Schiff Moments Of Xe Isotopes In The Nuclear Shell Model

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Outline of talk

1. What are Schiff moment and EDM

- Backgrounds
- Definition of the Schiff moment

2. Framework

- Framework of the Shell Model calculations
- Framework of the Schiff Moment calculations

3. Results

- Schiff moments
- Evaluation of EDM

4. summary

1. What are Schiff moment and electric dipole moment?

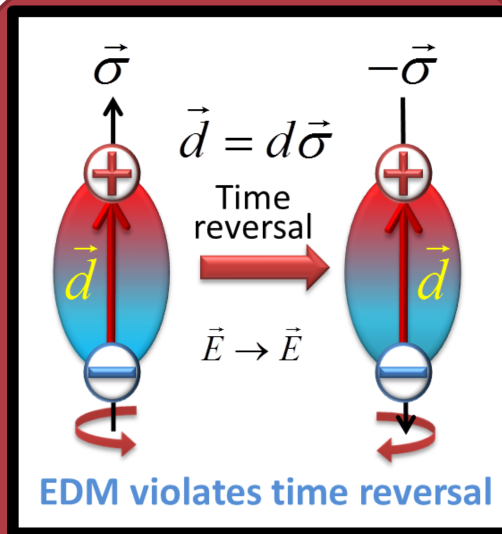
What is the electric dipole moment : EDM

- For a point-like particle at rest,
  EDM $\vec{d}$ is proportional to the spin $\vec{\sigma}$

$$\vec{d} = d\vec{\sigma}$$

- Interaction between the EDM and the external electric field $\vec{E}$ is

$$H = -\vec{d}\cdot\vec{E} = -d\vec{\sigma}\cdot\vec{E}$$



Existence of the EDM indicates violation of time-reversal invariance

T violation relates with CP invariance from the CPT theorem

Atomic EDMs and Schiff moments

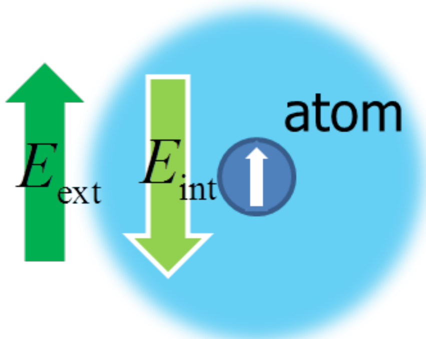
- For EDMs of diamagnetic atoms, Nuclear Schiff moment is important.

- The EDM of a neutral diamagnetic atom is induced mainly by the **nuclear Schiff moment**

  , which gives the leading order contribution to the EDM that is not canceled if the system consists of finite size particles

- **Schiff theorem** L. I. Schiff, Phys. Rev. **132**, 2194 (1963).

  We cannot observe the EDM of a neutral system that consists of point-like particles such as a nucleus and electrons, even though they have EDMs themselves.



$$\vec{E}_{\rm int} + \vec{E}_{\rm ext} = 0$$

Schiff moment : definition

Effective electric potentials for electrons surrounding the atomic nucleus

$$e\Phi_{eff}(\vec{r}) = \langle 0_N | e\Phi(\vec{r}) + \frac{1}{Z}\langle e\vec{r}\rangle_N \cdot \Phi(\vec{r}) | 0_N \rangle \equiv -\frac{Ze^2}{r} + 4\pi e \vec{S}\cdot\vec{\nabla}\delta(\vec{r}) + \cdots$$

Schiff moment operator

$$k = 1,2,3, = x,y,z$$

$$\hat{S}_k = \frac{1}{10}\int\left(r^2 r_k - \frac{5}{3}\langle r^2\rangle_{ch} r_k - \frac{2}{3}\langle Q_{kk'}\rangle r_{k'}\right)\rho(\vec{r})d\vec{r}$$

$\vec{r}_i$: position of the *i*th nucleon

$\langle Q_{kk'}\rangle$: quadrupole moment

$\langle r^2\rangle_{ch}$: nuclear charge mean square radius

$\rho(\vec{r})$: nuclear charge distribution

$\langle Q_{kk'}\rangle = 0$ for the spin $I = \frac{1}{2}$ ground state

Schiff moment operator in terms of nucleons

Schiff moment operator (for *I* =1/2 states)

$$\hat{\mathbf{S}} = \frac{1}{10}\sum_{i}^{A} e_i \left(r_i^2 \mathbf{r}_i - \frac{5}{3}\langle r^2 \rangle_{ch} \mathbf{r}_i \right)$$

$\vec{r}_i$: position of the *i*th nucleon

$\langle r^2 \rangle_{ch}$: nuclear charge mean square radius

$$e_i = \begin{cases} 0 & \text{(for neutrons)} \\ e & \text{(for protons)} \end{cases}$$

