Magnetic properties of quark matter with the inhomogeneous chiral condensate

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collaborators：Kazuya Nishiyama, Toshitaka Tatsumi

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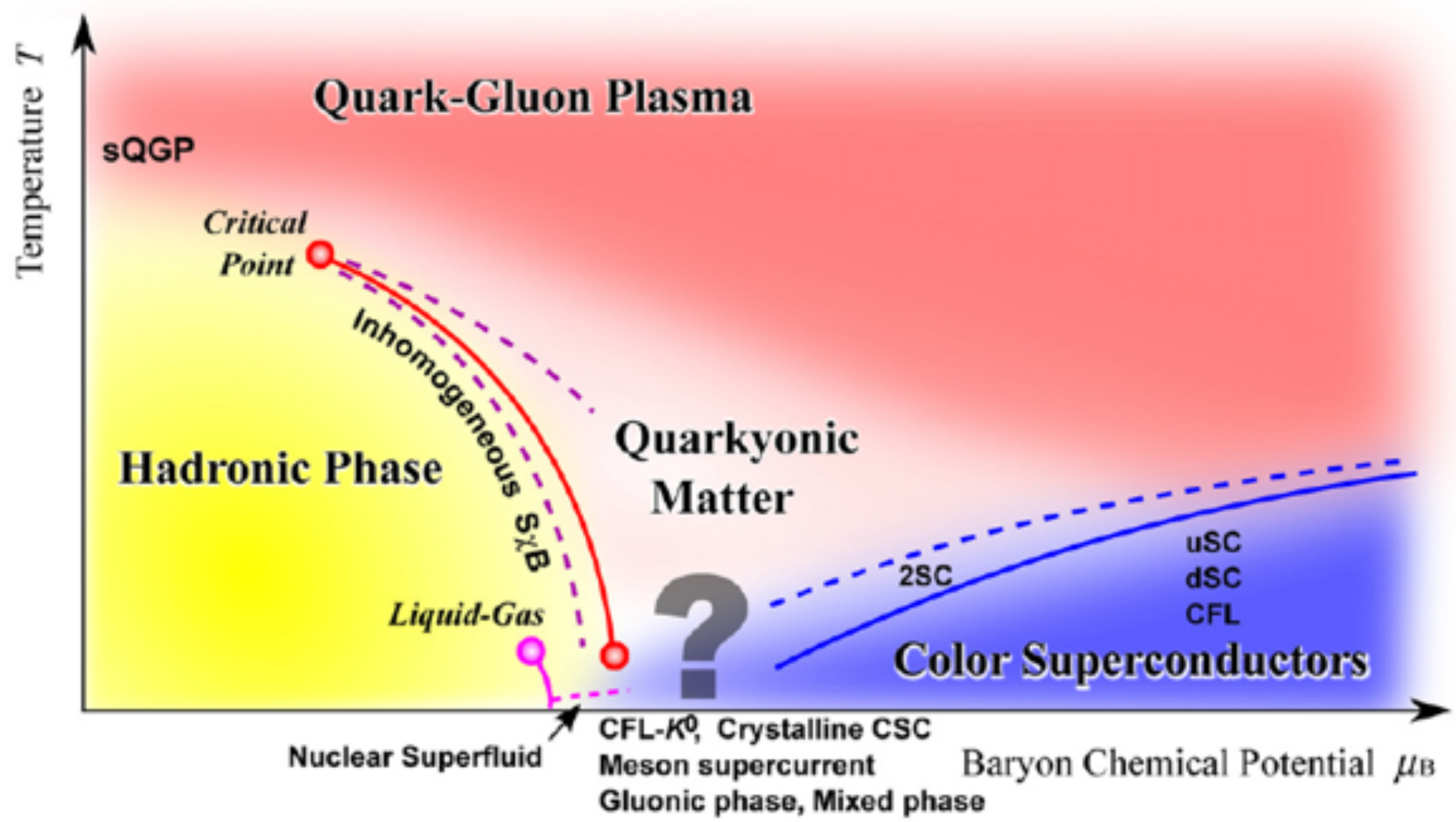
QCD phase diagram

Phase structure of quark matter



- Hadron phase
- Quark-gluon plasma
- Color superconductivity etc…
- Inhomogeneous chiral phase

K. Fukushima, T. Hatsuda,
Rep. Prog. Phys. 74, 014001 (2011)

Chiral symmetry

※chiral limit

Restored phase

$$\langle \bar{\psi}\psi \rangle = 0$$

chiral condensate

SSB

Broken phase

$$\langle \bar{\psi}\psi \rangle \neq 0$$

spatially modulating

cf. FFLO superconductivity, CDW, SDW

QCD phase diagram

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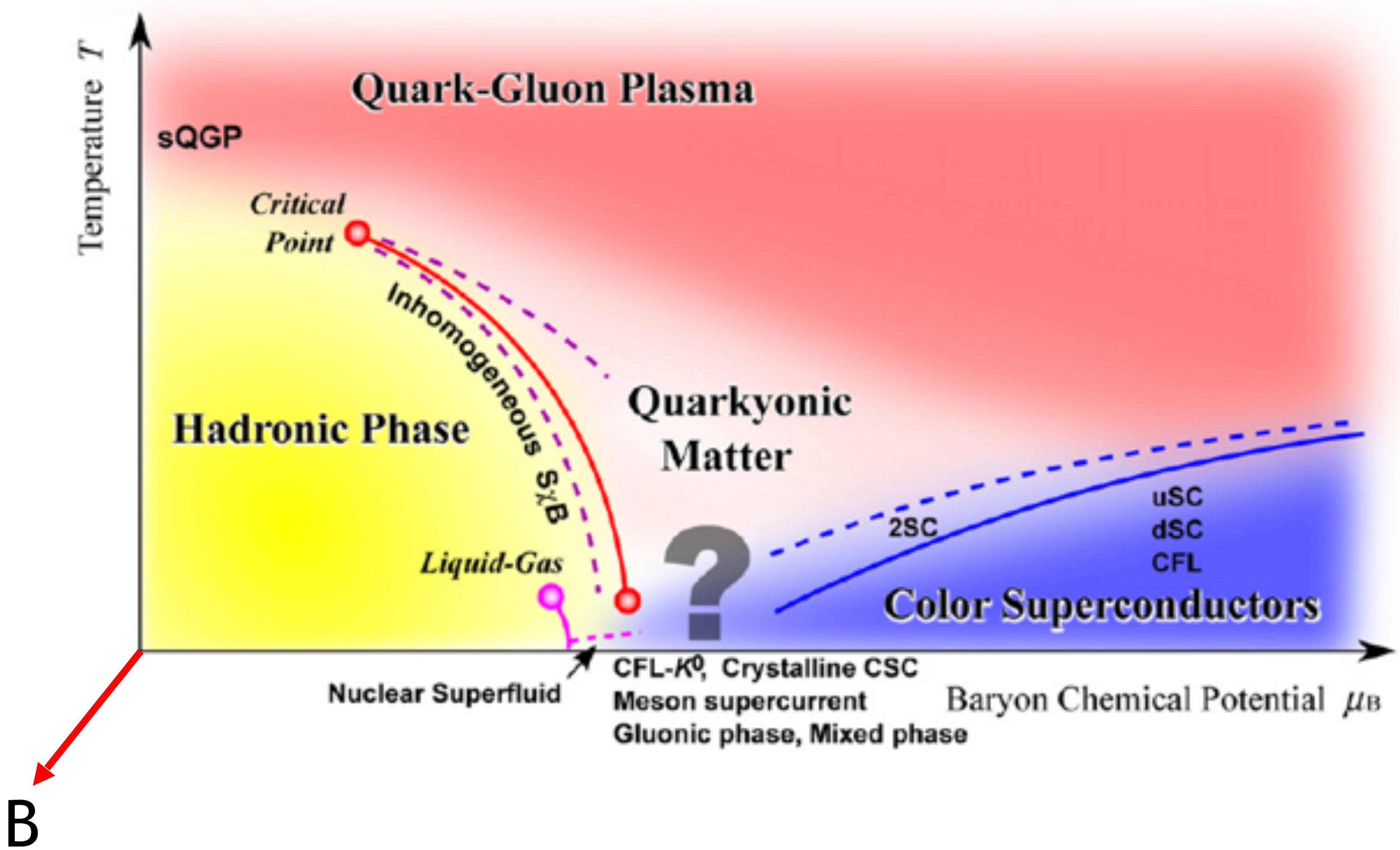

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Magnetic properties in the inhomogeneous chiral phase

What is the inhomogeneous chiral phase

"New phase where the chiral condensate spatially modulates"

NJL model in the mean field approximation(2-flavor) ※chiral limit

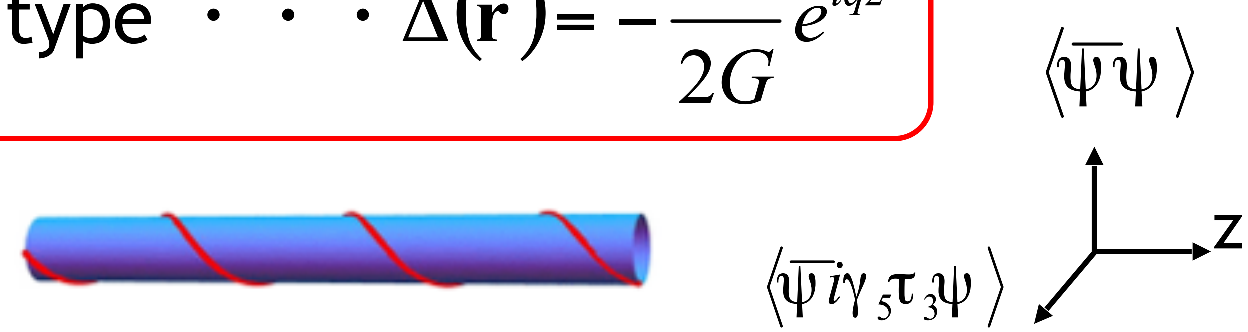
$$\mathcal{L}_{\rm MF} = \bar{\psi}\left[i\partial\!\!\!/ + 2G\left(\langle\bar{\psi}\psi\rangle + i\gamma_5\tau_3\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle\right)\right]\psi + G\left(\langle\bar{\psi}\psi\rangle^2 + \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle^2\right)$$

Inhomogeneous chiral condensate

$$\Delta(\mathbf{r}) = \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle$$

cf. conventional "homogeneous" condensate : $\Delta(\mathbf{r}) = const.$

dual chiral density wave(DCDW) type ・・・ $\Delta(\mathbf{r}) = -\frac{m}{2G}e^{iqz}$



Order parameter: m, q

$\Delta(\mathbf{r})$ is complex ⟷ Magnetic field

➡ Chiral anomaly

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

T. Tatsumi, K. Nishiyama, S. Karasawa, Phys. Lett. B 743, 66 (2015)

The DCDW phase in the phase diagram (B=0)

$$\Delta(\mathbf{r}) = -\frac{m}{2G} e^{iqz}$$

order parameter: m, q

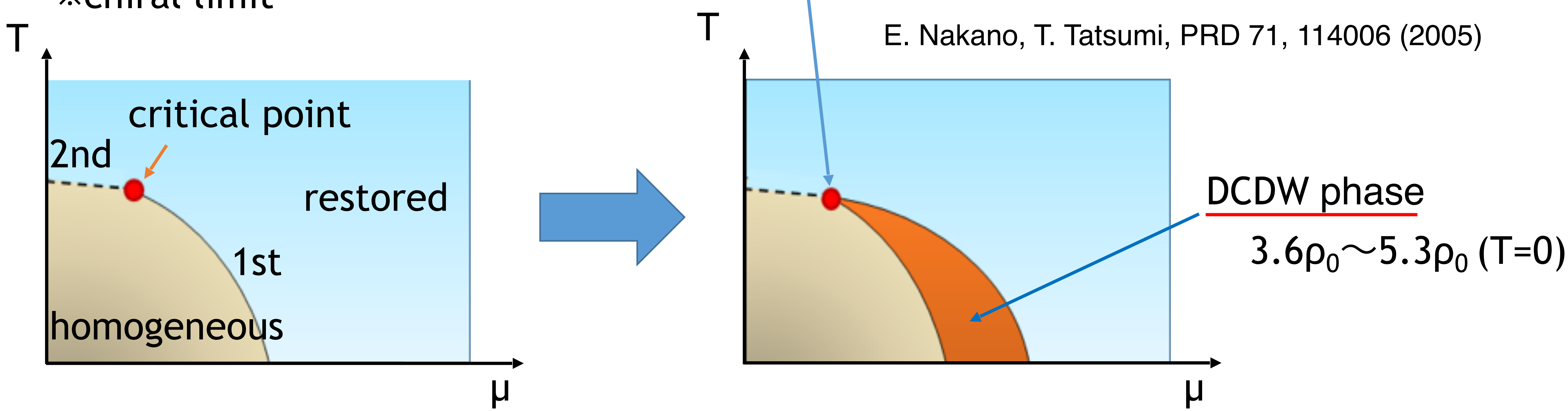
Homogeneously broken phase・・・ q=0 , m≠0

DCDW phase・・・ q≠0 , m≠0

Restored phase・・・ m=0

Lifshitz point・・・Three phases meet at this point

※chiral limit



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Homogeneously broken phase・・・ $q=0$, $m\neq0$

DCDW phase・・・ $q\neq0$, $m\neq0$

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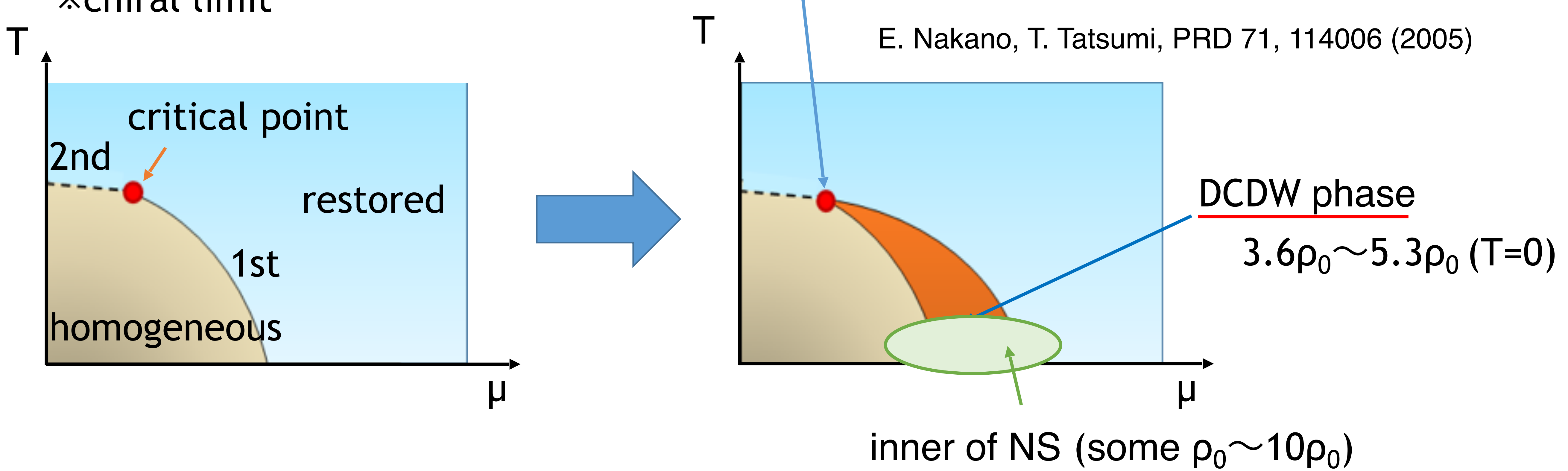

The inhomogeneous chiral phase can realize in neutron stars

Response to the weak B in the DCDW phase

Lagrangian in the external B

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m e^{i\gamma_5 \tau^3 q z}\right)\psi - \frac{m^2}{4G} \qquad \mathbf{B} \,//\, \mathbf{q} \,//\, \hat{\mathbf{z}}$$

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Thermodynamic potential

$$\Omega(\mu, T, B; m, q)$$ ※sufficiently weak B

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Determination of the order parameters minimizing Ω : $m(\mu, T, B)$ $q(\mu, T, B)$

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Response to B of the minimized Ω

$$\Omega_{\min}(\mu, T, B) = \Omega^{(0)}_{\min}(\mu, T) + eB\Omega^{(1)}_{\min}(\mu, T) + (eB)^2\Omega^{(2)}_{\min}(\mu, T) + \cdots$$

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Spontaneous magnetization

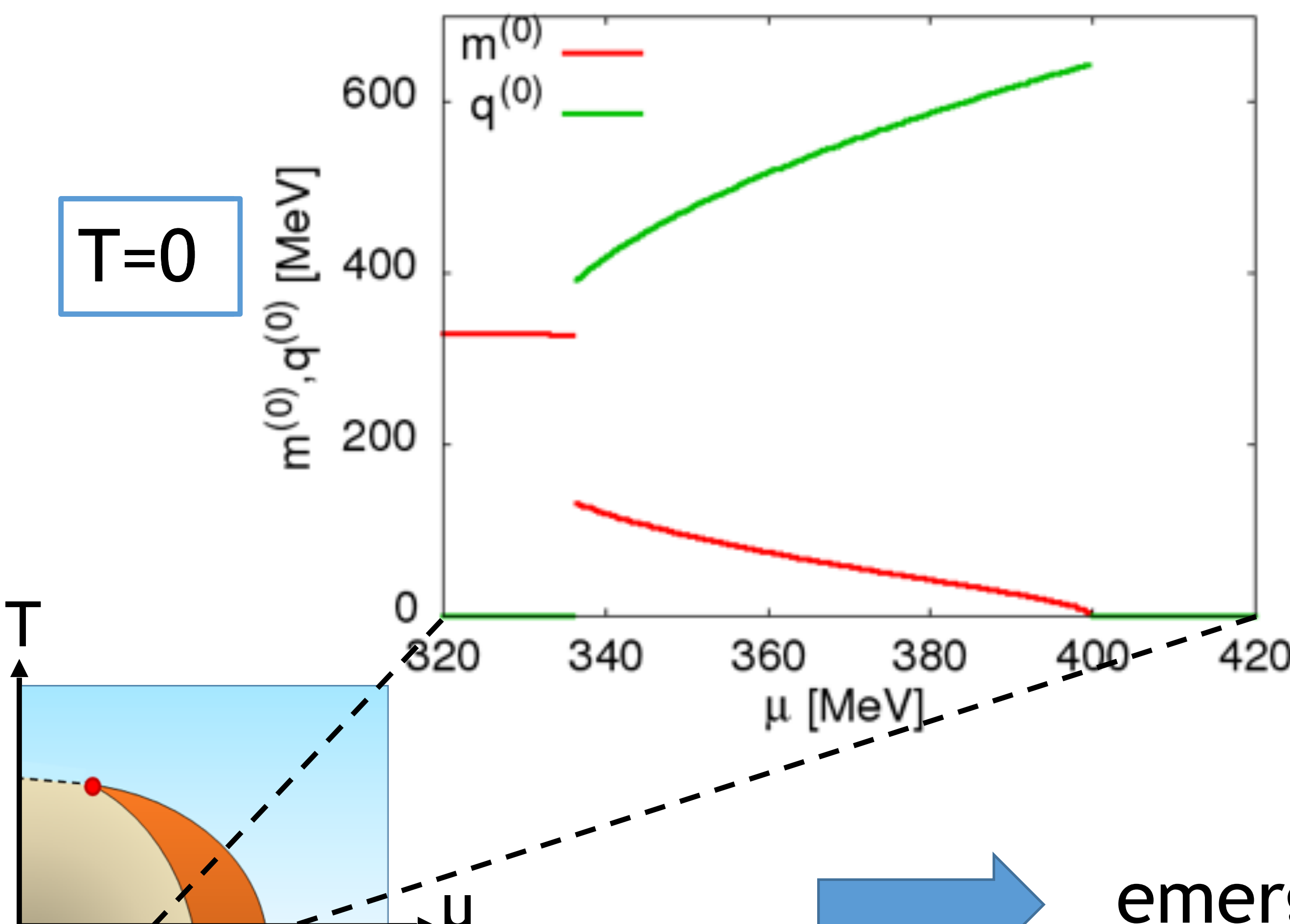
Magnetic susceptibility

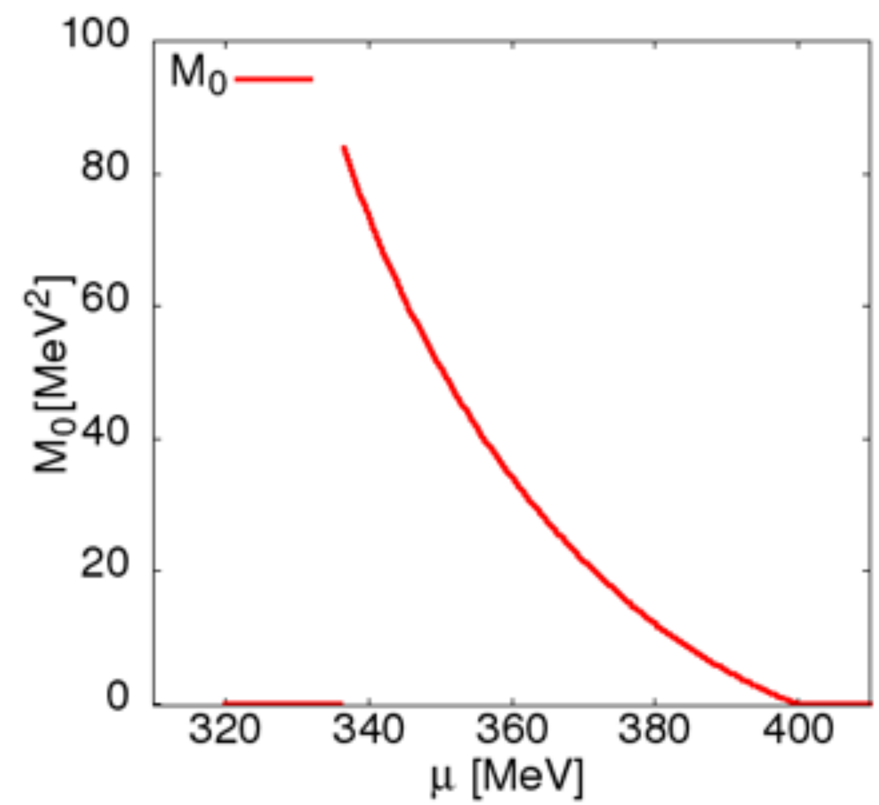
Spontaneous magnetization in the DCDW phase

$$M_0 = -\left.\frac{\partial \Omega_{\min}}{\partial B}\right|_{B=0}$$

RY, K. Nishiyama and T. Tatsumi, Phys. Lett. B751, 123 (2015)

