Probing the structures of exotic and halo nuclei

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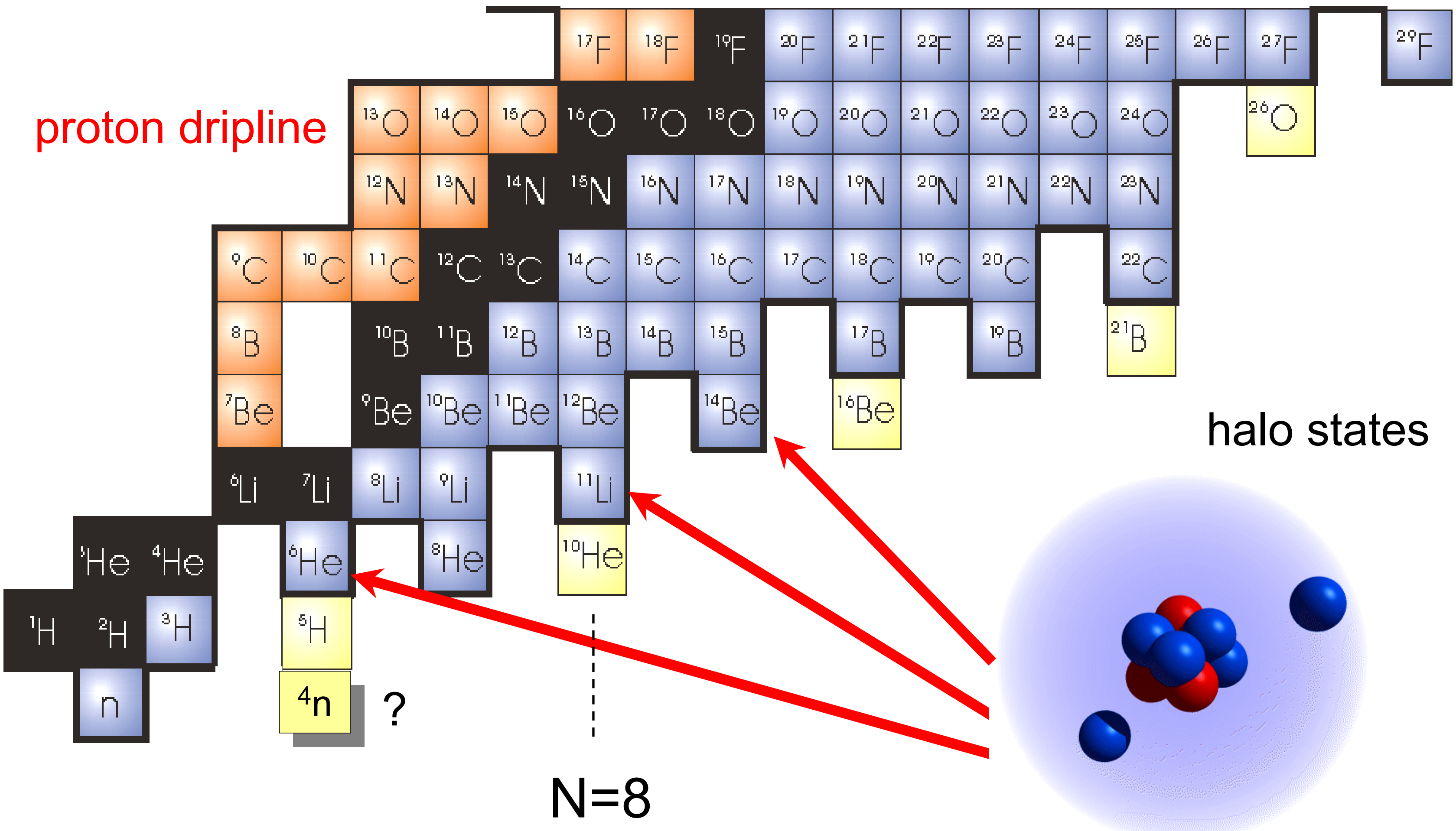
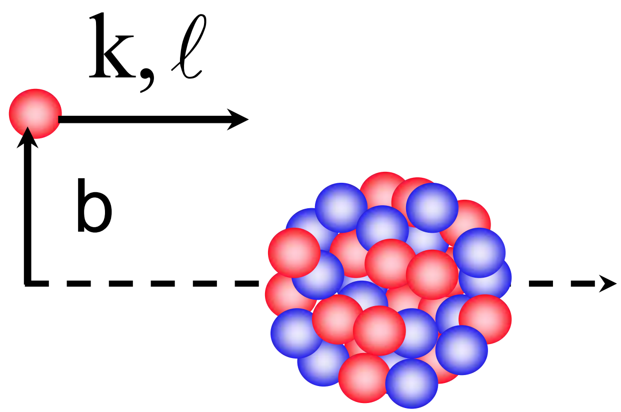
The neutron dripline in light nuclei



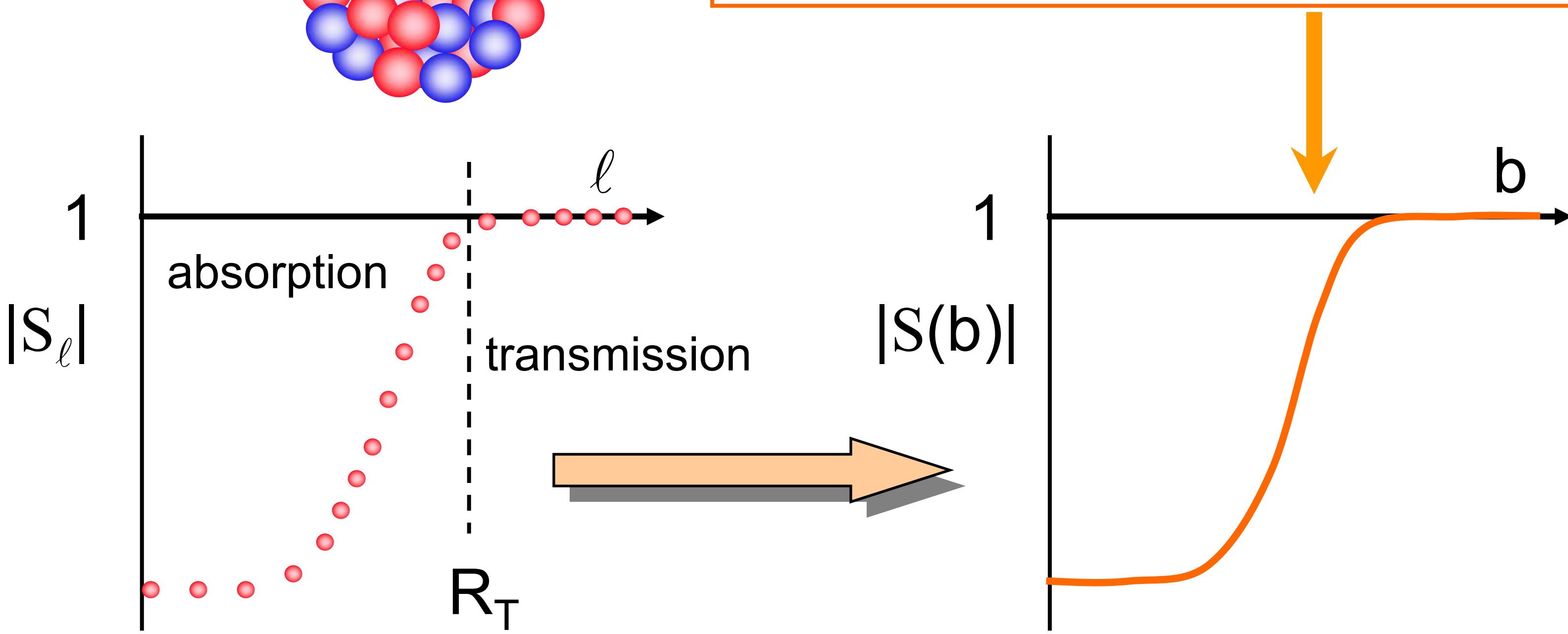
The semi-classical S-matrix - S(b)



b=impact parameter

for high energy/or large mass, semi-classical ideas are good

$kb \simeq \ell$, actually $\Rightarrow \ell + 1/2$



Eikonal S-matrix in the point particle case

$$\Psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\omega(\mathbf{r})$$

$$\omega(\mathbf{r}) = \exp\left\{-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} dz' V(r')\right\}$$

So, after the interaction and as $z \to \infty$

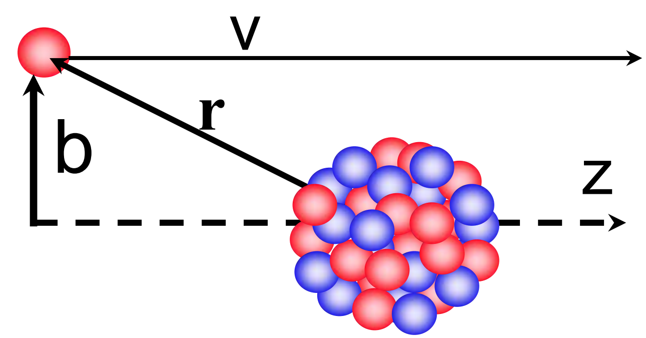
$\hbar k/\mu =$ classical velocity v

$$\Psi_{\mathbf{k}}(\mathbf{r}) = S(b)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

S(b) is amplitude of the outgoing waves from the scattering at b

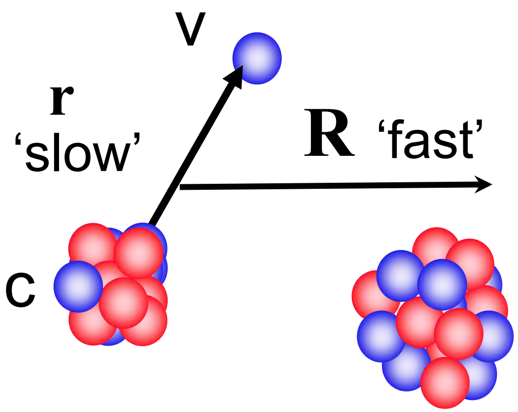
Eikonal approximation to the S-matrix $S(b)$

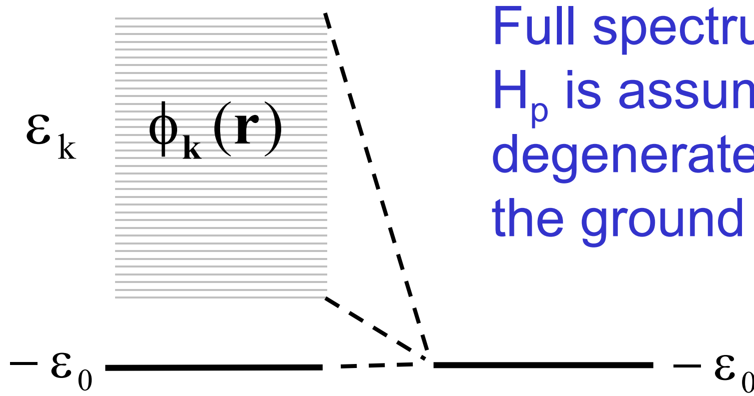
$$S(b) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz' V(r')\right\}$$



Moreover, the structure of the theory generalises to few-body projectiles

Adiabatic (sudden) model for few-body projectiles





Full spectrum of H_p is assumed degenerate with the ground state

Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute $f(\theta,\mathbf{r})$ for all required fixed values of $\mathbf{r}$

Physical amplitude for breakup to state $\phi_{\mathbf{k}}(\mathbf{r})$ is then,

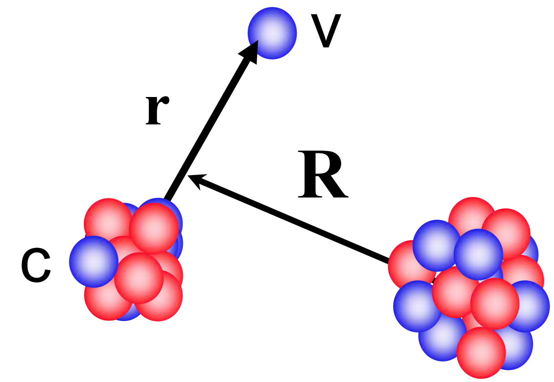
$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