Schiff moment

$$S \equiv \langle I^\pi | \hat{S}_z | I^\pi \rangle$$

$|I^\pi\rangle$: Ground state wavefunction with spin *I* and parity *π*, including PT violating components

Evaluation of Schiff Moment

- If P and T violating interaction V^{PT} exists

$$H = H_0 + V^{PT}, \qquad H_0 \left| I_i^{\pi} \right\rangle = E_i^{\pi} \left| I_i^{\pi} \right\rangle$$

, using perturbation theory for the ground state $\left| I_1^{+} \right\rangle$

$$S = \sum_{k \neq 1} \frac{\left\langle I_1^{+} \middle| \hat{S}_z \middle| I_k^{-} \right\rangle \left\langle I_k^{-} \middle| \hat{V}^{PT}_{\pi(T)} \middle| I_1^{+} \right\rangle}{E_1^{+} - E_k^{-}} + c.c.$$

- isoscalar $\quad V^{PT}_{\pi(0)} = F_0 \bar{g}^{(0)} g \left(\vec{\tau}_1 \cdot \vec{\tau}_2 \right) \left(\vec{\sigma}_1 - \vec{\sigma}_2 \right) \cdot \vec{r} f(r)$
- isovector $\quad V^{PT}_{\pi(1)} = F_1 \bar{g}^{(1)} g \left[\left(\tau_{1z} + \tau_{2z} \right) \left(\vec{\sigma}_1 - \vec{\sigma}_2 \right) + \left(\tau_{1z} - \tau_{2z} \right) \left(\vec{\sigma}_1 + \vec{\sigma}_2 \right) \right] \cdot \vec{r} f(r)$
- isotensor $\quad V^{PT}_{\pi(2)} = F_2 \bar{g}^{(2)} g \left(3\tau_{1z}\tau_{2z} - \vec{\tau}_1 \cdot \vec{\tau}_2 \right) \left(\vec{\sigma}_1 - \vec{\sigma}_2 \right) \cdot \vec{r} f(r)$

$$f(r) = \frac{\exp\left(-m_\pi r\right)}{m_\pi r^2} \left(1 + \frac{1}{m_\pi r} \right)$$

P. Herczeg, Hyperfine Interact. 75, **127** (1992)

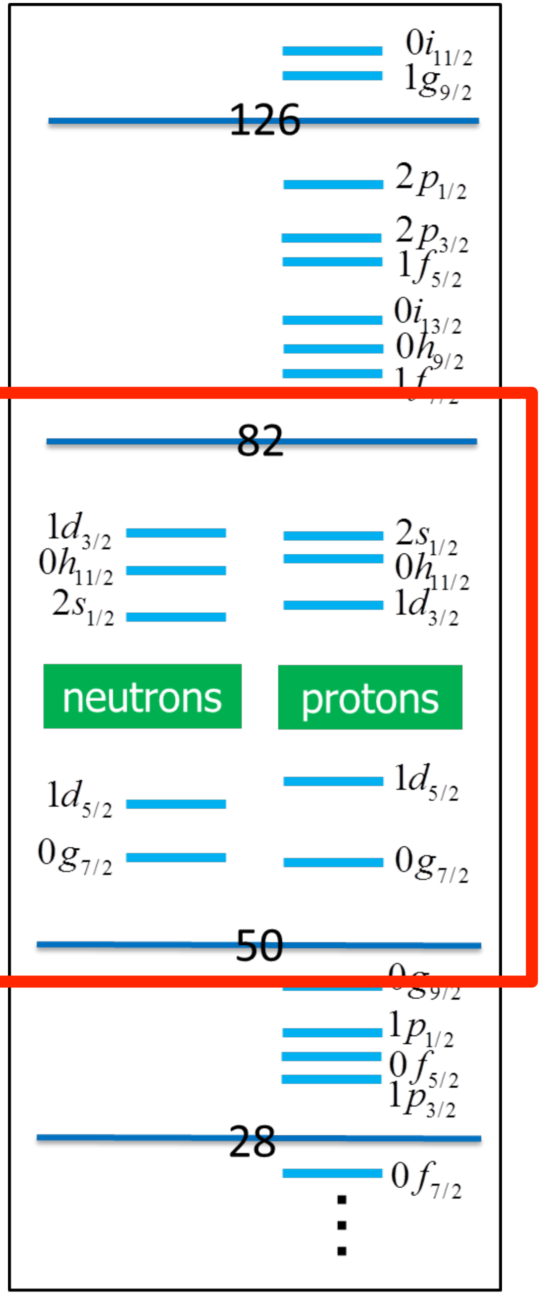
2. framework

- Shell model framework
- Schiff moment framework

Shell model frame work

- For the ground state $\left|I_1^\pi\right\rangle$ and excited states for Xe isotopes