T=0





emergence of spontaneous magnetization

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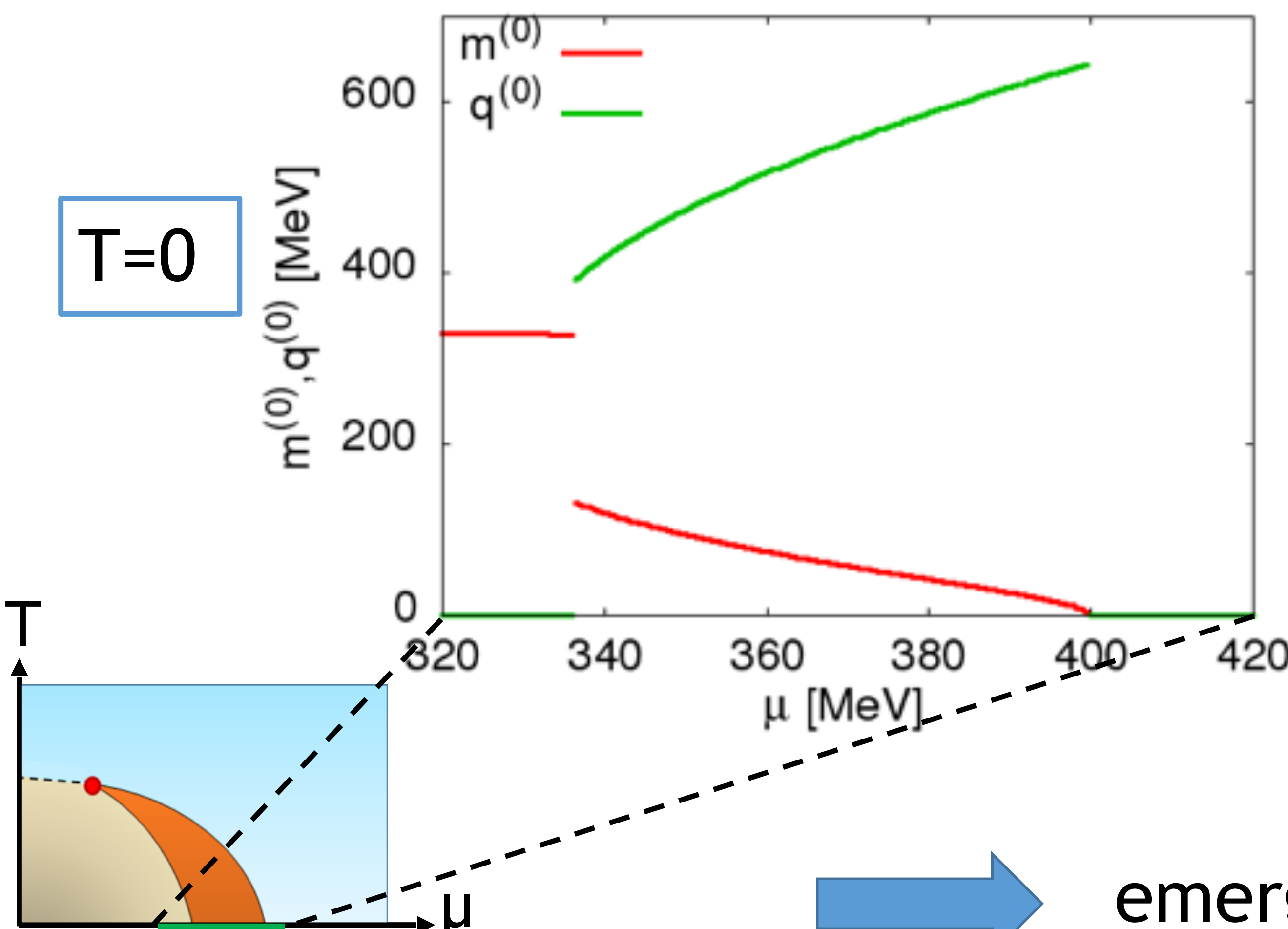
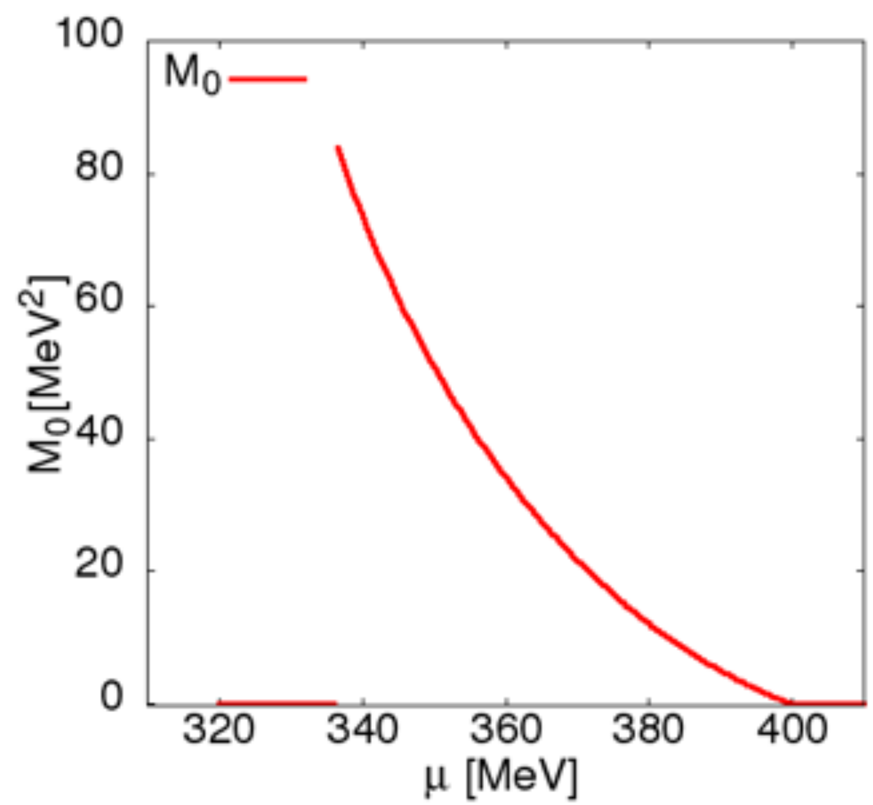




emergence of spontaneous magnetization

Magnetic susceptibility → Does it diverge at the 2nd order phase transition point?

NG mode → Magnon?

Magnetic susceptibility

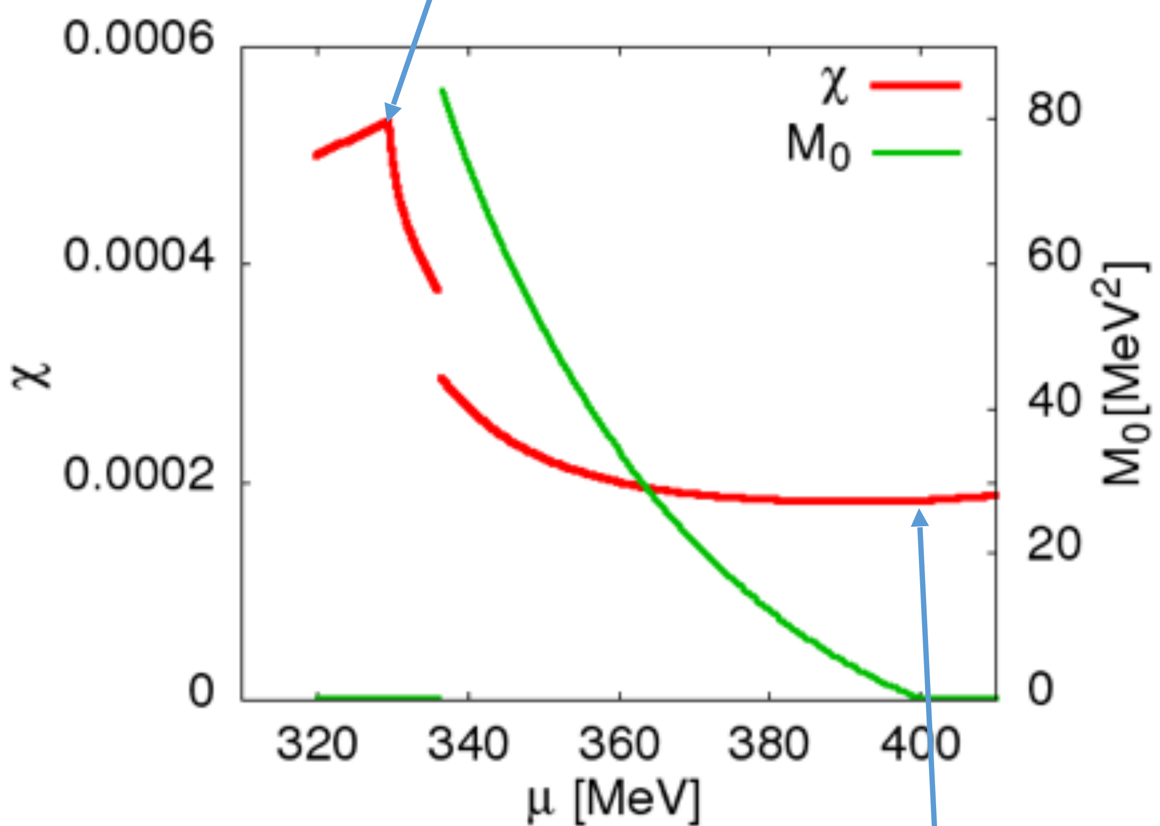
$$\chi \equiv \left.\frac{\partial M}{\partial B}\right|_{B=0} = -\left.\frac{\partial^2 \Omega_{min}}{\partial B^2}\right|_{B=0}$$

※Normalization

$$\chi(\mu = 0, T = 0) = 0$$

T=0

Phase transition to the finite density



continuous at transition point

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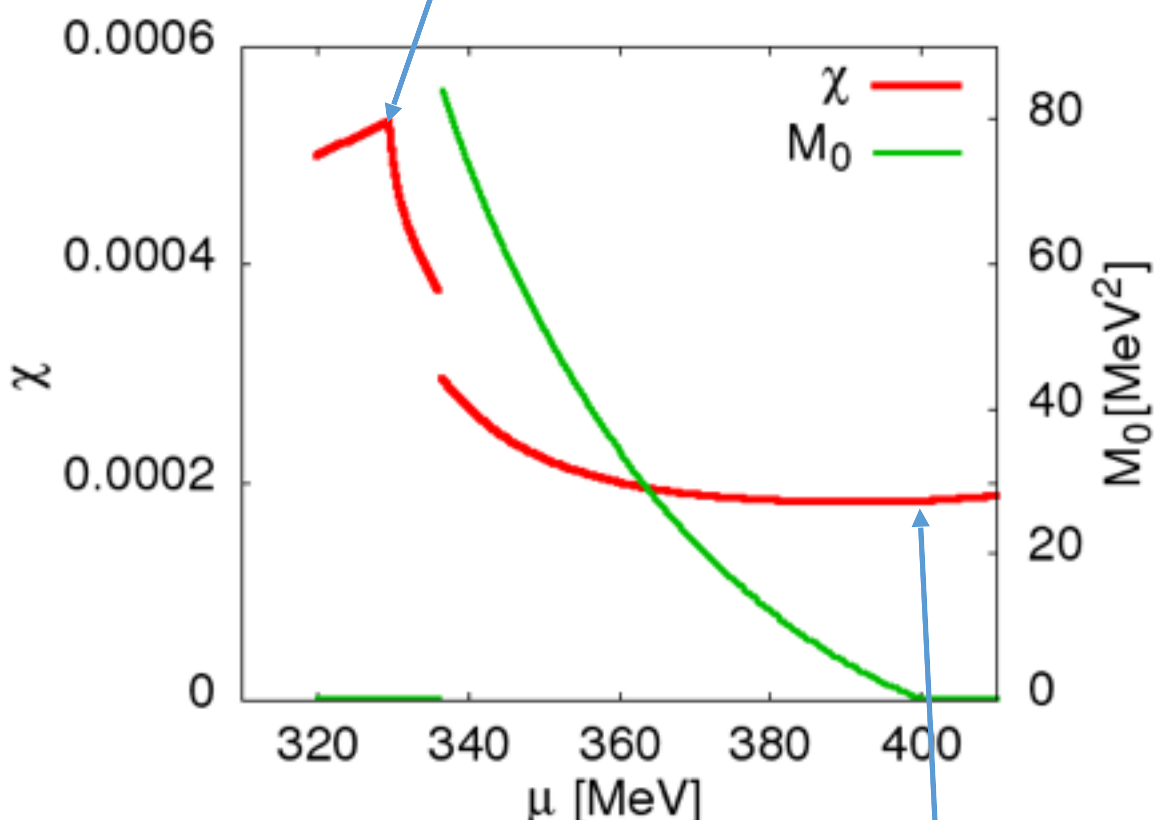
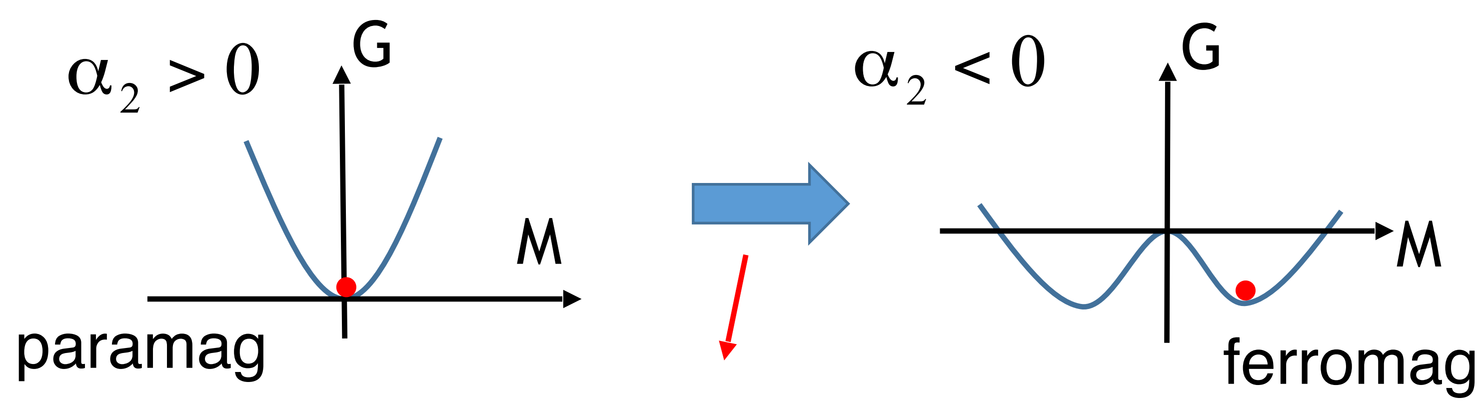


continuous at transition point

cf. Ising model

$$G = \frac{1}{2}\alpha_2 M^2 + \frac{1}{4}\alpha_4 M^4$$

$\alpha_2 > 0$ $\alpha_2 < 0$

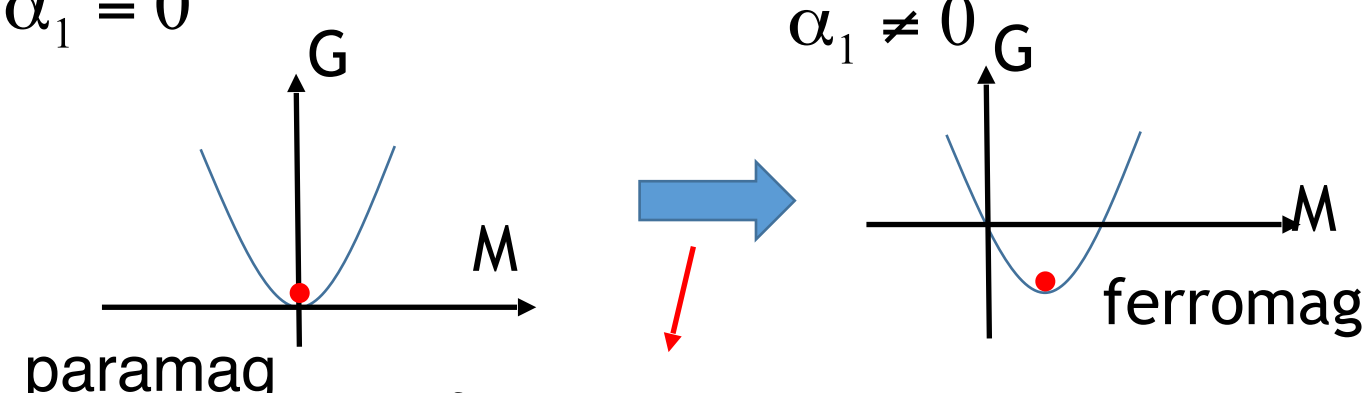


The curvature becomes 0. → χ diverges.

DCDW

$$G = \Omega_{\min} + BM\big|_{B=B(M)} = \alpha_1 M + \frac{1}{2}\alpha_2 M^2$$

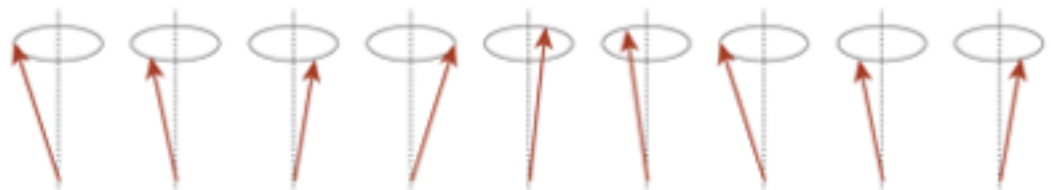
$\alpha_1 = 0$ $\alpha_1 \neq 0$



χ does not diverge.

