Constraints on the $s - \bar{s}$ asymmetry of the proton in chiral effective theory

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Strange sea in nucleon

- The nature of the quark-antiquark sea complements the valence structure of the proton;
- Strange contribution to the proton spin, electroweak form factors
  J. Ashman *et al.*, Nucl. Phys. B 328 (1989) 1
  R.D. Young *et al.*, Phys. Rev. Lett. 97 (2006) 102002
- Strange distribution from global fit to Deep Inelastic Scattering (DIS) data, most of which assume $s(x) = \bar{s}(x)$
  MMHT14: L.A. Harland-Lang *et al.*, EPJC **75** (2015) 204;
  MRST, Eur. Phys. J. C **4** (1998) 463.

Strange asymmetry

- Perturbative contribution:

$$< x\,[s - \bar{s}] >= -5 \times 10^{-4} \tag{1}$$

S. Catani *et al.*, Phys. Rev. Lett. 93 (2004) 152003

- Non-perturbative contribution:

$$|p\rangle = |p\rangle_0 + |K\Lambda\rangle + \cdots \tag{2}$$

Signal and Thomas, Phys. Lett. B 191 (1987) 205;
H. Holtmann, *et al.*, Nucl. Phys. **A** 569 (1996) 631

While the existence of $s - \bar{s}$ is not surprising, the magnitude and even the sign of the asymmetry has been far more difficult to determine

- Phenomenological analysis subjects to sizeable uncertainties, because of various approximations made about nuclear corrections and functional forms for the PDFs

$$\langle x\,[s - \bar{s}]\rangle = (0 \pm 2) \times 10^{-3}\ ,\ \ Q^2 = 16\ \mathrm{GeV}^2 \qquad (3)$$

W. Benz *et al.*, Phys. Lett. B 693 (2010) 462

- Model calculations also leads to fairly wide range predications

$$\langle x\,[s - \bar{s}]\rangle = (-1 \mathrm{to} + 5) \times 10^{-3} \qquad (4)$$

T.J. Hobbs *et al.*, Phys. Rev. C 91 (2015) 035205

- More systematic approach is needed, which has direct connection to the underlying QCD theory
- Chiral effective theory is such an approach, which preserves chiral symmetry, gauge invariance, and the model independent leading nonanalytic (LNA) behavior

Effective field theory

The $(n-1)$th spin independent (SI) Mellin moments of the quark distribution functions are defined as

$$\langle x^{n-1}\rangle_q^B = \int_0^1 dx x^{n-1}\left(q^B(x) + (-1)^n \bar{q}^B(x)\right) \ . \qquad (5)$$

The operator product expansion (OPE) allows these moments to be related to the matrix elements of local twist-two operators $\mathcal{O}_q$ by

$$\langle N|\mathcal{O}_q|N\rangle = 2\langle x^{n-1}\rangle_q p^{\{\mu_1}\dots p^{\mu_n\}} \ , \qquad (6)$$

where the operators are given by quark bilinears

$$\mathcal{O}_q^{\mu_1\dots\mu_n} = i^{n-1}\bar{q}\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\dots\overleftrightarrow{D}^{\mu_n\}}q \ , \qquad (7)$$

with $\overleftrightarrow{D} = \frac{1}{2}\left(\overrightarrow{D} - \overleftarrow{D}\right)$.

In an effective field theory (EFT), these quark operators are matched to hadronic operators with the same quantum numbers Chen and Ji, Phys. Rev. Lett. **87** (2001) 152002.,

$$\mathcal{O}_q^{\mu_1\cdots\mu_n} = \sum_{j=1}^{\infty} c_{q/j}^{(n)} \mathcal{O}_j^{\mu_1\cdots\mu_n} \ , \tag{8}$$

where j labels different types of hadronic operators.

$$q(x) \ = \ \sum_j \left(f_j \otimes q_j^v\right)(x) \ \equiv \sum_j \int_0^1 dy \int_0^1 dz\, \delta(x-yz)\, f_j(y)\, q_j^v(z)$$

