Neutrinoless double-beta decay rates around mass 80 in the nuclear shell model

K. Yanase[1], N.Yoshinaga[1], K. Higashiyama[2],
E. Teruya[1], and D. Taguchi[1]

[1]Department of Physics, Saitama University
[2]Department of Physics, Chiba Institute of Technology

INPC 2016 in Adelaide

Saitama University
埼玉大学

CHIBA INSTITUTE OF TECHNOLOGY

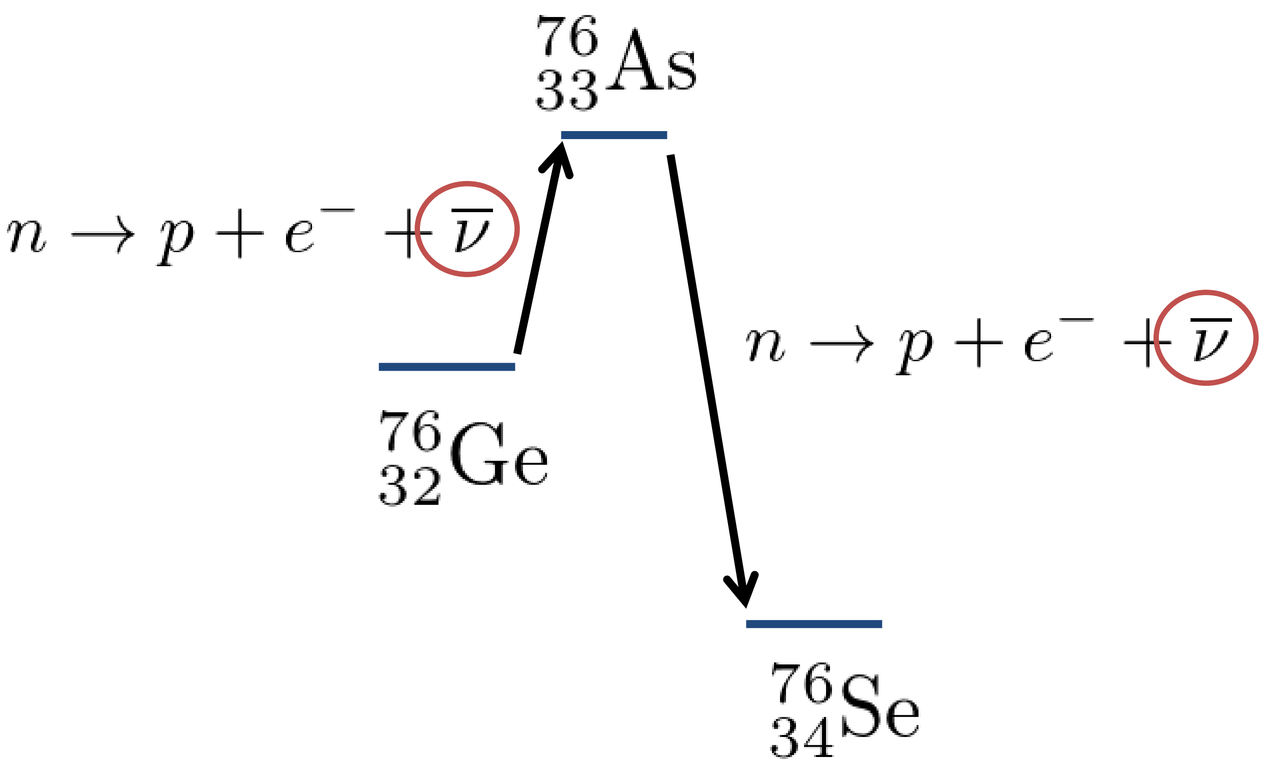
Outline

- We discuss neutrino-less double beta decay
- We perform shell model calculations

  for mass 76 and 82 nuclei
- We calculate nuclear matrix elements

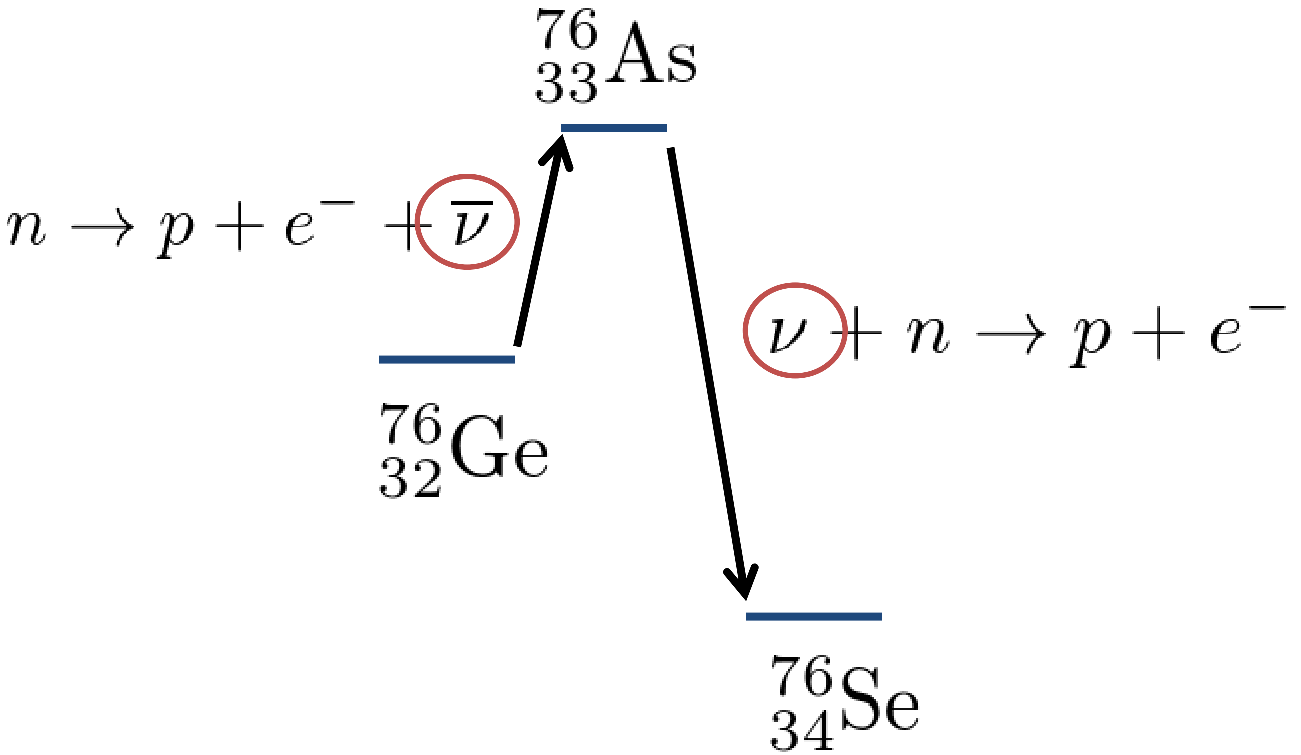
Two modes of double beta decay

1) 2ν mode, characterized by emitting two neutrinos, expected within the Standard Model



Two modes of double beta decay

2) 0ν mode can take place if the neutrino is a massive Majorana particle $(\nu = \overline{\nu})$



This process emits no neutrinos in total

half-life for 0νββ

$$\left[T_{1/2}^{0\nu}\right]^{-1} = \left[\frac{\langle m_\nu \rangle}{m_e}\right]^2 G_{0\nu} \left|M^{0\nu}\right|^2$$