NG mode associated with the ferromagnetic transition

Conventional spin wave



$$\mathbf{S} \rightarrow R(\mathbf{r})\mathbf{S} = \mathbf{S} + \delta\,\mathbf{S}(\mathbf{r})$$

Local rotation

NG mode (magnon)

"Spin wave" in the DCDW phase

$$\Delta(\mathbf{r}) = -\frac{m}{2G} e^{i\mathbf{q}\cdot\mathbf{r}}$$

q can be pointed to any direction without B.

Applying B ➡ $\mathbf{B} \,//\, \mathbf{q}$ Ω has the terms $\sim \mathbf{B}\cdot\mathbf{q}$.

Therefore, $\mathbf{M} \sim \mathbf{q} \leftrightarrow \mathbf{S}$

➡ "Spin wave" $\mathbf{q} \rightarrow R(\mathbf{r})\mathbf{q} = \mathbf{q} + \delta\,\mathbf{q}(\mathbf{r})$

Independent NG mode in the DCDW phse

'Spin wave' $\mathbf{q} \rightarrow R(\mathbf{r})\mathbf{q} = \mathbf{q} + \underline{\delta\, \mathbf{q}(\mathbf{r})}$

$$\Delta(\mathbf{r}) \rightarrow -\frac{m}{2G} e^{iR(\mathbf{r})\mathbf{q}\cdot\mathbf{r}} = -\frac{m}{2G} \underline{e^{i\mathbf{q}\cdot R^{-1}(\mathbf{r})\mathbf{r}}}$$

Equivalent to the space rotation

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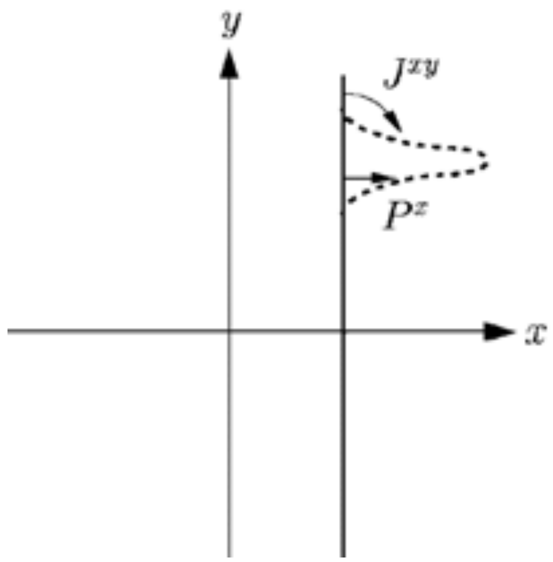

Space rotation is equivalent to the local translation.

I. Low, A. V. Manohar, PRL 88, 10 (2002)

Independent NG mode in the DCDW phse

'Spin wave' $\mathbf{q} \to R(\mathbf{r})\mathbf{q} = \mathbf{q} + \underline{\delta\,\mathbf{q}(\mathbf{r})}$

$$\Rightarrow \quad \Delta(\mathbf{r}) \to -\frac{m}{2G} e^{iR(\mathbf{r})\mathbf{q}\cdot\mathbf{r}} = -\frac{m}{2G} \underline{e^{i\mathbf{q}\cdot R^{-1}(\mathbf{r})\mathbf{r}}}$$

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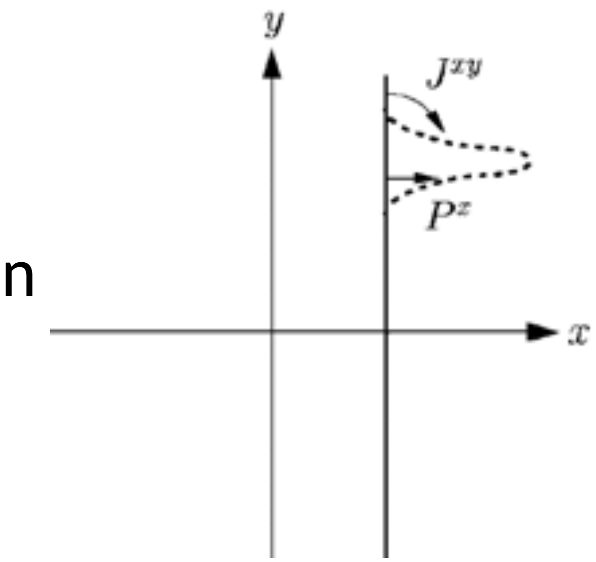

⇒ Space rotation is equivalent to the local translation.

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⇒ Local translation is equivalent to the local chiral transformation

T-G Lee, et al, PRD 92, 034024(2015) $e^{i\mathbf{q}\cdot(\mathbf{r}+\mathbf{s}(\mathbf{r}))} \Leftrightarrow e^{i\mathbf{q}\cdot\mathbf{r}+\alpha(\mathbf{r})} \left(\psi \to e^{i\gamma_5\alpha(\mathbf{r})/2}\psi\right)$

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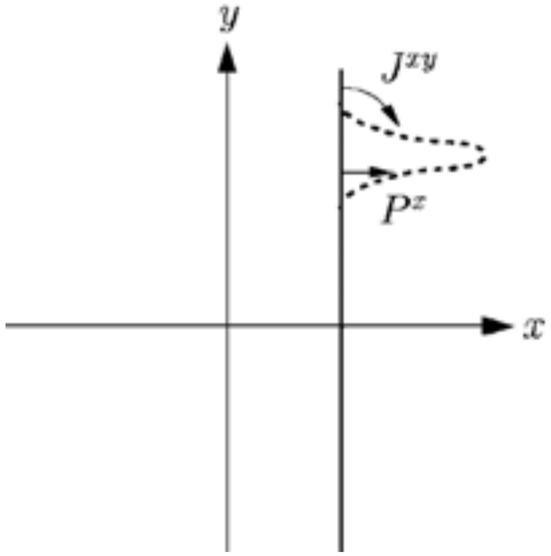

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Candidates of the NG mode

SSB of
rotation of q, space rotation, translation, chiral symmetry

Independent NG modes are only pions

Summary

Quark matter has ferromagnetism in the DCDW phase…

However,

・Magnetic susceptibility does not diverge on the 2nd order phase transition point.

・There are only pions as the independent NG modes in the DCDW phase.

Appendix

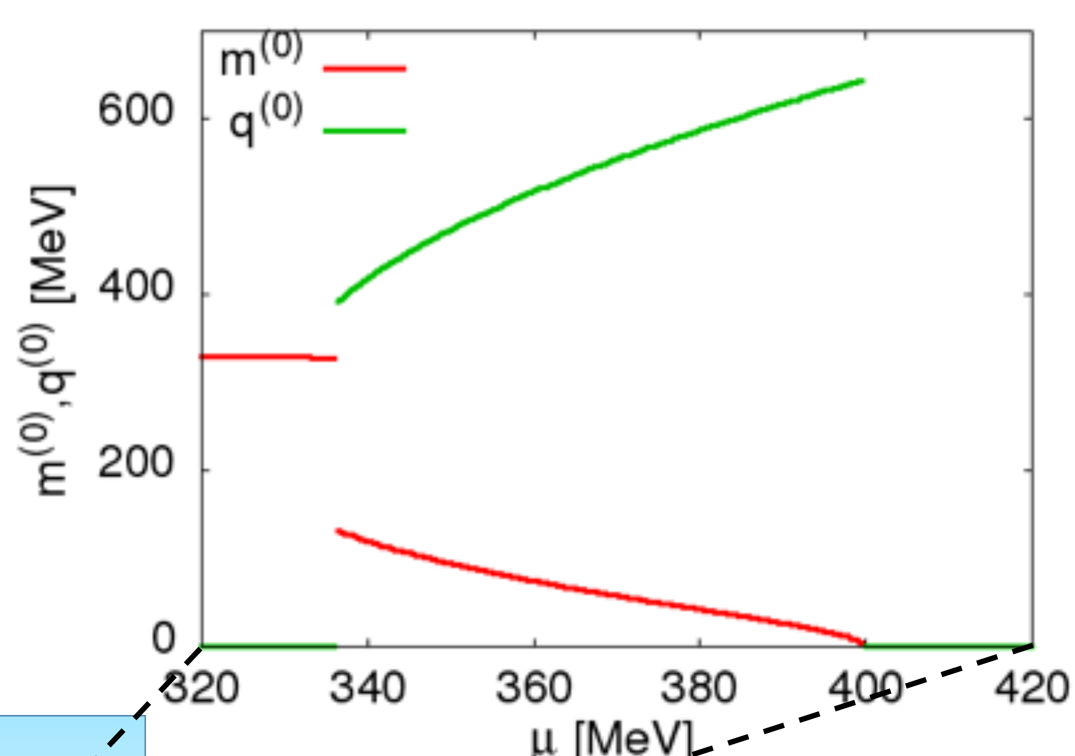
The existence of spontaneous magnetization

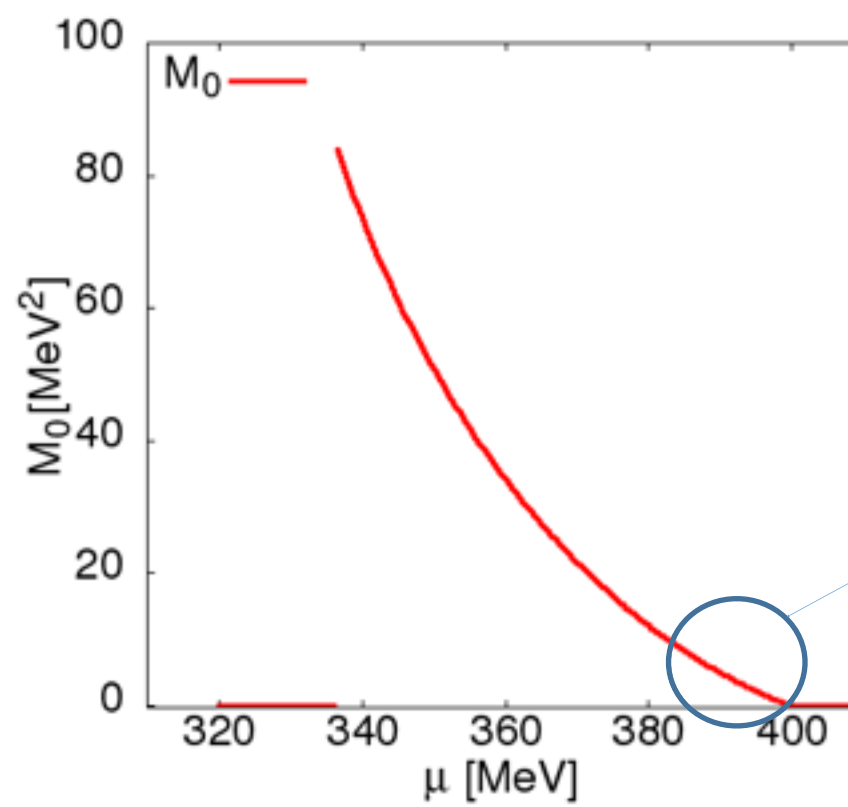
Spontaneous magnetization

$$M_0 = -e\delta\Omega^{(1)}\left(\mu, T; q = q^{(0)}, m = m^{(0)}\right)$$

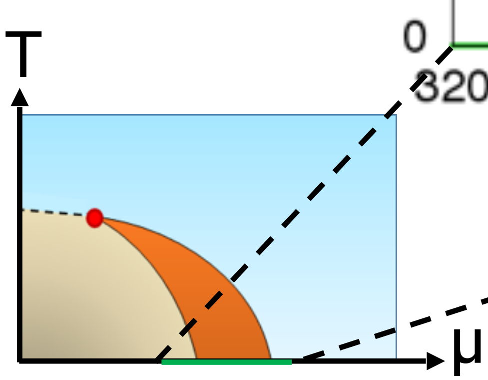
$q^{(0)}, m^{(0)} \neq 0 \rightarrow M_0 \neq 0$ ➡ spontaneously magnetized in the DCDW phase

T=0

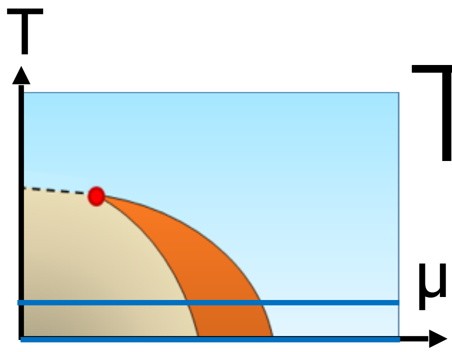




$\sim \left(m^{(0)}\right)^2$

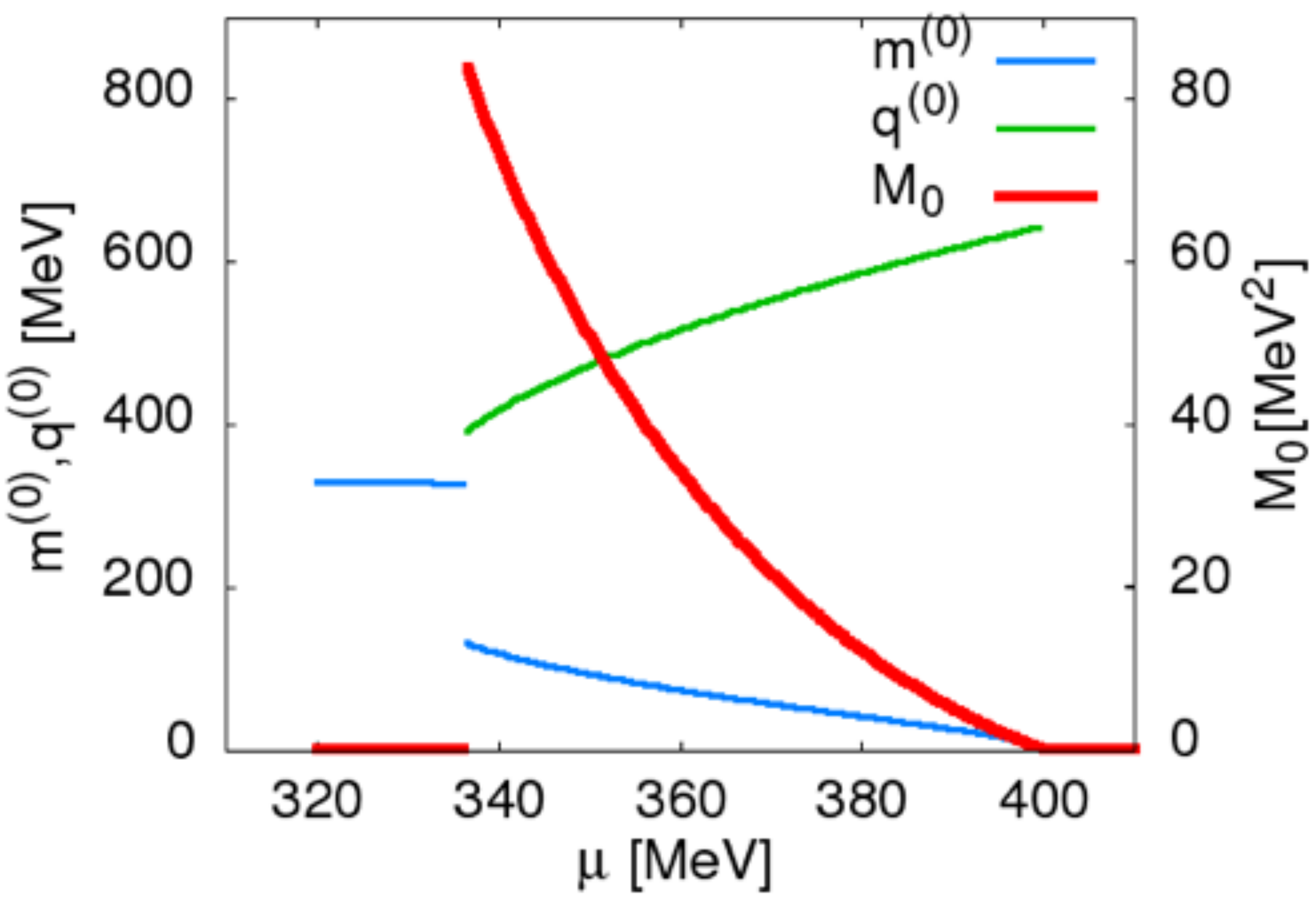


assume a sphere (constant density) ➡ $B_{mag} = \frac{8\pi}{3} M_0 \sim 10^{16} G$

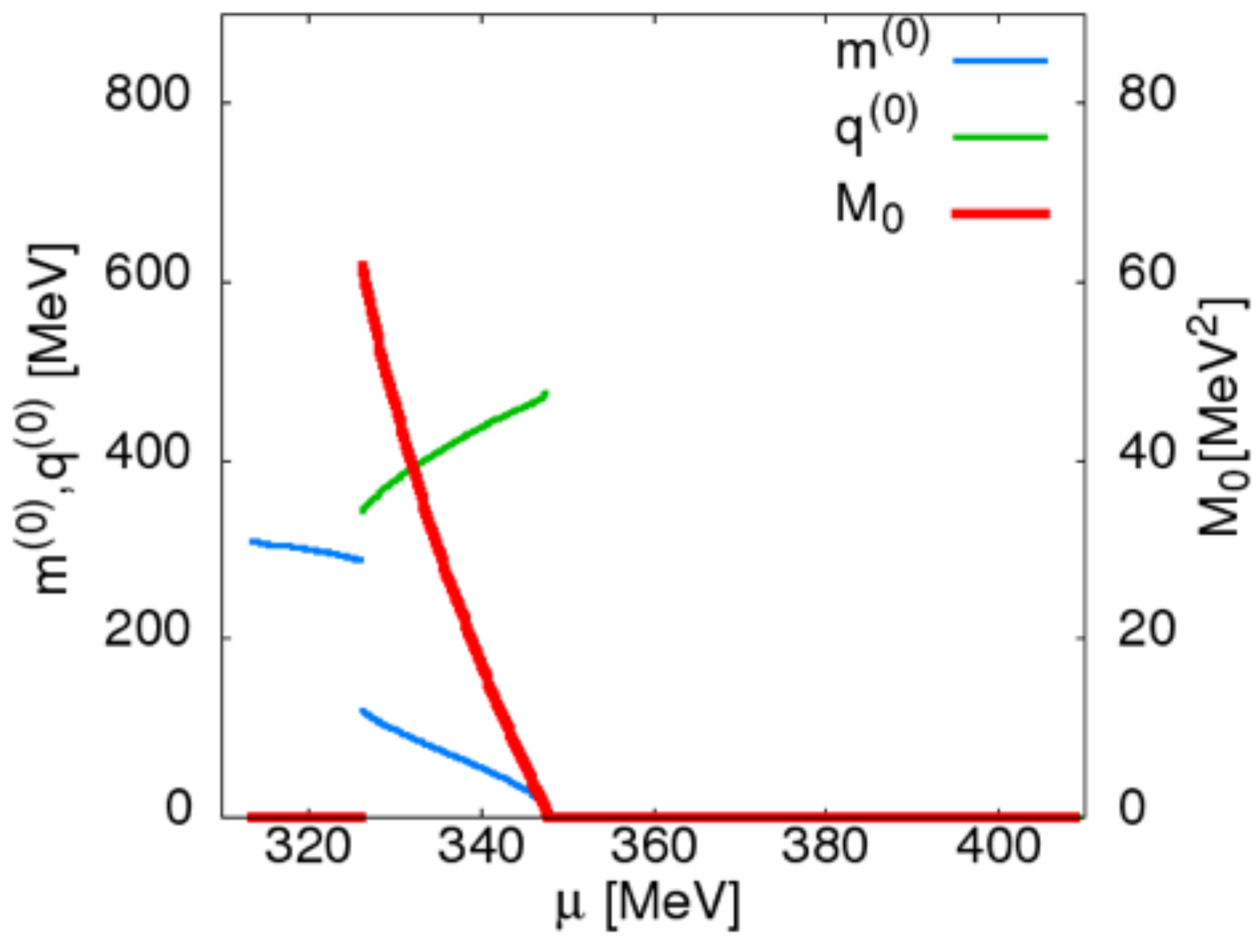


The temperature dependence

T=0



T=30MeV



As T increases, $m^{(0)}$, $q^{(0)}$ and M_0 decrease

Energy spectrum in the magnetic field

Landau level

I. E. Frolov, et al., PRD 82, 076002 (2010)

$\mathbf{B} = B\hat{\mathbf{z}}$: constant

$$E_{n,p,\zeta=\pm 1,\varepsilon=\pm 1} = \begin{cases} \varepsilon\sqrt{\left(\zeta\sqrt{m^2+p^2}+\dfrac{q}{2}\right)^2+2|e_f B|n} & (n=1,2,\cdot\cdot\cdot) \\ \varepsilon\sqrt{m^2+p^2}+\dfrac{q}{2} & (\text{lowest Landau level (LLL), } n=0) \end{cases}$$

spin (↑ ζ) branch (↑ ε)

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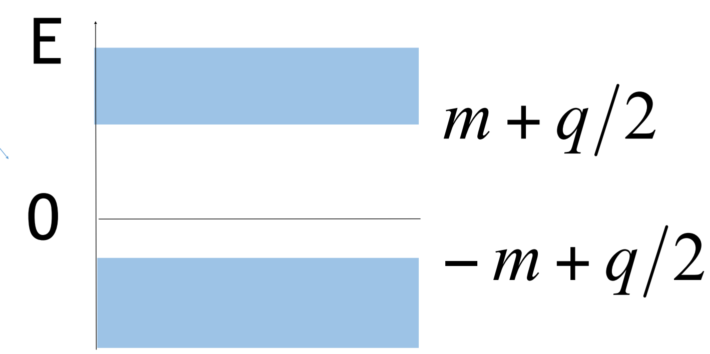
spin branch

asymmetric about zero



※ Complex $\Delta(\mathbf{r})$ is necessary condition of the asymmetric spectrum.

