{E}{GeV}\right)^{\alpha}$$

WHERE TYPICALLY:
$A=(1-10)\ 10^{27}\ cm^2/s$
$\alpha=0.3-0.5$

$$E_{\max,GeV} \approx \left(0.31 u_8^2 \tau_{1000} / A_{27}\right)^{1/\alpha}$$

FOR ALL CHOICES OF PARAMETERS THE MAX ENERGY OBTAINED IN THIS WAY IS FRACTIONS OF GeV, THEREFORE IRRELEVANT !!!

…BUT IT WOULD BE HIGHER IF D(E) WERE MUCH SMALLER…CAN IT HAPPEN?

LECTURE III:
NON LINEAR THEORY OF PARTICLE ACCELERATION

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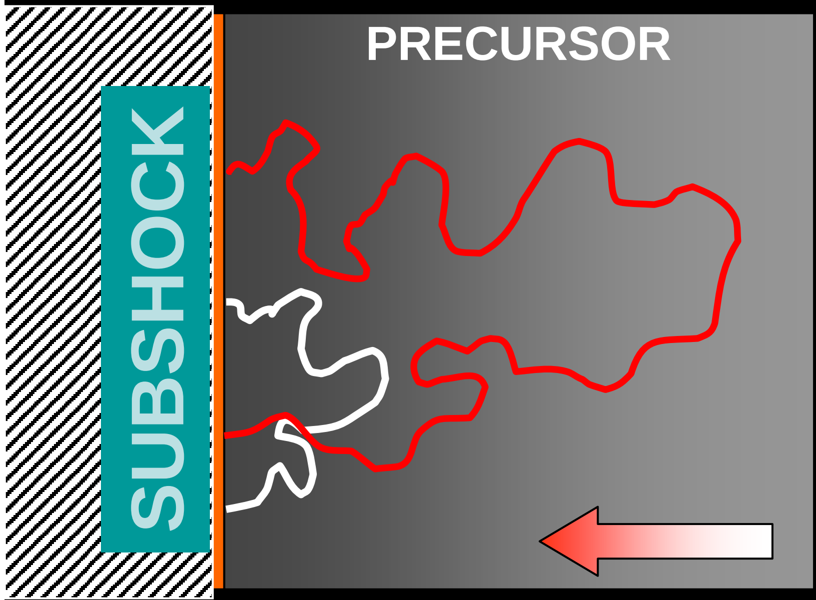
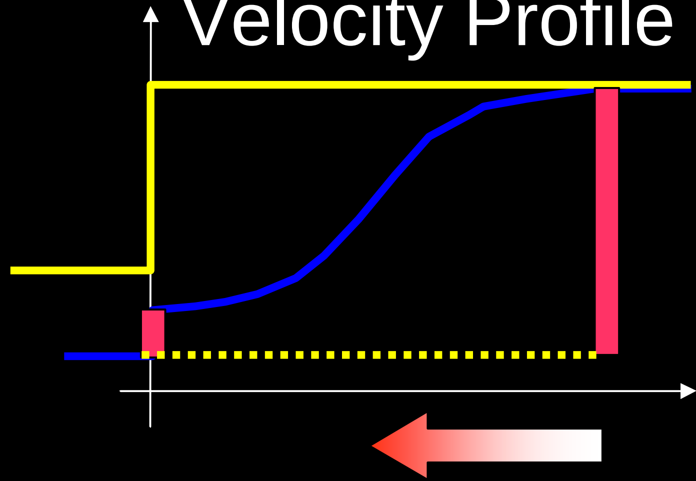
BEYOND THE TEST PARTICLE THEORY: NON LINEAR DSA

- AIM OF THE THEORY:
  - PREDICT THE EFFICIENCY OF ACCELERATION
  - ACCOUNT FOR THE DYNAMICAL REACTION OF THE ACCELERATED PARTICLES ON THE SHOCK
  - EXPLAIN WHY PARTICLES RETURN TO THE SHOCK
  - DETERMINATION OF THE SPECTRUM OF CR SEEN AT THE EARTH

Dynamical Reaction of Accelerated Particles





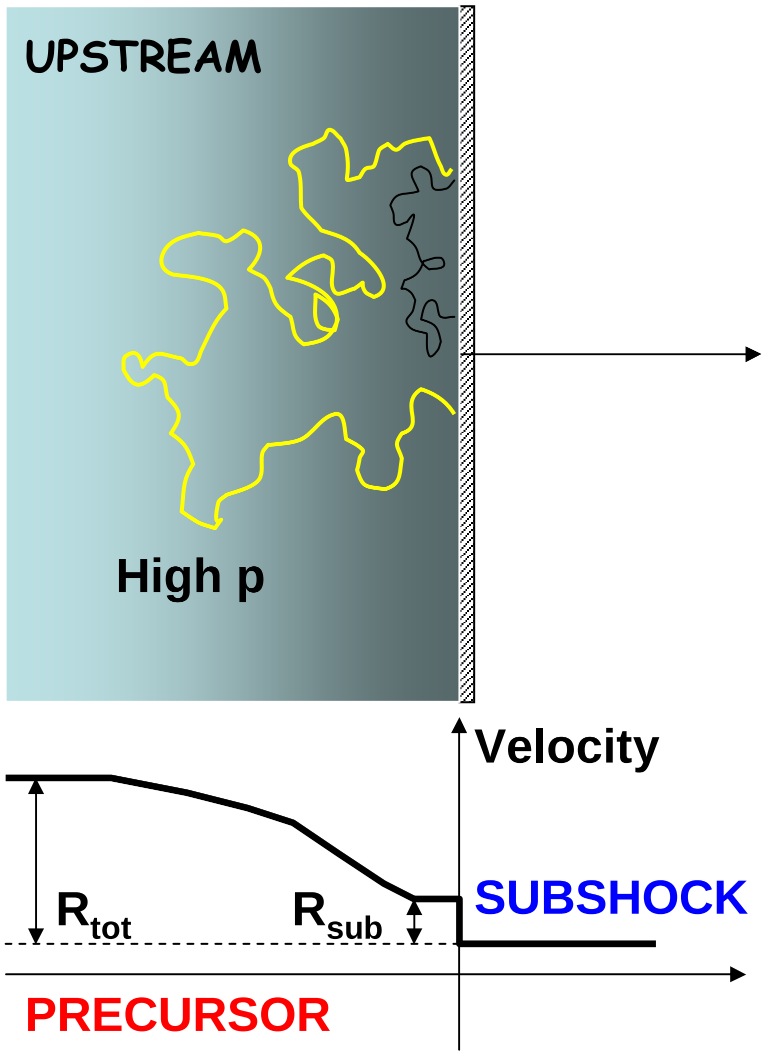
$$\rho_0 u_0^2 + P_{g,0} = \rho(x)\,u(x)^2 + P_g(x) + \boxed{P_{CR}(x)}$$

Conservation of Momentum

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

Transport equation for cosmic rays

WHAT SHOULD BE EXPECT?

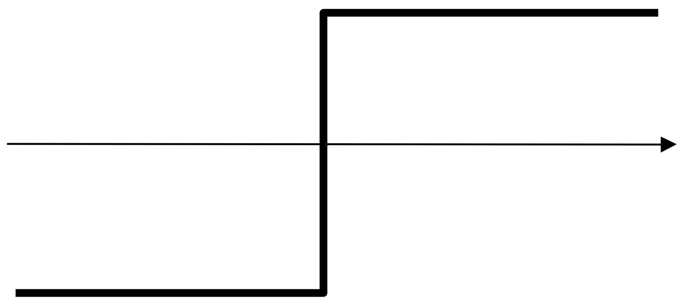


1. PARTICLES WITH HIGH P FEEL LARGER COMPRESSION FACTOR OF LOW P PARTICLES
2. THE TOTAL COMPRESSION BECOMES LARGER THAN 4
3. THE SPECTRUM SHOULD BE NO LONGER A POWER LAW!
4. AT HIGH P THE SLOPE SHOULD BE FLATTER THAN 2 !!!
5. THE GAS BEHIND THE SHOCK SHOULD BE COOLER THAN FOR AN ORDINARY SHOCK

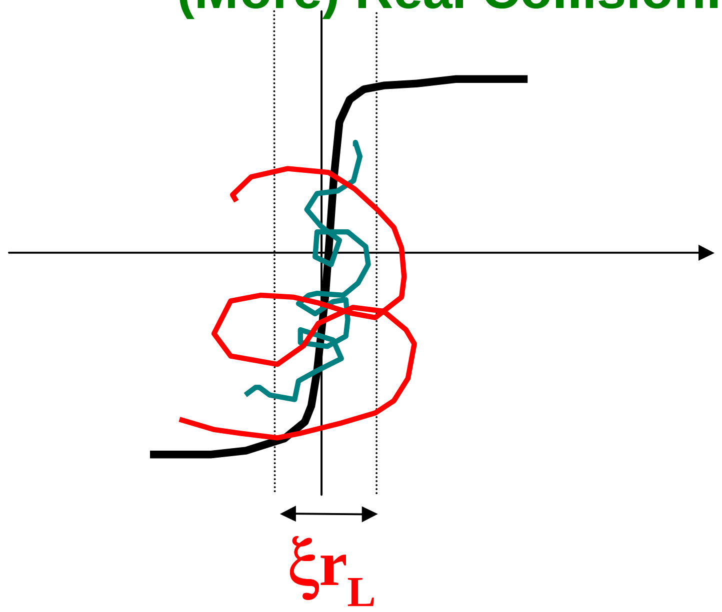
INJECTION

THIS IS THE LEAST UNDERSTOOD ASPECTS OF ALL !

Ideal Collisionless Shock

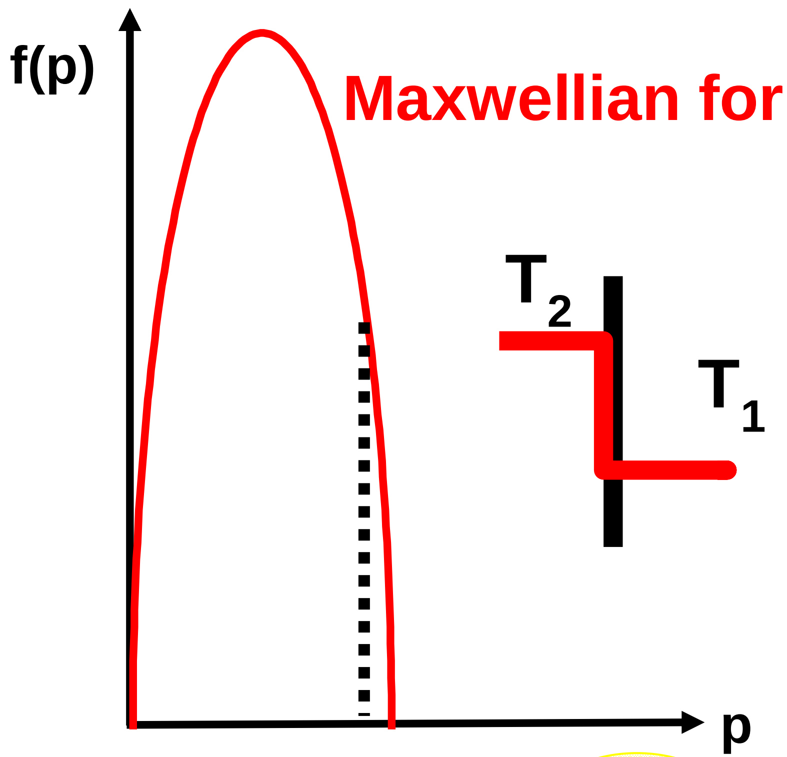


(More) Real Collisionless Shock



WHICH PARTICLES AND HOW MANY PARTICLES ENTER THE ACCELERATION CYCLE?

