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Accessing high momentum nucleons in lattice QCD

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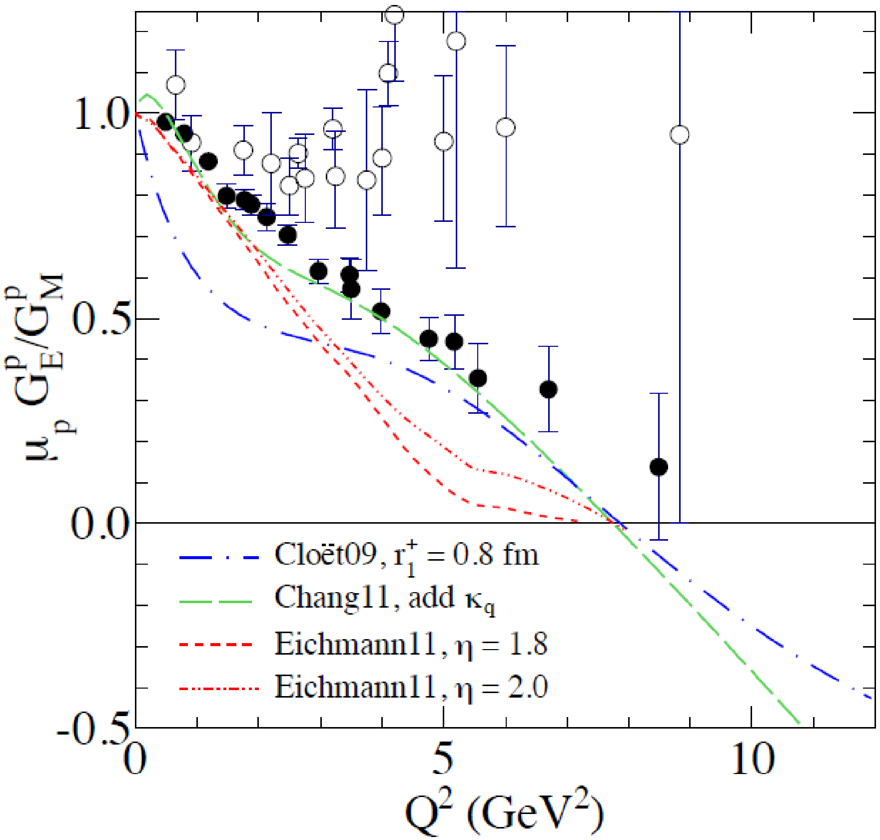
Outline

- Motivation
- Z3 Noise Dilution Source
- Momentum phase in the smearing
- Comparison
- Outlook

Motivation

- Form factor is important for us to understand the properties of hadron.

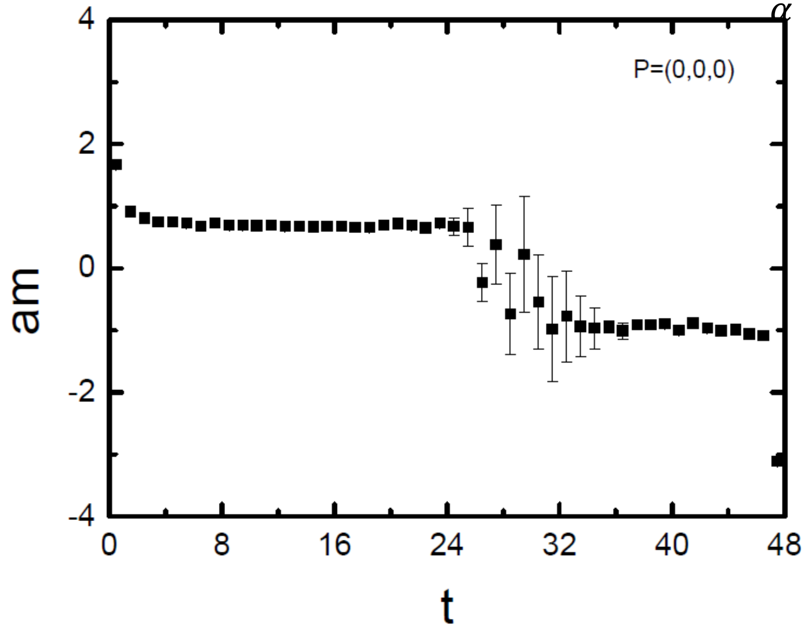


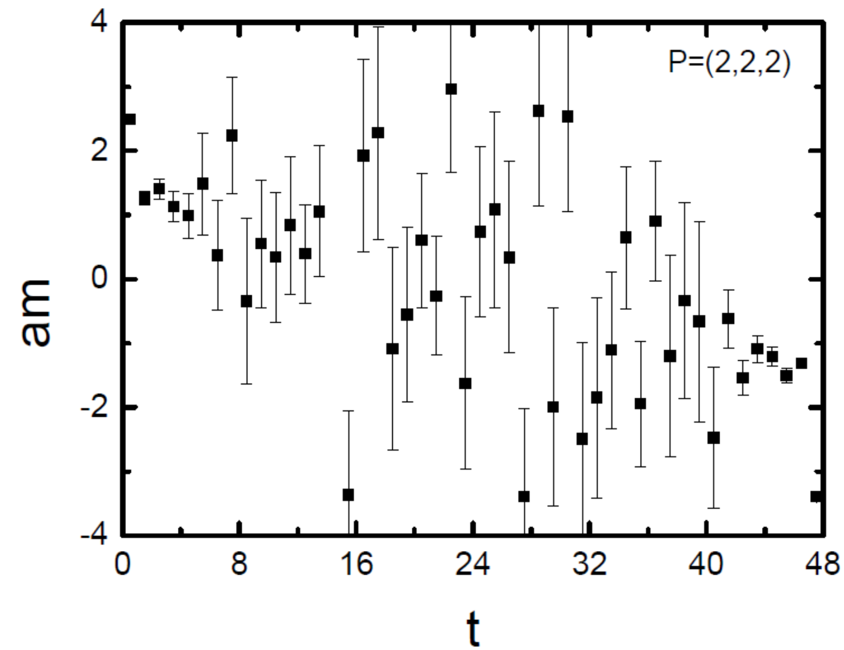
JLab, Hall A, PRC85 (2012) 045203

Motivation

- The form factor extracted from Lattice need the information of high momentum hadron.
- The high momentum spectrum suffers a large error.

$$G(\vec{p};t) \sim \sum_{\alpha} e^{-E_\alpha t} \qquad V = 24^3 \times 48$$





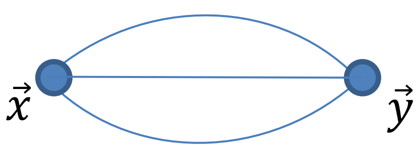
Correlation function of Proton

- Operator for Proton:

$$\chi = \epsilon^{abc}\left(u^{aT} C\gamma_5 d^b\right)u^c$$

- The correlation function:

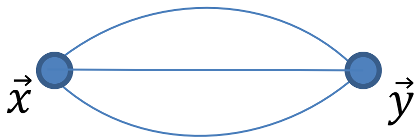
$$G(\vec{p};\vec{x},t) = \sum_{\vec{y}} \Gamma e^{i\vec{p}.(\vec{y}-\vec{x})}\langle\Omega|\chi(\vec{y},t)\bar{\chi}(\vec{x},0)|\Omega\rangle$$



$$= \sum_{\vec{y}} e^{i\vec{p}.(\vec{y}-\vec{x})}\mathrm{f}\left(S(\vec{y},\vec{x}),S(\vec{y},\vec{x}),S(\vec{y},\vec{x})\right)$$

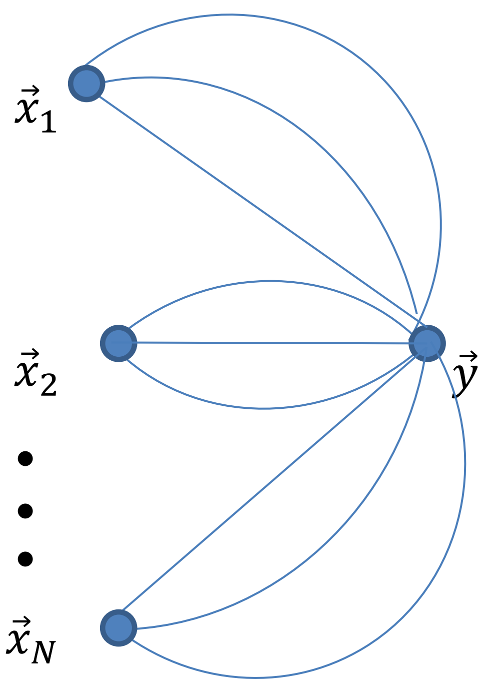
Only one source location is calculated

Single Source



$$e^{i\vec{p}.(\vec{y}-\vec{x})}\mathrm{f}\big(S(\vec{y},\vec{x}),S(\vec{y},\vec{x}),S(\vec{y},\vec{x})\big)$$