$M^{0\nu}$: Nuclear matrix element

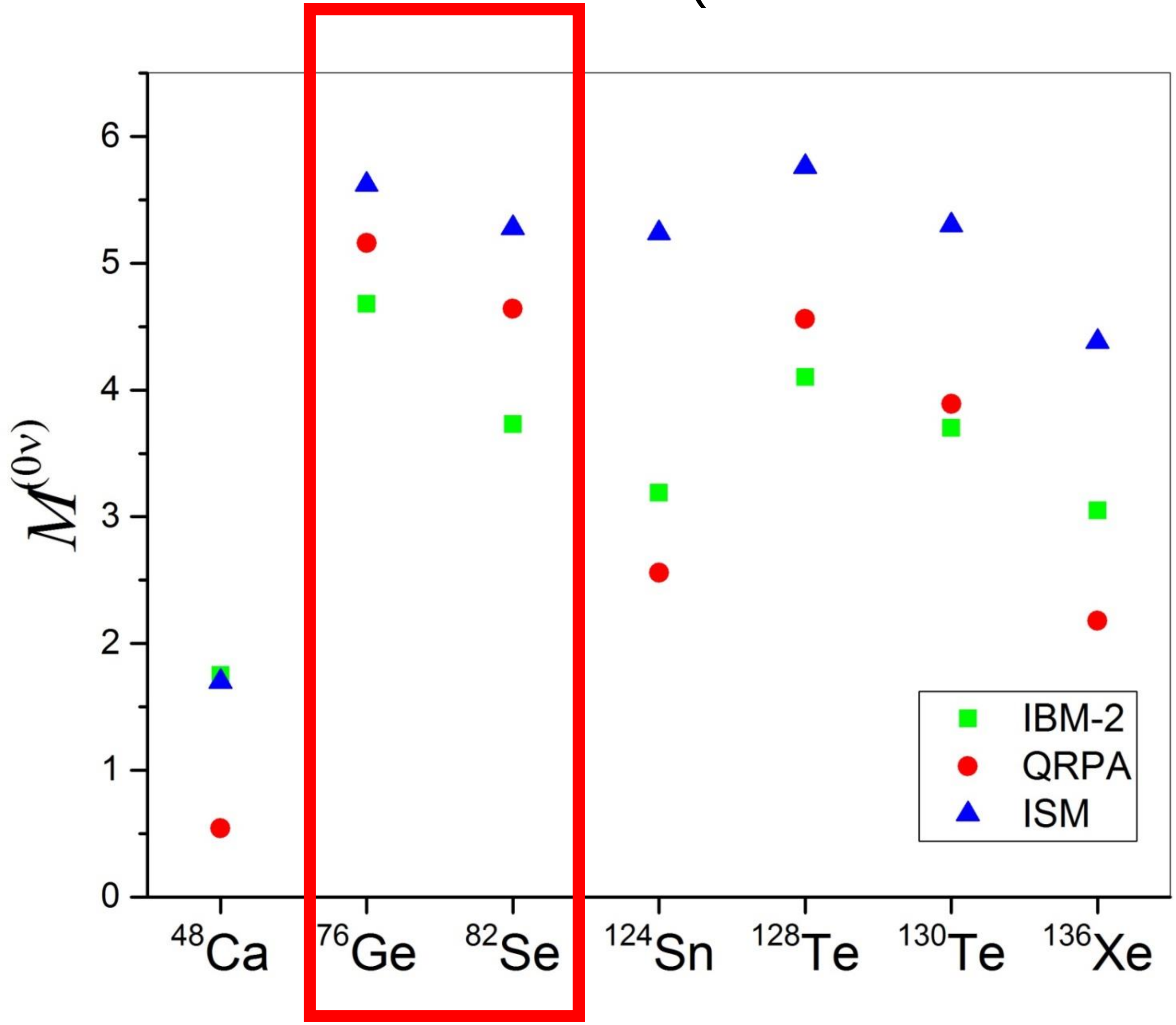
$\langle m_\nu \rangle$: Effective mass for neutrino

m_e : mass for electron

$G_{0\nu}$: kinematical phase space factor

Matrix elements in various models

(in dimensionless by $2R$)



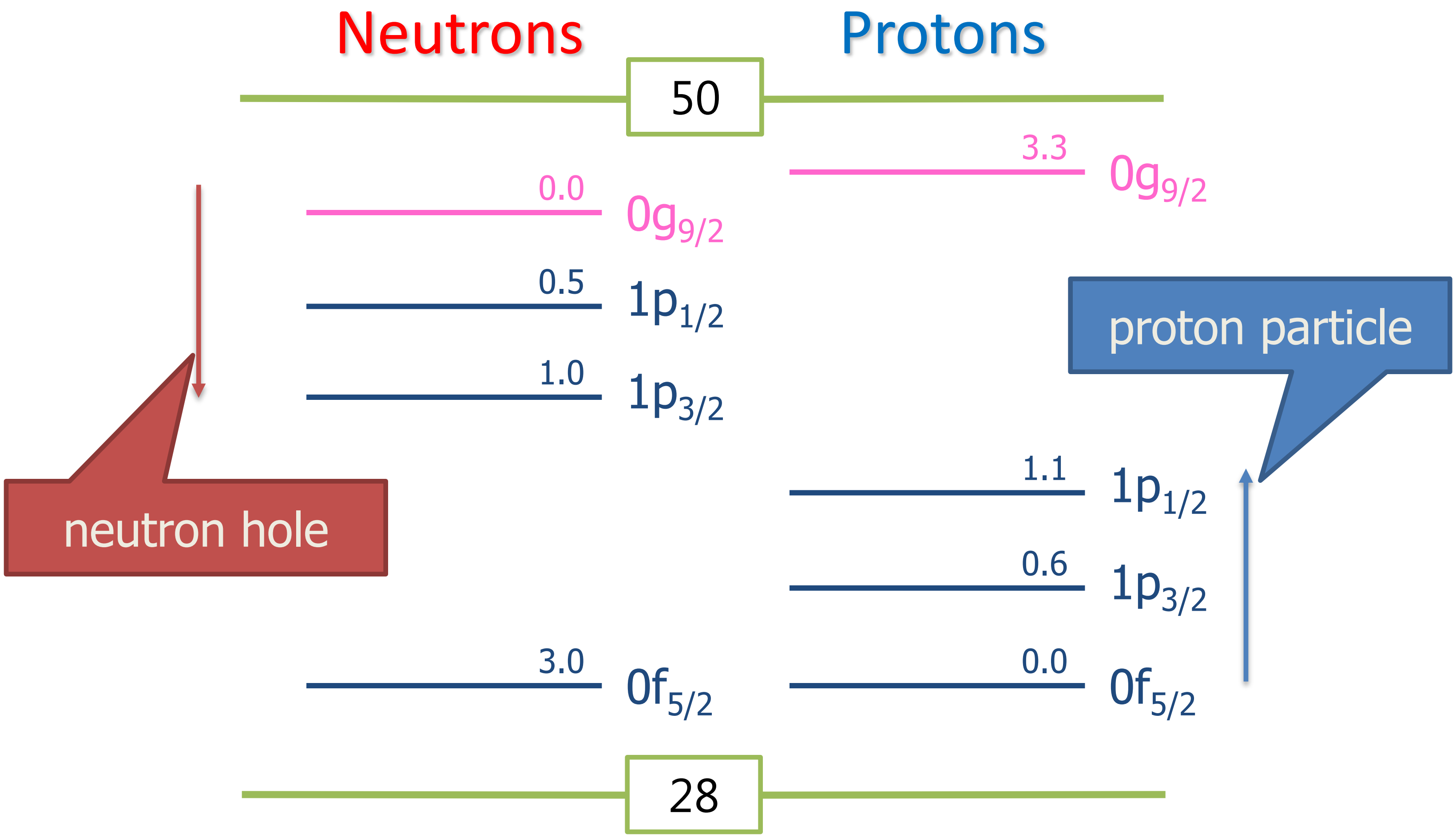
Shell model calculations

- Systematic study carried out for mass around 80 nuclei within the Shell Model
- Pairing + QQ effective interaction

