Quark spin polarization and Spontaneous magnetization in high density quark matter

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Introduction

- Quark and/or hadronic matter at finite density

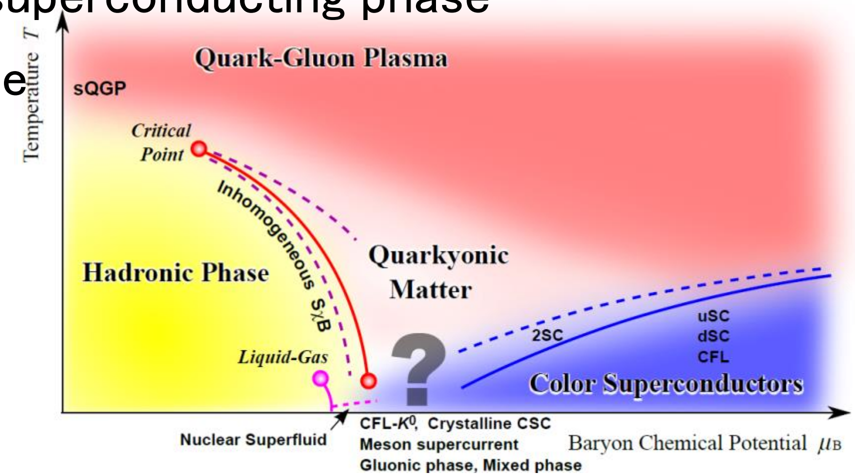
  ー low density・・・hadronic phase : chiral broken phase

  ー high density・・・quark-gluon or color superconducting phase

  : chiral symmetric phase

- Possibility of quark spin polarization ?

  → Spontaneous quark magnetization ?



@K.Fukushima and T. Hatsuda

We expect that the spin polarization leads to spontaneous magnetization.

Consider a possibility of spontaneous spin polarization in quark matter at high baryon density

Introduction

- We show ・・・

  By using the NJL model with tensor–type four–point interaction between quarks (if there exists the tensor–type four–point interaction) ,

  ♪ quark spin polarization may occurs at high density quark matter

  even in chiral symmetric phase (quark mass is zero),

  cf. pseudovector interaction

  (cf, E.Nakano, T.Maruyama and T.Tatsumi, PRD 68 (2003) 105001)

  — spin polarization disappears in chiral symmetric phase

  due to quarks being massless (S.Maedan, PTP 118 (2007) 729)

  ♪ spontaneous magnetization may occurs at high density quark matter

  due to quark spin polarization and anomalous magnetic moment of quark

Spin polarized phase --- two-flavor case

- Consider high density (and low temperature) quark matter in two-flavor

Is the spin polarized phase occurs at high density ?

- Model used here is the NJL model with tensor-type interaction :

$$L = \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi + G_S \left((\bar{\psi}\psi)^2 + \cdots \right) - \frac{G_T}{4} \left((\bar{\psi}\gamma^{\mu}\gamma^{\nu}\vec{\tau}\psi)(\bar{\psi}\gamma_{\mu}\gamma_{\nu}\vec{\tau}\psi) + \cdots \right)$$

Here, $\gamma^1\gamma^2 = -i\Sigma_3 = -i \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

Then, $\langle \bar{\psi}\gamma^1\gamma^2\vec{\tau}\psi \rangle \neq 0 \quad \longrightarrow$ quark spin polarization occurs

- Hereafter, $\mu = 1, \nu = 2$ are taken into account.

Mean field approximation

- Lagrangian density under the mean field approximation :

$$L_{MFA} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M\right)\psi - F\left(\bar{\psi}\Sigma_3\psi\right) - \frac{M^2}{4G_S} - \frac{F^2}{2G_T}$$

Here, $M = -2G_S\langle\bar{\psi}\psi\rangle$, dynamical quark mass (chiral cond.)

$F = -G_T\langle\bar{\psi}\Sigma_3\psi\rangle$, spin polarized (tensor) condensate

- Energy eigenvalue :

$$E_{\vec{p}}^{(\eta)} = \sqrt{p_3^2 + \left(\sqrt{p_1^2 + p_2^2 + M^2} + \eta F\right)^2}, \quad (\eta = \pm 1)$$

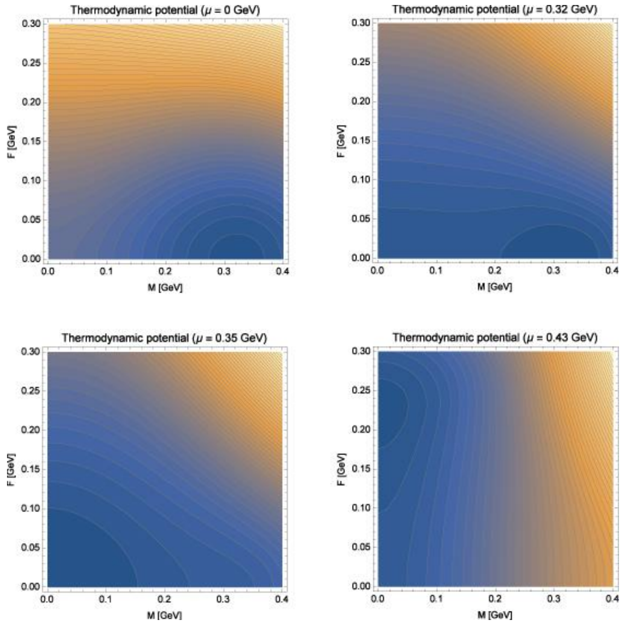
Thermdynamic potential

- Thermodynamic potential ：

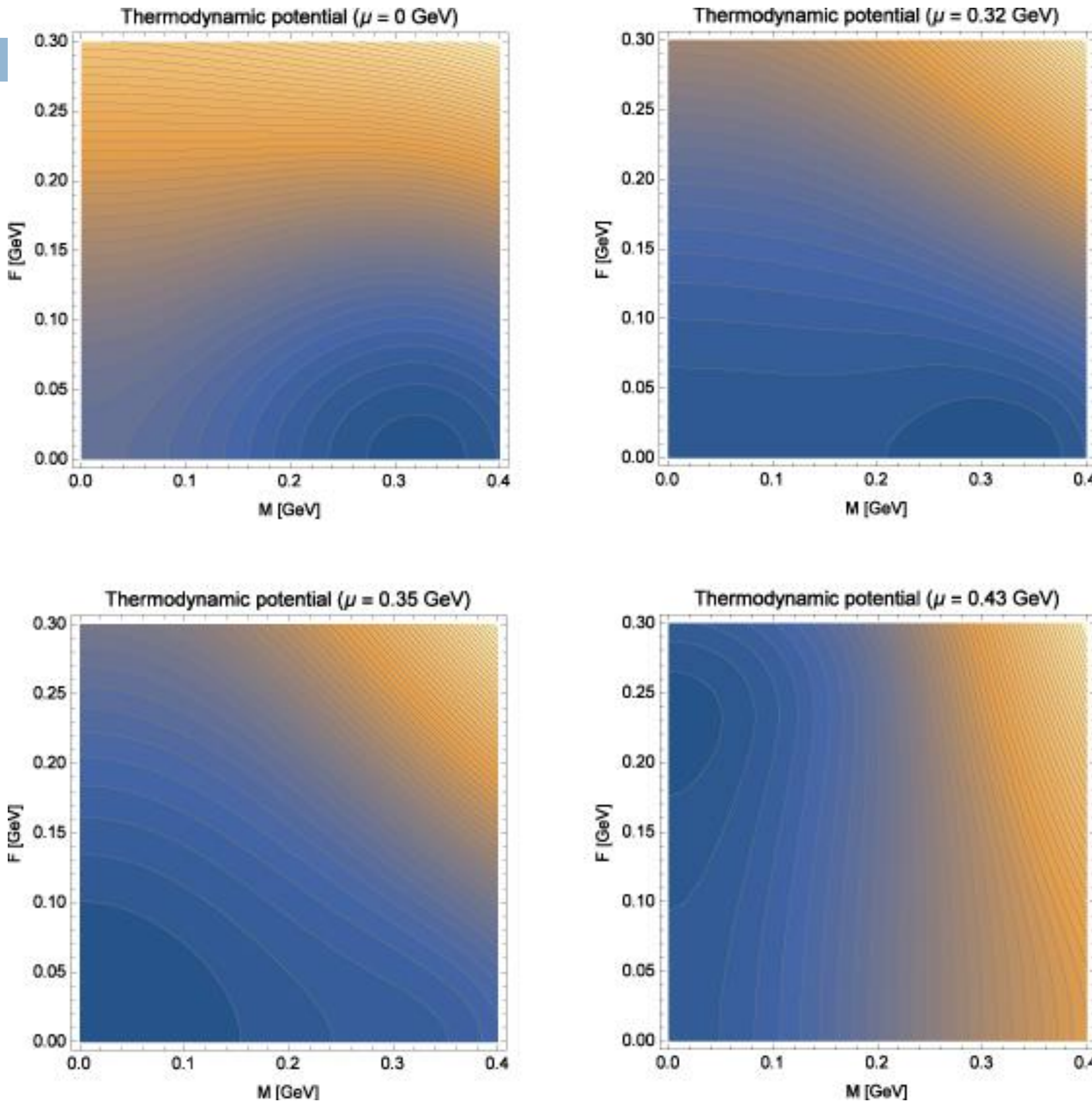
$$\Omega = H - \mu N - TS$$

$$= -\sum_{\vec{p},\eta,\tau,\beta}\left[E_{\vec{p}}^{(\eta)} + T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right)\right) + T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}+\mu}{T}\right)\right)\right] + \frac{M^2}{4G_S} + \frac{F^2}{2G_T}$$