1 source location



$$\sum_{i=1,N} e^{i\vec{p}.(\vec{y}-\vec{x}_i)}\mathrm{f}\big(S(\vec{y},\vec{x}_i),S(\vec{y},\vec{x}_i),S(\vec{y},\vec{x}_i)\big)$$

N source location

Problem: Cost more inversions

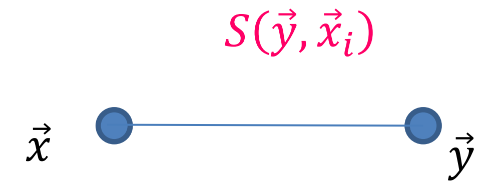
Z3 Noise Dilution Source

- Z3 noise vector:

  $\boldsymbol{\eta}(\vec{x}_n) = e^{i\,2r(n)\boldsymbol{\pi}/3}$; n = 0, 1, 2, ….;

  r(n) is random number of "0,1,2"

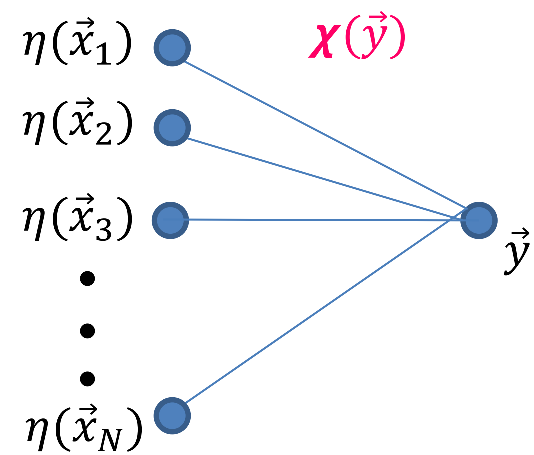
  $\langle \boldsymbol{\eta}(\vec{x}_i)\boldsymbol{\eta}(\vec{x}_j)\boldsymbol{\eta}(\vec{x}_k)\rangle = \delta_{ij}\delta_{jk}$



- Dilution Source:

  pick out N source locations

$$\boldsymbol{\chi}(\vec{y}) = \sum_{i=1,N} \boldsymbol{\eta}(\vec{x}_i) S(\vec{y},\vec{x}_i)$$



- The correlation function:(at rest $\vec{p} = 0$)

$$G_N(\vec{0};t) = \sum_{\vec{y}} \mathrm{f}\big(\chi(\vec{y}),\chi(\vec{y}),\chi(\vec{y})\big)$$

Including information of N source locations, still only one inversion, but in the correlation function it will bring a lot noise terms.

Z3 Noise Dilution Source

- The correlation function:(at rest $\vec{p} = 0$)

$$G_N(\vec{p}; t)$$

$$= \sum_{\vec{y}} \sum_{i=1,N} f\big(S(\vec{y}, \overrightarrow{x_i}), S(\vec{y}, \overrightarrow{x_i}), S(\vec{y}, \overrightarrow{x_i})\big)$$

$$+ \sum_{\vec{y}} \sum_{i,j,k=1,N} \boldsymbol{\eta}(\overrightarrow{x_i})\boldsymbol{\eta}(\overrightarrow{x_j})\boldsymbol{\eta}(\overrightarrow{x_k})(1 - \delta_{ij}\delta_{ik}) f\left(S(\vec{y}, \overrightarrow{x_i}), S\big(\vec{y}, \overrightarrow{x_j}\big), S(\vec{y}, \overrightarrow{x_k})\right)$$

Z3 Noise Dilution Source

- The correlation function:(at rest $\vec{p} = 0$)

$$G_N(\vec{p};t)$$

$$= \sum_{\vec{y}} \sum_{i=1,N} f\left(S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i})\right)$$

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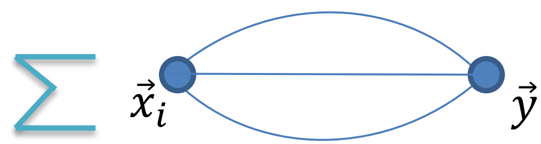
Signal Term's Error: σ_S



Z3 Noise Dilution Source

- The correlation function:(at rest $\vec{p} = 0$)

$$G_N(\vec{p};t)$$

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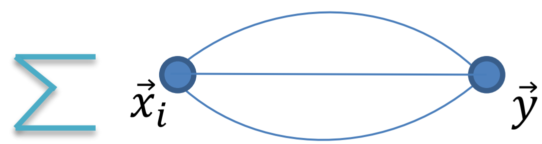
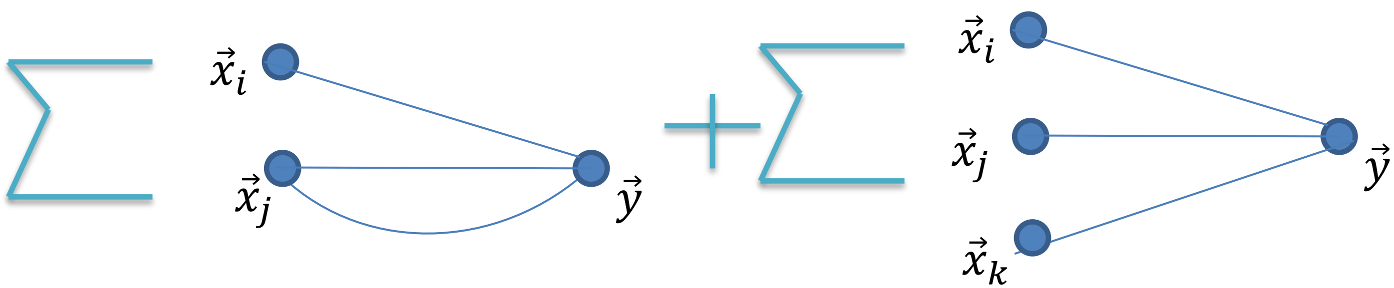
$$+ \sum_{\vec{y}} \sum_{i,j,k=1,N} \boldsymbol{\eta}(\vec{x_i})\boldsymbol{\eta}(\vec{x_j})\boldsymbol{\eta}(\vec{x_k})(1-\delta_{ij}\delta_{ik}) f\left(S(\vec{y},\vec{x_i}), S(\vec{y},\vec{x_j}), S(\vec{y},\vec{x_k})\right)$$

Signal Term's Error: σ_S



Noise Term's Error: $\sigma_N \propto N(N^2-1)$ and $(1-\delta_{ij}\delta_{ik}) f\left(S(\vec{y},\vec{x_i}), S(\vec{y},\vec{x_j}), S(\vec{y},\vec{x_k})\right)$



Z3 Noise Dilution Source

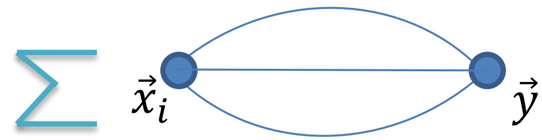
- The correlation function:(at rest $\vec{p}=0$)

$$G_N(\vec{p};t)$$

$$=\sum_{\vec{y}}\sum_{i=1,N} f\left(S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i})\right)$$

$$+\sum_{\vec{y}}\sum_{i,j,k=1,N} \boldsymbol{\eta}(\overrightarrow{x_i})\boldsymbol{\eta}(\overrightarrow{x_j})\boldsymbol{\eta}(\overrightarrow{x_k})(1-\delta_{ij}\delta_{ik})f\left(S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_j}),S(\vec{y},\overrightarrow{x_k})\right)$$

