HYBRID QUARK STARS WITH STRONG MAGNETIC FIELD

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Refs: H. Sotani and T. T., MNRAS 447,3155 (2015);
in preparation.

I.Introduction

Some important problems in compact star physics, related to modern nuclear physics:

- Origin of the strong magnetic fields in pulsars and magnetars
- Massive neutron stars beyond $2M_{solar}$
- Possible existence of quark stars

One may give answers to these problems from the microscopic viewpoint.

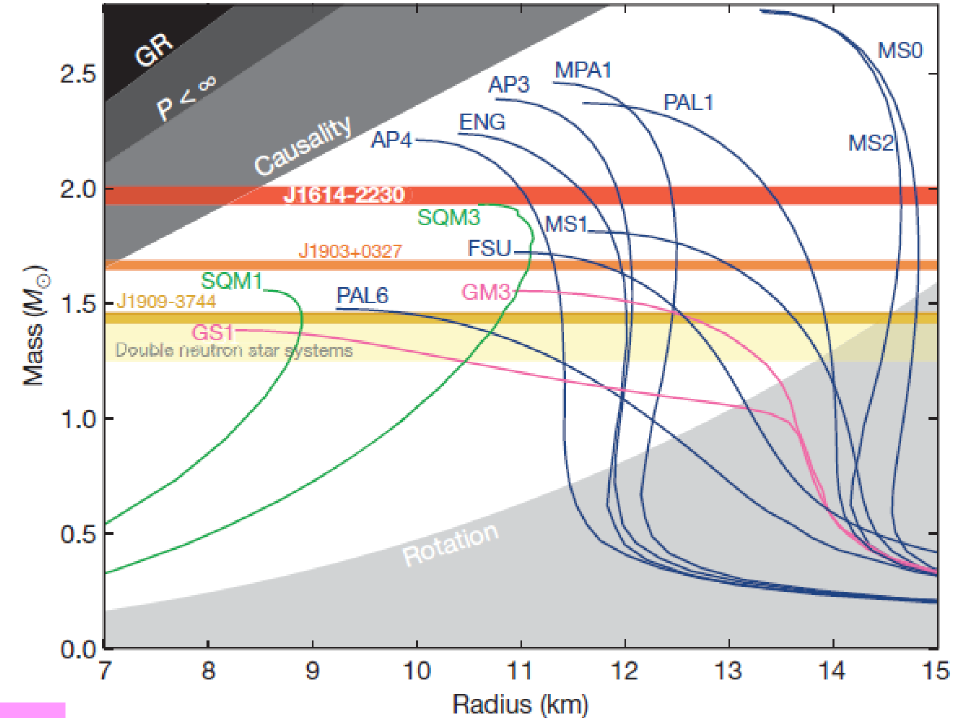
→ EOS at supranuclear density

Nucleon,hyperon or quark degree of freedom?

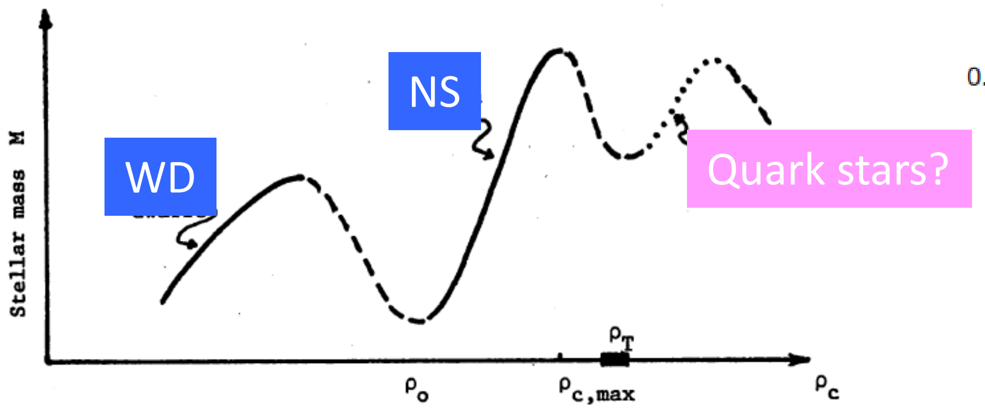
- We consider a possibility of massive neutron stars beyond two solar mass

- Possibility of third family of compact stars

WD → NS → 3[rd] family?



