Probing the structures of exotic and halo nuclei

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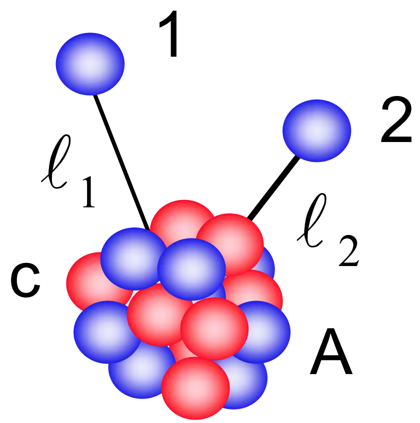
Two nucleon removal - what are useful regimes?

$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1-|S_1|^2)(1-|S_2|^2)\,|\,\phi_0\rangle$$



Estimate assuming removal of a pair of uncorrelated nucleons -

$$\phi_0(A,\mathbf{r}_1,\mathbf{r}_2) = \Phi_c(A)\phi_{\ell_1}(\mathbf{r}_1)\phi_{\ell_2}(\mathbf{r}_2)$$

$$\sigma_{\text{strip}} \Rightarrow \sigma_{\text{strip}}(\ell_1\ell_2)$$

contribution from direct 2N removal σ_{-2N}

p particles ℓ_α

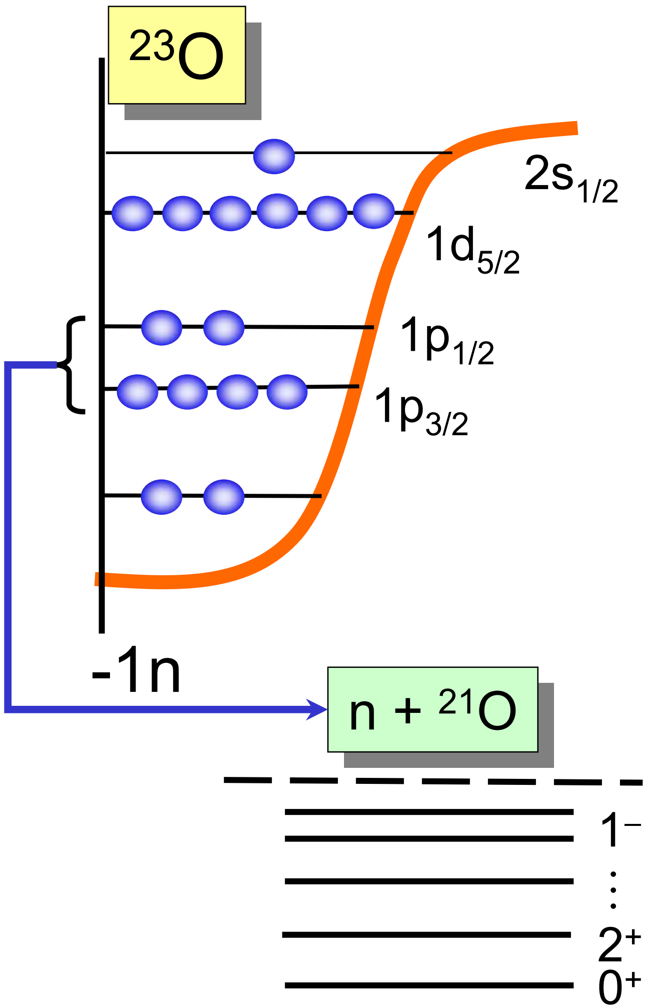
q particles ℓ_β

$$\boxed{\sigma_{-2N} = \frac{p(p-1)}{2}\sigma_{\text{strip}}(\ell_\alpha\ell_\alpha) + \frac{q(q-1)}{2}\sigma_{\text{strip}}(\ell_\beta\ell_\beta) + pq\,\sigma_{\text{strip}}(\ell_\alpha\ell_\beta)}$$

D. Bazin et al., MSU preprint, submitted

Complications of 2 neutron removal reactions

B.A. Brown, P.G. Hansen and J.A. Tostevin, submitted



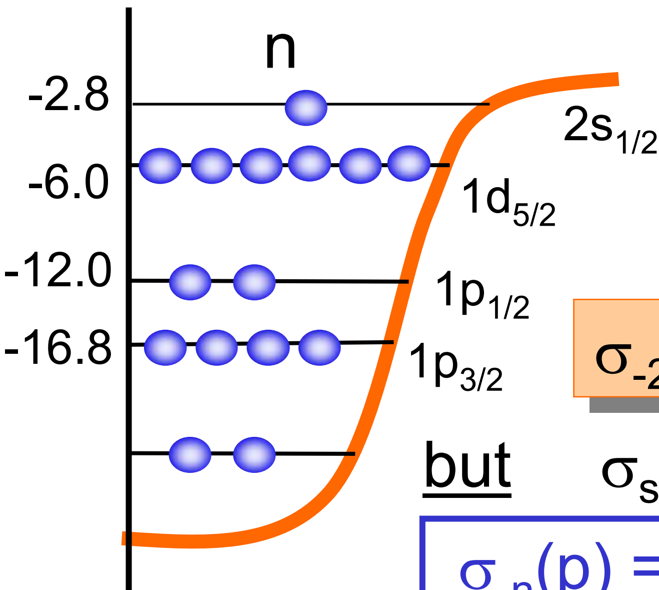
^{22}O

^{22}O final states below n-threshold from the shell model (B.A. Brown)

Energy (MeV)	I^π	ℓ	C^2S
0	0^+	0	0.797
3.38	2^+	2	2.130
4.62	0^+	0	0.115
4.83	3^+	2	3.079
5.32	1^-	1	0.851
5.93	0^-	1	0.332
6.50	2^+	2	0.242

Two neutron knockout from neutron rich nuclei

e.g. $^{23}O \rightarrow {}^{21}O$ (RIKEN measurement at 72A MeV on ^{12}C)



$\sigma \quad = 82(25)$ mb - is large!!

$\sigma_{strip}(02) = 0.9$ mb

$\sigma_{strip}(22) = 0.6$ mb

$\sigma_{-2n} = 6\sigma_{strip}(02) + 15\sigma_{strip}(52) = 14$ mb

<u>but</u> $\sigma_{sp}(p_{1/2}) = 12$ mb, $\sigma_{sp}(p_{3/2}) = 11$ mb

$\sigma_{-n}(p) = 2\,\sigma_{sp}(p_{1/2}) + 4\,\sigma_{sp}(p_{3/2}) = 68$ mb

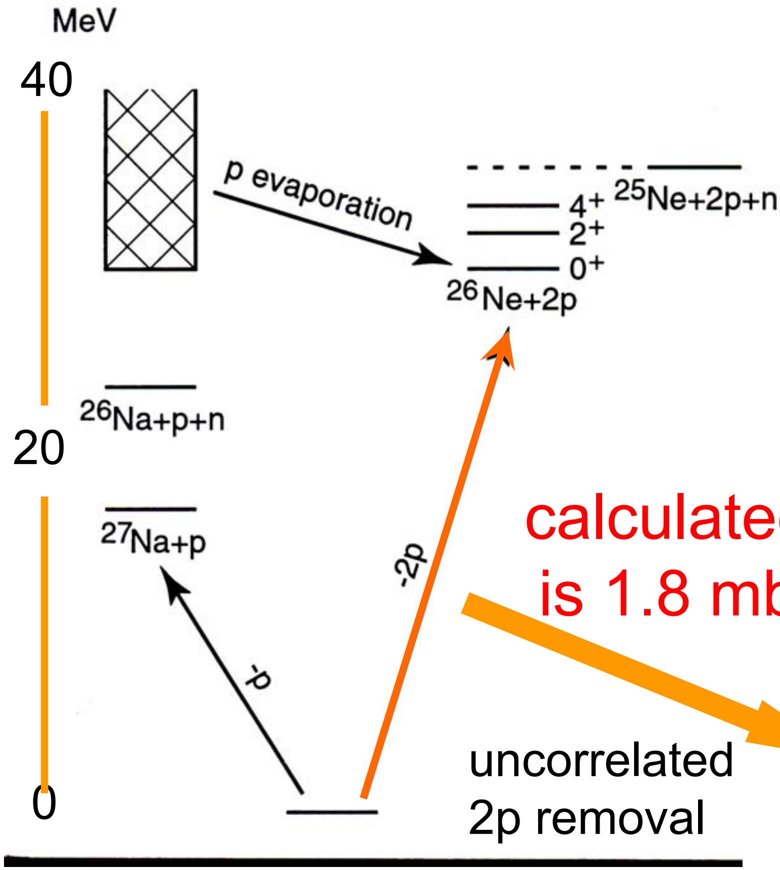
leading to the ^{22}O continuum - n evaporation

Shell model - 1 unit of p-strength leads to bound ^{22}O

$\sigma_{-n}(p) = 57$ mb

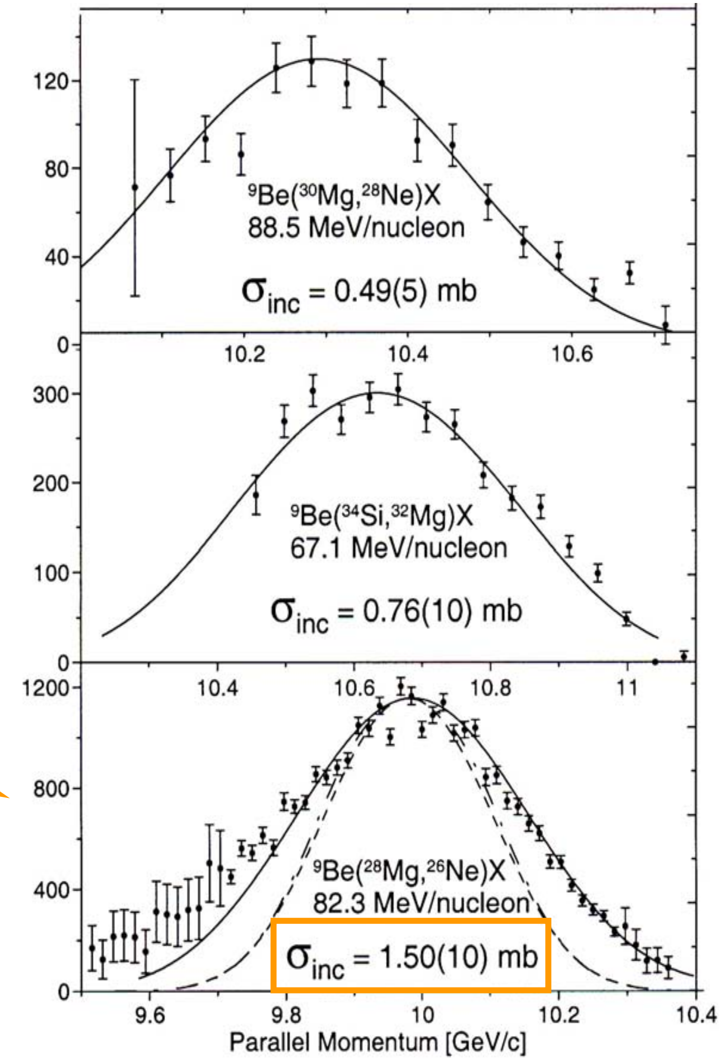
Measurement: Kanungo et al, PRL **88** (2002) 142502