Achieved by replacing $H_p \rightarrow -\varepsilon_0$ in Schrödinger equation

Adiabatic approximation - time perspective

The time-dependent equation is

$$H\Psi(\mathbf{r},\mathbf{R},t) = i\hbar\frac{\partial\Psi}{\partial t}$$



and can be written

$$\Psi(\mathbf{r},\mathbf{R},t) = \Lambda\,\Phi(\mathbf{r}(t),\mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+}\,\mathbf{r}\,\Lambda$$

$\Lambda = \exp\{-i(H_p+\varepsilon_0)t/\hbar\}$ and where

$$[T_R + U(\mathbf{r}(t),\mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t),\mathbf{R}) = i\hbar\frac{\partial\Phi}{\partial t}$$

Adiabatic step assumes

$\mathbf{r}(t) \approx \mathbf{r}(0)=\mathbf{r}=$fixed

or $\Lambda=1$ for the collision time t_{coll}

Adiabatic equation

$$[T_R + U(\mathbf{r},\mathbf{R})]\Phi(\mathbf{r},\mathbf{R}) = \left(E+\varepsilon_0\right)\Phi(\mathbf{r},\mathbf{R})$$

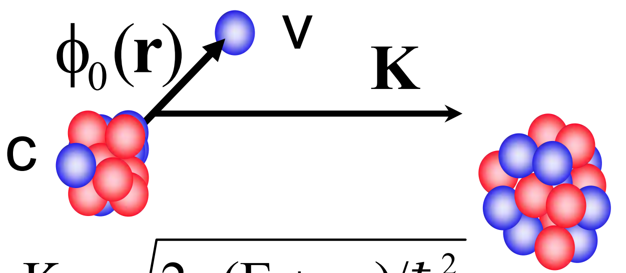
requires

$$(H_p+\varepsilon_0)t_{coll}/\hbar << 1$$

Eikonal solution of the few-body model

Practical application of adiabatic approximation: $H_p \rightarrow -\varepsilon_0$

$$\boxed{H = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_p} \longrightarrow \boxed{H^{AD} = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - \varepsilon_0}$$



$$K = \sqrt{2\mu(E+\varepsilon_0)/\hbar^2}$$

substituting the eikonal form solution

$$\Psi_{\mathbf{K}}^{AD}(\mathbf{r},\mathbf{R}) = \underbrace{e^{i\mathbf{K}\cdot\mathbf{R}}\phi_0(\mathbf{r})}_{\text{incident wave}}\,\underbrace{\omega(\mathbf{r},\mathbf{R})}_{\text{modulating function}}$$

$$\boxed{[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - (E+\varepsilon_0)]\,\Psi_{\mathbf{K}}^{AD}(\mathbf{r},\mathbf{R}) = 0}$$

and neglecting the curvature term $\nabla_R^2\omega(\mathbf{r},\mathbf{R}) << 2\nabla_R\omega\cdot\mathbf{K}$

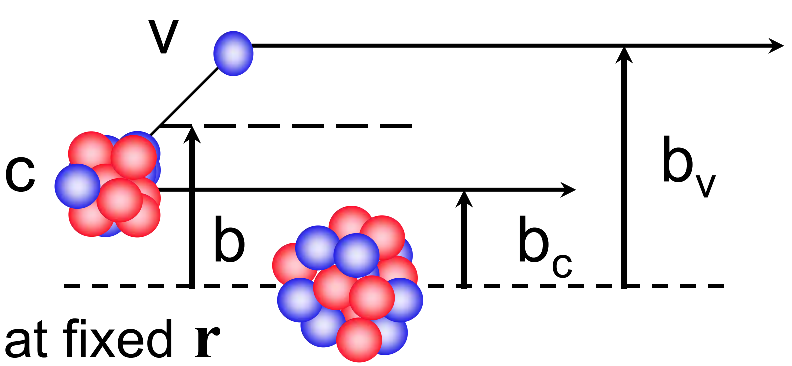
$$\omega(\mathbf{r},\mathbf{R}) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ'\, U(\mathbf{r},\mathbf{R}')\right\} \longrightarrow V_{cT} + V_{vT}$$

Few-body eikonal model amplitudes

So, after the collision, as $Z\rightarrow\infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\Psi_{\mathbf{K}}^{\mathrm{Eik}}(\mathbf{r},\mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\, S_v(b_v)\, \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



at fixed $\mathbf{r}$

adiabatic

So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v $\leftarrow$ amplitude that c,v survive interaction with b_c and b_v

Take stock of things - where have we got to?

Wish to test spectroscopy with weakly bound systems - coupling to the continuum is strong - few-body models

Single particle properties - direct reactions with minimal rearrangement of other than active nucleon(s) needed

Adiabatic approximation - high E, small ε, slow internal motions - will be increasingly accurate as E increases

Eikonal few-body models - make clear the role of the dynamics and the structure input required - transparent

$$\boxed{S_{\alpha\beta}(b) = \langle \phi_\beta \mid S_c(b_c)\, S_v(b_v) \mid \phi_\alpha \rangle}$$

Approximate description - accuracy will need to be tested as and when data are good enough to warrant

Dynamics and structure - formal transparency

Independent scattering information of c and v from target

$$S_{\alpha\beta}(b) = \langle \underbrace{\phi_\beta \mid \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{dynamics}} \mid \phi_\alpha}_{\text{structure}} \rangle$$

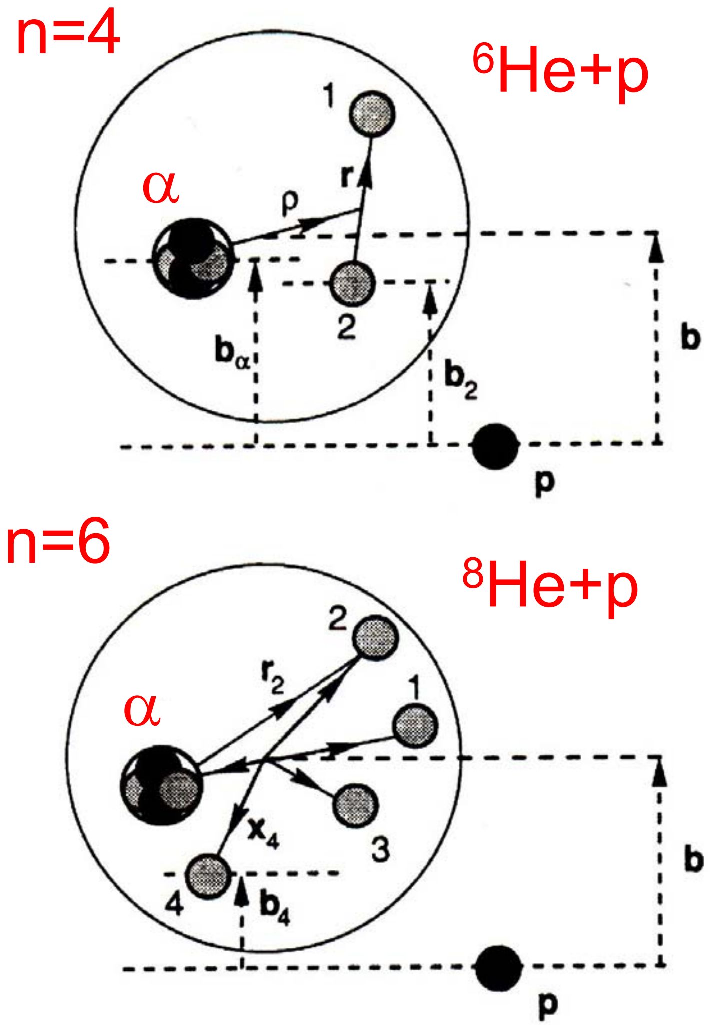
Use the best available few- or many-body wave functions

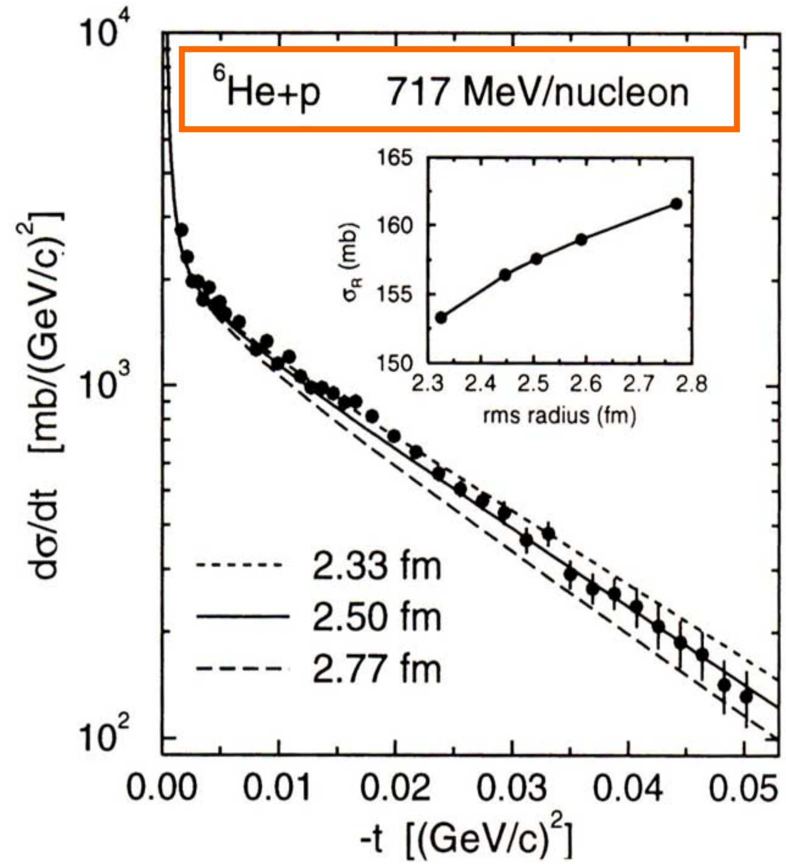
More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \mid S_1(b_1)\, S_2(b_2) \ldots\ldots\, S_n(b_n) \mid \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a realistic wave function φ is available

Four and six-body reaction calculations





Al-Khalili and Tostevin, PRC **57** (1998) 1846
Tostevin et al., PRC **56** (1997) R2929

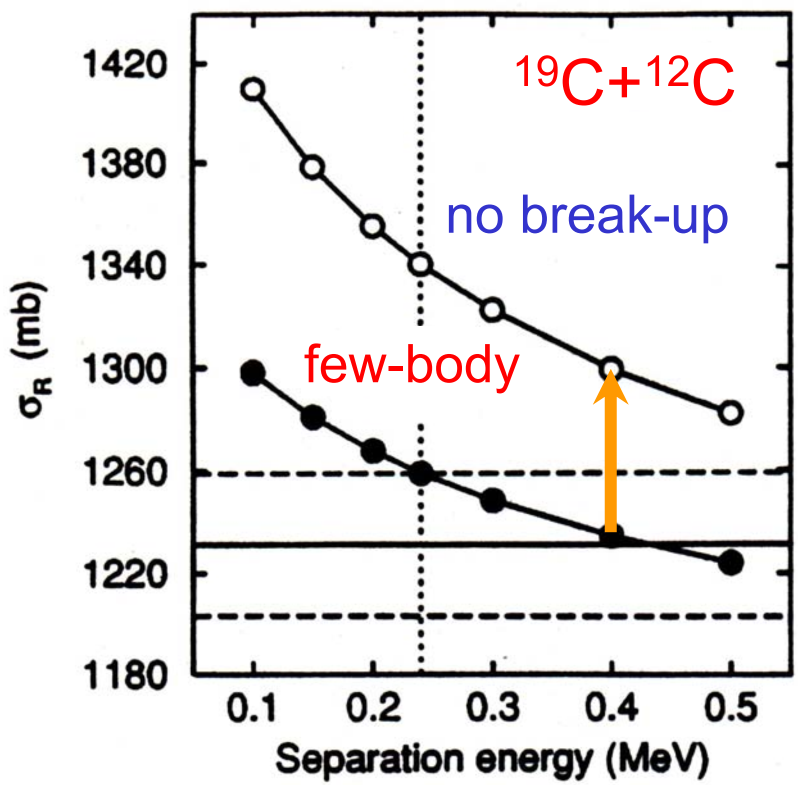
Reaction observables for composite systems

Elastic S-matrix is $S_p(b) = \langle \phi_0 \mid S_c(b_c)\, S_v(b_v) \mid \phi_0 \rangle$ so the total elastic and reaction cross sections are

$$\sigma_{el} = \int d\mathbf{b}\, |1 - S_p(b)|^2$$

$$\sigma_R = \int d\mathbf{b}\, [1 - |S_p(b)|^2]$$

The latter expression includes breakup effects to all orders and is being used extensively to study the (structure) effects of nuclear halos on reaction cross sections σ_R - these few-body breakup effects are very important.