P.B.Demorest et al., Nature 467 (2010) 1081



G. Baym,Lecture notes (NORDITA, 1977)

J.Antoniadis et al., Science 340 (2013) 6131.

$2.01 \pm 0.04 M_{SUN}$ (PSR J0348+0432)

$$\frac{dM}{d\rho_c} < 0$$ Gravitationally unstable

Large stiffening effect on EOS in the core is needed to support such massive and dense stars!

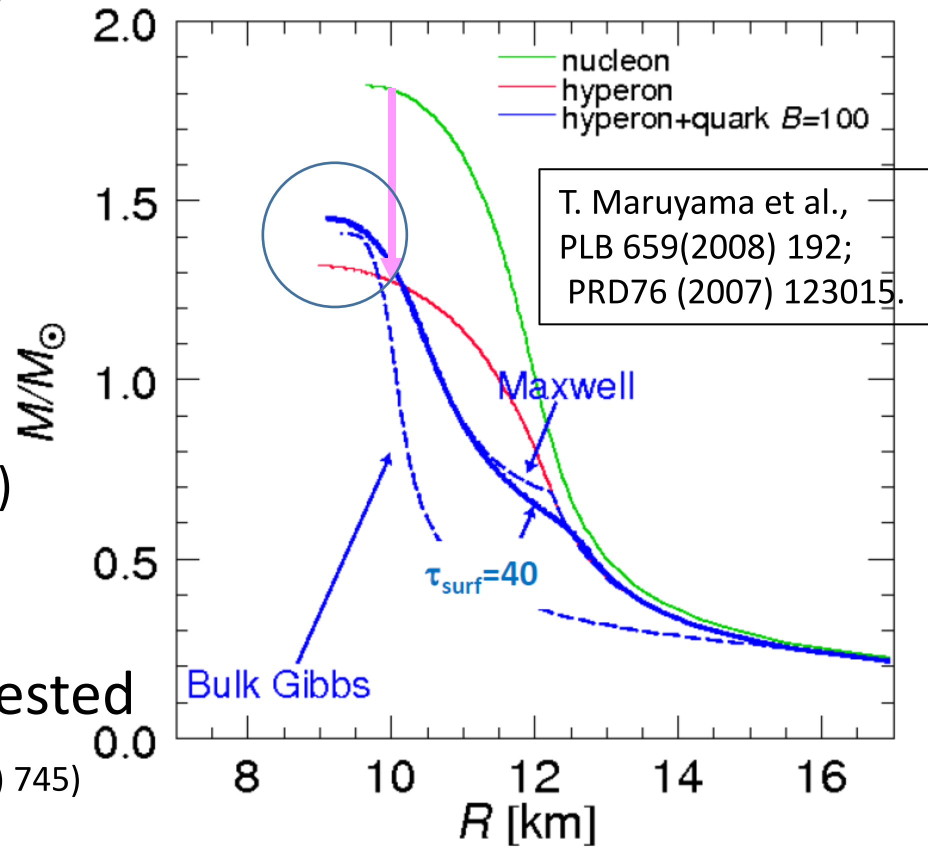
There are many studies about these issues.

➡ Some important ingredients

Hyperon degree of freedom

Quark degree of freedom

Naïve thinking may lead us to the softening of EOS due to the phase transition, without any additional mechanism such as repulsive three-body force among hadrons or repulsive interaction between quarks.