INJECTION…cont'd



Maxwellian for the Downstream gas

$$\eta = \frac{4}{3\pi^{1/2}}\left(R_{sb} - 1\right)\xi^3 e^{-\xi^2}$$

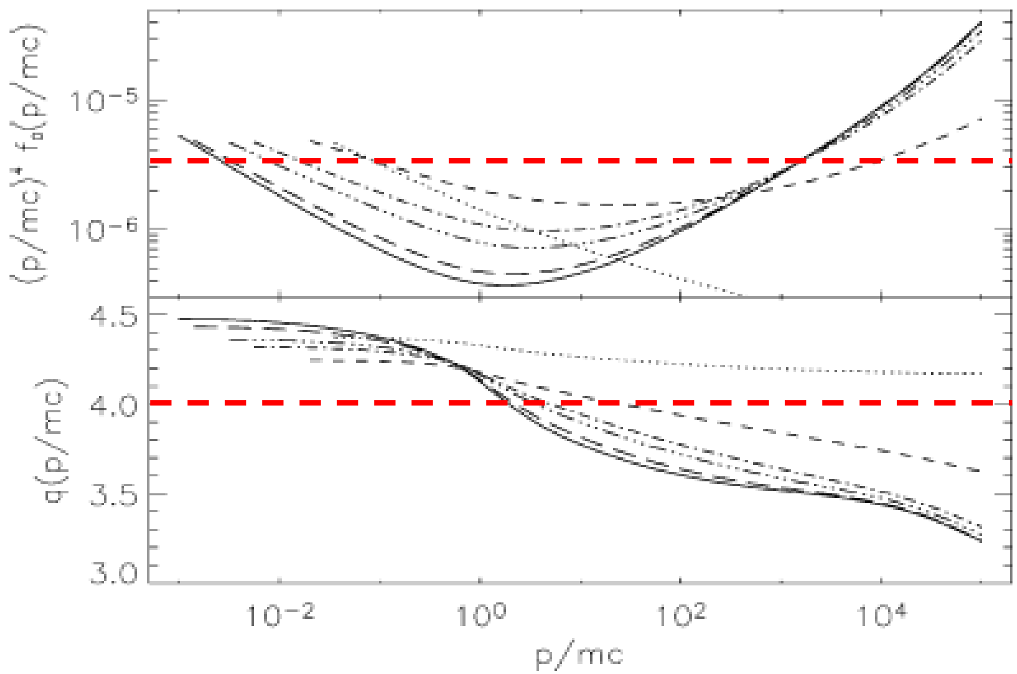
Relation between

R_{sub}, ξ and η

$P_{inj} = \xi\, p_{th,2}$

OUTPUT OF THE CALCULATIONS

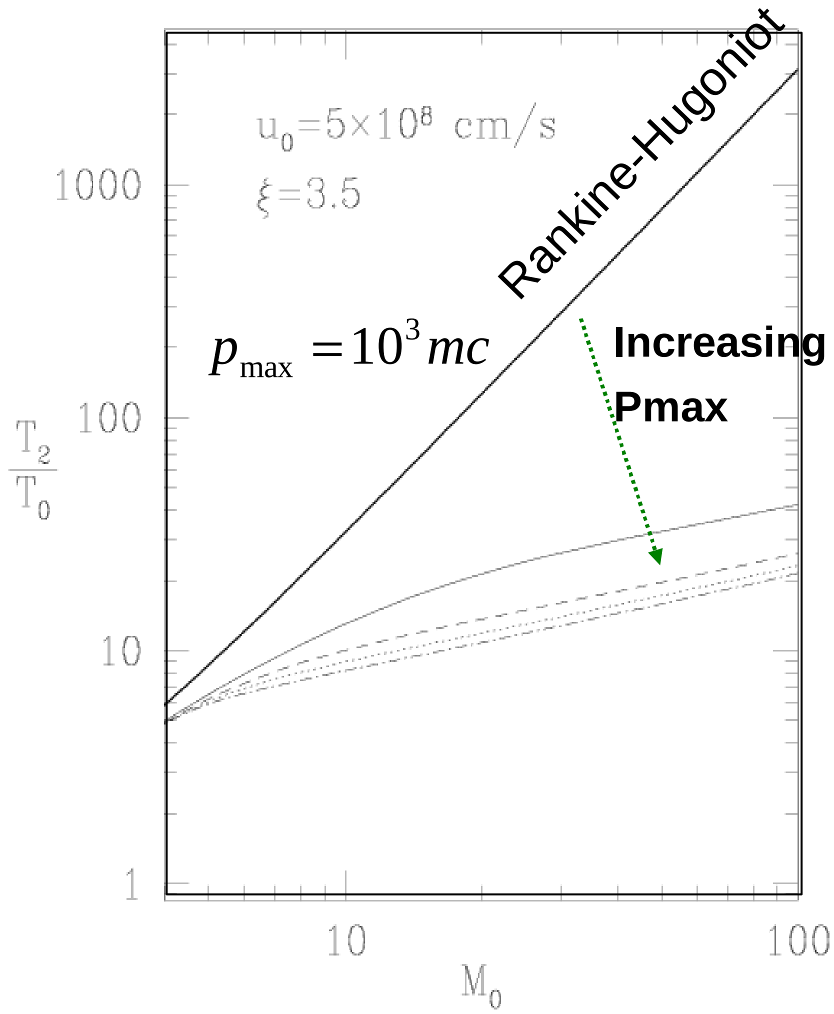
SPECTRA OF ACCELERATED PARTICLES

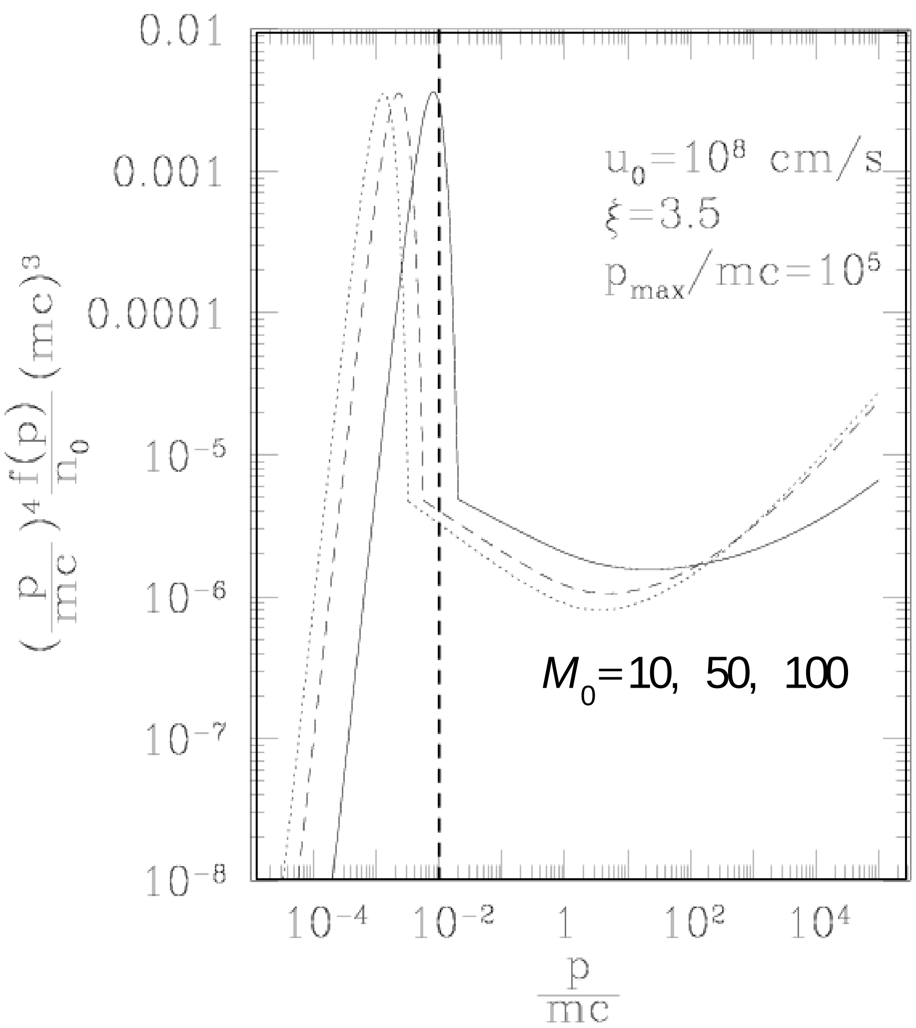


Amato&PB 2005

Mach number M_0	R_{sub}	R_{tot}	$\xi_c(0)$	p_{inj}	η
4	3.19	3.57	0.1	0.035	3.4×10^{-4}
10	3.413	6.57	0.47	0.02	3.7×10^{-4}
50	3.27	23.18	0.85	0.005	3.5×10^{-4}
100	3.21	39.76	0.91	0.0032	3.4×10^{-4}
300	3.19	91.06	0.96	0.0014	3.4×10^{-4}
500	3.29	129.57	0.97	0.001	3.5×10^{-4}

SHOCK HEATING

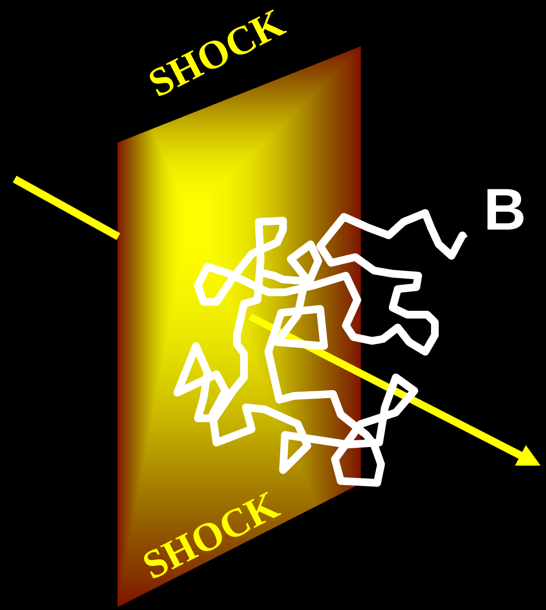




Cosmic Ray self-induced scattering: a primer

Small perturbations in a magnetized medium made of electrons and protons simply give ALFVEN WAVES

WHAT HAPPENS WHEN THERE IS A SHOCK AND IT IS *ACCELERATING COSMIC RAYS*?



$$\frac{dP_{CR}}{dt} = n_{CR}\, m\, \Gamma_{CR}\, (v_S - v_A)\Omega$$

$$\frac{dP_W}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{v_A}$$

$$\boxed{\gamma_W = \frac{n_{CR}}{n_{gas}} \Omega_{cyc} \left(\frac{v_S - v_A}{v_A}\right)}$$

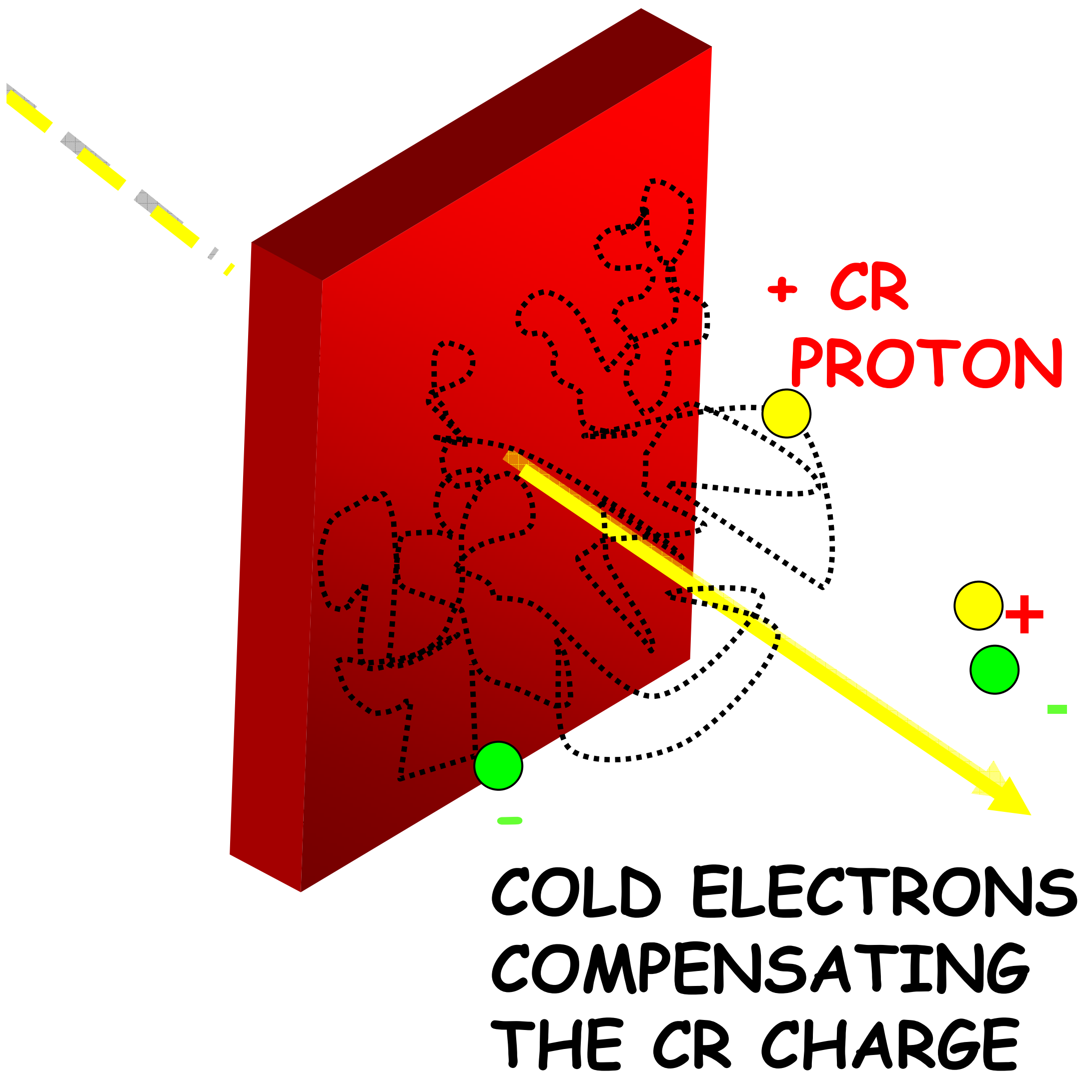
A HINT TO HOW THIS SHOULD BE DONE

WE HAVE ALREADY DISCUSSED A DERIVATION OF THE RESONANT MODES THAT LEAD TO WAVE PRODUCTION. HERE IS THE MORE GENERAL AND FORMALLY CORRECT APPROACH:

ONE SHOULD PERTURB THE FOLLOWING EQUATIONS:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{c}\left(\vec{v} \times \vec{B}\right)_\beta \frac{\partial f_\alpha}{\partial p_\beta} = 0$$

+ Maxwell Equations



$$
\begin{aligned}
f_i(p,\mu) &= \frac{n_i}{2\pi p^2}\delta(p - m_i v_s)\delta(\mu - 1) \\
f_e(p,\mu) &= \frac{n_e}{2\pi p^2}\delta(p - m_e v_s)\delta(\mu - 1) \\
f_e^{cold}(p) &= \frac{N_{CR}}{4\pi p^2}\delta(p) \\
f_{CR}(p) &= \frac{N_{CR}}{4\pi}g(p).
\end{aligned}
$$