Two proton knockout from neutron rich nuclei



calculated
is 1.8 mb

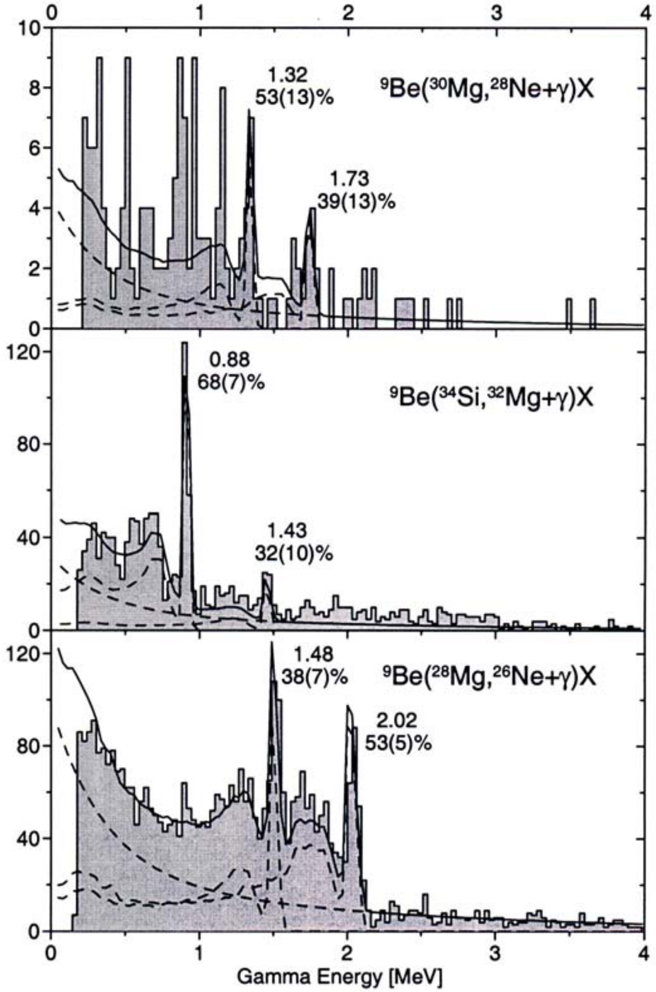
uncorrelated
2p removal



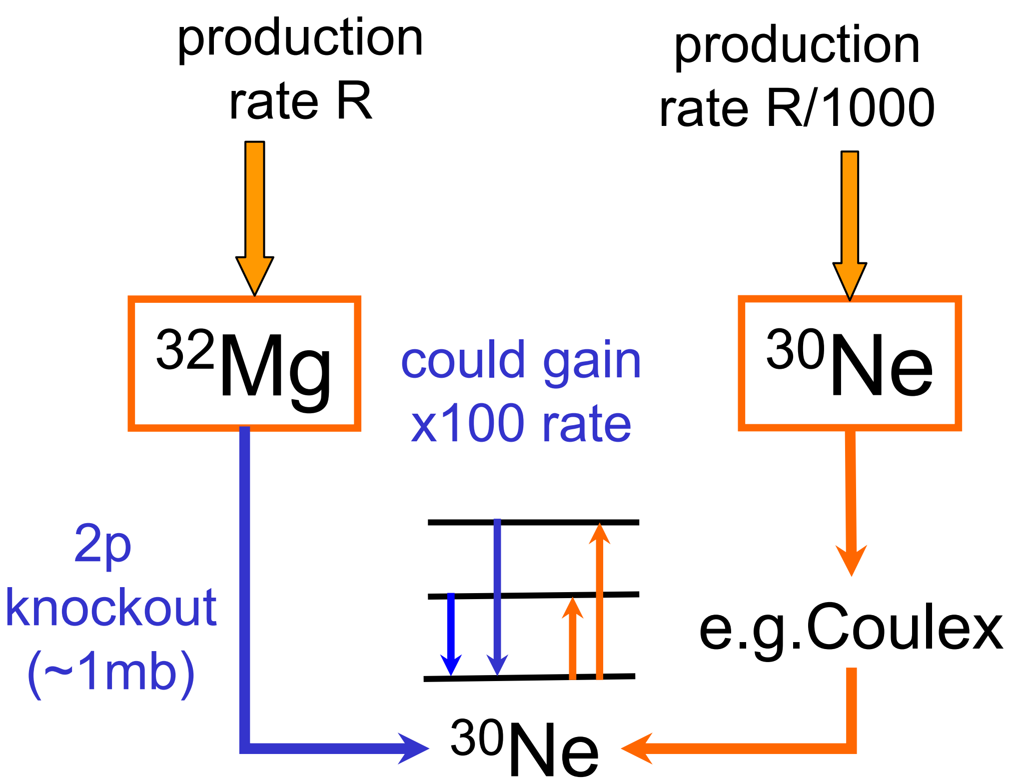
D. Bazin et al., MSU preprint, submitted

UniS

Two proton knockout - a useful option?



D. Bazin et al., MSU preprint, submitted.



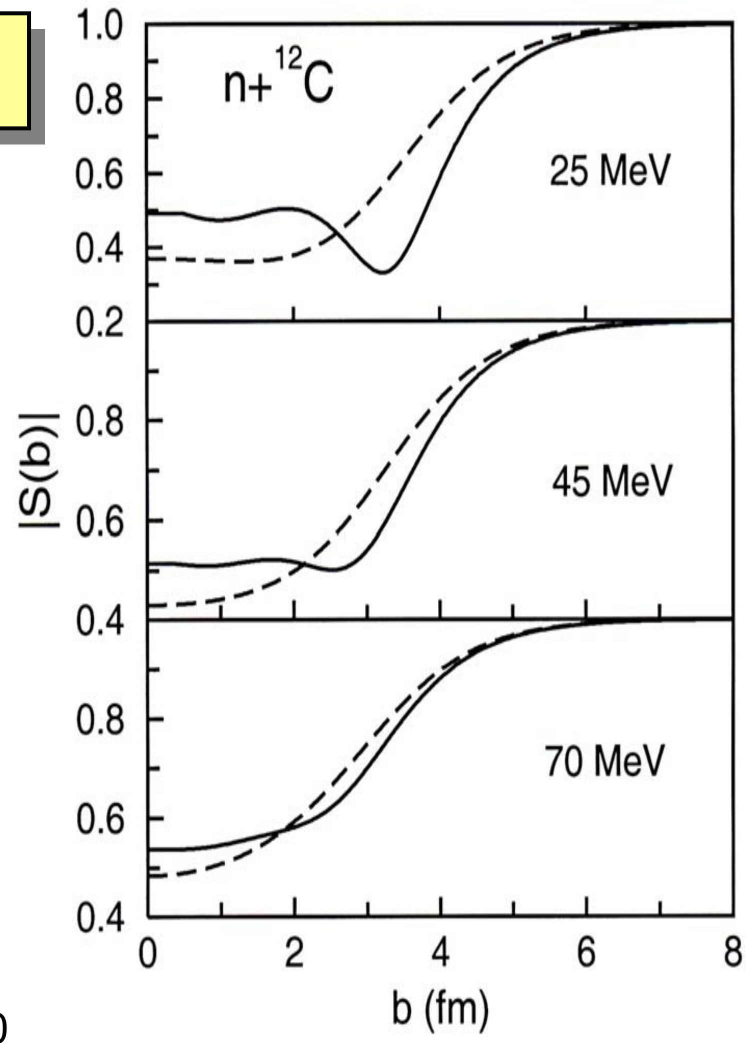
Improving the eikonal approximation

$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, S_c(b_c)\, S_v(b_v) \,|\, \phi_\alpha \rangle$$

$$u_\ell(r) \rightarrow (i/2)\{H_\ell^-(kr) - S_\ell\, H_\ell^+(kr)\}$$

eikonal S(b) are poor for lower energy and light particle - small k

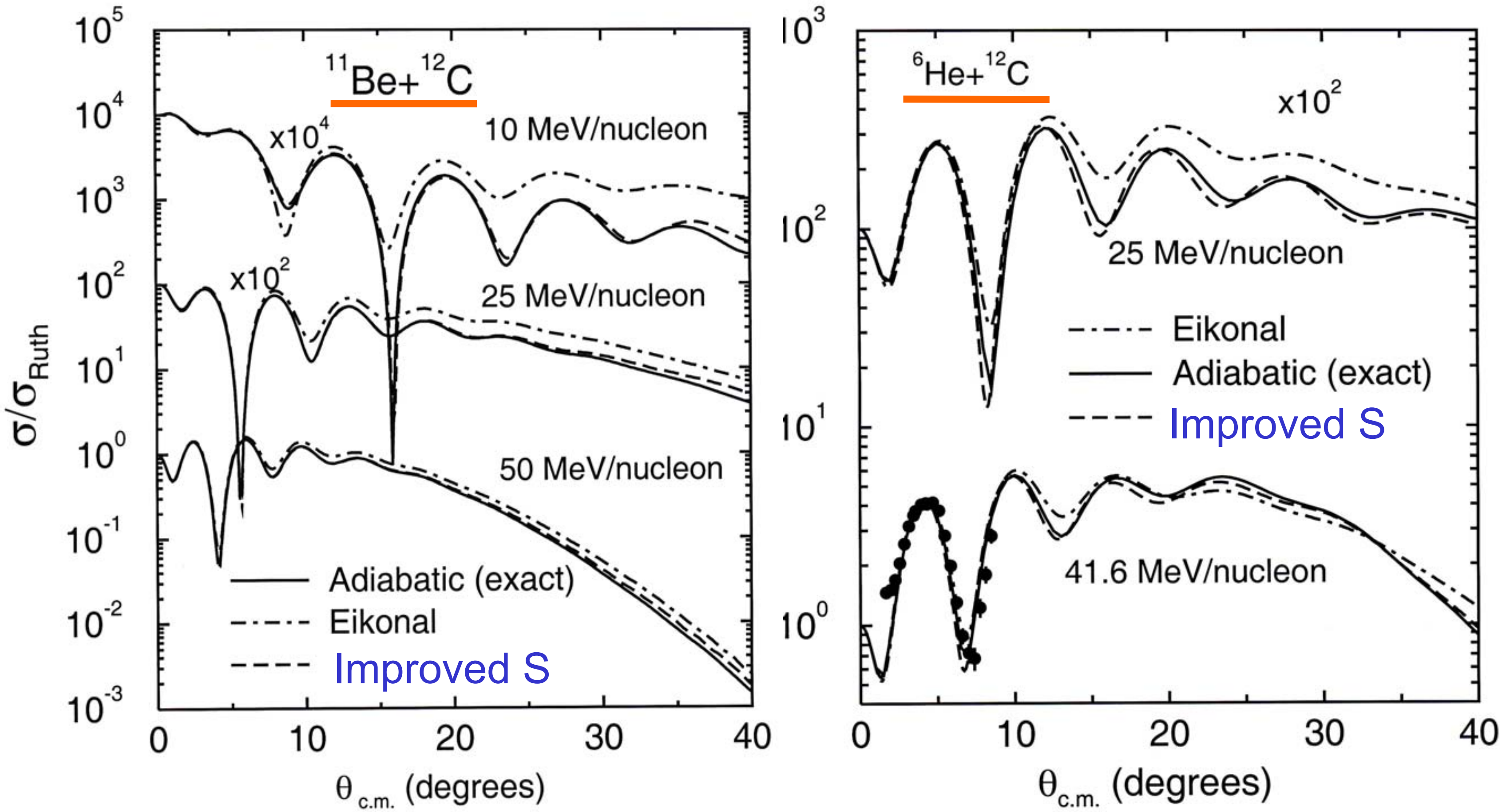
dashed - eikonal
solid - exact



So, use instead the exact S_ℓ, analytically continued to non-integer ℓ, or b, in $S_{\alpha\beta}$