J.A. Tostevin and J.S. Al-Khalili, PRC **59** (1999) R5

Diffractive dissociation of composite systems

The total cross section for removal of the valence particle from the projectile due to the break-up (or diffractive dissociation) mechanism is the break-up amplitude, summed over all final continuum states

$$\sigma_{\text{diff}} = \int d\mathbf{k} \int d\mathbf{b}\, |\langle \phi_{\mathbf{k}} \mid S_c(b_c)\, S_v(b_v) \mid \phi_0 \rangle|^2$$

but, using completeness of the break-up states

$$\int d\mathbf{k}\, |\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}| = 1 - |\phi_0\rangle\langle\phi_0| \boxed{- |\phi_1\rangle\langle\phi_1| \;\ldots\ldots}$$

If > 1 bound state

can (for a weakly bound system with a single bound state) be expressed in terms of only the projectile ground state wave function as:

$$\sigma_{\text{diff}} = \int d\mathbf{b} \left\{ \langle \phi_0 \,|\, |S_c\, S_v|^2 \,|\, \phi_0 \rangle - |\langle \phi_0 \,|\, S_c\, S_v \,|\, \phi_0 \rangle|^2 \right\}$$

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels → $|S(b)|^2 \leq 1$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 \,|\, \underbrace{1 - |S_c S_v|^2}\, |\, \phi_0 \rangle$$

$$\left\{ \begin{array}{l} |S_v|^2 (1-|S_c|^2) + \\ |S_c|^2 (1-|S_v|^2) + \\ (1-|S_v|^2)(1-|S_v|^2) \end{array} \right.$$

v survives, c absorbed

v absorbed, c survives

v absorbed, c absorbed

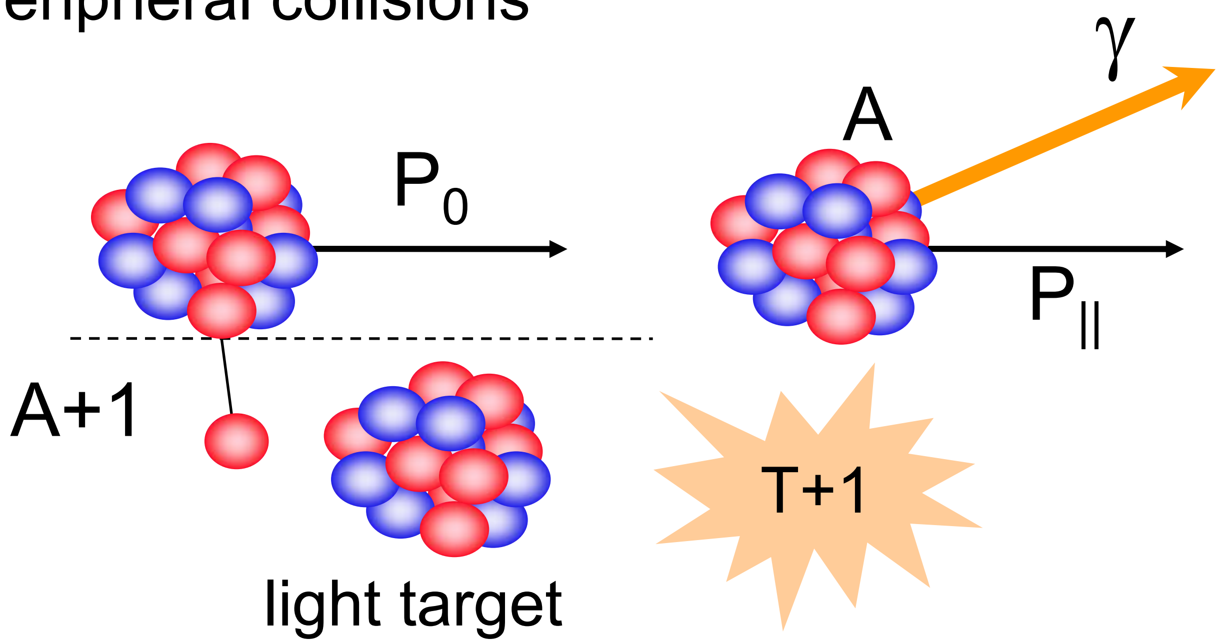
$$\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1-|S_v|^2) \,|\, \phi_0 \rangle$$

stripping of v from projectile exciting the target. c scatters at most elastically with the target

Related equations exist for the differential cross sections, etc.

Use of formalism for single-nucleon knockout

Peripheral collisions

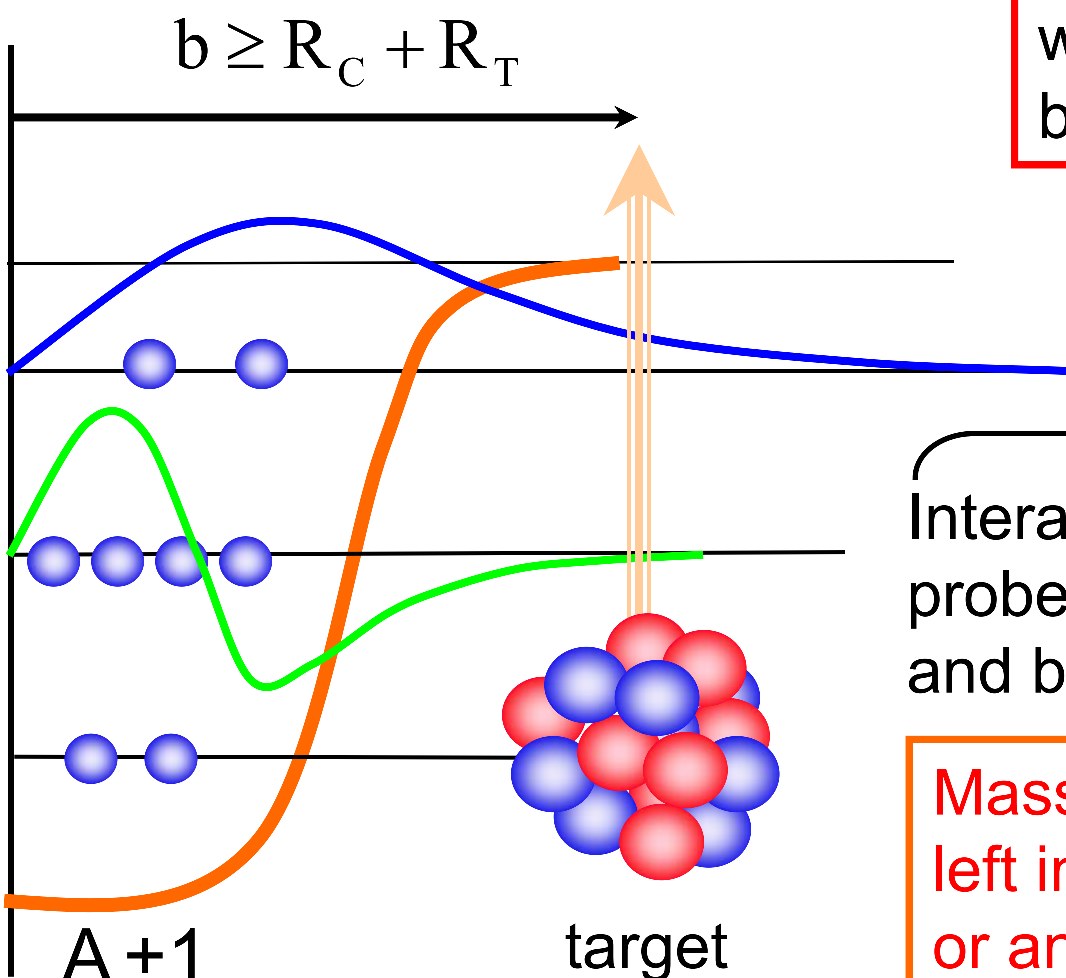


light target
T=^{9}Be,^{12}C

only mass A residue is detected, with or without a coincident γ-ray

Events contributing will be both stripping and break-up both of which leave a mass A residue in the final state

Probing the surface and tails of wave functions



with sensitivity to both ℓ and κ

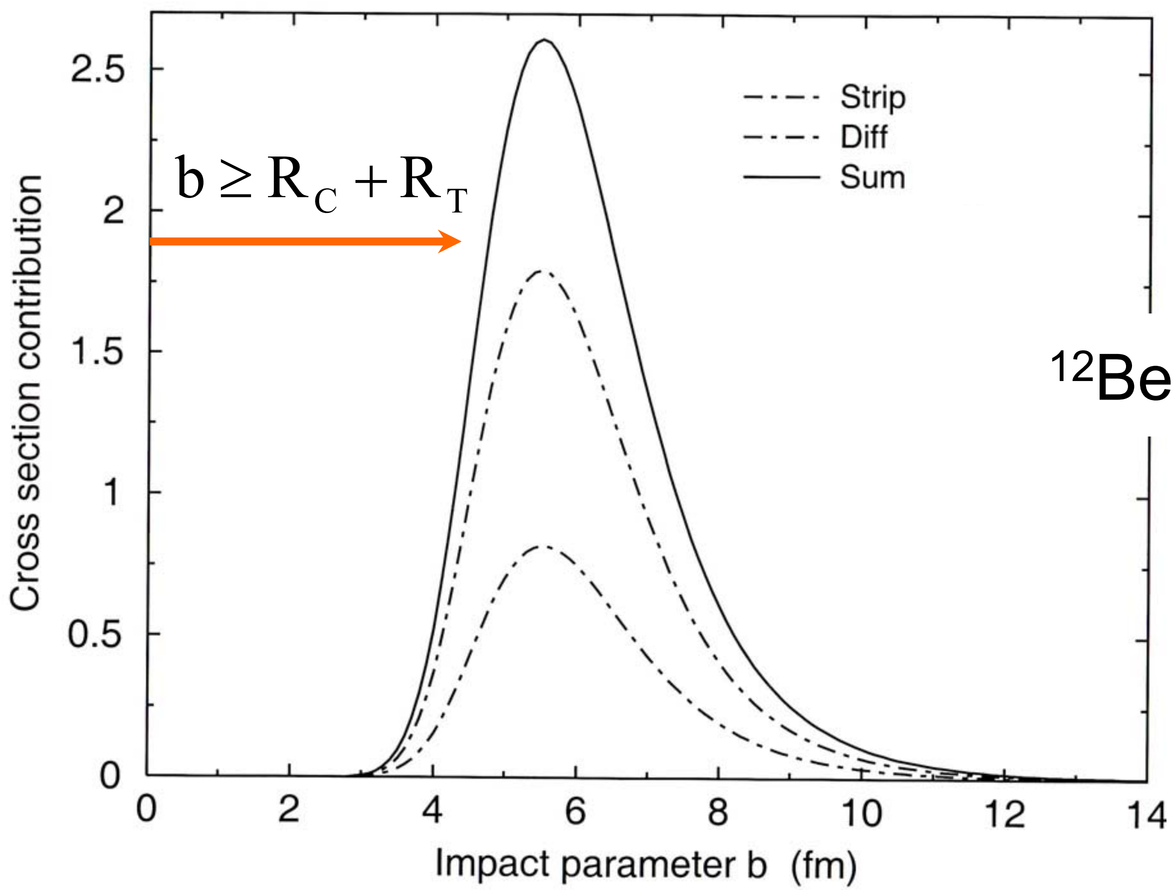
$h_\ell(i\kappa r)$

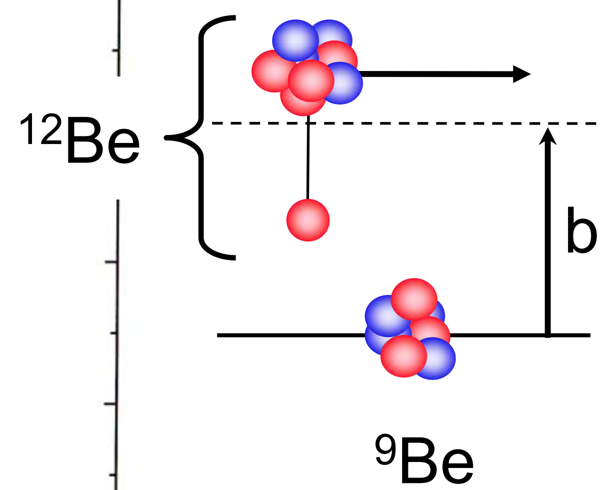
Interaction with the target probes $\phi_0(\mathbf{r})$ at surface and beyond $b \geq R_C + R_T$