The PDFs $q_j(x)$ in the hadronic configuration j, is related to the coefficients $c_{q/j}^{(n)}$ through their moments

$$\int_{-1}^{1} dx\, x^{n-1}\, q_j(x) \quad = \quad c_{q/j}^{(n)} \tag{9}$$

The hadronic $N \to j$ splitting functions $f_j(y)$ are related to the nucleon matrix elements of the hadronic operators $\mathcal{O}_j^{\mu_1 \cdots \mu_n}$,

$$\int_{-1}^{1} y^{n-1} f_j(y) = \frac{1}{2(P^+)^n} \langle N(p)|\mathcal{O}_j^{+\cdots+}|N(p)\rangle \tag{10}$$

Our work

In our work X.G. Wang *et al.*, arXiv: 1602. 06646 [nucl-th]:

- We construct hadronic operators in $SU(3)$ case;
- We calculate s and $\bar{s}$ distributions within convolution model
- We try to extract its correction to the NuTeV anomaly

Building Block

- Meson field:

$$U = u^2 = \exp(\frac{\sqrt{2}i\phi}{f_P}) \tag{11}$$

where

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

- Baryon field:

$$\mathcal{B}_{ijk} = \frac{1}{\sqrt{6}}\left(\epsilon_{ijk'}B_k^{k'} + \epsilon_{ikk'}B_j^{k'}\right), \tag{12}$$

where

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \tag{13}$$

Hadronic Operator

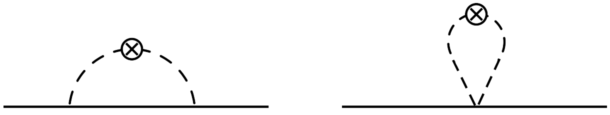
The local twist-two quark operators can be matched to hadronic operators P.E. Shanahan *et al.*, Phys. Rev. D 87 (2013) 114515

$$
\begin{aligned}
&\mathcal{O}_q^{\mu_1\cdots\mu_n} \\
&= a^{(n)} i^n \frac{f_\phi^2}{4}\left\{\mathrm{Tr}\left[U^\dagger \lambda_+^q \partial_{\mu_1}\cdots\partial_{\mu_n} U\right] + \mathrm{Tr}\left[U \lambda_+^q \partial_{\mu_1}\cdots\partial_{\mu_n} U^\dagger\right]\right\} \\
&+ \left[\alpha^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\mathcal{B}\lambda_+^q) + \beta^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\lambda_+^q\mathcal{B}) + \sigma^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\mathcal{B})\,\mathrm{Tr}[\lambda_+^q]\right] p^{\mu_2}\cdot\cdot p^{\mu} \\
&+ \left[\bar{\alpha}^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\gamma_5\mathcal{B}\lambda_-^q) + \bar{\beta}^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\gamma_5\lambda_-^q\mathcal{B})\right. \\
&\left. + \bar{\sigma}^{(n)}(\overline{\mathcal{B}}\gamma^{\mu_1}\gamma_5\mathcal{B})\,\mathrm{Tr}[\lambda_-^q]\right] p^{\mu_2}\cdots p^{\mu_n} \\
&+ \text{permutations} - \mathrm{Tr}
\end{aligned}
$$

Hadronic Operator

The local twist-two quark operators can be matched to hadronic operators P.E. Shanahan *et al.*, Phys. Rev. D 87 (2013) 114515