Signal Term's Error: σ_S



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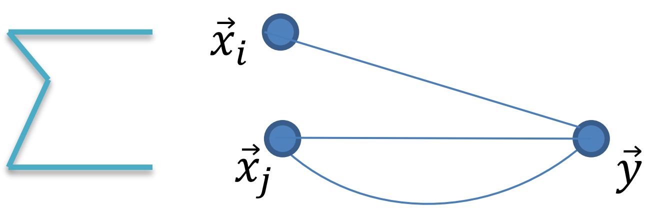
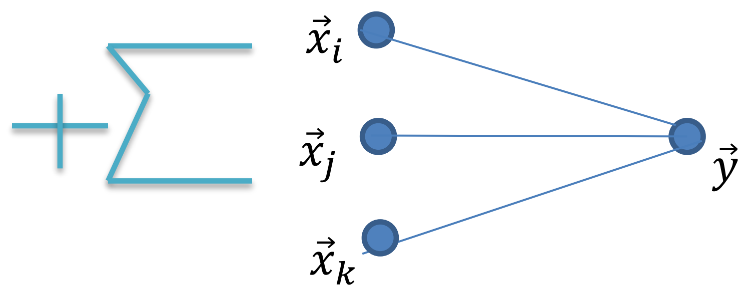




$G_N(\vec{0};t)$ σ_S^2 σ_N^2

$G(\vec{0};\overrightarrow{x_i},t)$ σ_C^2

Z3 Noise Dilution Source

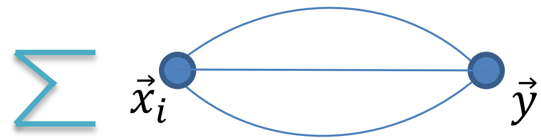
- The correlation function:(at rest $\vec{p} = 0$)

$$G_N(\vec{p};t)$$

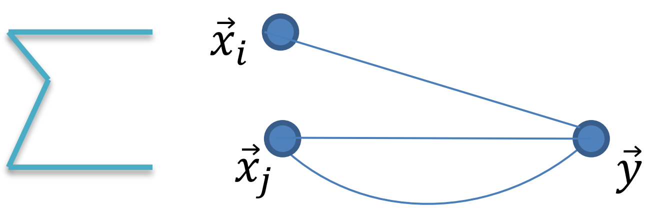
$$= \sum_{\vec{y}} \sum_{i=1,N} f\left(S(\vec{y},\vec{x_i}),S(\vec{y},\vec{x_i}),S(\vec{y},\vec{x_i})\right)$$

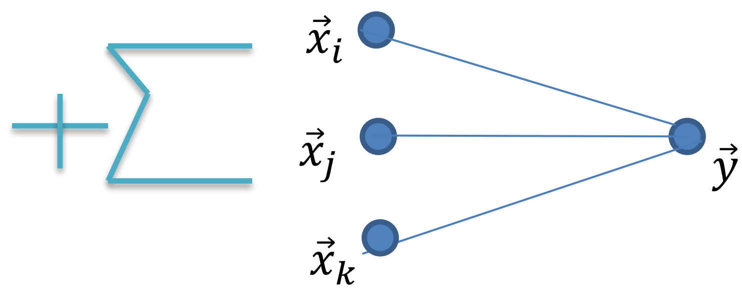
$$+ \sum_{\vec{y}} \sum_{i,j,k=1,N} \boldsymbol{\eta}(\vec{x_i})\boldsymbol{\eta}(\vec{x_j})\boldsymbol{\eta}(\vec{x_k})(1-\delta_{ij}\delta_{ik}) f\left(S(\vec{y},\vec{x_i}),S(\vec{y},\vec{x_j}),S(\vec{y},\vec{x_k})\right)$$

Signal Term's Error: σ_S



Noise Term's Error: $\sigma_N \propto N(N^2-1)$ and $(1-\delta_{ij}\delta_{ik}) f\left(S(\vec{y},\vec{x_i}),S(\vec{y},\vec{x_j}),S(\vec{y},\vec{x_k})\right)$





$G_N(\vec{0};t)$ σ_S^2 σ_N^2

$G(\vec{0};\vec{x_i},t)$ σ_c^2 $\sigma_S^2 + \sigma_N^2 < \sigma_c^2$

- $f\left(S(\vec{y},\vec{x_i}),S(\vec{y},\vec{x_j}),S(\vec{y},\vec{x_k})\right)$ smaller and smaller
- N can not be too large, i.e., Dilution

Z3 Noise Dilution Source

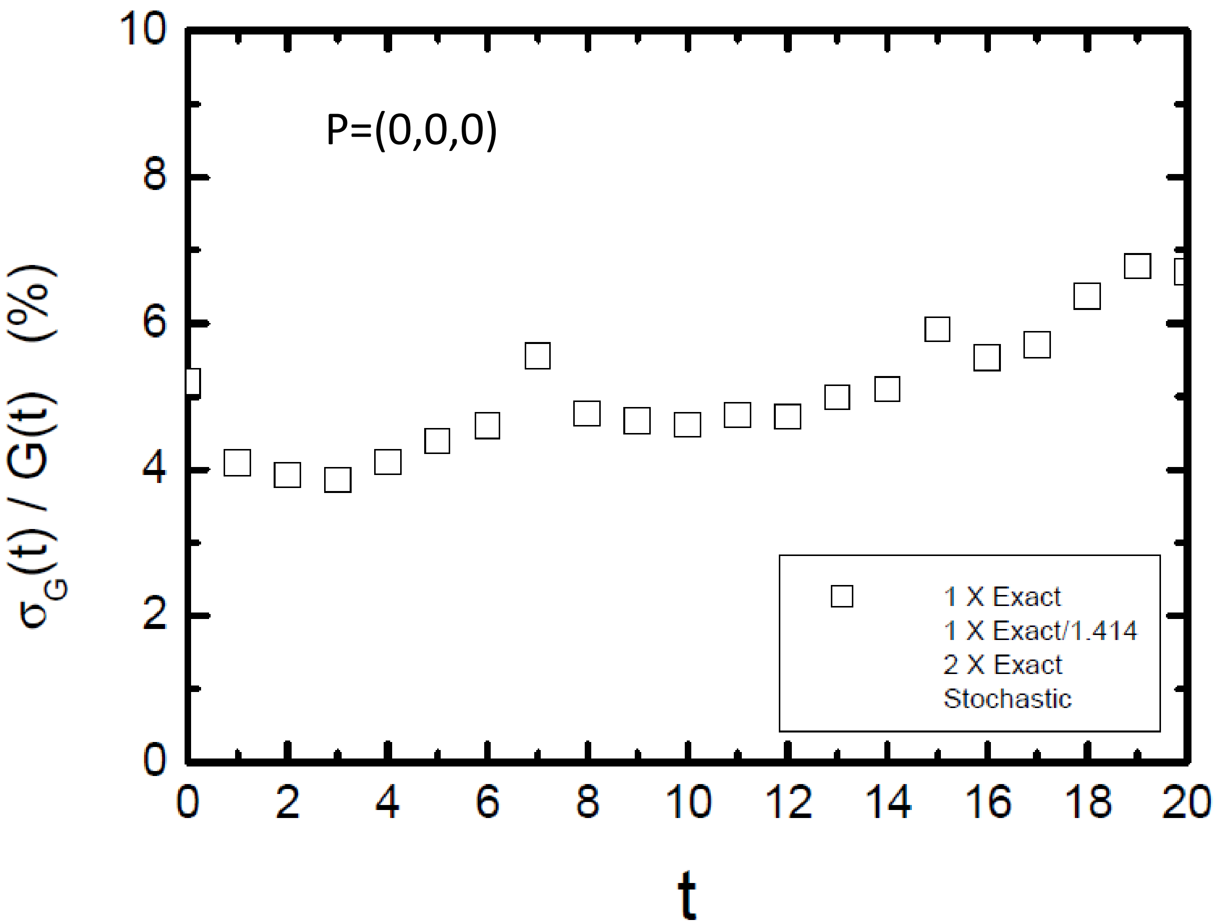
$\sigma_s^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\overrightarrow{x_i}), S(\vec{y},\overrightarrow{x_j}), S(\vec{y},\overrightarrow{x_k})\right)$ smaller and smaller