Neutrons and protons in five oribitals between magic numbers 50 and 82



Nuclear ground and excited states

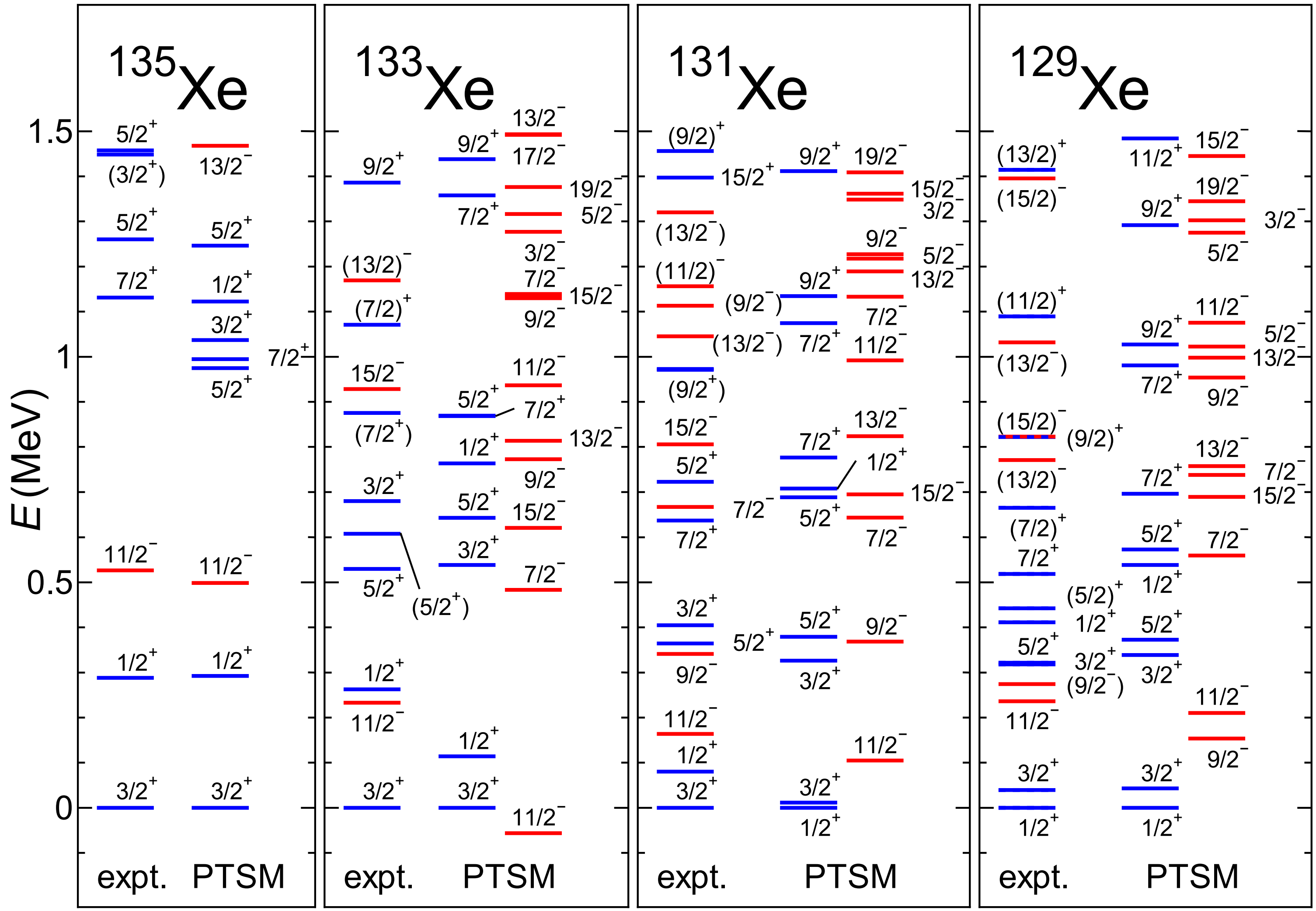
◆ In order to obtain the ground and excited state $\left|I_i^\pi\right\rangle$ of Xe isotopes, we diagonalize the hamiltonian which does not break P and T.

