Four- and three-body dynamics in ^{6}Li scattering

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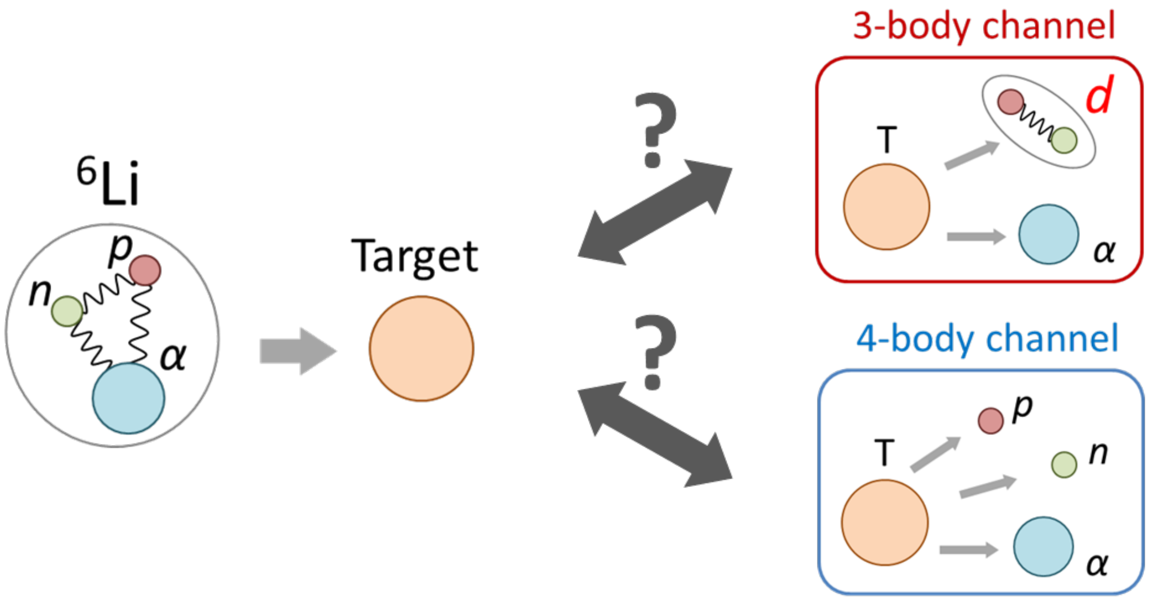

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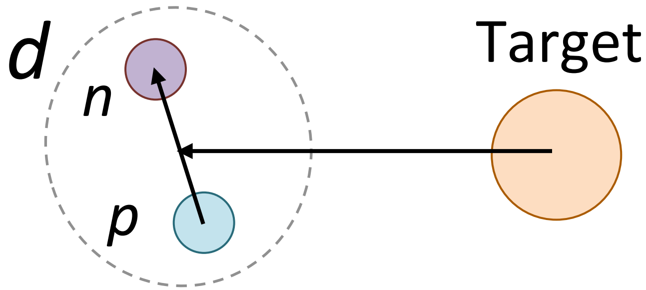
Adelaide Convention Centre, Australia

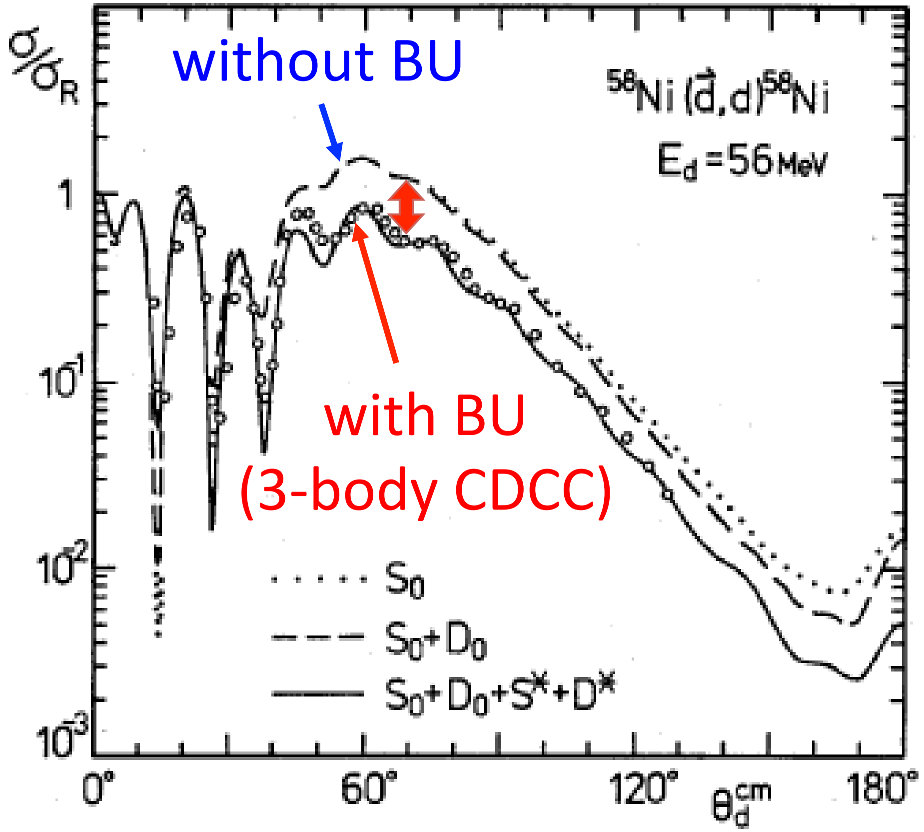
CDCC & Breakup effects

CDCC Continuum Discretized Coupled Channels

- ✓ CDCC is a fully quantum mechanical method for treating BU effects.
- ✓ CDCC was born as a theory for *d*-scattering ⇒ 3-body CDCC



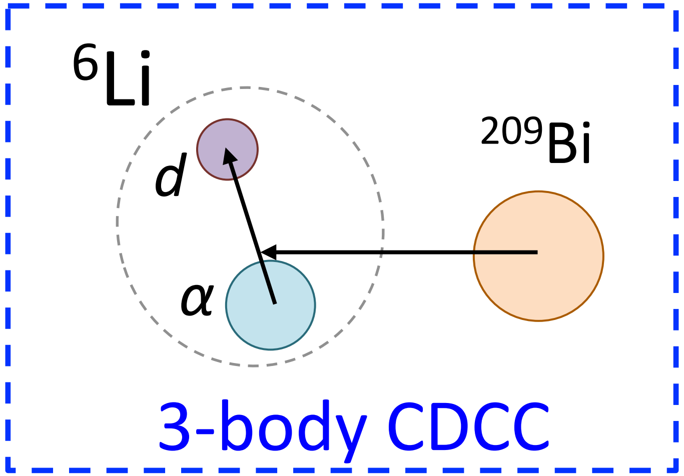
Three-body CDCC has been widely applied to many kinds of scattering.



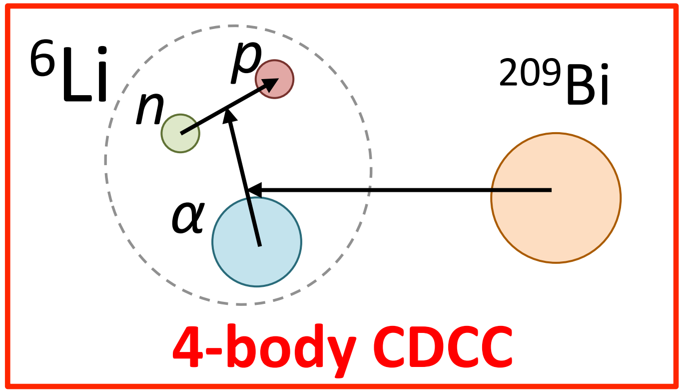
First application of 3-body CDCC

M. Yahiro, Y. Iseri, H. Kameyama, M. Kamimura, and M. Kawai, Prog. Theor. Phys. Suppl. No. 89 (1986), 32.

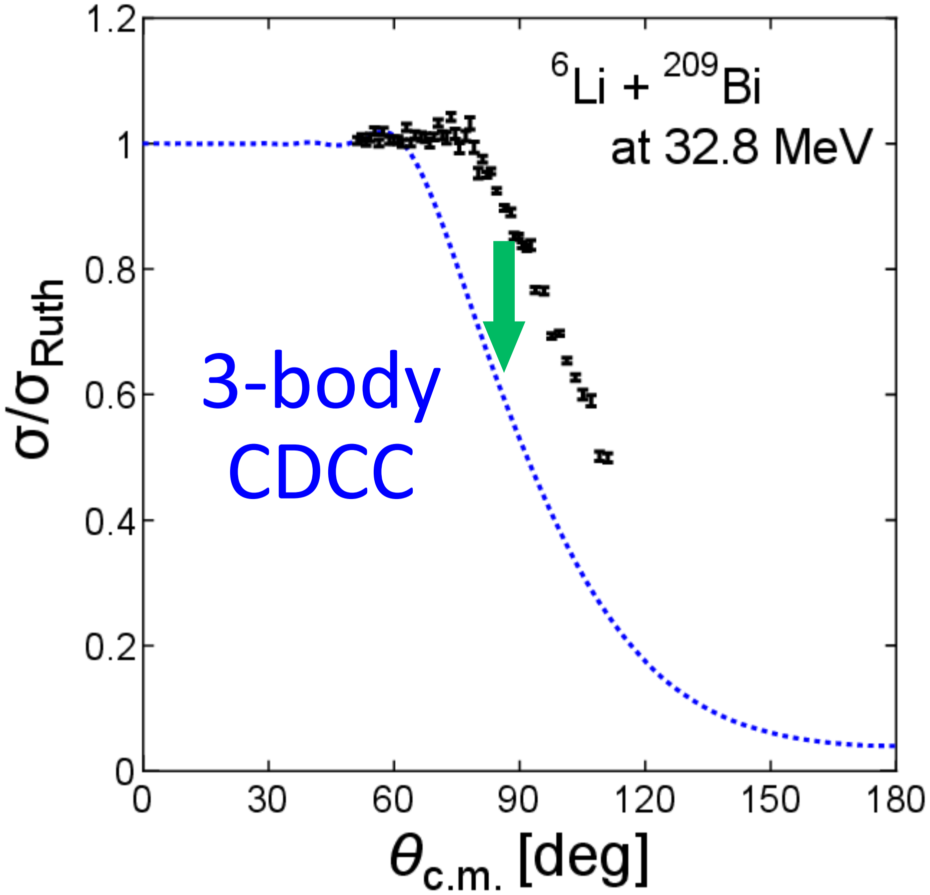
Problem of 3-body CDCC in ^{6}Li scattering