We consider here the effect of strong magnetic field on EOS

$B_{surface} \sim 10^{12-13} G$ (Radio Pulsars)

$\sim 10^{14-15} G$ (Magnetars)

Much stronger field has been suggested Inside the star (D. Lai ,S.L.Shapiro, Ap.J.383 (1991) 745)

II. Model

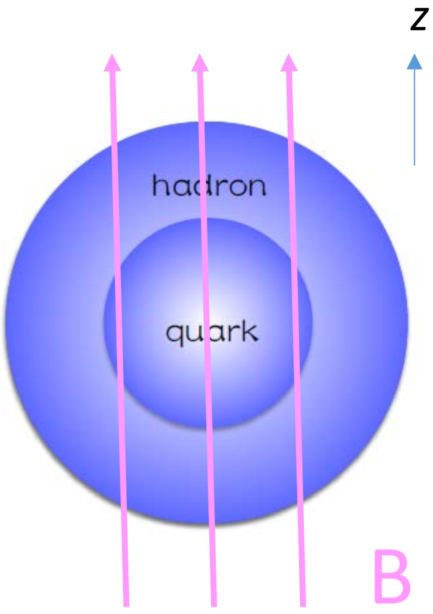
Almost free quark matter
Strong magnetic field
in the core region

Similar idea has been applied for white dwarfs,
Super-Chandrasekhar-mass WD

B. Mukhopadhyay et al., arXiv:1507.05439;1509.00936

Many studies of EOS with the magnetic field
(R. Casali et al., arXiv:1307.2651)

Landau quantization



$$E(n,k_z)=\sqrt{k_z^2+\left(\sqrt{m_q{}^2+2\nu\,|qB|}-s\mu_N\kappa B\right)^2}\ :\ \text{Landau levels}$$

$$\nu=n+1/2-\mathrm{sign}(q)\mathrm{s}/2,\ \kappa:\text{anomalous magnetic moment}$$

$$\psi_{n,k_z}(\mathbf{r}):\ \text{Hermite ploynomial (localization) in x-y plane}$$

$$e^{ik_z z}\ \text{along } z \text{ axis}$$

Quarks produce one-dim. Fermi sea for each Landau level *n*

Remarks:

- All quarks are charged ⟷ Neutrons are neutral
- Mass is very small, compared with $\sqrt{qB}$ $\quad m_q\approx 0\ll M_n$

$B > B_c \sim O(10^{19} G)$

only LLL is occupied.

Number density

$$n_f = \frac{3|e_f B|}{2\pi^2} p_{fF}$$

Energy density

$$\varepsilon_f = \frac{3|e_f B|}{4\pi^2} p_{fF}^2$$

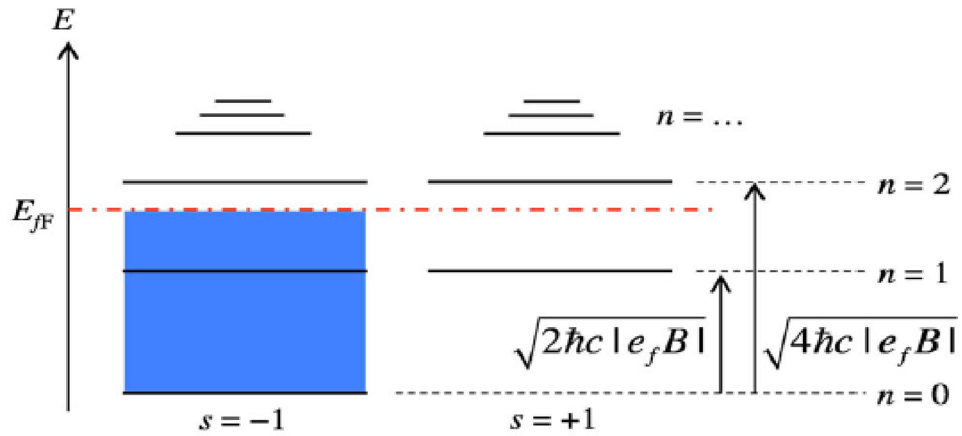
EOS for quark matter
(MIT bag model)

$$\varepsilon = \frac{5\pi^2}{2eB} n_B^2, n_B = \sum_f n_f$$

$$P = n_B^2 \frac{\partial(\varepsilon / n_B)}{\partial n_B} = \frac{5\pi^2}{2eB} n_B^2 - \mathrm{B}_{bag}$$