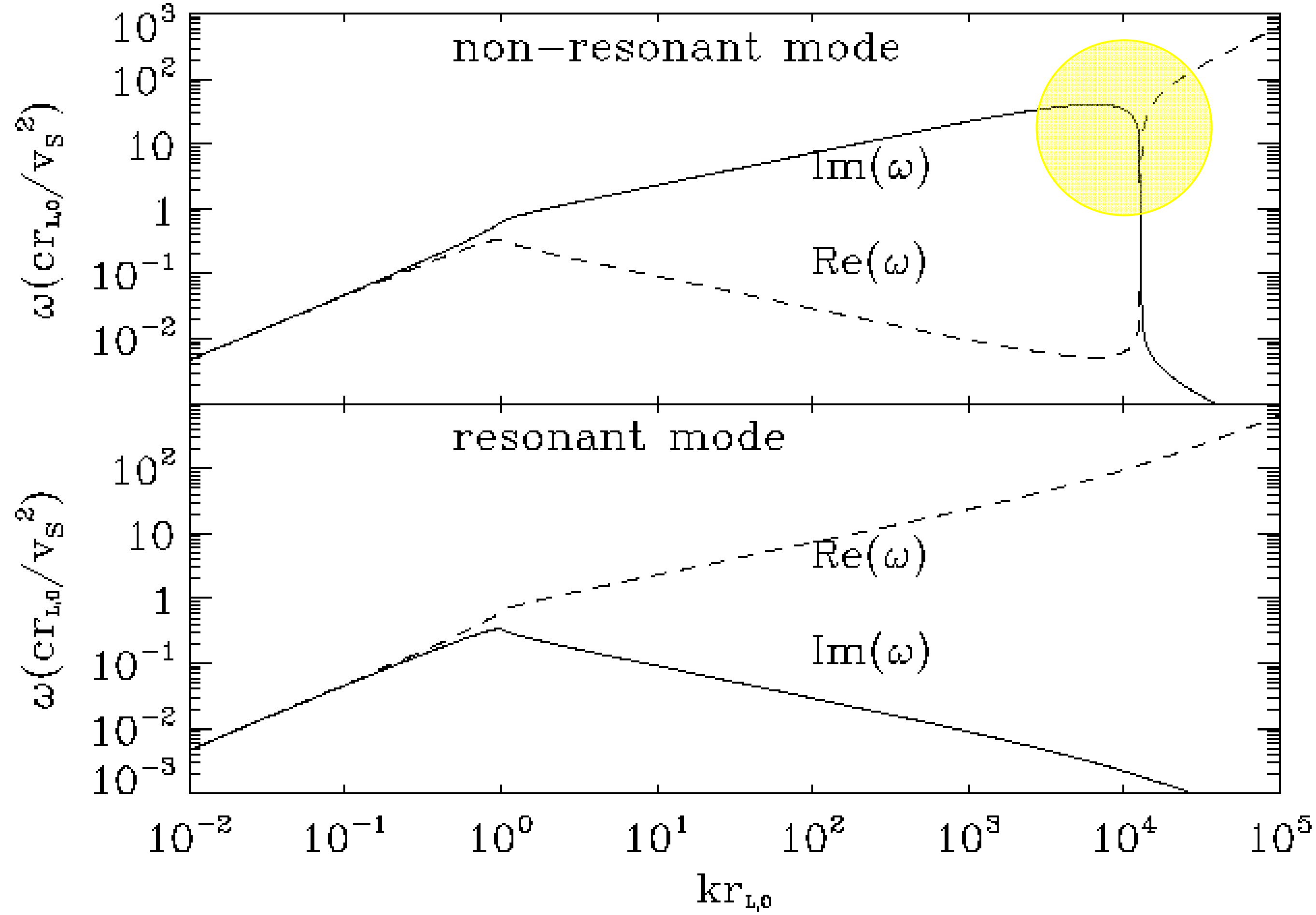
THE DISPERSION RELATION

$$v_A^2 k^2 = \tilde{\omega}^2 \pm \frac{N_{CR}}{n_i}(\tilde{\omega} - k v_s)\Omega_i^* \left[1 \pm I_1^{\pm}(k) \mp i I_2(k)\right]$$

$$\begin{aligned} I_1^{\pm}(k) &= \frac{p_{min}(k)}{4} \int_{p_0}^{p_{max}} dp \frac{dg}{dp} \left[(p^2 - p_{min}(k)^2) \ln \left| \frac{1 \pm p/p_{min}}{1 \mp p/p_{min}} \right| \pm 2 p_{min} p \right] \\ I_2(k) &= \frac{\pi}{4} p_{min}(k) \int_{Max[p_0, p_{min}(k)]}^{p_{max}} dp \frac{dg}{dp} (p^2 - p_{min}(k)^2). \end{aligned}$$

AT FIRST SIGHT THE EXCITED WAVES ARE BASICALLY MODIFIED ALFVEN WAVES WITH A SMALL FORCING PROPORTIONAL TO THE COSMIC RAY DENSITY

REAL AND IMAGINARY PARTS OF FREQUENCY



GROWTH OF THE WAVES

THE WAVES ARE GENERATED UPSTREAM AND EVENTUALLY ADVECTED WITH THE FLUID. THE EQUATION FOR THE NORMALIZED WAVE ENERGY IS THEREFORE

$$\frac{\partial \mathcal{F}_w(k,x)}{\partial x} = u(x)\frac{\partial \mathcal{P}_w(k,x)}{\partial x} + \sigma(k,x)\mathcal{P}_w(k,x) - \Gamma(k,x)\mathcal{P}_w(k,x)$$

ADVECTION AMPLIFICATION DAMPING

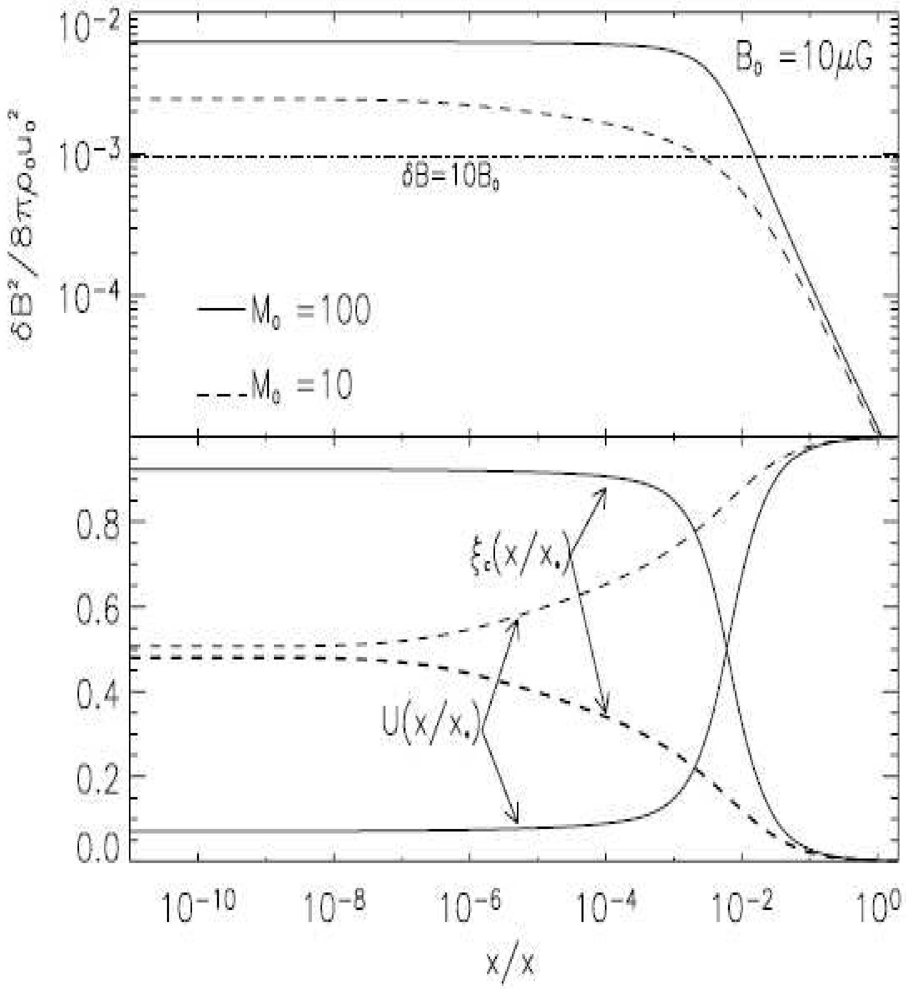
$$\boxed{\sigma = \frac{P_C(x) - P_C(x-dx)}{dx/v_A} = \frac{4\pi}{3} v_A c \left[p^4 \frac{\partial f}{\partial x} \right]_{p_{res}(k)} \frac{1}{U_M}}$$

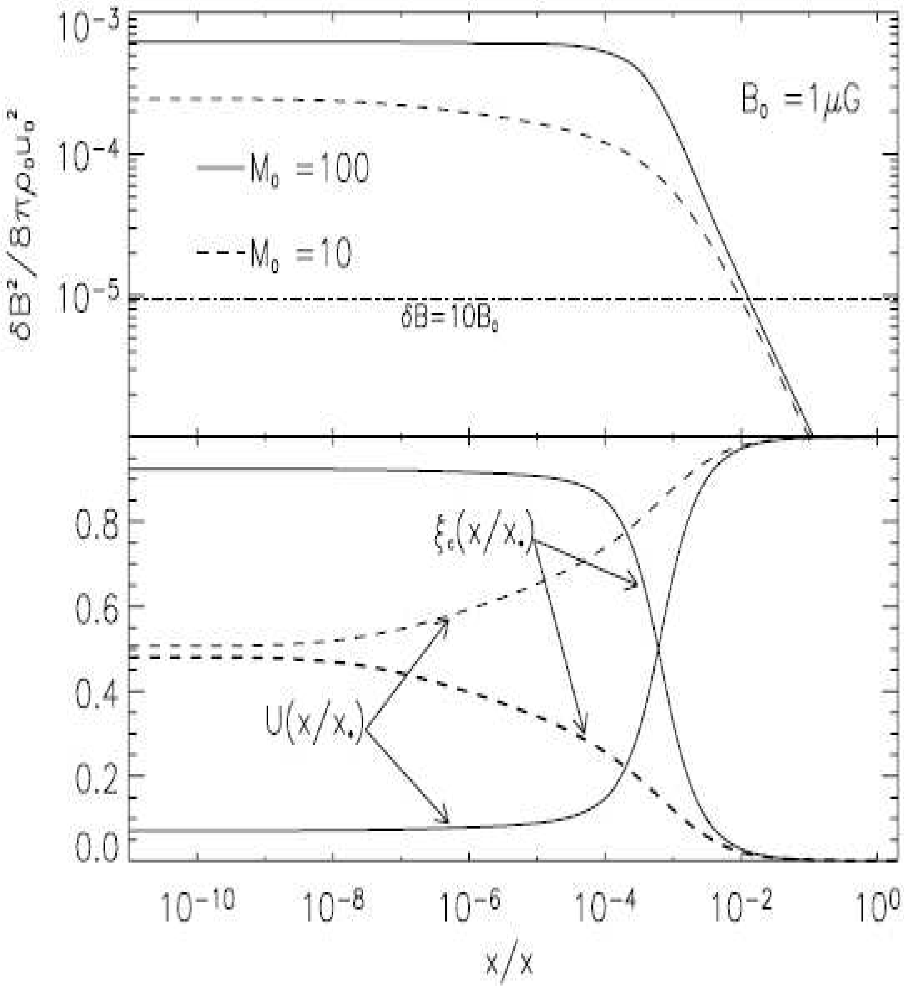
INTEGRATING IN K:

$$\frac{dF_w(x)}{\partial x} = u(x)\frac{dp_w(x)}{\partial x} + v_A \frac{dp_c(x)}{dx} \qquad F_w(x) \simeq 3u(x)p_w(x).$$

TYPICALLY: $\delta B=[10^{-2}\,\rho u^2]^{1/2}$ = 30-50 μG UPSTREAM

And further compressed to about 100 μG downstream





Amato&PB 2006

SOME POINTS TO MAKE

- THE IMPLICATIONS FOR THE ORIGIN OF COSMIC RAYS ARE EVIDENT: LARGER MAX MOMENTA BECOME POSSIBLE (see below)

- THE PREDICTION OF LARGE TURBULENT FIELDS AT THE SHOCK HAS VERY IMPORTANT OBSERVATIONAL CONSEQUENCES ON RADIATION FROM THE ACCELERATORS (see below)

- BUT A LOT OF CARE SHOULD BE USED TOO: REMEMBER THAT WE OBTAINED A LARGE FIELD BY ASSUMING A PERTURBATIVE APPROACH…WHAT DO REALITY AND SIMULATIONS TELL US?

IMPLICATIONS FOR THE ORIGIN OF GALACTIC COSMIC RAYS

WE ARE NOW IN THE UNCHARTED WATERS OF $\delta B/B>>1$. WE DO NOT KNOW HOW TO CALCULATE D(E) IN THESE CONDITIONS. BUT REMEMBER WHAT WE OBTAINED:

$$D(E) \approx v^2 \tau = \frac{v^2}{\Omega\, G(k_{res})} \approx r_L(E)c\frac{1}{G(k(E))}$$

IF G=1 FOR ALL K's then

$$D(E) \approx r_L(E)c$$

BOHM DIFFUSION

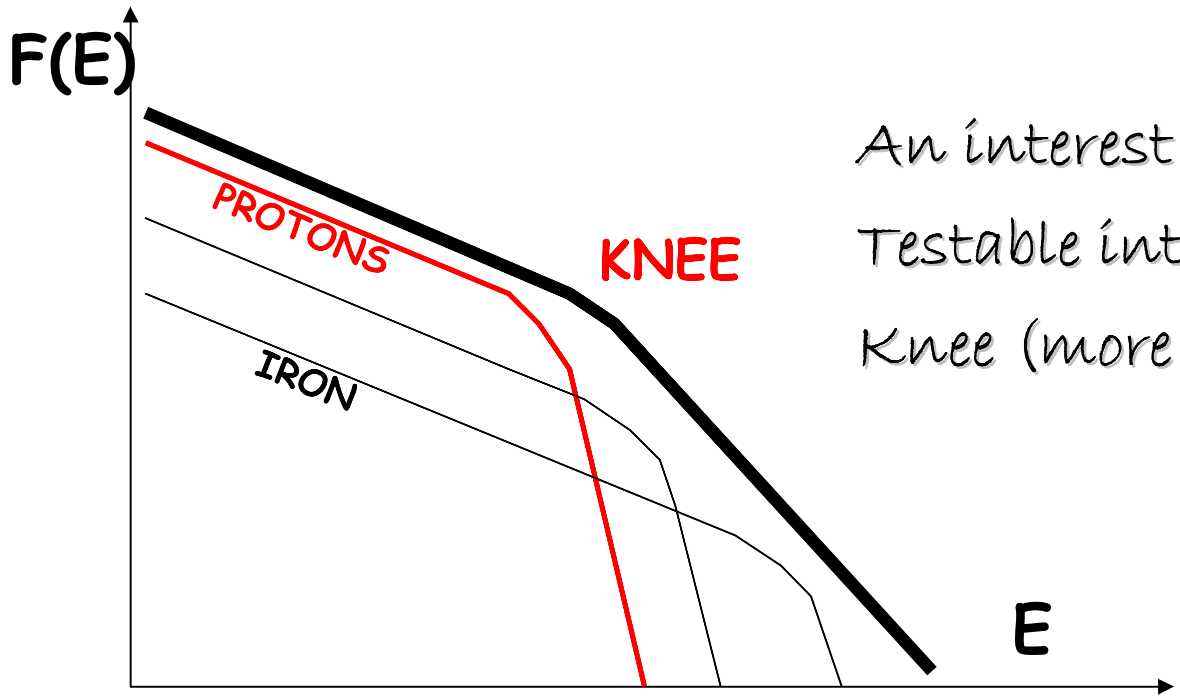
$$\sigma = \frac{P_C(x)-P_C(x-dx)}{dx/v_A} = \frac{4\pi}{3} v_A c\left[p^4 \frac{\partial f}{\partial x}\right]_{p_{res}(k)} \frac{1}{U_M}$$

INDEPENDENT OF K IF $f \sim p^{-4}$

IF WE ASSUME BOHM DIFFUSION IN THE SELF-GENERATED MAGNETIC FIELD:

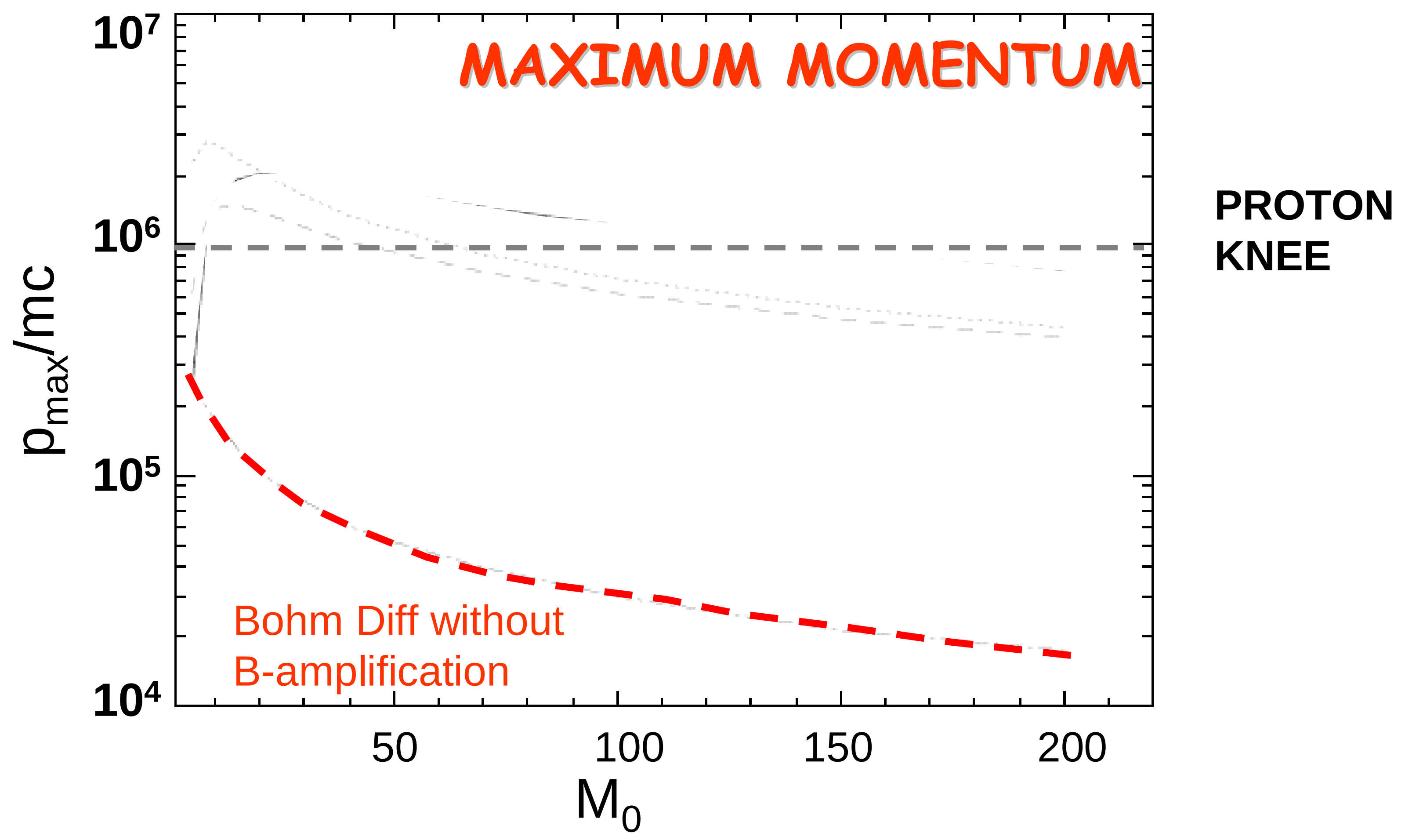
$$\boxed{E_{MAX} \approx 3\times10^{15}\, Z\, u_{3000}^{2}\, \tau_{1000}\, B_{100}\ \ \mathrm{eV}}$$

WHIGH IS TANTALIZINGLY CLOSE TO THE KNEE REGION, AND EVEN HIGHER FOR HEAVIER (HIGHER Z) NUCLEI



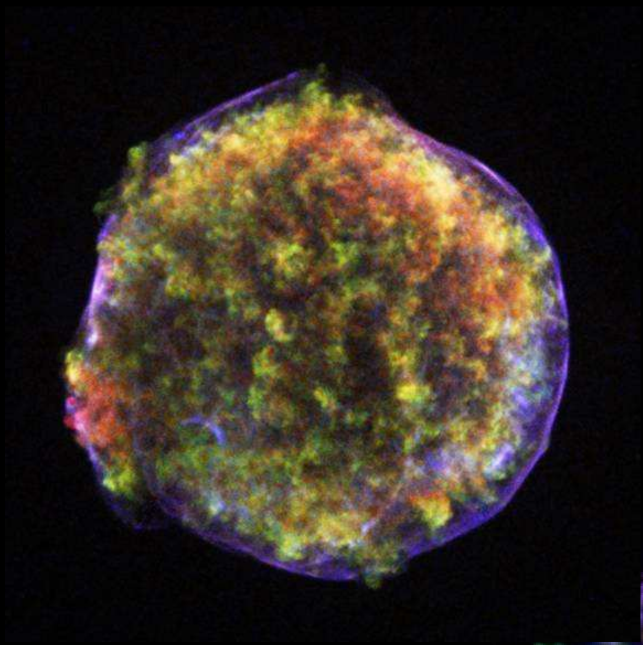
An interest new possible and Testable interpretation of the Knee (more later!)

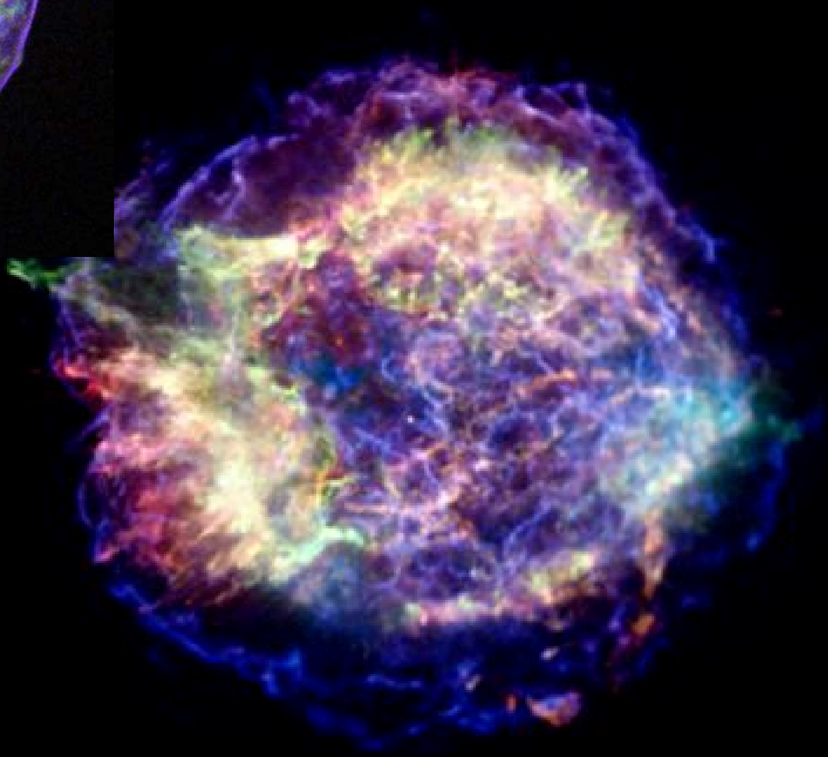
Implications of B-field amplification

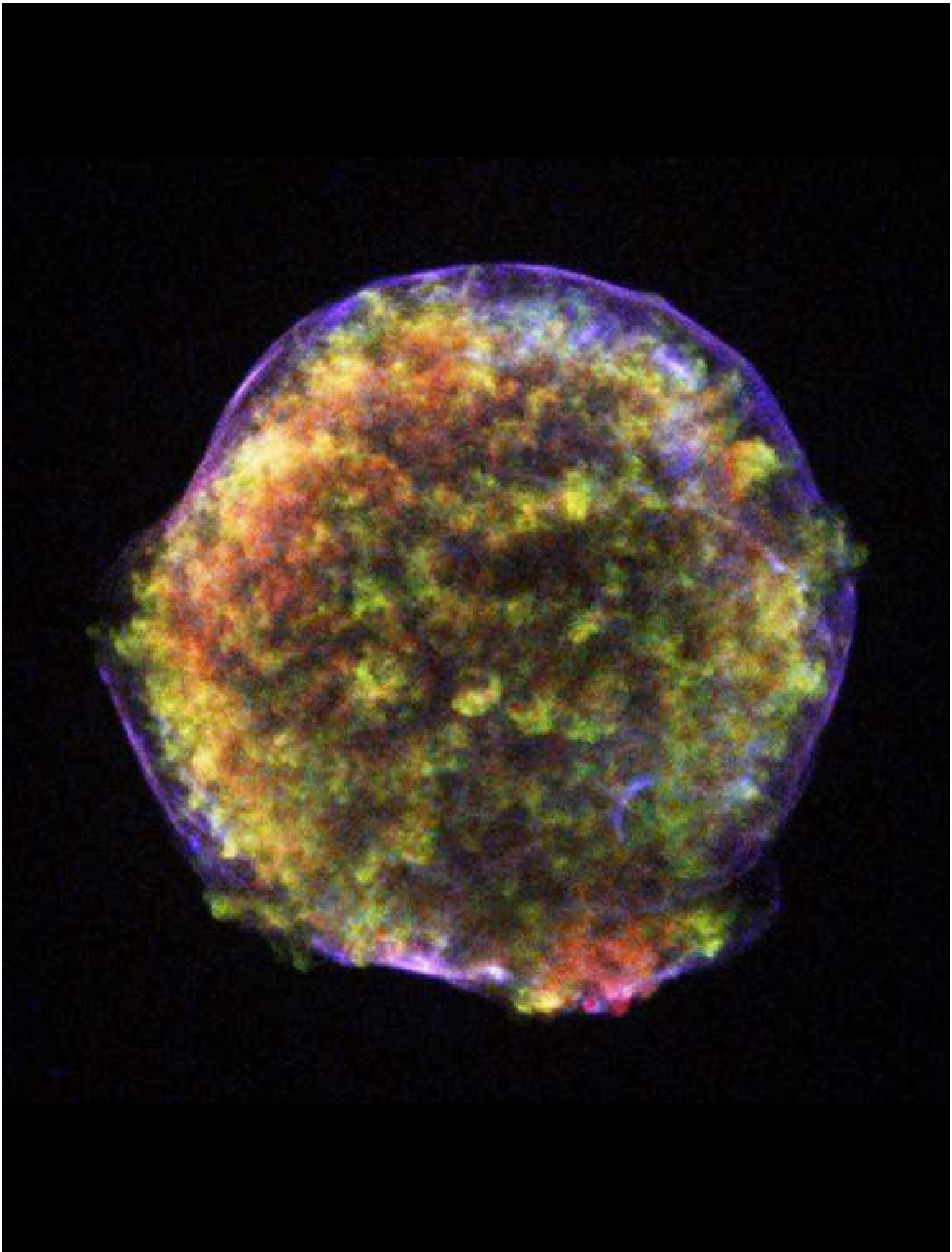


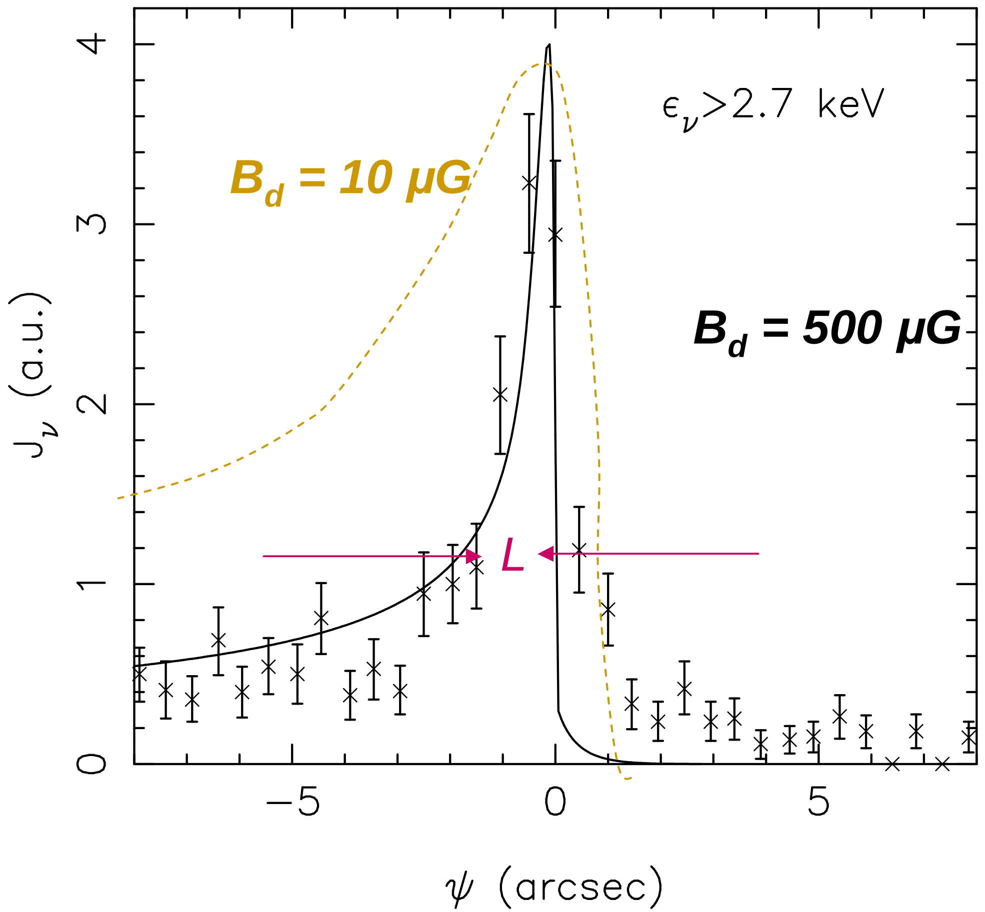
PB, Amato & Caprioli, 2007

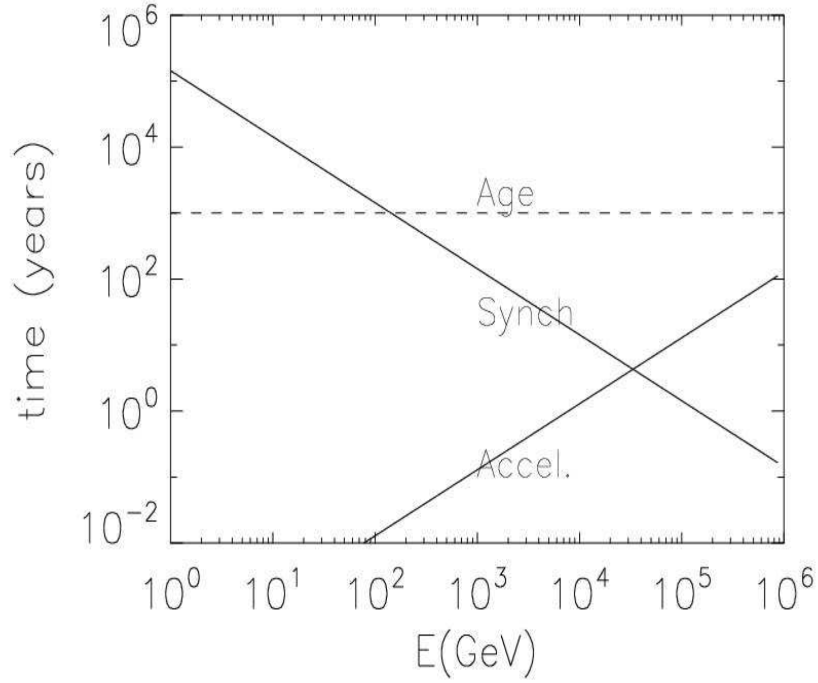
HOW DOES THIS GENERAL THEORY CONFRONT NATURE?