N can not be too large, i.e., Dilution

$V = 24^3 \times 48$

$\vec{x} = (0,0,0)$



Z3 Noise Dilution Source

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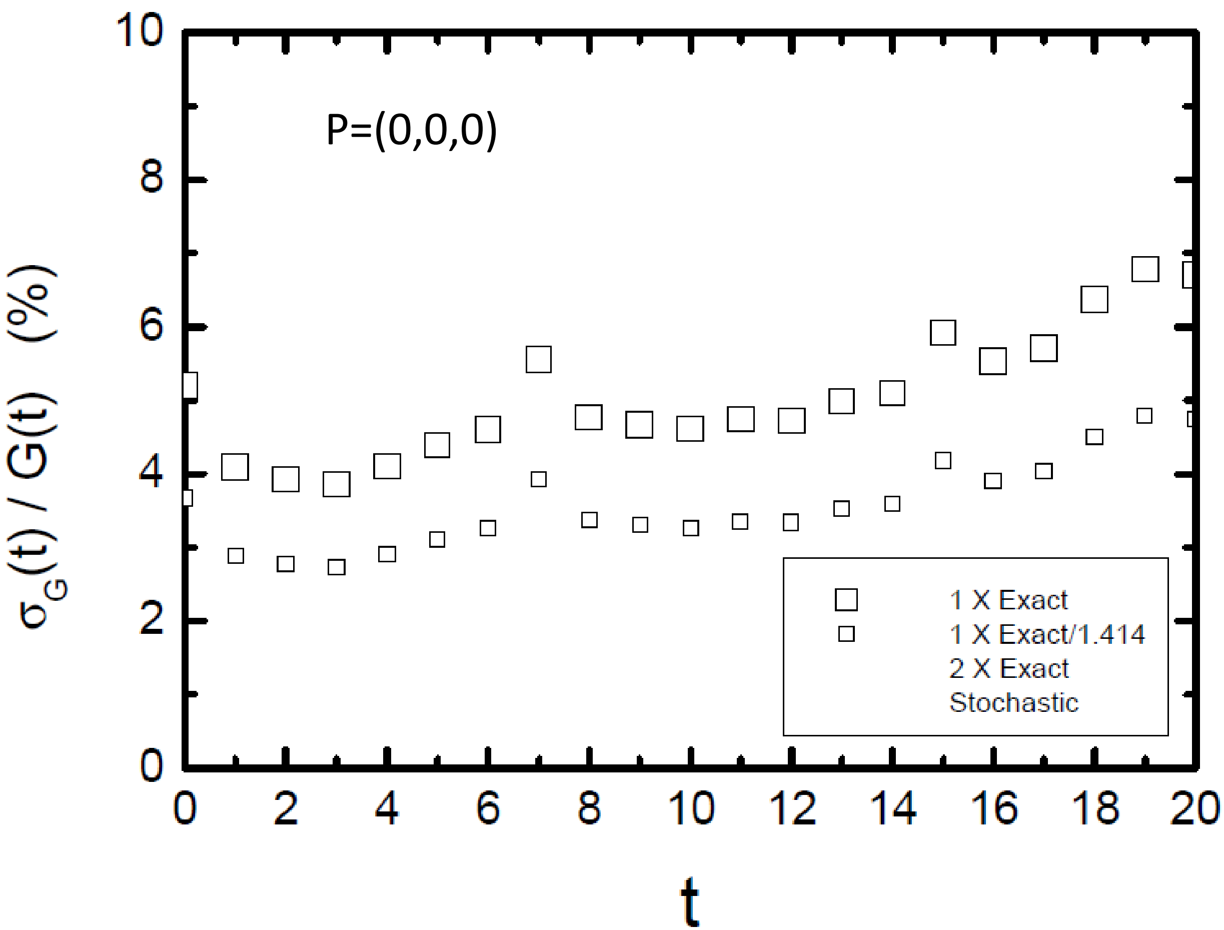

$V = 24^3 \times 48$

$\vec{x} = (0,0,0) \quad (2,2,2)$

Z3 Noise Dilution Source

$\sigma_s^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\vec{x_i}), S(\vec{y},\vec{x_j}), S(\vec{y},\vec{x_k})\right)$ smaller and smaller

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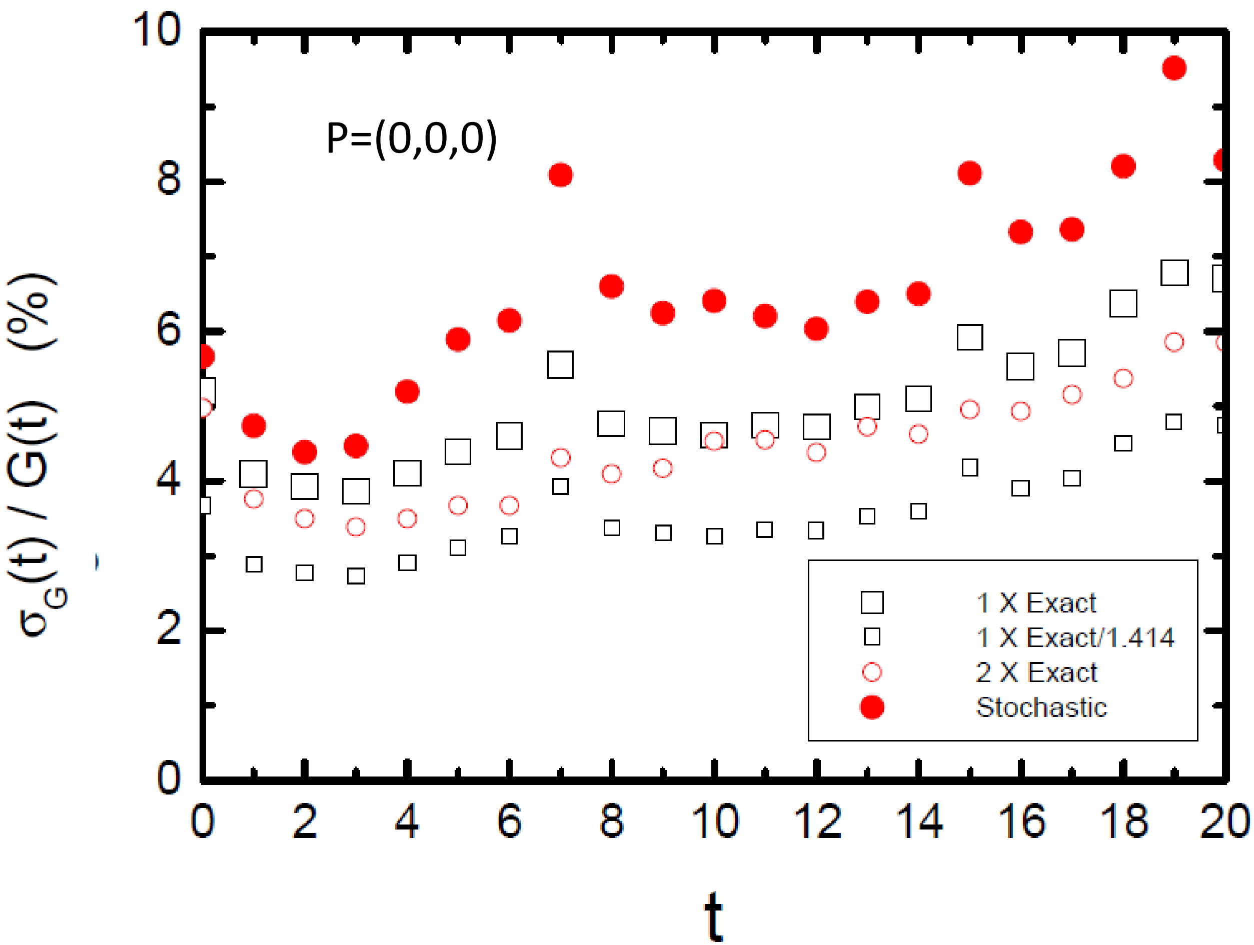