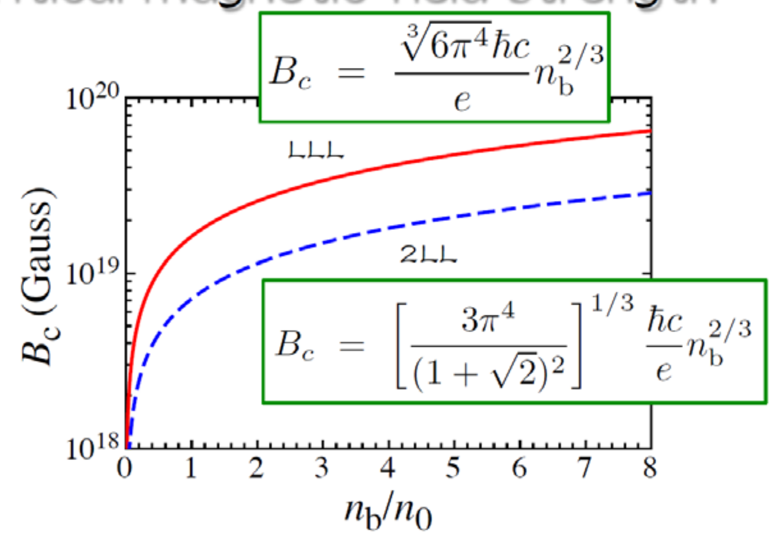
$$= \varepsilon - 2\mathrm{B}_{bag}$$

(B independent!)



(LLL)

critical magnetic field strength



strong magnetic field such as ~10^{19}G is necessary in the quark phase

Energy scale

$$(\mu_B, \sqrt{eB}) \sim O(\text{several hundred MeV})$$

Some specific features of our EOS

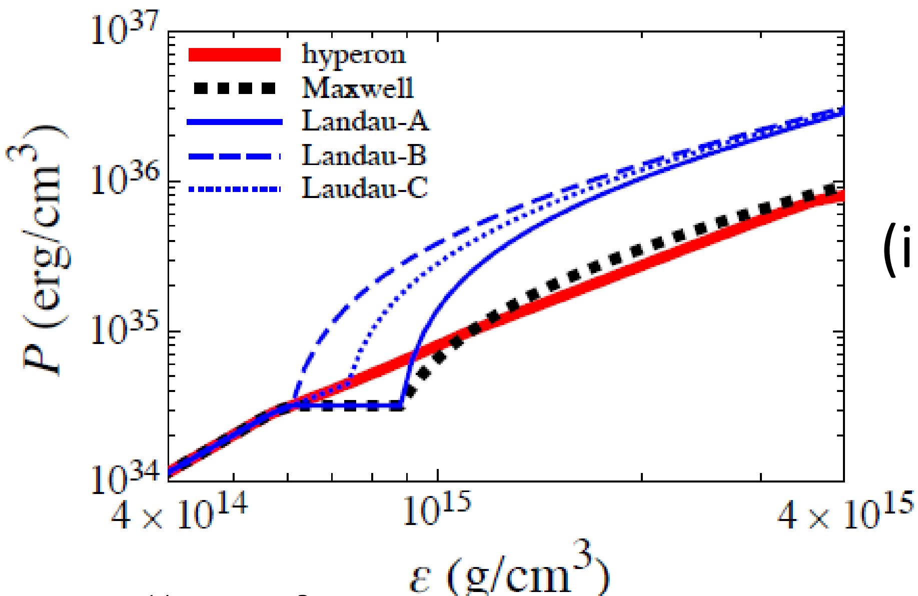
(i) It gives the "stiffest" one

$$c_s^2 = \frac{dP}{d\varepsilon} = 1$$ (Causality limit)

→ 1/3 (ultra-relativistic gas) (B → 0)

cf Massive vector interaction also realizes the causality limit $\varepsilon \propto n_B^2$

(Ya.B. Zeldovich, Sov. Phys. JETP 14, 1143 (1962))