Mass A residue will be left in the ground state or an excited state

Contributions from different impact parameters

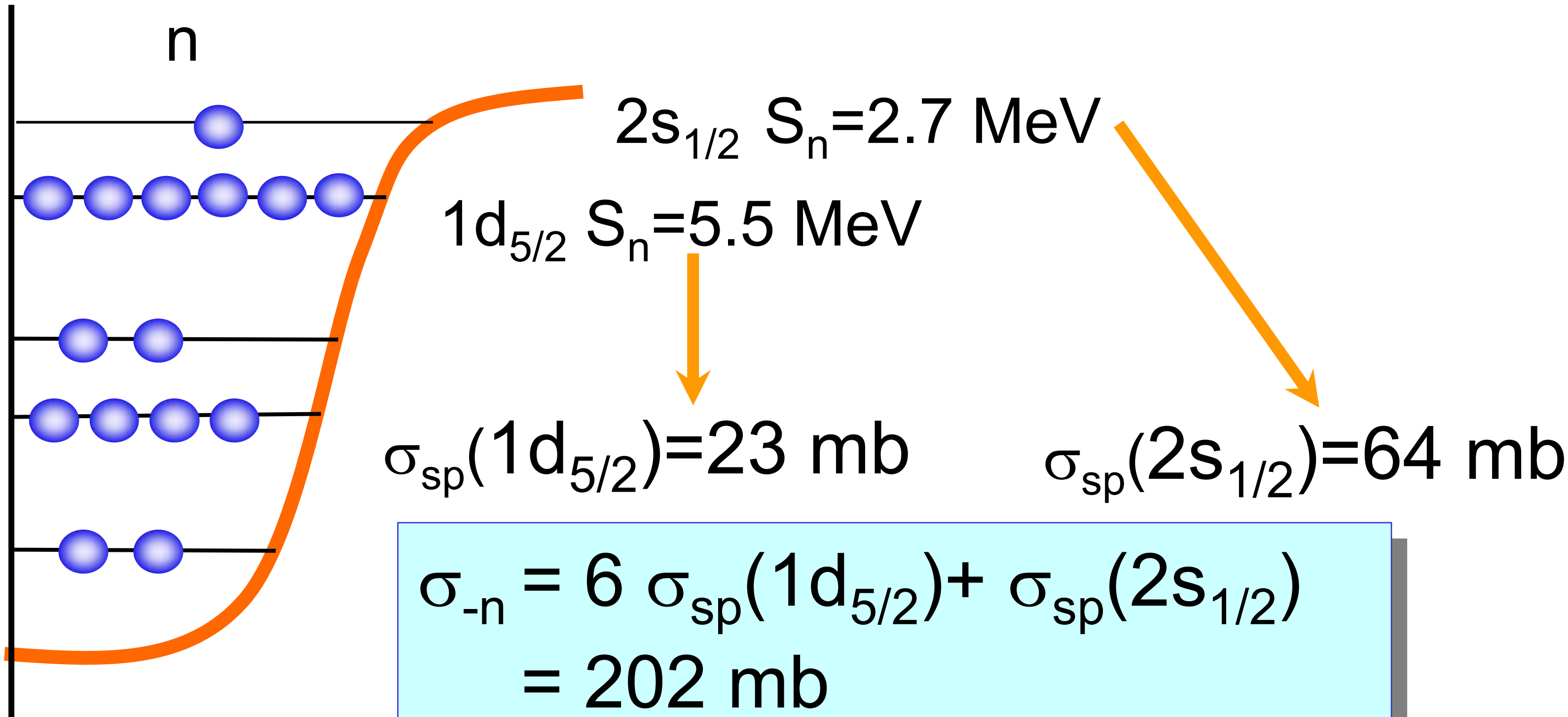
$^{12}Be + ^{9}Be \rightarrow ^{11}Be(gs) + X$, 80A MeV





Example for orientation - extreme sp model

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$$\sigma_{-n} = 6\,\sigma_{sp}(1d_{5/2}) + \sigma_{sp}(2s_{1/2})$$
$$= 202 \text{ mb}$$

Measurement at RIKEN [Kanungo et al PRL **88** ('02) 142502] at 72 MeV/nucleon on a ^{12}C target; σ_{-n} = 233(37)mb

Of course we need to do this carefully

Energy (MeV)	I^π	ℓ	C^2S	σ_{sp} (mb)	σ_{1n} (mb)
0	0^+	0	0.797	64.2	51.2
♦ 3.38	2^+	2	2.130	22.8	48.6
4.62	0^+	0	0.115	32.0	3.7
♦ 4.83	3^+	2	3.079	20.4	62.9
5.32	1^-	1	0.851	17.8	15.2
5.93	0^-	1	0.332	16.9	5.6
6.50	2^+	2	0.242	18.0	4.4
				Sum:	191

p

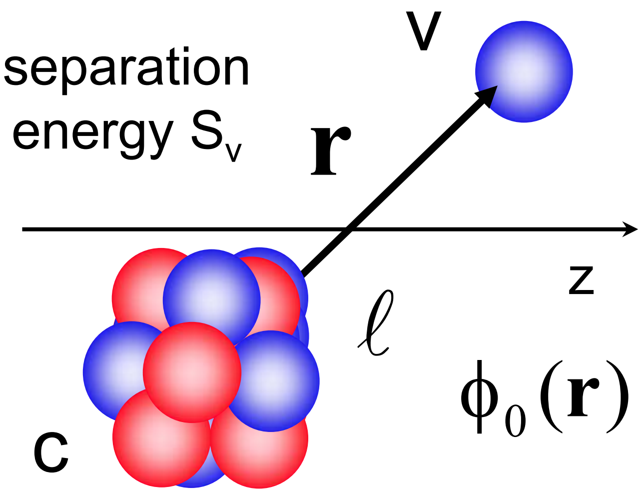
datum 233(37)mb

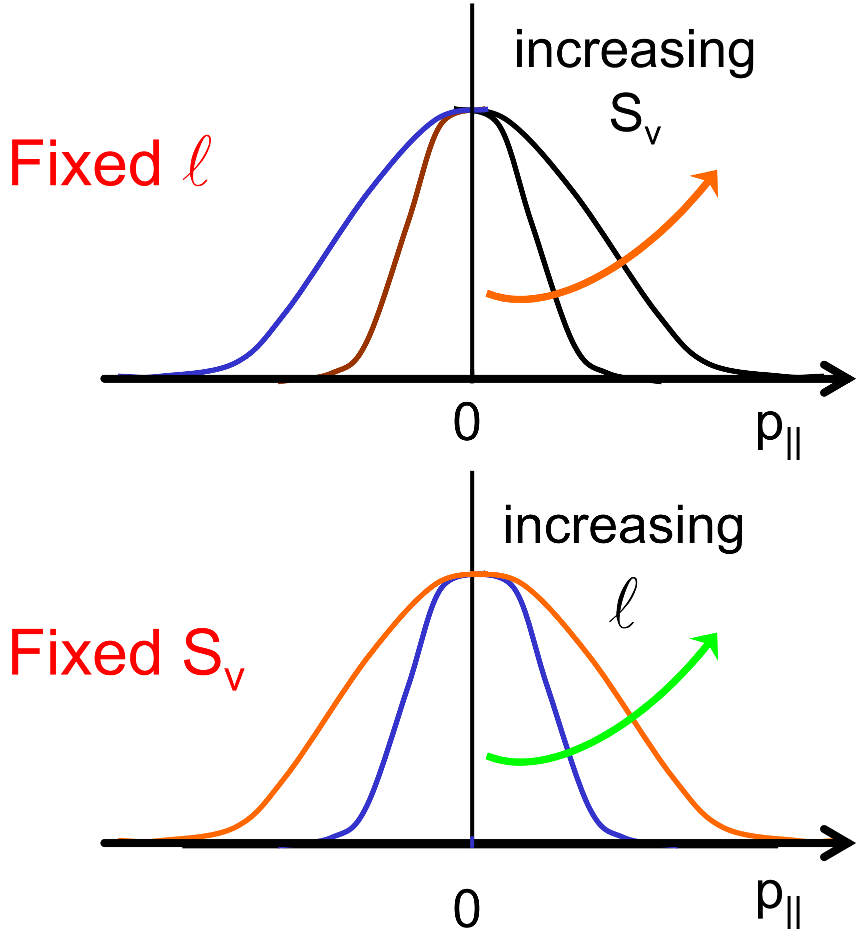
Shell model (Brown)

^{22}O final states below n-threshold

♦ $[d_{5/2} \times s_{1/2}]_J$ $\begin{cases} C^2S(2^+)=2.5 \\ C^2S(3^+)=3.5 \end{cases}$

Measurement of the momentum components

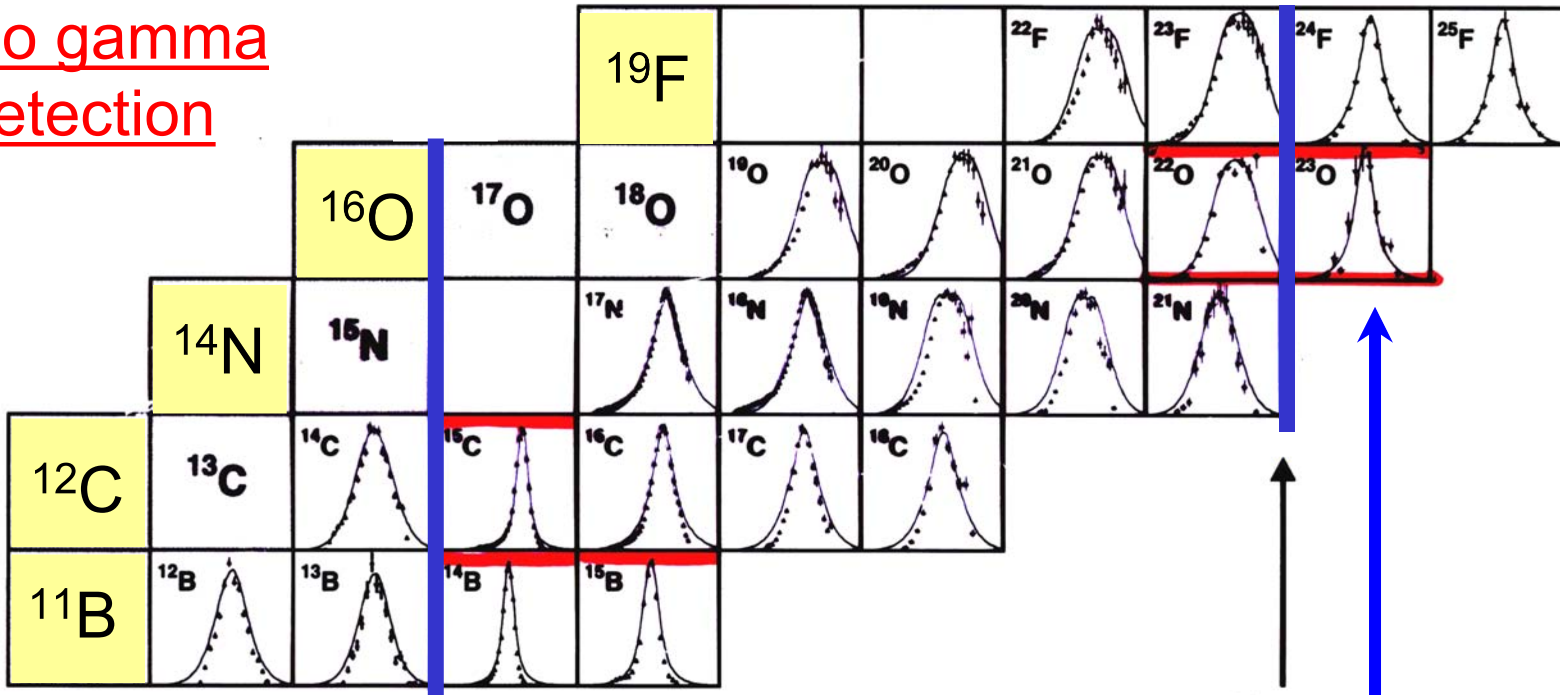




consider momentum components $p_{||}$ of the core parallel to the z direction, in the projectile rest frame