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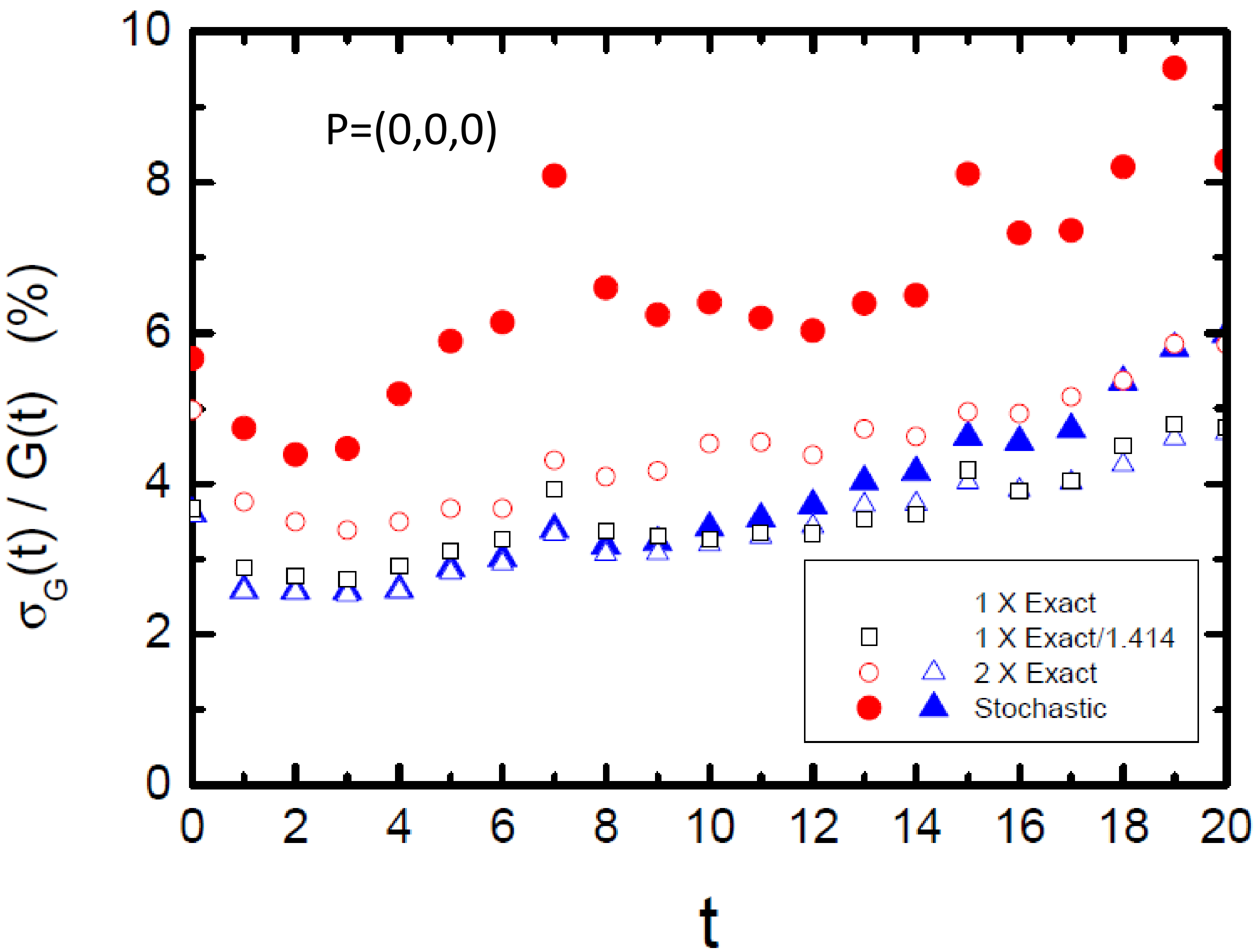
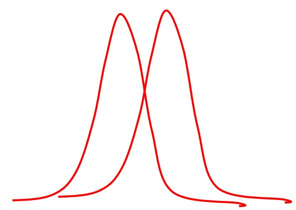
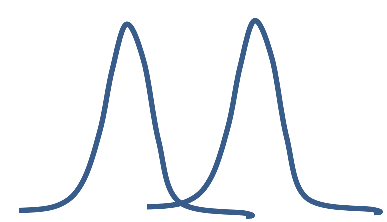


$V = 24^3 \times 48$



$\vec{x} = (0,0,0) \quad (2,2,2)$



$\vec{x} = (0,0,0) \quad (12,12,12)$

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Z3 Noise Dilution Source

$\sigma_S^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_j}),S(\vec{y},\overrightarrow{x_k})\right)$ smaller and smaller

N can not be too large, i.e., Dilution

- The correlation function:(at rest $\vec{p}=0$)

$$G_N(\vec{p};t)$$

$$=\sum_{i=1,N} f\left(S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i}),S(\vec{y},\overrightarrow{x_i})\right)$$

$$+\sum_{i,j,k=1,N} \boldsymbol{\eta}(\overrightarrow{x_i})\boldsymbol{\eta}(\overrightarrow{x_j})\boldsymbol{\eta}(\overrightarrow{x_k})(1-\delta_{ij}\delta_{ik})f\left(S(\vec{y},\overrightarrow{x_i}),S\left(\vec{y},\overrightarrow{x_j}\right),S(\vec{y},\overrightarrow{x_k})\right)$$

Signal Terms: $\propto$ N

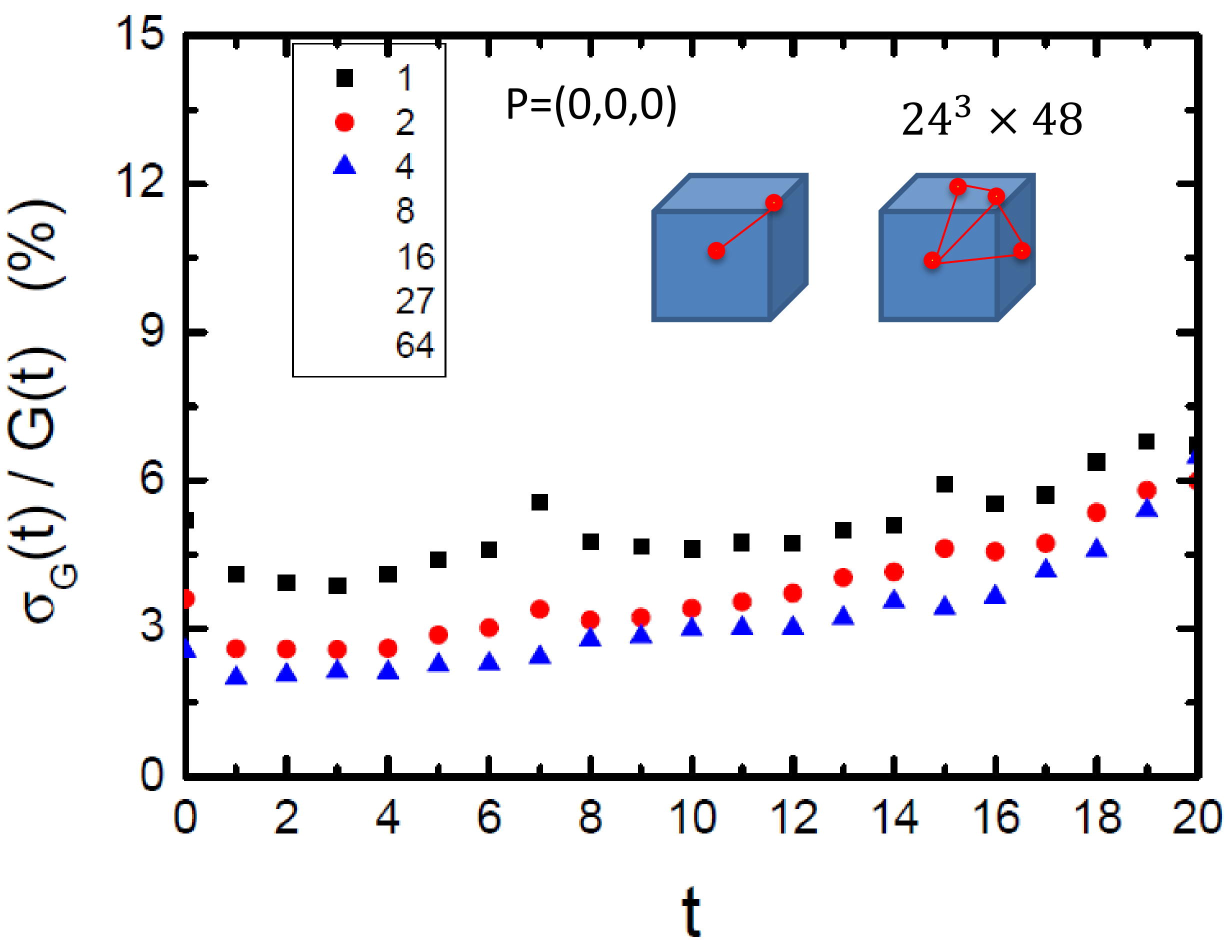
Noise Terms: $\propto$ N(N^2-1)