$H(\Delta(\boldsymbol{r}))$ has pairs of the eigenvalues, $E_k(\Delta)$ and $E_k(\Delta^*)$.

Therefore, the energy spectrum cannot be asymmetric with real Δ.

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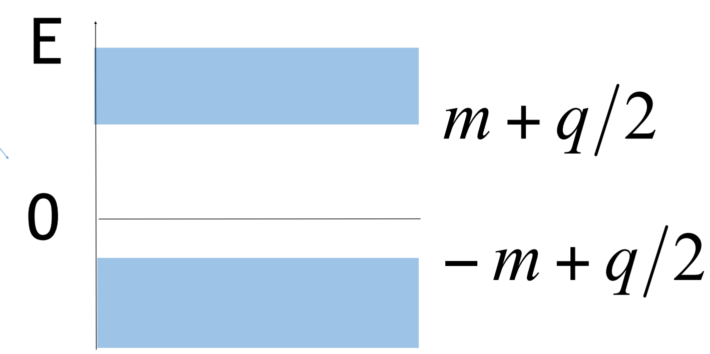
spin (ζ) branch (ε)

asymmetric about zero



→ anomaly

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Anomaly due to the asymmetric spectrum

Anomalous particle number A. J. Niemi, G. W. Semenoff, Phys. Rep. 135, 99 (1986)

cf. chiral bag model : J. Goldstone, R. L. Jaffe, PRL 51, 1518 (1983)

$$N = \frac{1}{2}\int d\mathbf{r} \left\langle \left[\psi^\dagger(\mathbf{r}), \psi(\mathbf{r}) \right] \right\rangle$$

= (particle number) − (anti-particle number)

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Energy eigenvalue

$$= (\text{particle number}) - (\text{anti-particle number}) - \frac{1}{2}\sum_k \mathrm{sign}(E_k)$$

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Energy eigenvalue

In this case

T. Tatsumi, K. Nishiyama, S. Karasawa, Phys. Lett. B 743, 66 (2015)

$$-\frac{eBN_c}{4\pi}\lim_{s\to+0}\int\frac{dp_z}{2\pi}\sum_{\epsilon=\pm1}|E_{p,\epsilon}^{\mathrm{LLL}}|^{-s}\mathrm{sign}(E_{p,\epsilon}^{\mathrm{LLL}})$$

Regularization about energy
not violating gauge invariant
※ It does not divergent

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Regularization about energy not violating gauge invariant

※ It does not divergent

Chiral anomaly (WZW term)

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

In DCDW $$\Omega_{\mathrm{WZW}} = -\frac{eB\mu q}{4\pi^2}$$

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$$N = \frac{1}{2}\int d\mathbf{r} \left\langle \left[\psi^{\dagger}(\mathbf{r}), \psi(\mathbf{r}) \right] \right\rangle$$

Energy eigenvalue

$$= \text{(particle number)} - \text{(anti-particle number)} - \frac{1}{2}\sum_{k} \text{sign}(E_k)$$

In this case

T. Tatsumi, K. Nishiyama, S. Karasawa, Phys. Lett. B 743, 66 (2015)

$$-\frac{eBN_c}{4\pi} \lim_{s \to +0} \int \frac{dp_z}{2\pi} \sum_{\epsilon = \pm 1} |E_{p,\epsilon}^{\text{LLL}}|^{-s} \text{sign}(E_{p,\epsilon}^{\text{LLL}})$$

Regularization about energy not violating gauge invariant

※ It does not divergent

Chiral anomaly (WZW term)

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

In DCDW

$$\Omega_{\text{WZW}} = -\frac{eB\mu q}{4\pi^2}$$

magnetization $M = \frac{e\mu q}{4\pi^2}$

number density $n_q = \frac{eBq}{4\pi^2}$

Construction of the thermodynamic potential

regularization of Ω

Model parameter : $\Lambda = 660 MeV,\ G\Lambda^2 = 6.35$

$$\Omega = \Omega_0(\mu, T, B; q = 0, m = 0) + \delta\Omega(\mu, T, B; q, m)$$
$$= \Omega_0 + \delta\Omega^{(0)}(\mu, T; q, m) + eB\,\delta\Omega^{(1)}(\mu, T; q, m) + (eB)^2\,\delta\Omega^{(2)}(\mu, T; q, m) + \boxed{?}$$

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constant (Ω₀)

proper time regularization (δΩ⁽⁰⁾)

finite (δΩ⁽¹⁾, δΩ⁽²⁾)

E. Nakano, T. Tatsumi, PRD 71, 114006 (2005)

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constant

proper time regularization

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E. Nakano, T. Tatsumi, PRD 71, 114006 (2005)

determination of order parameters

Stationary condition : $\dfrac{\partial\Omega}{\partial m, \partial q} = 0$ ⇨

$$m(\mu, T, B) = m^{(0)}(\mu, T) + m^{(1)}(\mu, T)B + m^{(2)}(\mu, T)B^2 + \boxed{?}$$

$$q(\mu, T, B) = q^{(0)}(\mu, T) + q^{(1)}(\mu, T)B + q^{(2)}(\mu, T)B^2 + \boxed{?}$$

minimizing Ω

Construction of the thermodynamic potential

regularization of Ω

Model parameter : $\Lambda = 660 MeV,\ G\Lambda^2 = 6.35$

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$$q(\mu, T, B) = q^{(0)}(\mu, T) + q^{(1)}(\mu, T)B + q^{(2)}(\mu, T)B^2 + \boxed{?}$$

minimizing Ω

Spontaneous magnetization

$$M_0 \equiv -\left.\frac{\partial\Omega_{\min}(\mu, T, B)}{\partial B}\right|_{B=0} \Rightarrow M_0 = -e\,\delta\Omega^{(1)}(\mu, T; q = q^{(0)}, m = m^{(0)})$$

The value at B=0

Properties about $\delta\Omega^{(1)}(\mu, T; m, q)$

$$\delta\Omega^{(1)} = \Omega_{\rm val}^{(1)} + \frac{\mu N_c}{4\pi} \lim_{s\to+0} \int \frac{dp_z}{2\pi} \sum_{\epsilon} \text{sign}\left(E_{p,\epsilon}^{\rm LLL}\right) \left|E_{p,\epsilon}^{\rm LLL}\right|^{-s}$$

Contribution of valence quarks

Spectral asymmetry

Properties about $\delta\Omega^{(1)}(\mu, T; m, q)$

$$\delta\Omega^{(1)} = \Omega^{(1)}_{\text{val}} + \frac{\mu N_c}{4\pi} \lim_{s\to+0} \int \frac{dp_z}{2\pi} \sum_{\epsilon} \text{sign}\left(E^{\text{LLL}}_{p,\epsilon}\right) \left|E^{\text{LLL}}_{p,\epsilon}\right|^{-s}$$

Contribution of valence quarks

Spectral asymmetry

Odd function about q → $\mathbf{B} \cdot \mathbf{q}$

Properties about $\delta\Omega^{(1)}(\mu, T; m, q)$

$$\delta\Omega^{(1)} = \Omega^{(1)}_{\mathrm{val}} + \frac{\mu N_c}{4\pi} \lim_{s \to +0} \int \frac{dp_z}{2\pi} \sum_{\epsilon} \mathrm{sign}\left(E^{\mathrm{LLL}}_{p,\epsilon}\right) \left|E^{\mathrm{LLL}}_{p,\epsilon}\right|^{-s}$$

Contribution of valence quarks (first term) — Spectral asymmetry (second term)

Odd function about q ⇒ $\mathbf{B} \cdot \mathbf{q}$

In the case of no valence quarks

$$m \to \infty \;\; \text{or} \;\; \mu < m - q/2 \;\; \text{at} \;\; T = 0$$

$$\delta\Omega^{(1)} = -\frac{N_c \mu q}{4\pi^2} \qquad \left(\Omega^{(1)}_{\mathrm{val}} = 0 \right)$$

This is consistent with Son's result of CPT.

※CPT is the low energy effective theory, where quarks are integrated out.

Properties about $\delta\Omega^{(1)}(\mu, T; m, q)$

$$\delta\Omega^{(1)} = \Omega^{(1)}_{\mathrm{val}} + \frac{\mu N_c}{4\pi} \lim_{s\to+0} \int \frac{dp_z}{2\pi} \sum_{\epsilon} \mathrm{sign}\left(E^{\mathrm{LLL}}_{p,\epsilon}\right) \left|E^{\mathrm{LLL}}_{p,\epsilon}\right|^{-s}$$

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Spectral asymmetry

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In the limit of $m \to 0$

$$\delta\Omega^{(1)} \to 0$$

The behavior may be physically correct.

The wave vector q should be redundant, because there is no condensate. ($m \to 0$)

Properties about $\delta\Omega^{(1)}(\mu, T; m, q)$

$$\delta\Omega^{(1)} = \Omega^{(1)}_{\text{val}} + \frac{\mu N_c}{4\pi} \lim_{s \to +0} \int \frac{dp_z}{2\pi} \sum_{\epsilon} \text{sign}\left(E^{\text{LLL}}_{p,\epsilon}\right) \left|E^{\text{LLL}}_{p,\epsilon}\right|^{-s}$$

Contribution of valence quarks (the $\Omega^{(1)}_{\text{val}}$ term)

Spectral asymmetry (the sum term)

Odd function about q ⇒ $\mathbf{B} \cdot \mathbf{q}$

In the case of no valence quarks

$$m \to \infty \text{ or } \mu < m - q/2 \text{ at } T = 0$$

$$\delta\Omega^{(1)} = -\frac{N_c \mu q}{4\pi^2} \quad \left(\Omega^{(1)}_{\text{val}} = 0 \right)$$

This is consistent with Son's result of CPT.

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In the limit of $m \to 0$

$$\delta\Omega^{(1)} \to 0$$

The behavior may be physically correct.

The wave vector q should be redundant, because there is no condensate. ($m \to 0$)

⇒ $\delta\Omega^{(1)} \neq 0$ only when m , $q \neq 0$

Summary

- constitute the thermodynamic potential in B with anomaly
- analyze the response to B of the thermodynamic potential

- Quark matter has the spontaneous magnetization in the DCDW phase.
- This magnetization includes the contribution of valence quarks and anomaly.
- Magnetic susceptibility does not diverge on the 2^{nd} order phase transition point.

Future work

Application to neutron stars

impose the charge neutrality and chemical equilibrium

➡ magnitude of magnetization, effect to EOS

Extension to finite current quark mass system

configuration of the inhomogeneous condensate, response to B

➡ 3-flavor (u,d,s) system, confirmation in the lattice QCD

Theoretical improvement

formulation above MFA ➡ functional regularization group

※treatment of quasi long range order cf. liquid crystal (smectic phase)

analyze based on QCD ➡ Schwinger-Dayson approach

stability of the inhomogeneous chiral phase etc…

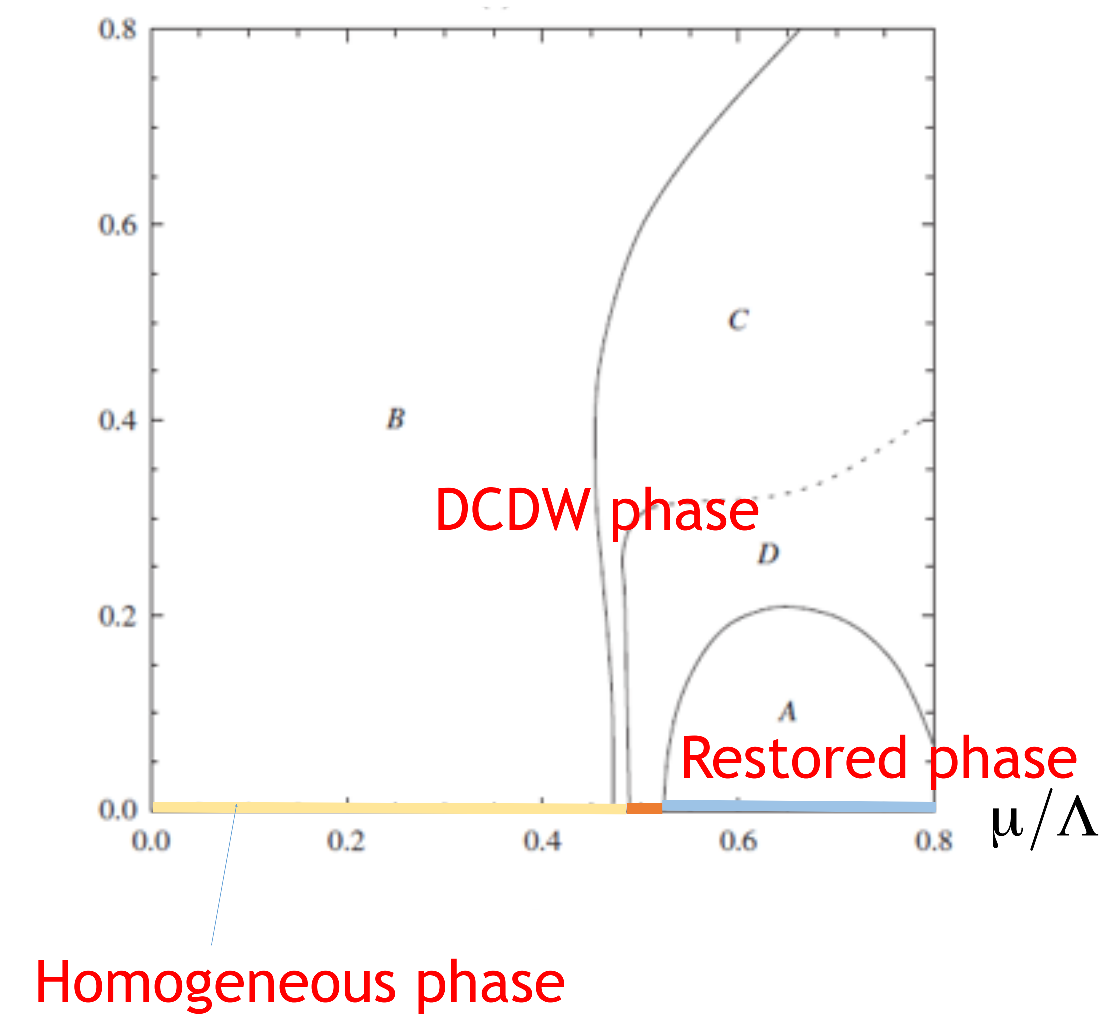
The DCDW phase in the magnetic field

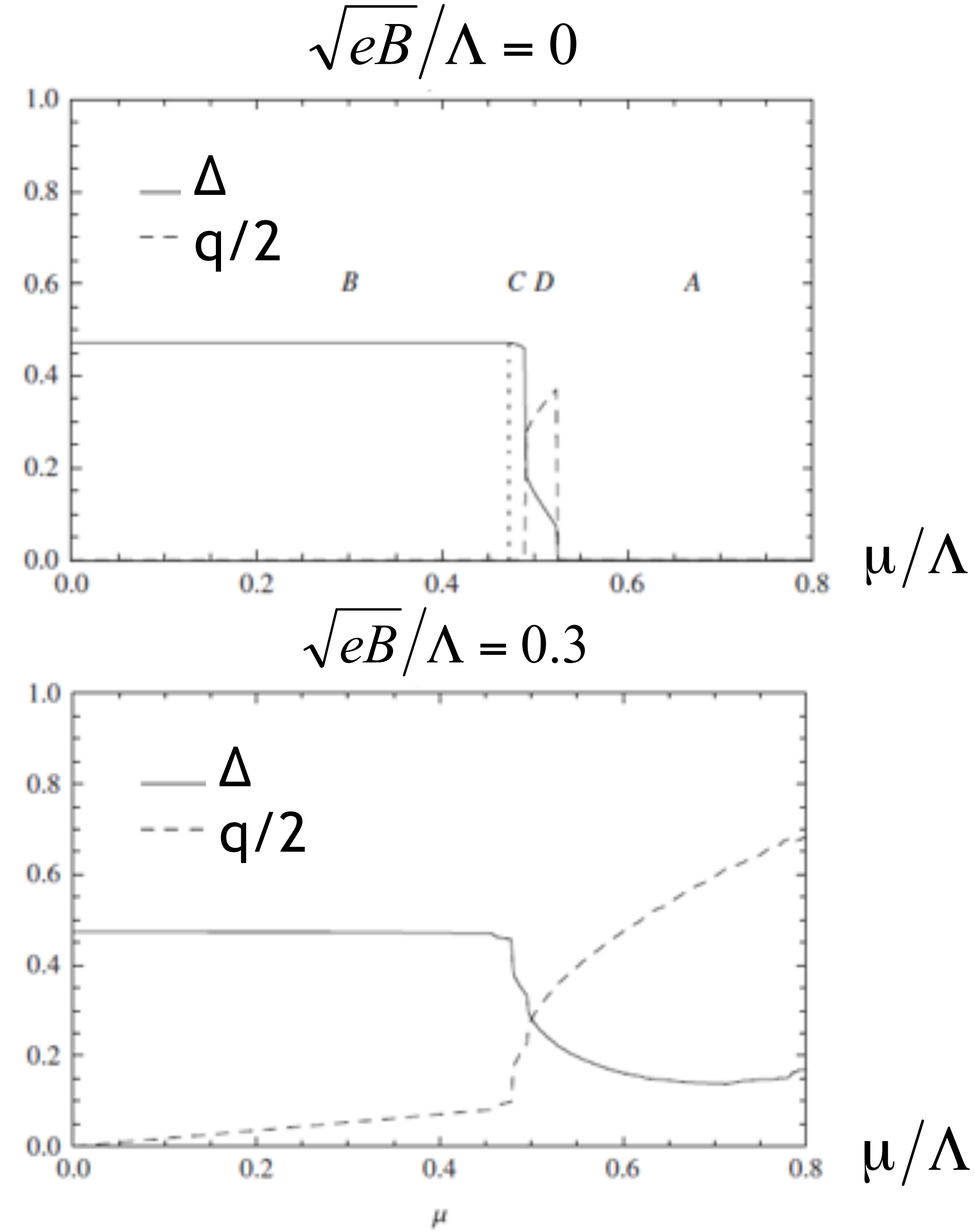
Analysis of the NJL model with the mean field approximation in the magnetic field

I. E. Frolov, et al., PRD 82, 076002 (2010)

DCDW-type condensate : $\Delta(\mathbf{r}) = \Delta e^{iqz}$

$\sqrt{eB}/\Lambda$ Phase diagram at T=0





Change of the order parameters

➡ The homogeneous phase at μ≠0 becomes the DCDW phase as soon as B is applied.