Systematic of momentum content in p-shell

No gamma detection



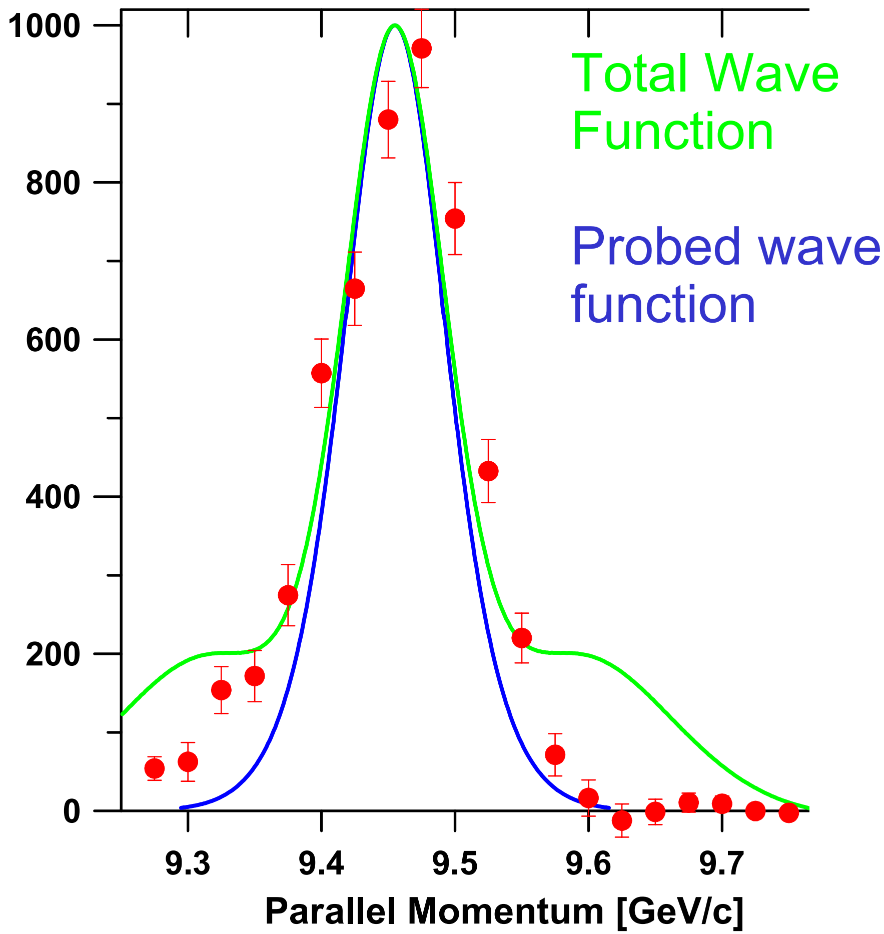
N=14

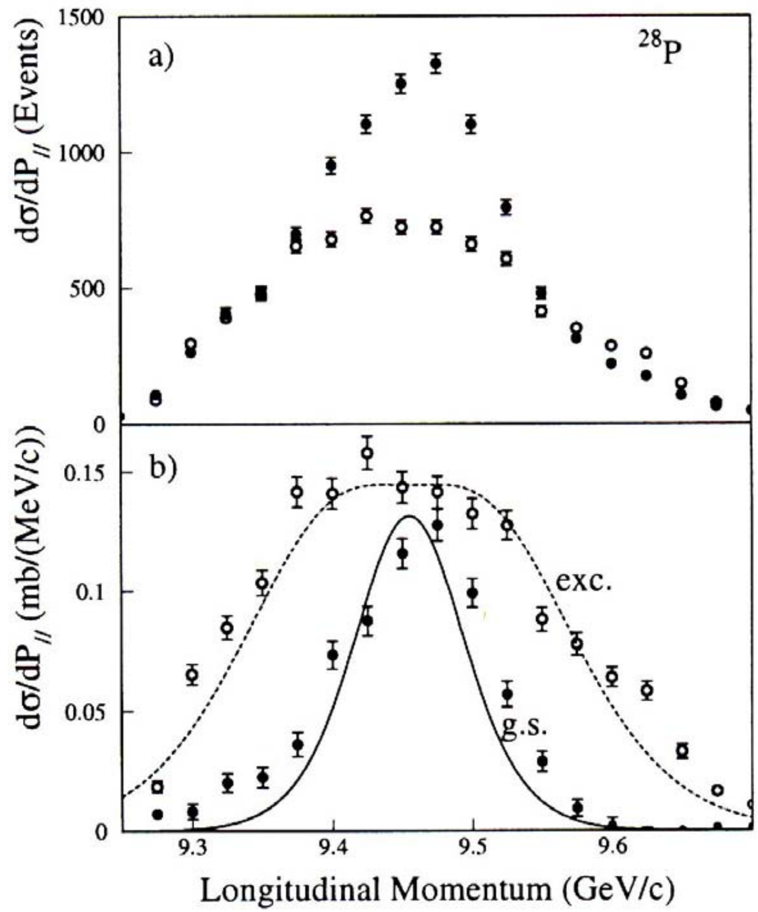
N=8

distributions narrow (weak binding or s-states as one crosses shell or sub-shell closures

E.Sauvan et al., Phys Lett B **491** (2000) 1

An s-state ground state in ^{28}P?





(^{28}P,^{27}Si(gs))X

Proton halo states in phosphorus isotopes?

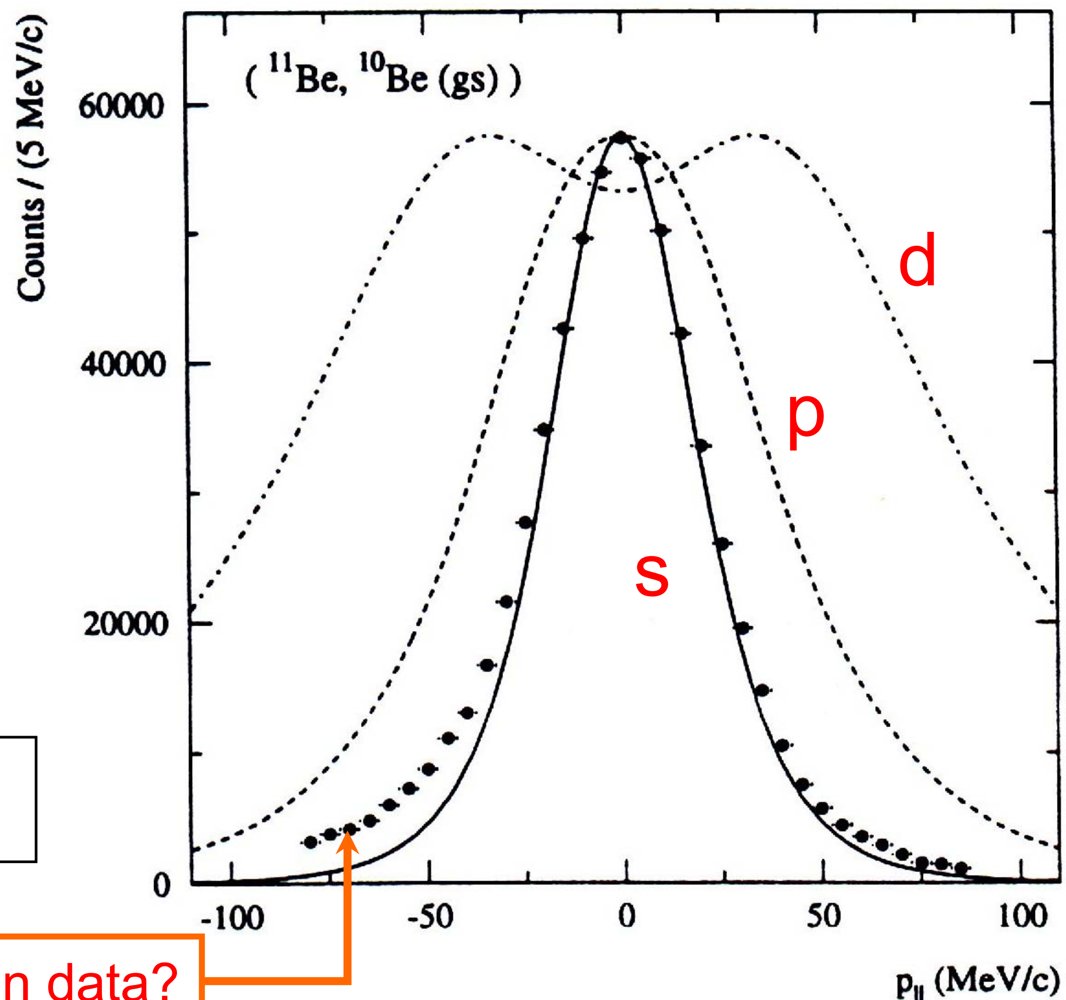
	S_n [MeV]	σ(tot,exp) [mb]	σ(exp) [mb]	σ (gs,theo) [mb]
^{26}P	0.14 (0.20)	72 (13)	40 (14)	36
^{27}P	0.90 (0.04)	74 (11)	22 (8)	23
^{28}P	2.07	70 (11)	21 (5)	23

Navin et al. PRL **81** (1998) 5089

Residue parallel momentum distributions

Calculations of ^{10}Be residue $p_{||}$ momentum distributions following neutron knockout from a ^{11}Be beam at 60A MeV/, with no coincident photon - ^{10}Be in its ground state.

T. Aumann et al. PRL **84** (2000) 35

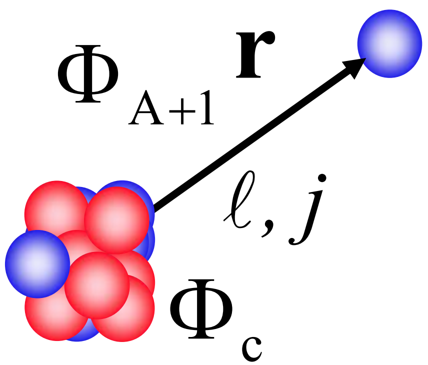


There is asymmetry in data?