N. Yoshinaga, K. Higashiyama, and P. H. Regan, Phys. Rev. C **78**, 044320 (2008).

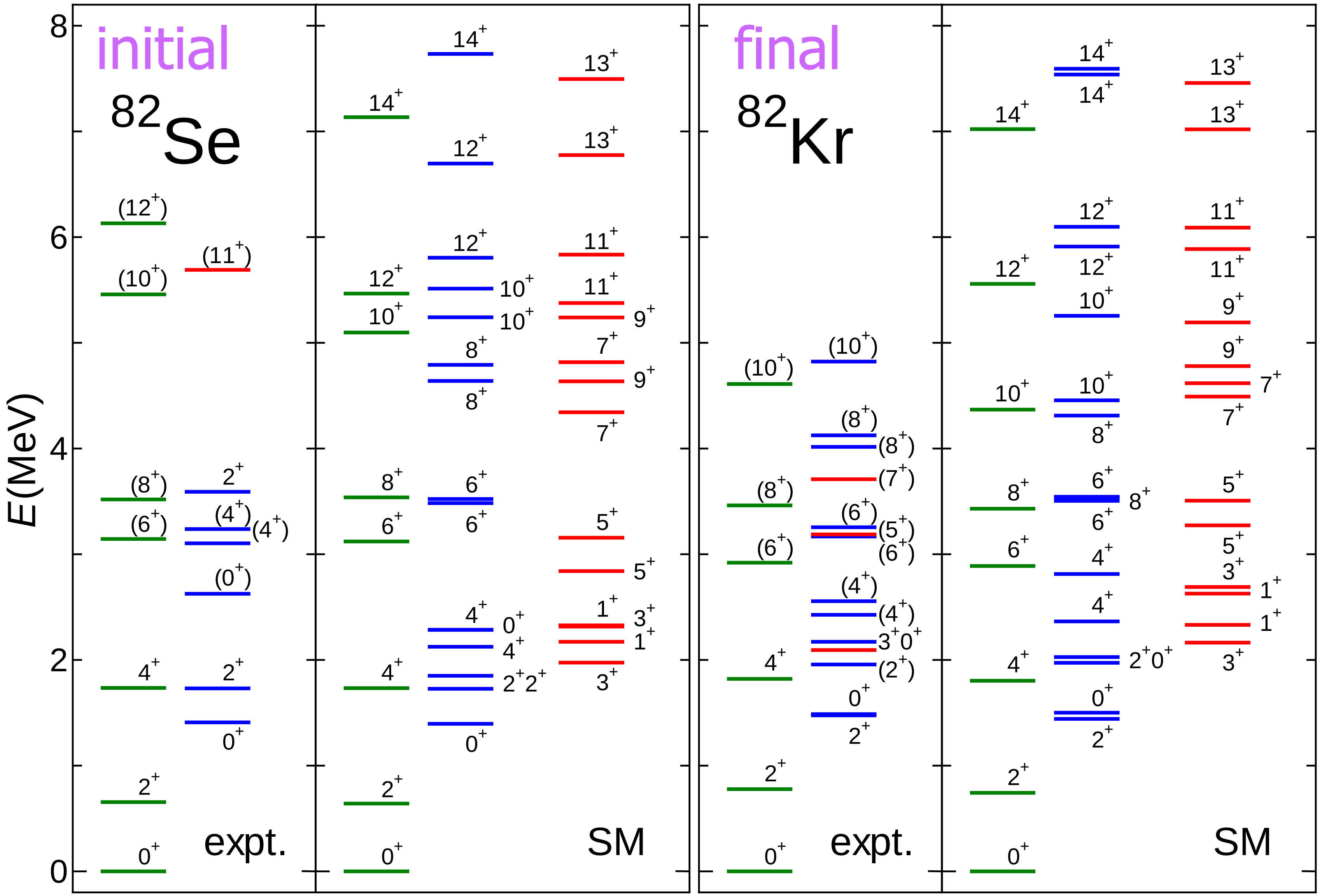
Single particle levels

Single particle energies are denoted on each level (in MeV).

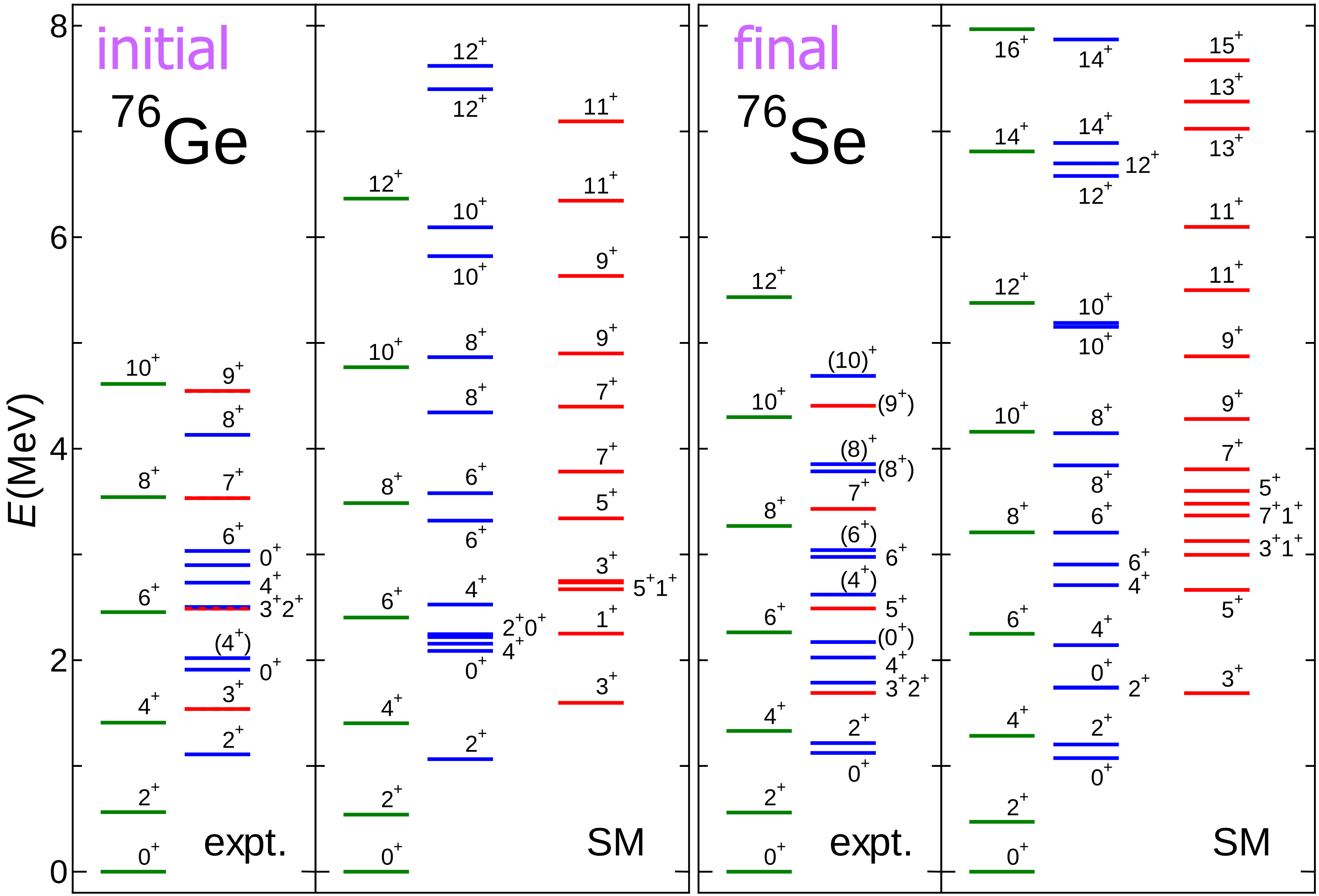


Single-particle energies are the same as previous shell model calculations.

Energy spectra for A=82



Energy spectra for A=76



Numerical results for NMEs

We use

1) Tomoda's formulation

2) Simkovic's formulation

T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991)

F. Simkovic, G. Pantis, J. D. Vergados, and A. Faessler,
Phys. Rev. C **60**, 055502 (1999)

Tomoda's formulation

Total nuclear matrix element

$$M^{(0\nu)} = -\left(\frac{1}{g_A^{(0)}}\right)^2 M_{\mathrm{F}}^{(0\nu)} + M_{\mathrm{GT}}^{(0\nu)}, \quad g_A^{(0)} = 1.25$$

Fermi type in momentum space:

$$H_{\mathrm{F}} = \frac{2}{\pi}\frac{g_V^2(p)}{p(p+\tilde{A})} \qquad g_V(p) = \frac{1}{\left(1+p^2/M_V^2\right)^2}$$

Gamow-Teller type:

$$H_{\mathrm{GT}} = \frac{2}{\pi}\frac{g_A^2(p)}{p(p+\tilde{A})} \qquad g_A(p) = \frac{1}{(1+p^2/M_A^2)^2}$$

closure energy

$$\tilde{A} = 10.08 \text{ MeV for } {}^{82}\mathrm{Se}, \quad \tilde{A} = 9.41 \text{ MeV for } {}^{76}\mathrm{Ge}$$

Results in Tomoda's formulation

(in units of fm^{-1})

Matrix elements for $^{82}Se \rightarrow {}^{82}Kr$

Model	M_F	M_{GT}	$M^{(0\nu)}$
Shell model (this work)	**−0.084**	**0.131**	**0.185**
IBM-2	−0.211	0.346	0.481
QRPA	−0.131	0.293	0.377

Matrix elements for $^{76}Ge \rightarrow {}^{76}Se$

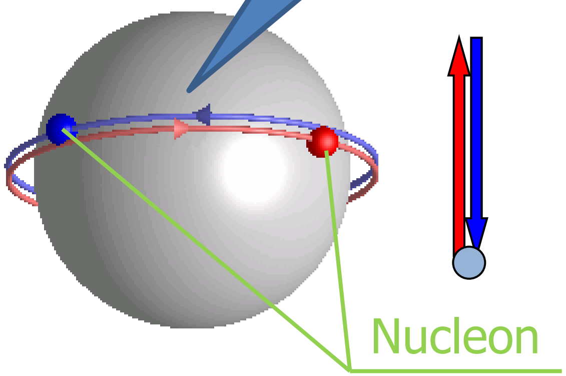
Model	M_F	M_{GT}	$M^{(0\nu)}$
Shell model (this work)	**−0.087**	**0.093**	**0.149**
IBM-2	−0.249	0.446	0.606
QRPA	−0.150	0.330	0.426

IBM-2: J. Barea and F. Iachello, Phys. Rev. C **79**, 044301 (2009)
QRPA: F. Simkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87, 045501 (2013)

Pair-truncated shell model (PTSM)

S-pair

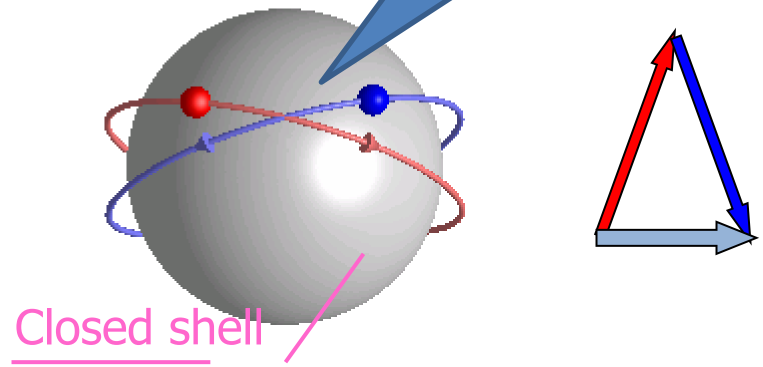
angular momentum 0



$$S^\dagger = \sum_j \alpha_j A_0^{\dagger(0)}(jj)$$

D-pair

angular momentum 2



$$D_M^\dagger = \sum_{j_1 j_2} \beta_{j_1 j_2} A_M^{\dagger(2)}(j_1 j_2)$$

G-pair

angular momentum 4

$$G_M^\dagger = \sum_{j_1 j_2} \gamma_{j_1 j_2} A_M^{\dagger(4)}(j_1 j_2)$$

Results in Tomoda's formulation

(in units of fm^{-1})