Spectral asymmetry in the DCDW phase

Number density

$$n = -\frac{\partial\Omega}{\partial\mu} = n_{nom} - \frac{eB}{4\pi}\int\frac{dp}{2\pi}\sum_{\varepsilon=\pm1}\text{sign}\left(E_{p,\varepsilon}^{LLL}\right)$$

(T=0)

Asymmetry (chiral anomaly?)

The number of the occupied states (E=0~μ)

Spectral asymmetry in the DCDW phase

Number density

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The number of the occupied states (E=0~μ)

Contribution of the spectral asymmetry

T. Tatsumi, K. Nishiyama, S. Karasawa,
Phys. Lett. B 743, 66 (2015)

$$\lim_{s \to +0} - \frac{eB}{4\pi}\int \frac{dp}{2\pi} \sum_{\varepsilon = \pm 1} \mathrm{sign}\left(E_{p,\varepsilon}^{LLL}\right)\left|E_{p,\varepsilon}^{LLL}\right|^{-s}$$

Regularization about the energy
not violating gauge invariance
※ It does not divergent

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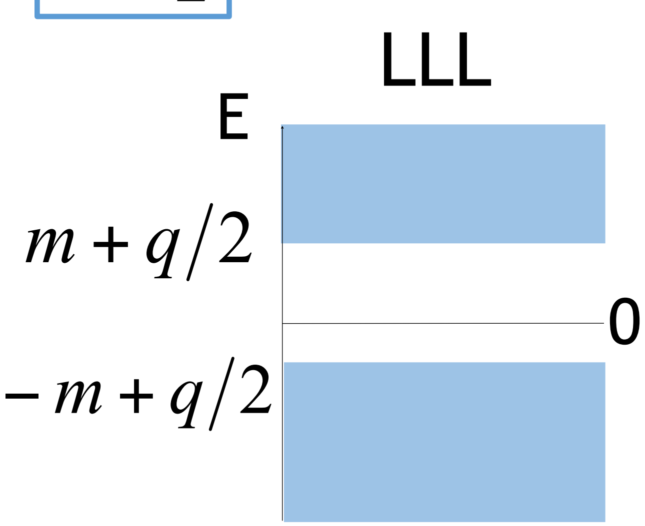
Regularization about the energy not violating gauge invariance
※ It does not divergent

$$m > \frac{q}{2}$$



$$\frac{eBq}{4\pi^2}$$

This is the surface term,
which vanish on the regularization violating gauge invariance

$$\lim_{\Lambda\to\infty} -\frac{eB}{4\pi}\int_{-\Lambda}^{\Lambda} \frac{dp}{2\pi}\sum_{\varepsilon=\pm 1}\mathrm{sign}\left(E^{LLL}_{p,\varepsilon}\right) = 0$$

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$$n = -\frac{\partial \Omega}{\partial \mu} = n_{nom} - \frac{eB}{4\pi}\int \frac{dp}{2\pi}\sum_{\varepsilon=\pm 1}\operatorname{sign}\left(E_{p,\varepsilon}^{LLL}\right)$$

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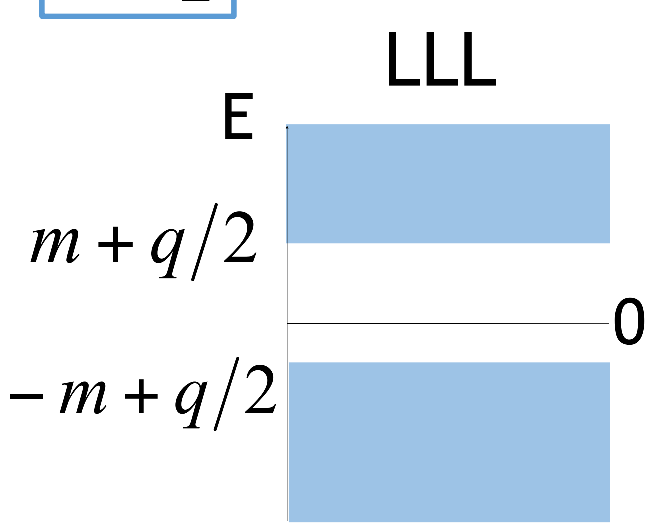

$$\frac{eBq}{4\pi^2} \leftrightarrow n_{\rm WZW}$$

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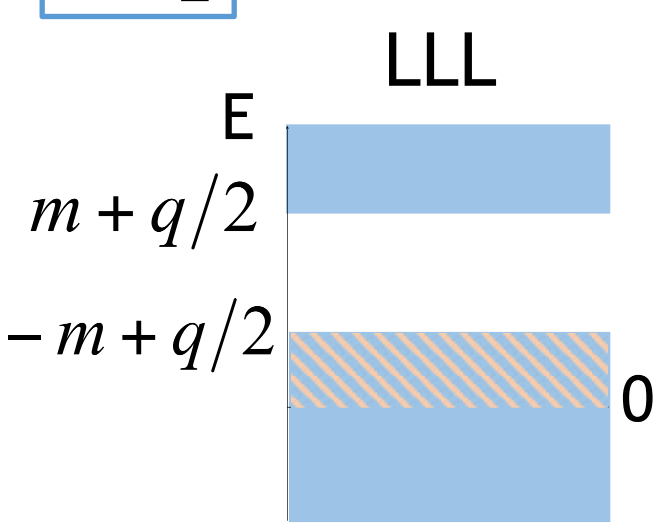

$$\frac{eBq}{4\pi^2} - \frac{eB}{2\pi^2}\sqrt{(q/2)^2 - m^2}$$

Surface term

Not surface term

Spectral asymmetry in the DCDW phase

Number density

$$n = -\frac{\partial\Omega}{\partial\mu} = n_{nom} - \frac{eB}{4\pi}\int\frac{dp}{2\pi}\sum_{\varepsilon=\pm1}\mathrm{sign}\left(E^{LLL}_{p,\varepsilon}\right)$$

(T=0)

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The number of the occupied states (E=0~μ)

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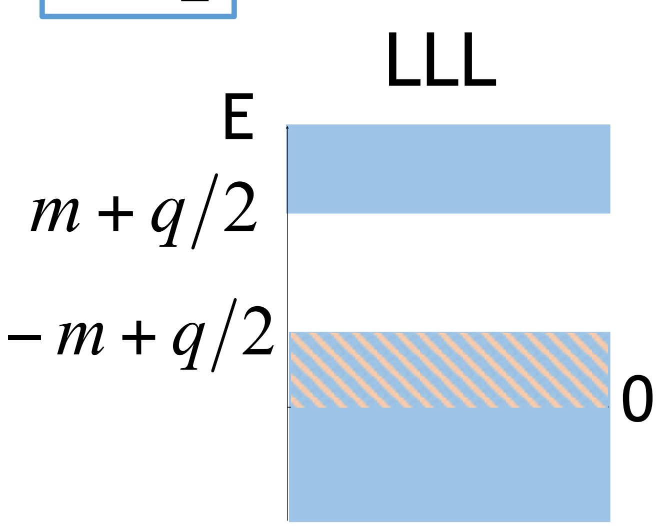
$m < \frac{q}{2}$



$$\frac{eBq}{4\pi^2} - \frac{eB}{2\pi^2}\sqrt{(q/2)^2 - m^2}$$

$\not\leftrightarrow$ n_{WZW}

Surface term

Not surface term

非対称スペクトルによるアノマリー

Anomalous particle number　A. J. Niemi, G. W. Semenoff, Phys. Rep. 135, 99 (1986)

固有関数で展開されたfermion場 $\psi(\mathbf{r}) = \sum_k b_k \varphi_k^{(+)}(\mathbf{r}) + \sum_{k'} d_{k'}^\dagger \varphi_{k'}^{(-)}(\mathbf{r})$

$$H\varphi_k = \lambda_k \varphi_k$$

エネルギー固有値

発散を避けるように反対称化して粒子数を定義する

$$N = \frac{1}{2}\int d\mathbf{r} \left\langle \left[\psi^\dagger(\mathbf{r}), \psi(\mathbf{r})\right] \right\rangle$$

$$= \sum_k \left\langle b_k^\dagger b_k \right\rangle - \sum_{k'} \left\langle d_{k'}^\dagger d_{k'} \right\rangle - \frac{1}{2}\int d\mathbf{r} \left(\sum_k \varphi_k^{(+)\dagger} \varphi_k^{(+)} - \sum_{k'} \varphi_{k'}^{(-)\dagger} \varphi_{k'}^{(-)} \right)$$

正エネルギー　負エネルギー

$$= N_{nom} - \frac{1}{2} \sum_k \mathrm{sign}(\lambda_k)$$

スペクトルが正負非対称ならば残る

cf. chiral bag model におけるパイオンの雲が持つバリオン数

M. Rho, Phys. Rep. 240, 1 (1994)

Chiral anomaly in the magnetic field

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

Chiral perturbative theory [SU(2)]

$$L = \frac{f_\pi^2}{4} tr\left(D^\mu \Sigma^\dagger D_\mu \Sigma\right) + tr\left(M\Sigma + H.c.\right) \qquad D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu \left[Q, \Sigma\right]$$

$$\Sigma = \exp\left(\frac{i\tau^a \phi^a}{f_\pi}\right) = \frac{1}{f_\pi}\left(\sigma + i\tau^a \pi^a\right) \qquad (\sigma^2 + \vec{\pi}^2 = f_\pi^2)$$

Wess-Zumino-Witten action Describing the chiral anomaly cf. $\pi^0 \to 2\gamma$

$$S_{WZW} = -\int d^4x \left(A_\mu^B + \frac{e}{2} A_\mu\right) j_B^\mu$$

(Auxiliary) gauge field of $U_B(1)$
(μ,**0**)

Gauge field of $U_{EM}(1)$

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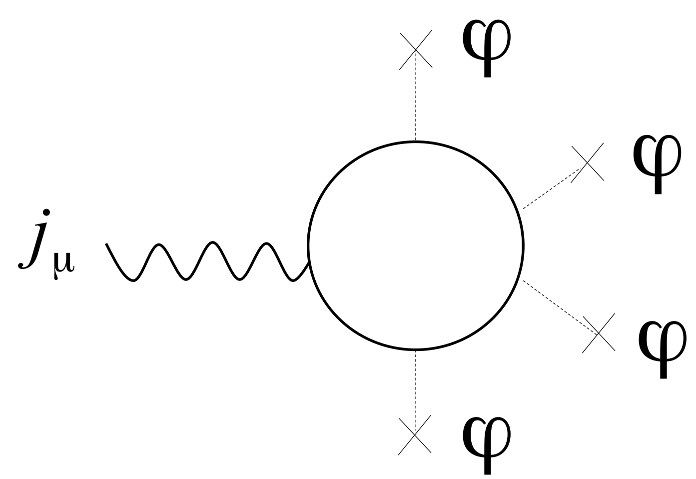
Baryon current $\langle \bar{\psi}\gamma^\mu \psi \rangle$

Linear sigma model

J. Goldstone, F. Wilczek, PRL 47, 986 (1981)

$$L = \bar{\psi}\left(i\not{D} + \sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi}\right)\psi$$



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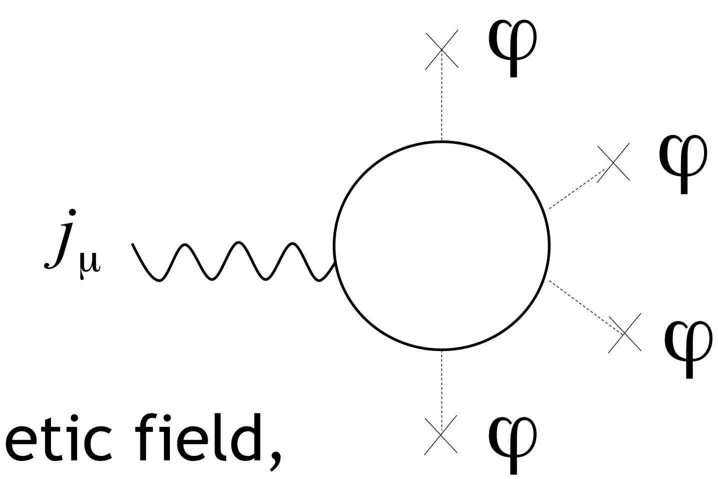


If the mesons take the form of the DCDW-type in the magnetic field,

$$\sigma = f_\pi \cos qz \ , \ \pi^3 = f_\pi \sin qz \qquad \boxed{S_{WZW} = \frac{\mu e B_z q}{4\pi^2} \int d^4x} \longrightarrow \text{Magnetization}$$

About $\delta\Omega^{(1)}$

The contribution of lowest Landau level

$$\delta\Omega^{(1),\mathrm{LLL}}_{\mathrm{vac}} = -\frac{N_c}{4\pi}\int\frac{dp}{2\pi}\sum_{\epsilon=\pm1}\left|E^{\mathrm{LLL}}_{p,\epsilon}\right|$$

Contribution of anomaly

$$\delta\Omega^{(1),\mathrm{LLL}}_{\mu} = -\frac{N_c}{2\pi}\int\frac{dp}{2\pi}\sum_{\epsilon=\pm1}\left(\mu - E^{\mathrm{LLL}}_{p,\epsilon}\right)\theta(E^{\mathrm{LLL}}_{p,\epsilon})\theta(\mu - E^{\mathrm{LLL}}_{p,\epsilon}) + \frac{\mu N_c}{4\pi}\eta_H$$

$$\delta\Omega^{(1),\mathrm{LLL}}_{T} = -\frac{N_c T}{2\pi}\int\frac{dp}{2\pi}\sum_{\epsilon=\pm1}\ln\left(1 + e^{-\beta|E^{\mathrm{LLL}}_{p,\epsilon}-\mu|}\right)$$

Contribution of valence quarks

The property of $\delta\Omega^{(1)}$

$$\delta\Omega^{(1)} = \delta\Omega^{(1),\mathrm{LLL}}_{q-\mathrm{odd}} + \delta\Omega^{(1),\mathrm{LLL}}_{q-\mathrm{even}} + \delta\Omega^{(1),\mathrm{hLLs}}$$

→ Odd-function about q

$$m\to\infty \ \text{ or } \ \mu < m - q/2, T = 0 \ : \ \delta\Omega^{(1)} = -\frac{N_c \mu q}{4\pi^2}$$

(no contribution of valence quarks)

Magnetization derived from chiral anomaly

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

$m \to 0 \ : \ \delta\Omega^{(1)} \to 0$ q-independent → physically correct

Anomaly due to the spectral asymmetry

Anomalous particle number A. J. Niemi, G. W. Semenoff, Phys. Rep. 135, 99 (1986)

Fermion field expanded about the eigenfunctions $\psi(\mathbf{r}) = \sum_k b_k \varphi_k^{(+)}(\mathbf{r}) + \sum_{k'} d_{k'}^\dagger \varphi_{k'}^{(-)}(\mathbf{r})$

$$H\varphi_k = \lambda_k \varphi_k$$

Energy eigenvalue

The particle number is defined avoiding the divergence

$$N = \frac{1}{2}\int d\mathbf{r} \left\langle \left[\psi^\dagger(\mathbf{r}), \psi(\mathbf{r})\right] \right\rangle$$

$$= \sum_k \left\langle b_k^\dagger b_k \right\rangle - \sum_{k'} \left\langle d_{k'}^\dagger d_{k'} \right\rangle - \frac{1}{2}\int d\mathbf{r} \left(\sum_k \varphi_k^{(+)\dagger} \varphi_k^{(+)} - \sum_{k'} \varphi_{k'}^{(-)\dagger} \varphi_{k'}^{(-)} \right)$$

$$= N_{nom} - \frac{1}{2}\sum_k \mathrm{sign}(\lambda_k)$$

Positive energy Negative energy

This does not vanish if the energy spectrum is asymmetric

cf. Baryon number of the pion cloud in the chiral bag model

M. Rho, Phys. Rep. 240, 1 (1994)

The case that the energy spectrum can be asymmetric

$$L_{MF} = \bar{\psi}\left[i\not{D} + \mu\gamma^0 + G\left(\left(1+\gamma^5\tau_3\right)\Delta(\mathbf{r}) + \left(1-\gamma^5\tau_3\right)\Delta(\mathbf{r})^*\right)\right]\psi + G\left|\Delta(\mathbf{r})\right|^2$$

where $\Delta(\mathbf{r}) = \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau_3\psi\rangle$

EOM: $\left(\alpha\cdot(-i\nabla + Q\mathbf{A}) - G\gamma^0\left[\left(1+\gamma^5\tau_3\right)\Delta(\mathbf{r}) + \left(1-\gamma^5\tau_3\right)\Delta(\mathbf{r})^*\right]\right)\psi_k = \lambda_k(\Delta)\psi_k$

transformation: $\psi \to i\gamma^0\gamma^5\psi^*$

$$\lambda_k(\Delta(\mathbf{r})) \to -\lambda_k\left(\Delta(\mathbf{r})^*\right)$$

Therefore, it is a necessary condition of the asymmetric spectrum that Δ(r) is complex number.

Component of the magnetization

In the homogeneous phase

Gordon id. $\bar{\psi}\gamma^{\mu}\psi = \frac{1}{2m}\bar{\psi}\left[\overset{\boxed{?}}{i\partial^{\mu}} - \overset{\boxed{?}}{i\partial^{\mu}} - i\sigma^{\mu\nu}\left(\overset{\boxed{?}}{i\partial_{\nu}} + \overset{\boxed{?}}{i\partial_{\nu}}\right)\right]\psi$

$$M_0 \equiv -\left.\frac{\partial\Omega(\mu,T,B)}{\partial B}\right|_{B=0} = e\langle\bar{\psi}\gamma^{2}x\psi\rangle = \frac{e}{2m}\left[\langle\bar{\psi}2ix\partial^{y}\psi\rangle + \langle\bar{\psi}\sigma^{21}\psi\rangle\right]$$

orbit — Landau diamagnetism

spin — Pauli paramagnetism

$$S \sim \int dx^4\bar{\psi}\gamma^{2}eBx\psi$$

In the DCDW phase

Modified Gordon id. $\bar{\psi}\gamma^{\mu}\psi = \frac{1}{2m}\bar{\psi}\left[e^{i\gamma^5 qz}\overset{\boxed{?}}{i\partial^{\mu}} - \overset{\boxed{?}}{i\partial^{\mu}}e^{i\gamma^5 qz} - i\sigma^{\mu\nu}\left(e^{i\gamma^5 qz}\overset{\boxed{?}}{i\partial_{\nu}} + \overset{\boxed{?}}{i\partial_{\nu}}e^{i\gamma^5 qz}\right)\right]\psi$

$$M_0 = \frac{e}{2m}\left[\langle\bar{\chi}2ix\partial^{y}\chi\rangle + \langle\bar{\chi}\sigma^{21}\chi\rangle + \langle\bar{\chi}\sigma^{23}i\gamma^{5}qx\chi\rangle\right]$$

where $\chi \equiv e^{i\gamma^5 qz/2}\psi$

The magnetization is constituted by the orbit and spin contribution described by x
and the additional term.