Structure information - nucleon formfactors

Nucleon removal from Φ_{A+1} will leave mass A residue in the ground or an excited state - even in extreme sp model

More generally: amplitude for finding nucleon with sp quantum numbers ℓ,*j*, about core state Φ_c in Φ_{A+1} is



$$F^{c}_{\ell j}(\mathbf{r}_c) = \langle \mathbf{r}, \Phi_c \,|\, \Phi_{A+1} \rangle, \quad S_N = E_{A+1} - E_c$$

$$\int d\mathbf{r}\, |F^{c}_{\ell j}(\mathbf{r})|^2 = C^2 S(\ell j)$$

Spectroscopic factor - occupancy of the state

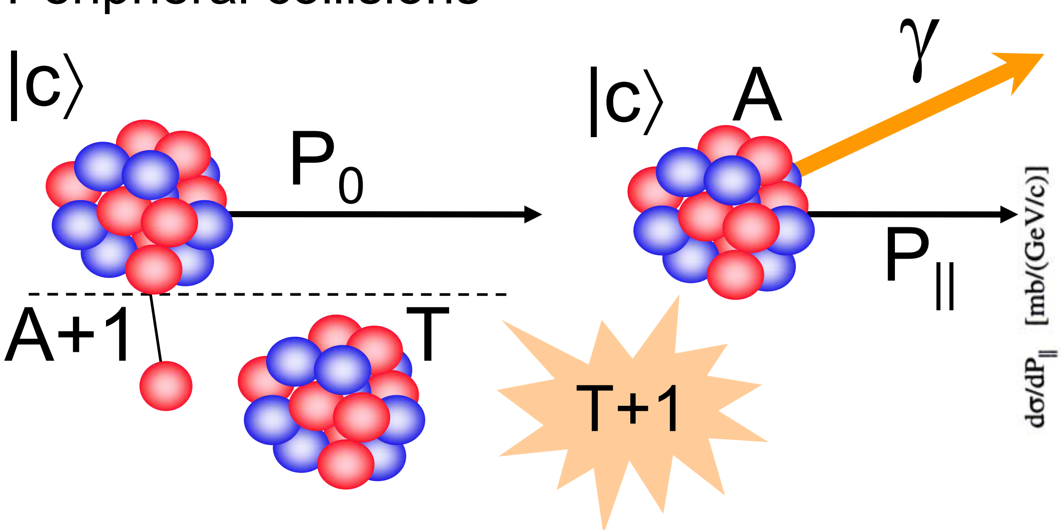
Usual to write

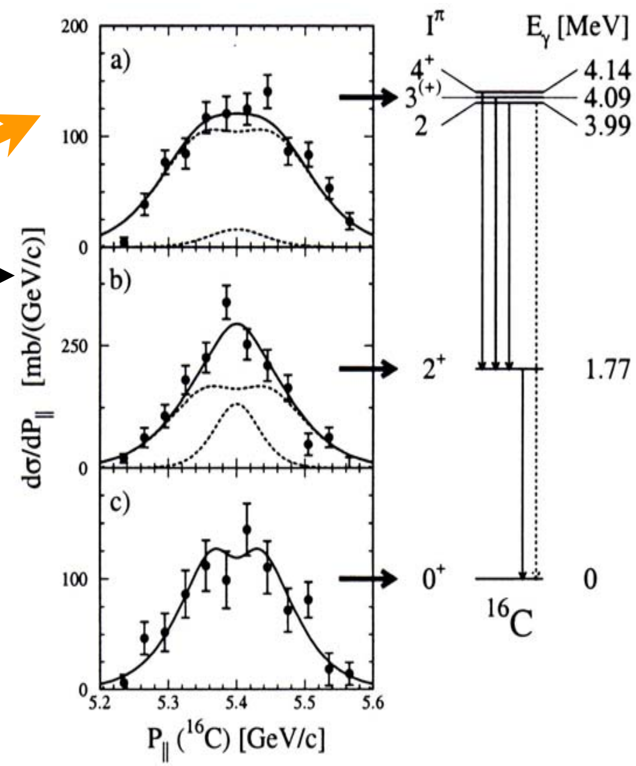
$$F^{c}_{\ell j}(\mathbf{r}) = \sqrt{C^2 S(\ell j)}\; \phi^{c}_{\ell j}(\mathbf{r})\,; \qquad \int d\mathbf{r}\, |\phi^{c}_{\ell j}(\mathbf{r})|^2 = 1$$

with $\phi(\mathbf{r})$ calculated in a potential model (Woods-Saxon)

Adiabatic spectator core model of knockout

Peripheral collisions





$$\sigma_{sp}(j,B_n) = \overbrace{\sigma_{sp}^{\text{diff}}(j,B_n)}^{\text{T(gs)+n}} + \overbrace{\sigma_{sp}^{\text{strip}}(j,B_n)}^{\text{T*}}$$

$$\sigma(nI^{\pi}) = \sum C^2 S(j,nI^{\pi})\,\sigma_{sp}(j,B_n)$$

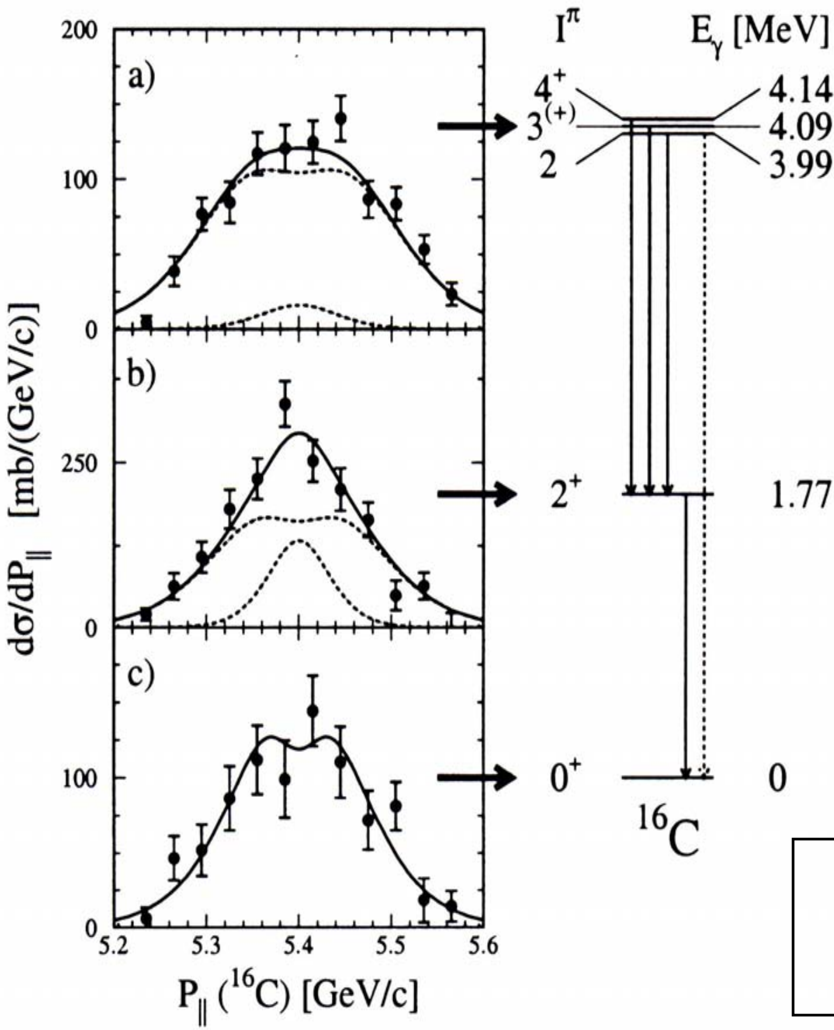
V. Maddalena et al. PRC **63** (2001) 024613

Single-neutron knockout from ^{17}C

ℓ=0,2
admixture

ℓ=0,2
admixture

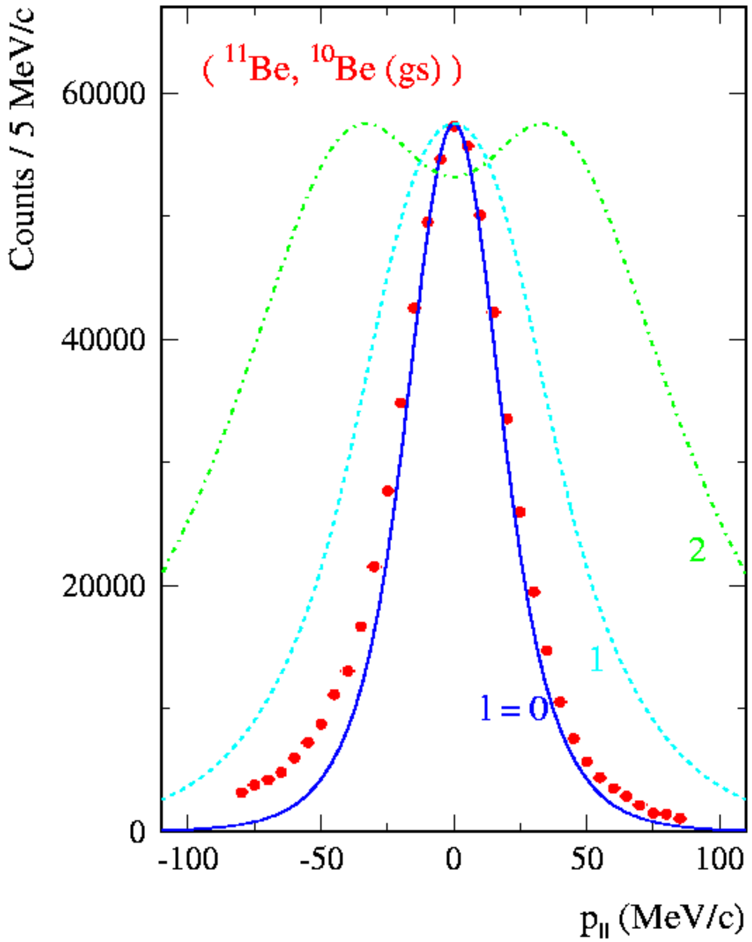
ℓ=2
pure



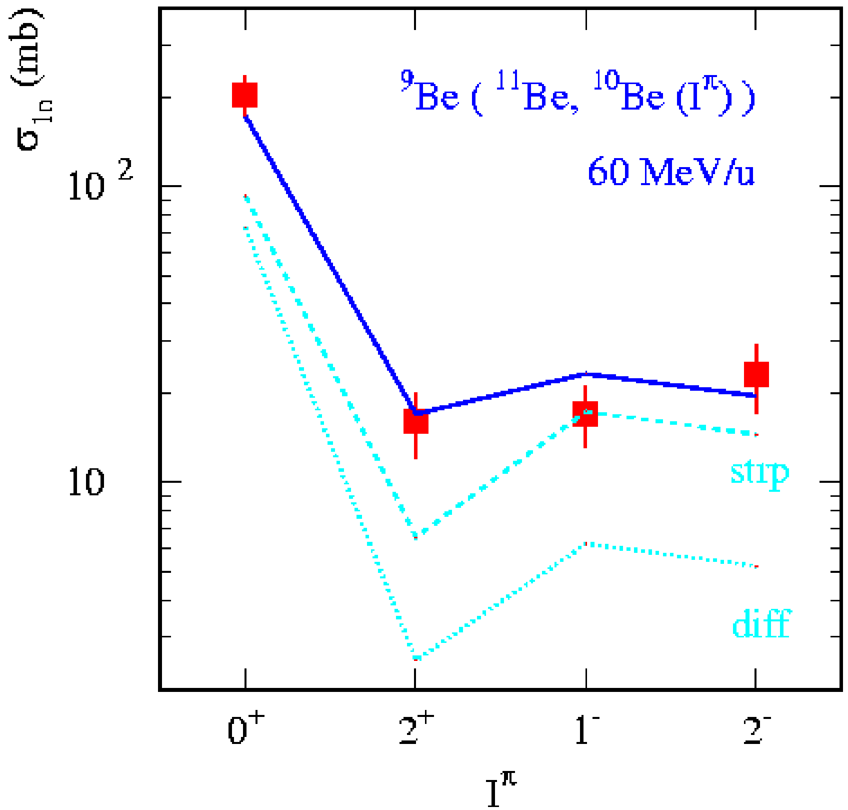
V. Maddalena et al.
PRC **63**
(2001) 024613