$$
\begin{aligned}
& \mathcal{O}_q^{\mu_1\cdots\mu_n} \\
= \quad & a^{(n)} i^n \frac{f_\phi^2}{4} \Big\{ \mathrm{Tr}\left[U^\dagger \lambda_+^q \partial_{\mu_1} \cdots \partial_{\mu_n} U \right] + \mathrm{Tr}\left[U \lambda_+^q \partial_{\mu_1} \cdots \partial_{\mu_n} U^\dagger \right] \Big\} \\
+ \quad & \left[\alpha^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \mathcal{B} \lambda_+^q) + \beta^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \lambda_+^q \mathcal{B}) + \sigma^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \mathcal{B}) \, \mathrm{Tr}[\lambda_+^q] \right] p^{\mu_2} \cdot\cdot p^{\mu} \\
+ \quad & \left[\bar{\alpha}^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \gamma_5 \mathcal{B} \lambda_-^q) + \bar{\beta}^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \gamma_5 \lambda_-^q \mathcal{B}) \right. \\
& \left. + \bar{\sigma}^{(n)} (\overline{\mathcal{B}} \gamma^{\mu_1} \gamma_5 \mathcal{B}) \, \mathrm{Tr}[\lambda_-^q] \right] p^{\mu_2} \cdots p^{\mu_n} \\
+ \quad & \text{permutations} - \mathrm{Tr}
\end{aligned}
$$



(a) **(b)**

Figure: Loop contributions to the $\bar{s}$ PDF from the (a) kaon rainbow and (b) kaon bubble diagrams. The crosses $\otimes$ represent insertions of the twist-2 operator.

$$\bar{s}(x) = \left(\sum_{KY} f_{KY}^{(\mathrm{rbw})} + \sum_{K} f_{K}^{(\mathrm{bub})} \right) \otimes \bar{s}_K \quad (14)$$

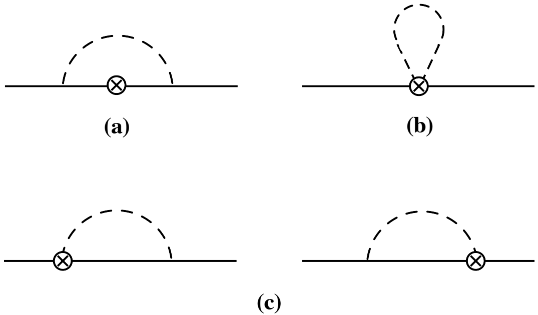


Figure: Loop contributions to the s PDF from the (a) hyperon rainbow, (b) tadpole, and (c) Kroll-Ruderman diagrams.

$$s(x) = \sum_{YK} \left(\bar{f}_{YK}^{(\mathrm{rbw})} \otimes s_Y + \bar{f}_{YK}^{(\mathrm{KR})} \otimes s_Y^{(\mathrm{KR})} \right) + \sum_K \bar{f}_K^{(\mathrm{tad})} \otimes s_K^{(\mathrm{tad})}$$

Hyperon rainbow diagram

The relevant operator is:

$$\mathcal{O}_s^{\mu_1\cdots\mu_n} \sim \left(\frac{1}{2}\alpha^{(n)} + \sigma^{(n)}\right)\left(\bar{\Lambda}\gamma^{\mu_1}\Lambda\right) p^{\mu_2}\cdots p^{\mu_n}$$

- $s_\Lambda(x) = \frac{1}{3}\left[2u(x) - d(x)\right]$
- Splitting function:

$$\begin{aligned} & f_{YK}^{(\mathrm{rbw})}(y) \\ = \; & \frac{C_{KY}^2}{f_P^2}\int\frac{d^4k}{(2\pi)^4}\bar{u}(p)\not{k}\gamma_5\frac{i(\not{p}-\not{k}+M_Y)}{D_Y}\gamma^+\frac{i(\not{p}-\not{k}+M_Y)}{D_Y} \\ & \times\gamma_5\not{k}u(p)\frac{i}{D_K}\delta(k^+ - yp^+) \end{aligned}$$