*N. Keeley et al., PRC **68** (2003), 054601.*



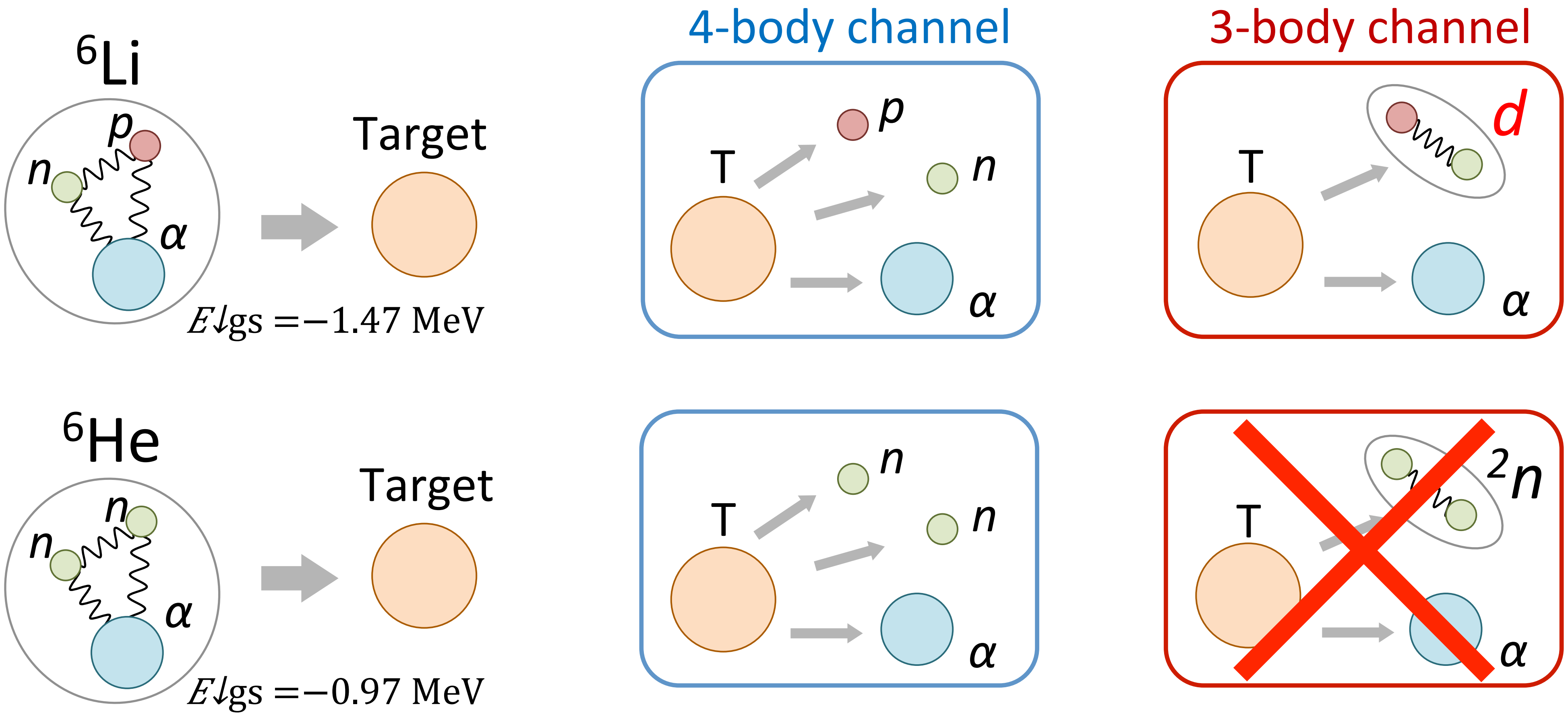
***4-body CDCC**: T. Matsumoto et al., PRC **70**, 061601(R) (2004).*



Exp.
*E. F. Aguilera et al., PRL **84**, 5058 (2000).*
*E. F. Aguilera et al., PRC **63**, 061603 (2001).*

→ Can 4-body CDCC solve this problem?

Competition: 4-body channel vs 3-body channel

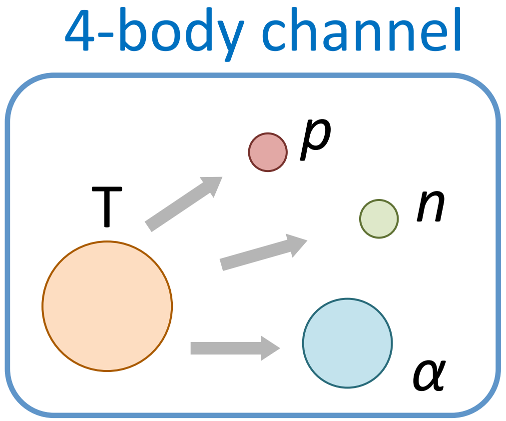


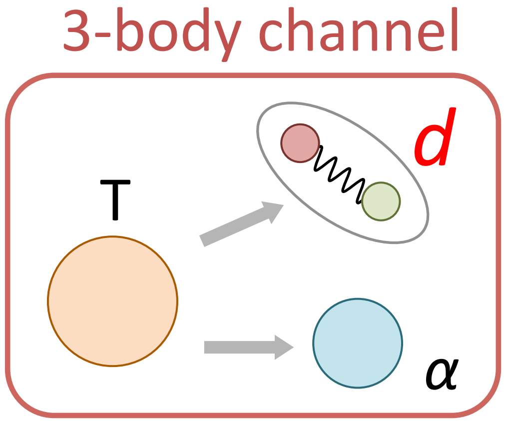
Which of these 4- and 3-body channels is favored in ^{6}Li scattering?

Purpose

Purpose 1

We apply 4-body CDCC to ^{6}Li scattering to treat both 4-body & 3-body channels explicitly.





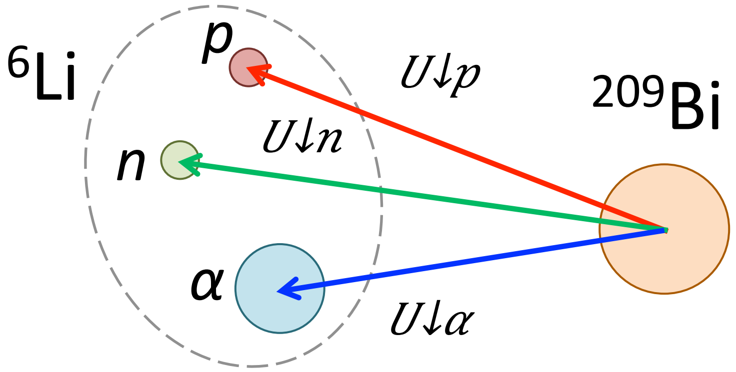
Purpose 2

We estimate 4-body and 3-body breakup channel-coupling effects, and clarify the reaction dynamics.

Model Hamiltonian of 4-body CDCC

■ 4-body Schrödinger equation

$$(H_{4b}-E)\Psi(\boldsymbol{R},\boldsymbol{\xi})=0$$
$$H_{4b}=K_R+U_n+U_p+U_\alpha+e^2 Z_{Li} Z_{Bi}/R+h_\xi$$



Phenomenological optical potentials

A. J. Koning et al., NPA 713 (2003), 231-310.
*A. R. Barnett et al., PRC **9** (1974), 2010.*

■ Internal Hamiltonian h_ξ

$$(h_\xi-\varepsilon)\phi_\varepsilon(\boldsymbol{\xi})=0$$

$\phi_\varepsilon(\boldsymbol{\xi})$: ^{6}Li internal wf.

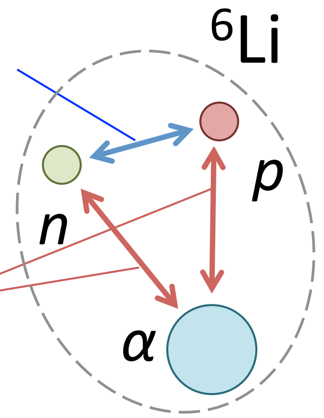
$$h_\xi=T_{\boldsymbol{r}_c}+T_{\boldsymbol{y}_c}+V_{np}+V_{n\alpha}+V_{p\alpha}+V_{3b}$$

Bonn-A interaction

*R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).*

KKNN interaction

*H. Kanada et al., Theor. Phys. **61**, 1327 (1979).*

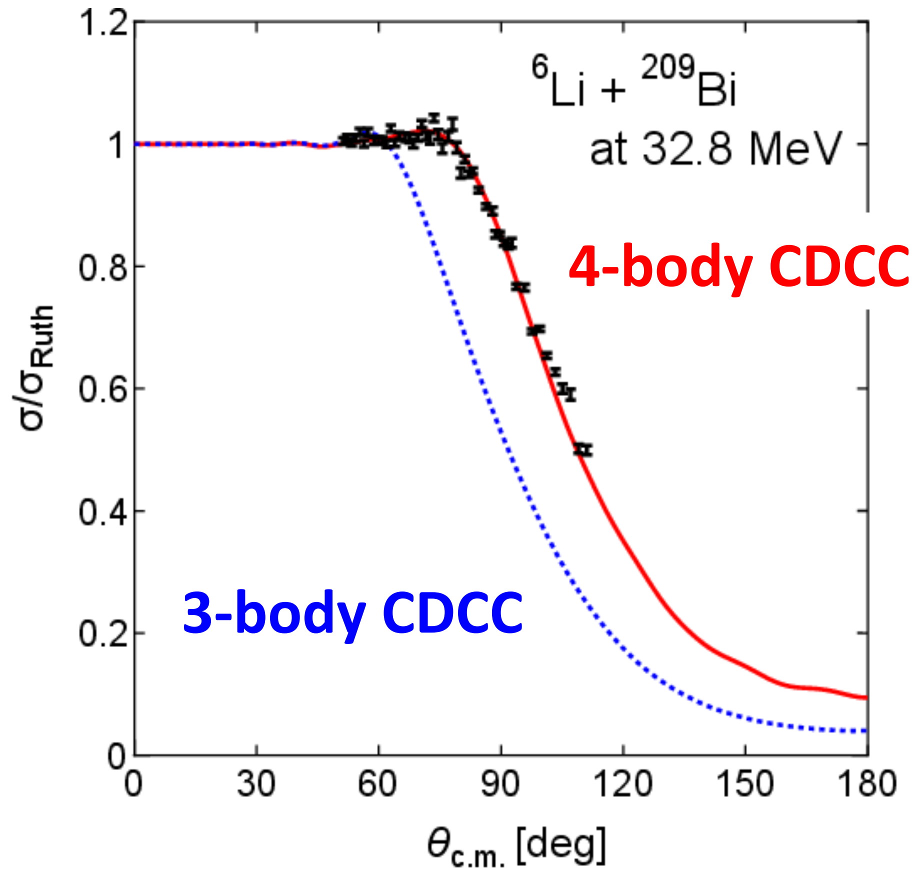


We have no adjustable parameter from now on.

Result 1: 3-body CDCC vs 4-body CDCC

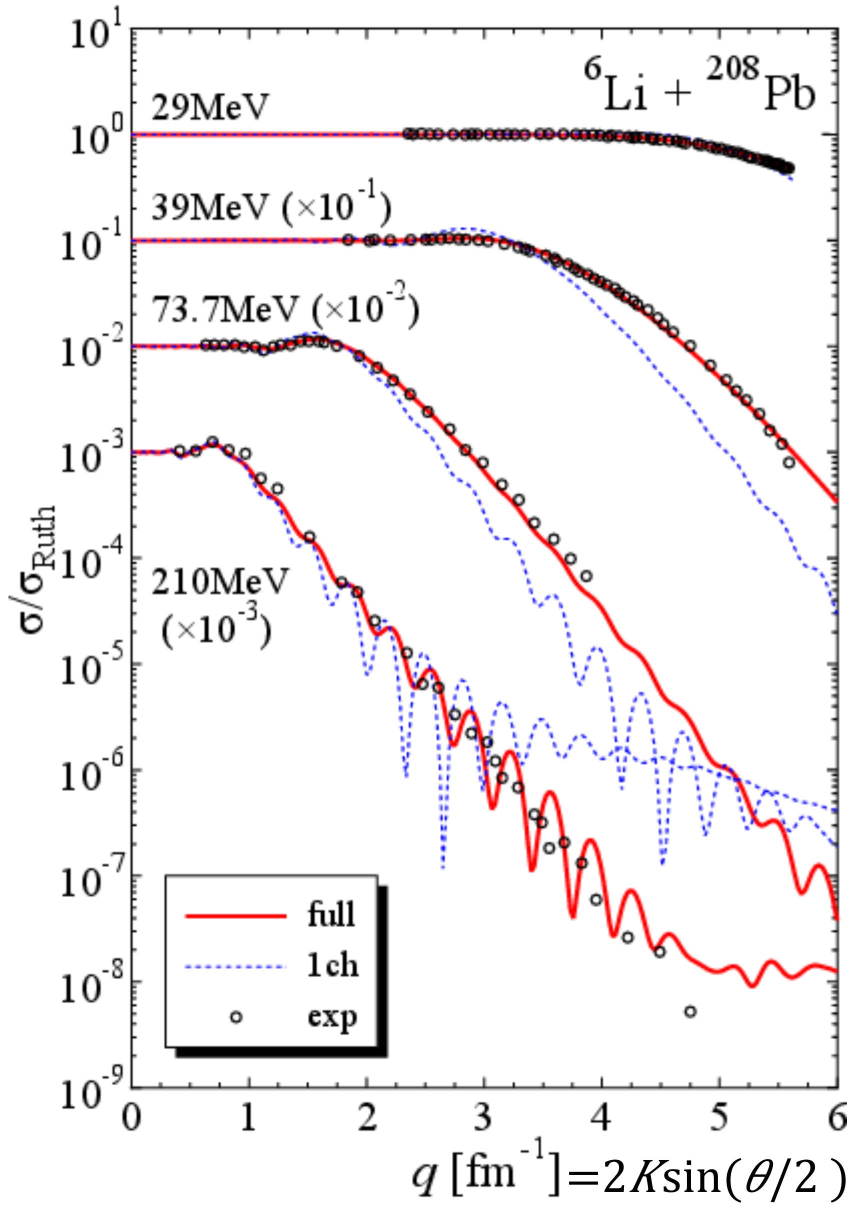
☹ 3-body CDCC cannot reproduce the data.