Z3 Noise Dilution Source

$\sigma_S^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\vec{x_i}), S(\vec{y},\vec{x_j}), S(\vec{y},\vec{x_k})\right)$ smaller and smaller

N can not be too large, i.e., Dilution



Signal Terms: ∝ N

Noise Terms: ∝ $N(N^2-1)$

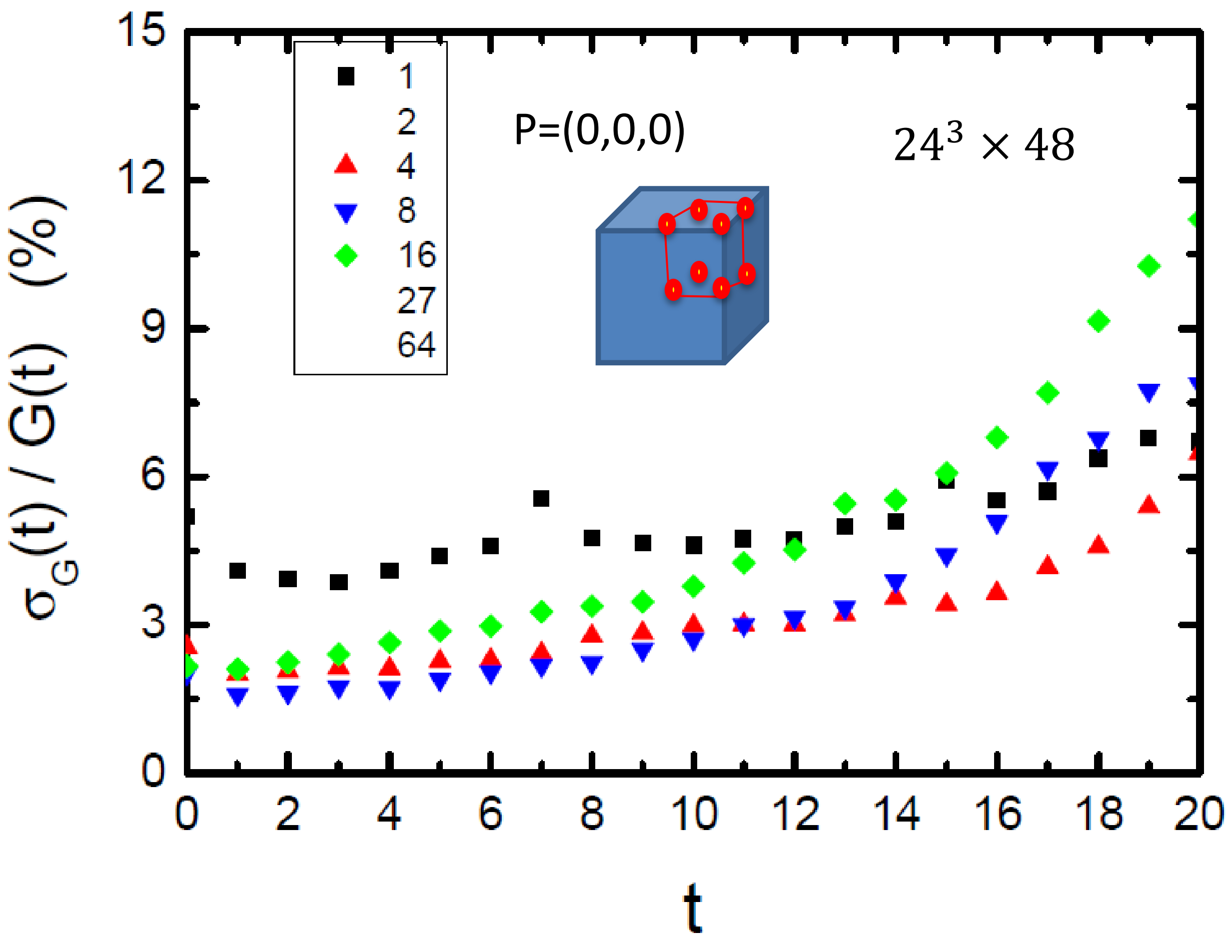
4 points better than
2 points better than
1 point

Z3 Noise Dilution Source

$\sigma_S^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\overrightarrow{x_i}), S(\vec{y},\overrightarrow{x_j}), S(\vec{y},\overrightarrow{x_k})\right)$ smaller and smaller

N can not be too large, i.e., Dilution



Signal Terms: $\propto$ N

Noise Terms: $\propto$ N(N^2-1)

4 points better than
2 points better than
1 point

8 points almost the same as 4 points

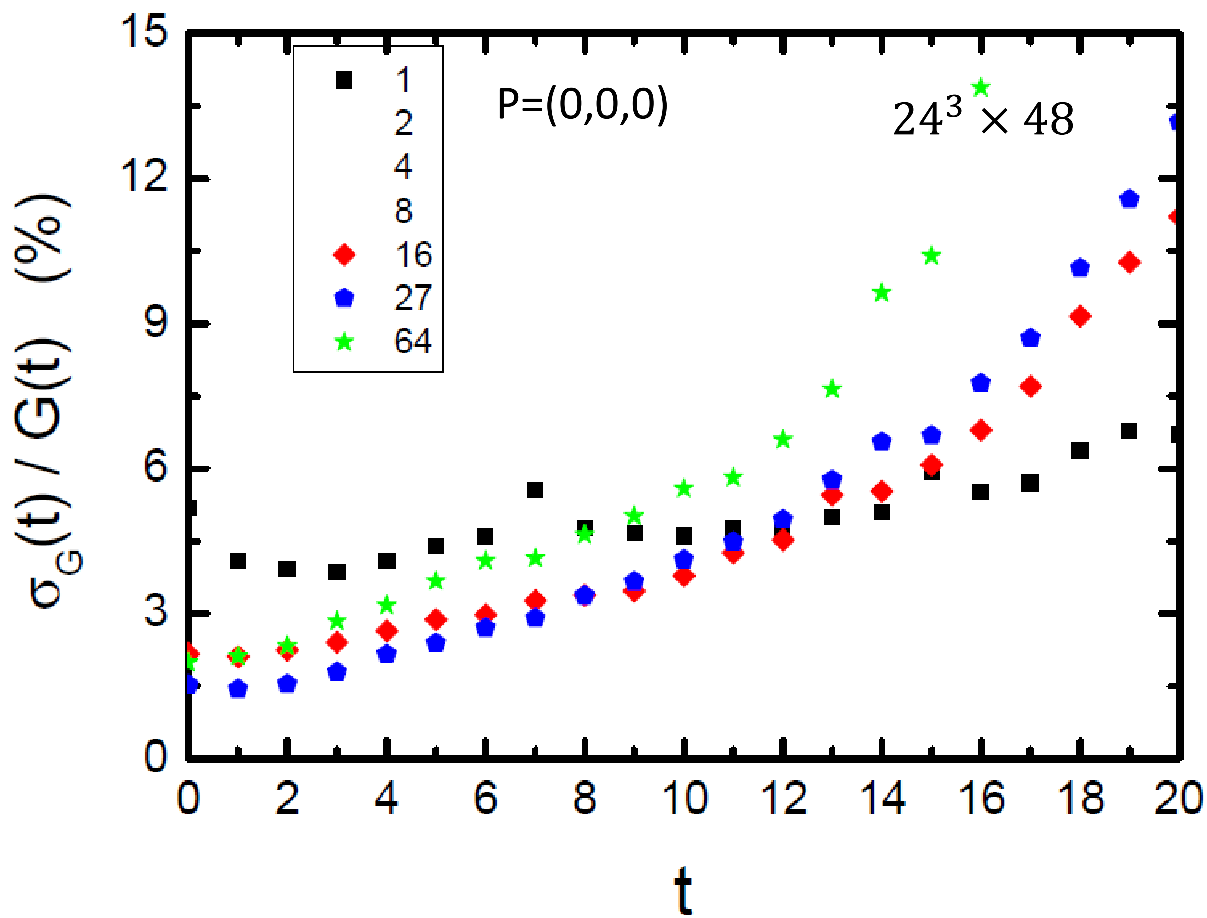
16 points worse than 8 points

Z3 Noise Dilution Source

$\sigma_S^2 + \sigma_N^2 < \sigma_c^2$

$f\left(S(\vec{y},\overrightarrow{x_i}), S(\vec{y},\overrightarrow{x_j}), S(\vec{y},\overrightarrow{x_k})\right)$ smaller and smaller