$(\varepsilon_0 \sim 2.8 \times 10^{14}\,\mathrm{g \cdot cm^{-3}})$

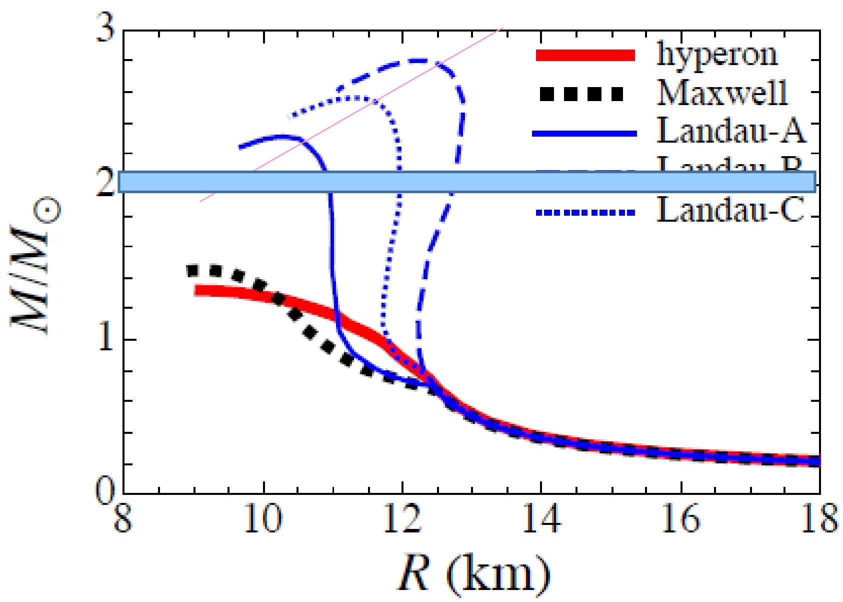
NJL model

(K. Masuda, T. Hatsuda and T. Takatsuka, Ap.J. 764 ,12(2013).
T. Kojo et al., PRD 91, 045003 (2015))

(ii) It is independent of B for B>Bc

(A) B_{bag}=237.3MeV/fm3
(B) B_{bag}=160.9MeV/fm3
(C) B_{bag}=192.8.3MeV/fm3

$M_{\max} = 2.8 M_\odot$



Hybrid quark stars

- Recall that the adiabatic index must satisfy the relation,

$$\gamma_{ad} > \frac{4}{3} + \kappa \frac{M}{R}, \kappa \sim O(1)$$

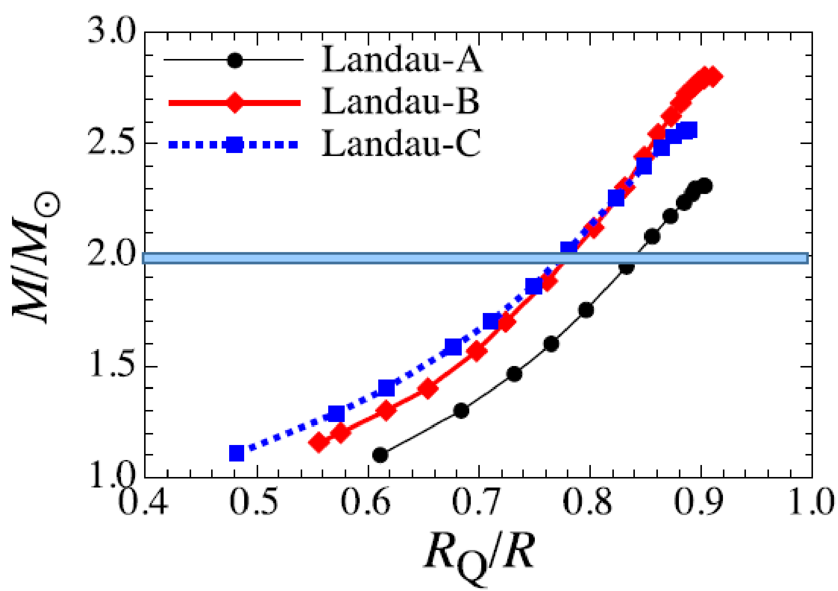
for the stability condition.