Matrix elements for $^{82}Se \rightarrow {}^{82}Kr$

Model	M_F	M_{GT}	$M^{(0\nu)}$
Shell model (this work)	-0.084	0.131	0.185
SDGH-pairs (this work)	**-0.107**	**0.142**	**0.211**
SDG-pairs (this work)	**-0.129**	**0.165**	**0.247**
SD-pairs (this work)	**-0.140**	**0.143**	**0.233**
S-pairs (this work)	**-0.200**	**0.256**	**0.384**
IBM	-0.211	0.346	0.481
QRPA	-0.131	0.293	0.377

H-pairs :

$$H_M^{\dagger(K)} = \left[c_{9/2}^{\dagger} c_{9/2}^{\dagger}\right]_M^{(K)}$$

Results in Tomoda's formulation

(in units of fm^{-1})

Matrix elements for $^{76}Ge \rightarrow {}^{76}Se$

Model	M_F	M_{GT}	$M^{(0\nu)}$
Shell model (this work)	−0.087	0.093	0.149
SDGH-pairs (this work)	**−0.149**	**0.151**	**0.246**
SDG-pairs (this work)	**−0.154**	**0.156**	**0.255**
SD-pairs (this work)	**−0.176**	**0.207**	**0.319**
S-pairs (this work)	**−0.265**	**0.319**	**0.489**
IBM	−0.249	0.446	0.606
QRPA	−0.150	0.330	0.426

Simkovic's formulation

Nuclear matrix elements

$$M^{(0\nu)} = -\left(\frac{1}{g_A^{(0)}}\right)^2 M_{\rm F}^{(0\nu)} + M_{\rm GT}^{(0\nu)} + M_{\rm T}^{(0\nu)}, \qquad g_A^{(0)} = 1.25$$

Transition potentials in momentum space

$$H(p) = -h_{\rm F}(p) + h_{\rm GT}(p)\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_{n'} + h_{\rm T}(p) S^p_{nn'}$$

$$S^p_{nn'} = 3\left(\boldsymbol{\sigma}_n \cdot \hat{\boldsymbol{p}}\right)\left(\boldsymbol{\sigma}_{n'} \cdot \hat{\boldsymbol{p}}\right) - \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_{n'}$$

Results in Simkovic's formulation

(in dimensionless)

Matrix elements for $^{82}Se \rightarrow {}^{82}Kr$

Model	M_F	M_{GT}	M_T	$M^{(0\nu)}$
Shell model (this work)	**−1.380**	**2.184**	**−0.208**	**2.860**
SDGH-pairs (this work)	**−1.760**	**2.220**	**−0.262**	**3.084**
SDG-pairs (this work)	**−2.106**	**2.582**	**−0.310**	**3.620**
SD-pairs (this work)	**−2.296**	**2.300**	**−0.356**	**3.412**
S-pairs (this work)	**−3.284**	**4.084**	**−0.436**	**5.748**
IBM-2	−0.60	3.59	−0.23	3.73
QRPA	−1.531	4.207	−0.516	4.642

IBM-2: J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C, **91**, 034304 (2015)

QRPA: F. Simkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C, 87, 045501 (2013)

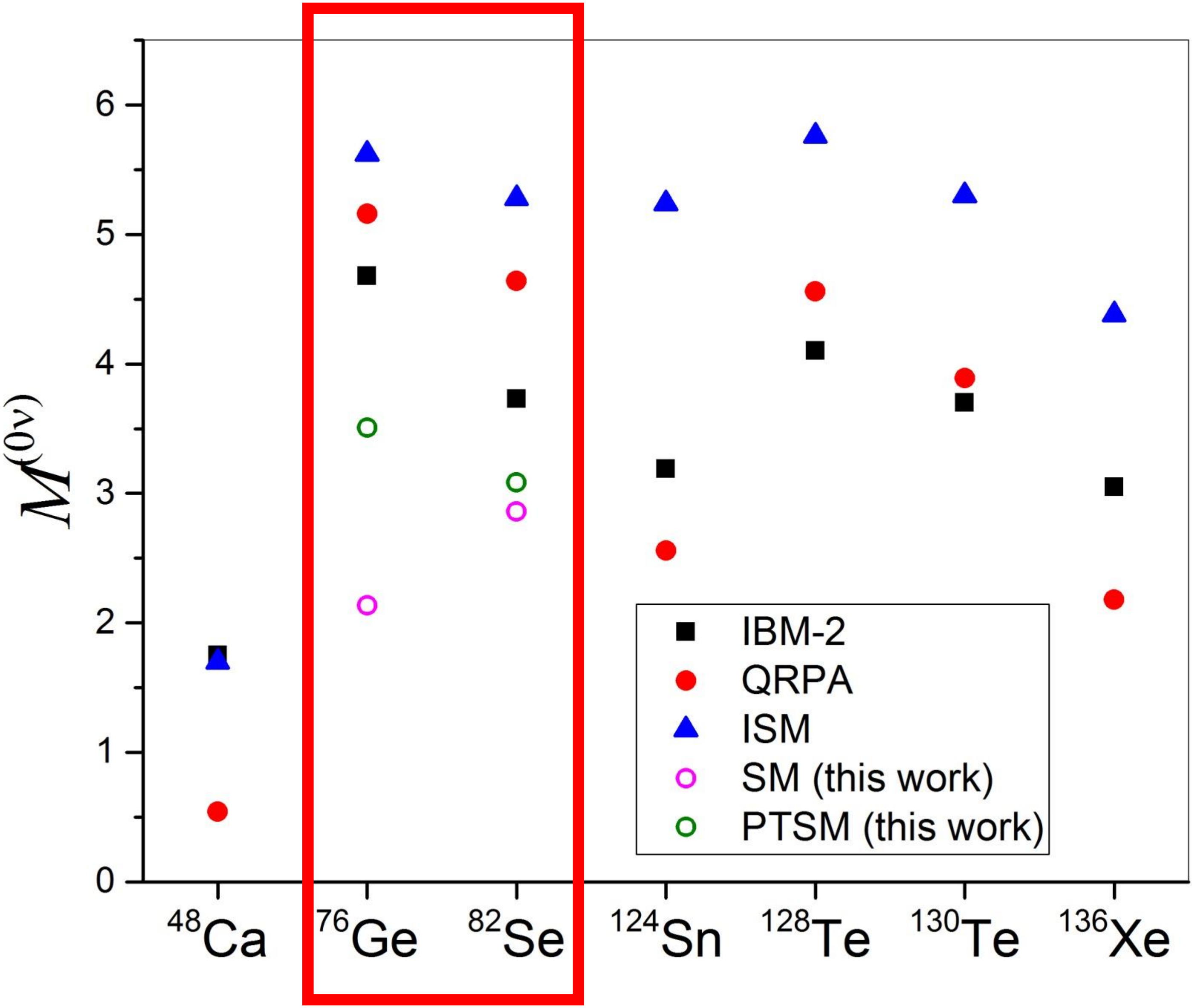
Results in Simkovic's formulation

(in dimensionless)