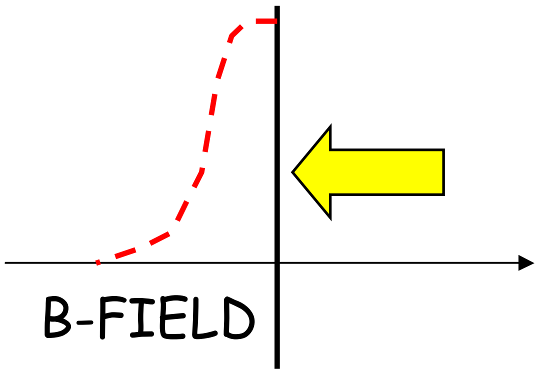
IN THE DOWNSTREAM PLASMA THE THICKNESS OF THE RIM IS:

$$\Delta L \approx \mathrm{Min}\left\{u_2 \tau_{loss}, \sqrt{4D(E)\tau_{loss}(E)}\right\}$$

AND FOR BOHM DIFFUSION THE EMISSION ALWAYS ENDS UP IN THE X-RAY BAND

AN ALTERNATIVE EXPLANATION: THE IMPORTANCE OF DAMPING

THE BRIGHT RIMS COULD BE PRODUCED IF THE MAGNETIC FIELD DOWNSTREAM WERE DAMPED



IT IS NOT CLEAR WHAT IS THE CORRECT INTERPRETATION, BUT THERE IS A SIMPLE DIAGNOSTIC:

IF THE FIELD IS DAMPED DOWNSTREAM THE FILAMENTS SHOULD APPEAR IN THE RADIO AS WELL.

AT THE PRESENT TIME THERE DOES NOT SEEM TO BE EVIDENCE OF FILAMENTS IN THE RADIO BAND

DYNAMICAL REACTION OF THE B-FIELD

BEFORE MOVING ON TO OTHER PHENOMENOLOGICAL TESTS OF THE THEORY, WE NEED TO ASSESS ONE LAST POINT, THE DYNAMICAL REACTION OF THE LARGE FIELD ON THE SHOCK

RECALL THAT WE HAVE SHOWED THAT

$$\frac{P_W(x)}{\rho_0 u_0^2} = \alpha(x) = \frac{1 - U(x)^2}{4M_A(x)U(x)} \ll 1$$

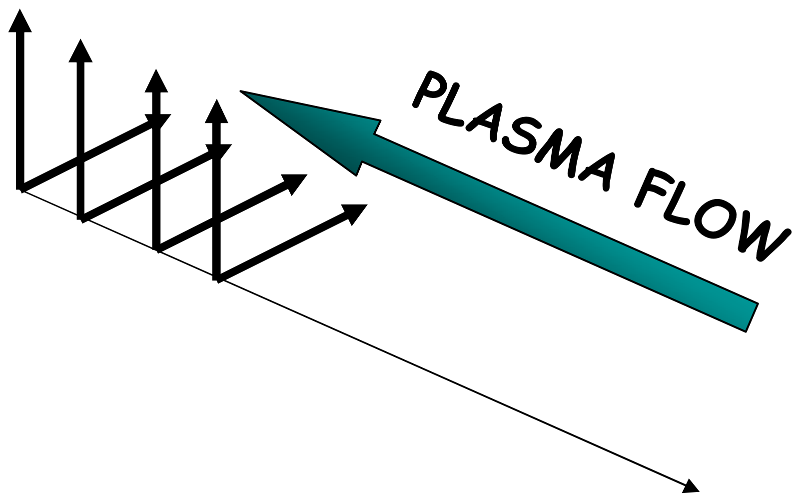
BUT THE PRESSURE TERM IS:

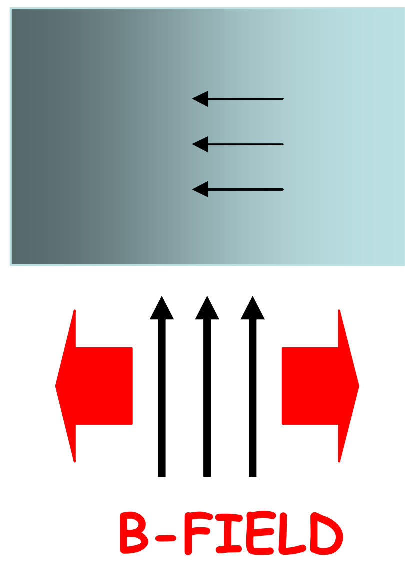
$$\frac{P_{gas}(x)}{\rho_0 u_0^2} = \frac{U(x)^{-\gamma}}{\gamma M_0^2}$$

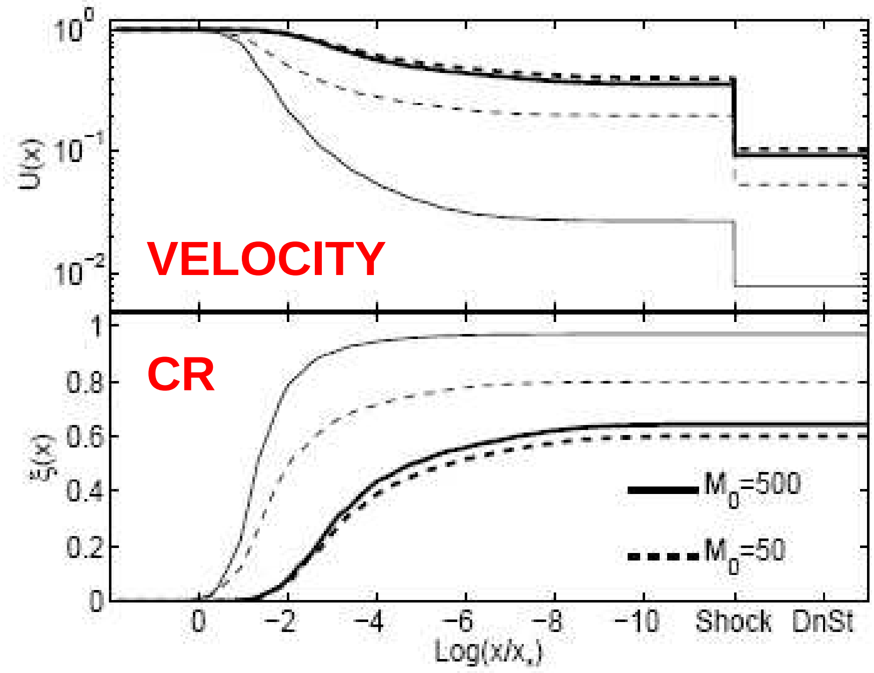
DYNAMICAL REACTION OF THE B-FIELD

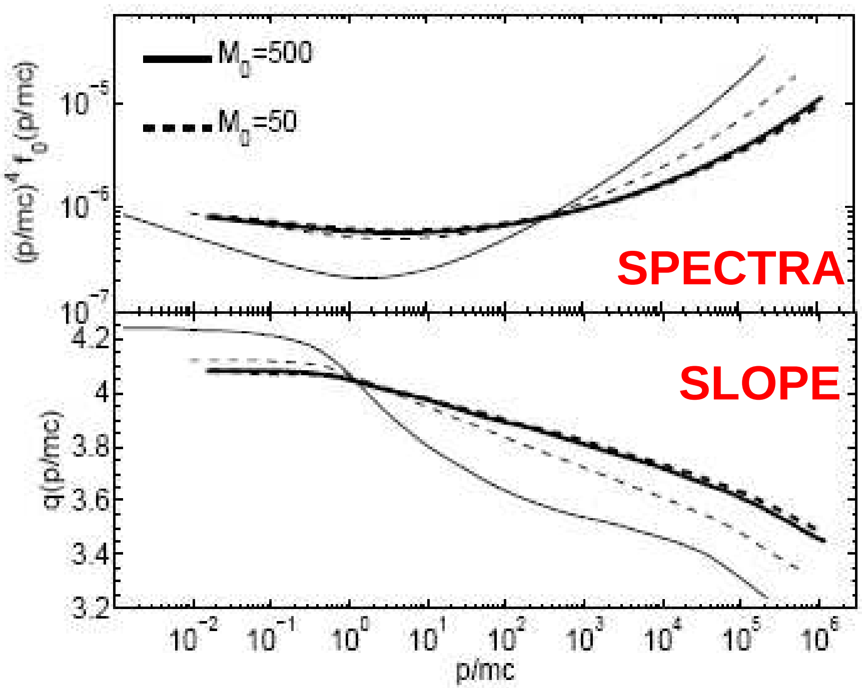
IT FOLLOWS THAT THE MAGNETIC TERM, THOUGH SMALL, BECOMES COMPARABLE WITH THE PRESSURE TERM WHEN:

$$M_{A,0} \approx M_0^2 \qquad v \approx 3800\, T_8\, n^{1/2}\, B_\mu^{-1} \quad \mathrm{km/s}$$