Using Dirac equation, we can get

$$f_{YK}^{(\mathrm{rb})}(y) = -i\frac{C_{KH}^2}{f_P^2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{A}{D_K D_Y^2}-\frac{B}{D_K D_Y}-\frac{1}{D_K}\right]\delta\left(y-\frac{k^+}{p^+}\right),$$

where

$$\begin{aligned} A &= (M+M_H)^2\left[k^2-2yp\cdot k+2yM(M-M_Y)-(M-M_H)^2\right], \\ B &= 2M(M+M_Y)y-2(M^2-M_Y^2). \end{aligned}$$

$$f_{YK}^{(\mathrm{rb})}(y) = \frac{C_{KY}^2\overline{M}^2}{(4\pi f_P)^2}\left[f_Y^{(\mathrm{on})}(y)+f_Y^{(\mathrm{off})}(y)-f_K^{(\delta)}(y)\right], \qquad (15)$$

By first integrating over k^- using residue theorem,

$$\begin{aligned} f_Y^{(\mathrm{on})}(y) &= y \int dk_\perp^2 \, \frac{k_\perp^2 + [M_Y - (1-y)M]^2}{(1-y)^2 D_{KY}^2} F^{(\mathrm{on})}(y, k_\perp^2) \\ f_Y^{(\mathrm{off})}(y) &= \frac{2}{\overline{M}} \int dk_\perp^2 \, \frac{[M_Y - (1-y)M]}{(1-y) D_{KY}} F^{(\mathrm{off})}(y, k_\perp^2) \\ f_K^{(\delta)}(y) &= \frac{1}{\overline{M}^2} \int dk_\perp^2 \, \log \Omega_K \, \delta(y) \, F^{(\delta)}(y, k_\perp^2) \end{aligned}$$

where

$$D_{KY} \equiv -\frac{1}{1-y}[k_\perp^2 + yM_Y^2 + (1-y)m_K^2 - y(1-y)M^2] \quad (16)$$

is the kaon virtuality for an on-shell hyperon intermediate state, and $\Omega_K = k_\perp^2 + m_K^2$.

Pauli-Villars Regularization

- To regulate the on-shell and off-shell terms, we make the following replacement,

$$\frac{1}{D_K} \to \frac{1}{D_K} - \frac{1}{D_{\mu_1}} \ . \tag{17}$$

where $D_{\mu_1} = k^2 - \mu_1^2$.

$$F^{\rm on} = 1 - \frac{D_{KY}^2}{D_{\mu_1}^2} \ , \ F^{\rm off} = 1 - \frac{D_{KY}}{D_{\mu_1}} \tag{18}$$

- To regulate the δ-function term, we need to introduce two subtraction terms to the kaon propagator,

$$\frac{1}{D_K} \to \frac{1}{D_K} - \frac{a_1}{D_{\mu_1}} - \frac{a_2}{D_{\mu_2}} \ , \tag{19}$$

where

$$a_1 = \frac{\mu_2^2 - m_K^2}{\mu_2^2 - \mu_1^2} \ , \quad a_2 = -\frac{\mu_1^2 - m_K^2}{\mu_2^2 - \mu_1^2} \ . \tag{20}$$

$$F^\delta = 1 - \frac{a_1 \Omega_{\mu_1} + a_2 \Omega_{\mu_2}}{\log \Omega_K} \tag{21}$$

with $\Omega_{\mu_i} = k_\perp^2 + \mu_i^2$.

The only free parameters are the cutoffs μ_1 and μ_2.

Net Strangeness

The rainbow and KR contributions satisfy

$$f_{YK}^{(\mathrm{rbw})} + f_{YK}^{(\mathrm{KR})} = f_{KY}^{(\mathrm{rbw})} \ . \tag{22}$$

Finally, the tadpole contribution is related to the bubble term,

$$f_{K}^{(\mathrm{tad})} = f_{K}^{(\mathrm{bub})} \ . \tag{23}$$

These two conditions guarantee that the net strangeness in the nucleon is zero,

$$\langle s - \bar{s} \rangle = \int_0^1 dx \, [s(x) - \bar{s}(x)] = 0 \ . \tag{24}$$