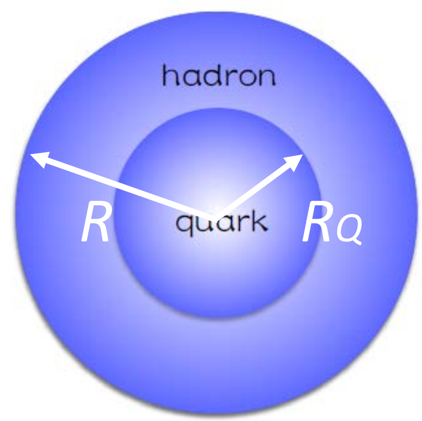
(Shapiro and Teukolsky, 1983)

$\gamma_{ad} \sim 2$ in our EOS.

- Virial theorem is cleared.

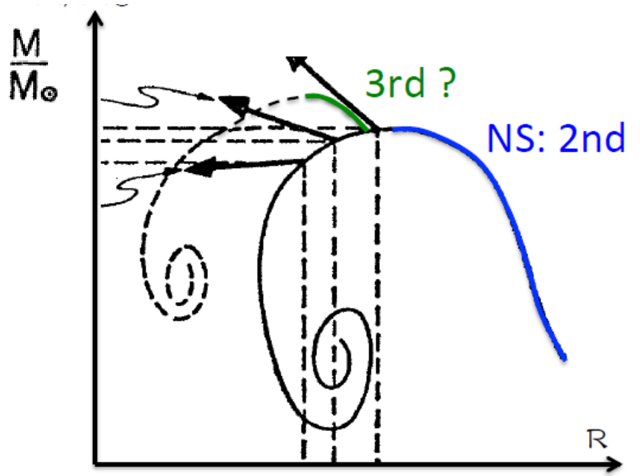
(D.Lai and S.L. Shapiro, ApJ. 383, 745 (1991))





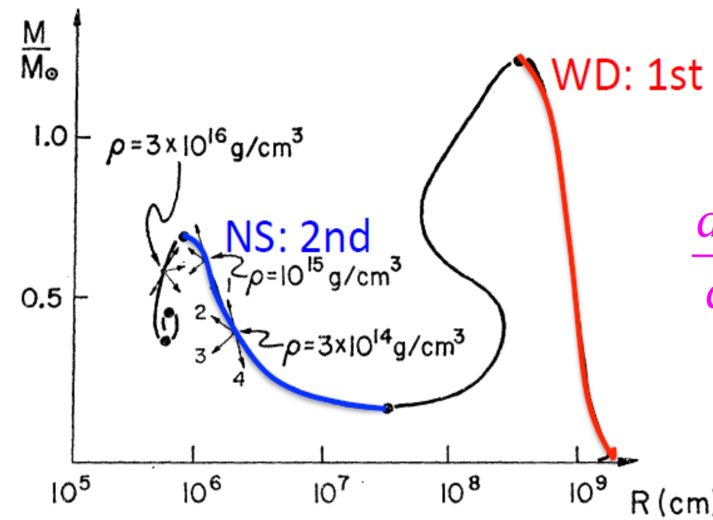
In case M=2Msolar is a hybrid quark star, RQ/R is around 0.8.

III Possibility of the third family of compact stars



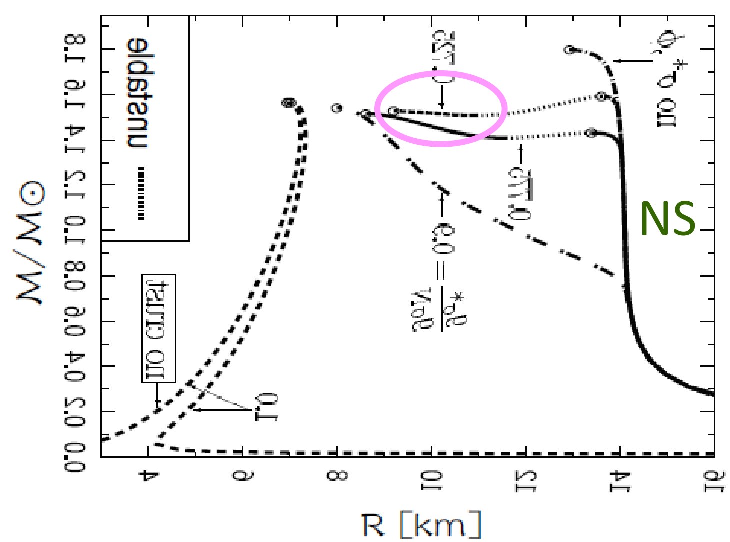
one needs an additional physics
to support the gravity

U.H. Gerlach, PR 172, 1325 (1968)



$$\frac{dM}{dR} < 0$$

Quark stars?