J.M. Brooke et al., Phys. Rev. C **59** (1999) 1560

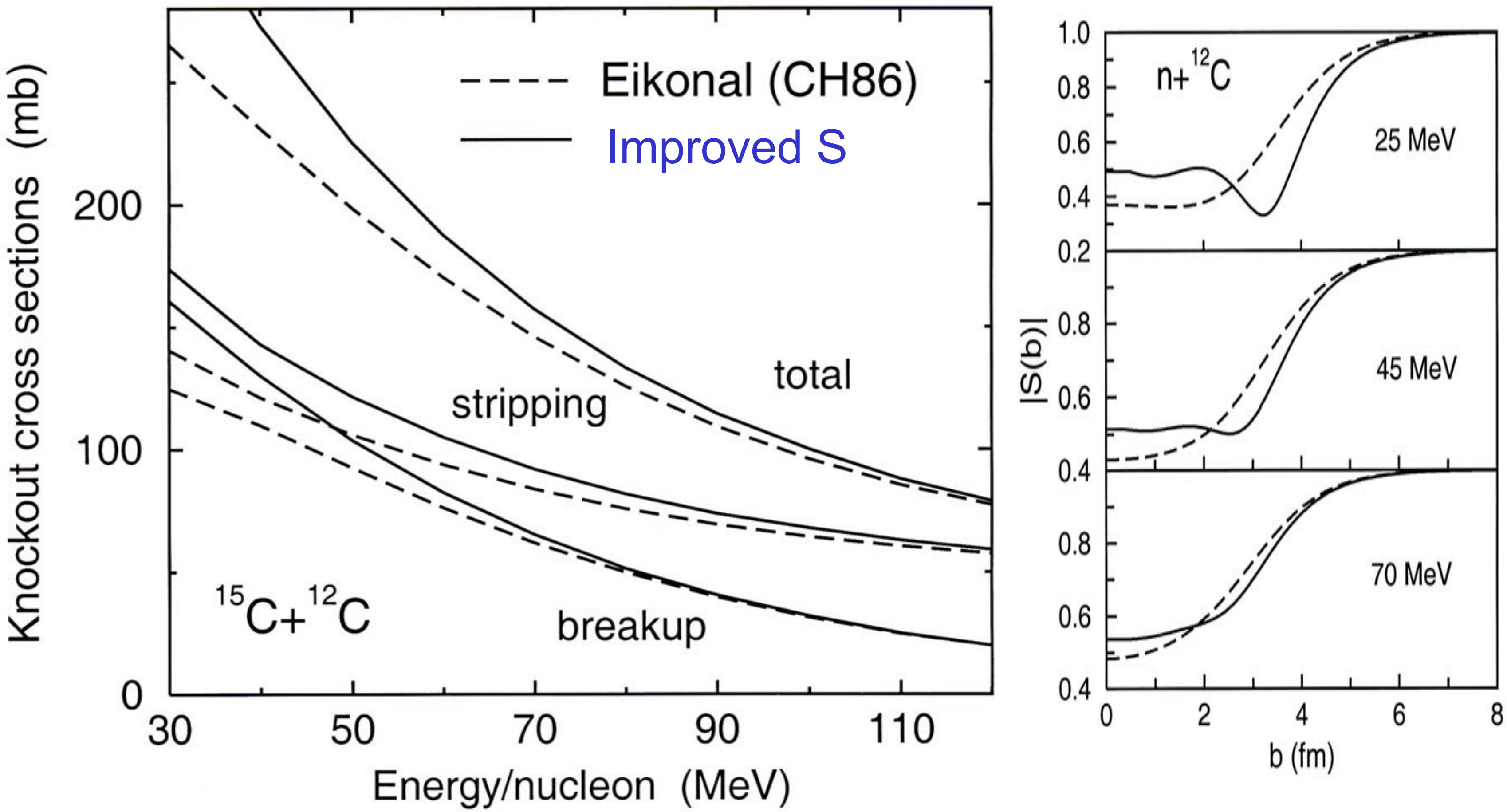
Beyond the eikonal approximation



J.M. Brooke et al., Phys. Rev. C **59** (1999) 1560

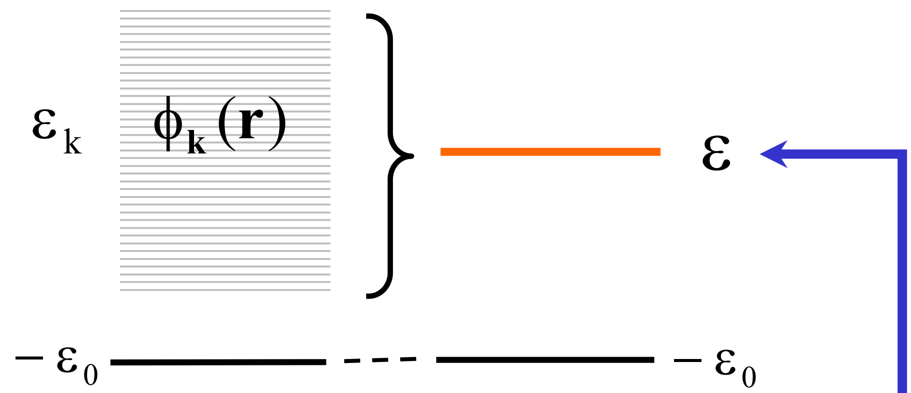
J.A. Christley et al., Nucl. Phys. A **624** (1997) 275

Nucleon removal cross sections also corrected



Beyond the adiabatic approximation

The adiabatic approximation treats all break-up configurations, but with no explicit reference to $\phi_k(\mathbf{r})$' by solution of:



$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - (E+\varepsilon_0)]\,\Psi^{AD}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) = 0$$

$$\Psi^{AD}_{\mathbf{K}} = \Psi^{AD}_{el} + \Psi^{AD}_{bu}$$

elastic part

$$\Psi^{AD}_{el} = |\phi_0\rangle\langle\phi_0|\Psi^{AD}_{bu}\rangle$$
$$H_p\Psi^{AD}_{el} = -\varepsilon_0\Psi^{AD}_{el}$$

is well approximated

breakup part is less well treated

$$\langle\phi_{\mathbf{k}}|\Psi^{AD}_{bu}\rangle \neq 0$$

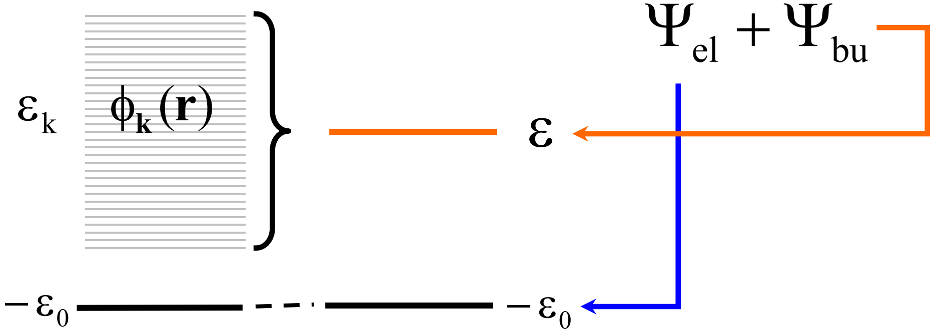
Quasi-adiabatic

continuum of H_p is assumed degenerate with a new energy ε which is a better representation of the states excited

Quasi-adiabatic type approximations

Using the non-adiabatic few-body model equation

$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_p - E]\,\underbrace{\Psi^{(+)}_{\mathbf{K}}(\mathbf{r},\mathbf{R})}_{\Psi_{el} + \Psi_{bu}} = 0$$



$$[T_{\mathbf{R}} + U + \varepsilon - E]\,\Psi^{QAD}_{bu} = \overbrace{[E + \varepsilon_0 - T_{\mathbf{R}} - U]\Psi_{el}}^{\text{inhomogeneous equation with source term}}$$

$$\approx [E + \varepsilon_0 - T_{\mathbf{R}} - U]\Psi^{AD}_{el}$$

$$\boxed{\varepsilon(R) \approx \langle \Psi^{AD}_{bu} \,|\, H_p \,|\, \Psi^{AD}_{bu} \rangle_{\mathbf{r}} \,/\, \langle \Psi^{AD}_{bu} \,|\, \Psi^{AD}_{bu} \rangle_{\mathbf{r}}}$$

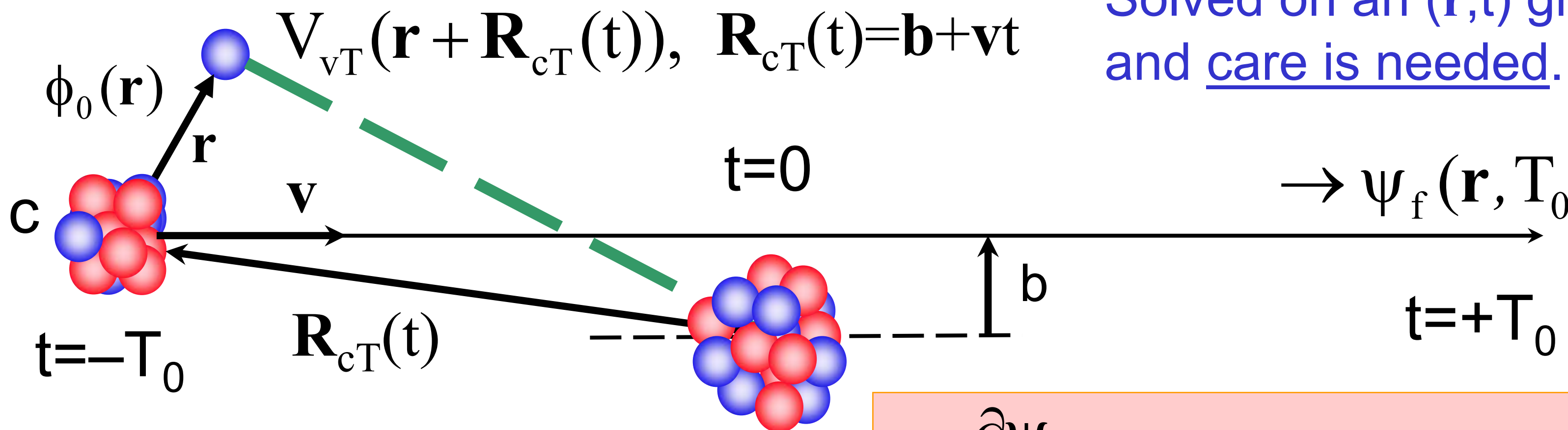
and then iterate if necessary

Important corrections in transfer reactions which are sensitive to near- and far-side interference effects

Non-adiabatic - but trajectory based

Time-dependent (finite difference) solution of the valence particle motion - assuming the heavy core, or c.m., follows a trajectory: [See: Bertsch and Esbensen, Baur and Typel, Suzuki, Melezhik and Baye]

Solved on an ($\mathbf{r}$,t) grid and care is needed.



Not exact - but non-adiabatic
Dynamics of V_{cT} is not included and no energy transfer/sharing between core and internal motion.
For heavy targets - Coulomb path

$$i\hbar\frac{\partial\psi}{\partial t}=(H_p+V_{vT})\psi(\mathbf{r},t)$$

as $t\rightarrow-\infty \quad \psi(\mathbf{r},t)\rightarrow\phi_0(\mathbf{r})$

$t\rightarrow+\infty \quad \psi(\mathbf{r},t)\rightarrow\psi_f(\mathbf{r},T_0)$

The time-dependent approach - observables

$$i\hbar \frac{\partial \psi}{\partial t} = (H_p + V_{vT})\psi(\mathbf{r}, t)$$

as $t \to -\infty \quad \psi(\mathbf{r}, t) \to \phi_0(\mathbf{r})$

$t \to +\infty \quad \psi(\mathbf{r}, t) \to \psi_f(\mathbf{r}, T_0)$

absorptive effects of target have to be put in ‘by hand’ - restricting impact parameters b to values

$b > b_{min} \approx R_T + R_c$

Only absorption/loss of flux in the equation is due to V_{vT} and so

At an impact parameter b then (for a neutron valence particle):

neutron removal probability $\quad P_{-n}(b) = 1 - |\langle \phi_0 | \psi_f \rangle|^2$