※ $\langle\bar{\psi}\sigma^{12}\psi\rangle \sim \cos qz$ vanishes after the spatial average

Gordon identity

Dirac eq. : $(i\not\partial - m)\psi = 0$ ⇒

$$\begin{cases} \bar{\psi}\gamma^{\mu}(i\not\partial - m)\psi = 0 \\ \bar{\psi}(i\not\partial + m)\gamma^{\mu}\psi = 0 \end{cases}$$

Subtracting the both sides

$$-2m\bar{\psi}\gamma^{\mu}\psi + \bar{\psi}(\gamma^{\mu} i\not\partial - i\not\partial\gamma^{\mu})\psi = 0 \qquad [\gamma^{\mu}\gamma^{\nu} = g^{\mu\nu} - i\sigma^{\mu\nu}]$$

⇒

$$\bar{\psi}\gamma^{\mu}\psi = \frac{1}{2m}\bar{\psi}\left[i\partial^{\mu} - i\partial^{\mu} - i\sigma^{\mu\nu}\left(i\partial_{\nu} + i\partial_{\nu}\right)\right]\psi$$

In the DCDW phase

Dirac eq. : $\left(i\not\partial - me^{i\gamma^{5}qz}\right)\psi = 0$ ⇒

$$\begin{cases} \bar{\psi}\gamma^{\mu}\left(e^{-i\gamma^{5}qz} i\not\partial - m\right)\psi = 0 \\ \bar{\psi}\left(i\not\partial e^{-i\gamma^{5}qz} + m\right)\gamma^{\mu}\psi = 0 \end{cases}$$

相境界上での自発磁化

DCDW相⇔回復相境界上では $m^{(0)}=0$, $q^{(0)}\neq 0$　（ただし B≠0 では m≠0）

$$\frac{\partial \Omega(\mu, T, B; q, m)}{\partial q} = 0$$ が解けたとする ➡ $$q = \underbrace{q^{(0)}(\mu, T, m^2)}_{\neq 0} + eBq^{(1)}(\mu, T, m^2) + (eB)^2 q(\mu, T, m^2) + ?$$

これを Ω に代入して相境界上なので m について展開

$$\Omega = \frac{1}{2}\left(\alpha_2 + eB\alpha_2' + (eB)^2\alpha_2'' + ?\right)m^2 + \frac{1}{4}\left(\alpha_4 + eB\alpha_4' + (eB)^2\alpha_4'' + ?\right)m^4 + ?$$

B=0 のとき2次相転移線なので$\alpha_2=0$

$$\frac{\partial \Omega}{\partial m} = m\left(eB\alpha_2' + (eB)^2\alpha_2'' + \left(\alpha_4 + eB\alpha_4' + (eB)^2\alpha_4''\right)m^2\right) = 0$$

➡ $$m \sim (eB)^{\frac{1}{2}}$$

よって　$$\Omega \sim (eB)^2 \rightarrow \left.\frac{\partial \Omega}{\partial B}\right|_{B=0} = 0$$

自己無撞着な磁化

自己無撞着な式

$$M = -\left.\frac{\partial \Omega'(\mu, T, B)}{\partial B}\right|_{B=\frac{8\pi}{3}M}$$

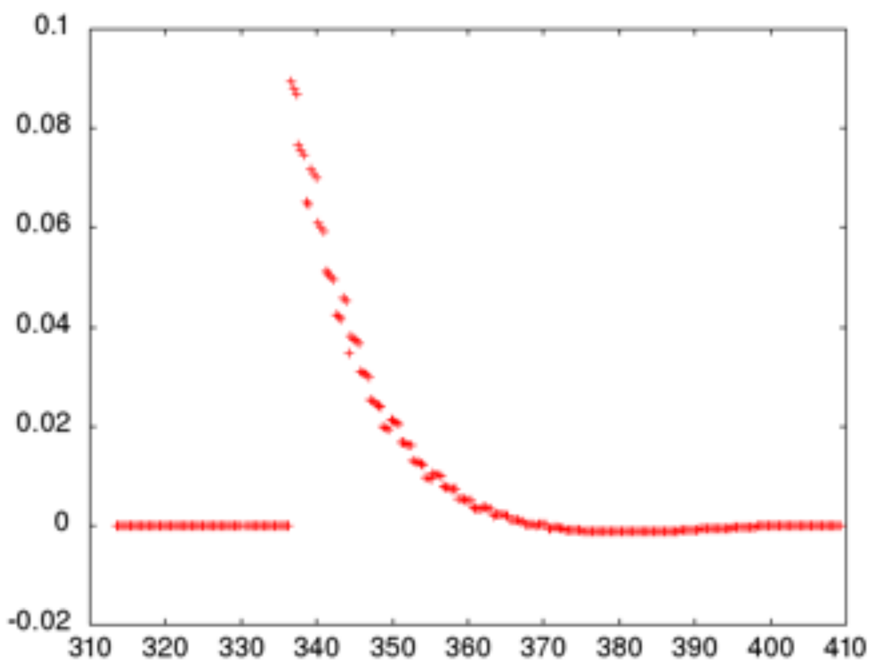
$$\Omega' = \Omega'^{(0)} - M_0 B + (eB)^2 \Omega'^{(2)}$$

2次まで

$$M = M_0 - \frac{16\pi}{3} e^2 \Omega'^{(2)} M$$

T=0

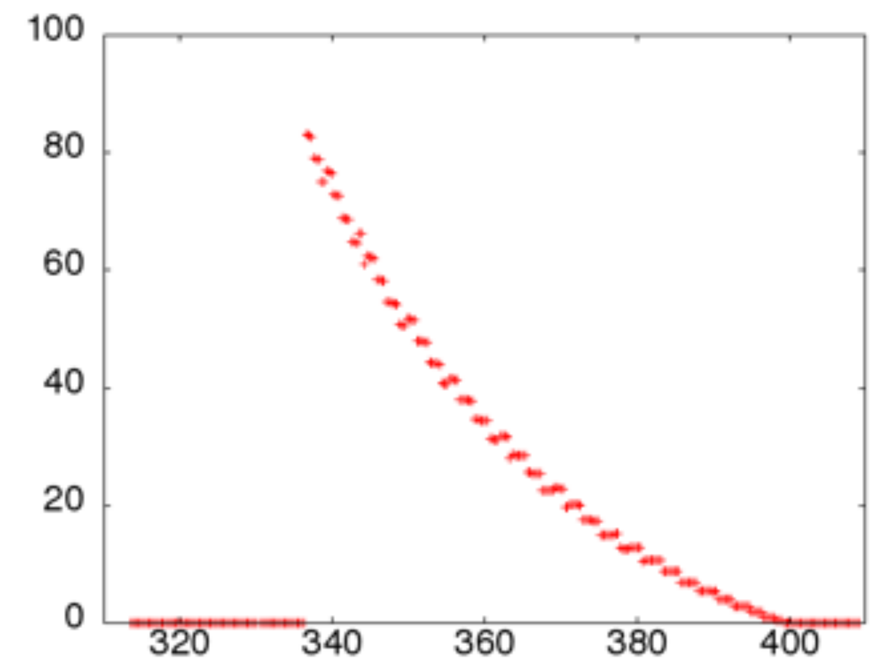
[MeV²] $\Delta M \equiv M - M_0$



補正は非常に小さい

$$\frac{\Delta M}{M_0} \sim 10^{-3}$$

M_0[MeV²]



展開パラメータ $eB/\mu^2 \to e\frac{8\pi}{3}M \Big/ \mu^2 \sim 10^{-3}$

→ M_0までの評価で十分

一般化Ginzburg-Landau展開

Lifshitz point 周りは秩序変数で展開した熱力学ポテンシャルで議論すれば十分

- 相転移の様子が理解しやすい
- 数値計算が容易

熱力学ポテンシャルをカイラル凝縮で展開する

※カイラル凝縮の微分も存在

$$\Omega(\Delta(\mathbf{r})) = \Omega(\Delta(\mathbf{r})=0) + \frac{\alpha_2}{2}|\Delta(\mathbf{r})|^2 + \frac{\alpha_3}{3}\mathrm{Im}\left(\Delta(\mathbf{r})\Delta'^*(\mathbf{r})\right) + \frac{\alpha_{4a}}{4}|\Delta(\mathbf{r})|^4 + \frac{\alpha_{4b}}{4}|\Delta'(\mathbf{r})|^2 + \boxed{?}$$

$$\Delta(\mathbf{r}) = \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle$$

3次の項について T. Tatsumi, K. Nishiyama, S. Karasawa, arXiv:1405.2155 (2014)

- Δが複素数でなければ現れない
- 磁場がなければ $\alpha_3=0$

各係数の求め方

D. Nickel, PRL 103, 072301 (2009)

$$\Omega = -\frac{T}{V}Tr\log\left[i\partial\!\!\!/ + \gamma^0\mu - \left(\frac{1+\gamma^5}{2}\Delta + \frac{1-\gamma^5}{2}\Delta^*\right)\right] + \frac{1}{V}\int d^3\mathbf{x}\frac{|\Delta|^2}{4G}$$

$$= -\frac{T}{V}Tr\log\left[S_0^{-1}\right] - \frac{T}{V}\sum_{n=1}\frac{1}{n}Tr\left[S_0\left(\frac{1+\gamma^5}{2}\Delta + \frac{1-\gamma^5}{2}\Delta^*\right)\right]^n + \frac{1}{V}\int d^3\mathbf{x}\frac{|\Delta|^2}{4G}$$

微分展開

Lifshitz point 周りの相図（α_4-α_2平面）

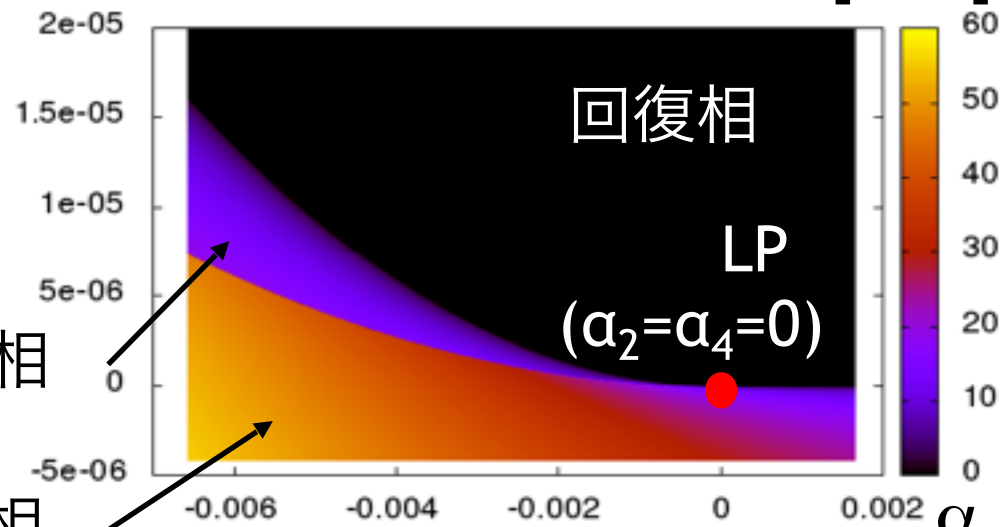
熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\qquad\right)m^2 + \frac{1}{4}\left(\alpha_4 \qquad\qquad\right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\qquad\right)\left(m^6 + 3m^4 q^2 + \frac{1}{2}m^2 q^4\right) + \boxed{?}$$

α_2 / α_6 [Λ^4]

m

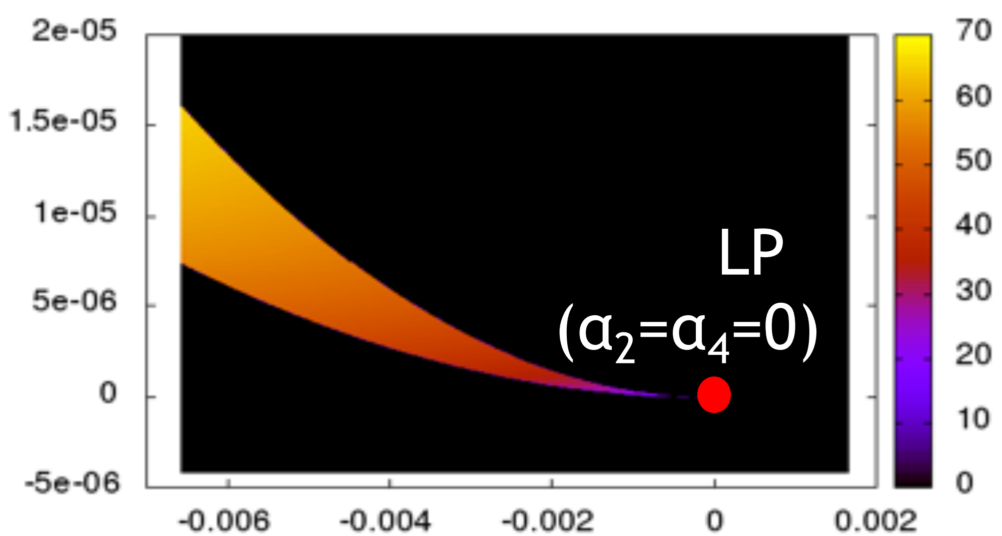


DCDW相

一様相

α_4 / α_6 [Λ^2]

q

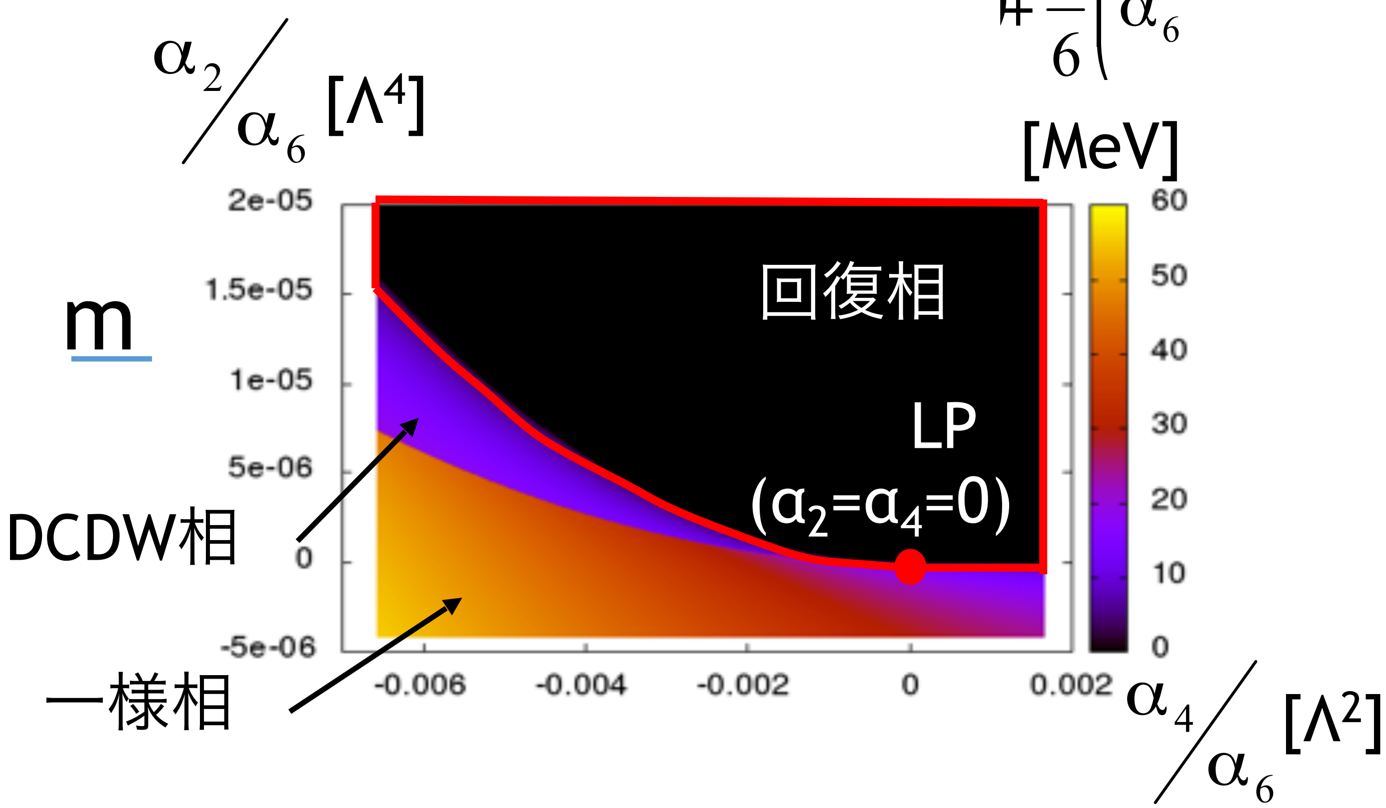


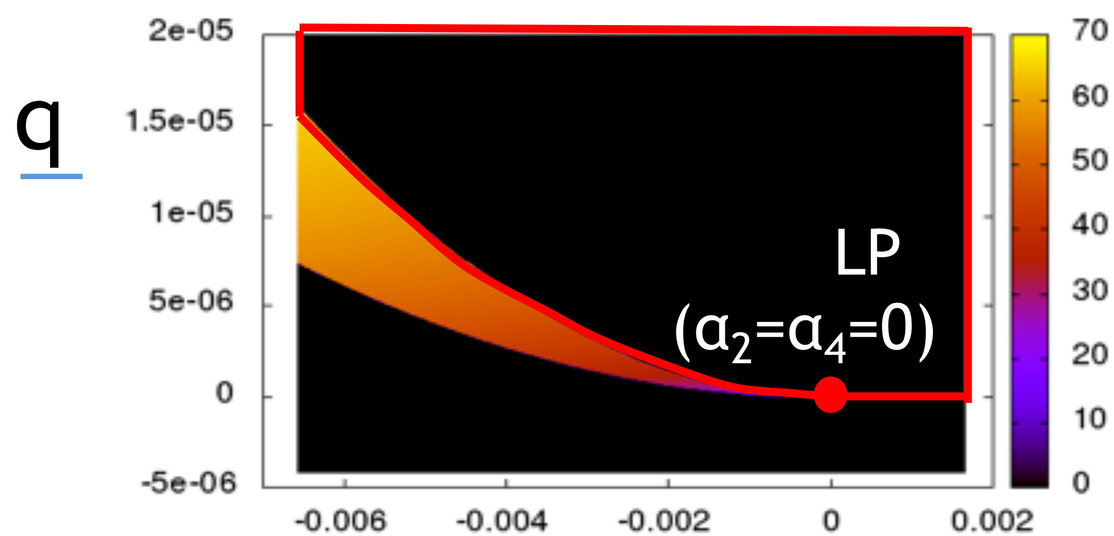
Lifshitz point 周りの相図（α_4-α_2平面）

熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\qquad \right)m^2 \qquad + \frac{1}{4}\left(\alpha_4 \qquad\qquad \right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\qquad \right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + \text{?}$$