Here, $\mu\cdots$ chemical potential, $T\cdots$ temperature



Thermdynamic potential

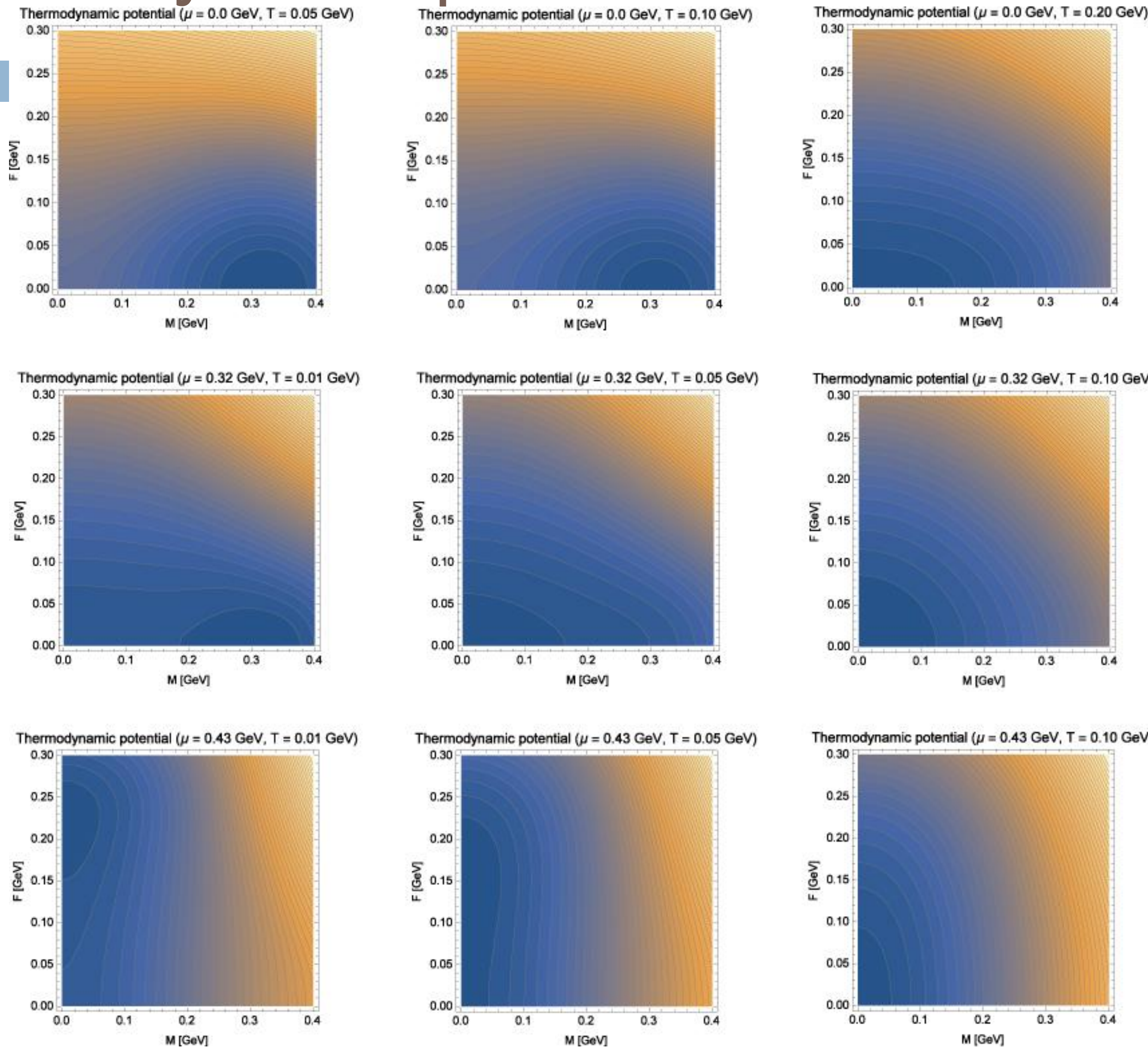


$$G = 5.5\,\mathrm{GeV}^{-2}\,,$$

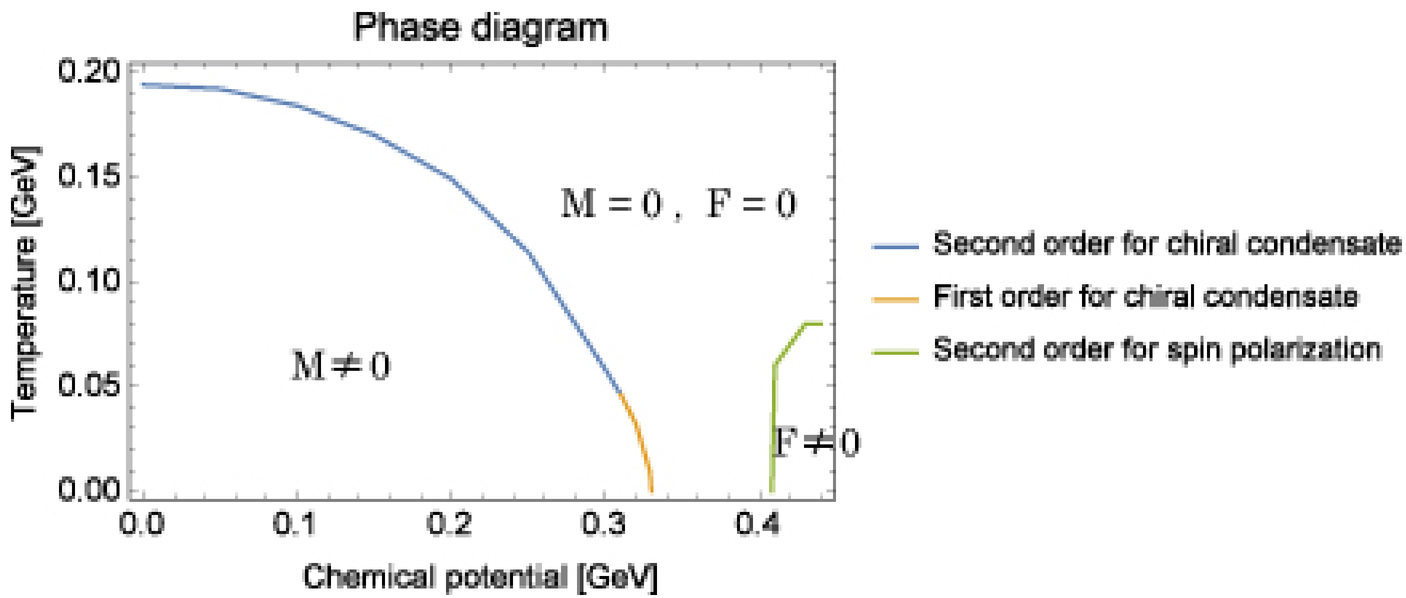
$$G_T = 11.0\,\mathrm{GeV}^{-2}\ (= 2G)$$

$$\Lambda = 0.631\,\mathrm{GeV}$$

Thermdynamic potential



Phase diagram



- In the region with low temperature and large chemical potential, the spin polarized phase ($F \neq 0$) may be realized.
- The order of the phase transition is of the second order from normal quark matter to the spin polarized phase.

Spontaneous Magnetization

Spontaneous magnetization

- Consider high density (and zero temperature) quark matter in two-flavor

Does the spin polarization leads to
The spontaneous magnetization ?

- Tensor-type four-point interaction in the NJL model

with the anomalous magnetic moment of quark :

$$L^{AMM} = L - \frac{i}{2}\bar{\psi}\mu_A \gamma^\mu \gamma^\nu F_{\mu\nu}\psi$$

$$\mu_A 1 = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix}, \quad \mu_u = 1.85\mu_N , \quad \mu_d = -0.97\mu_N ; \quad \mu_N = \frac{e}{2m_p} = 3.15\times 10^{-17} \text{ GeV/T}$$

(R.G.Felipe, A.P.Martinez, H.P.Rojas and M.Orsaria, Phys. Rev. C 77 (2008), 015807.)

Spontaneous magnetization

- Let us adopt the mean field approximation under the external magnetic field B : ($\langle \bar{\psi}\psi \rangle = 0$)

$$L_{MFA}^{AMM} = \bar{\psi} i\gamma^{\mu} D_{\mu}\psi - \bar{\psi}(F + \mu_A B)\Sigma_3 \psi - \frac{F^2}{2G_T}$$

$$D_{\mu} = \partial_{\mu} + iQA_{\mu}\ , \qquad A_{\mu} = \left(0, \frac{By}{2}, -\frac{Bx}{2}, 0\right) = (0, -\vec{A}), \quad (M = 0)$$

$$Q = Q_f = \frac{2}{3}e \quad \text{for} \quad (f =)\, u\text{-quark} \ \text{ or } \ -\frac{1}{3}e \quad \text{for} \quad (f =)\, d\text{-quark}$$

- Energy eigenvalue

$$E_{p_3, f, \nu}^{(\eta)} = \sqrt{\left(\tilde{F}_f + \eta\sqrt{2|Q_f|B\nu}\right)^2 + p_3^2}\ , \quad \nu = 0, 1, 2, \cdots \ \text{ for } \ \eta = 1$$

$$\nu = 1, 2, \cdots \quad \text{for} \ \ \eta = -1$$

$$\tilde{F}_f = F + \mu_f B$$

Spontaneous magnetization

- Thermodynamic potential :

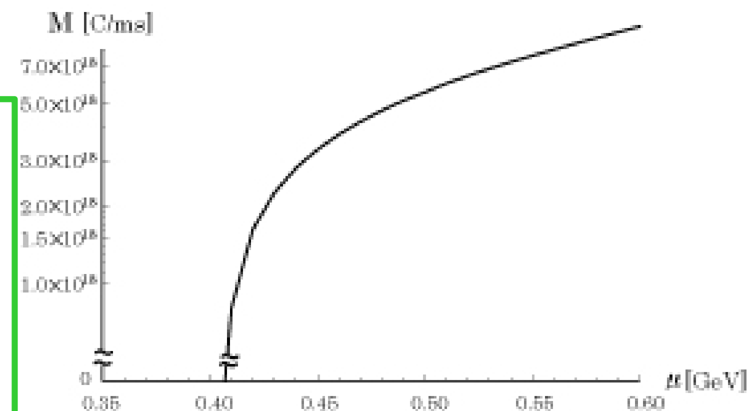
$$\Phi = 3\int_{-p_F}^{p_F}\frac{dp_3}{2\pi}\sum_{f=u,d,\eta=\pm}\frac{|Q_f|B}{2\pi}\sum_{\nu=\nu_{\min}^{f,\eta}}^{\nu_{\max}^{f,\eta}}\left[\sqrt{\left(\tilde{F}_f+\eta\sqrt{2|Q_f|B\nu}\right)^2+p_3^2}-\mu\right]+\frac{F^2}{2G_T}$$

$$\left(E_{\vec{p},\nu,\eta}^{f}\leq\mu\right)$$

- Spontaneous magnetization M can be derived by the thermodynamic relation:

$$M=-\left.\frac{\partial\Phi}{\partial B}\right|_{B=0}=\frac{F}{2G_T}\left(\mu_u+\mu_d\right)\qquad \text{for}\quad 0\leq\mu\leq F$$

$$M=-\left.\frac{\partial\Phi}{\partial B}\right|_{B=0}=\frac{\mu^3}{4\pi}\left(\mu_u+\mu_d\right)\qquad \text{for}\quad F\geq\mu$$



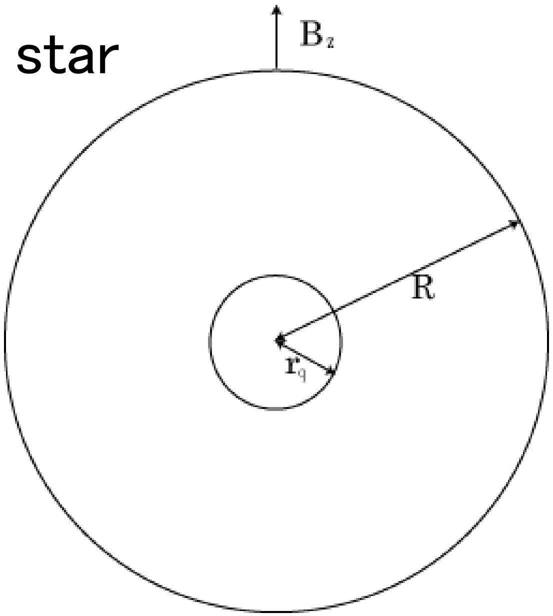
Spontaneous Magnetization

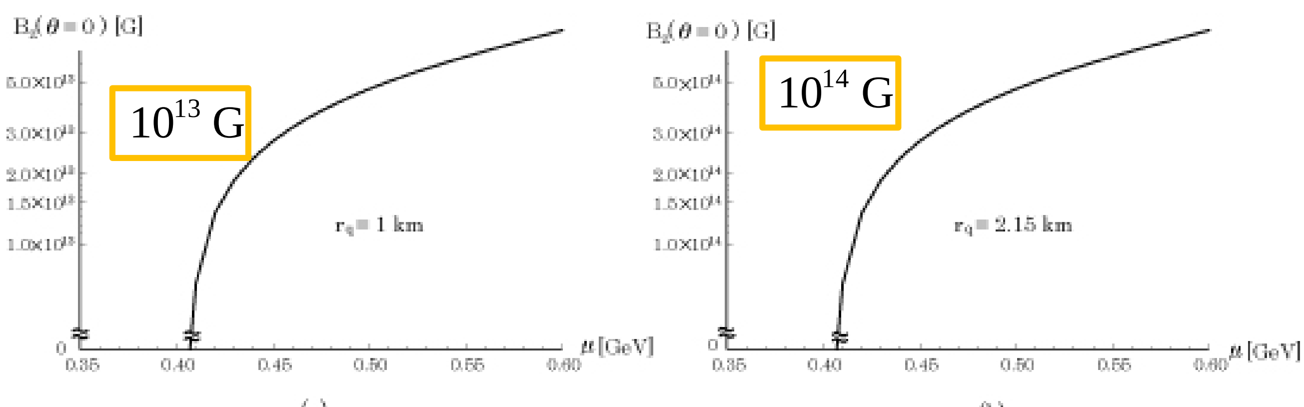
Spontaneous magnetization

- Application : Magnetic field of hybrid compact star
  - magnetic flux density

$$\vec{B} = \frac{\mu_0}{4\pi}\left[-\frac{\vec{m}}{r^3} + \frac{3\vec{r}\left(\vec{m}\cdot\vec{r}\right)}{r^5}\right] \quad , \quad \left(\mu_0 = 4\pi\times10^{-7}\ \mathrm{N/A}^2\right)$$

$$B_z = \frac{\mu_0}{4\pi}\left(-\frac{M}{R^3} + \frac{3z^2M}{R^5}\right)\times\frac{4}{3}\pi r_q^3 = \mu_0\frac{2Mr_q^3}{3R^3}$$