Single-neutron knockout spectroscopy of ^{11}Be

Momentum distributions



Partial cross sections



T. Aumann et al. PRL **84** (2000) 35

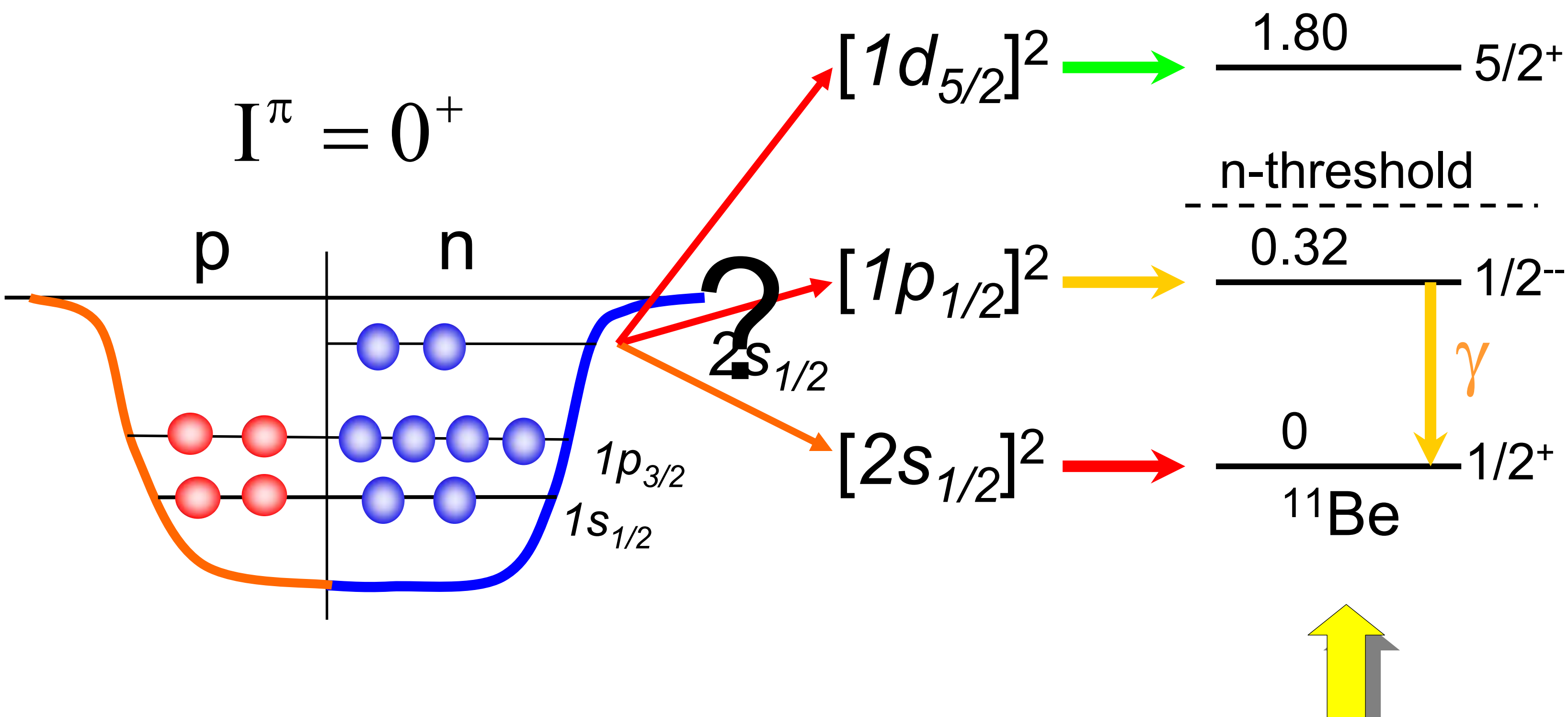
Single-neutron knockout spectroscopy of ^{11}Be

Partial cross sections in mb to the final states I^π in ^{10}Be

I^π	l	S	σ_{sp}^{strp}	σ_{sp}^{diff}	σ^{coll}	σ^{theo}	σ^{exp}
0^+	0	0.74	125	98	10[a]	172	203(31)
2^+	2	0.18	36	14	11[b]	17	16(4)
1^-	1	0.69	25	9		23	17(4)
2^-	1	0.58	25	9		20	23(6)
Σ						224	259(39)

[a] Coulomb dissociation
[b] Rotational excitation, Spectroscopic factor is that of the 0^+ state

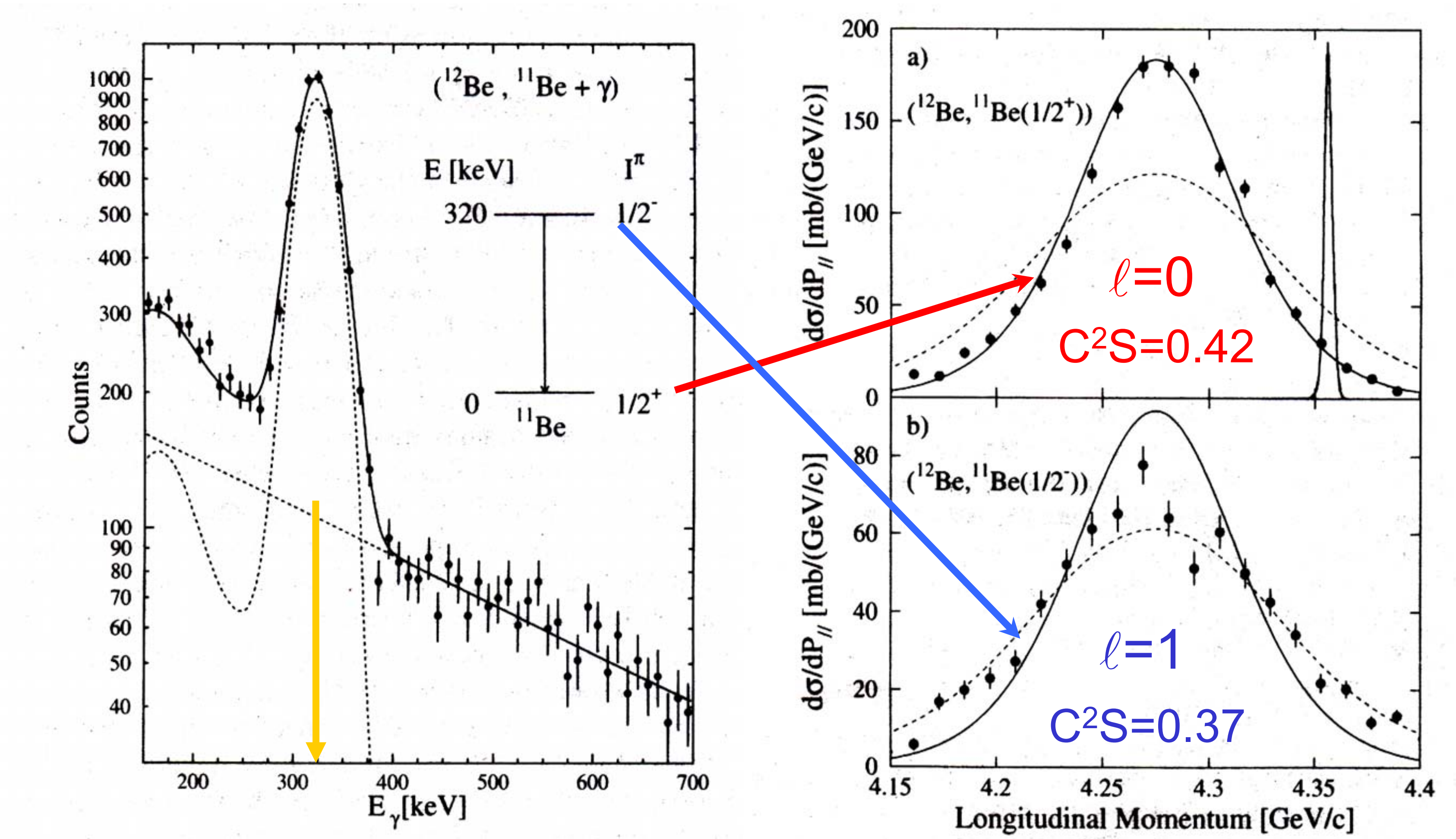
N=8 neutron shell closure (magic no.) in ^{12}Be?



is psd-shell ordering restored in ^{12}Be?

level inversion in ^{11}Be established - halo state

N=8 neutron shell closure in ^{12}Be?

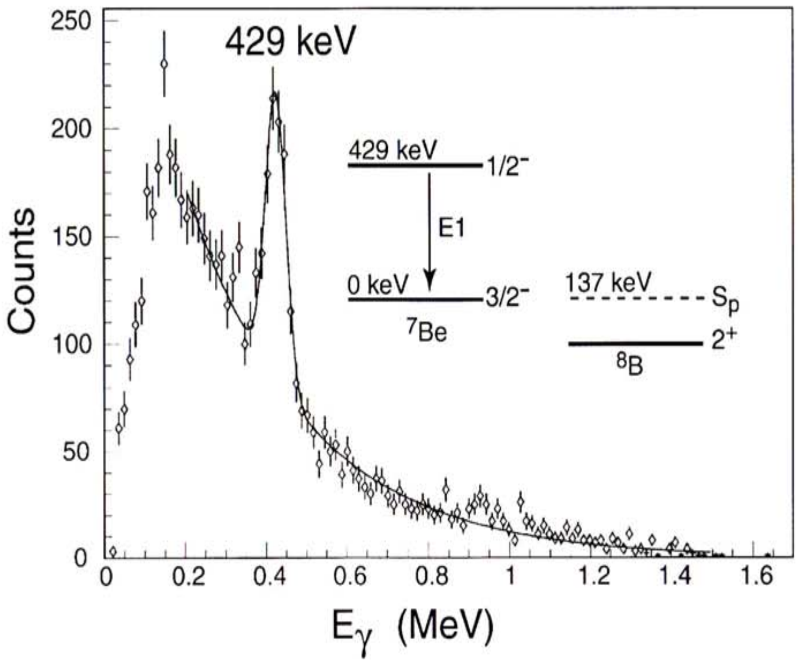


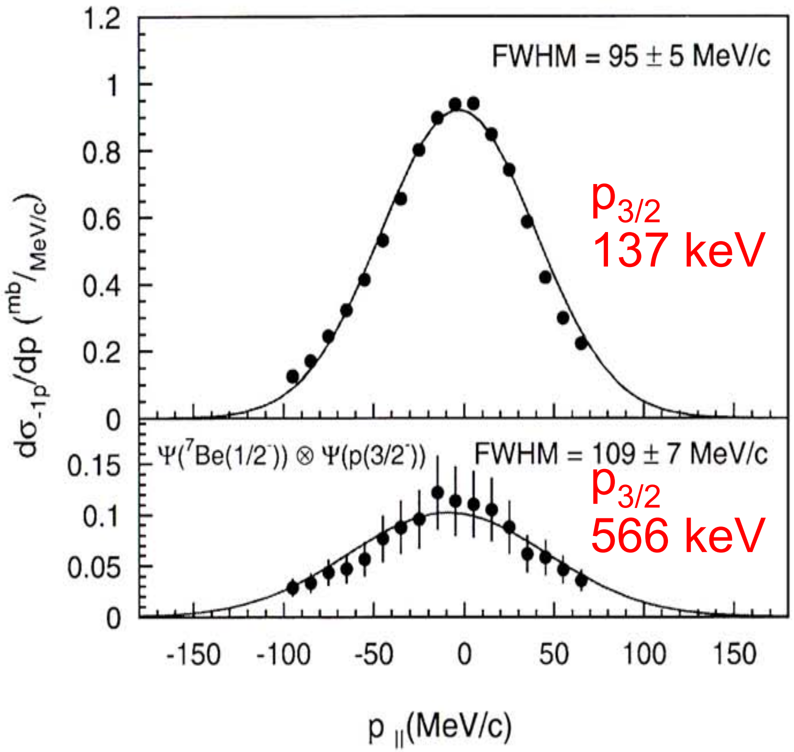
A. Navin et al., PRL **85** (2000) 266

Spectroscopic factors for $^{12}Be \rightarrow {}^{11}Be+n$

j^π	E (MeV)	σ_{exp} (mb)	σ_{sp} (mb)	S_{exp}	S^*_{exp}	S_{th} WBT	WBT2
$1/2^+$	0	32.0 ± 4.7	75.9	0.42 ± 0.10	0.53 ± 0.13	0.51	0.69
$1/2^-$	0.32	17.5 ± 2.6	47.2	0.37 ± 0.10	0.45 ± 0.12	0.91	0.58
$5/2^+$	1.8	...	...	...	...	0.40	0.55