$$H_0 \left|I_i^\pi\right\rangle = E_i^\pi \left|I_i^\pi\right\rangle$$

$$H_0 = \text{Pairing} + \text{QQ} + \text{Multipole-interaction}$$

K. Higashiyama, *et. al.*, Phys. Rev. C. **83**, 034321 (2011).

Shell model results

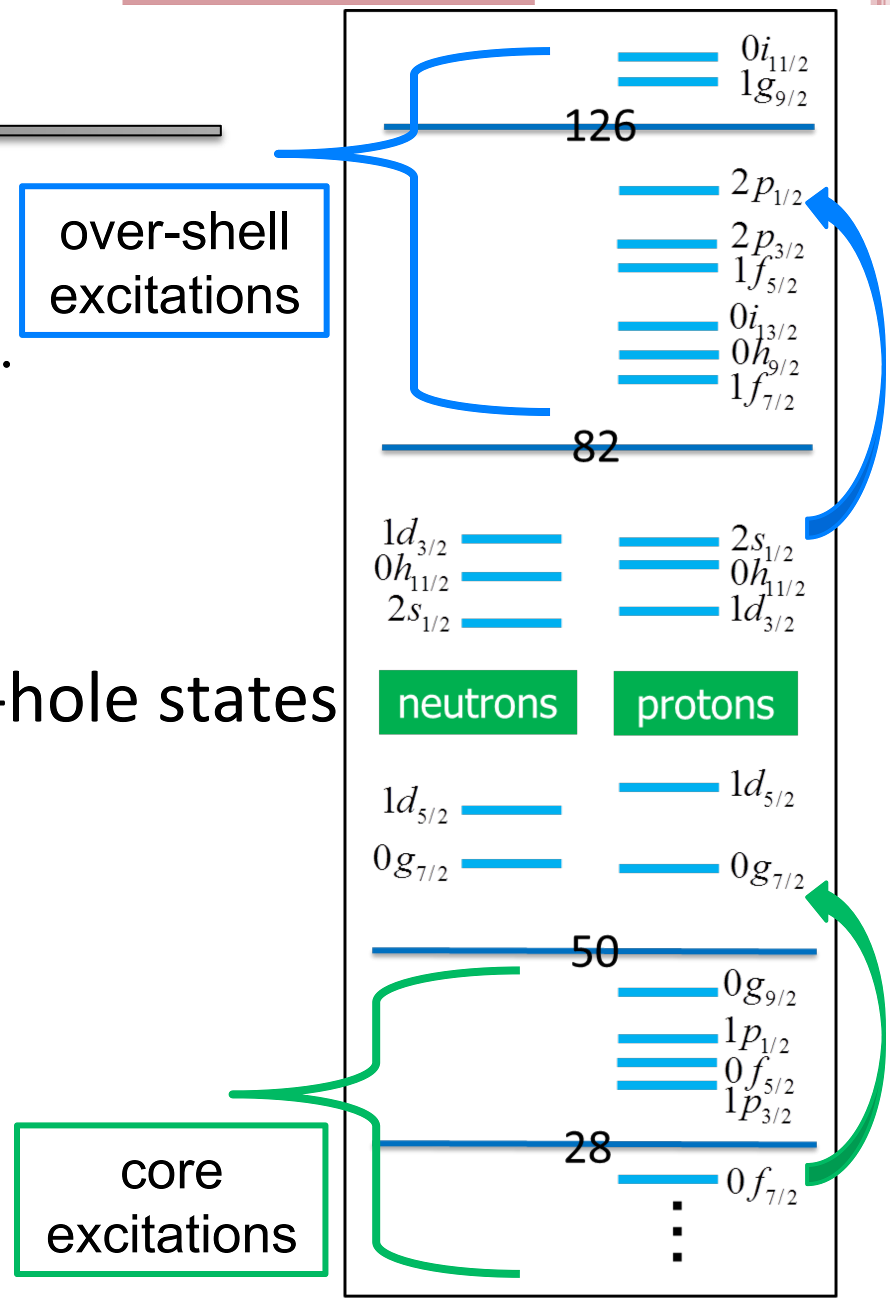


K. Higashiyama et al, Phys. Rev. C **83**, 034321 (2011).

Intermediate states

$$S=\sum_{k\neq 1}\frac{\langle I_1^+|\hat{S}_z|I_k^-\rangle\langle I_k^-|\hat{V}^{PT}_{\pi(T)}|I_1^+\rangle}{E_1^+-E_k^-}+c.c.$$

- As for intermediate states, we take one-particle and one-hole states
- For the intermediate states negative parity orbitals
  over-shell excitation (82<)
  core excitation (>50)



1p-1h states (explicitly)

- Intermediate states (approximately as 1p-1h states

$$\left|I_k^-\right\rangle ; \left|(ph)L;I^-\right\rangle = N_{ph}^{(L)}\left[\left[c_{p\pi}^\dagger \tilde{c}_{h\pi}\right]^{(L)}\left|I_1^+\right\rangle\right]_M^{(I)}$$