Matrix elements for $^{76}Ge \rightarrow {}^{76}Se$

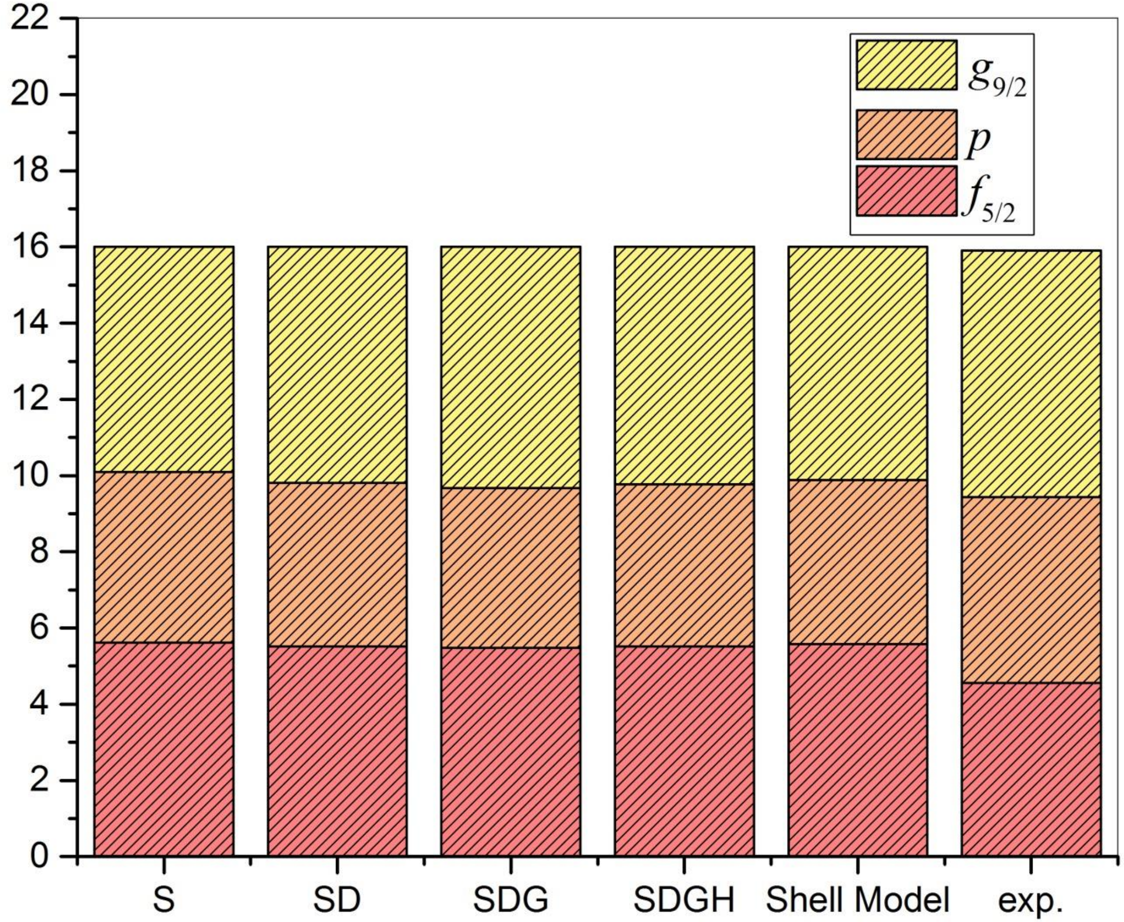
Model	M_F	M_{GT}	M_T	$M^{(0\nu)}$
Shell model (this work)	**−1.382**	**1.550**	**−0.298**	**2.136**
SDGH-pairs (this work)	**−2.380**	**2.380**	**−0.392**	**3.508**
SDG-pairs (this work)	**−2.462**	**2.456**	**−0.406**	**3.628**
SD-pairs (this work)	**−2.804**	**3.206**	**−0.420**	**4.582**
S-pairs (this work)	**−4.232**	**4.960**	**−0.580**	**7.086**
IBM-2	−0.68	4.49	−0.23	4.68
QRPA	−1.615	4.715	−0.561	5.157

Comparison (Simkovic's formulation)



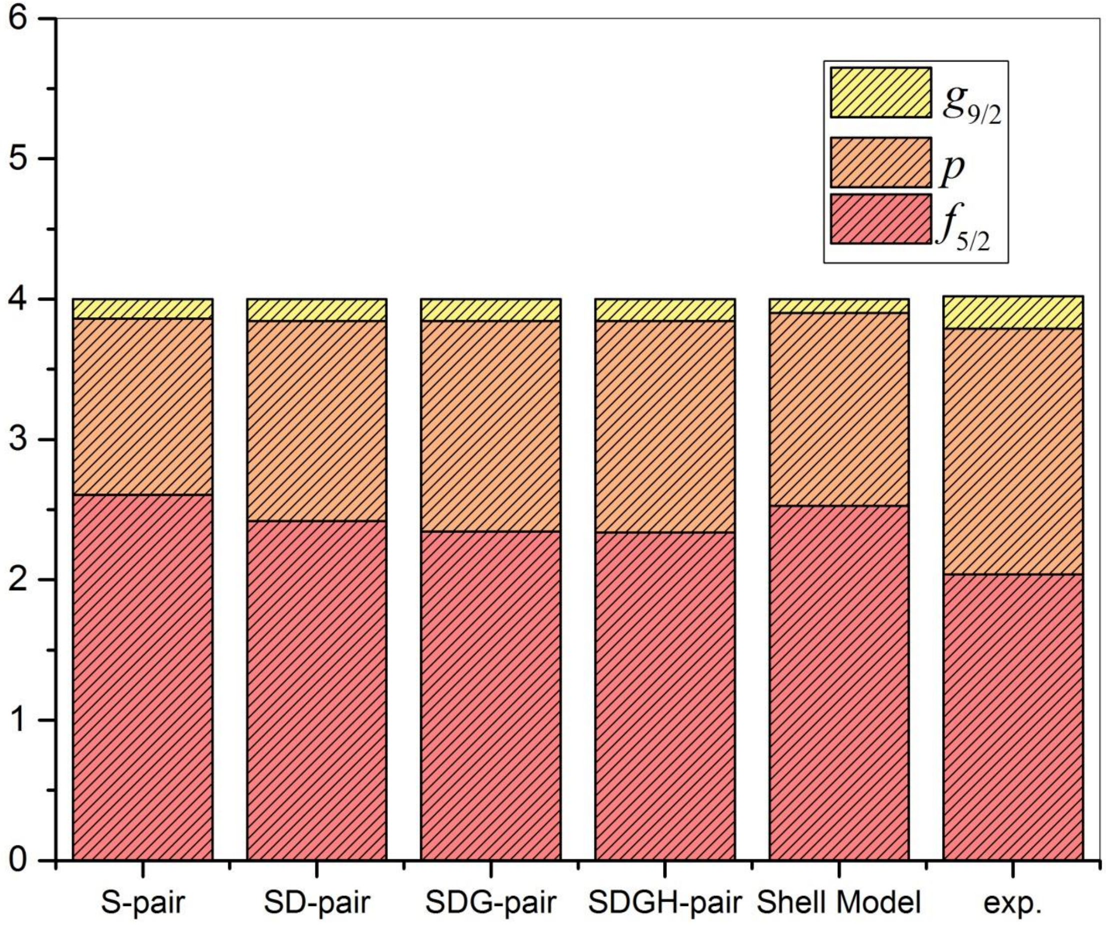
Occupancies in 0^+(^{76}Ge, neutron)

$$(\text{occupancy of orbital } j) = \left\langle {}^{76}\text{Ge}\,(0^+_{\text{g.s.}}) \left| c_j^\dagger c_j \right| {}^{76}\text{Ge}\,(0^+_{\text{g.s.}}) \right\rangle$$



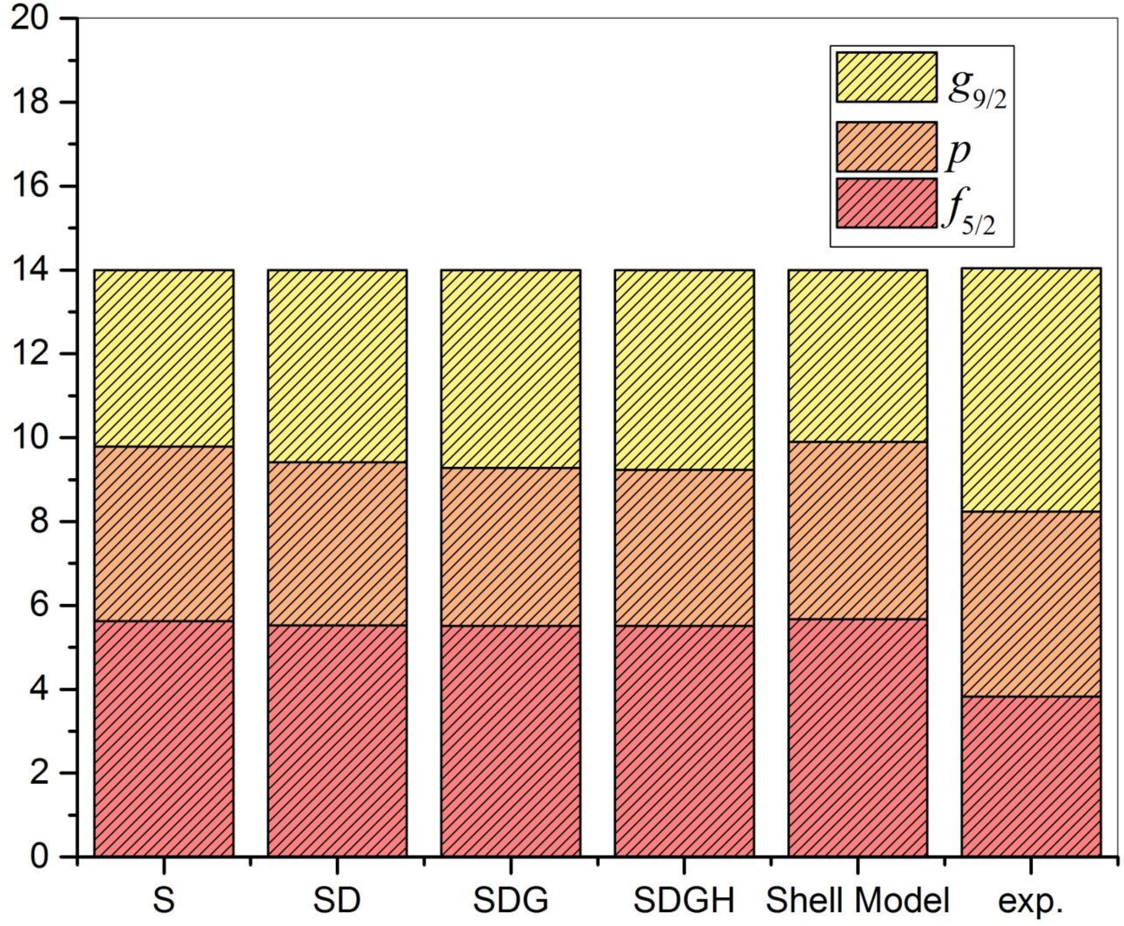
	$M^{(0\nu)}$
SM	**2.136**
SDGH	**3.508**
SDG	**3.628**
SD	**4.582**
S	**7.086**