Magnete flux density

Summary

- We have shown ・・・

  ♪ tensor–type four–point interaction between quarks leads to

  — the spin polarization of quark matter in the region of high baryon density and low temperature

  ♪ spontaneous magnetization may occur due to the anomalous magnetic moment of quark under the existence of the quark spin polarization

Back up

Spin polarization versus color superconducting phase

Spin polarized phase --- two-flavor case

- Tensor-type four-point interaction in the NJL) model :

$$L = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi + G_S\left(\left(\bar{\psi}\psi\right)^2 + \cdots\right) - \frac{G_T}{4}\left(\left(\bar{\psi}\gamma^{\mu}\gamma^{\nu}\vec{\tau}\psi\right)\left(\bar{\psi}\gamma_{\mu}\gamma_{\nu}\vec{\tau}\psi\right) + \cdots\right)$$
$$+ \frac{G_C}{2}\sum_{A=2,5,7}\left(\left(\bar{\psi} i\gamma_5\tau_2\lambda_A\psi^C\right)\left(\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi\right) + \cdots\right)$$

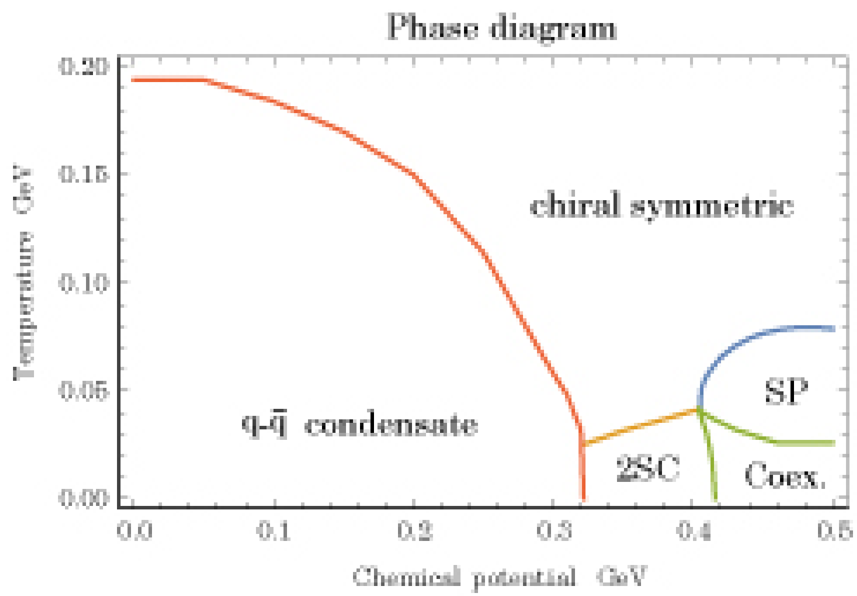
- Energy eigenvalue :

$$E_j = \sqrt{\varepsilon_j^2 + \Delta^2}$$

$$\varepsilon_j = \sqrt{p_3^2 + \left(\sqrt{p_1^2 + p_2^2} + \eta F\right)^2} \pm \mu$$

$$\Delta = \Delta_2 = \Delta_5 = \Delta_7 = -G_C\left\langle \bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi \right\rangle$$

Phase diagram



note

2SC : 2-flavor colorcuperconducting phase

SP : spin polarized phase

Coex. : coexisting phase with 2SC and SP condensates

Possible phases
under the strong external magnetic field

Possible phases under strong external magnetic field

- Consider high density (and zero temperature) quark matter in two-flavor under the strong external magnetic field

What phases do appear ?

- The case of

  (i) Tensor-type four-point interaction in the NJL model

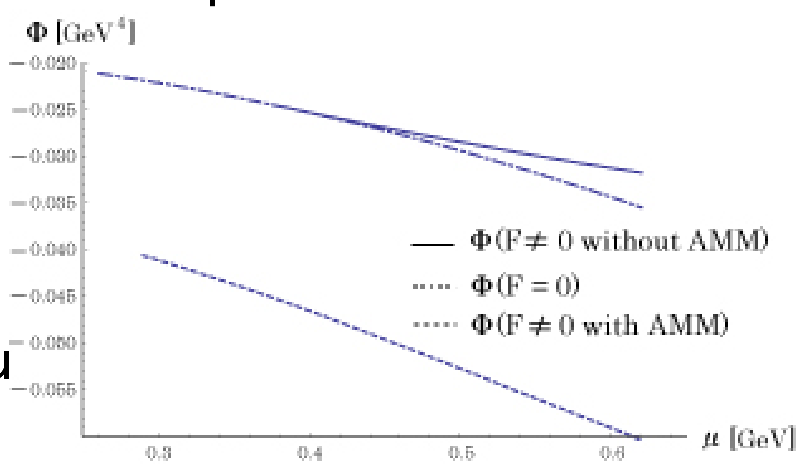
  with the anomalous magnetic moment of quark :

$$eB = 0.597\ \mathrm{GeV}^2 \ \left(\approx m_\rho^2\right)$$

$$\left(B \approx 10^{20}\ \mathrm{G}\right)$$

↓

Only Lowest Landau Level contribu



Possible phases under strong external magnetic field

- The case of

  (ii) Pseudovector-type four-point interaction in the NJL model

  with the anomalous magnetic moment of quark :

$$L = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi + G_S\left((\bar{\psi}\psi)^2 + \cdots\right) - \frac{G_P}{2}\left((\bar{\psi} i\gamma_5\gamma^{\mu}\vec{\tau}\psi)(\bar{\psi} i\gamma_5\gamma_{\mu}\vec{\tau}\psi) + \cdots\right)$$

$$L_{MFA}^{AMM} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - M\right)\psi - U\psi^{+}\Sigma_3\psi - \frac{M^2}{4G_S} - \frac{U^2}{2G_p}$$

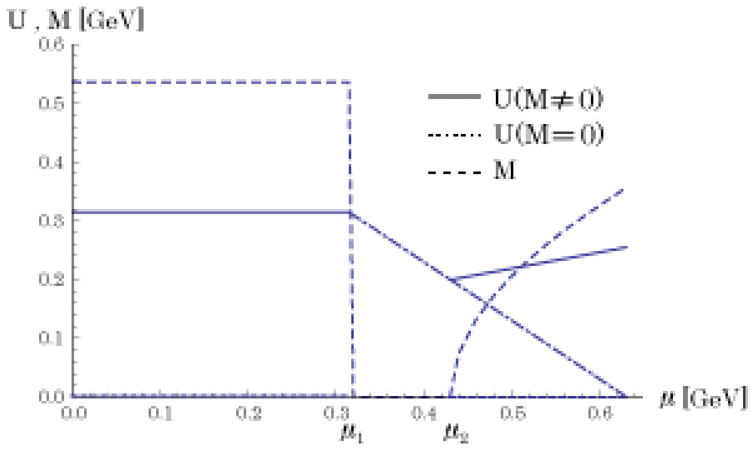
$$U = G_p\langle\psi^{+}\Sigma_3\psi\rangle$$

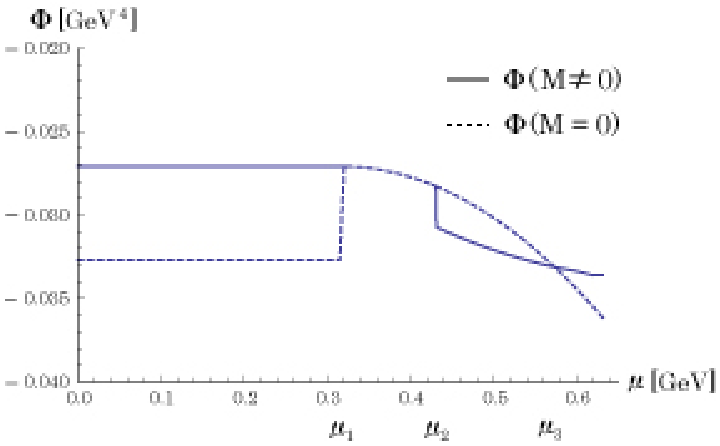
$$E^{(\eta)}_{p_3,f,\nu} = \sqrt{2|Q_f|B\nu + \left(\eta U + \sqrt{p_3^2 + M^2}\right)^2}\ , \quad \nu = 0, 1, 2, \cdots \ \text{for}\ \eta = 1$$

$$\nu = 1, 2, \cdots \quad \text{for}\ \eta = -1$$

Possible phases under strong external magnetic field

- Only Lowest Landau Level contributes





Origin of the tensor-type four-point interaction between quarks

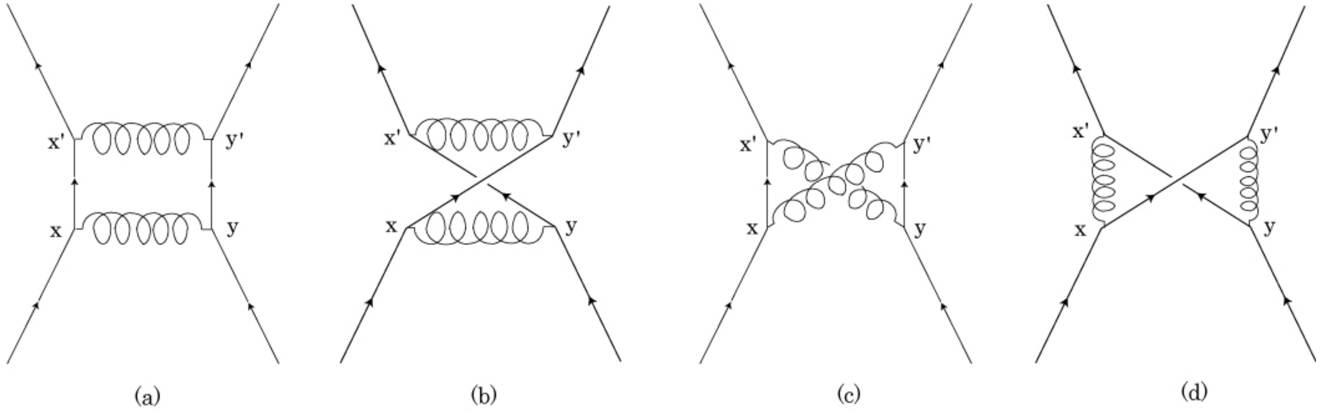
Two-gluon exchange

- Gauge interaction : $L_{QCD}^{\text{int}} = g\overline{\psi}(x)\gamma^{\mu}A_{\mu}(x)\psi(x)$

- Two-gluon exchange : fourth-order perturbation

$$\int d^4x\, L_{QCD}^{\text{int}}(x)\int d^4y\, L_{QCD}^{\text{int}}(y)\int d^4z\, L_{QCD}^{\text{int}}(z)\int d^4u\, L_{QCD}^{\text{int}}(u) = g^4\times\cdots$$

- Proper two-gluon exchange diagram : (c) and (d)