$c_{p\pi}^\dagger$: proton creation operator for orbital p

$\tilde{c}_{h\pi}$: proton annihilation operator for orbital h

$N_{ph}^{(L)}$: Normalization constant

$$S = \sum_{k=1} \frac{\left\langle I_1^+ \middle| \hat{S}_z \middle| I_k^- \right\rangle \left\langle I_k^- \middle| \hat{V}_{\pi(T)}^{PT} \middle| I_1^+ \right\rangle}{E_1^+ - E_k^-} + c.c.$$

$$; \sum_{Lph} \frac{\left\langle I_1^+ \middle| \hat{S}_z \middle| (ph)L;I^- \right\rangle \left\langle (ph)L;I^- \middle| \hat{V}_{\pi(T)}^{PT} \middle| I_1^+ \right\rangle}{\varepsilon_h^+ - \varepsilon_p^-} + c.c.$$

Energy denominator is approximated as one-particle one-hole energies

$\varepsilon_h^+, \varepsilon_p^-$: single particle energies of hole and particle states

Tabulation of Schiff moment

$$S = a_0 \, \bar{g}^{(0)} g + a_1 \, \bar{g}^{(1)} g + a_2 \, \bar{g}^{(2)} g$$

$\bar{g}^{(T)}$: $T = 0, 1, 2$

unknown parameters
breaking *P* and *T*

$g \approx 13.5$ Strong coupling constant

3. results

- Numerical results for Schiff moment
- The upper limit of atomic EDMs

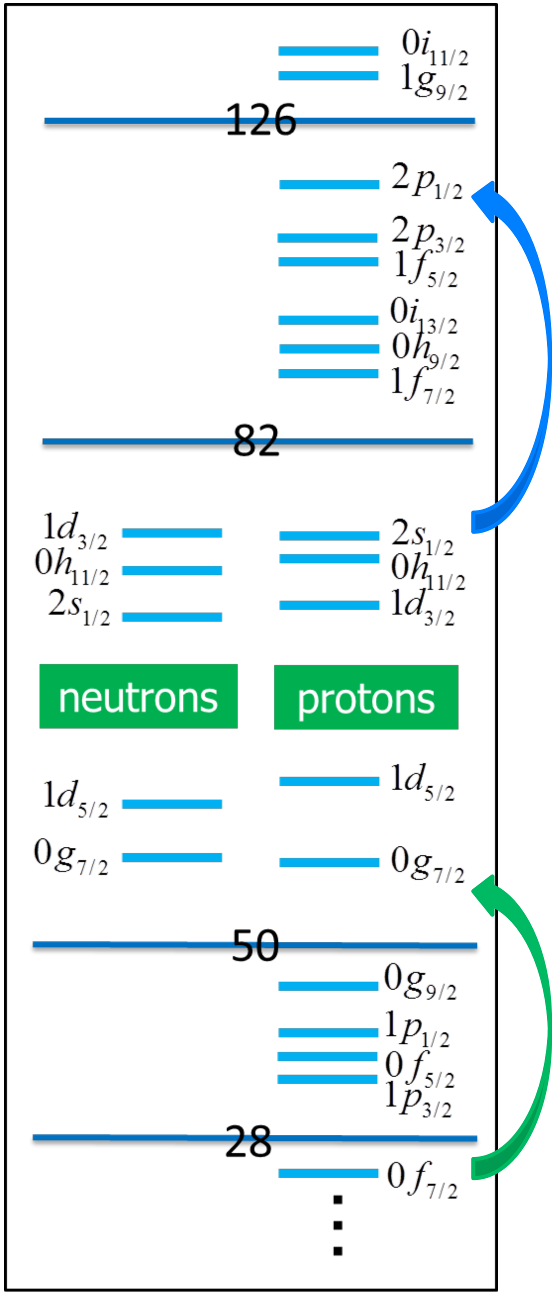
Schiff moment results

◆ Schiff moment of ^{129}Xe

isospin	over-shell	core	sum
a_0	0.929	2.910	3.838
a_1	0.392	1.146	1.538
a_2	1.426	3.963	5.389

in units of 10^{-3}

・contribution from core excitations are larger



Results (over-shell excitations)