4-body CDCC is in excellent agreement with the experimental data.



Experimental data
E. F. Aguilera et al., Phys. Rev. Lett. ***84****, 5058 (2000).*
E. F. Aguilera et al., Phys. Rev. C ***63****, 061603 (2001).*

Result 2: Breakup channel-coupling effects



The experimental data is available in a wide energy range (24-210MeV).

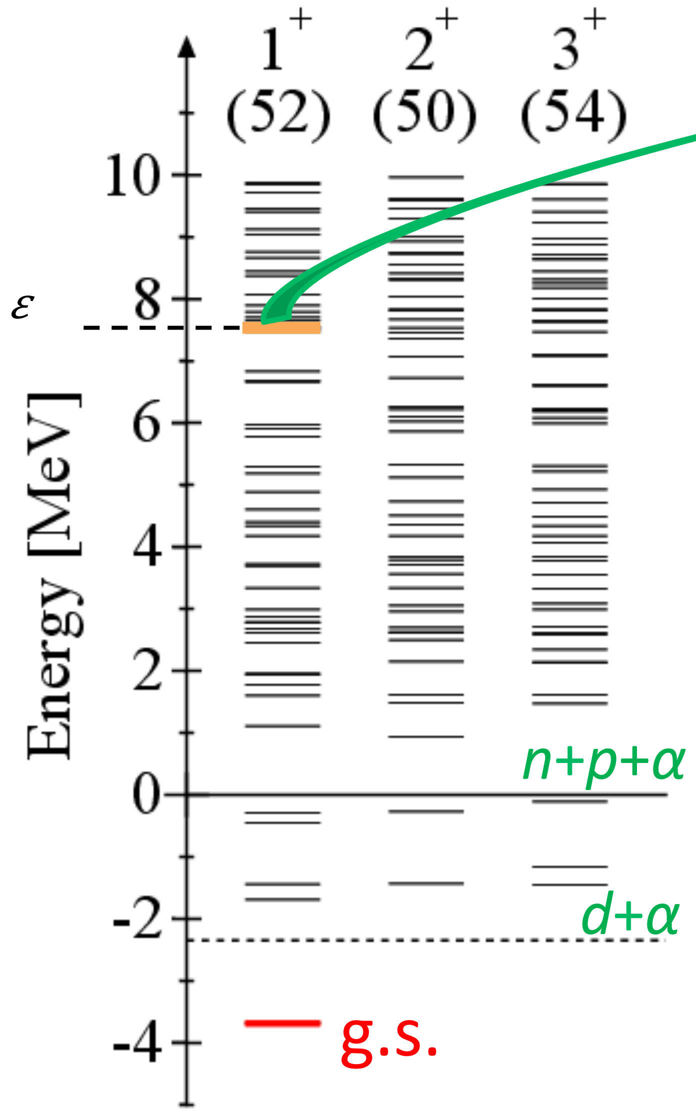
Exp

N. Keeley et al., Nucl. Phys. A ***571****, 326 (1994).*
R. Huffman et al., Phys. Rev. C ***22****, 1522 (1980).*
A. Nadasen et al., Phys. Rev. C ***39****, 536 (1989).*

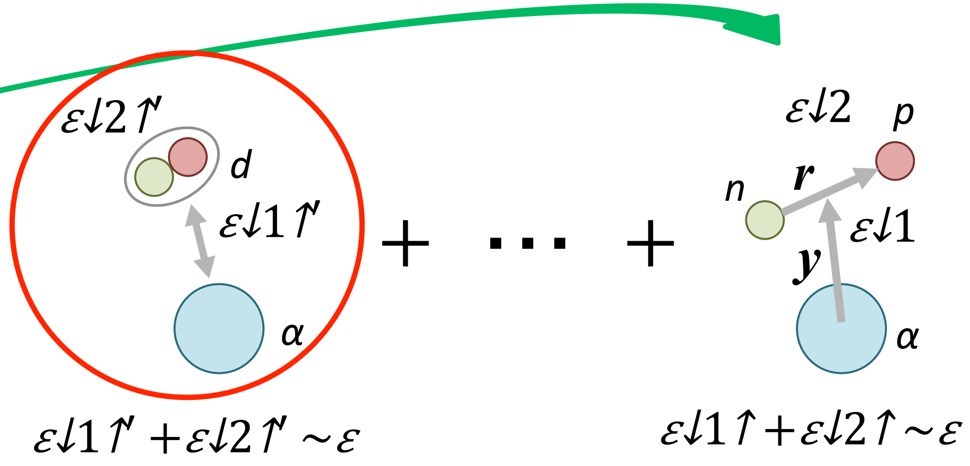
BU channel-coupling effects are quite important for all the incident energies.

4-body BU channel?
3-body BU channel?

Difficulty: In 4-body CDCC, both four- and three-body channels are mixed with each other.



Energy spectrum of ^{6}Li



$$\Psi^{^6\mathrm{Li}}(\boldsymbol{r},\boldsymbol{y}) = \phi_d(\boldsymbol{r})\varphi(\boldsymbol{y}) + \phi_{d}^{*}(\boldsymbol{r})\varphi^{*}(\boldsymbol{y}) + \cdots$$

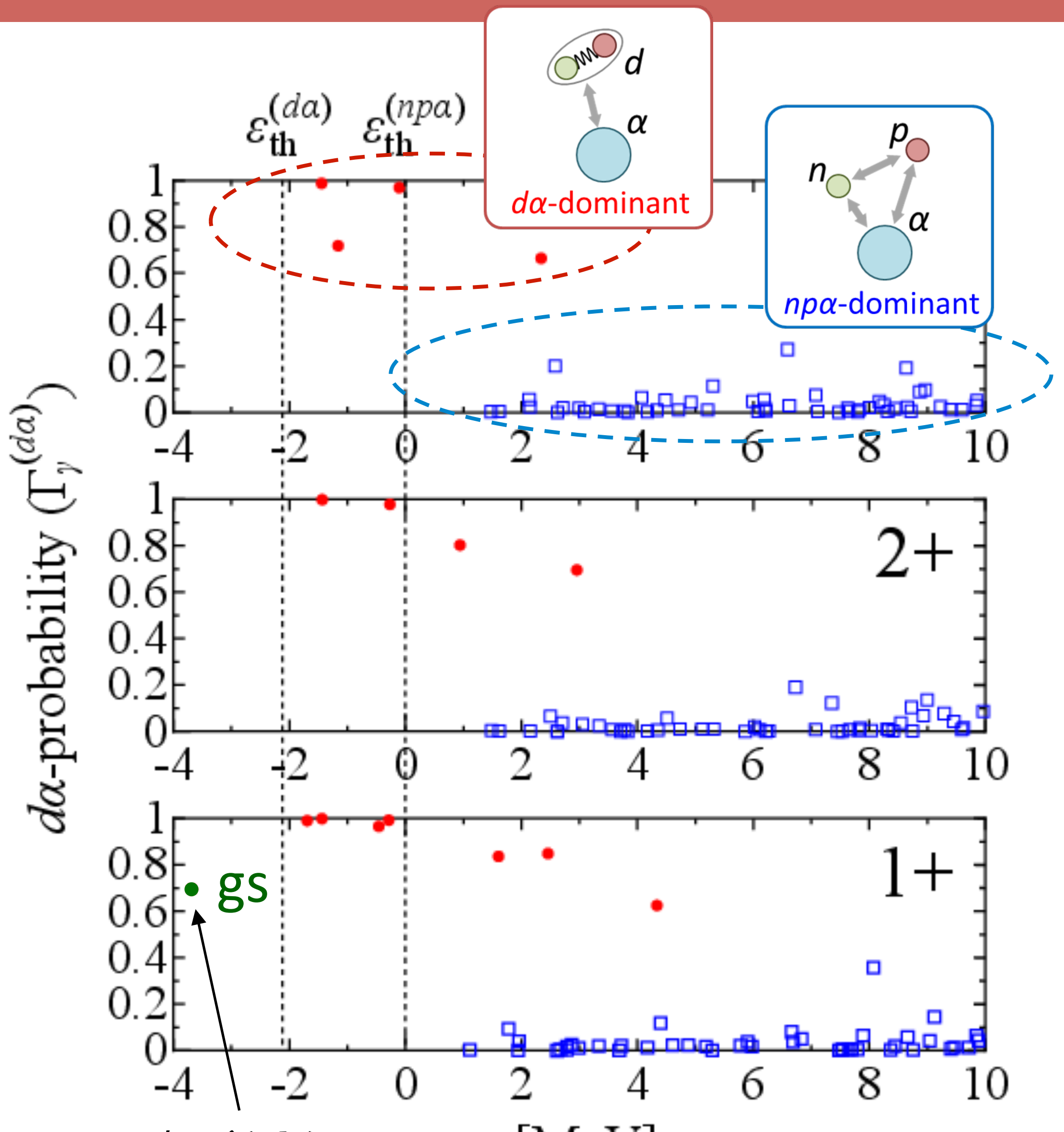
- Different types of 3-body motions are superposed (Pseudostate).

dα-probability

$$\Gamma(d\alpha) = |\phi_d(\boldsymbol{r})\Psi^{^6\mathrm{Li}}(\boldsymbol{r},\boldsymbol{y})|^2$$

- How the *dα* configuration is included in the scattering state $\Psi^{^6\mathrm{Li}}(\boldsymbol{r},\boldsymbol{y})$.

dα probability ($\Gamma^{(d\alpha)}$)



$\Gamma_{gs}^{(d\alpha)} = 0.70$

- **Categorize BU states**
  - *dα*-dominant state $|d\alpha\rangle_i$
    $|\text{BU}\rangle_i$ with $\Gamma_i^{(d\alpha)} > 0.5$
  - *npα*-dominant state $|np\alpha\rangle_j$
    $|\text{BU}\rangle_j$ with $\Gamma_j^{(d\alpha)} \leq 0.5$

The number of ***npα-dominant states*** is much more than that of ***dα-dominant states***.