Two-gluon exchange

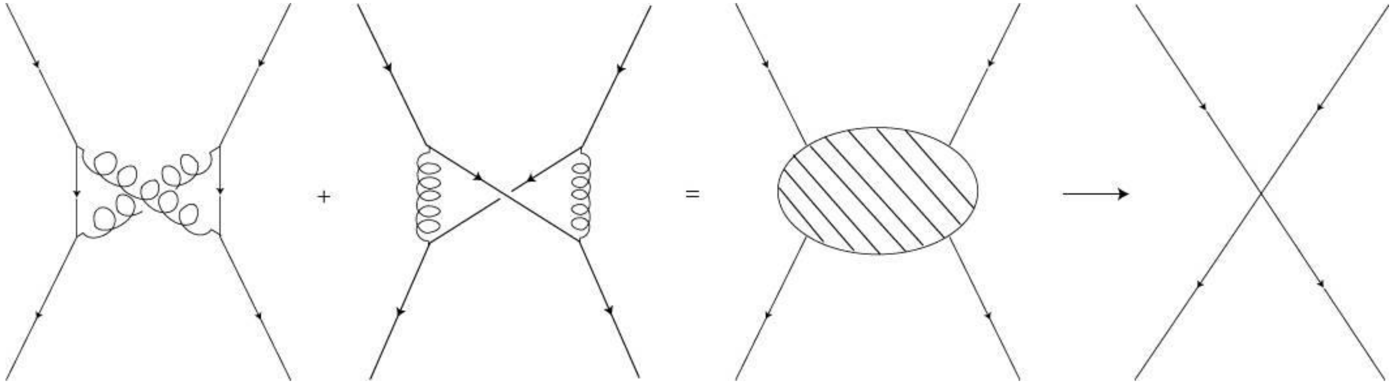
- "Approximation" to NJL model :

$$\left\langle \psi_i(x)\overline{\psi}_j(y)\right\rangle \approx \int \frac{d^4p}{i(2\pi)^4}\frac{1_{ij}}{M_q}e^{-ip(x-y)} = \frac{1}{iM_q}\delta_{ij}\delta^4(x-y)$$

$$\left\langle A_\mu^a(x)A_\nu^b(y)\right\rangle = \delta^{ab}\int \frac{d^4p}{i(2\pi)^4}\frac{1}{p^2+i\varepsilon}\left[g_{\mu\nu}-(1-\alpha)\frac{p_\mu p_\nu}{p^2}\right]e^{-ip(x-y)}$$

$$\approx \int \frac{d^4p}{i(2\pi)^4}\frac{g_{\mu\nu}}{-M_g^2}e^{-ip(x-y)} = \frac{1}{iM_g^2}\delta^{ab}g_{\mu\nu}\delta^4(x-y)$$

- Two-gluon exchange to four-point interaction :



Two-gluon exchange

- "Approximation" to NJL model :

$$\int d^4x\, L_{eff}^{NJL} = g^4 \int d^4x \int d^4y \int d^4z \int d^4u \left(L_{(c)} + L_{(d)} \right)$$

$$= \int d^4x \left(-\frac{G_T}{4} \bar{\psi}\gamma^\mu\gamma^\nu\psi \cdot \bar{\psi}\gamma_\mu\gamma_\nu\psi + 2G_T\bar{\psi}\psi \cdot \bar{\psi}\psi \right)$$

$$G_T = \frac{8g^4\delta^4(0)\left(\sum_a T^a T^a \right)^2}{M_q^2 M_g^4}$$

Interplay between spin polarization and color superconductivity at zero temperature

Interplay between spin polarization and color superconductivity

- Lagrangian density with 2-flavor color superconductivity

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_T + \mathcal{L}_c$$

$$\mathcal{L}_c = \frac{G_c}{2} \sum_{A=2,5,7} \left((\bar{\psi} i\gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i\gamma_5 \tau_2 \lambda_A \psi) + (\bar{\psi} \tau_2 \lambda_A \psi^C)(\bar{\psi}^C \tau_2 \lambda_A \psi) \right)$$

$$L_0 = \bar{\psi} i\gamma^\mu \partial_\mu \psi \ , \quad L_T = -\frac{G}{4}\left(\left(\bar{\psi}\gamma^\mu\gamma^\nu\tau_k^f\psi\right)\left(\bar{\psi}\gamma_\mu\gamma_\nu\tau_k^f\psi\right) + \left(\bar{\psi}i\gamma_5\gamma^\mu\gamma^\nu\psi\right)\left(\bar{\psi}i\gamma_5\gamma_\mu\gamma_\nu\psi\right) \right)$$

- Mean field approximation

$$\mathcal{L}^{MF} = \mathcal{L}_0 + \mathcal{L}_T^{MF} + \mathcal{L}_c^{MF} \ ,$$

$$\mathcal{L}_T^{MF} = -F(\bar{\psi}\Sigma_3\tau_3\psi) - \frac{F^2}{2G} \ , \quad (F = -G\langle\bar{\psi}\Sigma_3\tau_3\psi\rangle)$$

$$\mathcal{L}_c^{MF} = -\frac{1}{2} \sum_{A=2,5,7} (\Delta\bar{\psi}^C i\gamma_5\tau_2\lambda_A + h.c.) - \frac{3\Delta^2}{2G_c} \ ,$$

$$\Delta_A = \Delta_A^* = -G_c\langle\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi\rangle \ , \qquad \Delta = \Delta_2 = \Delta_5 = \Delta_7$$

Interplay between spin polarization and color superconductivity

- Hamiltonian formalism

$$\mathcal{H}_{MF} - \mu\mathcal{N} = \mathcal{K}_0 + \mathcal{H}_T^{MF} + \mathcal{H}_c^{MF} \ ,$$
$$\mathcal{K}_0 = \bar{\psi}(-\gamma \cdot \nabla - \mu\gamma_0)\psi \ ,$$
$$\mathcal{H}_T^{MF} = -\mathcal{L}_T^{MF} \ , \qquad \mathcal{H}_c^{MF} = -\mathcal{L}_c^{MF}$$

- Hamiltonian for quark and antiquark

$$H_{MF} - \mu N = \sum_{\mathbf{p}\eta\tau\alpha} \left[(p-\mu)c^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}\eta\tau\alpha} - (p+\mu)\tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}\eta\tau\alpha}\right]$$
$$+ F\sum_{\mathbf{p}\eta\tau\alpha}\phi_\tau\left[\frac{\sqrt{p_1^2+p_2^2}}{p}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}-\eta\tau\alpha} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}-\eta\tau\alpha}\right) - \eta\frac{p_3}{p}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}\eta\tau\alpha} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}\eta\tau\alpha}\right)\right]$$
$$+ \frac{\Delta}{2}\sum_{\mathbf{p}\eta\alpha\alpha'\alpha''\tau\tau'}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}c^\dagger_{-\mathbf{p}\eta\tau'\alpha'} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}^\dagger_{-\mathbf{p}\eta\tau'\alpha'} + c_{-\mathbf{p}\eta\tau'\alpha'}c_{\mathbf{p}\eta\tau\alpha} + \tilde{c}_{-\mathbf{p}\eta\tau'\alpha'}\tilde{c}_{\mathbf{p}\eta\tau\alpha}\right)\phi_\tau\epsilon_{\alpha\alpha'\alpha''}\epsilon_{\tau\tau'}$$
$$+ V\cdot\frac{F^2}{2G} + V\cdot\frac{3\Delta^2}{2G_c}$$

where $\eta = \pm 1 \cdots$ helicity , $\tau = \pm 1 \cdots$ isospin $(\phi_\pm = \pm 1)$, $\alpha \cdots$ color

Mean Field Approximation – for color-superconducting gap Δ and spin polarization F

Mean field approximation

$$H_{\mathrm{MF}} - \mu N = \sum_{p\eta\tau\alpha} \left[(\varepsilon_p^{(\eta)} - \mu) a^\dagger_{p\eta\tau\alpha} a_{p\eta\tau\alpha} - (\varepsilon_p^{(\eta)} + \mu) \tilde{a}^\dagger_{p\eta\tau\alpha} \tilde{a}_{p\eta\tau\alpha} \right]$$
$$+ \frac{\Delta}{2} \sum_{p\eta\tau\tau'\alpha\alpha'\alpha''} f(\eta) \left[a^\dagger_{p\eta\tau\alpha'} a^\dagger_{-p\eta\tau'\alpha''} - \tilde{a}^\dagger_{p\eta\tau\alpha'} \tilde{a}^\dagger_{-p\eta\tau\alpha''} + h.c. \right] \epsilon_{\alpha\alpha'\alpha''} \epsilon_{\tau\tau'} \phi_\tau$$
$$+ V \cdot \frac{F^2}{2G} + V \cdot \frac{3\Delta^2}{2G_c}.$$

$$\varepsilon_p^{(\pm)} = \sqrt{p_3^2 + \left(F \pm \sqrt{p_1^2 + p_2^2} \right)^2} \quad , \quad f(\eta) = \frac{p + \eta e}{\varepsilon_p^{(\eta)}}, \quad \left(e = F \frac{\sqrt{p_1^2 + p_2^2}}{p} \right)$$

$$\Delta_A = \Delta_A^* = -G_c \langle \bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi \rangle, \quad \Delta = \Delta_2 = \Delta_5 = \Delta_7$$

$$F\,\tau_k = -G \langle \bar{\psi} \Sigma^3 \tau_k \psi \rangle$$

$a_{p\eta\tau\alpha} \cdots$ positive energy states, $\tilde{a}_{p\eta\tau\alpha} \cdots$ negative energy states

Quark matter ・・・ positive energy particles are retained

Interplay between spin polarization and color superconductivity

- BCS state for positive energy particles

$$|\Psi\rangle = e^{S}|\Psi_0\rangle, \qquad |\Psi_0\rangle = \prod_{p\eta\tau\alpha\ (\varepsilon_p^{(\eta)}<\mu)} a^{+}_{p\eta\tau\alpha}|0\rangle,$$

$$S = \sum_{p\eta(\varepsilon_p^{(\eta)}>\mu)} \frac{K_{p\eta}}{2} \sum_{\alpha\alpha'\alpha''\tau\tau'} a^{+}_{p\eta\tau\alpha}\, a^{+}_{-p\eta\tau'\alpha'}\, \varepsilon_{\alpha\alpha'\alpha''}\, \varepsilon_{\tau\tau'}\phi_\tau + \sum_{p\eta(\varepsilon_p^{(\eta)}\le\mu)} \frac{\tilde{K}_{p\eta}}{2} \sum_{\alpha\alpha'\alpha''\tau\tau'} a_{p\eta\tau\alpha}\, a_{-p\eta\tau'\alpha'}\, \varepsilon_{\alpha\alpha'\alpha''}\, \varepsilon_{\tau\tau'}\phi_\tau$$

where $K_{p\eta} = K_{-p\eta}, \qquad \tilde{K}_{p\eta} = \tilde{K}_{-p\eta}$

and $\sin\theta_{p\eta} = \dfrac{\sqrt{3}K_{p\eta}}{\sqrt{1+3K_{p\eta}^2}}, \qquad \sin\tilde{\theta}_{p\eta} = \dfrac{\sqrt{3}\tilde{K}_{p\eta}}{\sqrt{1+3\tilde{K}_{p\eta}^2}}$

Interplay between spin polarization and color superconductivity

- Variational equations determine θ (K)

$$\frac{\partial}{\partial \theta_{p\eta}} \langle \Psi | H_{MF} - \mu N | \Psi \rangle = 0 \ , \qquad \frac{\partial}{\partial \tilde{\theta}_{p\eta}} \langle \Psi | H_{MF} - \mu N | \Psi \rangle = 0$$