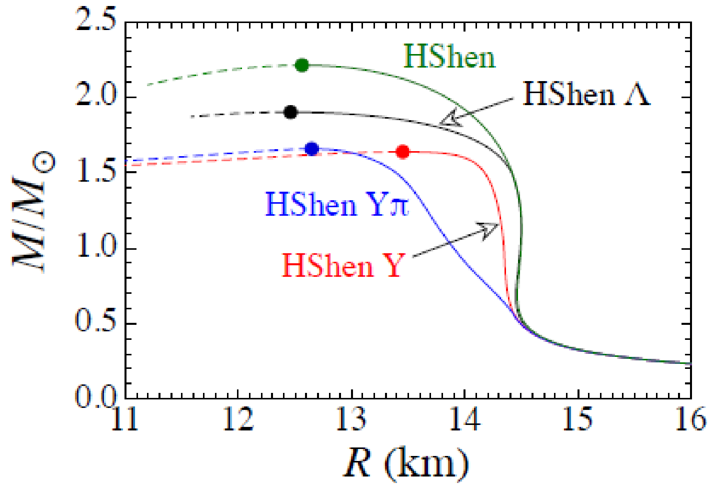


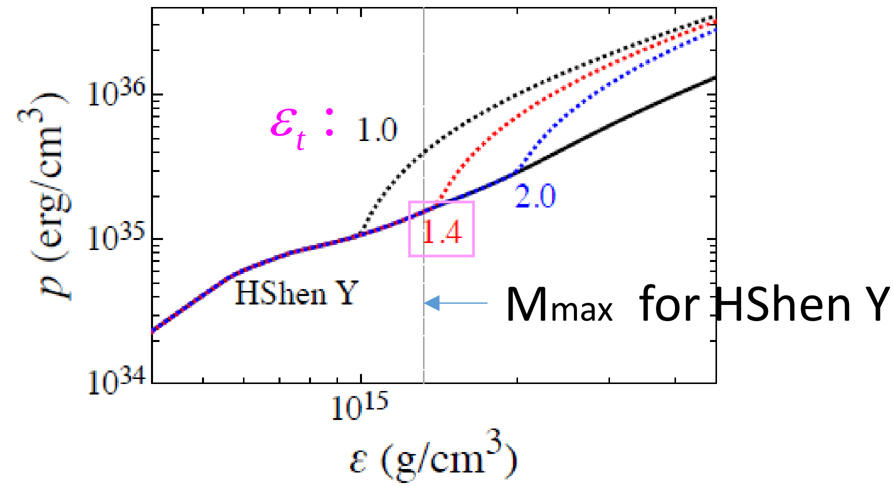
J. Schaffner-Bielich et al., PRL89 ,171101 (2002)

There has been discussed a possibility by the phase transition to pion, hyperon or quark matter