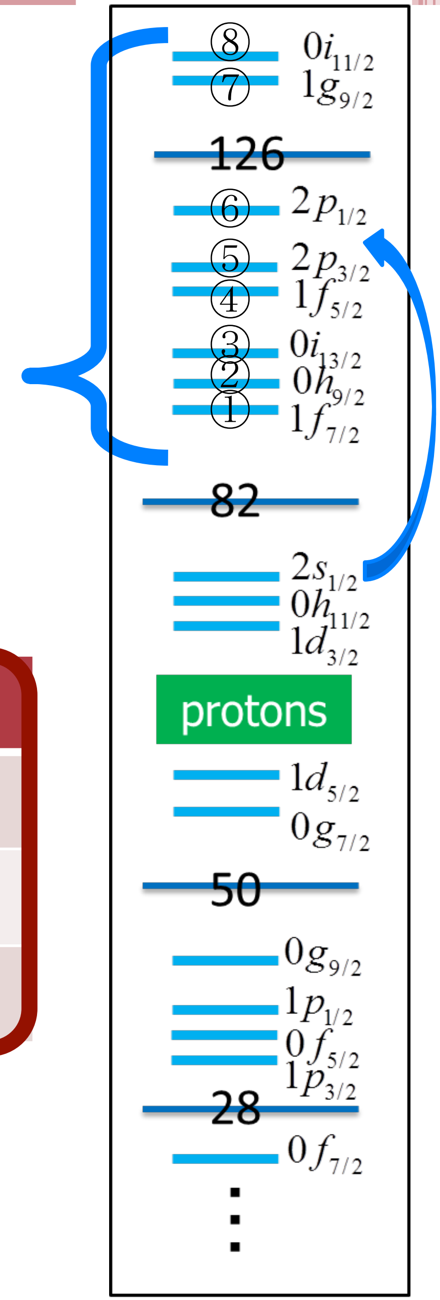
◆ effects of each state

in units of 10^{-3}

isospin	① $1f_{7/2}$	② $0h_{9/2}$	③ $0i_{13/2}$	④ $2p_{3/2}$
a_0	+0.133	+0.265	- 0.007	+0.079
a_1	+0.061	+0.107	- 0.005	+0.049
a_2	+0.235	+0.380	- 0.025	+0.214

isospin	⑤ $1f_{5/2}$	⑥ $1p_{1/2}$	⑦ $1g_{9/2}$	⑧ $0i_{11/2}$	SUM
a_0	+0.385	+0.055	+0.018	+0.001	+0.929
a_1	+0.153	+0.020	+0.008	+0.000	+0.392
a_2	+0.531	+0.063	+0.027	+0.000	+1.426

- Orbital difference: $1f_{7/2}$, $1f_{5/2}$, $0h_{9/2}$ are large
- Same isospin dependence



Results (core excitations)

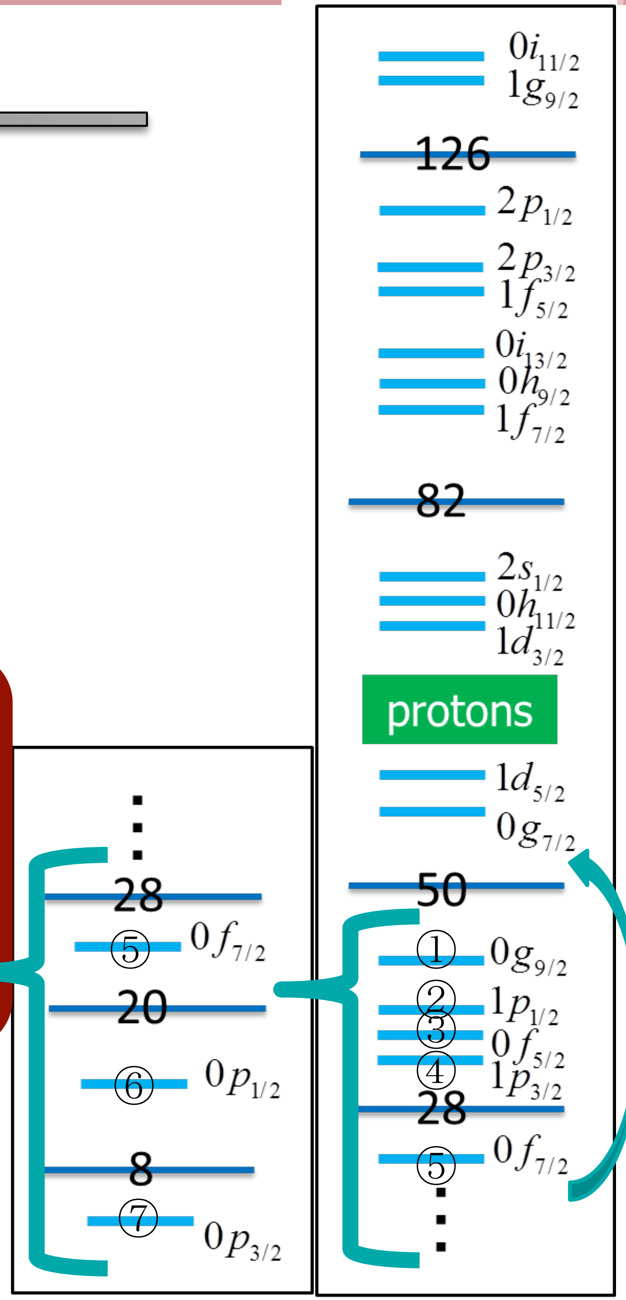
◆ effects of each state

in units of 10^{-3}

isospin	① $0g_{9/2}$	② $1p_{1/2}$	③ $0f_{5/2}$	④ $1p_{3/2}$
a_0	- 3.642	+0.621	+1.961	+1.443
a_1	- 2.022	+0.511	+0.866	+0.502
a_2	- 8.488	+2.444	+3.233	+1.569

isospin	⑤ $0f_{7/2}$	⑥ $0p_{1/2}$	⑦ $0p_{3/2}$	SUM
a_0	+0.897	+0.648	+0.982	+2.910
a_1	+0.480	+0.273	+0.537	+1.146
a_2	+1.980	+0.988	+2.237	+3.963