- Thermodynamic potential

$$\Phi(\Delta, F, \mu) = \frac{1}{V} \langle \Phi | H_{MF} - \mu N | \Phi \rangle$$

$$= 2 \cdot \frac{1}{V} \sum_{p\eta(\varepsilon_p^{(\eta)} \le \mu)} \left[2(\varepsilon_p^{(\eta)} - \mu) - \sqrt{(\varepsilon_p^{(\eta)} - \mu)^2 + 3\Delta^2 f(\eta)^2} \right]$$

$$+ 2 \cdot \frac{1}{V} \sum_{p\eta(\varepsilon_p^{(\eta)} > \mu)} \left[(\varepsilon_p^{(\eta)} - \mu) - \sqrt{(\varepsilon_p^{(\eta)} - \mu)^2 + 3\Delta^2 f(\eta)^2} \right] + \frac{F^2}{2G} + \frac{3\Delta^2}{2G_c}$$

Interplay between spin polarization and color superconductivity

Gap equation

$$\frac{\partial}{\partial \Delta}\langle \Phi | H_{MF} - \mu N | \Phi \rangle = 0$$

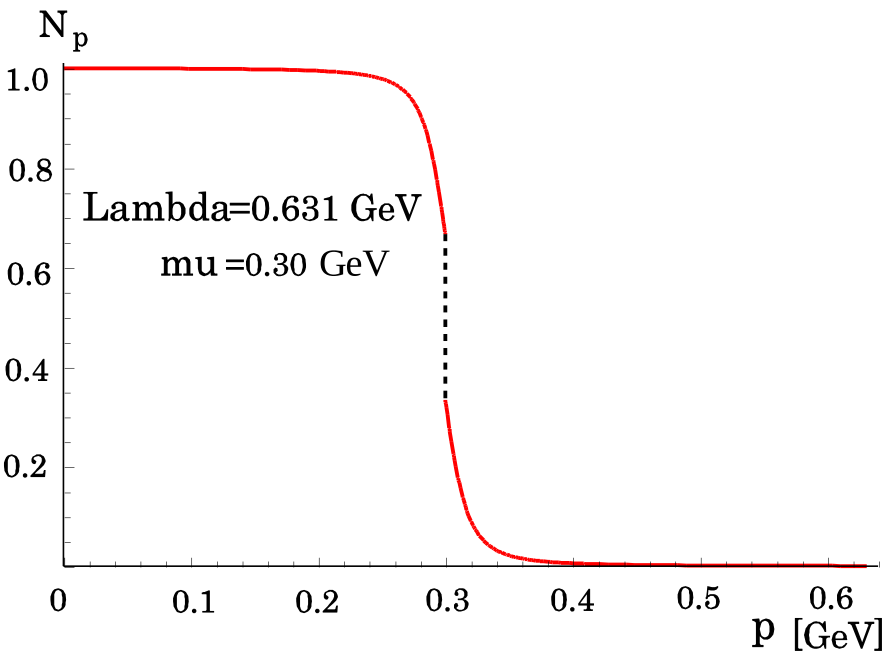
Namely,

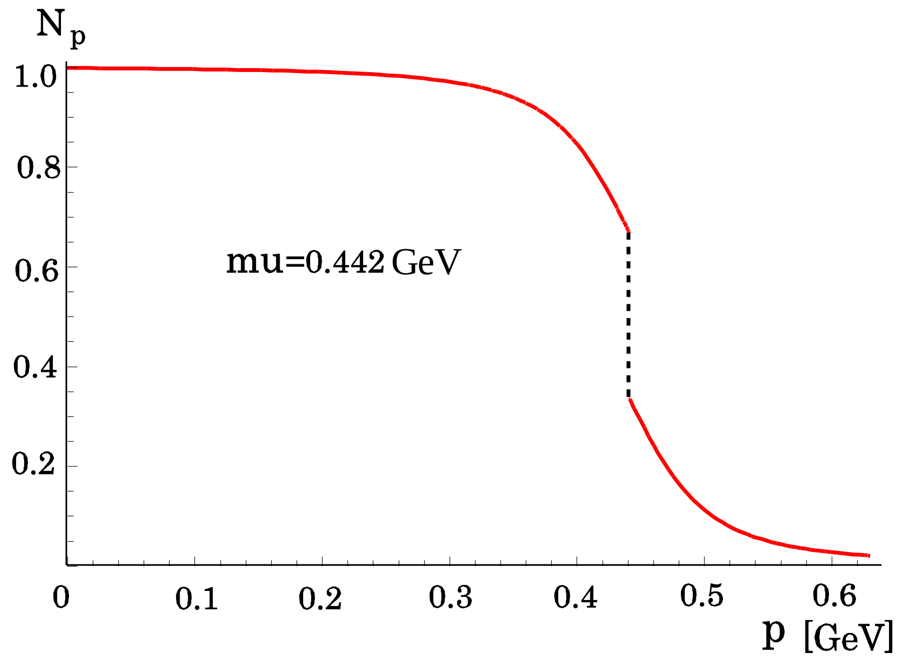
$$\Delta\left[2\cdot\frac{1}{V}\sum_{p\eta=\pm}^{\Lambda}\frac{f(\eta)^2}{\sqrt{(\varepsilon_p^{(\eta)}-\mu)^2+3\Delta^2 f(\eta)^2}}-\frac{1}{G_c}\right]=0$$

Numerical results

− for interplay between spin polarization and 2SC

- Occupation number for 2SC phase





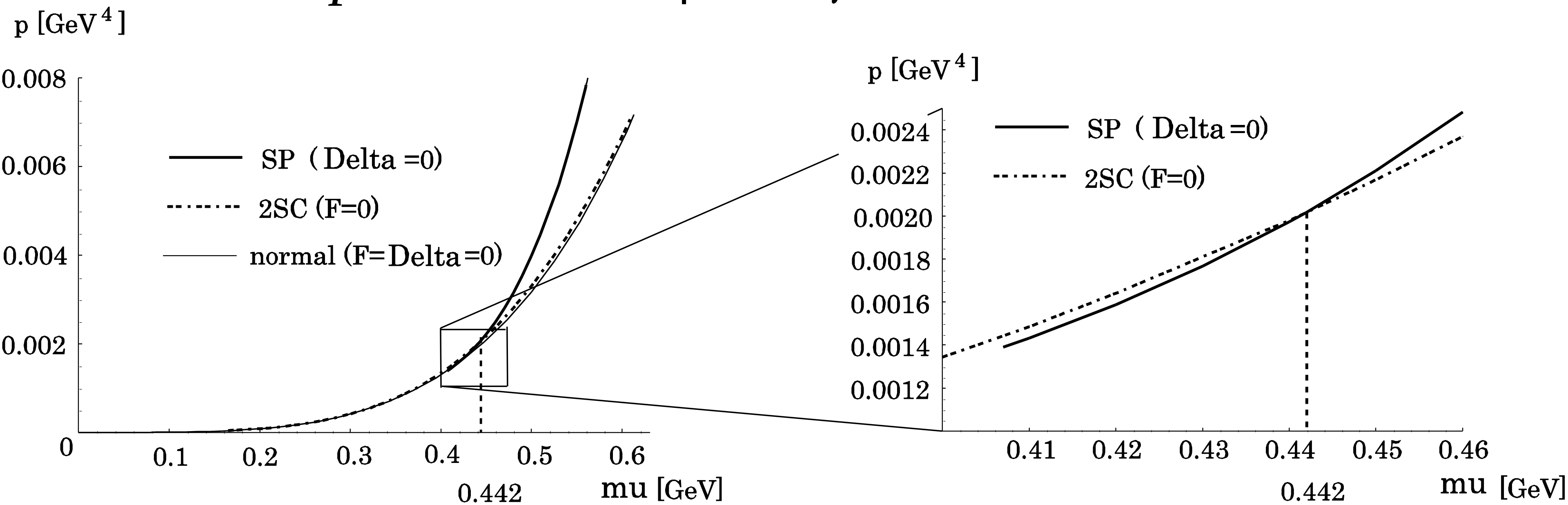
Usual cutoff Λ=0.631 GeV is valid for this calculation

Λ / GeV	G / GeV^{-2}	G_c / GeV^{-2}
0.631	20.0	6.6

Numerical results

− for interplay between spin polarization and 2SC

- Pressure p vs chemical potential μ



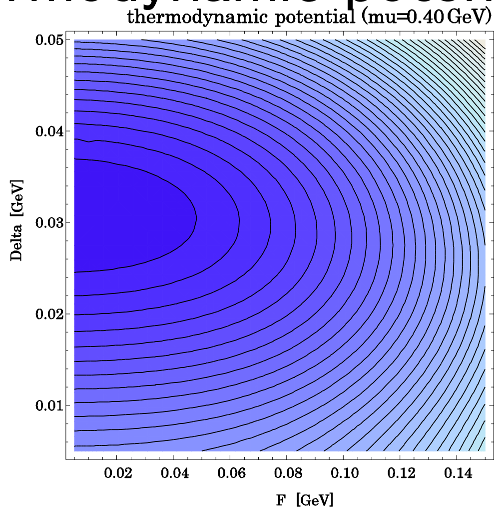
- Parameters used here

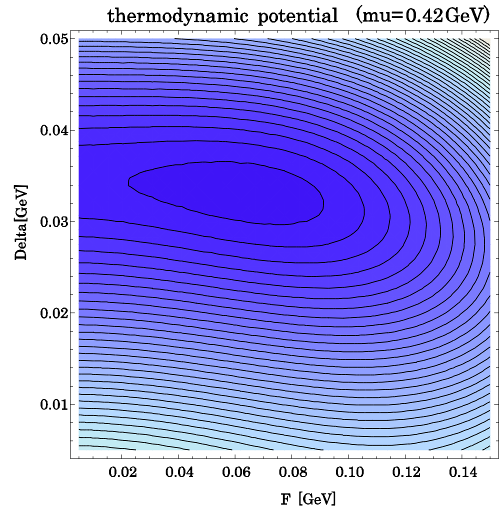
Λ / GeV	G / GeV^{-2}	G_c / GeV^{-2}
0.631	20.0	6.6

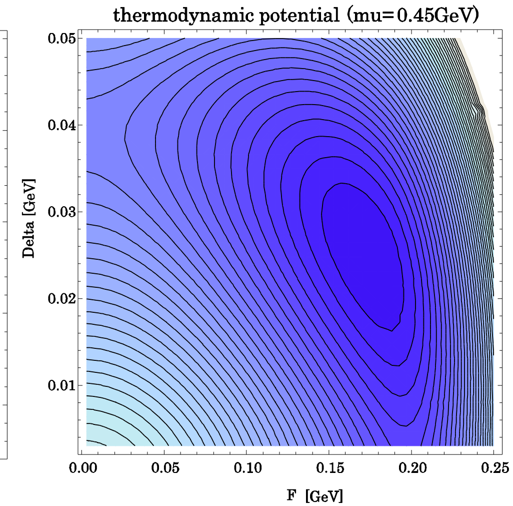
Numerical results

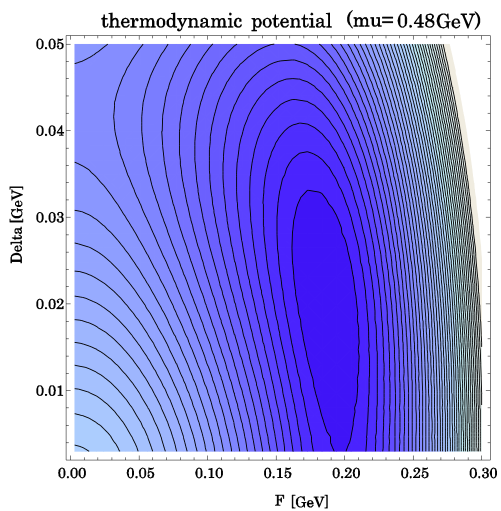
– for interplay between spin polarization and 2SC