Occupancies in 0^+ (^{76}Ge, proton)



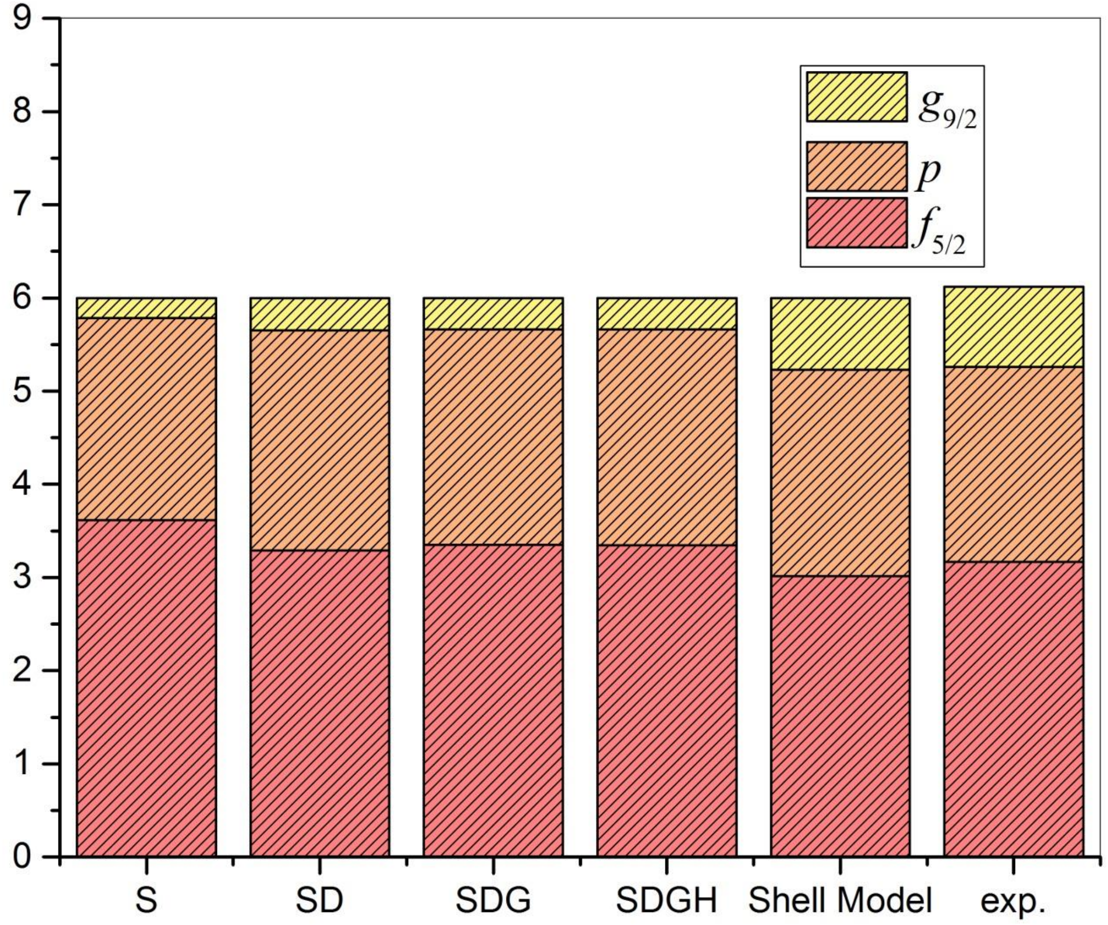
	$M^{(0\nu)}$
SM	**2.136**
SDGH	**3.508**
SDG	**3.628**
SD	**4.582**
S	**7.086**

Occupancies in 0^+ (^{76}Se, neutron)



	$M^{(0\nu)}$
SM	**2.136**
SDGH	**3.508**
SDG	**3.628**
SD	**4.582**
S	**7.086**

Occupancies in 0^+ (^{76}Se, proton)



	$M^{(0\nu)}$
SM	**2.136**
SDGH	**3.508**
SDG	**3.628**
SD	**4.582**
S	**7.086**

Summary

1) SM and PTSM calculations are performed for ^{82}Se, ^{82}Kr, ^{76}Ge, and ^{76}Se nuclei.

Good agreements between theory and experiment are obtained for the low-lying states.

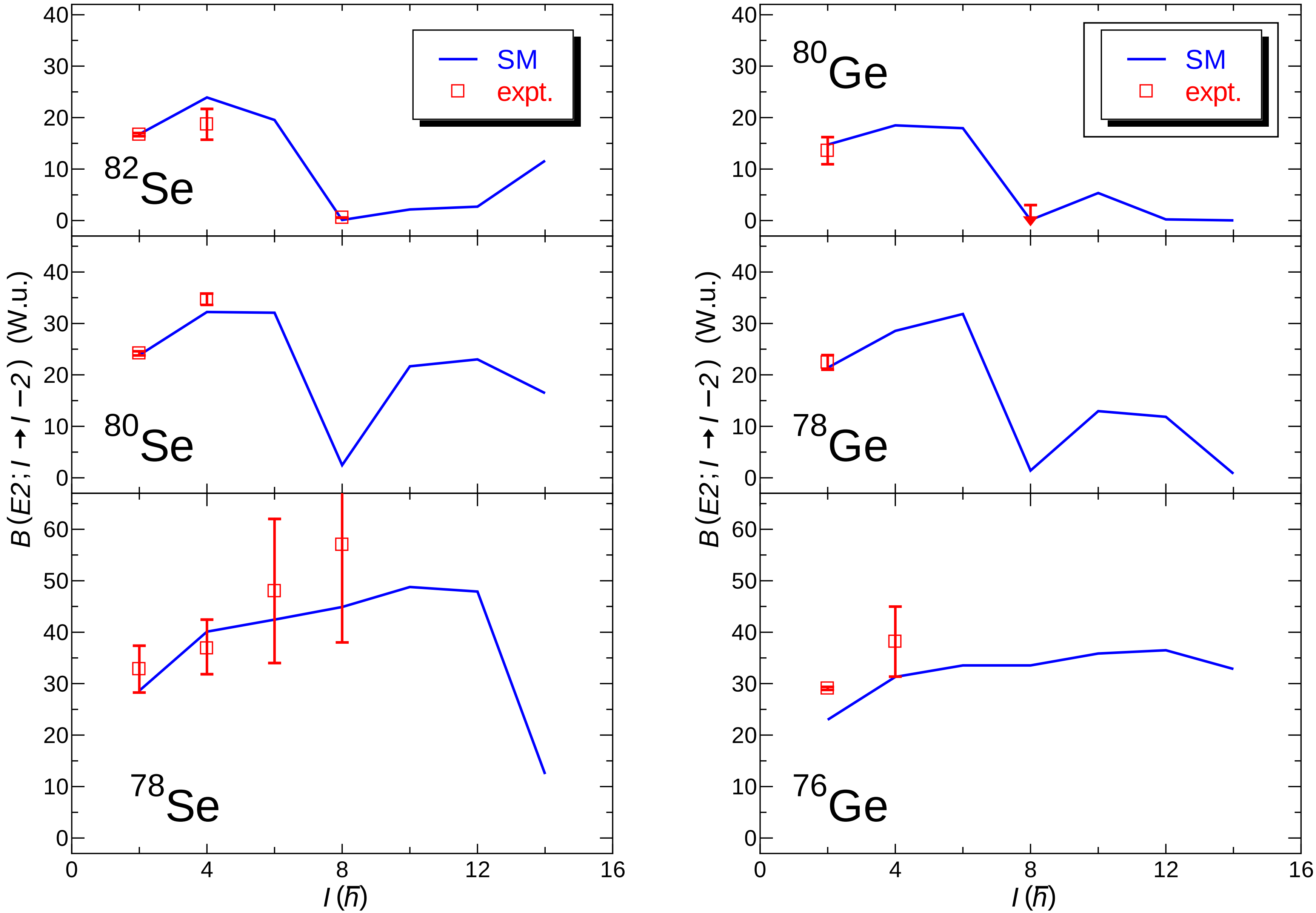
2) Using the wavefunctions thus obtained, we calculate nuclear matrix elements for $0\nu\beta\beta$ decays.