Determination on μ_1

The differential cross section for $pp \to \Lambda X$ with kaon-exchange is given by H. Holtmann, *et al.*, Nucl. Phys. **A** 569 (1996) 631,

$$E\frac{d^3\sigma(pp\to\Lambda X)}{d^3p}$$

$$= \frac{g^2_{\Lambda NK}}{16\pi^3}\frac{\bar{y}(1-\bar{y})[k_\perp^2+(m_\Lambda-\bar{y}m_N)^2]\mathrm{F}^{(\mathrm{on})}(1-\bar{y},k_\perp^2)}{[k_\perp^2-\bar{y}(1-\bar{y})m_N^2+(1-\bar{y})m_\Lambda^2+\bar{y}m_K^2]^2}\sigma^{Kp}_{tot}(s(1-\bar{y}))$$

where $\bar{y}=1-y$ is the momentum fraction carried by hyperon.

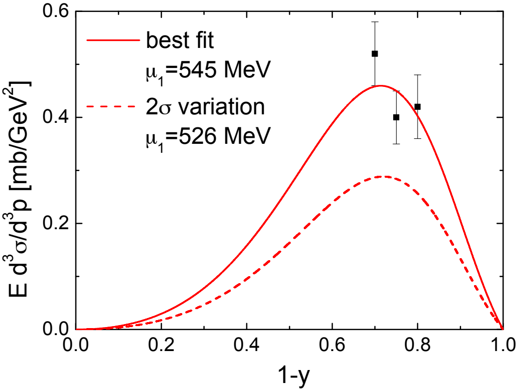


Figure: Differential cross section for the best fit to the $pp \to \Lambda X$ data in the region $y < 0.35$ (solid curve), as a function of $1 - y$ for $k_\perp = 75$ MeV, and for a fit 2σ from the central values (dashed curve).

Determination on μ_2

- For a fixed μ_1, the allowed range for μ_2 with the PV regularization is

$$m_K \leq \mu_2 \leq \mu_2^{\max} \tag{25}$$

- $\mu_2^{\max}$ is fixed by requiring the calculated $s + \bar{s}$ does not exceed the errors on the total phenomenological PDFs

$$(s + \bar{s})_{\rm loops} \leq (s + \bar{s})_{\rm tot} \tag{26}$$

  for any value of x.

- $(s + \bar{s})_{\rm tot}$ is taken from recent global fit MMHT14: L.A. Harland-Lang *et al.*, EPJC **75** (2015) 204; NNPDF3.0: R.D. Ball *et al.*, JHEP **04** (2015) 040.

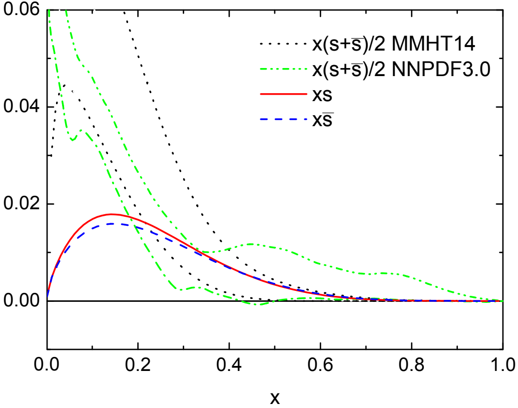


Figure: Comparison between the strange xs (solid red curve) and antistrange $x\bar{s}$ (dashed blue curve) PDFs from kaon loops, for the cutoff parameters ($\mu_1 = 545$ MeV and $\mu_2^{\max} = 600$ MeV), with the global fits.

The Second Moment

At $Q^2 = 1\ \mathrm{GeV}^2$,

- $\mu_1 = 545\ \mathrm{MeV}$, $m_K \leq \mu_2 \leq 600\ \mathrm{MeV}$:
- $\mu_1 = 526\ \mathrm{MeV}$, $m_K \leq \mu_2 \leq 894\ \mathrm{MeV}$:

Within these limits, the second moment

$$S^- = \int_0^1 x\,[s(s) - \bar{s}]\,dx \quad (27)$$

lies in the range

$$-0.07 \times 10^{-3} \leq S^- \leq 1.12 \times 10^{-3} \quad (28)$$

Asymmetric distribution function

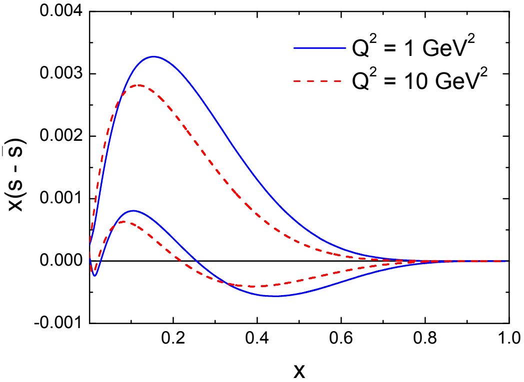


Figure: Strange quark asymmetry $x(s - \bar{s})$ at $Q^2 = 1$ GeV^2 (solid blue curves) and evolved to $Q^2 = 10$ GeV^2 (dashed red curves).

New features

- The contributions from off-shell and δ-function
- δ-function contribution to $\bar{s}$ at $x = 0$:

$$\bar{s}(x) \sim \delta(y) \otimes \bar{s}_K(x/y) \sim \delta(x) \tag{29}$$

Valence like component of s quark PDF:

$$s(x) \sim \delta(1-y) \otimes s_\Lambda(x/y) \sim s_\Lambda(x) \tag{30}$$

- For $s - \bar{s}$, there may be no zero crossing at $x > 0$; conservation of strangeness is ensured by the presence of the nonzero contribution from the δ-function term

NuTeV anomaly

- Possible strange quark asymmetry is of great importance in connection with its contribution to the NuTeV anomaly:

$$\sin^2\theta_W = 0.2277 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst}) \qquad (31)$$

which is approximately 3σ deviation from the average electroweak measurements:

$$\sin^2\theta_W = 0.2227 \pm 0.0004 \qquad (32)$$

G.P. Zeller *et al.*, Phys. Rev. Lett. 88 (2002) 091802

The corrections Δs_W^2 to s_W^2 arising from strange quark asymmetry is given by G. P. Zeller *et al.*, Phys. Rev. D **65** (2002) 111103(R),

$$\Delta s_W^2|_{\rm strange} = \int_0^1 F[s_W^2, s(x) - \bar{s}(x); x] s^-(x) dx \qquad (33)$$

at $Q^2 = 10\ \mathrm{GeV}^2$, where

$$s^-(x) = x\Big[s(x) - \bar{s}(x)\Big] \qquad (34)$$

Varying the μ_1 and μ_2 parameters over their maximally allowed range, we find a correction, $\Delta(\sin^2\theta_W)$, to the weak angle from the strange asymmetry of

$$-7.7 \times 10^{-4} \leq \Delta(\sin^2\theta_W) \leq -6.7 \times 10^{-7} \tag{35}$$

Summary

- We calculate the full set contributions to $s - \bar{s}$ asymmetry within effective field theory,
- Our analysis reveals new contribution to $\bar{s}$ PDF (proportional to $\delta(x)$), as well as small but nonzero valence-like component of s PDF
- Its effect on the NuTeV anomaly is extracted, which only reduces the NuTeV anomaly by less than 0.5 σ.

Thanks for your patience

For each quark flavor q, we introduce

$$\bar{\lambda}^u = \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix}, \quad \bar{\lambda}^d = \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}, \quad \bar{\lambda}^s = \begin{pmatrix} & & \\ & & \\ & & 1 \end{pmatrix}. \tag{36}$$

$\lambda^q_\pm$ is defined as

$$\lambda^q_\pm = \frac{1}{2}\left(u\bar{\lambda}^q u^\dagger \pm u^\dagger \bar{\lambda}^q u\right), \tag{37}$$