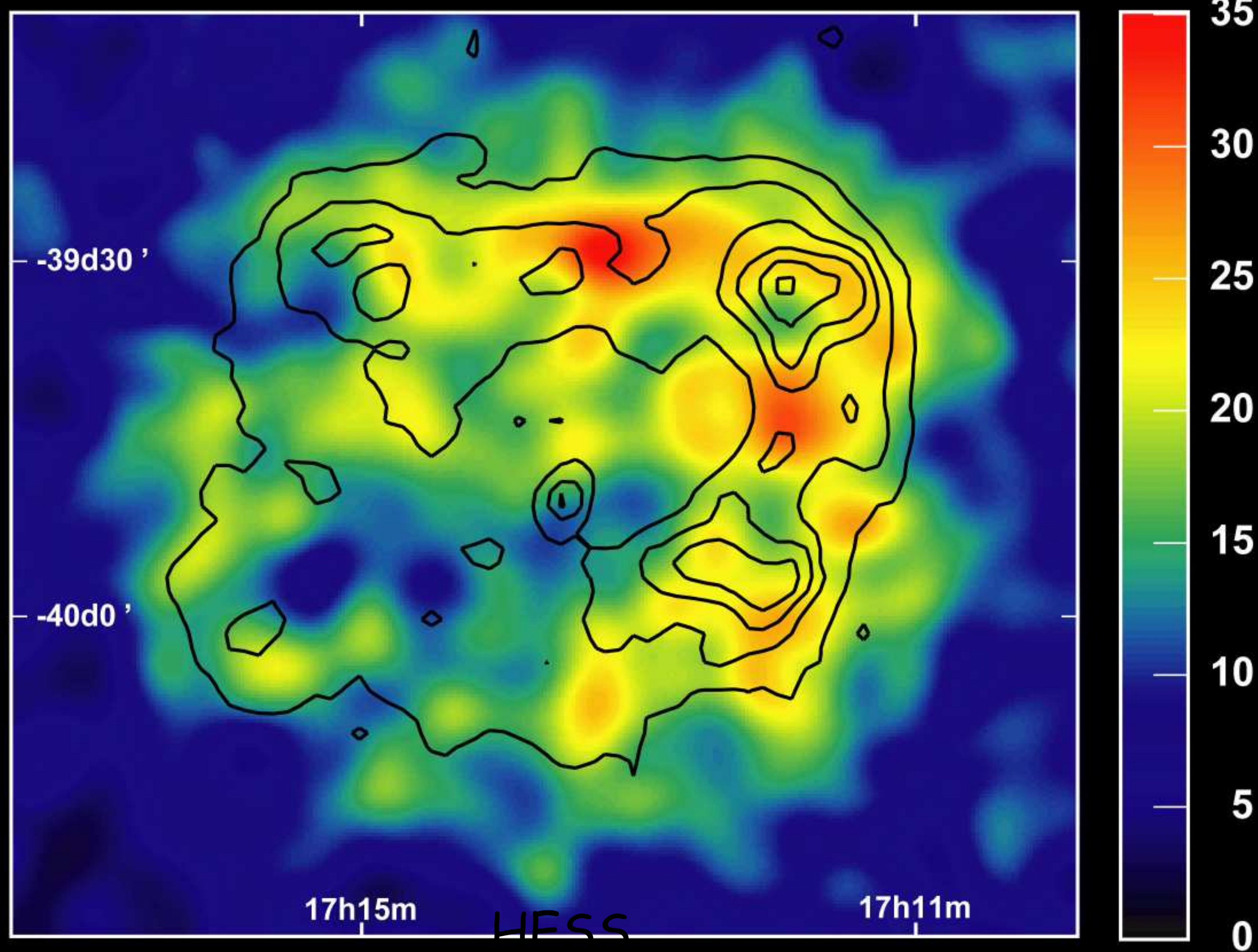




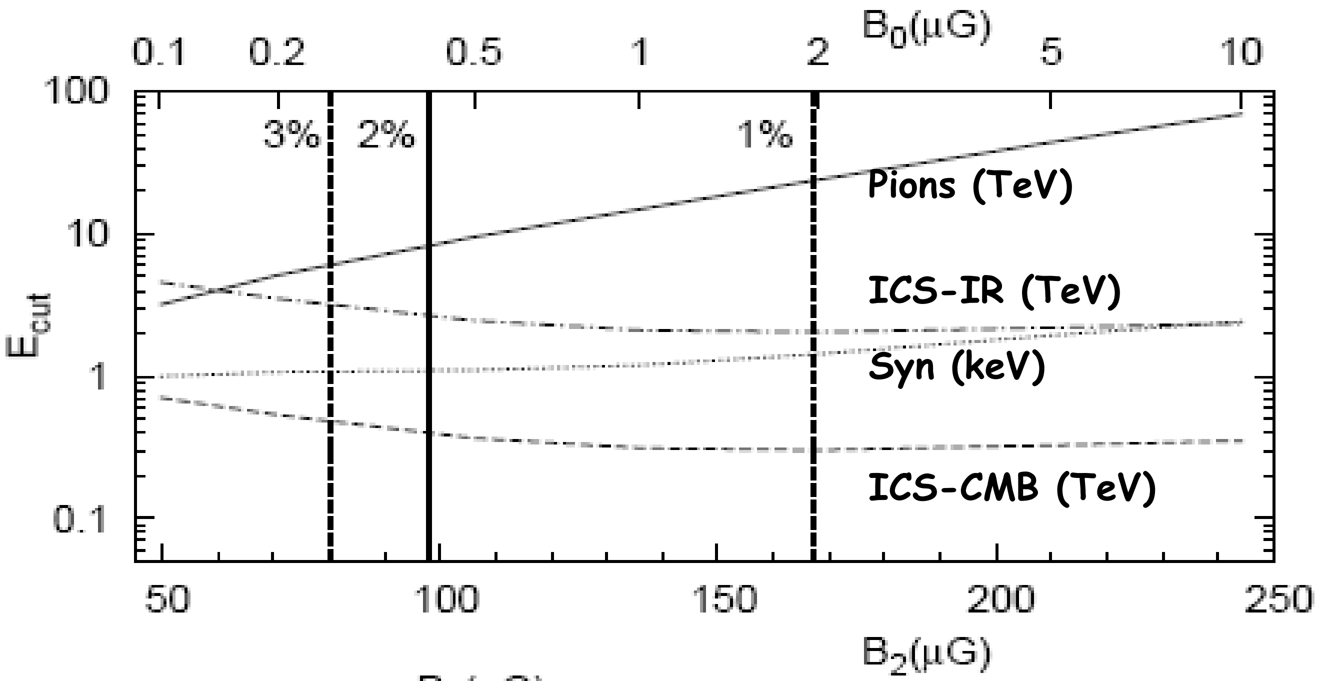
T_0(K)	Λ_B	ξ_1	$p_{max}(10^6 GeV)$	R_{sub}	R_{tot}	S_{sub}	S_{tot}	$B_2(\mu G)$	$T_2(10^6$ K)
10^4	No	0.97	0.22	3.38	125.1	3.43	128.6	679.4	0.67
10^4	Yes	0.64	1.19	3.75	10.6	3.76	10.7	525.0	87.7
10^6	No	0.80	0.53	3.67	18.6	3.69	18.7	236.7	33.1
10^6	Yes	0.60	1.17	3.76	9.52	3.77	9.57	475.6	114.6

Caprioli, PB, Amato & Vietri 2008

THE CASE OF RX-J1713



Streaming Instability ON



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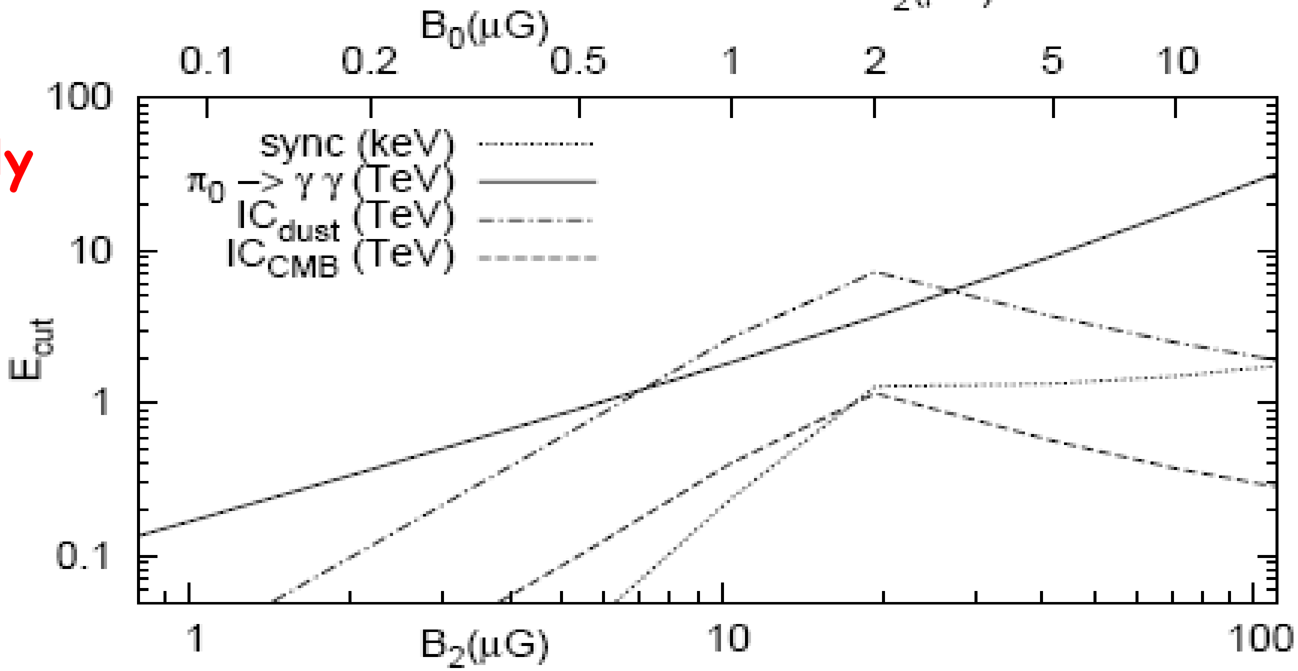
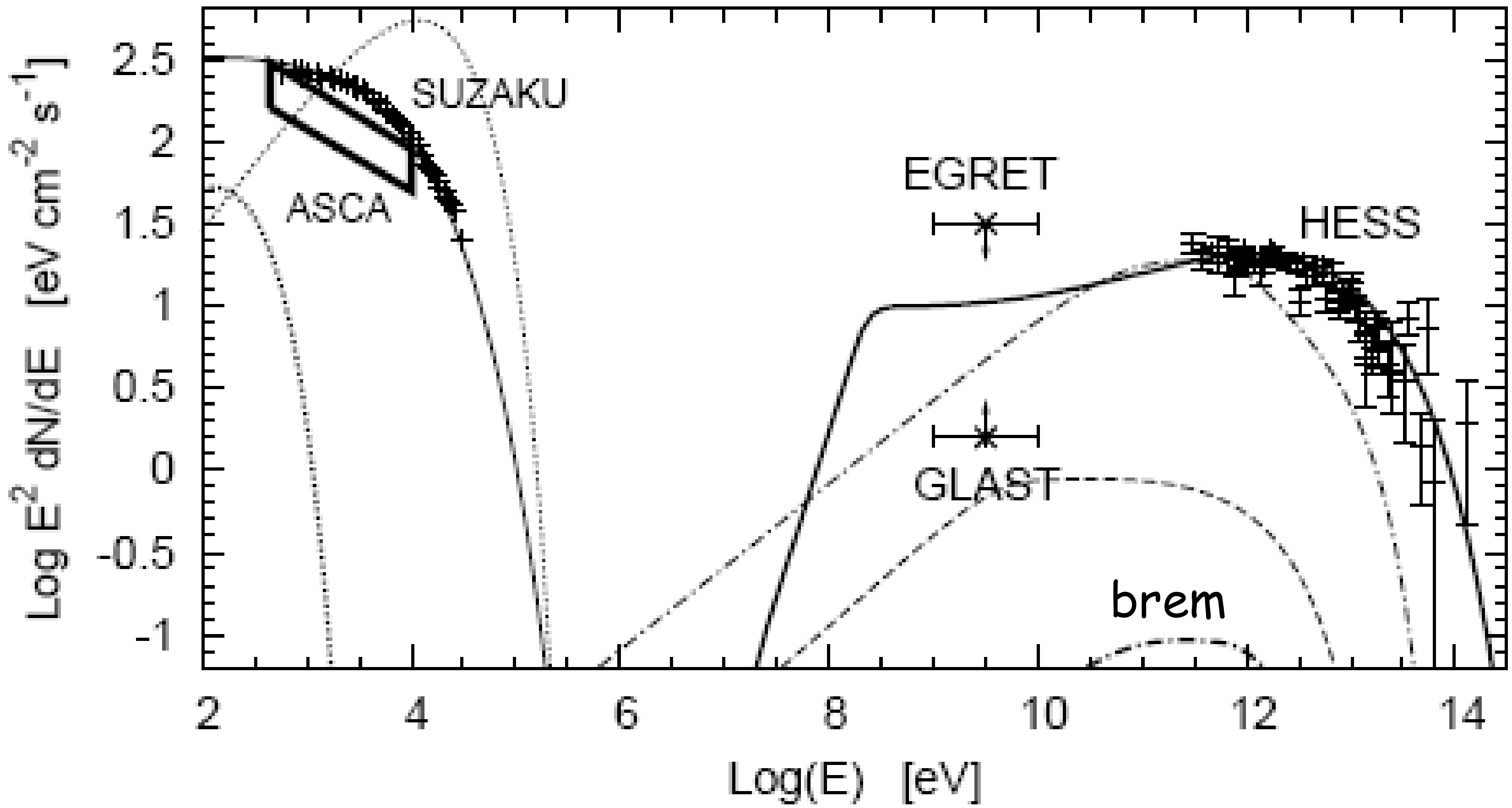


Morlino, PB & Amato 2008

RXJ1713 Morlino, PB & Amato 2008

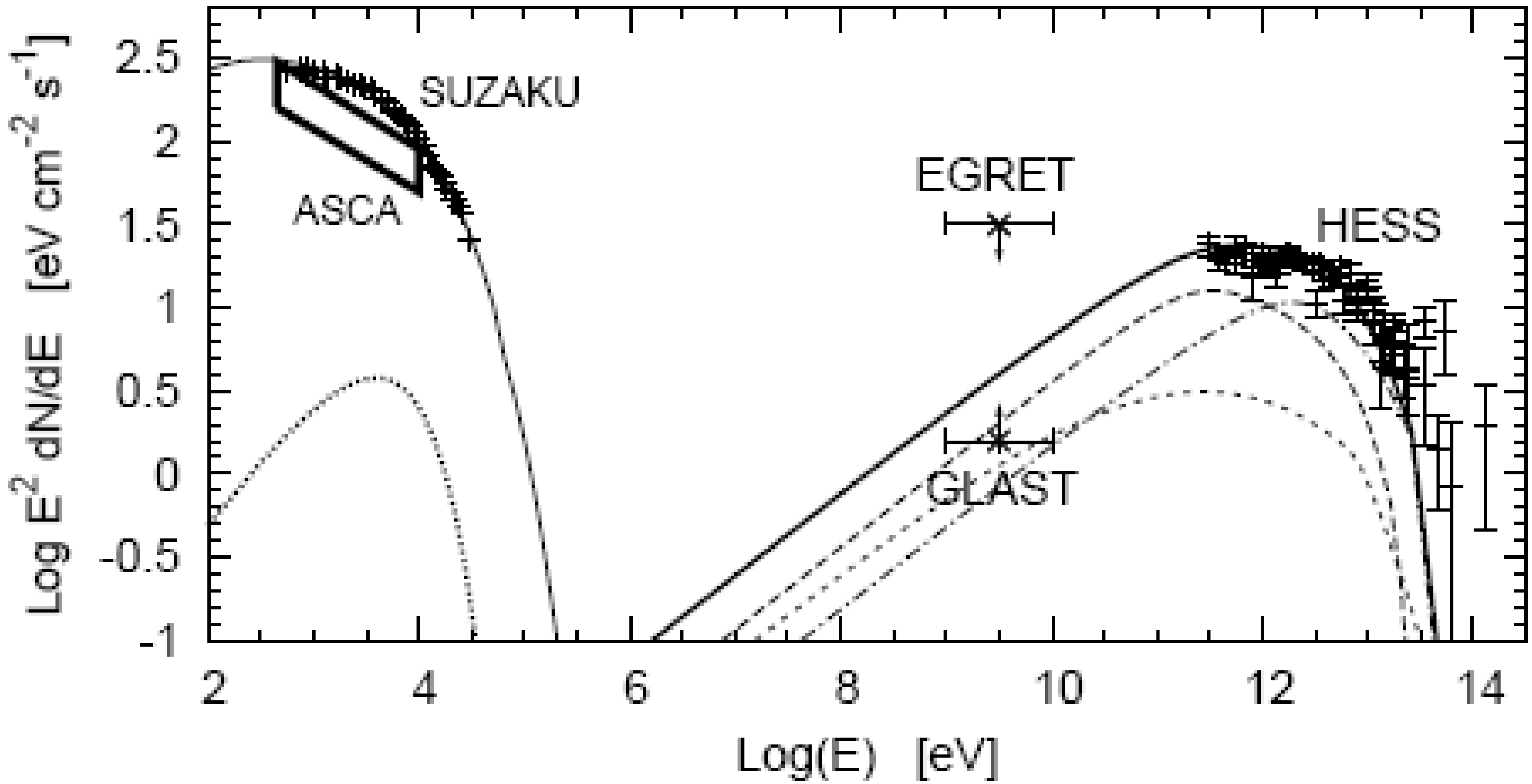
HADRONIC FIT – LARGE B FIELDS



PROBLEMS: 1. LARGE THERMAL X-RAYS (BUT…)

2. VERY LOW RATIO OF ELECTRONS AND PROTONS

LEPTONIC FIT – LOW B FIELDS



PROBLEMS:
1. VERY HIGH PHOTON DENSITY FOR ICS
2. LOW B FIELDS (IGNORES X-RAY OBSERVATIONS)
3. BAD FIT TO HIGHEST-E HESS DATA POINTS

WHICH ACCELERATED PARTICLES BECOME COSMIC RAYS?