Our calculated matrix elements are smaller in magnitude compared to other models.

3) Occupancies obtained by SM and PTSM calculations in this work have close values. However, NMEs are different about 3 times at most.

Back Up

E2 transitions of yrast states



Pairing + QQ interaction

Pairing + QQ interaction :

$$H = H_\nu + H_\pi + H_{\nu\pi}$$

Table: Single-particle energies

j	$g_{9/2}$	$p_{1/2}$	$p_{3/2}$	$f_{5/2}$
ε_ν	0.0	0.5	1.0	3.0
ε_π	3.3	1.1	0.6	0.0

Neutron (Proton) interaction :

$$H_\tau = \sum_{jm} \varepsilon_{j\tau} c^\dagger_{jm\tau} c_{jm\tau} - G_{0\tau} P^{\dagger(0)}_\tau P^{(0)}_\tau - G_{2\tau} P^{\dagger(2)}_\tau \cdot \tilde{P}^{(2)}_\tau - \kappa_\tau : Q_\tau \cdot Q_\tau :$$

Neutron-Proton interaction :

$$H_{\nu\pi} = -\kappa_{\nu\pi} Q_\nu \cdot Q_\pi$$

Table: Couplings

	$G_{0\nu}$	$G_{2\nu}$	κ_ν	$G_{0\pi}$	$G_{2\pi}$	κ_π	$\kappa_{\nu\pi}$
^{82}Se	0.20	0.060	0.24	0.20	0.02	0.08	−0.20
^{82}Kr	0.22	0.045	0.22	0.20	0.00	0.10	−0.20
^{76}Ge	0.34	0.095	0.19	0.22	0.03	0.14	−0.20
^{76}Se	0.32	0.065	0.07	0.18	0.03	0.21	−0.20

Neutrino potentials

$$h_{VV}^{F}(p)=\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}\frac{g_V^2}{(1+p^2/M_V^2)^4},$$

$$h_{AA}^{GT}(p)=\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}\frac{1}{(1+p^2/M_A^2)^4},$$

$$h_{AP}^{GT}(p)=\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}\left[-\frac{2}{3}\frac{1}{(1+p^2/M_A^2)^4}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right],$$

$$h_{PP}^{GT}(p)=\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}\left[\frac{1}{\sqrt{3}}\frac{1}{(1+p^2/M_A^2)^2}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]^2,$$

$$h_{MM}^{GT}(p)=\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}\left[\frac{2}{3}\frac{g_V^2}{g_A^2}\frac{1}{(1+p^2/M_V^2)^4}\frac{\kappa_\beta^2 p^2}{4m_p^2}\right],$$

$$h_{AP}^{T}(p)=-3h_{AP}^{GT}(p),\quad h_{PP}^{T}(p)=-3h_{PP}^{GT}(p),\quad h_{MM}^{T}(p)=\frac{3}{2}h_{MM}^{GT}(p).$$

Cutoff parameters:

$$M_V^2=0.71\ \mathrm{GeV}^2,\qquad M_A=1.09\ \mathrm{GeV}$$

Finite size & Short range correlation

Finite nucleon size effect:

$$g_V(p) = \frac{1}{(1 + p^2/M_V^2)^2}$$

$$g_A(p) = \frac{1}{(1 + p^2/M_A^2)^2}$$

Cutoff parameters:

$$M_V^2 = 0.71\ \mathrm{GeV}^2, \qquad M_A = 1.09\ \mathrm{GeV}$$

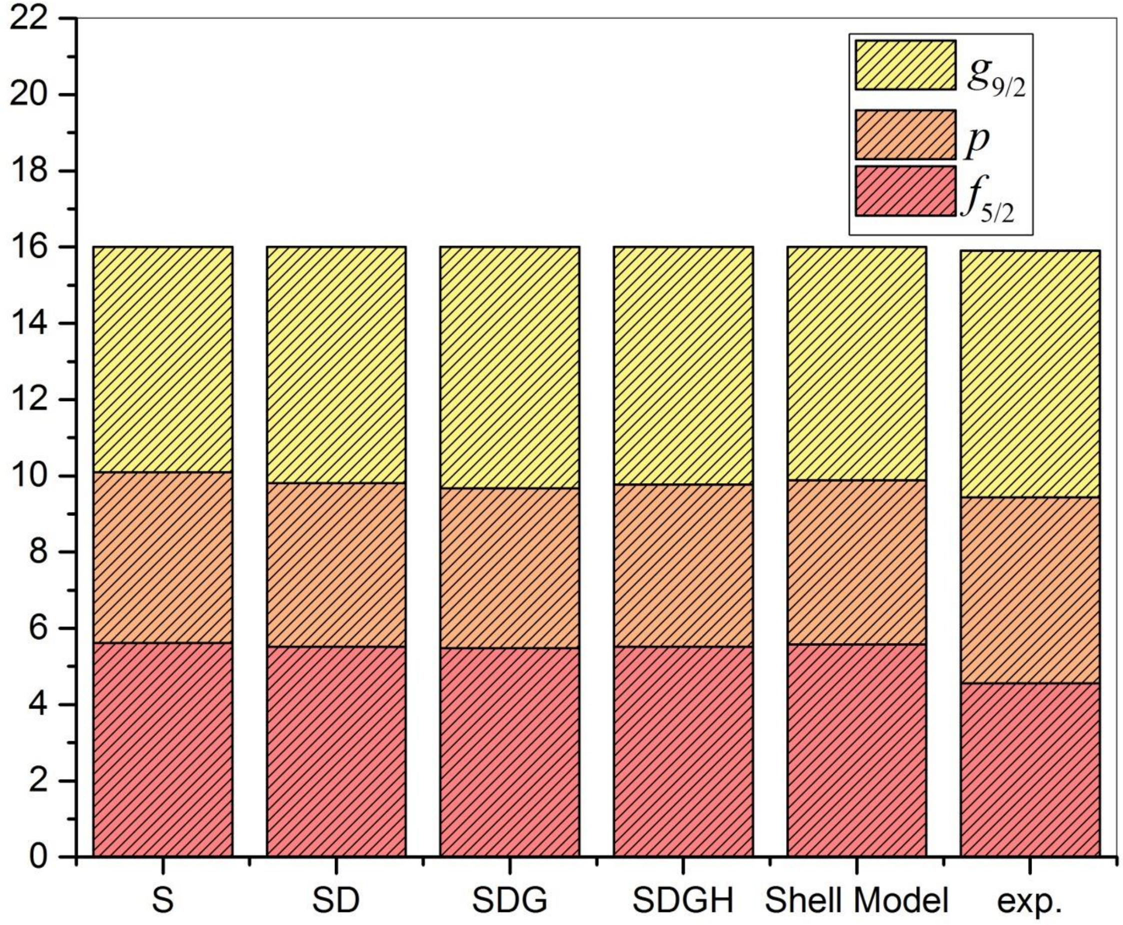
Short range correlation: $f(r)^2$

$$f(r) = 1 - e^{-ar^2}(1 - br^2)$$

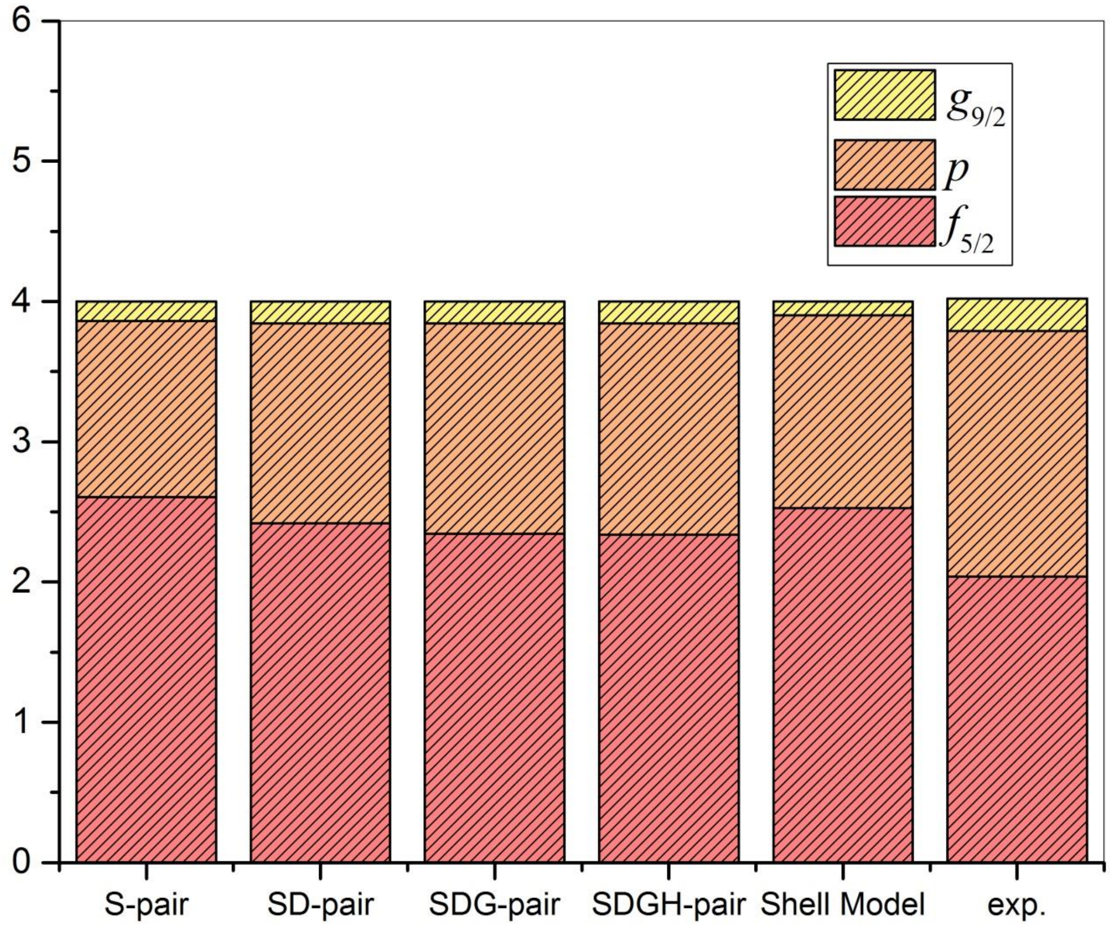
$$a = 1.1\ \mathrm{fm}^{-2}, \quad b = 0.68\ \mathrm{fm}^{-2}$$