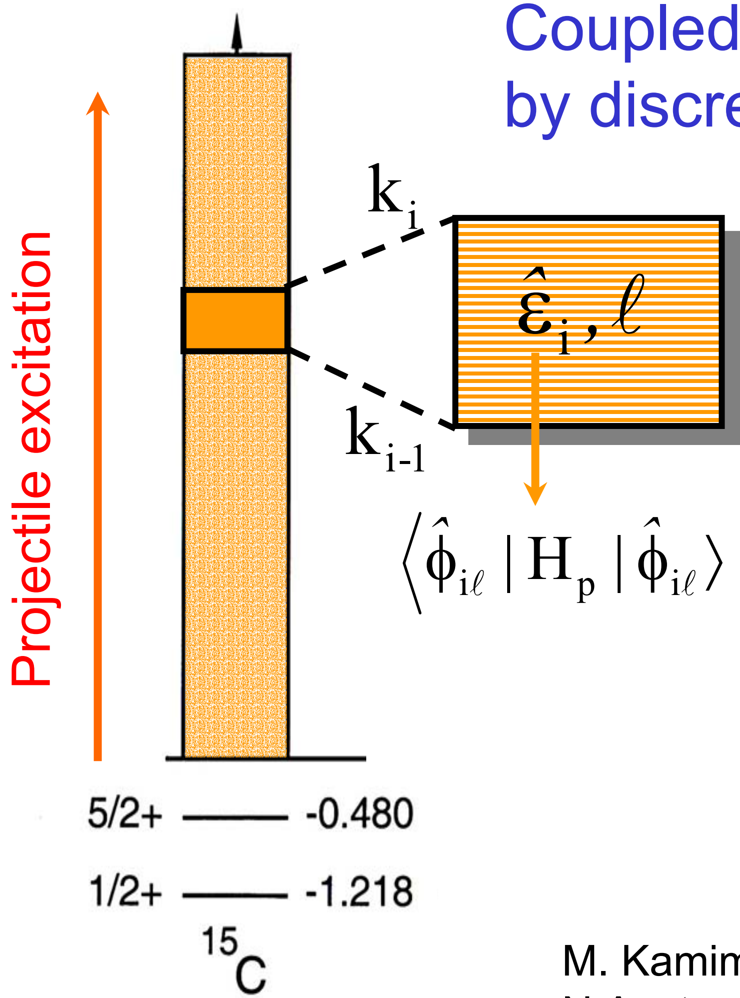
neutron stripping probability $\quad P_{str}(b) = 1 - \langle \psi_f | \psi_f \rangle$

diffractive break-up probability $\quad P_{diff}(b) = \langle \psi_f | \psi_f \rangle - |\langle \phi_0 | \psi_f \rangle|^2$

with cross sections $\quad \sigma_\alpha = 2\pi \int_{b_{min}}^{\infty} db\, b\, P_\alpha(b)$

Beyond the adiabatic limit - the CDCC

Coupled channels solution of break-up by discretisation of the continuum



$$\hat{\phi}_{i\ell m}(\mathbf{r}) = u_{i\ell}(r) Y_{\ell m}(\hat{\mathbf{r}})$$

complete
orthonormal
set of states
in relative motion
co-ordinate $\mathbf{r}$

$$u_{i\ell}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} dk \, f_i(k) \phi_\ell(k, r)$$

$$\Psi_{\mathbf{K}}^{(+)}(\mathbf{r}, \mathbf{R}) = \sum_i \hat{\chi}_i(\mathbf{R}) \hat{\phi}_i(\mathbf{r})$$

M. Kamimura et al, Prog Theor Phys (Suppl) **89** (1986), 1
N.Austern et al., Phys. Rep. **154** (1987), 125

Properties of CDCC bin (basis) states

bin states

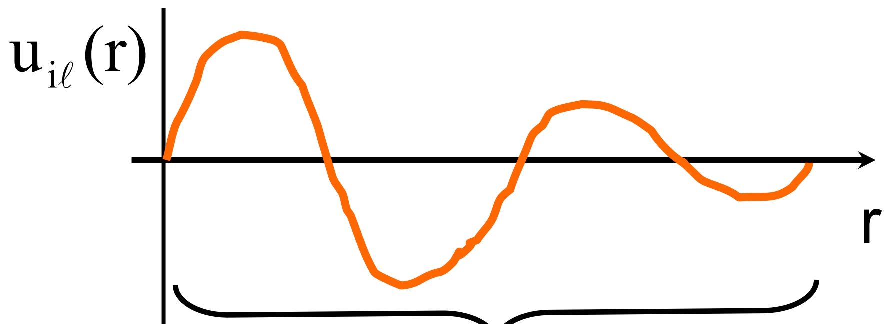
$\hat{\phi}_{i\ell m}(\mathbf{r})$

$$u_{i\ell}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{\Delta k_i} dk \; f_i(k)\,\phi_\ell(k,r)$$

normalised and orthogonal

$\hat{\varepsilon}_i\,,\ell$

Δk_i



r_{bin}

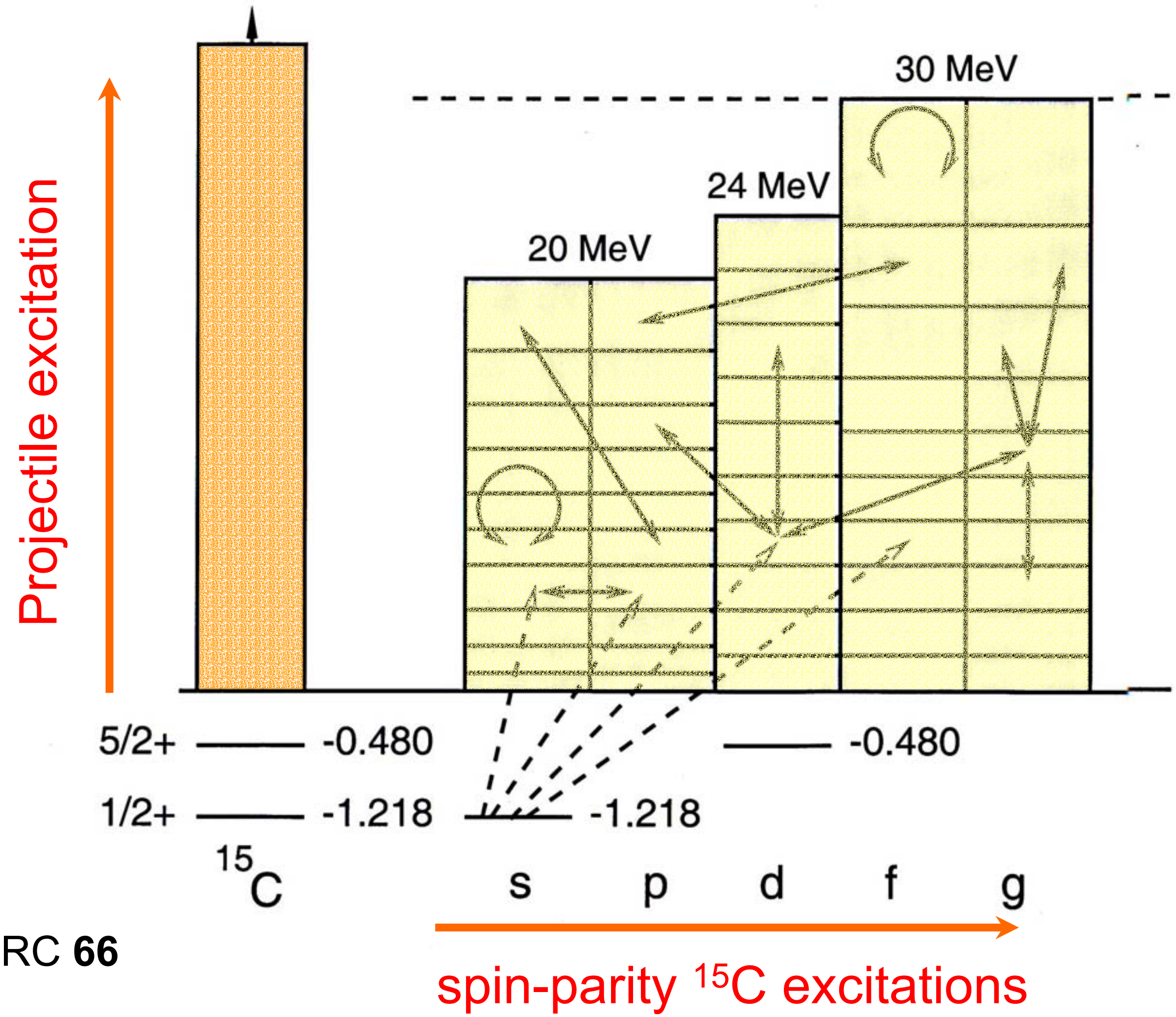
Uncertainty principle: so these must be chosen carefully

Couplings between bin states (channels) are

$$V_{ij}(\mathbf{R}) = \langle \hat{\phi}_i \,|\, U(\mathbf{r},\mathbf{R}) \,|\, \hat{\phi}_j \rangle_{\mathbf{r}}$$

Coupled channels model space is needed

Example of a coupled channel (CDCC) model space for ^{15}C break-up on a ^{9}Be target at E= 54A MeV

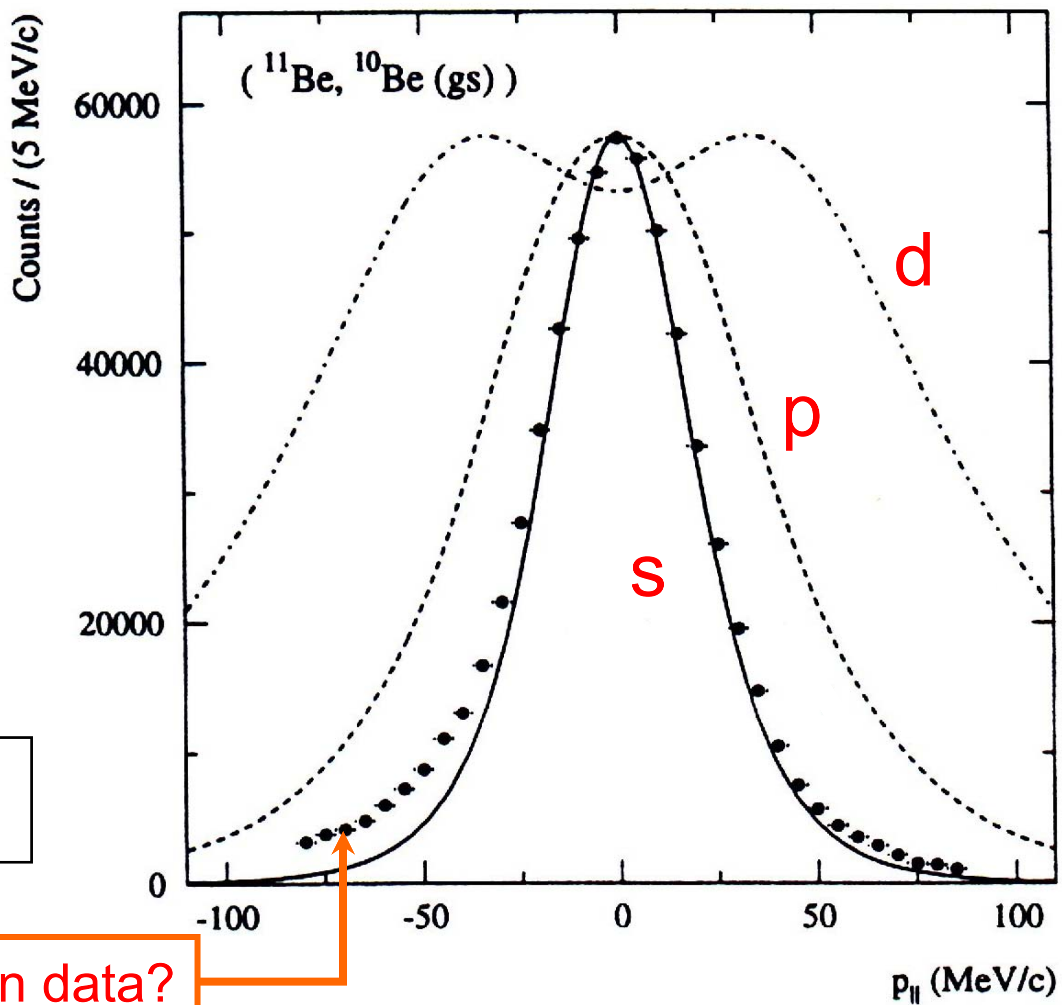


J.A. Tostevin et al, PRC **66** (2002) 024607

Residue parallel momentum distributions

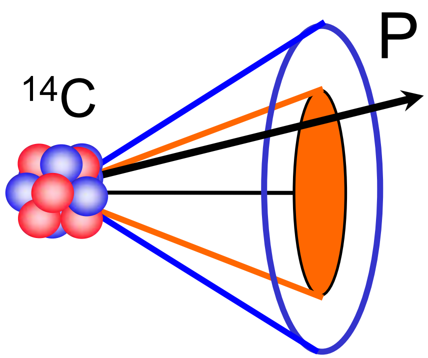
Calculations of ^{10}Be residue $p_{||}$ momentum distributions following neutron knockout from a ^{11}Be beam at 60A MeV/, with no coincident photon - ^{10}Be in its ground state.