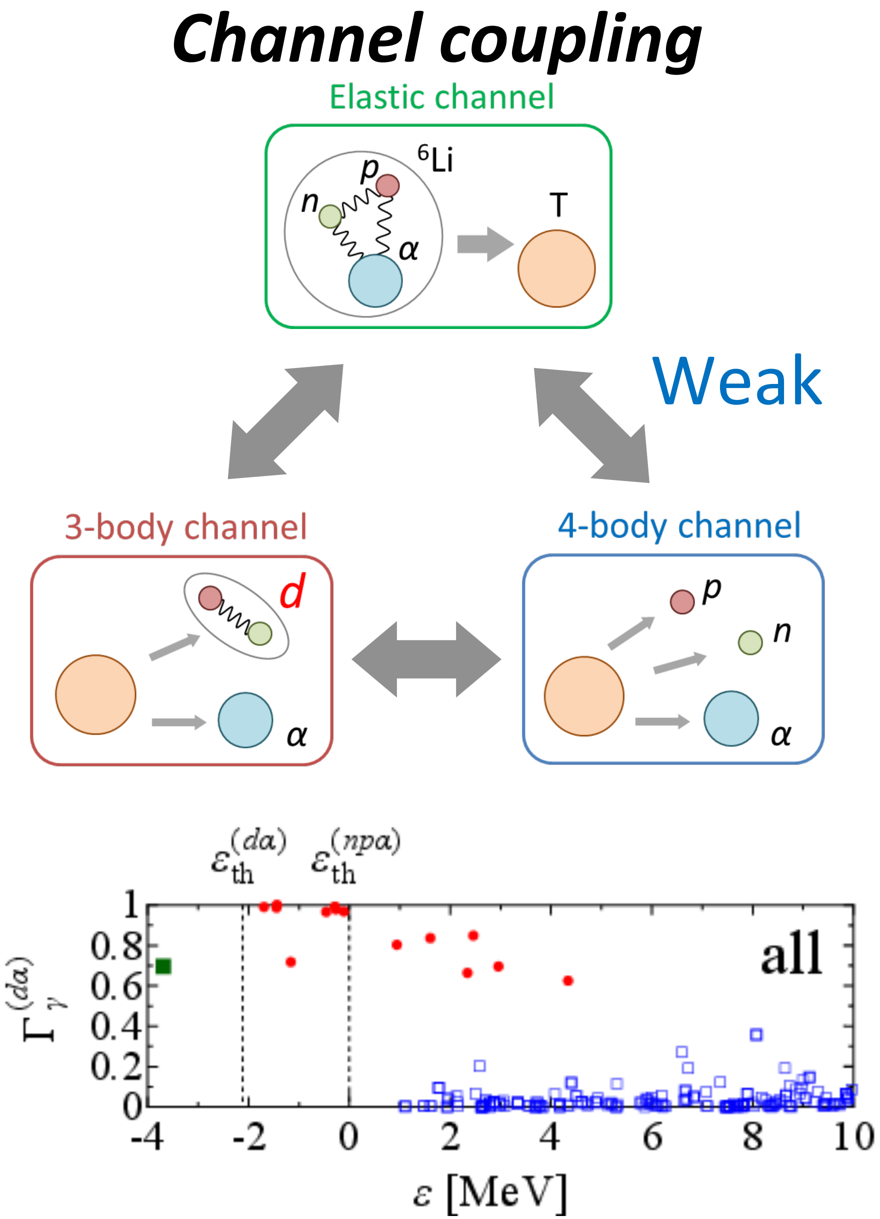
- **Decompose CDCC model space**

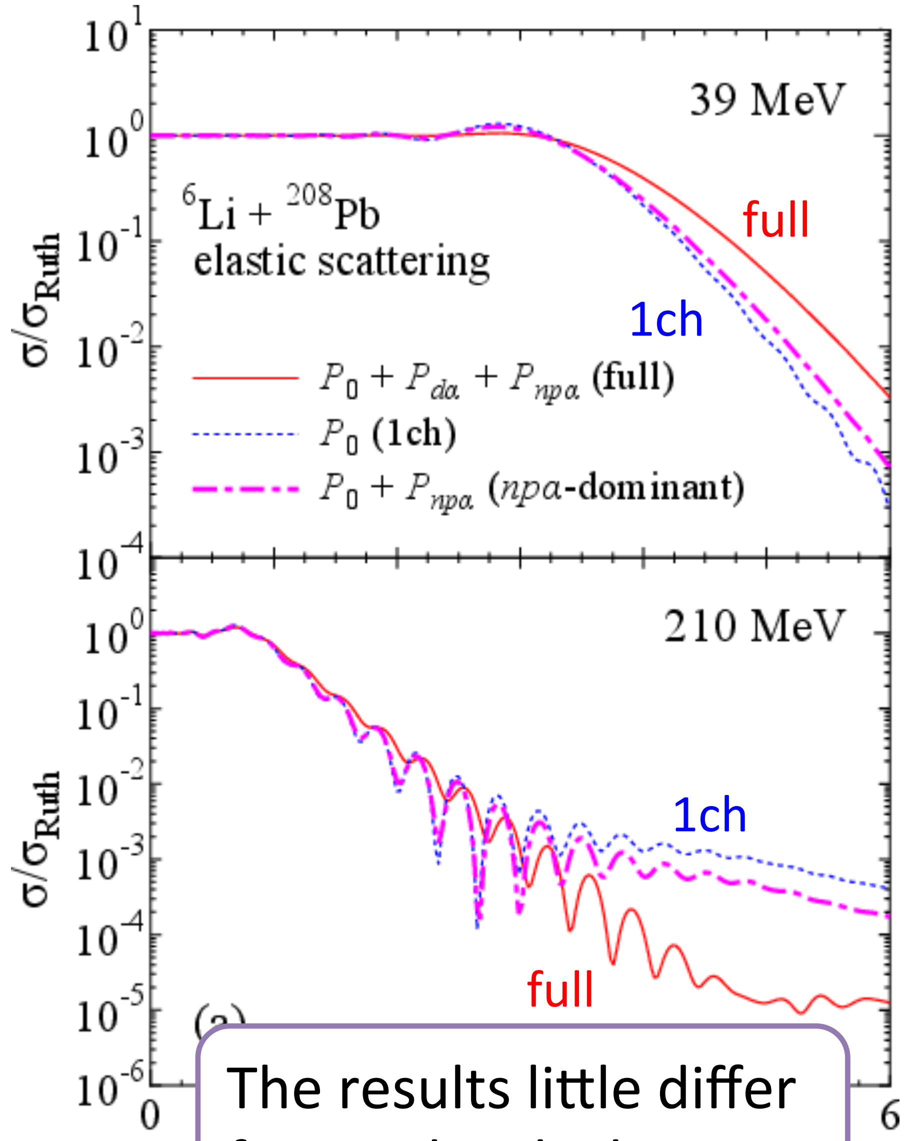
$$P = P_0 + P^*$$

$$= P_0 + P_{d\alpha} + P_{np\alpha}$$

$$\sum_i |d\alpha\rangle_{i\,i}\langle d\alpha| \qquad \sum_j |np\alpha\rangle_{j\,j}\langle np\alpha|$$

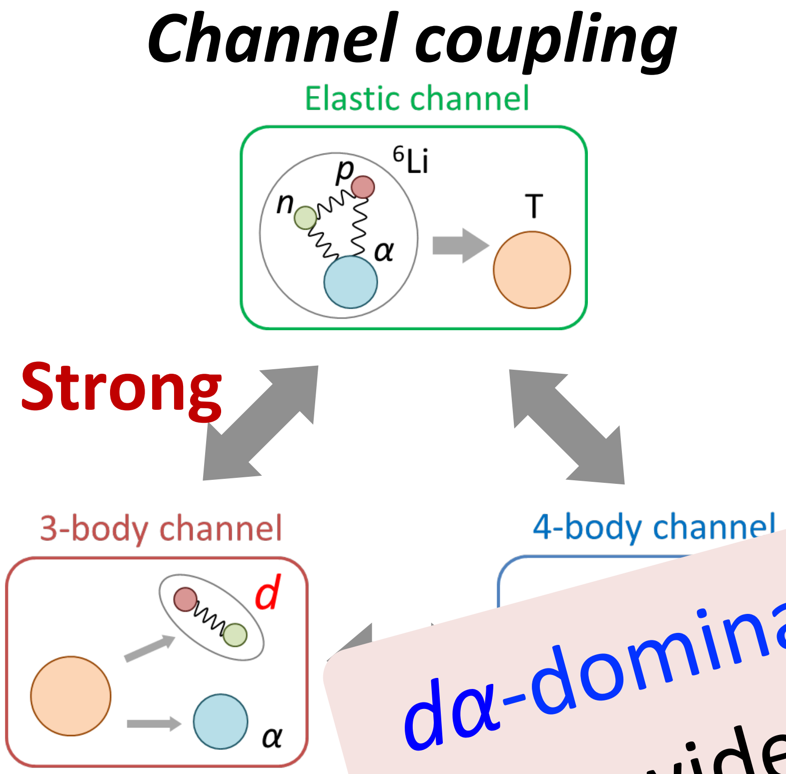
Four-body channel-coupling effect

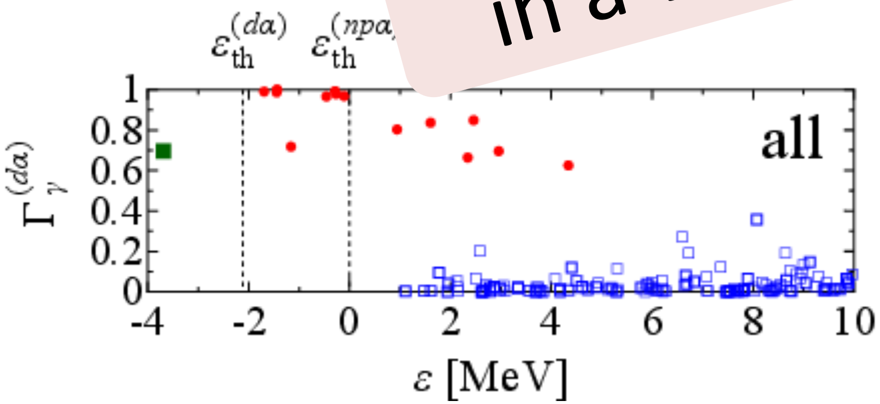


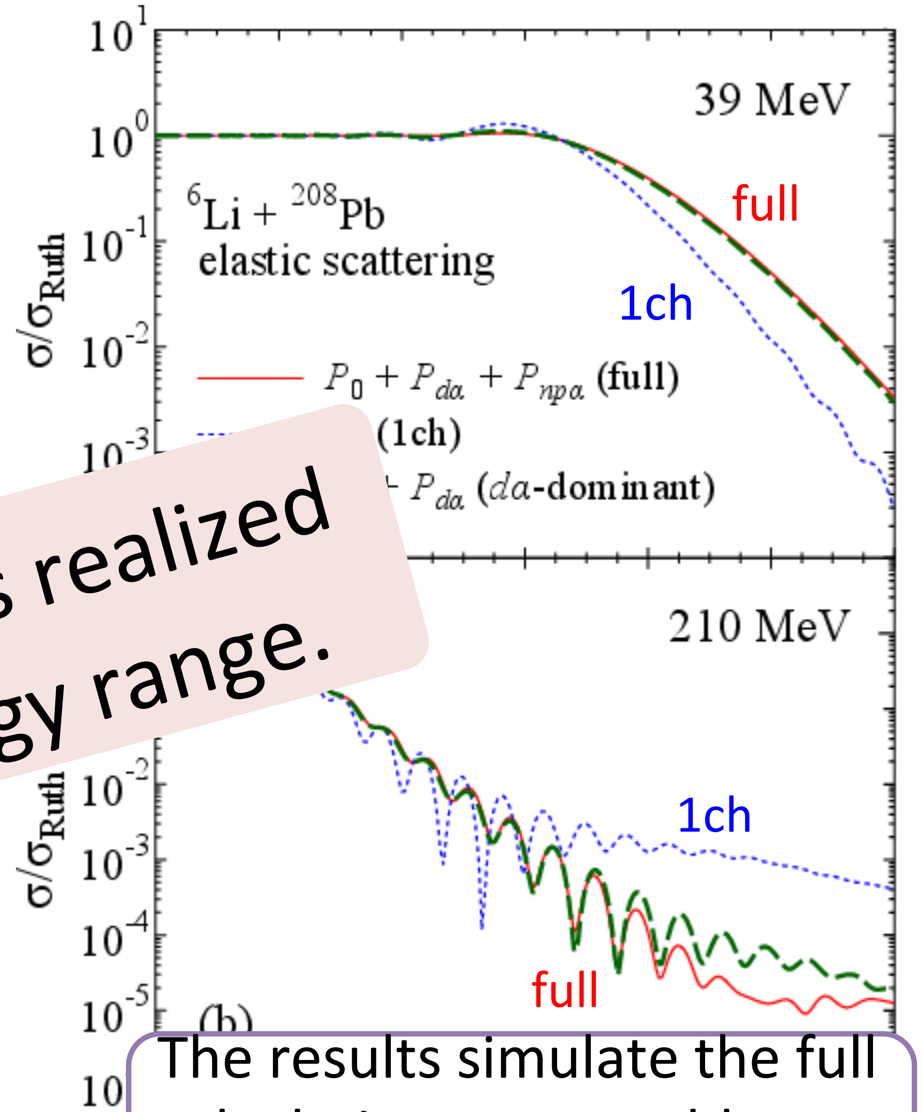


The results little differ from 1ch calculations.