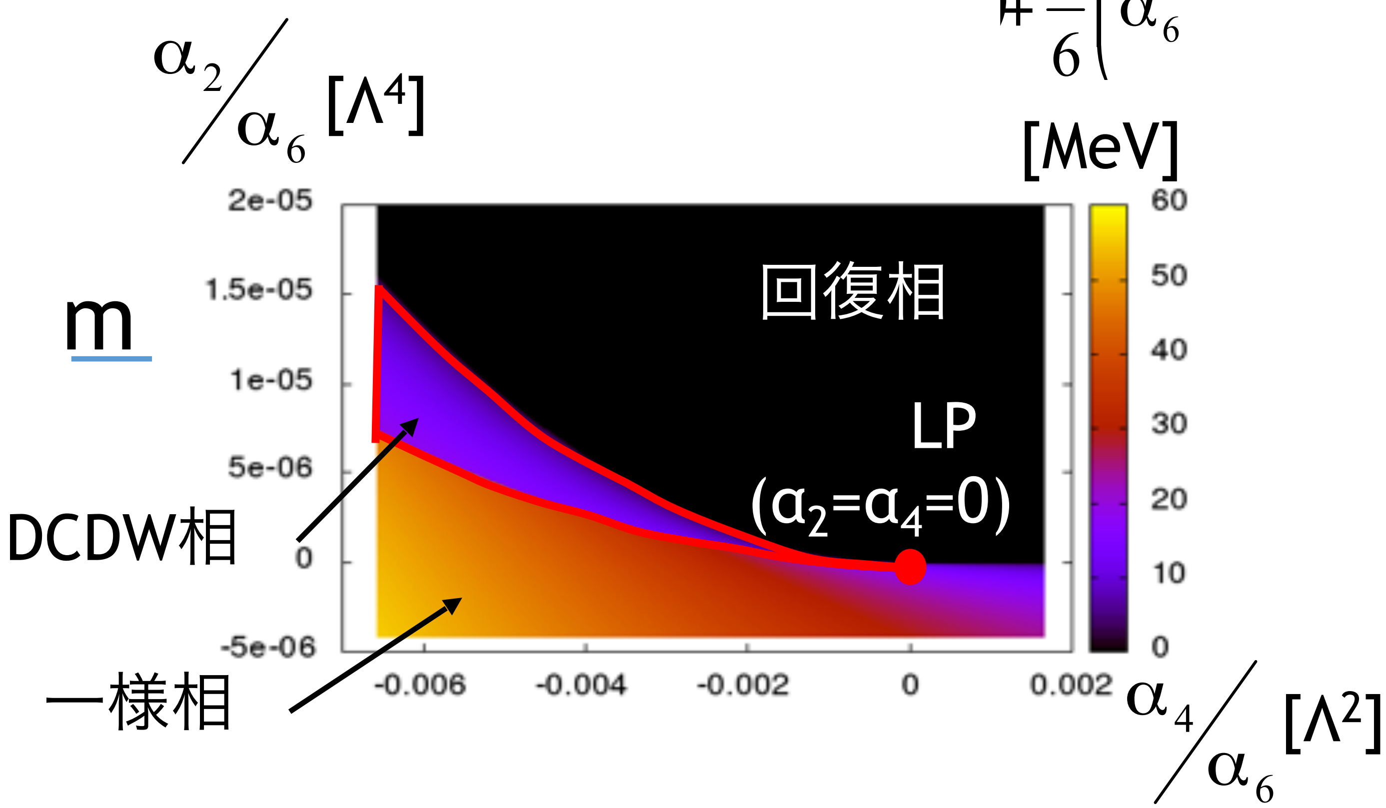


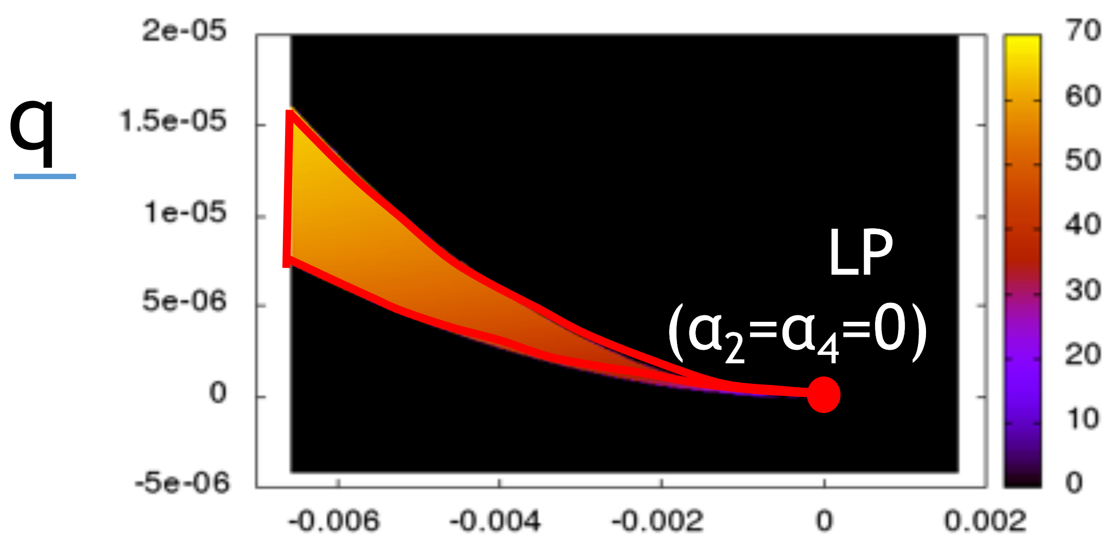


Lifshitz point 周りの相図（α_4-α_2平面）

熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad \right) m^2 + \frac{1}{4}\left(\alpha_4 \qquad \right)\left(m^4 + m^2 q^2\right) + \frac{1}{6}\left(\alpha_6 \qquad \right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + ?$$



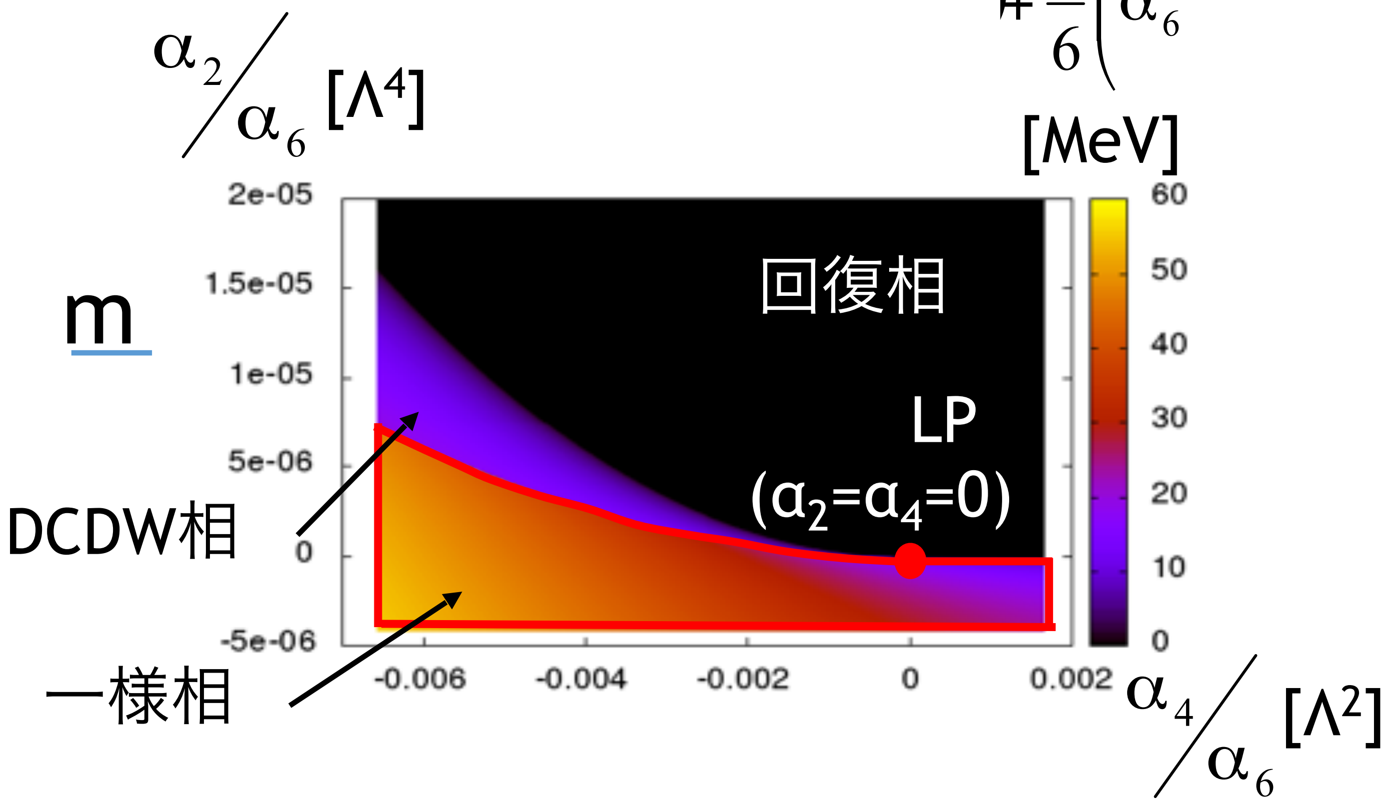


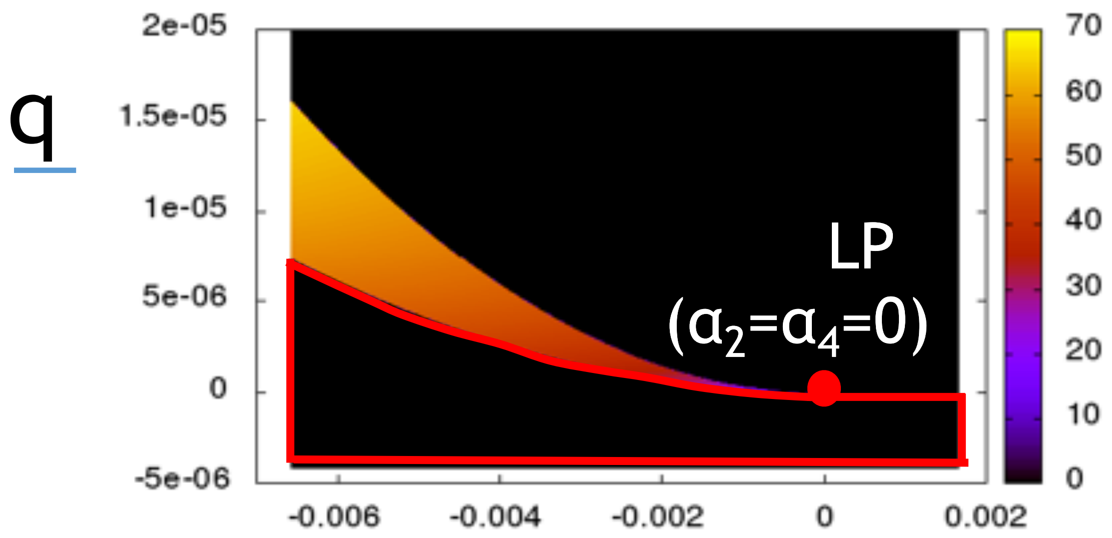
Lifshitz point 周りの相図（α_4-α_2平面）

熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\qquad \right)m^2 + \frac{1}{4}\left(\alpha_4 \qquad\qquad \right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\qquad \right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + \boxed{?}$$



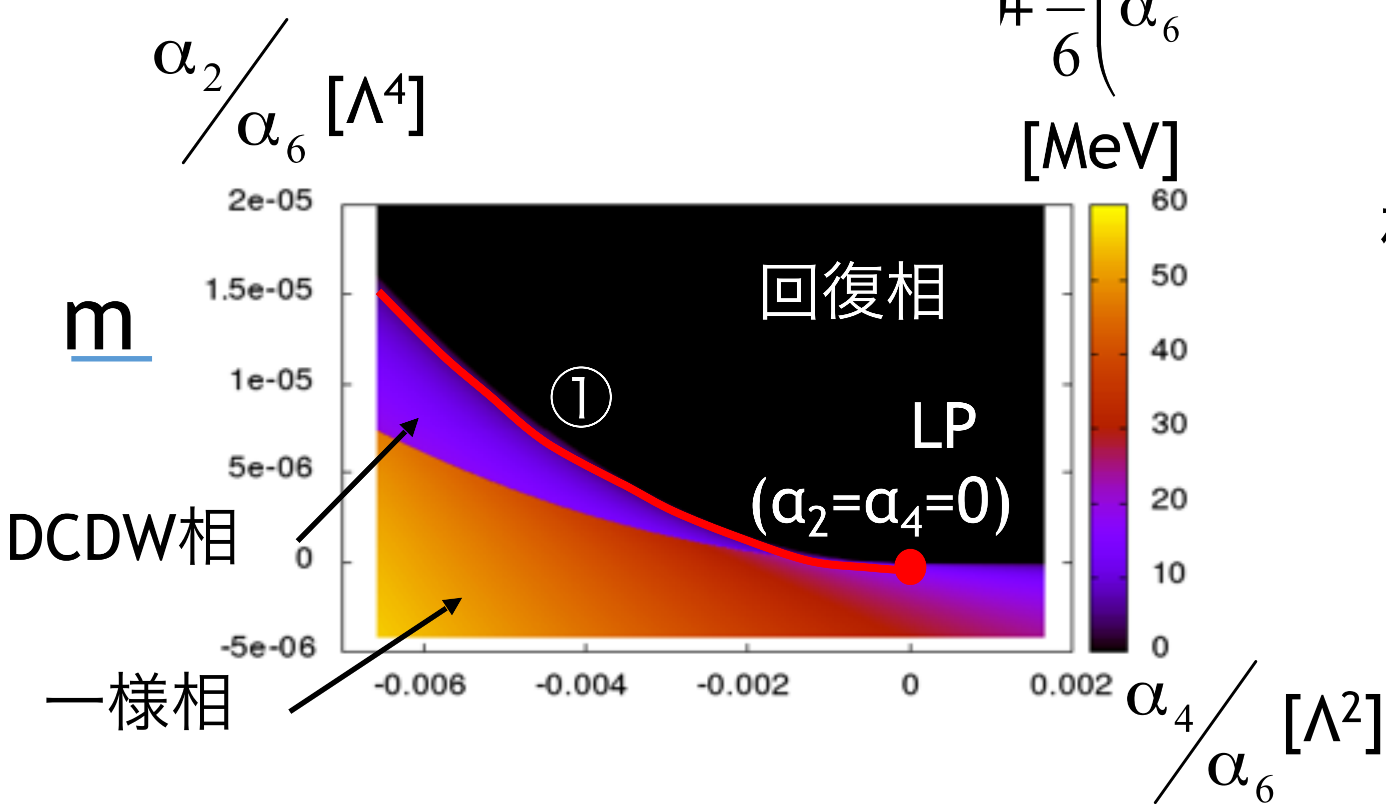


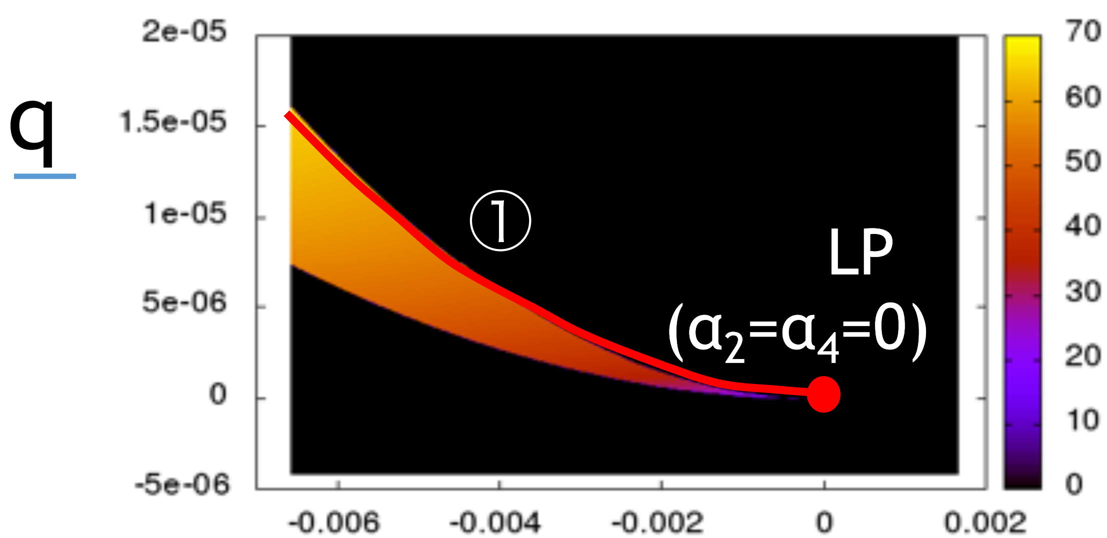
Lifshitz point 周りの相図（α_4-α_2平面）

熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\qquad\right)m^2 + \frac{1}{4}\left(\alpha_4 \qquad\qquad\right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\qquad\right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + ?$$





相境界

① $$\frac{\alpha_2}{\alpha_6} = -\frac{3}{8}\left(\frac{\alpha_4}{\alpha_6}\right)^2$$

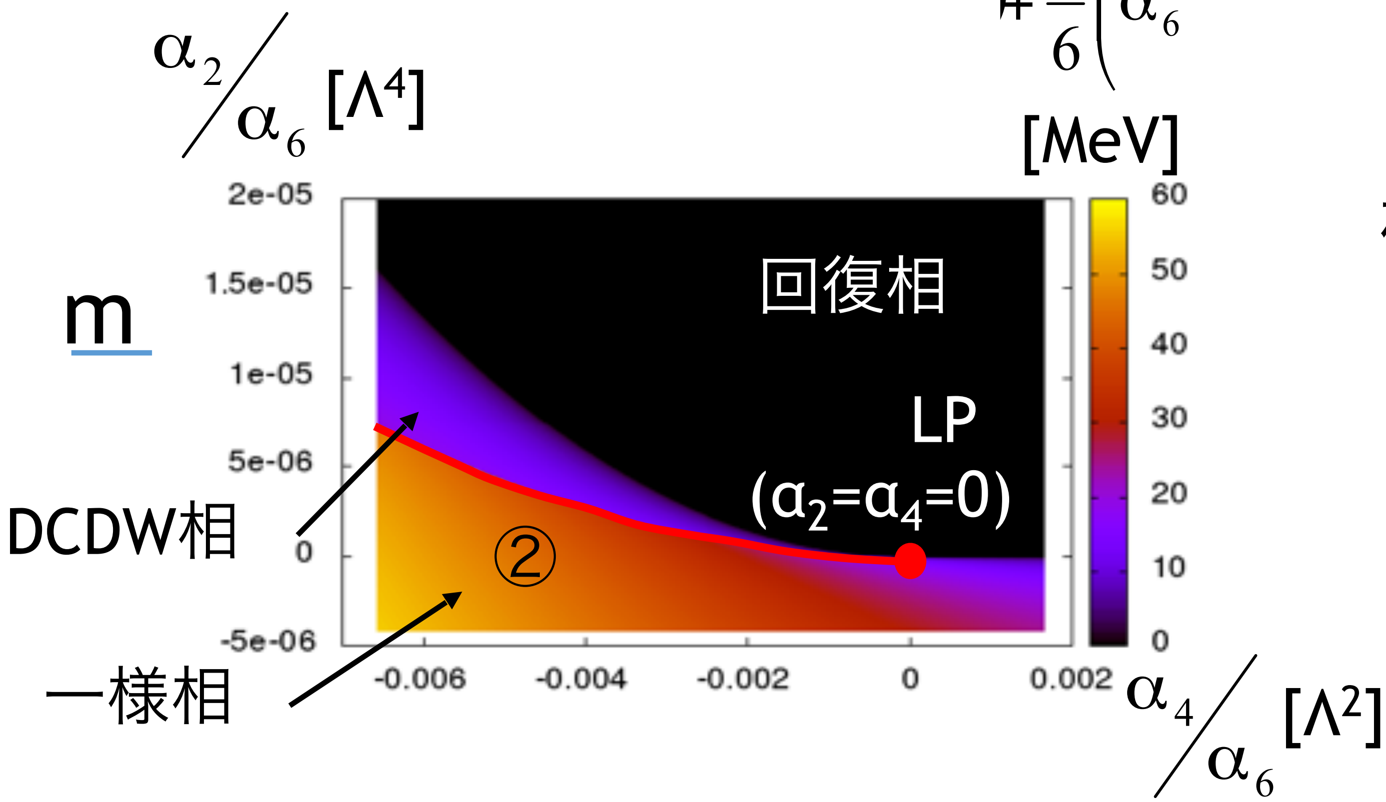
2次相転移線

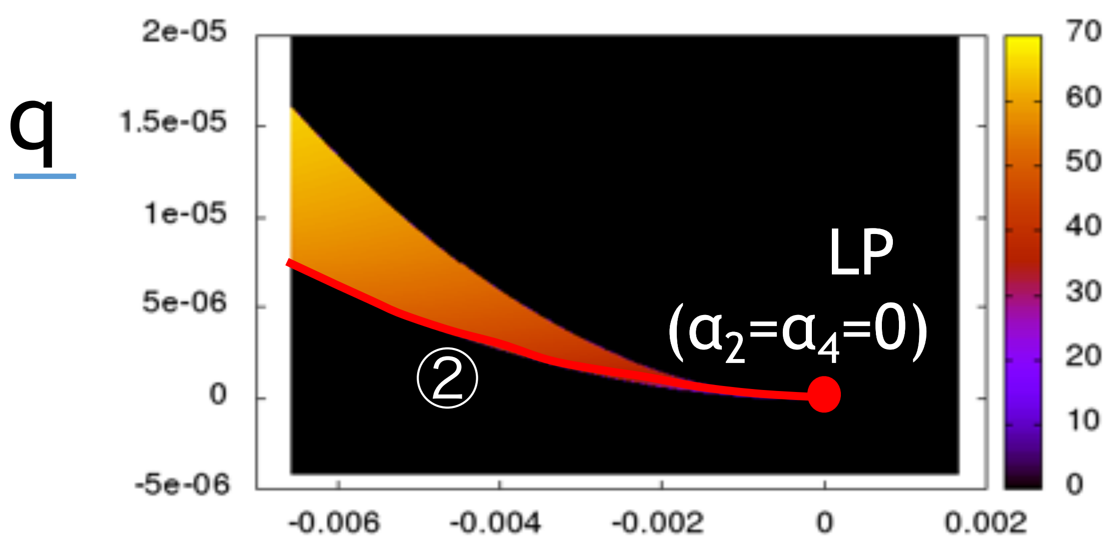
Lifshitz point 周りの相図（α_4-α_2平面）

熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\qquad\right)m^2 + \frac{1}{4}\left(\alpha_4 \qquad\qquad\right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\qquad\right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + \boxed{?}$$