WE HAVE SEEN THAT COSMIC RAYS ACCELERATED IN SNR HAVE CONCAVE SPECTRA

WE HAVE ALSO SEEN THAT THE RETURN PROBABILITY FROM UPSTREAM IS UNITY! ALL PARTICLES COME BACK, WITH THE POSSIBLE EXCEPTION OF THOSE AT P_{MAX}

WE HAVE SEEN THAT THE SPECTRUM OF THE ESCAPING PARTICLES AT UPSTREAM INFINITY IS CLOSE TO A DELTA FUNCTION

SO…WHICH ONES ARE THE COSMIC RAYS?

DIFFERENT PHASES OF THE SNR

THERE IS AN INITIAL PERIOD DURING WHICH THE SHELL OF THE SN EXPANDS FREELY (FREE EXPANSION PHASE -BALLISTIC MOTION):

MASS OF THE EJECTA: M_{ej}

TOTAL KINETIC ENERGY: E_{51}

FREE EXPANSION VELOCITY:

$$V_s = \sqrt{\frac{2E_{ej}}{M_{ej}}} = 10^9 \; E_{51}^{1/2} \; M_{ej,\Theta}^{-1/2} \; \text{cm/s}$$

BUT THE SHOCK SWEEPS THE MATERIAL IN FRONT OF IT AND AT SOME POINT IT ACCUMULATES ENOUGH MATERIAL TO SLOW DOWN THE EXPANDING SHELL:

SEDOV PHASE: $T_{Sedov} = 300\, E_{51}^{-1/2}\, n^{-1/3}\, M_{\Theta}^{5/6}$ years

$$R_{sh}(t) = 2.7 \times 10^{19}\mathrm{cm}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{2/5}$$

$$V_{sh}(t) = 4.7 \times 10^{8}\mathrm{cm/s}\left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{-3/5}$$

The sound speed in the ISM is about 10^6 cm/s

$\Downarrow$

Mach number $\approx 100 - 1000$ **STRONG SHOCK**

MAX ENERGY DURING SEDOV

THE MAXIMUM ENERGY OF ACCELERATED PARTICLES INCREASES DURING THE FRE EXPANSION PHASE AND REACHES A MAXIMUM AT THE BEGINNING OF THE SEDOV PHASE.

IN THE SEDOV PHASE:

$$\delta B(t) = 65\, n_1^{1/4}\, B_{0,\mu G}^{1/2} \left(\frac{E_{51}}{n_1}\right)^{1/10} t_{kyr}^{-3/10}\, \xi_c(t)^{1/2}\, \mu G$$

$$E_{max}(t) = 2.5 \times 10^6 \left(\frac{E_{51}}{n_1}\right)^{1/2} n_1^{1/4} B_{0,\mu G}^{1/2}\, \xi_c(t)^{1/2}\, t_{kyr}^{-1/2}\, GeV$$

OVERLAP OF ESCAPE FLUXES: A SIMPLE ESTIMATE

$$E_{MAX}(t) \propto \xi_c(t)\, t^{-1/2}$$

$$R_{sh}(t) = 2.7 \times 10^{19}\mathrm{cm} \left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{2/5}$$

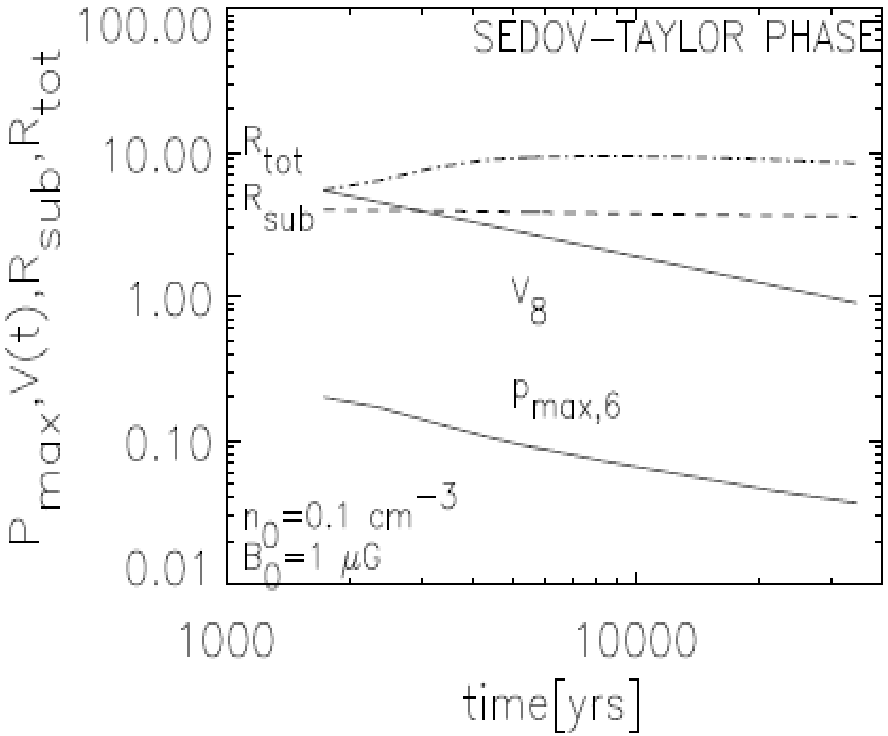
$$V_{sh}(t) = 4.7 \times 10^{8}\mathrm{cm/s} \left(\frac{E_{51}}{n_1}\right)^{1/5} t_{kyr}^{-3/5}$$

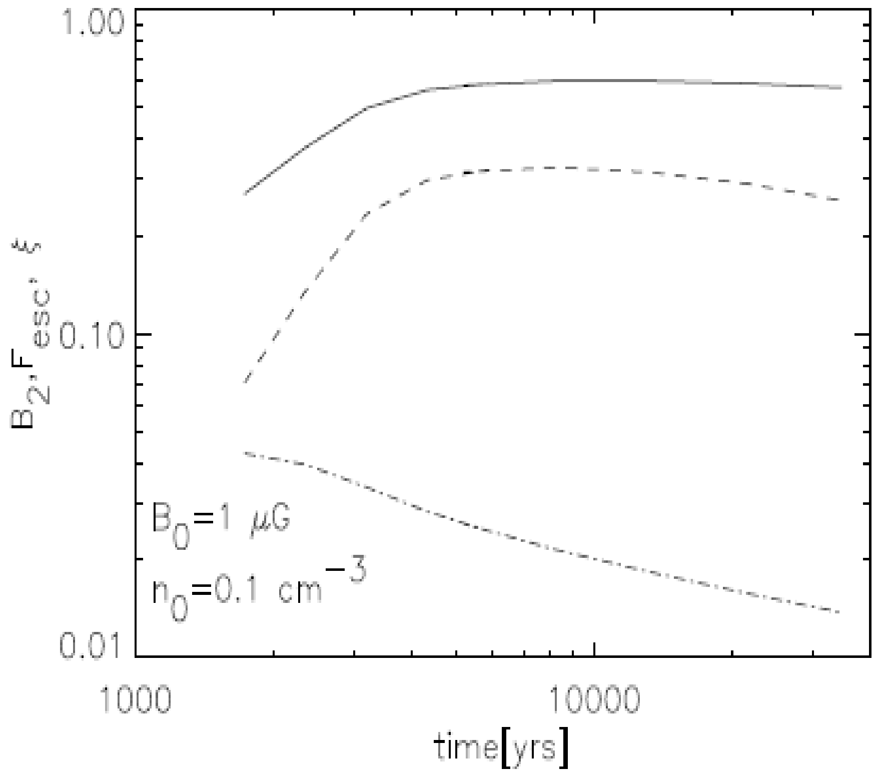
$$\boxed{\mathrm{EQ(E)dE} \approx \mathrm{F_{esc}(t)}\ \frac{1}{2}\rho \mathrm{V_s^3}\ 4\pi\pi\,_{\mathrm{sh}}^{2}\ \frac{\mathrm{dE_{max}}}{\mathrm{dt}}\mathrm{dE} \propto\ \mathrm{t^{1/2}\,dE} \propto\ \mathrm{E^{-1}\,dE}}$$

BE VERY CAREFUL…THIS IS JUST A WAY TO SHOW HOW YOU GET ROUGHLY A POWER LAW BUT SUMMING NON-POWER LAWS.

MORE DETAILED CALC'S SHOW DEPARTURES FROM THIS SIMPLE TREND

ESCAPE FLUX IN NON LINEAR REGIME





Caprioli, PB & Amato (2008)

A LOT OF PHYSICS IN THESE PLOTS !!!

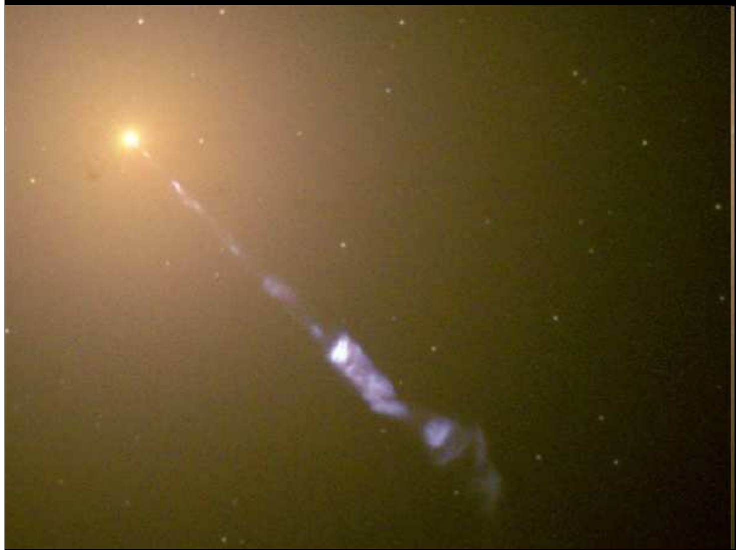
RECALL THAT OUR IGNORANCE OF HOW THINGS EVOLVE WHEN THE MAGNETIC FIELD BECOMES AMPLIFIED TO NON LINEAR LEVELS IS HUGE:

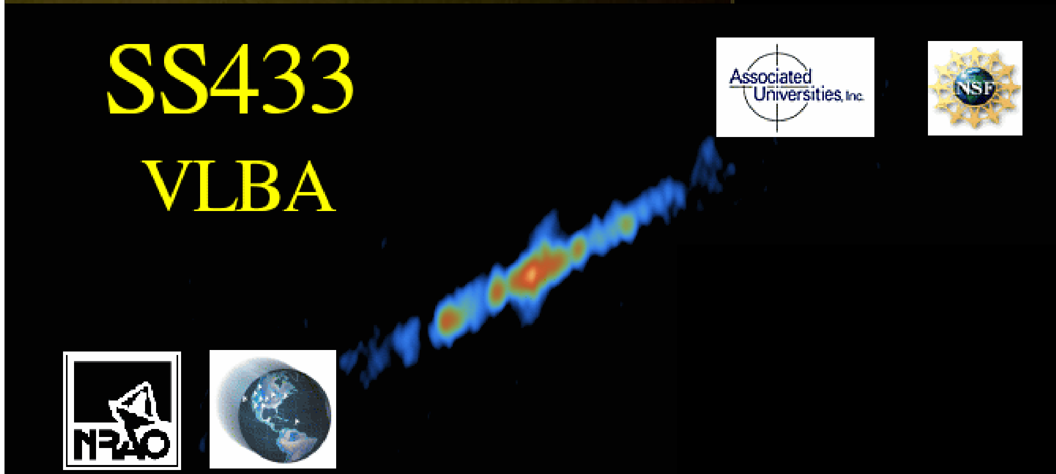
- WE DO NOT KNOW HOW B GROWS WHEN $\Delta B/B>1$
- WE DO NOT KNOW IF IT GROWS ANISOTROPICALLY
- WE DO NOT KNOW HOW TO EXTRACT A DIFFUSION COEFFICIENT FROM IT (PARTICLE-WAVE INTERACTION)
- EVEN THE TRANSPORT EQUATION ITSELF COULD BE CHANGED IN THE REGIME OF STRONG TURBULENCE

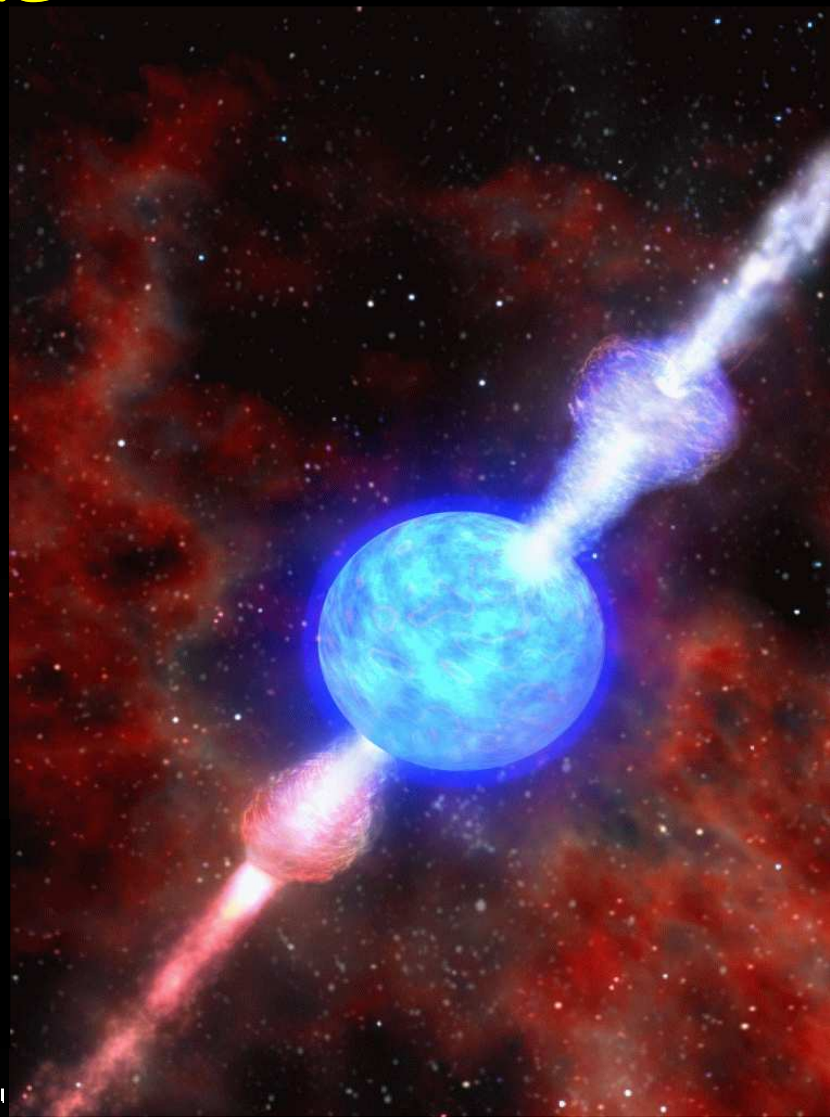
THIS IGNORANCE REFLECTS IN MANY OF OUR FINDINGS WHICH ARE STRONGLY AFFECTED BY ASSUMING BOHM DIFFUSION...