- Orbital difference: $0g_{9/2}$ is large
- Same isospin dependence



The upper limit of the atomic EDM

- Theoretical result (isoscalar part)

$$S\left({}^{129}\mathrm{Xe}\right)=3.84\times10^{-3}\,\bar{g}^{(0)}g\,e\mathrm{fm}^3$$

- The relation between the Schiff moment and the atomic EDM

$$d_{Atom}\left({}^{129}\mathrm{Xe}\right)=0.38\times10^{-17}\left(\frac{S}{e\mathrm{fm}^3}\right)e\mathrm{cm}$$

V. A. Dzuba, V. V. Flambaum, J. S. M. Ginges and M. G. Kozlov, Phys. Rev. A **66** (2002),012111.

- from the ^{199}Hg experiment

$$\left|\bar{g}^{(0)}g\right|<1.1\times10^{-10}$$

W. C. Griffith, *et.* al., Phys. Rev. Lett. **102**, 101601 (2009).

The upper limit of atomic EDM :

$$\left|d_{Atom}\left({}^{129}\mathrm{Xe}_{\mathrm{g.s.}}\right)\right|<1.61\times10^{-30}\,e\mathrm{cm}$$

cf. experimental upper limits :

$$\left|d_{Atom}\left({}^{129}\mathrm{Xe}\right)\right|<4.1\times10^{-27}\ e\mathrm{cm}$$

$$\left|d_{Atom}\left({}^{199}\mathrm{Hg}\right)\right|<7.4\times10^{-30}\ e\mathrm{cm}$$

The upper limit of the atomic EDM by SM

PT-violating strong πNN coupling constants in the Standard Model estimated by *Yamanaka* et al.

N. Yamanaka and E. Hiyama, JHEP 02 (2016) 067.

$$\bar{g}^{(0)} = -1.1\times 10^{-17}$$

$$\bar{g}^{(1)} = -1.3\times 10^{-17}$$

$$\bar{g}^{(2)} = +3.3\times 10^{-21}$$

$$\left|d_{SM}\left({}^{129}\mathrm{Xe}\right)\right| = 3.0\times 10^{-36}\, e\mathrm{cm}$$

cf. an experimental upper limit :

$$\left|d_{Exp.}\left({}^{129}\mathrm{Xe}\right)\right| < 4.1\times 10^{-27}\ e\mathrm{cm}$$

Observation of larger EDM than this value indicates New Physics beyond the Standard Model.

Summary

◆ **Nuclear Schiff moments of $1/2^+$ states for Xe isotopes are estimated. With obtained Schiff moments,**
upper limits of atomic EDMs are evaluated.

- The orbital dependence on Schiff moments is large
- contribution of core excitations are lager than over-shell excitations

$$S\left({}^{129}\mathrm{Xe}\right) = 3.84\times10^{-3}\,\bar{g}^{(0)}g + 1.54\times10^{-3}\,\bar{g}^{(1)}g + 5.39\times10^{-3}\,\bar{g}^{(2)}g$$

Future researches

◆ Other regions (Octupole correlations)

${}^{225}\mathrm{Ra}$ (A~220)

Our results

$$S\left({}^{129}\mathrm{Xe}\right) = 3.84\times10^{-3}\,\overline{g}^{(0)}g + 1.54\times10^{-3}\,\overline{g}^{(1)}g + 5.39\times10^{-3}\,\overline{g}^{(2)}g$$

Results by Dmitriev

V. F. Dmitriev, *et. al.*, Phys. Rev. C. **71**, 035501 (2005).

With core polarization (many body effects from even-even core)

$$S\left({}^{129}\mathrm{Xe}\right) = 8.00\times10^{-3}\,\overline{g}^{(0)}g + 6.00\times10^{-3}\,\overline{g}^{(1)}g - 9.00\times10^{-3}\,\overline{g}^{(2)}g$$

Note that definitions of tensor type V_{PT} has a sign difference

Our results are consistent with Dmitriev's results !

TABLE V. Effects of core polarization on the Schiff moment for neutron-odd nuclei, for finite range weak interaction. The bare values of the Schiff moment, without core polarization, are listed in the first line for each nucleus. The units are $e\,\mathrm{fm}^3$.