N can not be too large, i.e., Dilution



Signal Terms: $\propto$ N

Noise Terms: $\propto N(N^2-1)$

4 points better than
2 points better than
1 point

8 points almost the same as 4 points

64 points worse than
27 points worse than
16 points worse than
8 points

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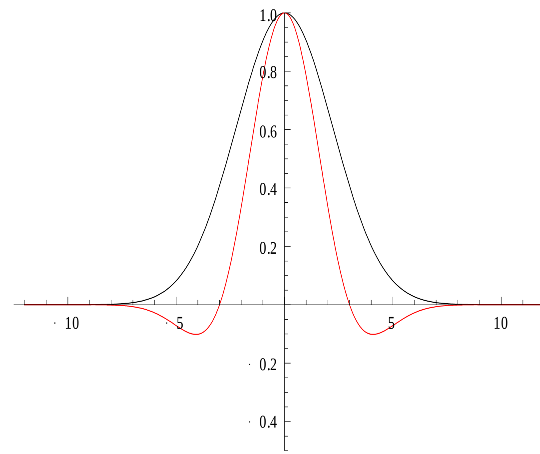
Momentum phase in the smearing

Gunnar S. Bali, Bernhard Lang, Bernhard U. Musch, and Andreas Schäfer   PRD 93, 094515 (2016)

- Source and Sink Smearing:

$$S^{smearing}(\vec{y},\vec{x}) = \sum_{\vec{x}_i,\vec{y}_i} f(\vec{y}_i - \vec{y}) f^*(\vec{x}_i - \vec{x})\, S(\vec{y},\vec{x})$$

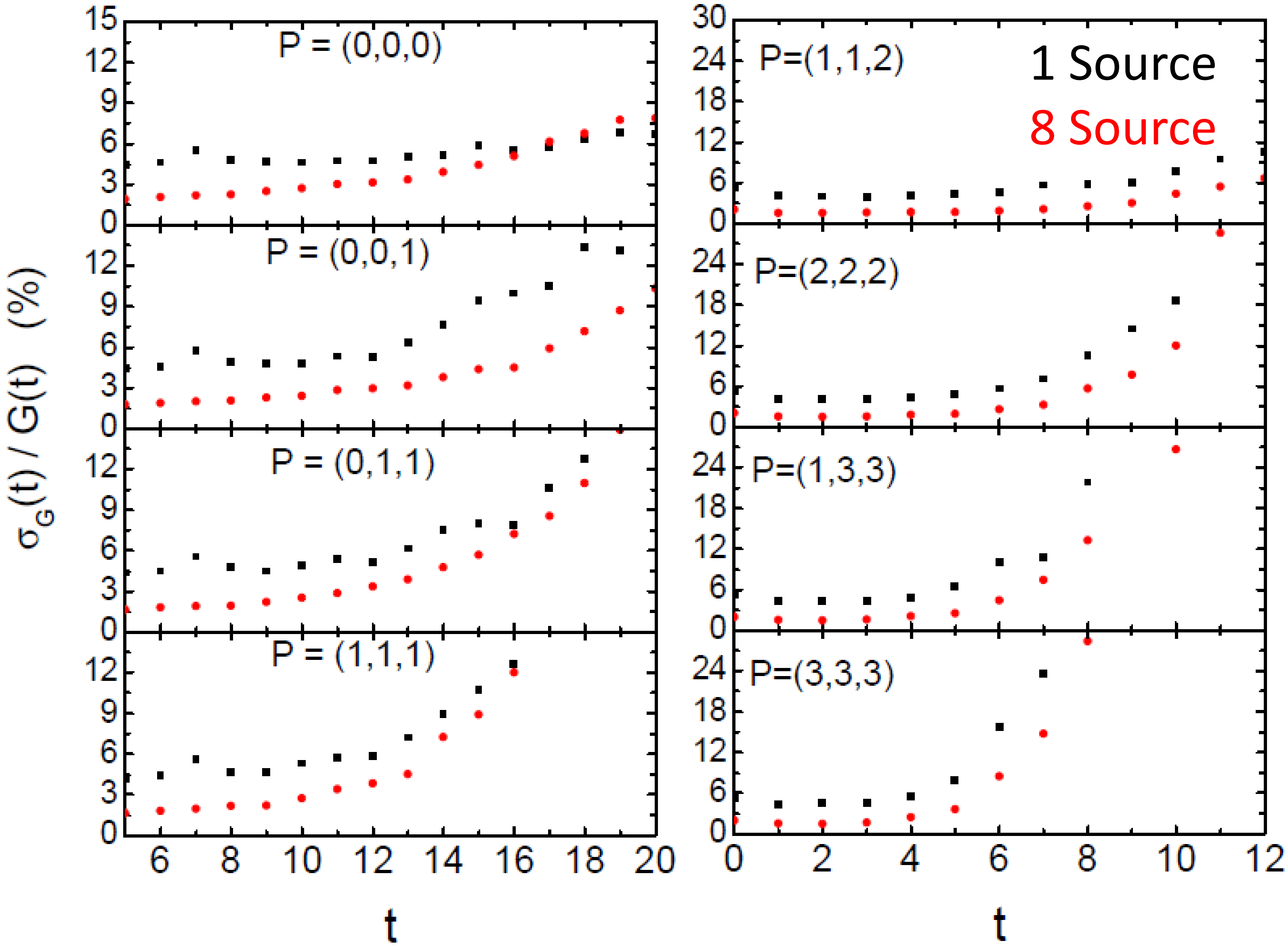


- Momentum phase:

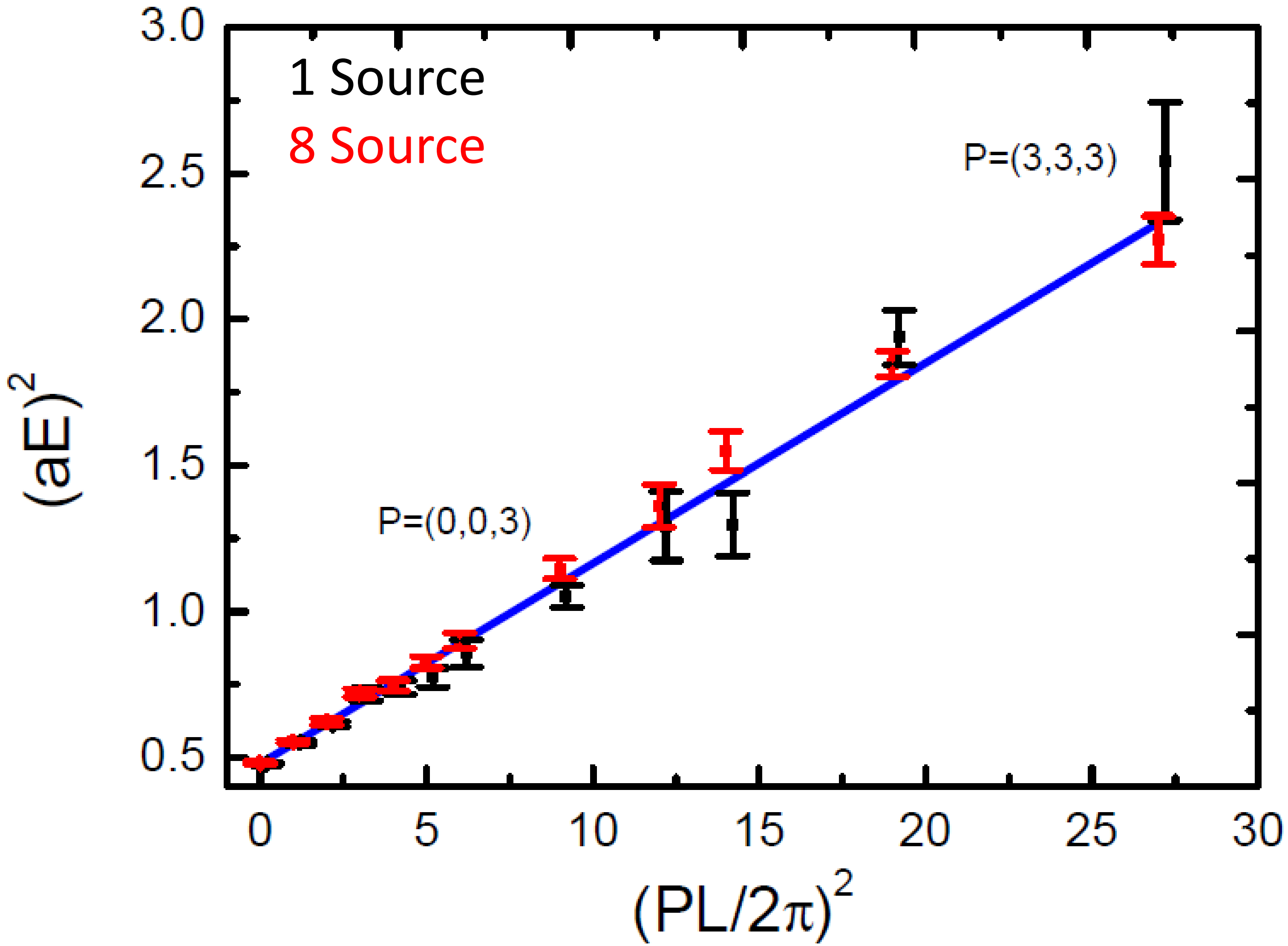
$$S^{smearing}_{\vec{p}}(\vec{y},\vec{x}) = \sum_{\vec{x}_i,\vec{y}_i} e^{i\,\vec{p}.(\vec{y}_i-\vec{y})} f(\vec{y}_i - \vec{y}) e^{-i\,\vec{p}.(\vec{x}_i-\vec{x})} f^*(\vec{x}_i - \vec{x})\, S(\vec{y},\vec{x})$$