Occupancies in 0^+(^{76}Ge, neutron)

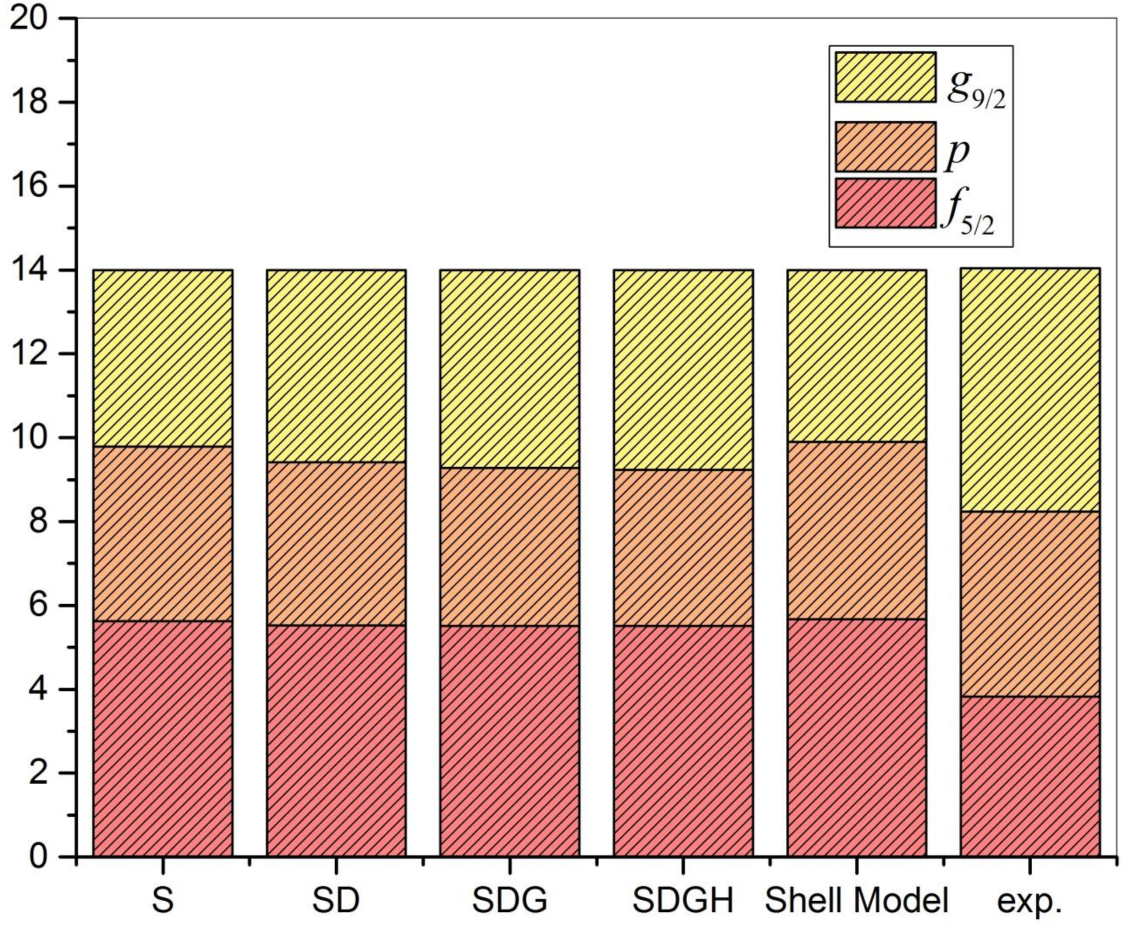
$$(\text{occupancy of orbital } j) = \left\langle {}^{76}\mathrm{Ge}\left(0^+_{\mathrm{g.s.}}\right) \left| c_j^\dagger c_j \right| {}^{76}\mathrm{Ge}\left(0^+_{\mathrm{g.s.}}\right) \right\rangle$$



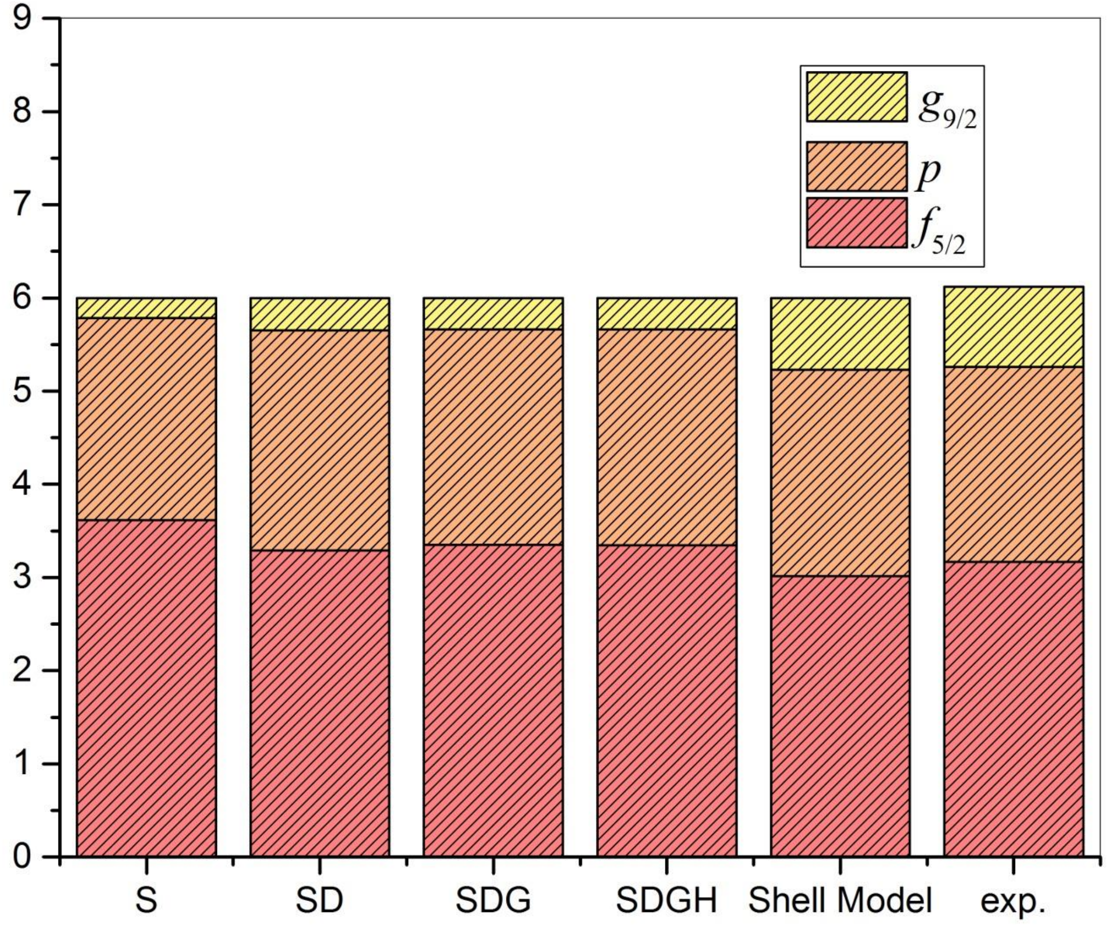
Occupancies in 0^+ (^{76}Ge, proton)



Occupancies in 0^+ (^{76}Se, neutron)



Occupancies in 0^+ (^{76}Se, proton)



	$M^{(0\nu)}$
SM	**2.136**
SDGH	**3.508**
SDG	**3.628**
SD	**4.582**
S	**7.086**