Three-body channel-coupling effect





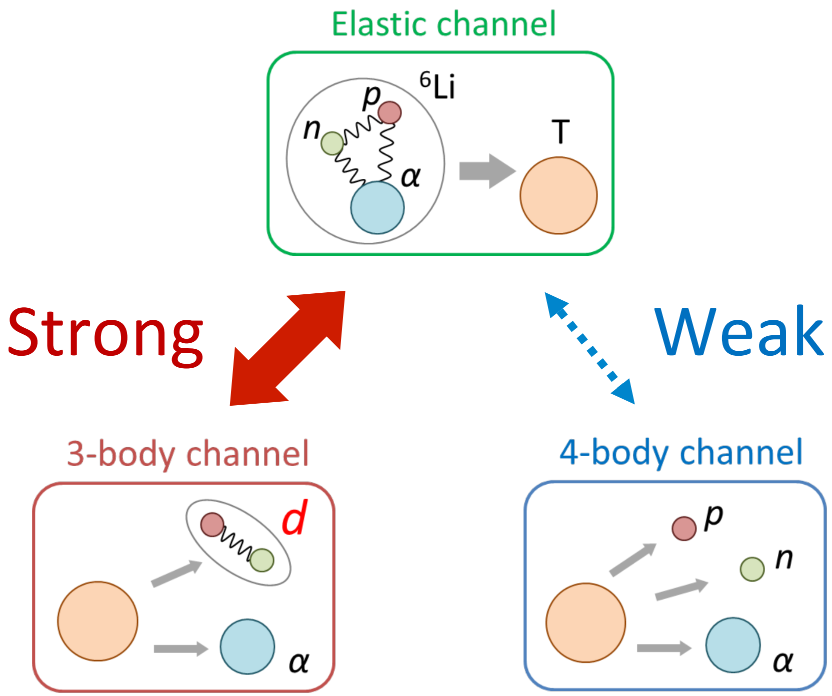


$d\alpha$-dominance is realized in a wide energy range.

The results simulate the full calculations reasonably well.

Proposal of an effective 3-body model

"deuteron" (*n*-*p* subsystem) of ^{6}Li hardly breaks up during scattering.

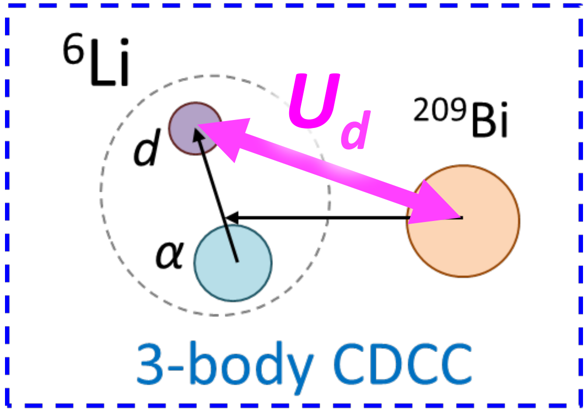


We can describe ^{6}Li scattering with three-body CDCC?

(Why traditional three-body CDCC does not work?)

Effective d+α+T three-body model

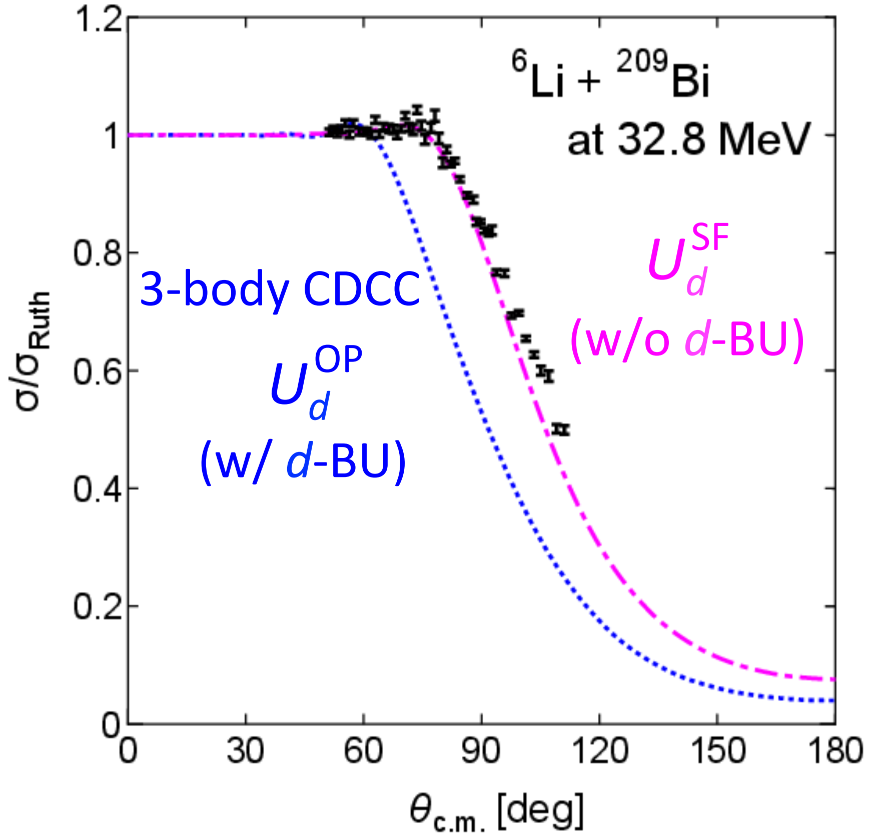
All we have to do is replace the d-T potential (U_d).



☹ U_d^{OP} : Optical potential (*includes d-BU effects*)

☺ U_d^{SF} : Single-folding potential (*NEVER includes d-BU effects*)

$$U_d^{SF} = \langle \phi_d^{(gs)} | U_n + U_p | \phi_d^{(gs)} \rangle$$



We can get the reasonable cross section with U_d^{SF}.

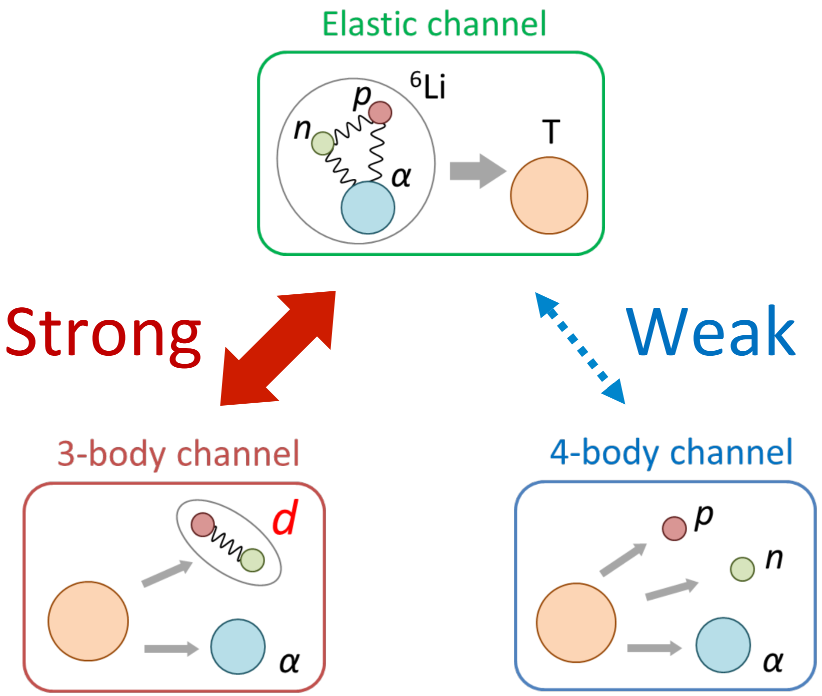
SW, T. Matsumoto, K. Minomo, K. Ogata, and M. Yahiro, Phys. Rev. C 86, 031601(R) (2012).

Summary

We have studied four-body dynamics (n+p+α+T) of ^{6}Li scattering in a wide energy range.

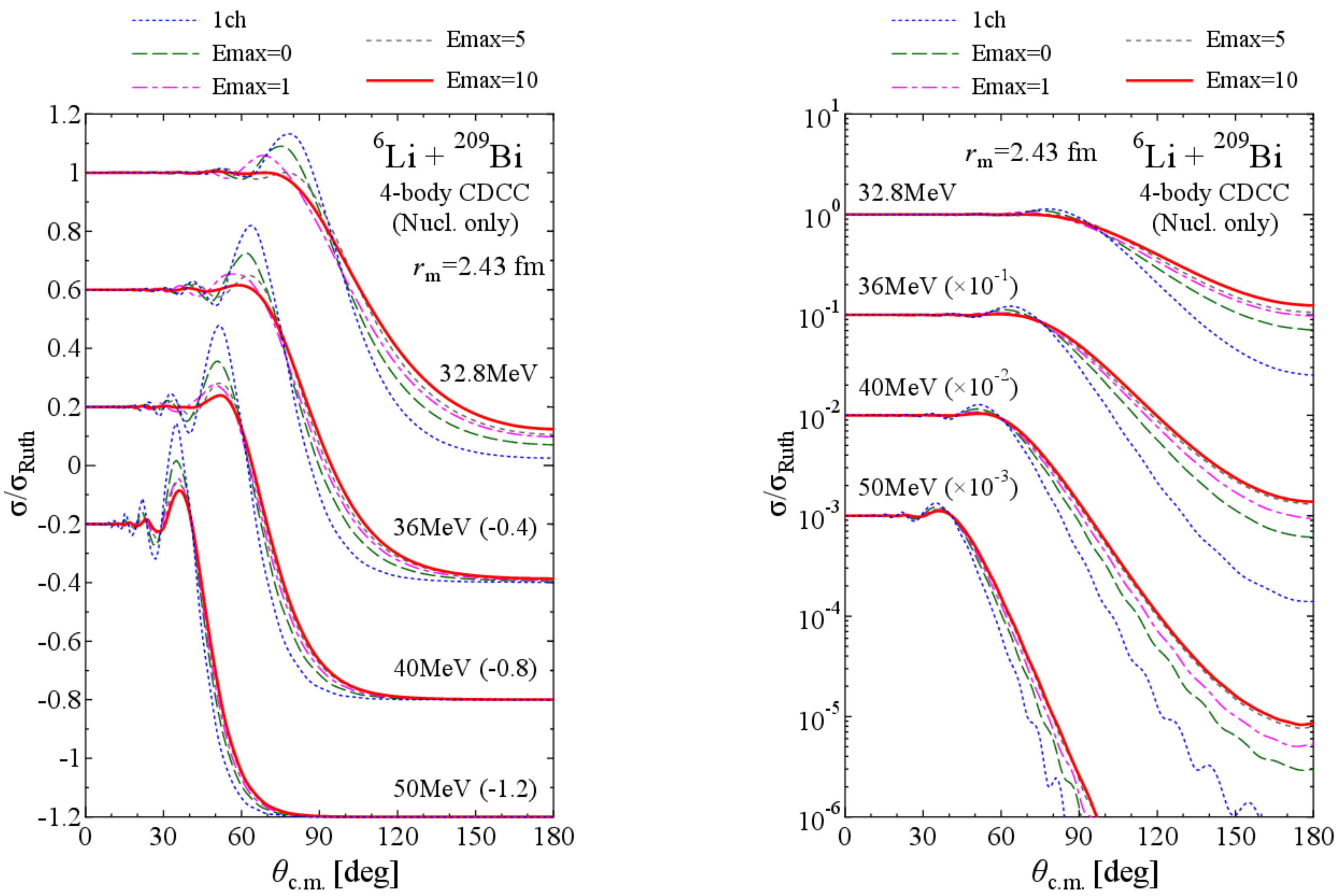
SW, T. Matsumoto, K. Ogata, and M. Yahiro, PRC 92, 044611 (2015).