- This smearing will help to get better signal at large momentum.

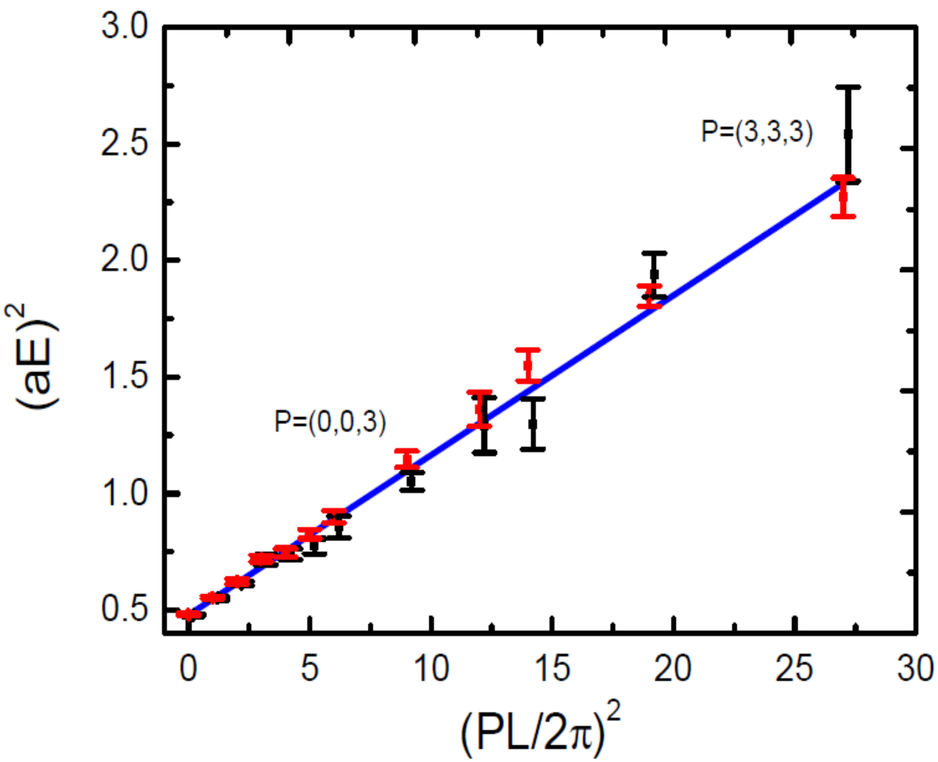
Comparison

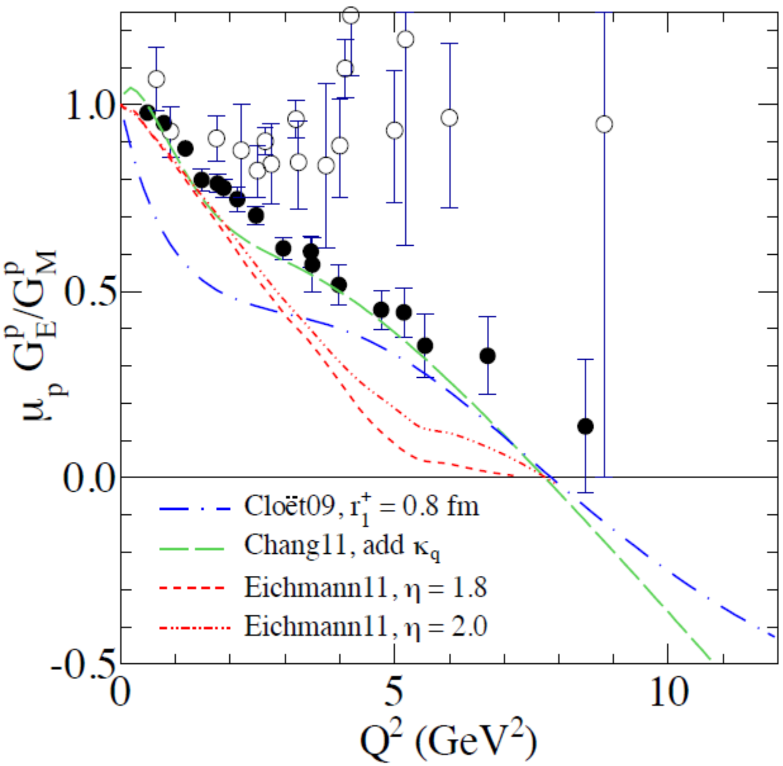


Momentum
Phase
Smearing


1 Source
8 Source
P=(3,3,3)
P=(0,0,3)
3.0
2.5
2.0
1.5
1.0
0.5
0
5
10
15
20
25
30
$(aE)^2$
$(PL/2\pi)^2$

Outlook





a ~ 0.07 fm   L ~ 1.7 fm   P=(3,3,3) ~ 3.8 GeV m_p ~ 2 GeV Q^2 ~ 57 GeV^2

L ~ 1.7 fm → Too small

m_p ~ 2 GeV → Too large → m_p ~ 1 GeV Q^2 ~ 12 GeV^2

Rough Estimate

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SUB CRUCE LUMEN

Thanks Very Much

Milt-Momentum method

$$\boldsymbol{\chi}_{\vec{p}}(\vec{y}) = \sum_{i=1,N} e^{-i\vec{p}.\vec{x}_i} \boldsymbol{\eta}(\overrightarrow{x_i}) S_{\vec{p}}(\vec{y},\overrightarrow{x_i})$$

$$G_N(\vec{p}_1+\vec{p}_2+\vec{p}_3;t) = \sum_{\vec{y}} e^{i(\vec{p}_1+\vec{p}_2+\vec{p}_3).\vec{y}} \epsilon^{abc}\epsilon^{a'b'c'} \left\{ \begin{array}{l} Tr\left[\chi_{\vec{p}_1}^{aa'}(\vec{y})(\gamma_5 C)\chi_{\vec{p}_2}^{bb'}(\vec{y})(\gamma_5 C)\right]\chi_{\vec{p}_3\,\gamma\gamma'}^{cc'}(\vec{y}) \\ +[\chi_{\vec{p}_1}^{aa'}(\vec{y})(\gamma_5 C)\chi_{\vec{p}_2}^{bb'}(\vec{y})(\gamma_5 C)\chi_{\vec{p}_3}^{cc'}(\vec{y})]_{\gamma\gamma'} \end{array} \right\}$$

- Four different Momentum Versions:

(0,0,0) (0,0,1)(0,1,1)(1,1,1)

→ generate 20 different total momentum from (0,0,0) to (3,3,3)