THEREFORE THERE IS A LOT FOR YOU TO DISCOVER!!!

SOME BASIC ASPECTS OF PARTICLE ACCELERATION AT RELATIVISTIC SHOCKS







BASICS OF ACCELERATION AT RELATIVISTIC SHOCKS

$$\gamma_1\beta_1 n_1 = \gamma_2\beta_2 n_2$$

$$\gamma_1^2\beta_1(\varepsilon_1 + p_1) = \gamma_2^2\beta_2(\varepsilon_2 + p_2)$$

$$\gamma_1^2\beta_1^2(\varepsilon_1 + p_1) + p_1 = \gamma_2^2\beta_2^2(\varepsilon_2 + p_2) + p_2$$

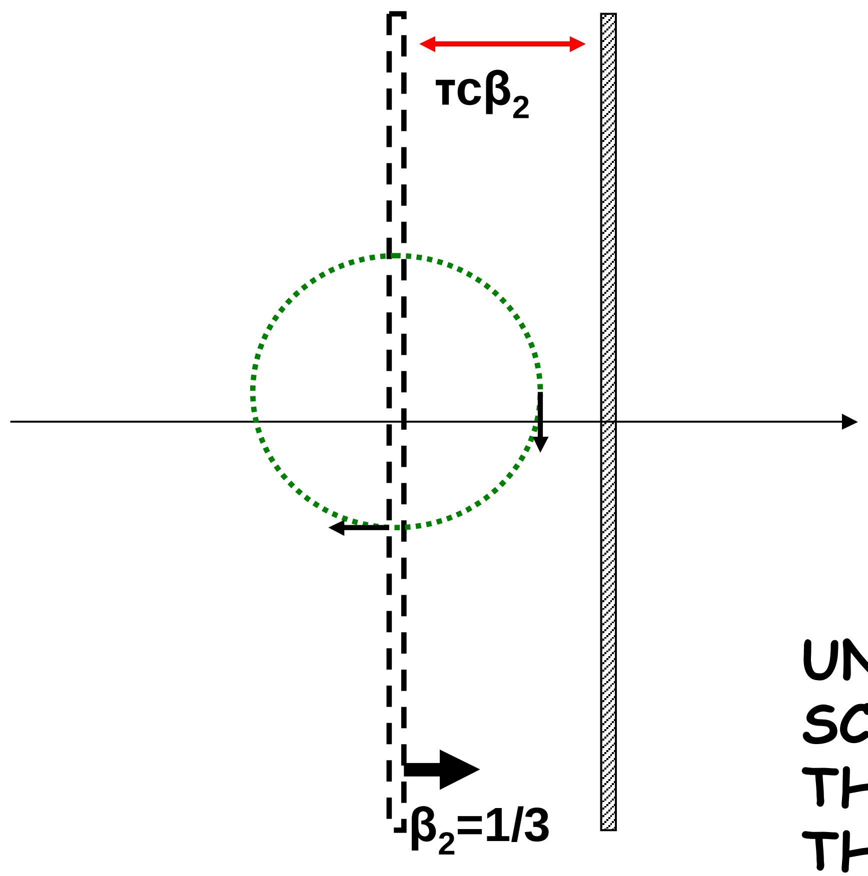
IN THE ASSUMPTION THAT:

$$\boxed{\frac{B_1^2}{4\pi} \ll (\varepsilon_1 + p_1)} \qquad \boxed{\gamma_1 \gg 1} \qquad \boxed{p_1 = 0}$$

No equipartition | ultrarelativistic | pressureless

WE FIND THAT:

$$p_2 = \frac{1}{3}\varepsilon_2 \qquad \beta_2 = \frac{1}{3}$$



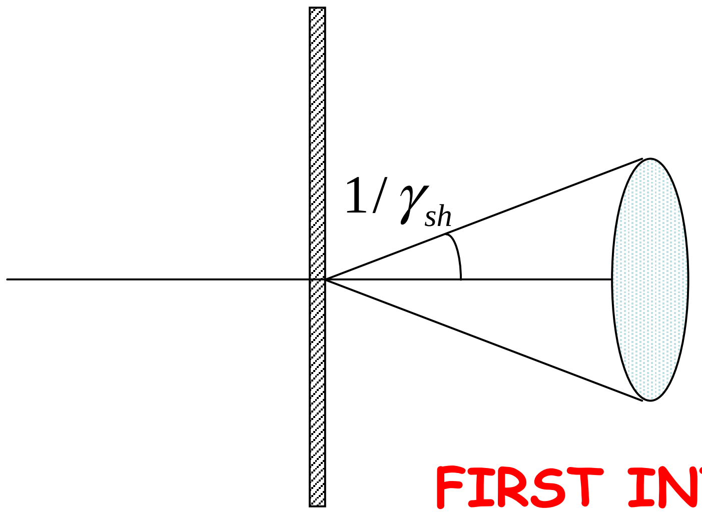
$$\tau \approx \frac{3}{4}\frac{2\pi\, r_L}{c}$$

$$\Delta x = \frac{1}{3} c\,\tau = \frac{\pi}{2} r_L > r_L$$

UNLESS THERE IS STRONG SCATTERING DOWSNTREAM THE PARTICLES ARE TRAPPED THERE

THE RETURN PROBABILITY FROM DOWNSTREAM IS EXPECTED TO BE SMALLER THAN FOR NEWTONIAN SHOCKS: STEEPER SPECTRA

ANISOTROPY



$$\delta\mu = \left[-1 + \frac{1+3\beta_{rel}}{3+\beta_{rel}}\right] \approx \frac{1}{4}\frac{1}{\gamma_{rel}^2}$$

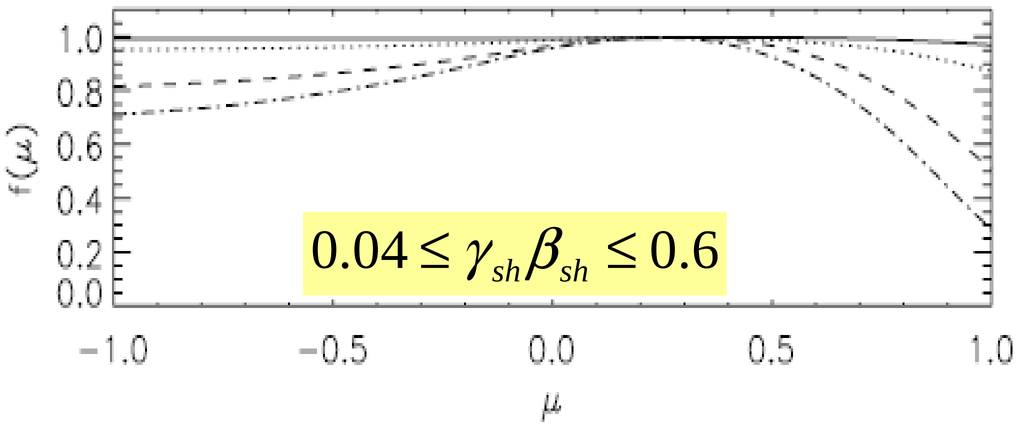
FIRST INTERACTION:

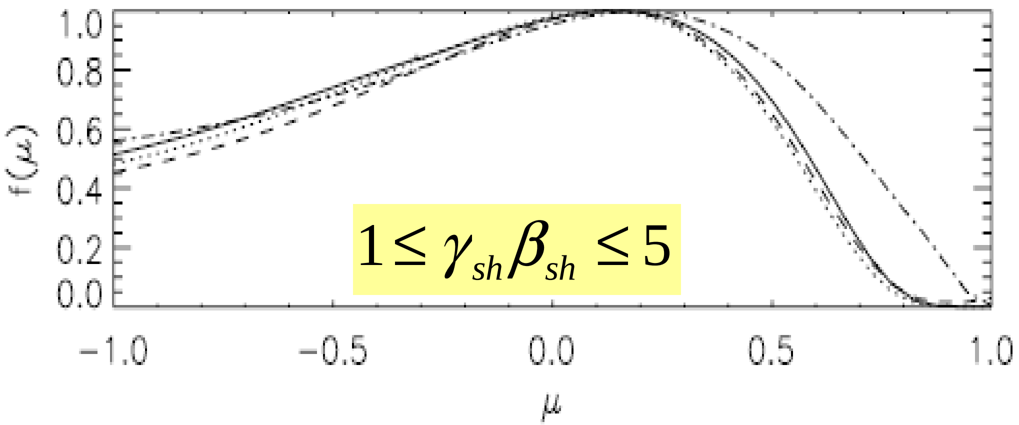
$$E_i \Rightarrow E_d = \gamma_{rel} E_i (1+\beta_{rel}) \Rightarrow E_f = \gamma_{rel}^2 E_i (1+\beta_{rel})^2 \approx 4\gamma_{rel}^2 E_i$$

FURTHER INTERACTIONS:

$$E_f \approx 2\ E_i$$

EFFECTS OF ANISOTROPY



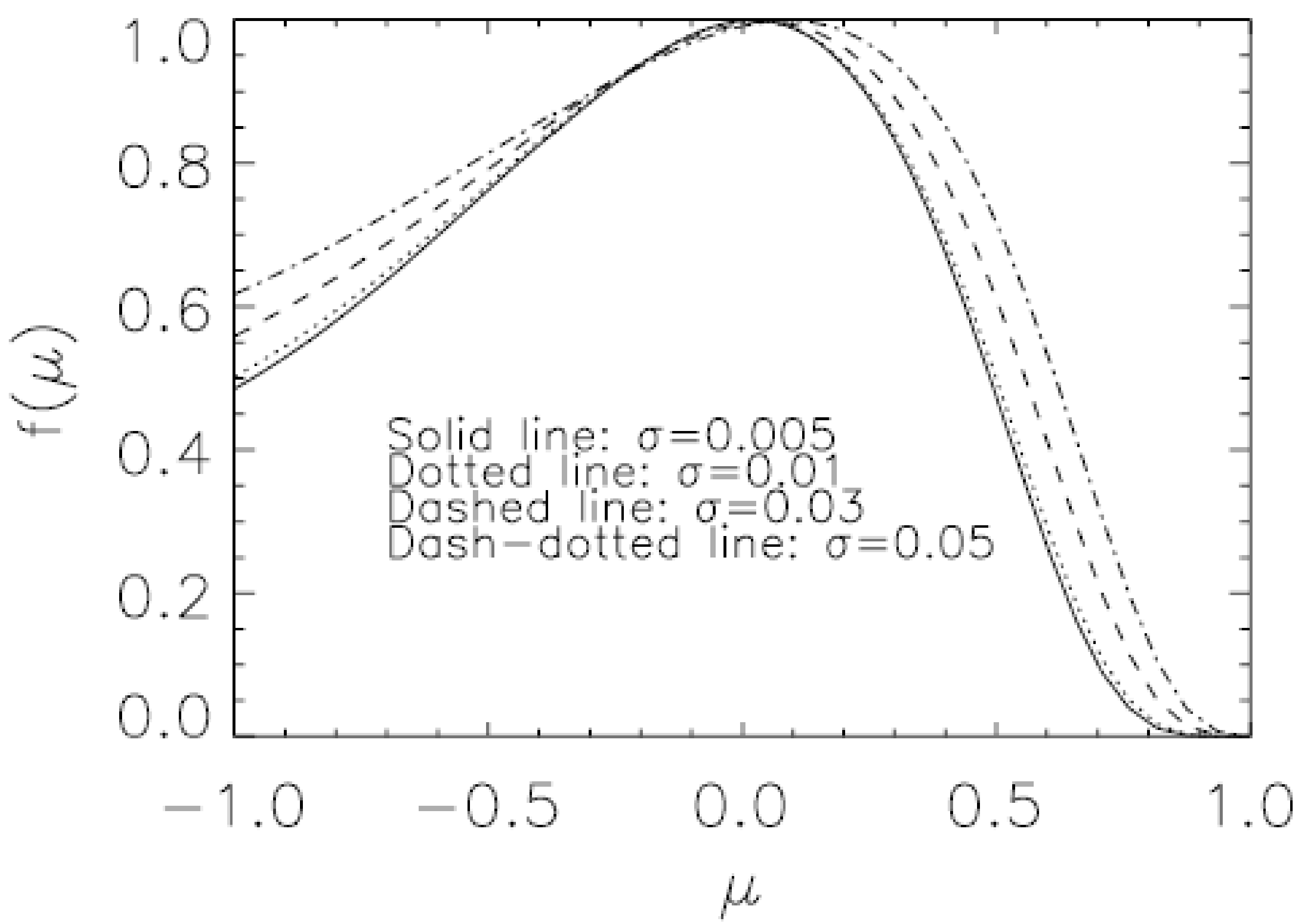


Particle Slopes for Shocks in the SPAS Limit

$\gamma_{sh}\beta_{sh}$	u	u_d	Slope
0.04	0.04	0.01	4.00
0.2	0.196	0.049	3.99
0.4	0.371	0.094	3.99
0.6	0.51	0.132	3.98
1.0	0.707	1.191	4.00
2.0	0.894	0.263	4.07
4.0	0.97	0.305	4.12
5.0	0.98	0.311	4.13

PB & Vietri 2005

EFFECTS OF THE SPAS



PB & Vietri 2005

SOME REMARKS

- THE SPECTRUM OF ACCELERATED PARTICLES IN THE RELATIVISTIC CASE IS STILL A POWER LAW
- THE SLOPE OF THIS POWER LOW IN THE ULTRAREL CASE IS ABOUT 2.3
- HOWEVER THE SLOPE CAN BECOME APPRECIABLY HARDER (FLATTER SPECTRA) FOR LARGE ANGLE SCATTERING
- OR APPRECIABLY SOFTER (STEEP SPECTRA) DUE TO …BASICALLY ANYTHING ELSE YOU DO (FOR INSTANCE COMPRESSION OF TURBULENCE AT THE SHOCK)
- SHOCK ACCELERATION AT RELATIVISTIC SHOCKS DEPENDS ALSO ON THE EQUATION OF STATE OF THE DOWNSTREAM PLASMA (PROTONS, PAIRS, B-FIELDS ALL CHANGE THE RESULTS DRAMATICALLY)
- THERE IS NO NON LINEAR THEORY OF PARTICLE ACCELERATION AT RELATIVISTIC SHOCKS