相境界

① $$\alpha_2/\alpha_6 = -\frac{3}{8}\left(\alpha_4/\alpha_6\right)^2$$

2次相転移線

② $$\alpha_2/\alpha_6 = -0.1717\boxed{?}\left(\alpha_4/\alpha_6\right)^2$$

1次相転移線

Lifshitz point 周りの相図（α_4-α_2平面）

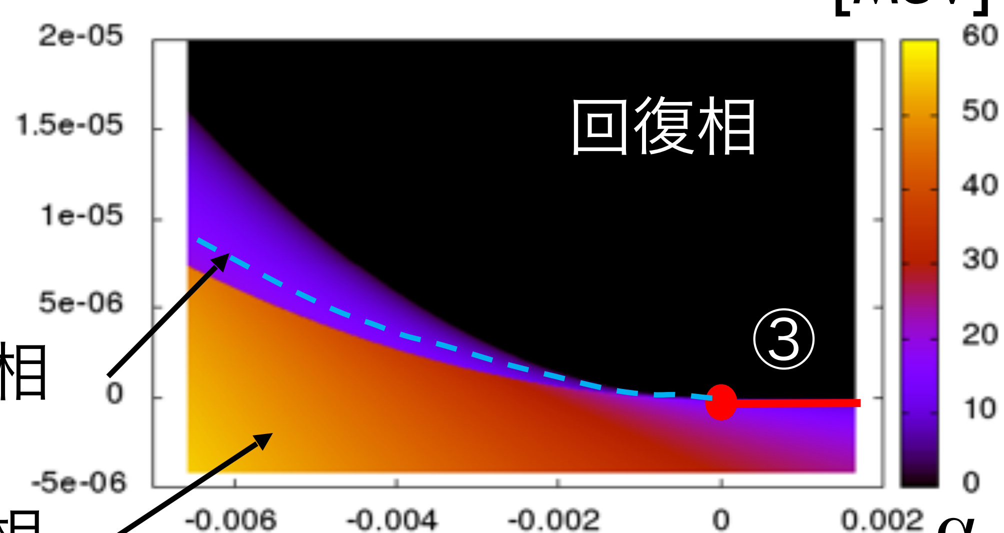
熱力学ポテンシャル (B=0)

$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 \qquad\right) m^2 + \frac{1}{4}\left(\alpha_4 \qquad\right)\left(m^4 + m^2 q^2\right)$$

$$+ \frac{1}{6}\left(\alpha_6 \qquad\right)\left(m^6 + 3m^4 q^2 + \frac{1}{2} m^2 q^4\right) + \boxed{?}$$

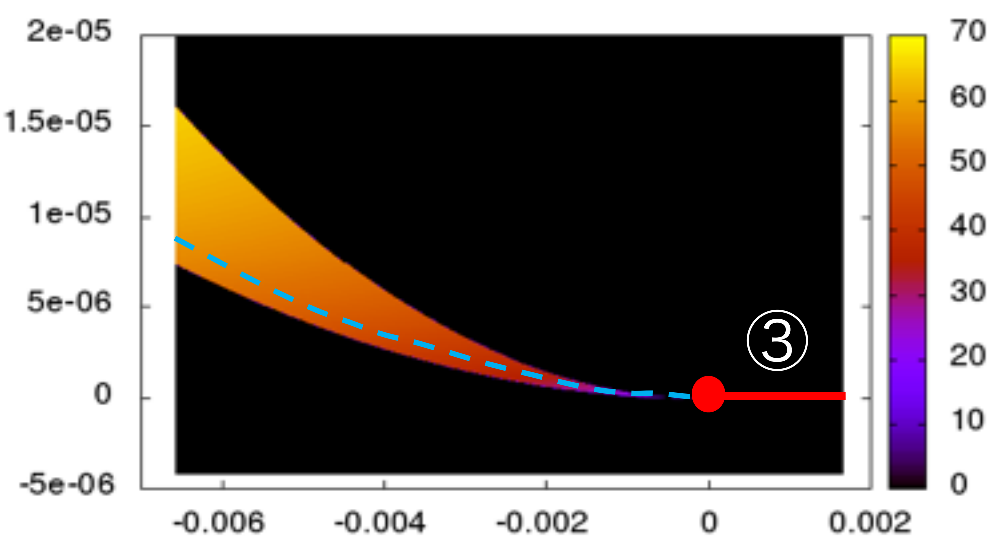
α_2/α_6 [Λ^4]

m



α_4/α_6 [Λ^2]

q



相境界

① $\alpha_2/\alpha_6 = -\frac{3}{8}\left(\alpha_4/\alpha_6\right)^2$

2次相転移線

② $\alpha_2/\alpha_6 = -0.1717\boxed{?}\left(\alpha_4/\alpha_6\right)^2$

1次相転移線

③ $\alpha_2 = 0$

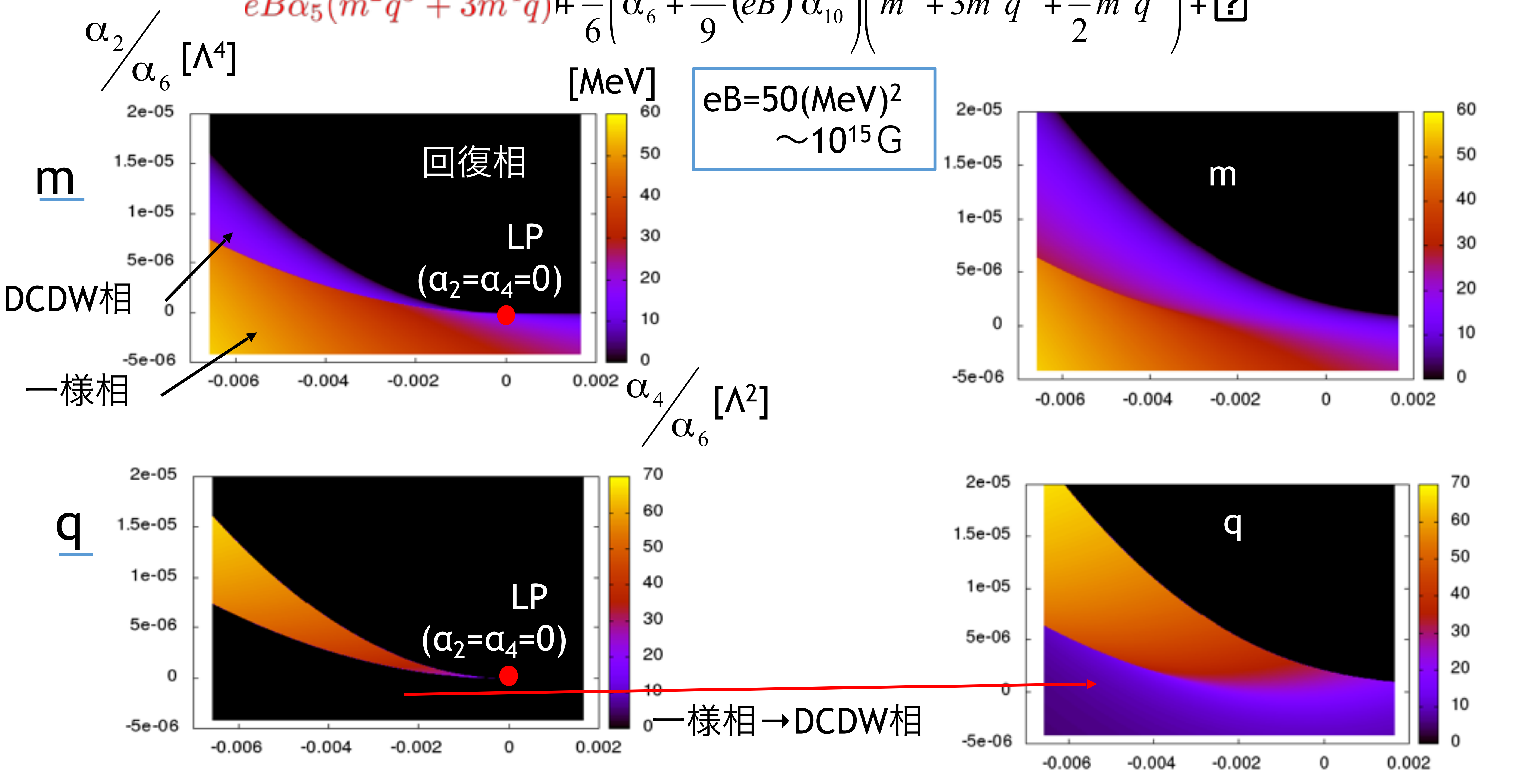
(従来の) 2次相転移線

(従来の1次相転移線 $\alpha_2/\alpha_6 = -\frac{3}{16}\left(\alpha_4/\alpha_6\right)^2$)

Lifshitz point 周りの相図（α_4-α_2平面）

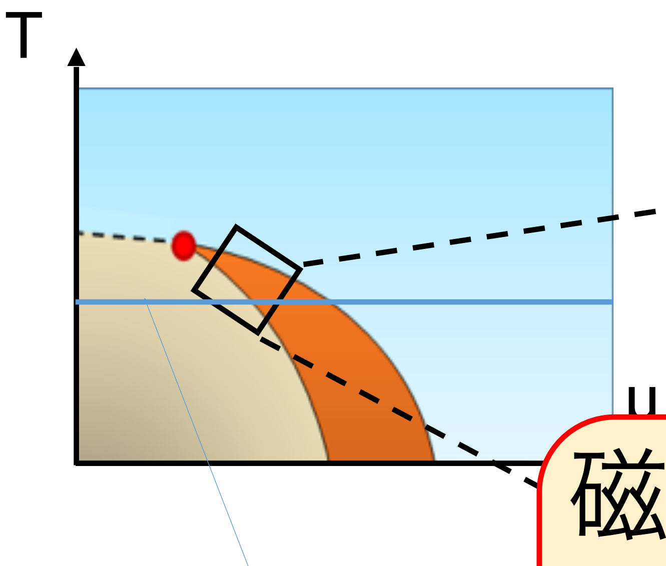
熱力学ポテンシャル

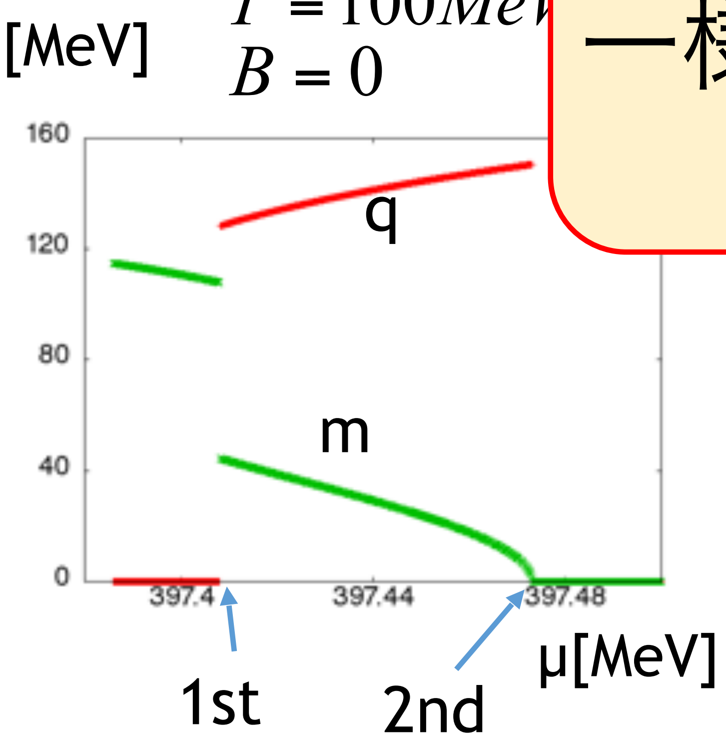
$$\Omega = \Omega_0 + \frac{1}{2}\left(\alpha_2 + \frac{5}{27}(eB)^2\alpha_6\right)m^2 + eB\alpha_3 m^2 q + \frac{1}{4}\left(\alpha_4 + \frac{5}{9}(eB)^2\alpha_8\right)\left(m^4 + m^2 q^2\right)$$
$$+ eB\alpha_5(m^2 q^3 + 3m^4 q) + \frac{1}{6}\left(\alpha_6 + \frac{10}{9}(eB)^2\alpha_{10}\right)\left(m^6 + 3m^4 q^2 + \frac{1}{2}m^2 q^4\right) + \boxed{?}$$

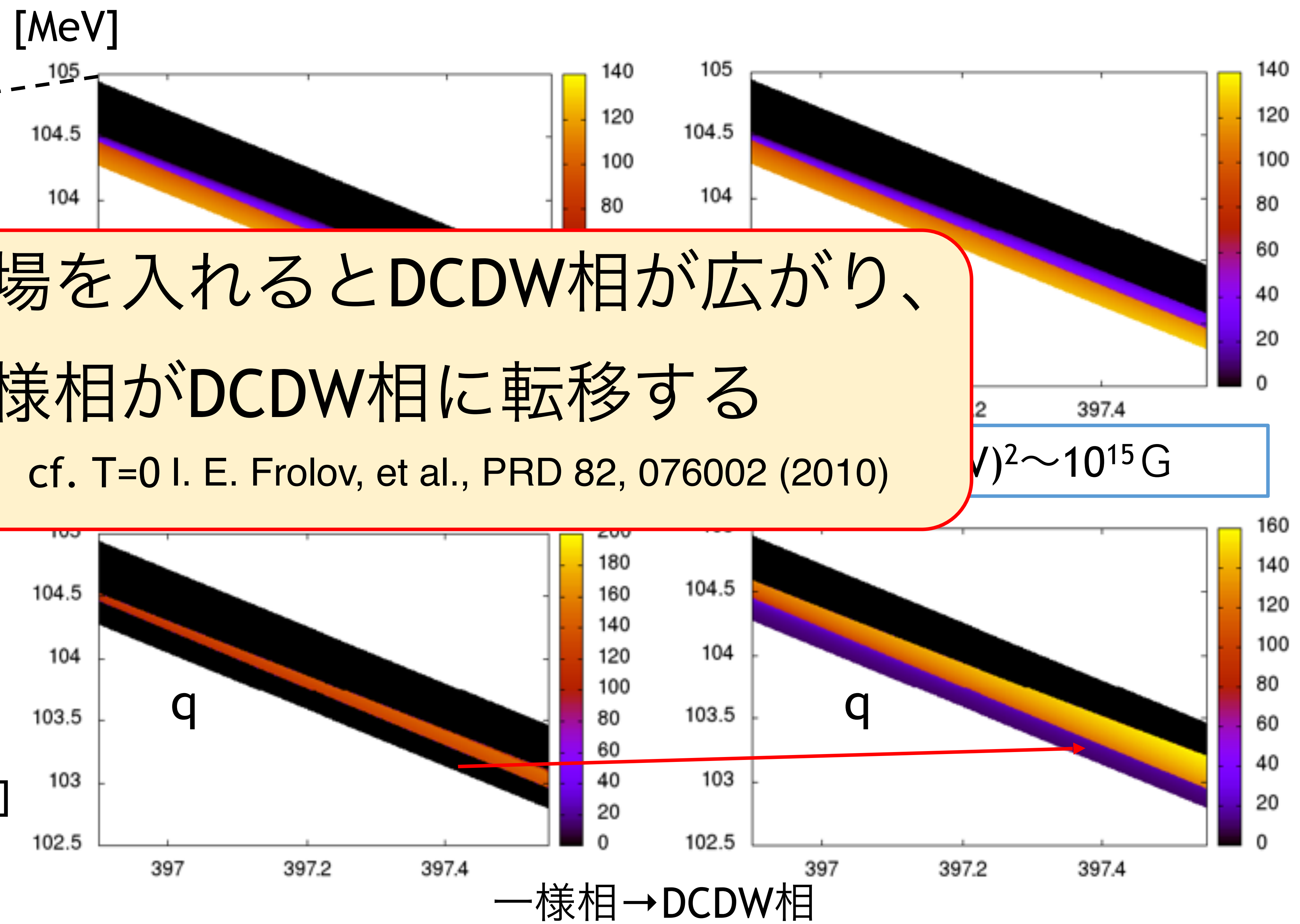


Lifshitz point 周りの相図（μ-T平面）

$LP(\alpha_2=0,\alpha_4=0) \quad \rightarrow \quad LP(\mu \approx 400MeV, T \approx 110MeV$







磁場を入れるとDCDW相が広がり、
一様相がDCDW相に転移する

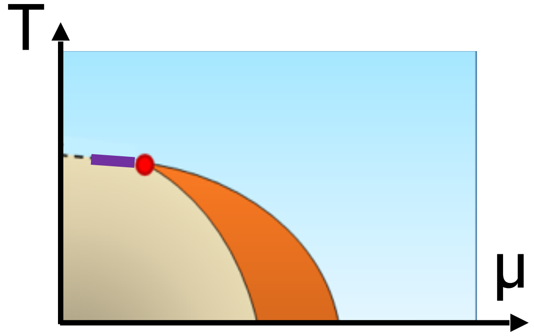
cf. T=0 I. E. Frolov, et al., PRD 82, 076002 (2010)

臨界指数の計算

$$q' \equiv q + 3\alpha_5 eB$$

$$\Delta\Omega = \frac{1}{2}\left(\alpha_2 + (eB)^2\tilde{\alpha}_2\right)m^2 + eB\tilde{\alpha}_3 m^2 q' + \frac{1}{4}\left(\alpha_4 + (eB)^2\tilde{\alpha}_4\right)\left(m^4 + m^2 q'^2\right) + \frac{1}{6}\left(m^6 + 3m^4 q'^2 + \frac{1}{2}m^2 q'^4\right) + \boxed{?}$$

$$\alpha_2 = 0 \ , \ \alpha_4 > 0$$

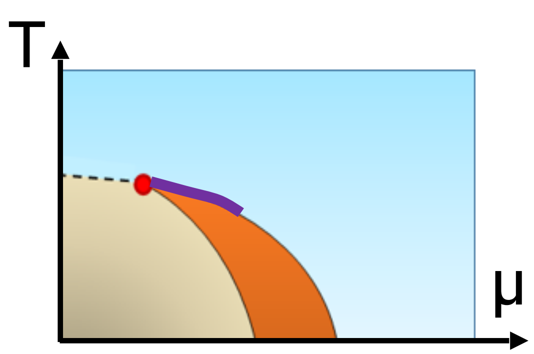


$$\frac{\partial\Omega(\mu, T, B; q', m)}{\partial q', \partial m} = 0$$

$$\begin{cases} (eB)^2\tilde{\alpha}_2 + 2eB\tilde{\alpha}_3 + \alpha_4\left(m^2 + \frac{1}{2}q'^2\right) = 0 \\ eB\tilde{\alpha}_3 + \frac{1}{2}\alpha_4 q' = 0 \end{cases}$$

$$m \sim eB \ , \ q \sim eB$$

$$\alpha_2 = \frac{3}{8}\alpha_4^2 \ , \ \alpha_4 < 0$$



$$q' = \sqrt{\frac{-3}{2}\alpha_4} + \delta q$$

$$\frac{\partial\Omega(\mu, T, B; q, m)}{\partial q, \partial m} = 0$$

$$\begin{cases} 2eB\tilde{\alpha}_3\sqrt{\frac{-3}{2}\alpha_4} - 2\alpha_4 m^2 = 0 \\ eB\tilde{\alpha}_3 - \alpha_4\delta q + \sqrt{\frac{-3}{2}\alpha_4}\,m^2 = 0 \end{cases}$$

$$m \sim (eB)^{\frac{1}{2}} \ , \ \delta q \sim eB$$