T. Aumann et al. PRL **84** (2000) 35

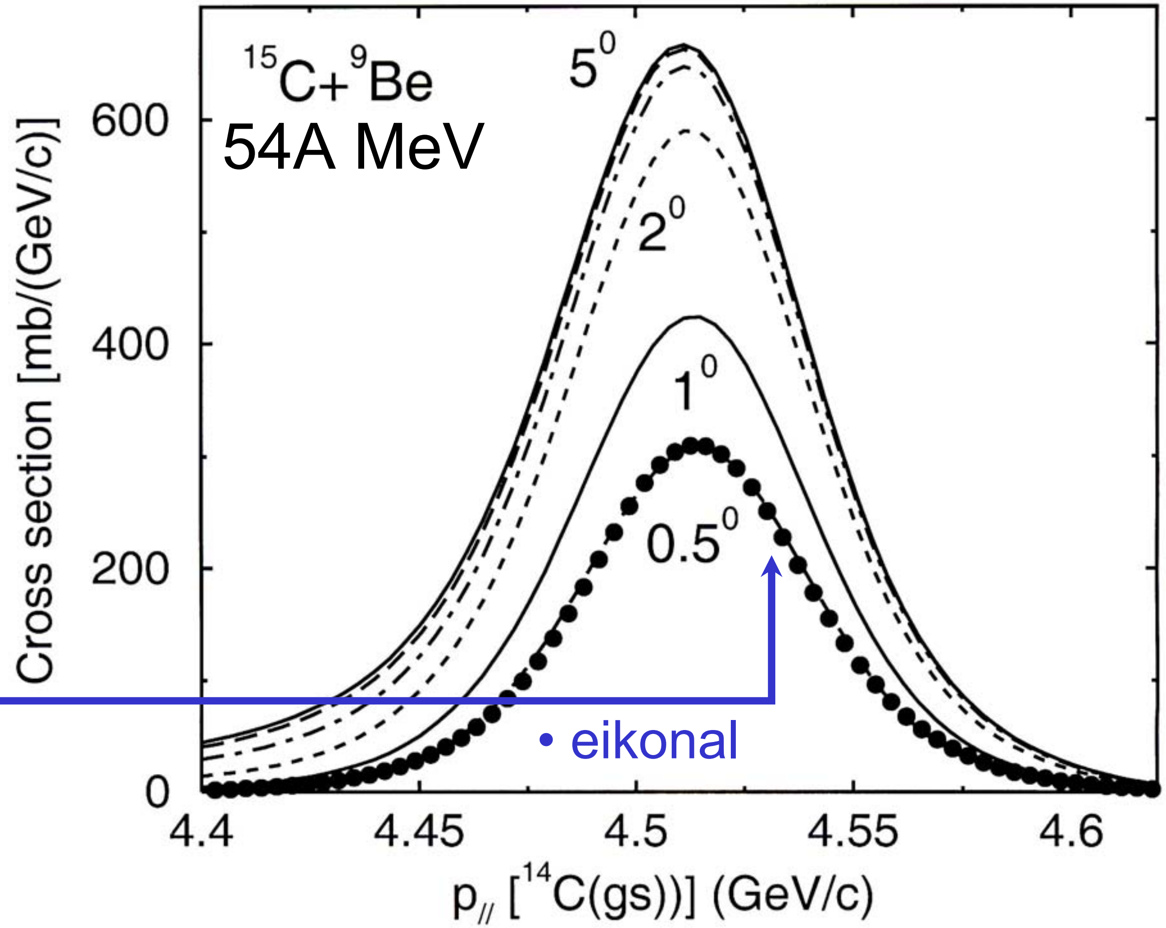


There is asymmetry in data?

Momentum distributions from the CDCC



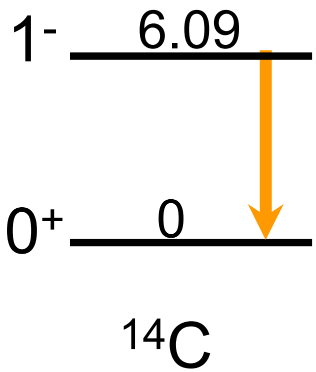
CDCC and eikonal calculations agree in most forward directions, but CDCC develops an asymmetry for deflected residues