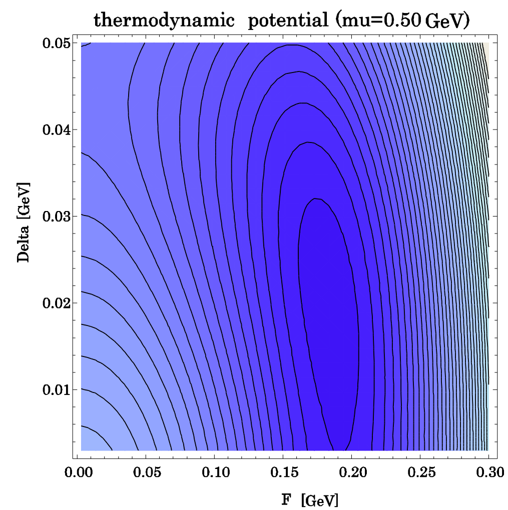
- Thermodynamic potential











Qualitative understanding
– qualitative feature of thermodynamic potential

Qualitative understanding

– qualitative feature of thermodynamic potential

- Relation to fermi surface

$$\Phi = -\int N d\mu \propto -\mu^4 \quad \left(N \propto \int^{\mu} d^3 p\, \theta(\mu - p) \right)$$:normal quark matter

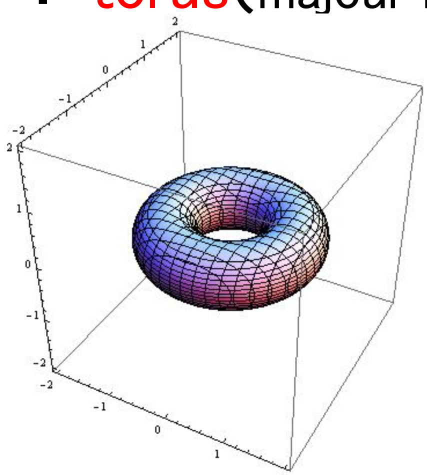
$$\Phi = -\int N d\mu \propto -\mu^6 \quad \left(N \propto \int^{\mu} d^3 p\, \theta(\mu - \varepsilon), \quad \varepsilon = \sqrt{\left(F - \sqrt{p_1^2 + p_2^2}\right)^2 + p_3^2} \right)$$:spin polarized phase

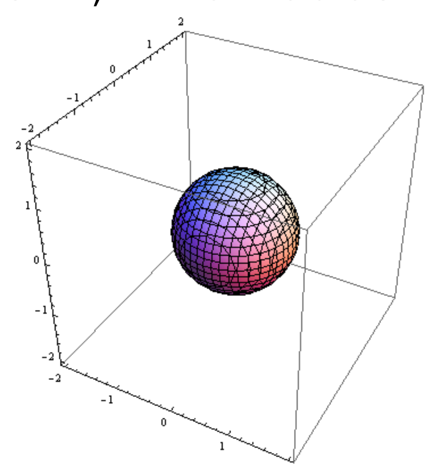
In spin polarized phase･･･ $F \neq 0$, fermi suraface has following form

$$\left(\sqrt{p_1^2 + p_2^2} - F\right)^2 + p_3^2 = \mu^2$$ ： torus(majour radius :F, minor radius: μ)

volume of torus:

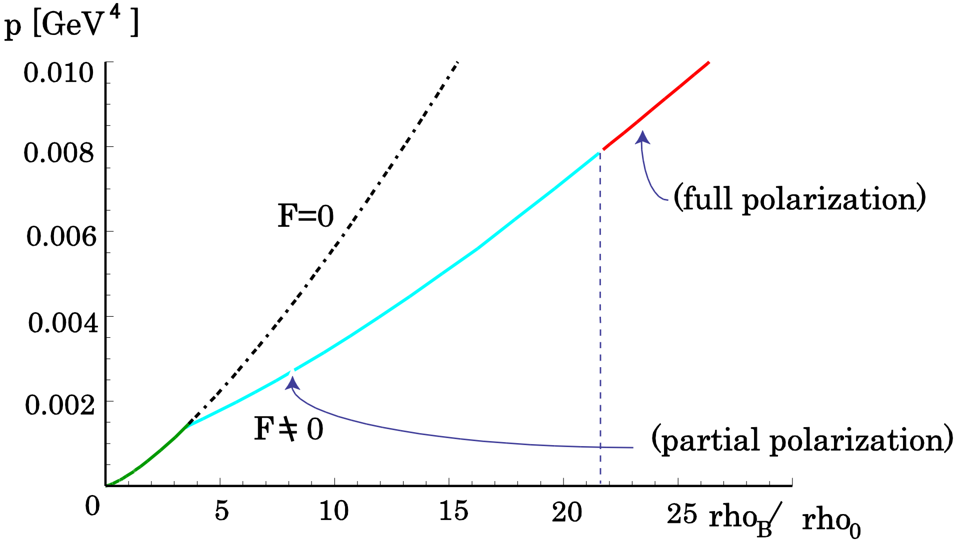
$$2\pi^2 \mu^2 F, \quad \left(F = \frac{G\mu^3}{2\pi} \right)$$



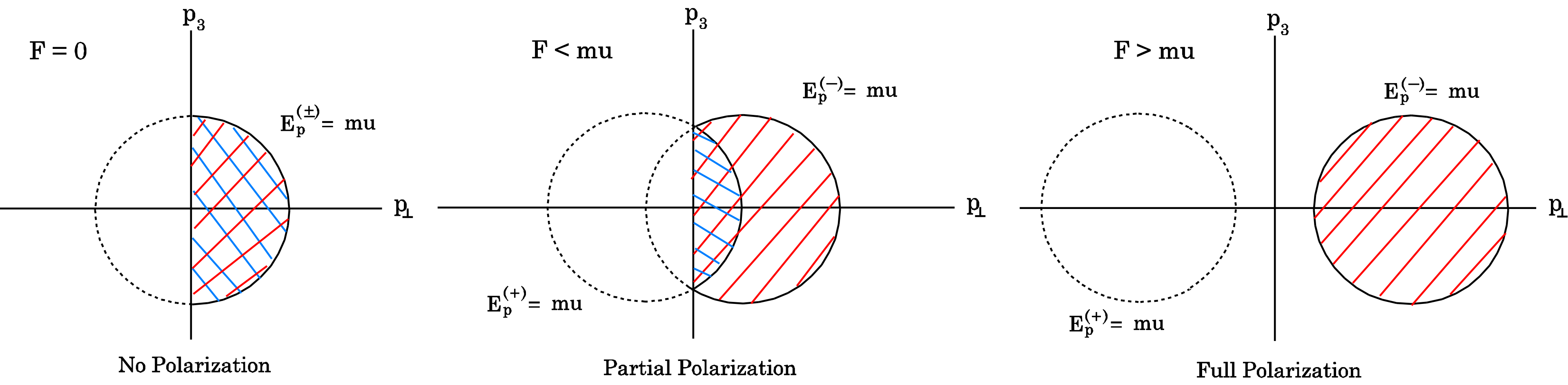


Numerical results – for spin polarization

- Relation to fermi surface



$$F < \mu \,, \quad \text{then,} \quad \epsilon_F = \epsilon_p^{(\pm)} \,,$$
$$F > \mu \,, \quad \text{then,} \quad \epsilon_F = \epsilon_p^{(-)}$$



Interplay between spin polarization and color-flavor locked phase at zero temperature

Stability of spin polarized phase
--- three-flavor case

- High density (and low temperature) quark matter in three-flavor:

  → color-flavor locked (CFL) phase may exist

Is the spin polarized phase survives
at high density against CFL phase ?

Interplay between spin polarization and color superconductivity

- Lagrangian density with 3-flavor color superconductivity

$$L=\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi-\frac{G}{4}\left(\bar{\psi}\gamma^{\mu}\gamma^{\nu}\lambda_k^f\psi\right)\left(\bar{\psi}\gamma_{\mu}\gamma_{\nu}\lambda_k^f\psi\right)+\frac{G_c}{2}\left(\bar{\psi}i\gamma_5\lambda_a^c\lambda_k^f\psi^C\right)\left(\bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi\right)$$

- Mean field approximation

$$L=\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi+L_T^{MF}+L_c^{MF}$$

$$L_T^{MF}=-\sum_{k=3,8}F_k\left(\bar{\psi}\Sigma_3\lambda_k^f\psi\right)-\frac{1}{2G}\sum_{k=3,8}F_k^2$$

$$\Sigma_3=-i\gamma^1\gamma^2=\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix},\quad F_3=-G\left\langle\bar{\psi}\Sigma_3\lambda_3^f\psi\right\rangle,\quad F_8=-G\left\langle\bar{\psi}\Sigma_3\lambda_8^f\psi\right\rangle$$

$$L_c^{MF}=-\frac{1}{2}\sum_{(a,k)=\{2,5,7\}}\left(\left(\Delta_{ak}^*\left(\bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi\right)+h.c.\right)+\frac{1}{2G_c}\left|\Delta_{ak}\right|^2\right)$$

$$\Delta_{ak}=-G_c\left\langle\bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi\right\rangle$$

Interplay between spin polarization and color superconductivity

- Hamiltonian formalism

$$\mathcal{H}_{MF} - \mu\mathcal{N} = \mathcal{K}_0 + \mathcal{H}_T^{MF} + \mathcal{H}_c^{MF} \ ,$$

$$\mathcal{K}_0 = \bar{\psi}(-\gamma \cdot \nabla - \mu\gamma_0)\psi \ ,$$

$$\mathcal{H}_T^{MF} = -\mathcal{L}_T^{MF} \ , \qquad \mathcal{H}_c^{MF} = -\mathcal{L}_c^{MF}$$

- Hamiltonian for quark and antiquark

$$H = H_0 - \mu N + V_{CFL} + V_{SP} + V \cdot \frac{1}{2G}\left(F_3^2 + F_8^2\right) + V \cdot \frac{3\Delta^2}{2G_c}$$

$$H_0 - \mu N = \sum_{p\eta\tau\alpha}\left[\left(|p| - \mu\right)c^+_{p\eta\tau\alpha}c_{p\eta\tau\alpha} - \left(|p| + \mu\right)\tilde{c}^+_{p\eta\tau\alpha}\tilde{c}_{p\eta\tau\alpha}\right]$$

$$V_{CFL} = \frac{\Delta}{2}\sum_{p\eta}\sum_{\alpha\alpha'\alpha''}\sum_{\tau\tau'}\left(c^+_{p\eta\tau\alpha}c^+_{-p\eta\tau'\alpha'} + c_{-p\eta\tau'\alpha'}c_{p\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}\tilde{c}^+_{-p\eta\tau'\alpha'} + \tilde{c}_{-p\eta\tau'\alpha'}\tilde{c}_{p\eta\tau\alpha}\right)\varepsilon_{\alpha\alpha'\alpha''}\varepsilon_{\tau\tau'}\phi_p$$

$$V_{SP} = \sum_{p\eta\tau\alpha}F_\tau\left[\frac{\sqrt{p_1^2 + p_2^2}}{|p|}\left(c^+_{p\eta\tau\alpha}c_{p-\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}\tilde{c}_{p-\eta\tau\alpha}\right) - \eta\frac{p_3}{|p|}\left(c^+_{p\eta\tau\alpha}\tilde{c}_{p\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}c_{p\eta\tau\alpha}\right)\right]$$

where

$\eta = \pm 1 \cdots$ helicity , $\tau = u, d, s \cdots$ flavor , $\alpha \cdots$ color $\left(\phi_p = -\phi_{\bar{p}} = 1\right)$

Mean Field Approximation – for color-superconducting gap Δ without spin polarization F (=0)