The ground state structure of ^{8}B



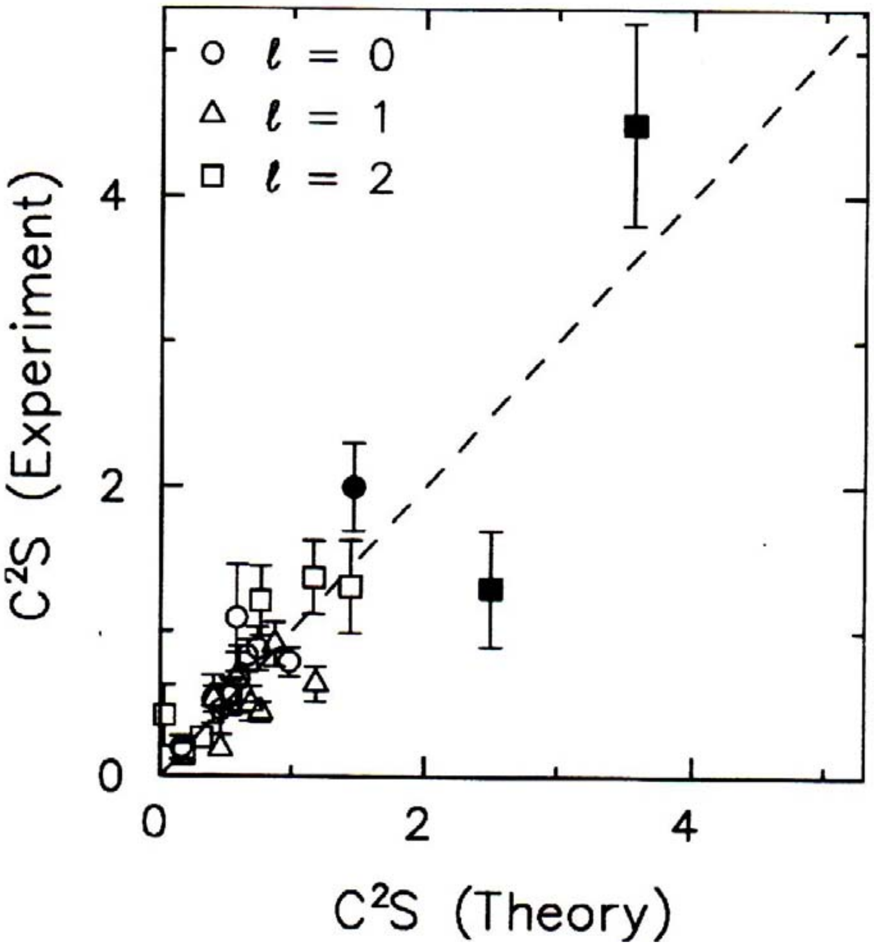


Proton removal from ^{8}B measured at the GSI with gamma coincidences, sees a (15%) branch from an excited ^{7}Be($1/2^-$) core component in the ^{8}B wave function.

D.Cortina-Gil et al., Phys Lett B **529** (2002) 36

Systematics of spectroscopic factors

Eikonal few-body reaction theory and experimental data



Shell model theory

Absolute spectroscopic factors using knockout

B.A Brown et al. PRC **65** (2002) 061601(R)

^{A-1}Z	E_B MeV/ nucleon	E^*	σ_{sp}(mb) [a] Strip.	Diffr.	σ_{th} (mb)	σ_{expt} (mb)	R_s		(e,e'p)
^{11}B	250	a	21.9	1.8	100.5	65.6(26) [b]	0.65(3)		
	1050	a	20.8	1.9	96.1	48.6(24) [c]	0.51(3)	0.53(2)	0.51(3)
15.96	2100	a	20.6	2.0	96.1	53.8(27) [c]	0.56(3)		
^{11}C	250	a	21.4	1.7	98.2	56.0(41) [b]	0.57(4)		
	1050	a	20.2	1.8	93.4	44.7(28) [c]	0.48(3)	0.49(2)	
18.72	2100	a	20.1	1.9	93.3	46.5(23) [c]	0.50(3)		
^{15}N	2100	0	15.40	1.77					
		6.324	12.95	1.30					
12.13		Sum			80.2	54.2(29) [b]	0.68(4)	0.68(4)	0.67(5)
^{15}O	2100	0	14.63	1.61					
		6.176	12.54	1.23					
15.66		Sum			76.9	42.9(23) [c]	0.56(3)	0.56(3)	

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Absolute spectroscopic factors for exotics?

B.A. Brown et al. PRC **65** (2002) 061601(R)

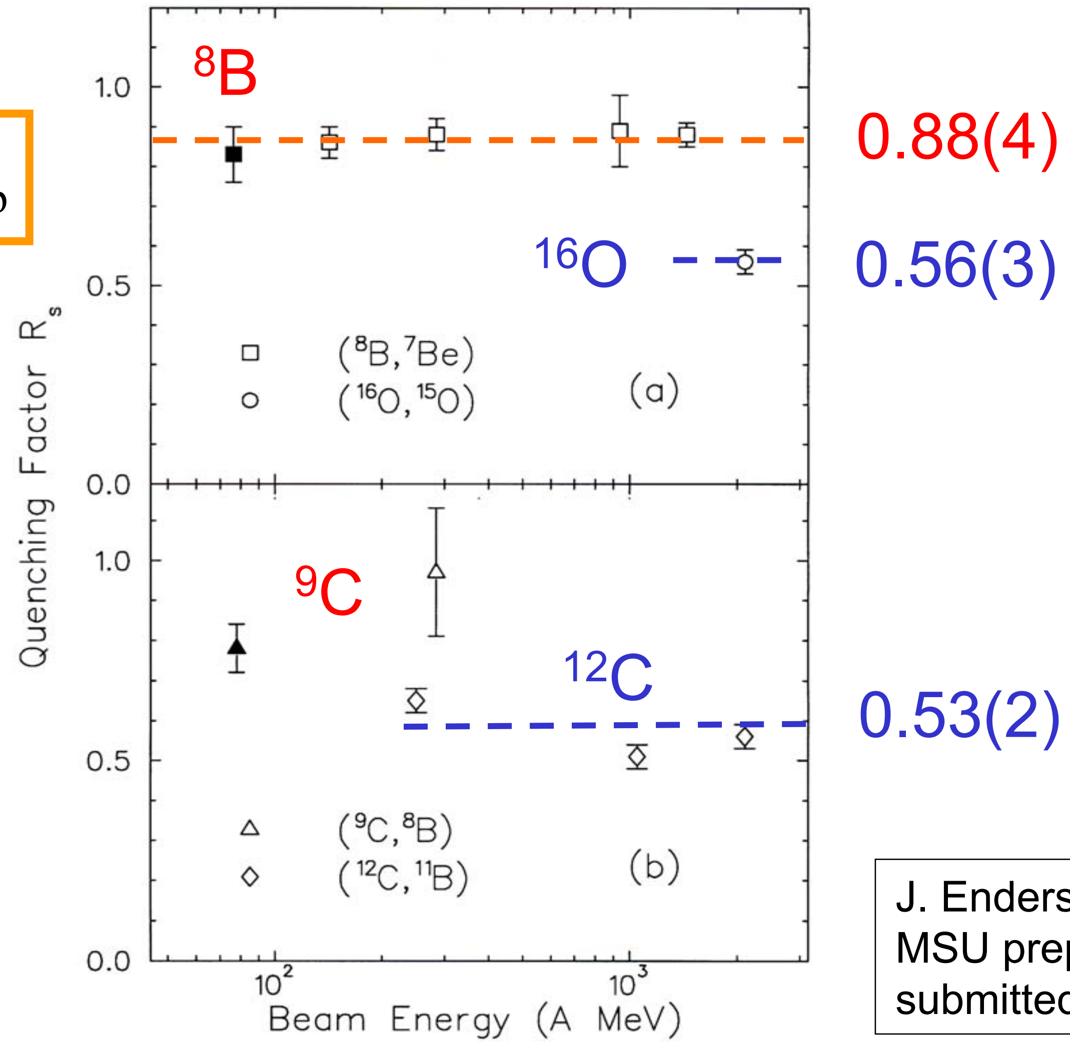
E_B MeV/nucleon	E^* MeV	σ_{sp} (mb) Str.	Dif.	Cou.	σ_{th} (mb)	σ_{expt} (mb)	R_s
142	0	59.8	26.6	4.0	107.1		
	0.429	53.6	20.6	1.5	19.0		
	Sum:				126.1	109(1) [b]	0.86(1)
285	0	57.3	11.9	2.6	85.0		
	0.429	51.8	9.2	1.0	15.6		
	Sum:				100.6	89(2) [b]	0.88(2)
936	0	59.4	14.5	1.6	89.4		
	0.429	52.8	11.1	0.6	16.2		
	Sum				105.6	94(9) [c]	0.89(9)
1440	0	60.5	15.9	1.4	92.1		
	0.429	53.6	12.1	0.6	16.7		
	Sum:				108.8	96(3) [d]	0.88(3)

$^9Be(^8B,^7Be)X$

0.88(4)

Spectroscopic factors at lower energy

$$R_s = \sigma_{th} / \sigma_{exp}$$



J. Enders et al., MSU preprint, submitted

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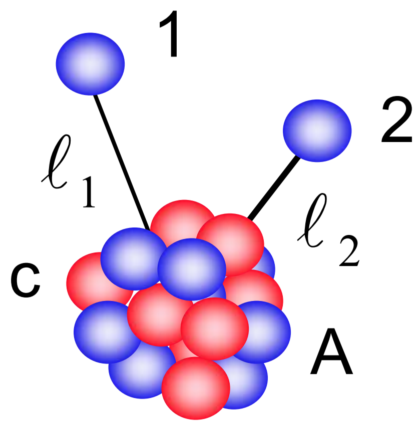
Two nucleon removal - what are useful regimes?

$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1-|S_1|^2)(1-|S_2|^2)\,|\,\phi_0\rangle$$

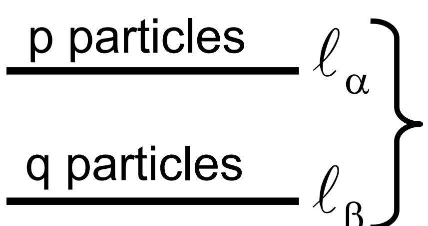


Estimate assuming removal of a pair of uncorrelated nucleons -

$$\phi_0(A,\mathbf{r}_1,\mathbf{r}_2) = \Phi_c(A)\phi_{\ell_1}(\mathbf{r}_1)\phi_{\ell_2}(\mathbf{r}_2)$$

$$\sigma_{\text{strip}} \Rightarrow \sigma_{\text{strip}}(\ell_1\ell_2)$$

contribution from direct 2N removal σ_{-2N}



$$\sigma_{-2N} = \frac{p(p-1)}{2}\sigma_{\text{strip}}(\ell_\alpha\ell_\alpha) + \frac{q(q-1)}{2}\sigma_{\text{strip}}(\ell_\beta\ell_\beta) + pq\,\sigma_{\text{strip}}(\ell_\alpha\ell_\beta)$$

D. Bazin et al., MSU preprint, submitted

Time for a coffee break …….