Expand the operators, we find

$$\mathcal{O}_u^{\mu_1\cdots\mu_n} \sim \left(\frac{5}{6}\alpha^{(n)} + \frac{1}{3}\beta^{(n)} + \sigma^{(n)}\right)\left(\bar{p}\gamma^{\mu_1}p\right) p^{\mu_2}\cdots p^{\mu_n}$$

$$\mathcal{O}_d^{\mu_1\cdots\mu_n} \sim \left(\frac{1}{6}\alpha^{(n)} + \frac{2}{3}\beta^{(n)} + \sigma^{(n)}\right)\left(\bar{p}\gamma^{\mu_1}p\right) p^{\mu_2}\cdots p^{\mu_n}$$

$$\mathcal{O}_s^{\mu_1\cdots\mu_n} \sim \sigma^{(n)}\left(\bar{p}\gamma^{\mu_1}p\right) p^{\mu_2}\cdots p^{\mu_n}$$

Within convolution model, the coefficients are related to the PDFs in bare proton,

$$\frac{5}{6}\alpha^{(n)} + \frac{1}{3}\beta^{(n)} + \sigma^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\, u(x),$$

$$\frac{1}{6}\alpha^{(n)} + \frac{2}{3}\beta^{(n)} + \sigma^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\, d(x),$$

$$\sigma^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\, s(x)$$

Solving the above equations, these coefficients can be obtained in terms of the proton PDFs,

$$\alpha^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\Big(\frac{4}{3}u(x) - \frac{2}{3}d(x)\Big),$$

$$\beta^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\Big(-\frac{1}{3}u(x) + \frac{5}{3}d(x)\Big)$$

$$\sigma^{(n)} = 0$$

Interaction Lagrangian

The lowest order effective chiral Lagrangian of pseudoscalar and octet baryons, consistent with chiral symmetry, can be written as

$$\mathcal{L} = i\langle \bar{B}\gamma_\mu [D^\mu, B]\rangle - \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle - \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle ,$$

where

$$u_\mu = i\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right) . \tag{38}$$

kaon rainbow diagram

- Splitting function:

$$f_{KY}^{(\mathrm{rbw})}(y) = \frac{C_{KY}^2 \overline{M}^2}{(4\pi f_P)^2} \left[f_Y^{(\mathrm{on})}(y) + f_K^{(\delta)}(y) \right], \qquad (39)$$

- Valence PDFs of kaon:

$$\bar{s}_{K^+} = \bar{s}_{K^0} = \bar{d}_{\pi^+} \ . \qquad (40)$$

$\bar{d}_{\pi^+}$ is taken from the recent fit by Aicher *et al.*, Phys. Rev. Lett. **105** (2010) 252003.

hyperon tadpole diagram

- Splitting function:

$$f_{K^+}^{(\mathrm{tad})} = 2f_{K^0}^{(\mathrm{tad})} = -\frac{\overline{M}^2}{(4\pi f_P)^2} f_K^{(\delta)} \tag{41}$$

- For the strange PDF at the *ppKK* vertex of the tadpole diagram, one has

$$s_{K^+}^{(\mathrm{tad})} = u/2 \ , \ s_{K^0}^{(\mathrm{tad})} = d \ . \tag{42}$$

MRST, Eur. Phys. J. C **4** (1998) 463.

Kroll-Ruderman diagram

- Splitting function:

$$f_{YK}^{(\mathrm{KR})}(y) = \frac{C_{KY}^2 \overline{M}^2}{(4\pi f_P)^2} \left[-f_Y^{(\mathrm{off})}(y) + 2 f_K^{(\delta)}(y) \right], \tag{43}$$

- For the KR diagrams, the strange PDFs at the KNY vertex can be related to the spin-dependent PDFs in the proton,

$$\begin{aligned} s_\Lambda^{(\mathrm{KR})} &= (2\Delta u - \Delta d)/(3F + D) , \\ s_{\Sigma^+}^{(\mathrm{KR})} &= s_{\Sigma^0}^{(\mathrm{KR})} = \Delta d/(F - D) , \end{aligned} \tag{44}$$

E. Leader *et al.*, Phys. Rev. D **82** (2010) 114018.