- 4-body CDCC reproduces experimental data well.
- 3-body channel coupling is dominant.
  - ✓ *"Deuteron"* in ^{6}Li hardly breaks up during scattering. (=*d*α dominance)
- We have proposed an effective three-body model.
  - ✓ We can treat ^{6}Li scattering easily and flexibly.

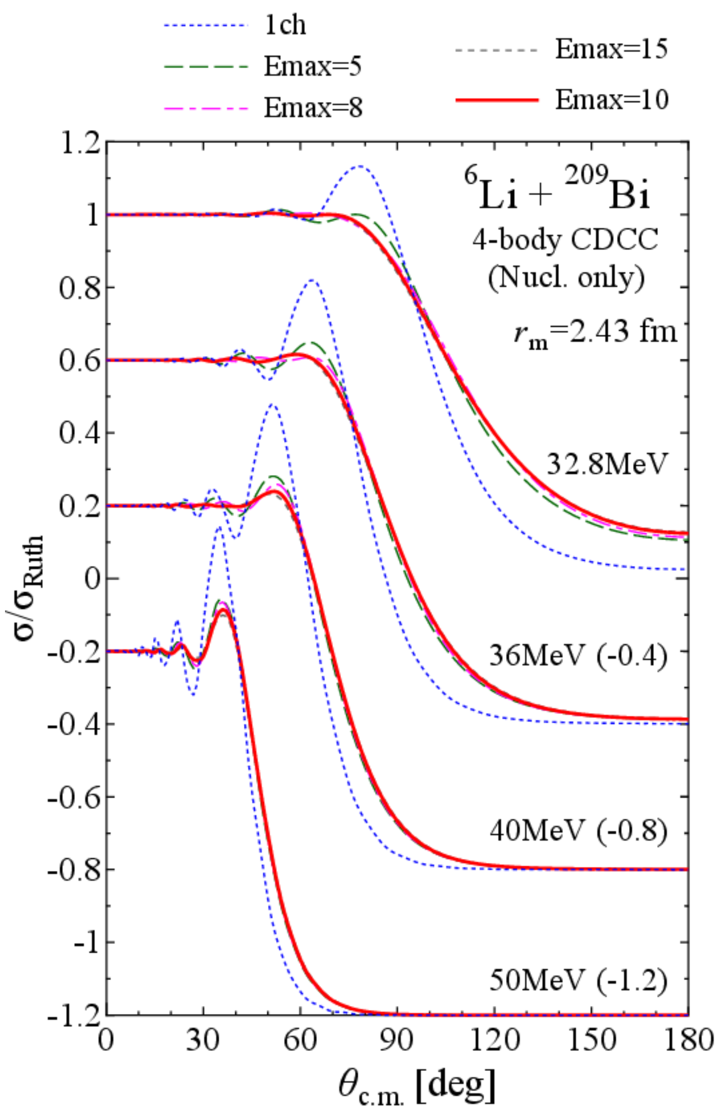


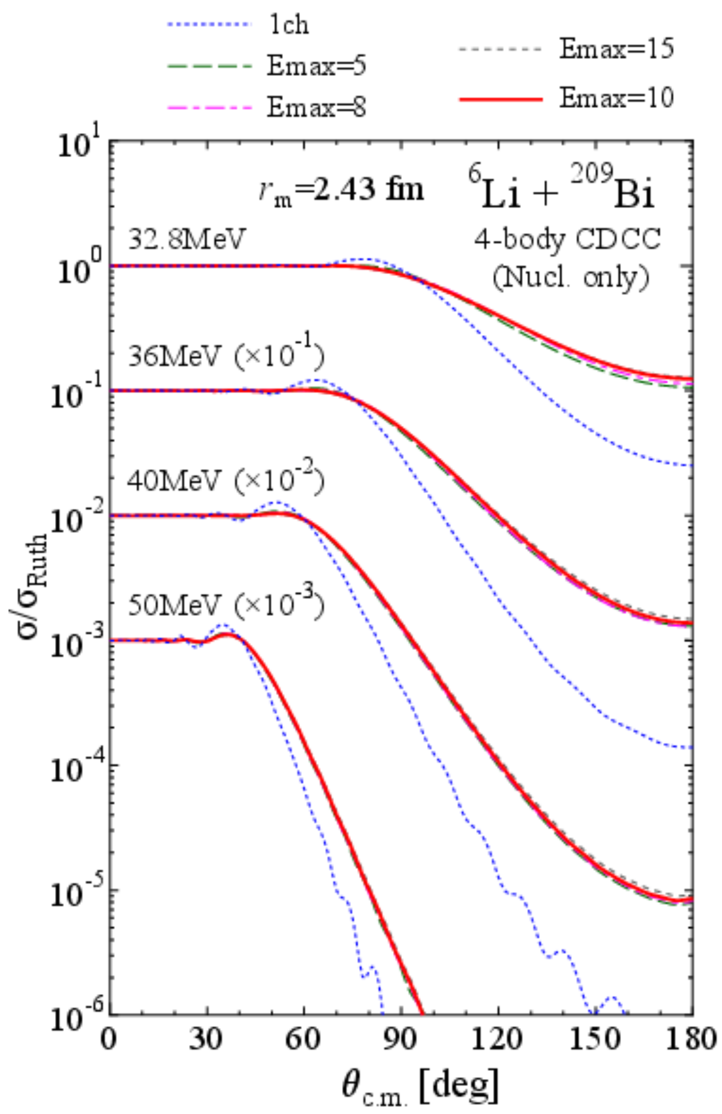
Backup

Convergence



Convergence 2





$d\alpha$-probability for the g.s.

Radius of the *n-p* system

"*d*" in ^{6}Li

$R_{n\text{-}p}/2$=1.63 [fm]

d in free space

R_d=1.97 [fm]

(by Bonn-A int.)

✓ The *n-p* system shrinks in ^{6}Li.

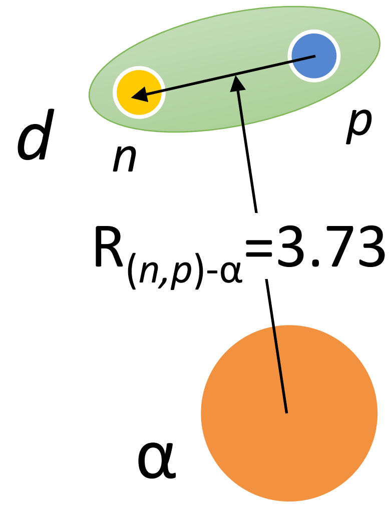
✓ Deuteron g.s. is still included with $\Gamma_{gs}(d\alpha)$=70%.

Evaluated value: 72±7% *D. R. Tilley et al., Nucl. Phys. A 708, 3 (2002).*

GEM calculation: 74% *Y. Kikuchi, N. Kurihara, A. Wana et al., Phys. Rev. C 84, 064610 (2011).*

^{6}Li ground state

$R_{n\text{-}p}/2$=1.63 [fm]



R_m=2.43 [fm]
(exp: 2.44±0.07)

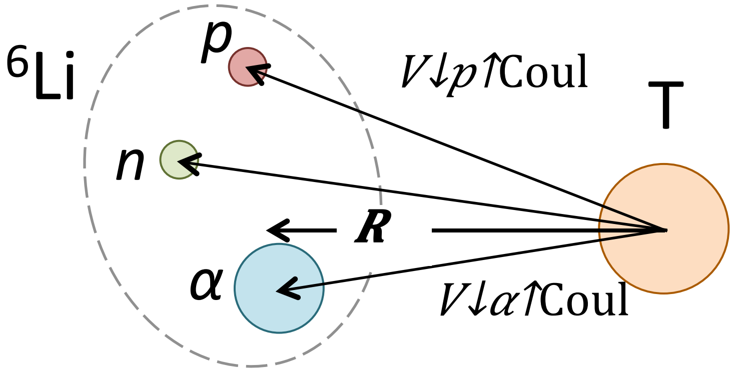
Discussion1: Coulomb breakup effects

Without Coulomb BU (present model)

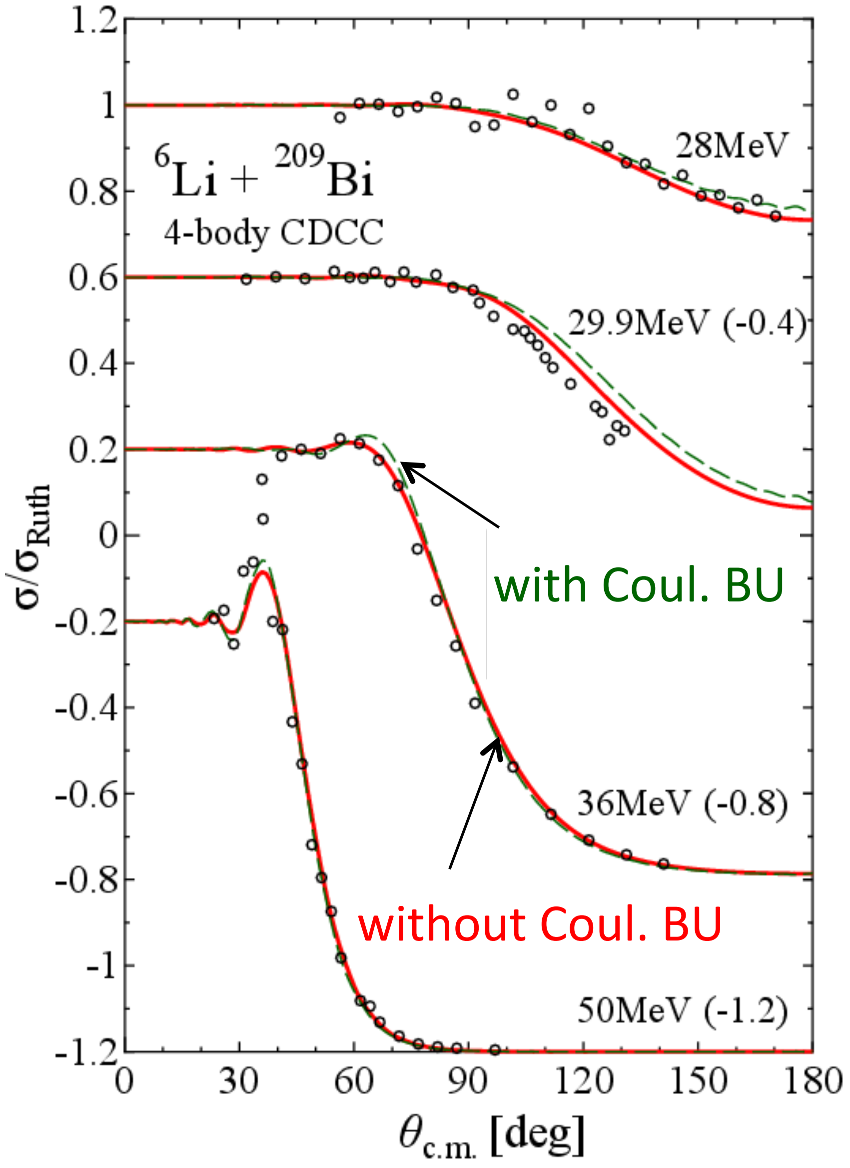
$$U_{Coul} = e^2 Z_{Li} Z_T / R$$

With Coulomb BU (more accurate)

$$U_{Coul} = e^2 Z_p Z_T / R_p + e^2 Z_\alpha Z_T / R_\alpha$$



Coulomb breakup effect is negligibly small.