Mean field approximation for quasi-particle operators

$$H_{CFL}=H_0-\mu N+V_{CFL}+V\cdot\frac{3\Delta^2}{2G_c}$$
$$=\frac{1}{2}\sum_{|p|>\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\sum_{|p|>\mu}\left[\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}\,d^+_{p;1}d_{p;1}+\sum_{a=2}^{9}\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\,d^+_{p;a}d_{p;a}\right]$$
$$+\frac{1}{2}\sum_{|p|<\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\sum_{|p|<\mu}\left[\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}\,d^+_{p;1}d_{p;1}+\sum_{a=2}^{9}\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\,d^+_{p;a}d_{p;a}\right]+V\cdot\frac{3\Delta^2}{2G_c}$$

$$\bar{\varepsilon}_p=p-\mu,\quad \Delta_{\alpha''\tau''}=G_c\sum_{p\eta\alpha\alpha'\tau\tau'}\left\langle c_{-p\eta\alpha'\tau'}c_{p\eta\alpha\tau}\right\rangle\varepsilon_{\alpha\alpha'\alpha''}\varepsilon_{\tau\tau'\tau_{\alpha''}}\phi_p;\quad \Delta=\Delta_{1u}=\Delta_{2d}=\Delta_{3s}$$

Thermodynamic potential for F=0

$$\Phi_0=\frac{1}{V}\left\langle H_{CFL}\right\rangle$$
$$=\frac{1}{2V}\sum_{|p|>\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\frac{1}{2V}\sum_{|p|<\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\frac{3\Delta^2}{2G_c}$$

Mean Field Approximation – for spin polarized gap F without CFL condensate Δ (=0)

Thermodynamic potential for Δ=0

$$\Phi_F = 3 \cdot \frac{1}{V} \sum_{p,\eta=\pm,\tau=u,d,s} \left(\varepsilon_{p\tau}^{(\eta)} - \mu\right) \theta\left(\mu - \varepsilon_{p\tau}^{(\eta)}\right) + \frac{1}{2G}\left(F_3^2 + F_8^2\right)$$

$$\varepsilon_{p\tau}^{(\eta)} = \sqrt{p_3^2 + \left(F_\tau + \eta\sqrt{p_1^2 + p_2^2}\right)^2}\ , \quad F_\tau = \left(F_3 + \frac{1}{\sqrt{3}}F_8\right)\delta_{\tau u} + \left(-F_3 + \frac{1}{\sqrt{3}}F_8\right)\delta_{\tau d} - \frac{2}{\sqrt{3}}F_8\delta_{\tau s}$$

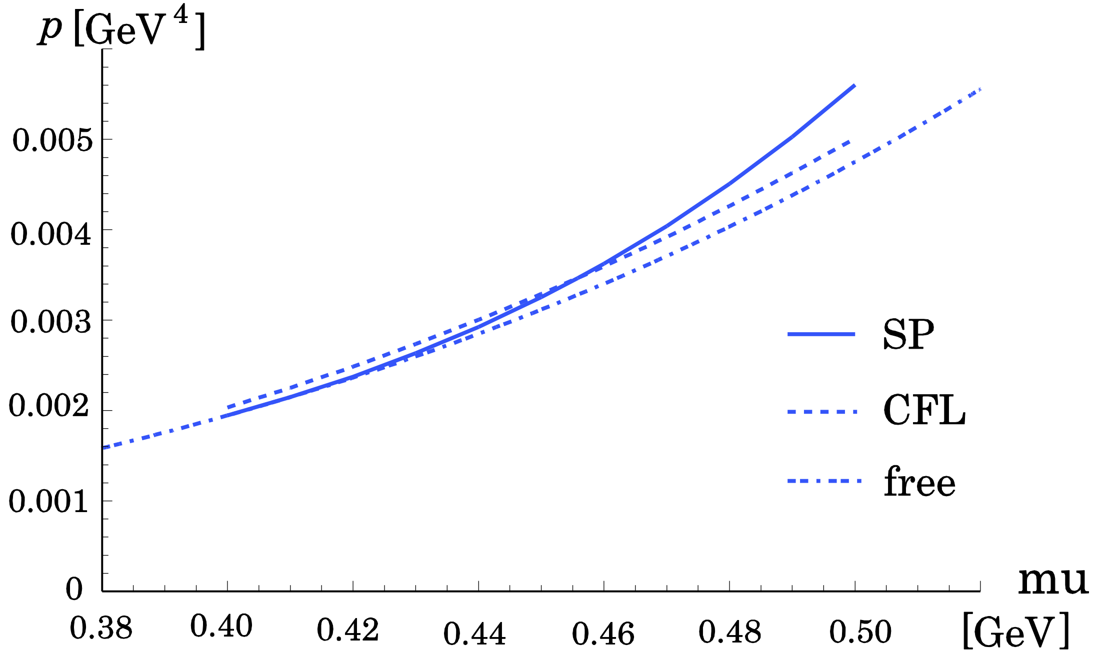
Gap equations for Φ_0 (CFL), Φ_F (SP)

$$\frac{\partial \Phi_0}{\partial \Delta} = 0\,, \text{ or } \frac{\partial \Phi_F}{\partial F_3} = \frac{\partial \Phi_F}{\partial F_8} = 0$$

Numerical results

– for interplay between spin polarization and CFL

- Pressure p vs chemical potential μ



- Parameters used here

Λ / GeV	G / GeV^{-2}	G_c / GeV^{-2}
0.631	20.0	6.6

Order of phase transition

– second order perturbation on CFL phase with respect to SP term

- Hamiltonian under consideration

$$H = H_{CFL} + H_{SP}\ , \qquad H_{SP} = \sum_{p\eta\alpha\tau} F_{\tau} \frac{\sqrt{p_1^2 + p_2^2}}{|p|} c^{+}_{p\eta\tau\alpha} c_{p-\eta\tau\alpha}$$

Here, H_{SP} is regarded as perturbation term

- First order perturbation = 0
- Second order perturbation

$$E_{corr} = \sum_{i} \frac{\langle \Phi | H_1 | i \rangle \langle i | H_1 | \Phi \rangle}{E_0 - E_i}$$

E_0; ground state energy , $|i\rangle$; intermediate (excited) state , E_i ; excited state energy

Order of phase transition

– second order perturbation on CFL phase with respect to SP term

- Thermodynamic potential

$$\Phi = \Phi_0 + \frac{1}{V} E_{corr} + \frac{1}{2G}\left(F_3^2 + F_8^2\right)$$

$$= \Phi_0 + \left(c_3 + \frac{1}{2G}\right) F_3^2 + \left(c_8 + \frac{1}{2G}\right) F_8^2$$

mu / GeV	c_3 + 1/(2G)	c_8 + 1/(2G)
0.40	0.015734	0.0076297
0.42	0.014873	0.0060047
0.44	0.014015	0.0043828
0.4558	0.0133487	0.0031934
0.46	0.013174	0.0027882
0.48	0.012367	0.0012519

coefficients of F_3 and F_8 are always positive

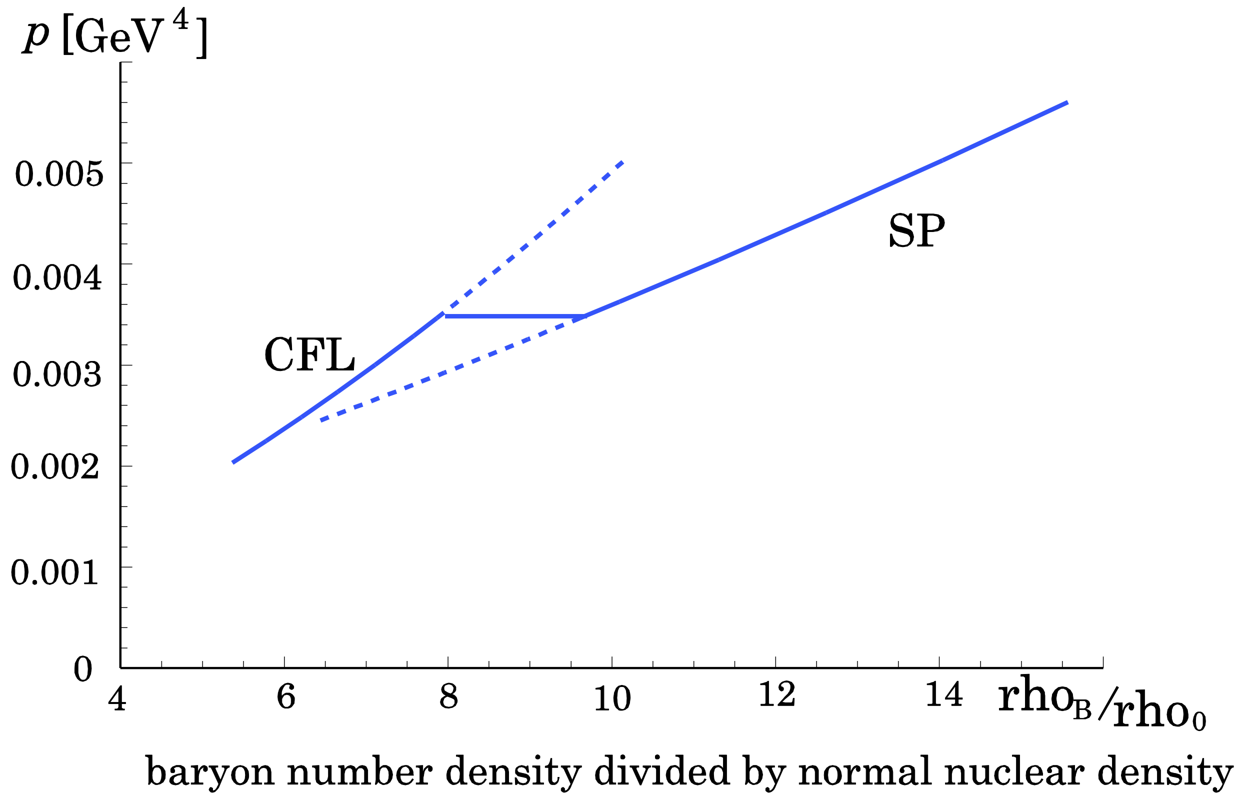
$\rightarrow \Delta \neq 0, \quad F_3 = F_8 = 0$ is local minimum

Order of phase transition

– second order perturbation on CFL phase with respect to SP term

- Thus, $\Delta \neq 0$ and $F_3 = F_8 = 0$ ···· stable

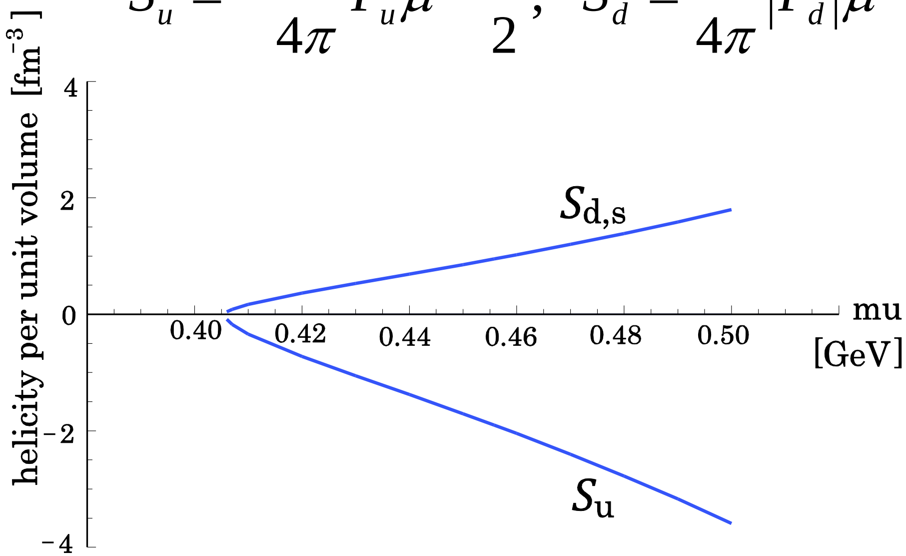
  Then, the phase transition may be the first order



Each and total helicity

- Helicity of each flavor for $F_u>0$, $F_d<0$, $F_s<0$

$$S_u = -\frac{3}{4\pi} F_u \mu^2 \cdot \frac{\hbar}{2}, \quad S_d = \frac{3}{4\pi} |F_d| \mu^2 \cdot \frac{\hbar}{2}, \quad S_s = \frac{3}{4\pi} |F_s| \mu^2 \cdot \frac{\hbar}{2},$$



Total helicity (spin) is zero because $F_3 \approx \sqrt{3} F_8$ is satisfied.