	$g_s g_0$	$g_s g_1$	$g_s g_2$
^{199}Hg	−0.09	−0.09	0.18
^{199}Hg	−0.00004	−0.055	0.009
^{129}Xe	0.06	0.06	−0.12
^{129}Xe	0.008	0.006	−0.009
^{211}Rn	−0.12	−0.12	0.24
^{211}Rn	−0.019	0.061	0.053
^{213}Ra	−0.12	−0.12	0.24
^{213}Ra	−0.012	−0.021	0.016
^{225}Ra	0.08	0.08	−0.16
^{225}Ra	0.033	−0.037	−0.046

Bare values

With core polarization

V. F. Dmitriev, *et. al.*, Phys. Rev. C. **71**, 035501 (2005).

Back ups

Our group: ^{129}Xe, shell model

Shell Model Estimate of Nuclear Electric Dipole Moments, ***"Intrinsic Schiff Moment"***
N. Yoshinaga, K. Higashiyama, and R. Arai, Prog. Theor. Phys. **124**, 1115 (2010).

Nuclear Schiff moments for the lowest 1/2$^+$ states in Xe isotopes, ***"PT-violating Schiff Moment"***
N. Yoshinaga, K. Higashiyama, R. Arai, and E. Teruya, Physical Review C **87**, 044332(2013).

Nuclear electric dipole moments for the lowest 1/2$^+$ states in Xe and Ba isotopes,
N. Yoshinaga, K. Higashiyama, R. Arai, and E. Teruya, Physical Review C **89**, 045501 (2014).
"Nuclear Electric Dipolemoment"

Global analysis for coefficients

The story can be reversed. Using the relation between Schiff moments and the PT-violating coupling constants and the experimental upper limits, we can estimate the coupling constants.

$$S = a_0\, \bar{g}^{(0)} g + a_1\, \bar{g}^{(1)} g$$

Our new results for ^{129}**Xe**

Experimental data from **TlF molecule**, ^{199}**Hg**, ^{129}**Xe**, and **neutron**.

	C_T **(10^7)**	$\bar{g}^{(0)}$ **(10^-10)**	$\bar{g}^{(1)}$ **(10^-10)**	d_n^{sr} **(10^-24)**
Chupp et al.	1.265	−6.687	1.431	9.878
Our calculation	1.409	−7.431	1.476	11.13

T. Chupp and M. Ramsey-Musolf, Phys. Rev. C 91, 035502 (2015).

In the isospin base, the general CP-odd one-kaon exchange nuclear force can be written as

$$H_{\not P \not T}^{K} = \frac{1}{2m_N}\Big\{ \bar{G}_K^{(0')}\, \sigma_- \cdot \nabla \mathcal{Y}_K(r) + \bar{G}_K^{(0)}\, \tau_1 \cdot \tau_2\, \sigma_- \cdot \nabla \mathcal{Y}_K(r)$$
$$+\frac{1}{2}\bar{G}_K^{(1)}\, (\tau_+^z\, \sigma_- + \tau_-^z\, \sigma_+) \cdot \nabla \mathcal{Y}_K(r) + \frac{1}{2}\bar{G}_K^{(1')}\, (-\tau_+^z\, \sigma_- + \tau_-^z\, \sigma_+) \cdot \nabla \mathcal{Y}_K(r)$$
$$+\bar{G}_K^{(2)}\, (3\tau_1^z \tau_2^z - \tau_1 \cdot \tau_2)\, \sigma_- \cdot \nabla \mathcal{Y}_K(r) \Big\}, \qquad (2.108)$$

$$G_K^{(0')} = 5.4 \times 10^{-16},$$
$$G_K^{(0)} = 9.9 \times 10^{-18},$$
$$G_K^{(1)} = -1.3 \times 10^{-16},$$
$$G_K^{(1')} = -4.0 \times 10^{-17},$$
$$G_K^{(2)} = -4.1 \times 10^{-21}.$$

Relativistic effect

V. V. Flambaum and A. Kozlov, PRC 85, 068502 (2012)

$$\mathbf{S}' = \frac{Ze}{10}\frac{1}{1-\frac{5}{14}Z^2\alpha^2}\left\{\left[\langle \mathbf{r}r^2\rangle - \frac{5}{3}\langle \mathbf{r}\rangle\langle r^2\rangle - \frac{2}{3}\langle r_i\rangle\langle q_{ij}\rangle\right]\right.$$
$$\left. -\frac{5}{28}\frac{Z^2\alpha^2}{R_N^2}\left[\langle \mathbf{r}r^4\rangle - \frac{7}{3}\langle \mathbf{r}\rangle\langle r^4\rangle - \frac{4}{3}\langle r_i\rangle\langle q_{ij}r^2\rangle\right]\right\}. \quad (23)$$

The magnetic moment effect

S. G. Porsev, J. S. M. Ginges, and V. V. Flambaum, PRA 83, 042507 (2011)