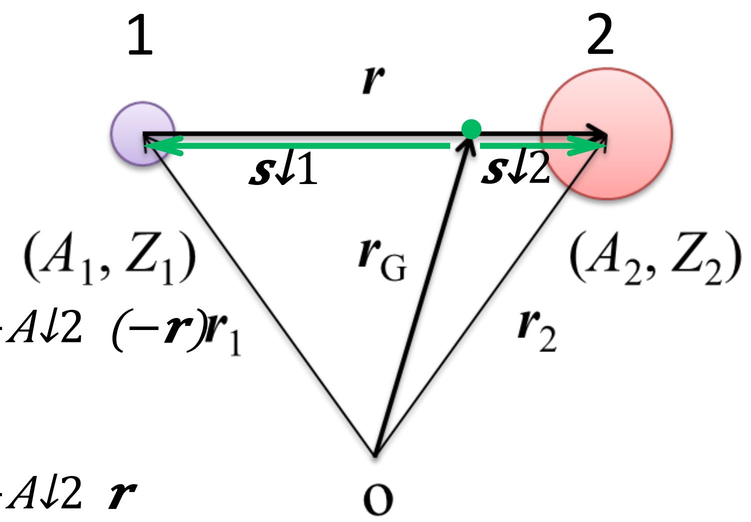


Why is the Coulomb BU so small?

$\sum_{i=1}^{2} e_i s_i Y_{1\mu}(\hat{\boldsymbol{s}}_i)$

(dipole operator)

$\boldsymbol{s}_i$: coordinate from c.m.



$$\boldsymbol{s}_1 = \boldsymbol{r}_1 - \boldsymbol{r}_G = \frac{A_2}{A_1 + A_2}(-\boldsymbol{r})$$

$$\boldsymbol{s}_2 = \boldsymbol{r}_2 - \boldsymbol{r}_G = \frac{A_1}{A_1 + A_2}\boldsymbol{r}$$

$$= Z_1 \frac{A_2}{A_1 + A_2} r Y_{1\mu}(-\hat{\boldsymbol{r}}) + Z_2 \frac{A_1}{A_1 + A_2} r Y_{1\mu}(\hat{\boldsymbol{r}})$$

$$= -\frac{Z_1 A_2 + Z_2 A_1}{A_1 + A_2} rY$$

$\equiv \beta$

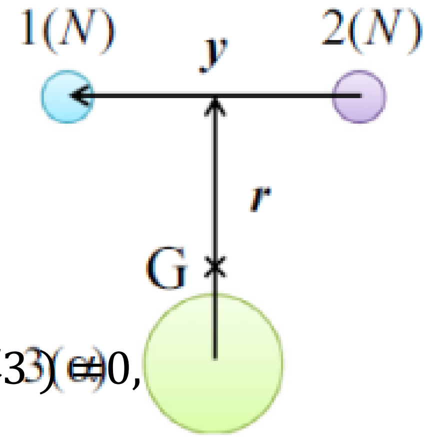
Nuclide	A_1	Z_1	A_2	Z_2	β
^{6}He	2	0	4	2	2/3
^{6}Li	2	1	4	2	0

Dipole operator becomes 0 for ^{6}Li (d+α).

Why is the Coulomb BU so small?

- **Three-cluster model**

$$D_\mu = \sum_{i=1}^{2} \left(\frac{1}{2} - \tau_{iz}\right) e x_i Y_{1\mu}(\boldsymbol{x}_i) + 2e x_3 Y_{1\mu}(\boldsymbol{x}_3)$$



$$|T T_z\rangle = |00\rangle = \frac{1}{\sqrt{2}} \left(|np\rangle - |pn\rangle\right)$$

The expectation value of D_μ for $|00\rangle$ is then

$$\langle 00|D_\mu|00\rangle = \frac{e}{2} \sum_{i=1}^{2} x_i Y_{1\mu}(\boldsymbol{x}_i) + 2e x_3 Y_{1\mu}(\boldsymbol{x}_3) = 0,$$

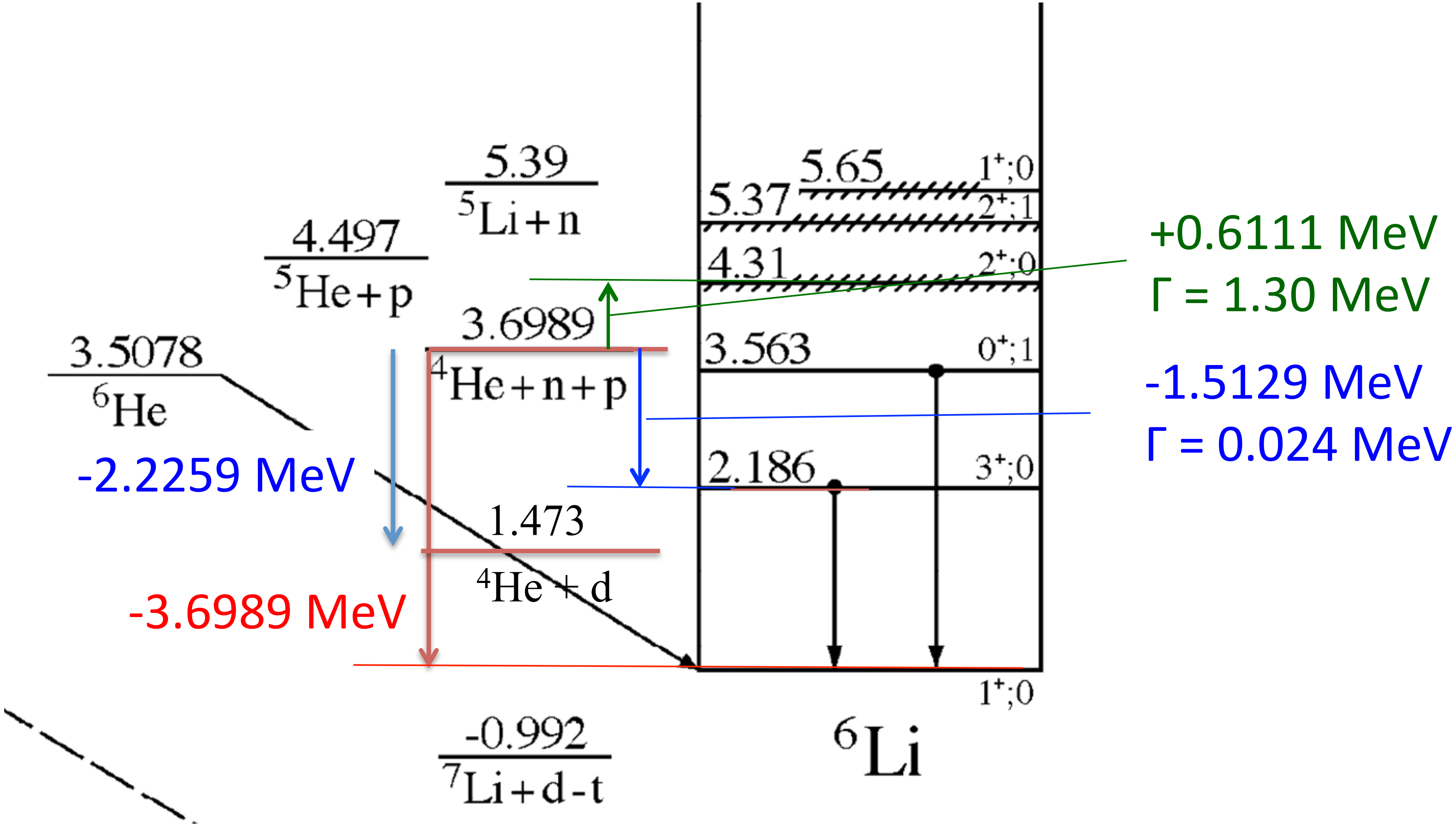
since

$$x_1 Y_{1\mu}(\boldsymbol{x}_1) = \frac{2}{3} r Y_{1\mu}(\boldsymbol{r}) + \frac{1}{2} y Y_{1\mu}(\boldsymbol{y}),$$

$$x_2 Y_{1\mu}(\boldsymbol{x}_2) = \frac{2}{3} r Y_{1\mu}(\boldsymbol{r}) - \frac{1}{2} y Y_{1\mu}(\boldsymbol{y}),$$

$$x_3 Y_{1\mu}(\boldsymbol{x}_3) = -\frac{1}{3} r Y_{1\mu}(\boldsymbol{r}).$$

Energy spectrum of ^6Li



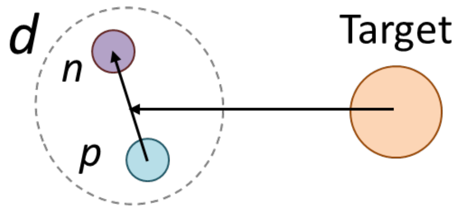
Feshbach theory 1

Problem setting

$H\Psi = E\Psi$, $\quad H = h + K + V$

$h|0\rangle = \varepsilon_0 |0\rangle \quad (\Psi = |0\rangle\chi_0 + |1\rangle\chi_1 + \cdots)$

$P = |0\rangle\langle 0|$, $\quad Q = 1 - P$



Problem: Find the effective Hamiltonian $H(P)$ for $P\Psi = |0\rangle\chi_0$.

That is $\quad H(P)P\Psi = EP\Psi \quad \cdots (A)$

Once, we have Eq.(A), we can get

$(K+U)\chi_0 = E_0\chi_0$, $\quad E_0 = E - \varepsilon_0$

$U \equiv \langle 0|V|0\rangle = \langle 0|V|0\rangle + \langle 0|PVQ(E^{+} - QHQ)^{-1}QVP|0\rangle$

Folding potential Dynamical polarization potential

Feshbach theory 2

$H\Psi = E\Psi, \qquad H = h + K + V \qquad (\Psi = |0\rangle\chi_0 + |1\rangle\chi_1 + \cdots)$

$P + Q = 1$

$PHP\Psi + PHQ\Psi = EP\Psi \cdots (1)$

$QHP\Psi + QHQ\Psi = EQ\Psi \cdots (2)$

From Eq. (2), we have

$Q\Psi = (E - QHQ)^{-1} QHP\Psi \cdots (3)$

By substituting Eq. (3) into Eq. (1), we can get

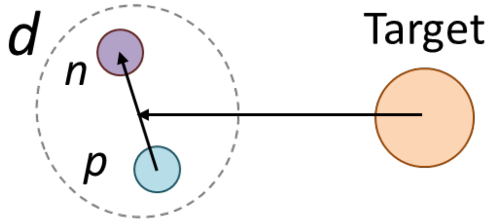
$$\underbrace{(PHP + PHQ(E - QHQ)^{-1} QHP)}_{\equiv H(P)} P\Psi = EP\Psi \cdots (4)$$

Feshbach theory 3

$H\Psi = E\Psi,\qquad H = h + K + V \qquad (\Psi = |0\rangle\chi_0 + |1\rangle\chi_1 + \cdots)$

$$(PHP + PHQ(E - QHQ)^{-1} QHP)P\Psi = EP\Psi \cdots (4)$$

Since P is commutable with h and K,



$$PHP = P(h + K + V)P \cdots (5)$$

$$PHQ = PVQ \cdots (6)$$

$$QHP = QVP \cdots (7)$$

are obtained. By substituting Eqs. (5)-(7) into Eq. (4), we have

ε_0

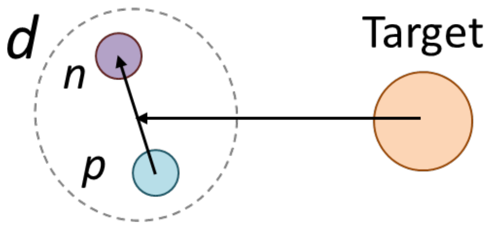
$$P(h + K + V)P + (PVQ(E - QHQ)^{-1} QVP)P\Psi = EP\Psi \cdots (8)$$

$$(K + PVP + PVQ(E - QHQ)^{-1} QVP)P\Psi = E_0 P\Psi \cdots (9)$$

$$E_0 = E - \varepsilon_0$$

Feshbach theory (Summary)