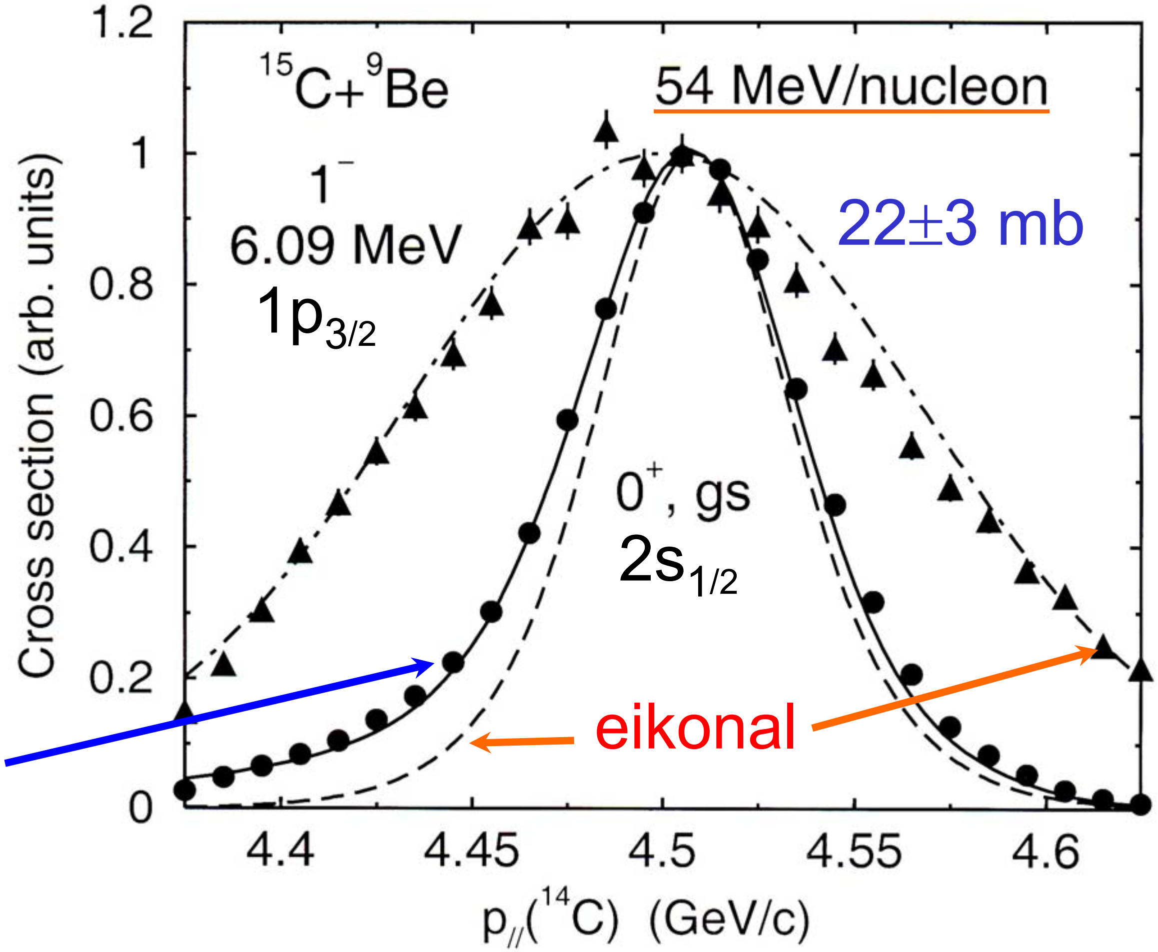


J.A. Tostevin et al, PRC **66** (2002) 024607

^{9}Be (^{15}C,^{14}C(gs)) X

Non-adiabatic and non-eikonal effects for ^{15}C



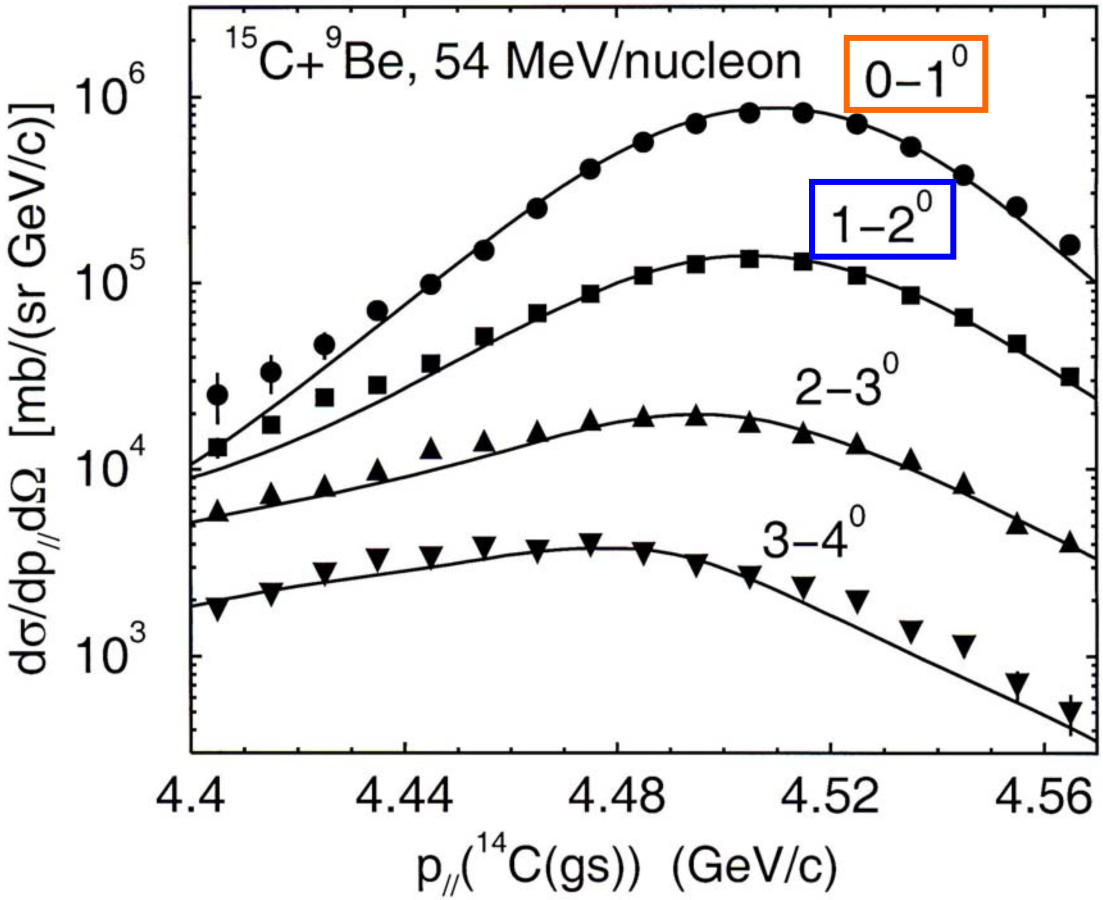


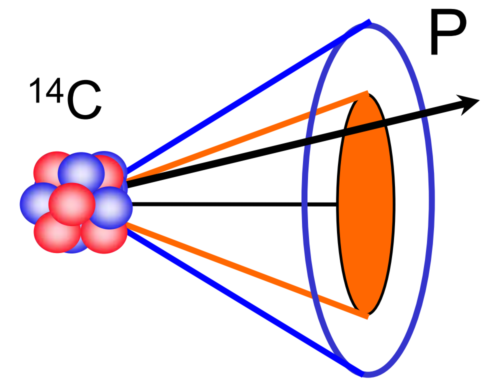
Coupled channels (CDCC) 109±13 mb

J.A. Tostevin et al, PRC **66** (2002) 024607

$^{9}Be\,(^{15}C,{}^{14}C(I^{\pi}))\,X$

Core fragment differential cross sections



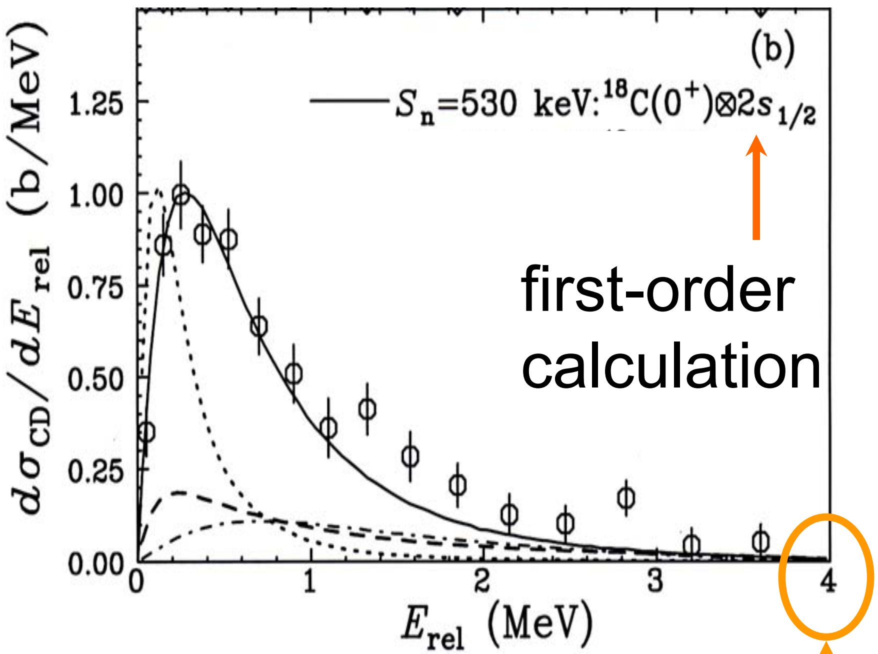


these yields almost entirely due to diffractive dissociation

$^{9}Be\ (^{15}C,^{14}C(gs))\ X$

J.A. Tostevin et al, PRC **66** (2002) 024607

Coupled channels and Coulomb break-up



T. Nakamura et al, PRL **83** (1998) 1112

Do CDCC calculations converge in the case of Coulomb couplings?

$\Delta k = k_i - k_{i-1}$ must be small

$\hat{\phi}_i(\mathbf{r})$ and associated couplings $\langle \hat{\phi}_i \,|\, U(\mathbf{r},\mathbf{R}) \,|\, \hat{\phi}_j \rangle$ of very long range

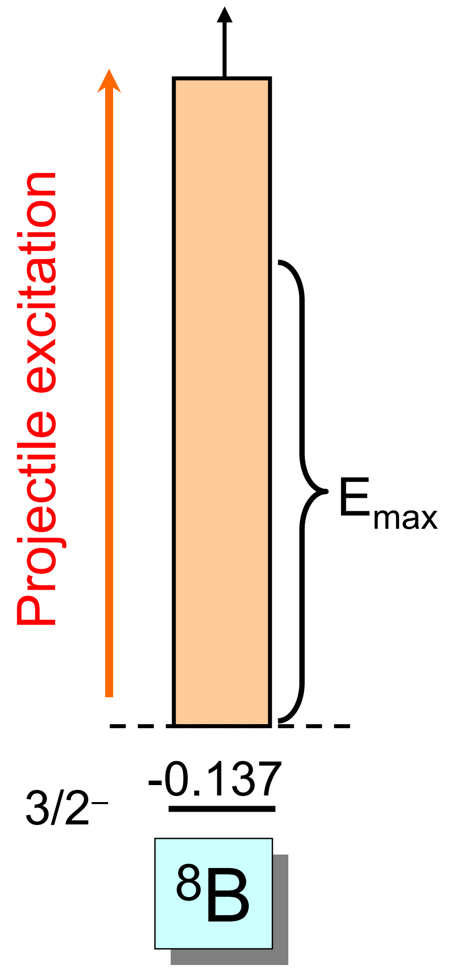
^{19}C + Pb $\rightarrow$ ^{18}C+n+X
E= 67A MeV Coulomb dominated

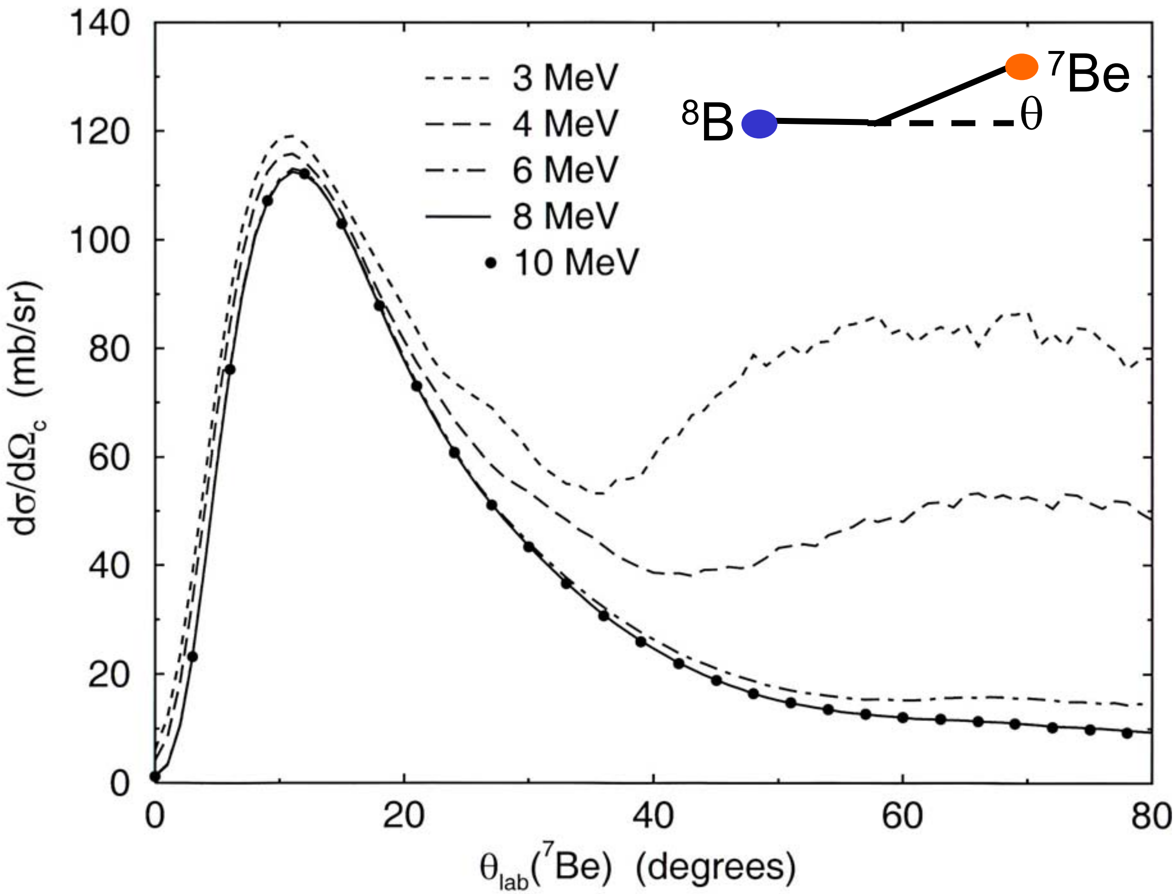
Convergence is not proven!

…… the foundation and general validity of the continuum-discretized-coupled-channel (CDCC) method (Sakuragi *et al* 1986) is under criticism. Clearly, it is just a model and does not provide a general solution of the three-body problem. The question remains whether it might be a general approximation that can converge in some sense to a three-body scattering theory. It has been revealed that 'CDCC is valid for special three-body models, constructed with absorptive phenomenological interactions' (Austern and Kawai 1988). In the particular situation of long-range Coulomb forces, absorptive interaction plays a small role and the applicability of the CDCC method is in serious doubt. Results of the calculations depend on the choice of the model-space and the way of discretization. The convergence is by no means convincingly demonstrated.

G.Baur and H. Rebel, J. Phys. G 20 (1994), 1

^8B - a weakly bound proton nucleus

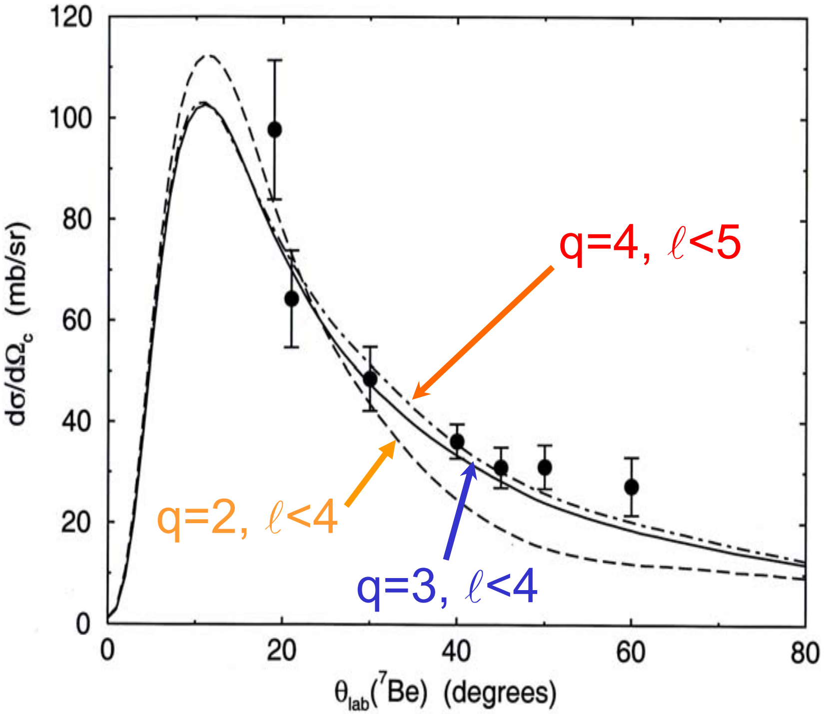


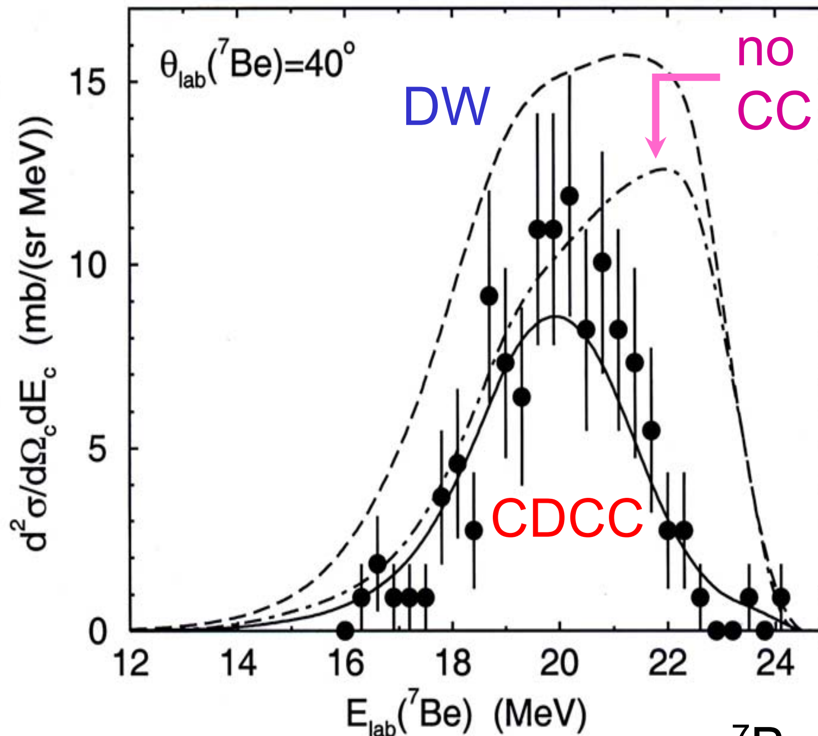


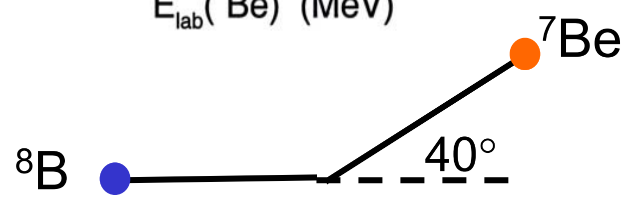
Convergence with E_{max}

CDCC can reproduce data at low energy

$^{8}B + ^{57}Ni \rightarrow ^{7}Be + X$, 25.8 MeV
(Notre-Dame)