Then, $F_u = -2F_d$ and $F_d = F_s$ is obtained.

Effective potential approach

Model – NJL model

- Nambu–Jona–Lasinio model with tensor–type interaction

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_T$$
$$\mathcal{L}_{\rm kin} = i\bar{\psi}\gamma^\mu\partial_\mu\psi$$
$$\mathcal{L}_S = -G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$
$$\mathcal{L}_V = -G_V[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2]$$
$$\mathcal{L}_T = -G_T[(\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\gamma^\mu\gamma^\nu\psi)^2]$$

At high baryon density, chiral symmetry is restored

$\rightarrow \langle\overline{\psi}\psi\rangle = 0$: quarks are massless

and then, (S.Maedan, PTP 118 (2007) 729)

$\rightarrow \langle\overline{\psi}\gamma_5\gamma^{\mu=3}\vec{\tau}\psi\rangle = 0$: pseudovector condensate is zero

due to quark being massless

Model – tensor interaction

- Tensor interaction is retained : $L = L_{kin} + L_T$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{G}{4}(\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)(\bar{\psi}\gamma_\mu\gamma_\nu\vec{\tau}\psi)$$

Here, $\gamma^1\gamma^2 = -i\Sigma_3 = -i\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

Then, $\langle \overline{\psi}\gamma^1\gamma^2\vec{\tau}\psi \rangle \neq 0 \quad \rightarrow$ quark spin polarization occurs

- Hereafter, μ =1, ν =2 are taken into account.

Effective potential – for spin polarization

- Generating functional Z

$$Z \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[i\int d^4x \left(\bar{\psi}i\gamma^\mu\partial_\mu\psi + \frac{G}{2}(\bar{\psi}\Sigma_3\tau_k\psi)(\bar{\psi}\Sigma_3\tau_k\psi)\right)\right]$$

- Inserting "auxiliary field" $F_k \left(= -G\langle\bar{\psi}\Sigma^3\tau_k\psi\rangle\right)$

$$1 = \int \mathcal{D}F_k \exp\left[-\frac{i}{2}\int d^4x \left(F_k + G(\bar{\psi}\Sigma_3\tau_k\psi)\right) G^{-1}\left(F_k + G(\bar{\psi}\Sigma_3\tau_k\psi)\right)\right]$$

Then, finally

$$Z \propto \int \mathcal{D}F_k \exp\left[i\int d^4x\left(-\frac{F_k^2}{2G} + \frac{1}{4i}\mathrm{tr}\ln\left(-p_0^2 + \epsilon_p^{(-)2}\right) + \frac{1}{4i}\mathrm{tr}\ln\left(-p_0^2 + \epsilon_p^{(+)2}\right)\right)\right]$$

$$\epsilon_p^{(\pm)} = \sqrt{\left((F_k\tau_k) \pm \sqrt{p_1^2 + p_2^2}\right)^2 + p_3^2}$$

: single-particle energy

Effective potential – for spin polarization

- Effective potential : $V[F]$

$$Z = \exp\left(i\Gamma[F]\right) \ , \qquad V[F] = -\frac{\Gamma[F]}{\int d^4x}$$

- At finite density, introduce chemical potential : μ

$$L \rightarrow L + \mu\bar{\psi}\gamma^0\psi$$

Then, finally

$$V[F] = \frac{F^2}{2G} + 2N_c \int^F dF \int \frac{d^3p}{(2\pi)^3} \left[\frac{F - \sqrt{p_1^2 + p_2^2}}{\epsilon_p^{(-)}} \theta(\mu - \epsilon_p^{(-)}) + \frac{F + \sqrt{p_1^2 + p_2^2}}{\epsilon_p^{(+)}} \theta(\mu - \epsilon_p^{(+)}) \right]$$

where

$$F_k\tau \rightarrow F\tau_3 = F\tau \ , \qquad \tau = \begin{cases} 1 & \text{for up quark} \\ -1 & \text{for down quark} \end{cases}$$

Effective potential – for spin polarization

- Effective potential : $V[F]$

$$F - \mu \leq \sqrt{p_1^2 + p_2^2} \leq F + \mu \qquad (\text{for} \quad \epsilon_p^{(-)}) \ ,$$

$$0 \leq \sqrt{p_1^2 + p_2^2} \leq \mu - F \qquad (\text{for} \quad \epsilon_p^{(+)})$$

Thus,

$$F < \mu \ , \quad \text{then,} \quad \epsilon_F = \epsilon_p^{(\pm)} \ ,$$

$$F > \mu \ , \quad \text{then,} \quad \epsilon_F = \epsilon_p^{(-)}$$

- Thermodynamic relation : pressure p

: quark number density ρ_q

$$p = -V[F] \ , \qquad \rho_q = -\frac{\partial V[F]}{\partial \mu}$$

Effective potential – for spin polarization

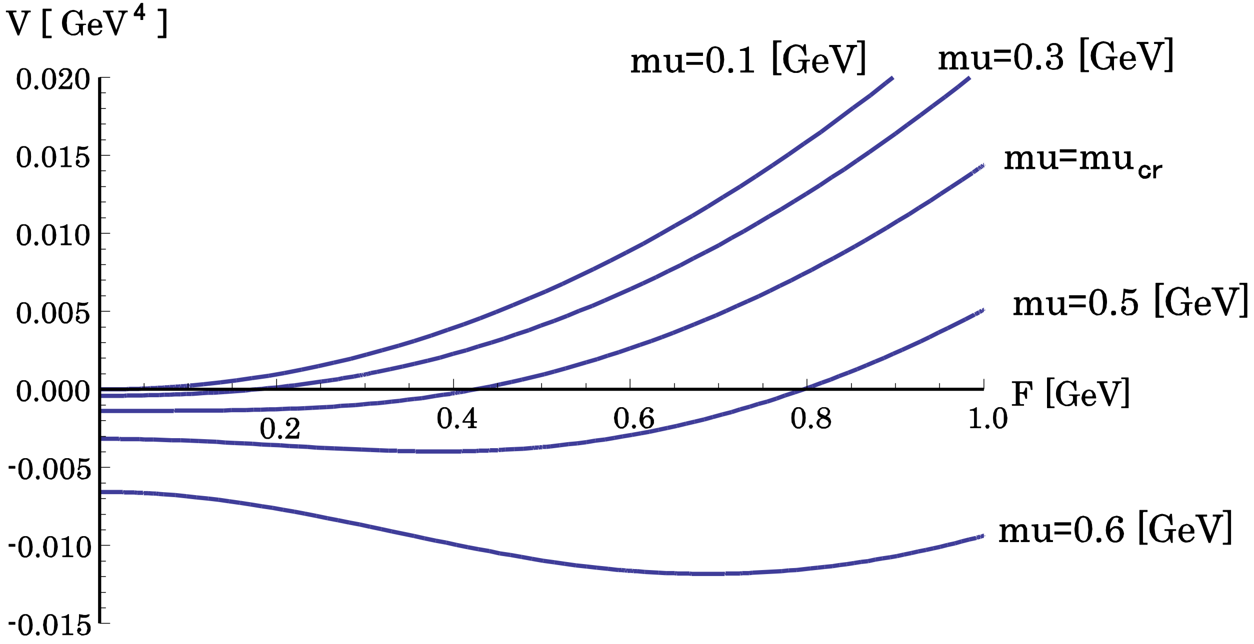
- Thermodynamic relation : pressure p

  : quark number density ρ_q

$$p = -V[F] \, , \qquad \rho_q = -\frac{\partial V[F]}{\partial \mu}$$

Numerical results – for spin polarization

- Effective potential : $V[F]$



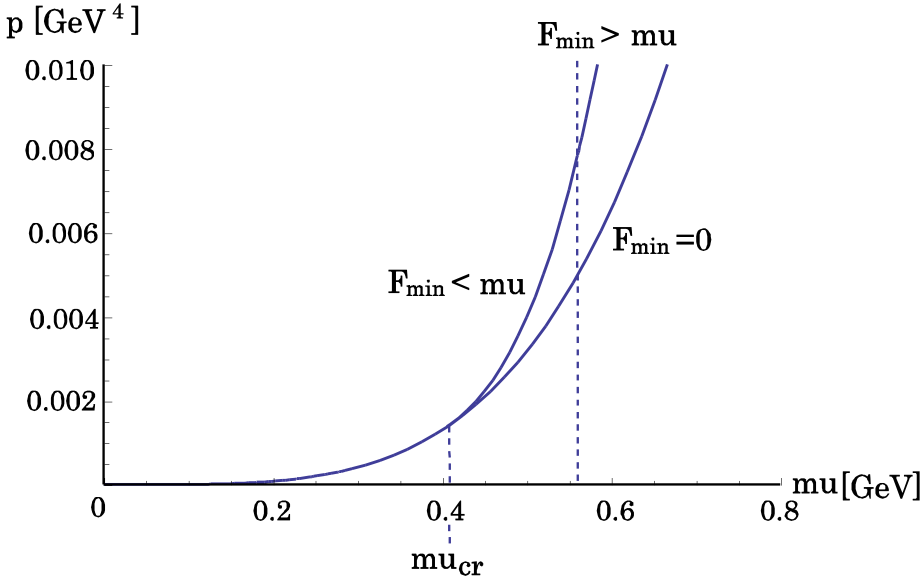
- Parameters used here

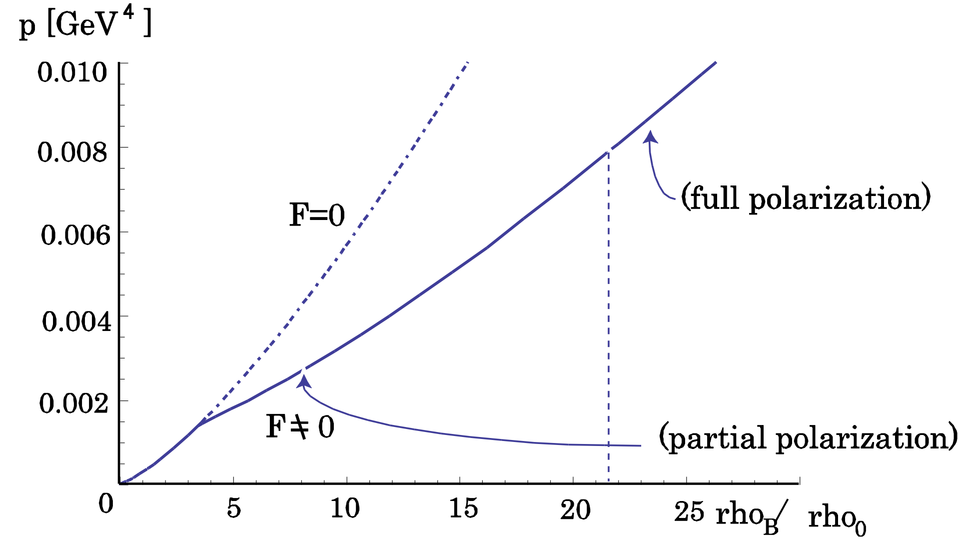
$$G = 20\,\mathrm{GeV}^{-2}$$

(with vaccume polarization, $G = 11.1\,\mathrm{GeV}^{-2}$ with cutoff $\Lambda = 0.631\,\mathrm{GeV}$)

Numerical results – for spin polarization

- Pressure vs chemical potential or baryon number density





- Critical density

G / GeV^{-2}	ρ_{cr}/ρ_0	μ_{cr} / GeV
15	5.34	0.468
20	3.47	0.406
25	2.48	0.363

$$\left(\rho_0 = 0.17\ \mathrm{fm}^{-3}\right)$$