$H\Psi = E\Psi,\qquad H = h + K + V \qquad (\Psi = |0\rangle\chi_0 + |1\rangle\chi_1 + \cdots)$



$$H(P)P\Psi = EP\Psi \ \cdots \text{(A)}$$

$$H(P) = PHP + PHQ(E - QHQ)^{-1}QHP$$

$$(K + PVP + PVQ(E - QHQ)^{-1}QVP)P\Psi = E_0 P\Psi \cdots (9)$$

$$\equiv V$$

$$(K + V)P\Psi = E_0 P\Psi$$

$\times\langle 0|$

$$(K + U)\chi_0 = E_0 \chi_0$$

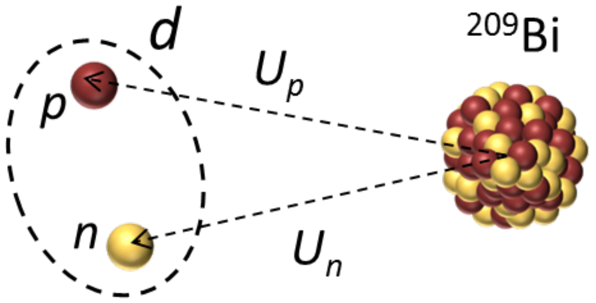
$U \equiv 0V0 = 0V$[illegible]$0 + 0PVQ(E^{+} - QHQ)^{-1}QVP0$

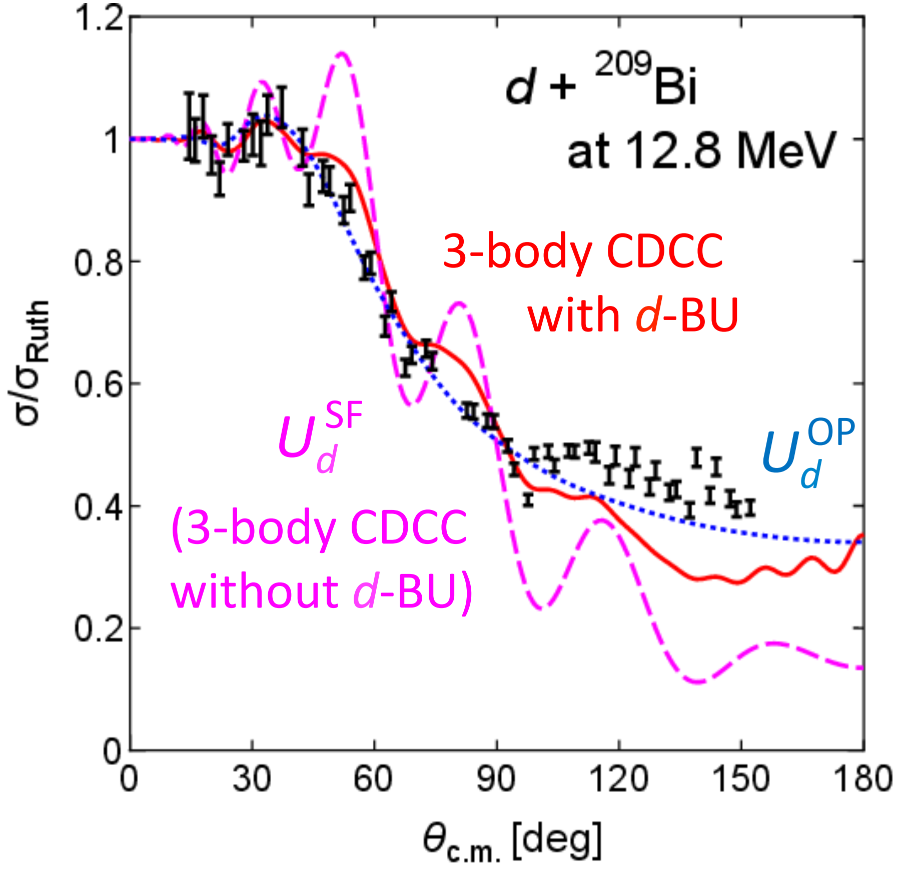
Folding potential Dynamical polarization potential

U: Generalized optical potential

d-breakup effects on d+^{209}Bi scattering

✓ We can check d-breakup effects directly with 3-body CDCC.





experimental data
A. Budzanowski *et al.*, Nuclear Physics **49**, 144 (1963).

■ Definition of U_d

U_d^{OP} : d-optical potential
(with d-breakup)

U_d^{SF} : Single folding potential
(without d-breakup)

d-breakup is significant for d + ^{209}Bi scattering

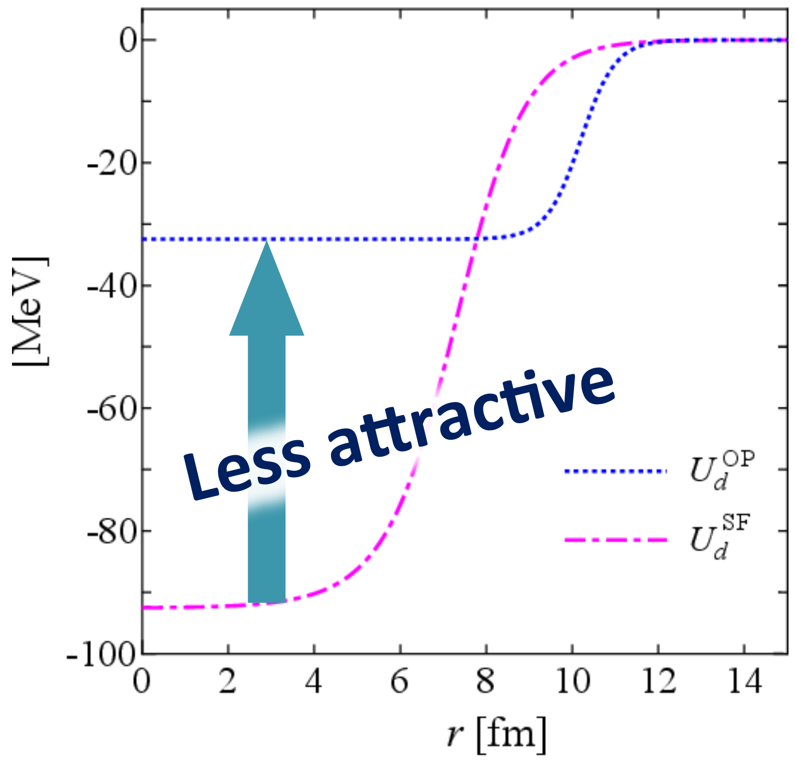
Direct comparison between U_d^{OP} and U_d^{SF}

$$U_d^{\mathrm{OP}} = U_d^{\mathrm{SF}} + 0\, PVQ(E^{+} - QHQ)^{-1}\, QVP\, 0$$

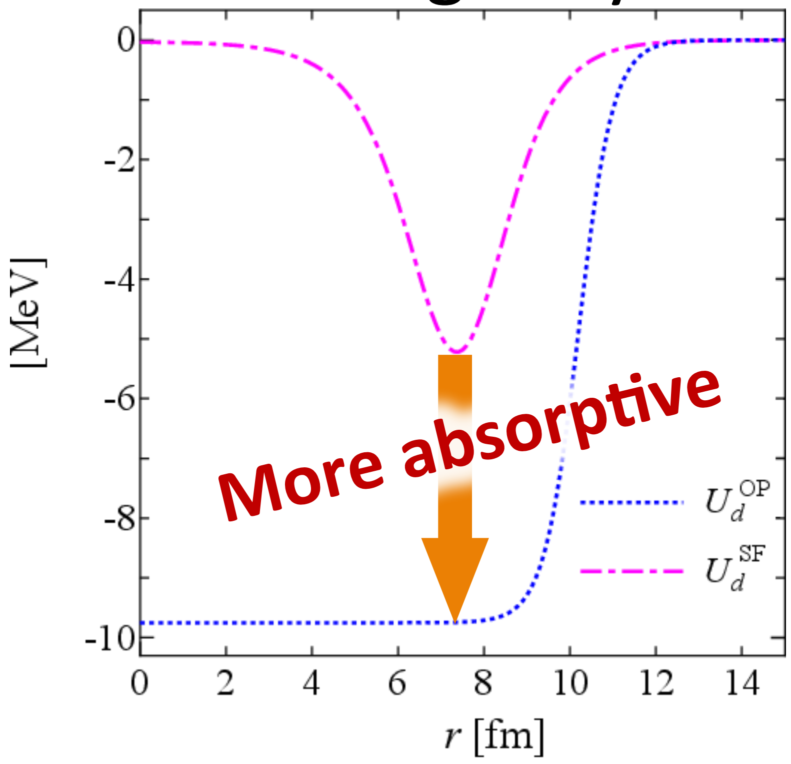
Dynamical polarization potential

⇒ *d-BU effects*

Real

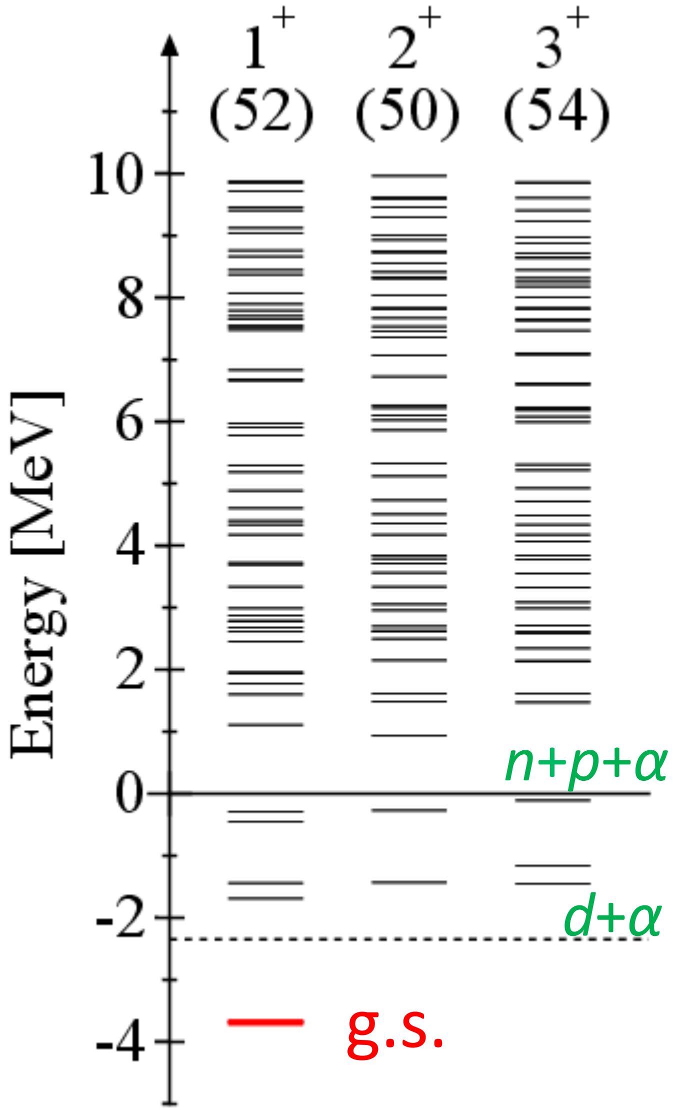


Imaginary



Results for ^{6}Li (Input for reaction calculations)

Energy spectrum obtained by GEM



	I^{π}	ε_0 [MeV]	R_{rms} [fm]
Calc.	1^+	-3.69	2.43
Exp.	1^+	-3.6989	2.44±0.07

Exp. A. V. Dobrovolsky et al., Nucl. Phys. A ***766****, 1 (2006).*
D. R. Tilley et al., Nucl. Phys. A ***708****, 3 (2002).*

✓ Introduce the effective 3-body force

(If $V_{3b} = 0$, $\varepsilon_0 = -2.94$ MeV)

We have no adjustable parameter from now on.