J.A. Tostevin et al., Phys Rev C **63** (2001) 024617
J. Kolata et al., Phys Rev C **63** (2001) 024616

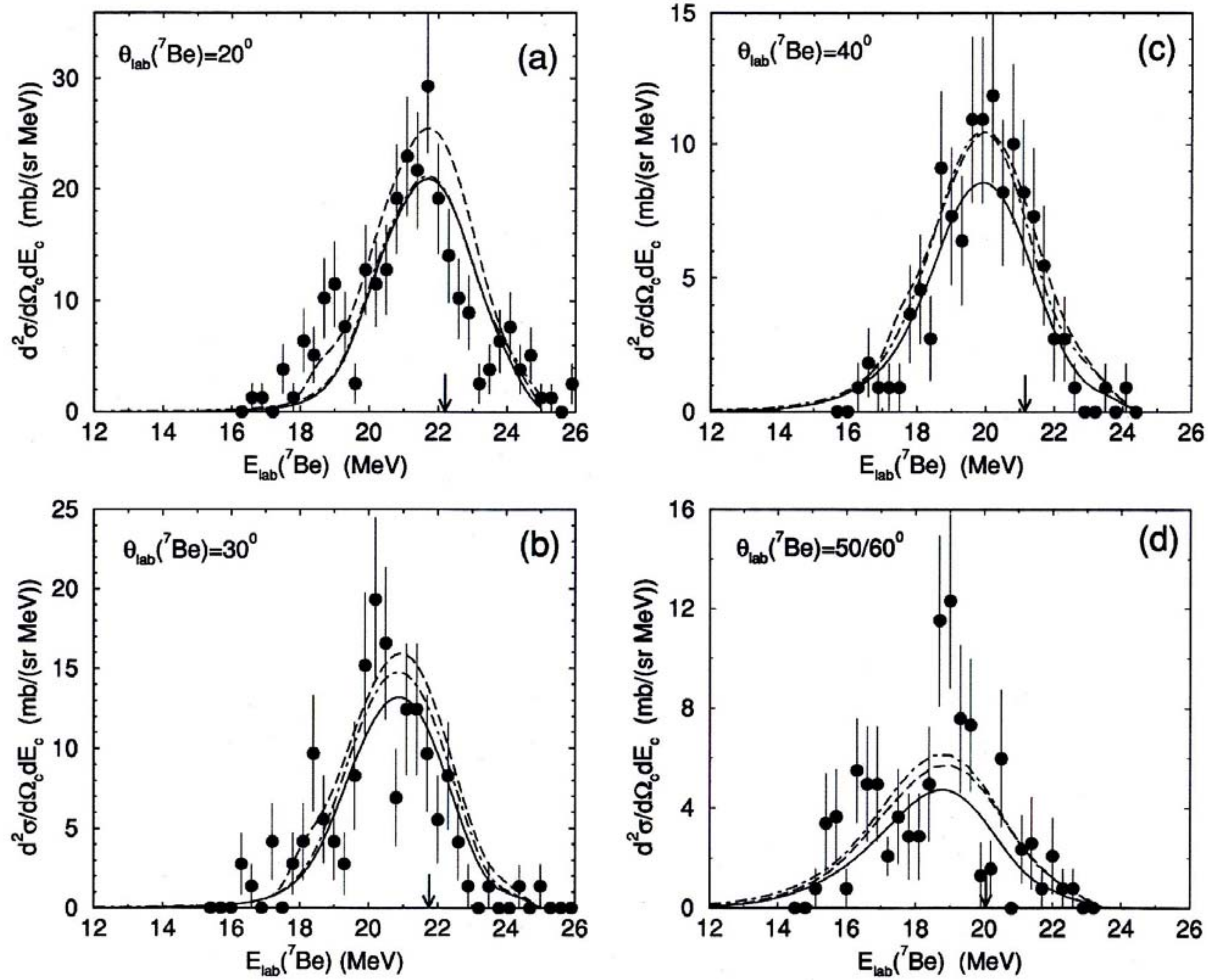
Double differential cross sections for breakup

$$\frac{d^2\sigma}{dE_c d\Omega_c}$$

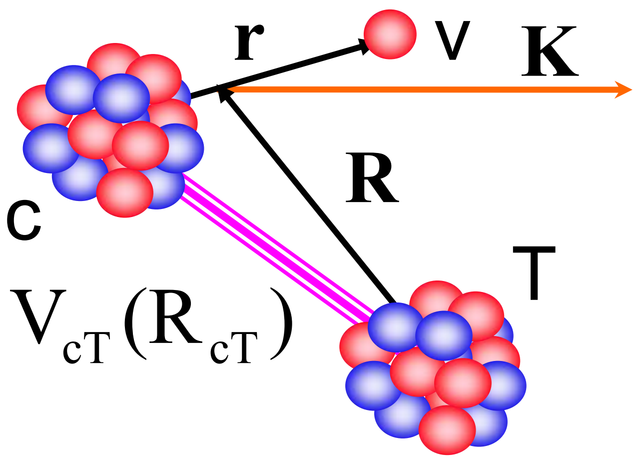
$^8B + {}^{57}Ni \rightarrow$
$^7Be + X$
25.8 MeV

J. Tostevin et al, Phys Rev C **63** (2001) 024617

J. Kolata et al., Phys Rev C **63** (2001) 024616



Recoil limit of the adiabatic few-body model



$$V_{vT}(r_{vT}) \approx 0$$

$V_{cT}(r_{cT})$ dominates

Removal of v is by core recoil or shake-off mechanism

closed-form solution in adiabatic approximation

$$\Psi_{\mathbf{K}}^{Ad}(\mathbf{r},\mathbf{R}) = \exp(i\alpha\mathbf{K}\cdot\mathbf{r})\phi_0(\mathbf{r})\underline{\chi_{\mathbf{K}}^{(+)}(\mathbf{R}_{cT})}, \quad \alpha = \frac{m_v}{(m_c + m_v)}$$

distorted wave for point projectile scattering from V_{cT}

and provides limit against which model calculations can be tested - e.g. CDCC

R.C. Johnson et al., PRL **79** (1997) 2771

Application to elastic scattering of composites

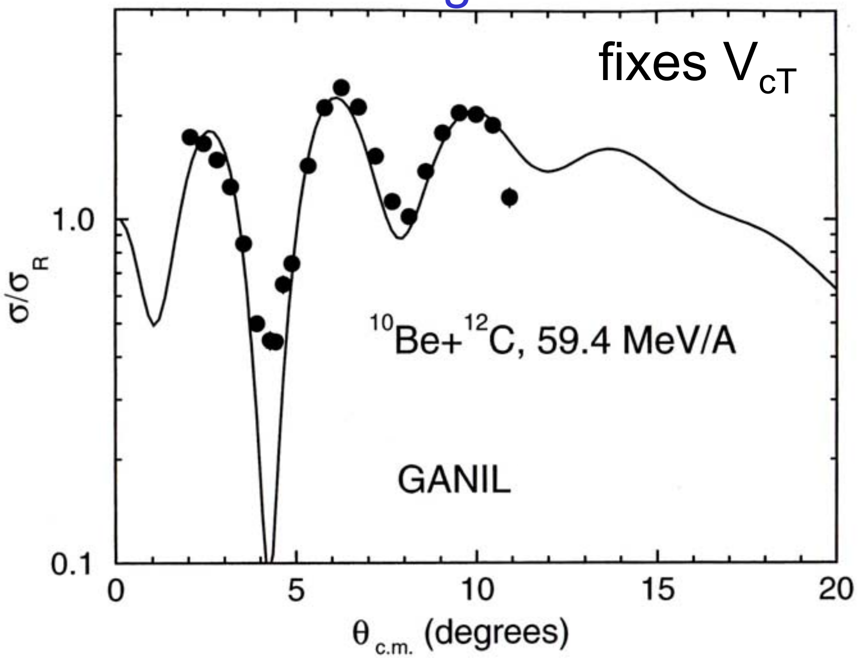
$$T_{el}(\mathbf{K}',\mathbf{K}) = \langle \mathbf{K}'\phi_0 \,|\, V_{cT} \,|\, \Psi^{Ad}_{\mathbf{K}}(\mathbf{r},\mathbf{R})\rangle = \langle \alpha\mathbf{Q},\phi_0 \,|\, \phi_0\rangle \langle \mathbf{K}' |\, V_{cT} \,|\, \chi^{(+)}_{\mathbf{K}}\rangle,$$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{el} = |\, F_{00}(\alpha\mathbf{Q})\,|^2 \left.\frac{d\sigma}{d\Omega}\right|_{point}} \qquad \mathbf{Q} = \mathbf{K}' - \mathbf{K}$$

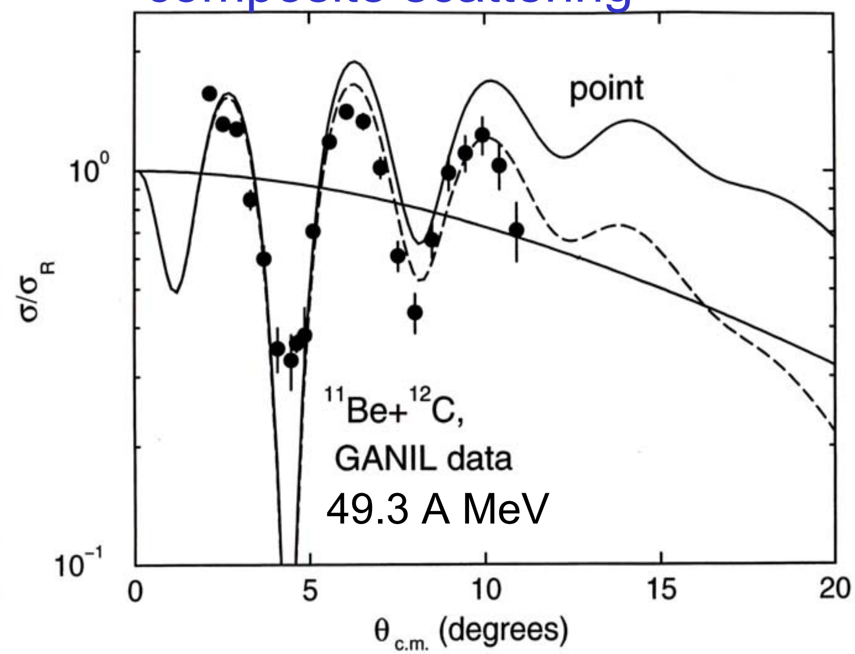
structure formfactor $F_{00}(\alpha\mathbf{Q})$

x point projectile scattering V_{cT}

core scattering



composite scattering

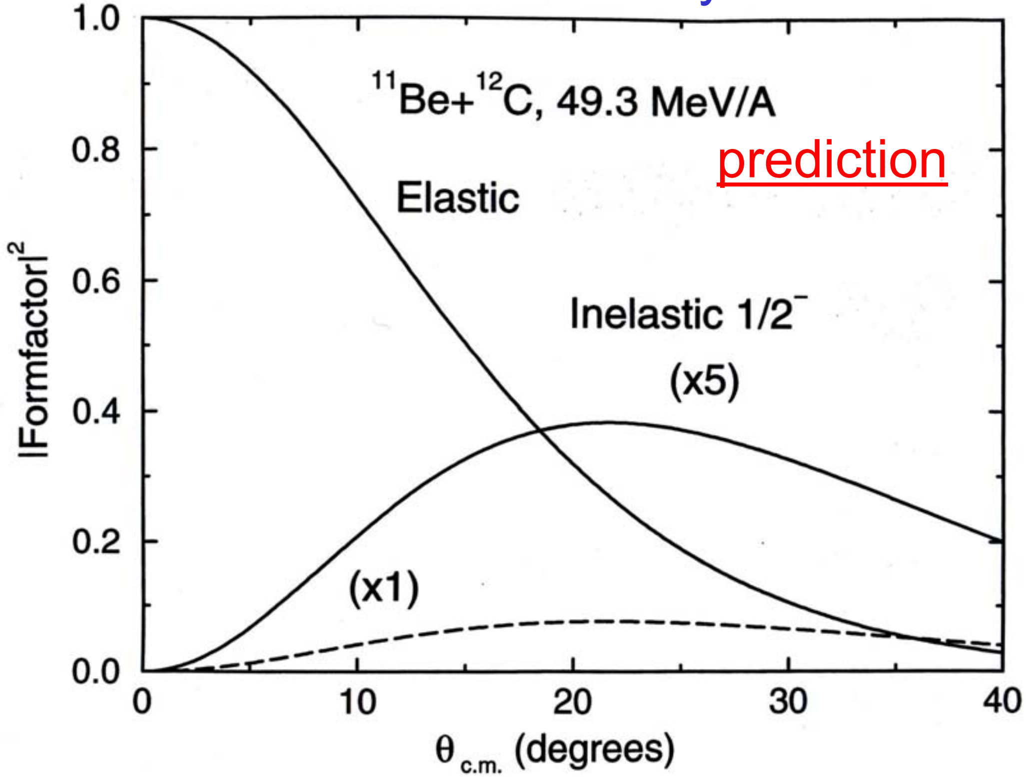


R.C. Johnson et al., PRL **79** (1997) 2771

Inelastic scattering, similarly

$$T_{inel}(\mathbf{K}',\mathbf{K}) = \langle \mathbf{K}'\phi_1 | V_{cT} | \Psi^{Ad}_{\mathbf{K}}(\mathbf{r},\mathbf{R})\rangle = \langle \alpha\mathbf{Q},\phi_1 | \phi_0\rangle\langle \mathbf{K}' | V_{cT} | \chi^{(+)}_{\mathbf{K}}\rangle$$

no measurement as of yet

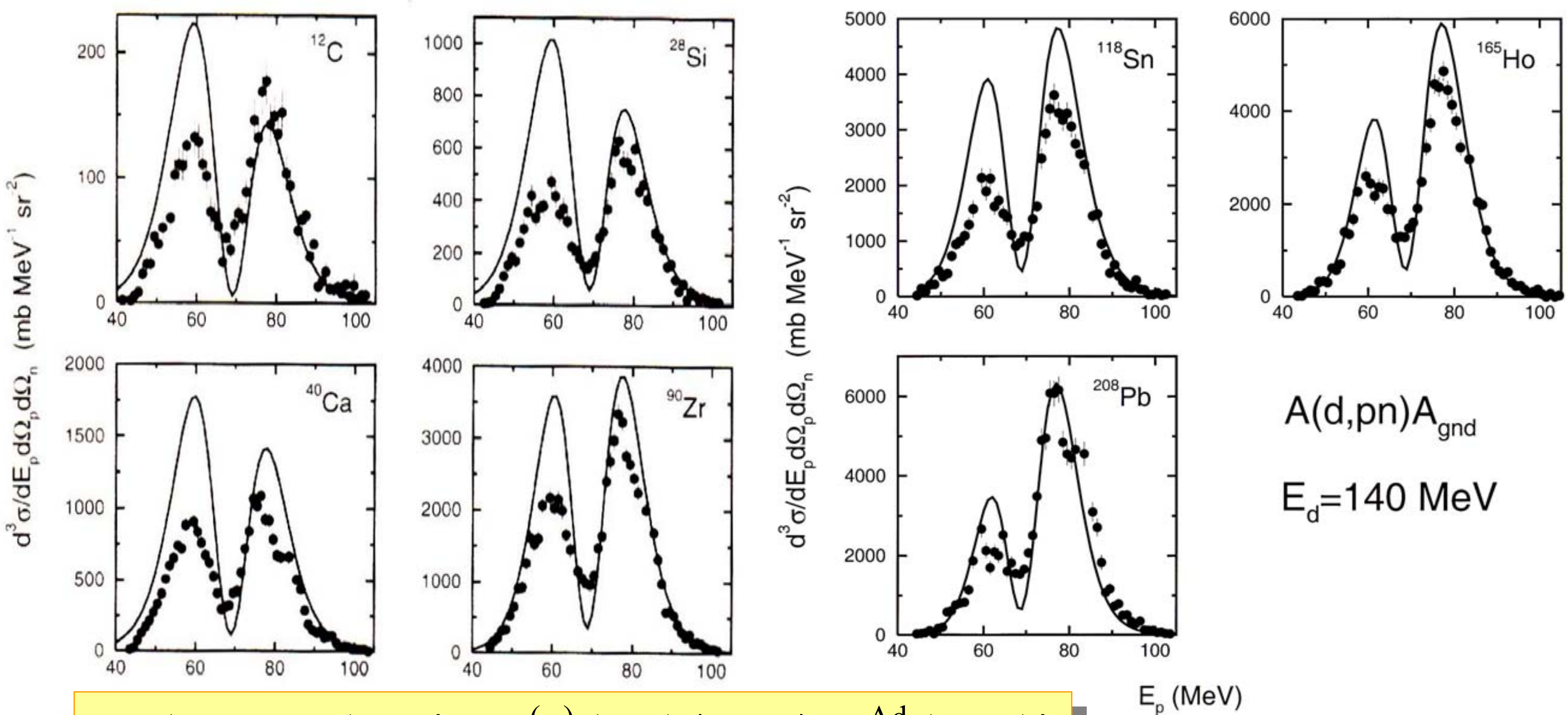


$$\left.\frac{d\sigma}{d\Omega}\right|_{inel} = |F_{10}(\alpha\mathbf{Q})|^2 \left.\frac{d\sigma}{d\Omega}\right|_{point}$$

elastic scattering cross section of point projectile by potential felt by the core c

Coulomb break-up of the deuteron

J.A. Tostevin et al., Phys. Rev. C **57** (1998) 3225



A(d,pn)A$_{gnd}$

E_d=140 MeV

$$T_{bu}(\mathbf{k}_v \mathbf{k}_c, \mathbf{K}) = \langle \mathbf{k}_v \chi^{(-)}_{\mathbf{k}_c}(\mathbf{R}_c) \,|\, V_{cv} \,|\, \Psi^{Ad}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) \rangle$$

$$= \langle \mathbf{P}_v \,|\, V_{cv} \,|\, \phi_0 \rangle \langle \mathbf{Q}_v \, \chi^{(-)}_{\mathbf{k}_c} \,|\, \chi^{(+)}_{\mathbf{K}} \rangle$$

Exact 3-body amplitude in the adiabatic limit

$$\Psi_{\mathbf{K}}^{\mathrm{Ad}}(\mathbf{r},\mathbf{R}) = \exp(i\alpha\mathbf{K}\cdot\mathbf{r})\phi_0(\mathbf{r})\chi_{\mathbf{K}}^{(+)}(\mathbf{R}_{\mathrm{cT}}), \quad \alpha = \frac{m_v}{(m_c + m_v)}$$

$$\mathrm{T_{el}}(\mathbf{K}',\mathbf{K}) = \langle \mathbf{K}' | \mathrm{V_{cT}} | \Psi_{\mathbf{K}}^{\mathrm{Ad}}(\mathbf{r},\mathbf{R})\rangle = \langle \alpha\mathbf{Q} | \phi_0\rangle\langle \mathbf{K}' | \mathrm{V_{cT}} | \chi_{\mathbf{K}}^{(+)}\rangle$$

includes effects of long range Coulomb couplings without partial wave decomposition or truncation

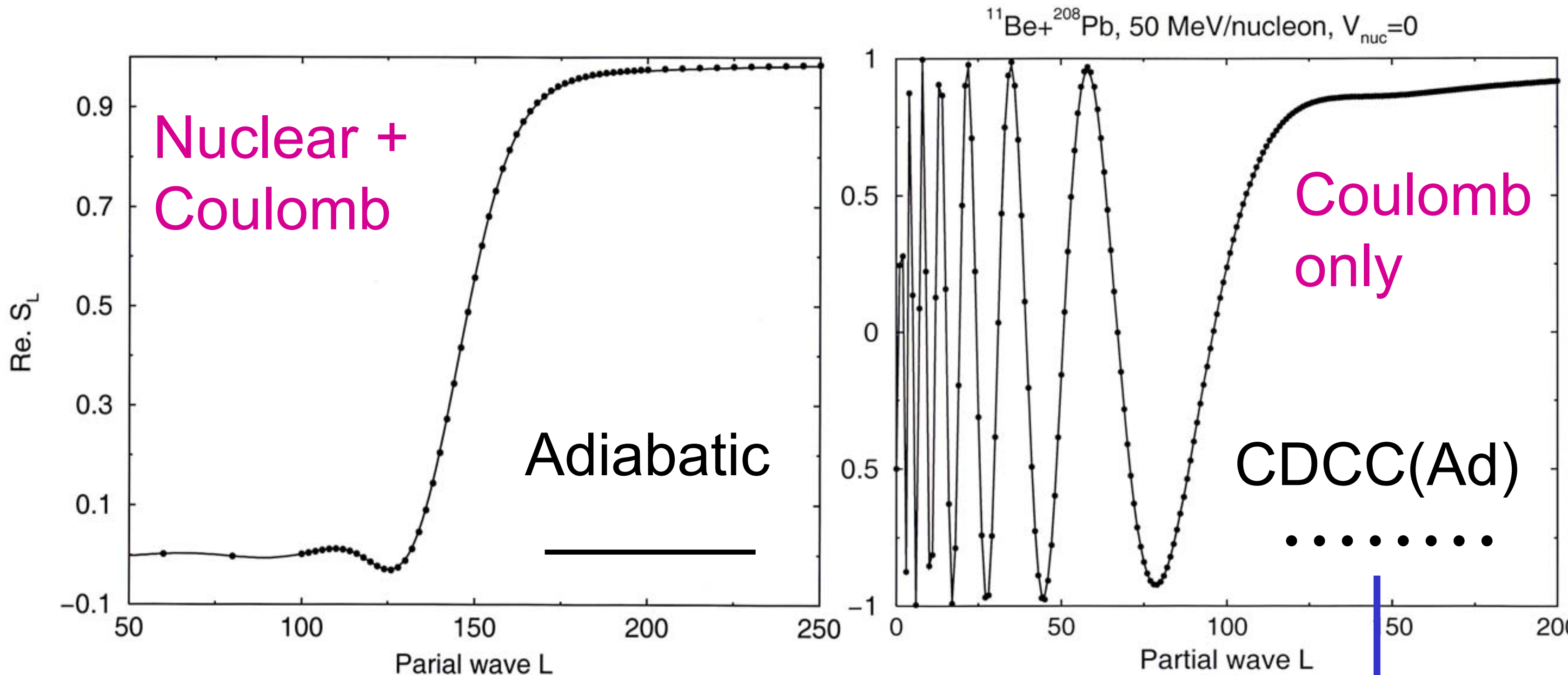
$$\mathrm{f_{el}}(\theta) = \mathrm{F}(\alpha\mathbf{Q})\,\mathrm{f_{pt}}(\theta)$$

$$\int_{-1}^{1} \mathrm{d}x\, \mathrm{P_L}(x)\left[\mathrm{f_{el}}(\theta) - \mathrm{f_C}(\theta)\right]$$

$$\mathrm{S_L}$$

Subtract point Coulomb amplitude and invert to give S_L to compare with that calculated using CDCC, in the limit that $H_p \rightarrow \varepsilon_0$

Coupled channels for Coulomb break-up?



Coupled channels (CDCC) calculations with all channel energies equal to that of the elastic channel

Messages to take away ……

Weak beams of rare weakly bound nuclei pose challenges to reaction theories – continuum of states, non-perturbative

Approximate schemes are being developed which allow sp spectroscopy on beams with of order 1pps – show Shell Model ideas are working away from stability

Apparently simple problems (the Coulomb interaction and its induced break-up) remain to be fully resolved.

Insight is being gained in the light nucleus domain and extended rapidly to heavier systems as new facilities are planned and commissioned (NSCL, RIKEN, GSI, RIA ..)


thanks for your attention
and hospitality .....
- and also for
the cricket!