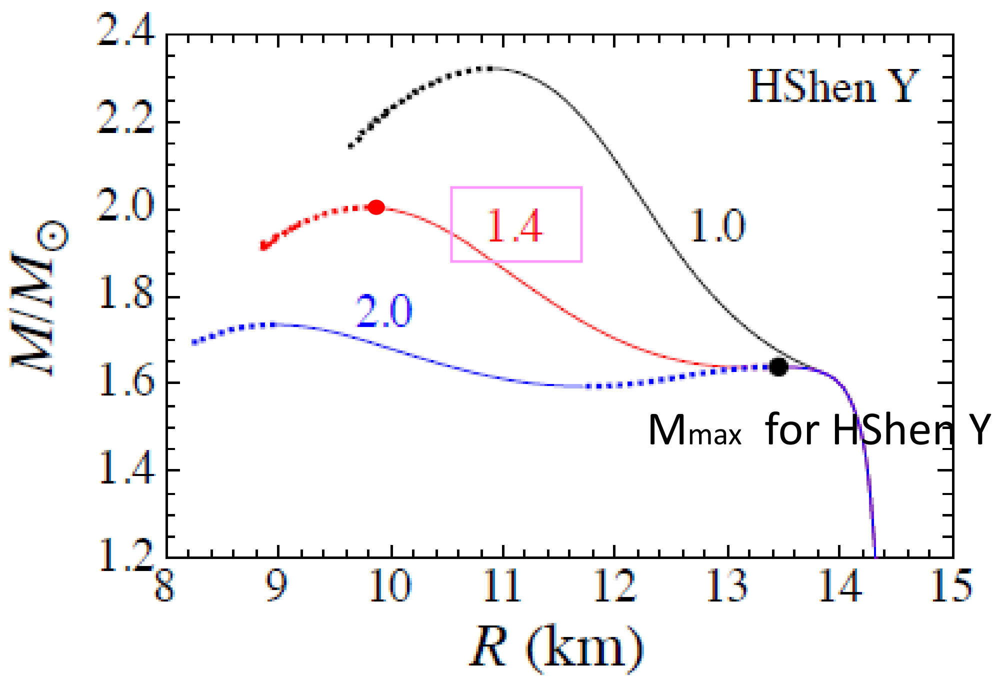
However, it is impossible to make the massive stars beyond 2Msolar due to the softening of EOS by the phase transition.

➡ One result





H. Shen et al.,NPA637, 435 (1998);ApJ.Sppl.197, 20 (2011)

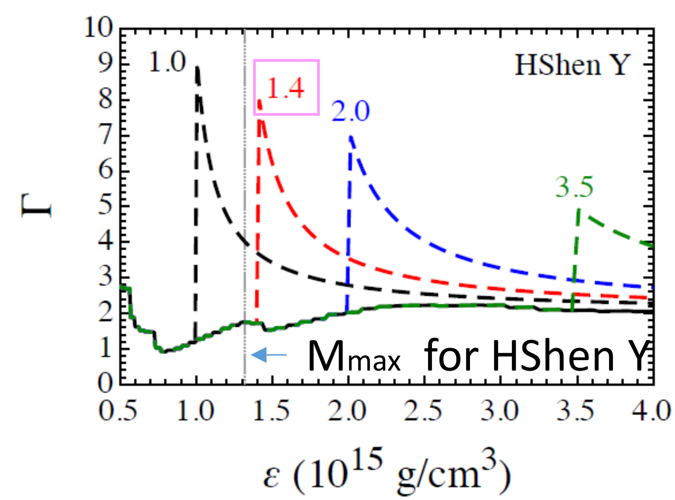


Quark Stars

Some conditions:

(i)H-Q transition occurs at low densities

(ii)Discontinuous jump of the adiabatic index is large.



IV. Summary and Concluding remarks

- We have discussed how the magnetic field stiffens EOS

  LLL gives rise to the stiffest EOS (Causality limit)

- Hybrid quark stars larger than $2M_{solar}$ is possible for B>Bc ＝O(10^19G)

  Quark matter content should be very large for hybrid quark stars

- We have demonstrated an example of third family of compact stars

  Second collapse of supernovae
  Gravitational wave after neutron star mergers

$$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu},$$

$$T_m^{\mu\nu} = \varepsilon_m u^\mu u^\nu - P_m \left(g^{\mu\nu} - u^\mu u^\nu\right),$$

$$T_f^{\mu\nu} = \frac{B^2}{4\pi}\left(u^\mu u^\nu - \frac{1}{2} g^{\mu\nu}\right) - \frac{B^\mu B^\nu}{4\pi}.$$

$$T^{\mu\nu} = \begin{bmatrix} \varepsilon_m + B^2/8\pi & 0 & 0 & 0 \\ 0 & P_m + B^2/8\pi - MB & 0 & 0 \\ 0 & 0 & P_m + B^2/8\pi - MB & 0 \\ 0 & 0 & 0 & (P_m + B^2/8\pi) - B^2/4\pi \end{bmatrix}$$

Anisotropic pressure

X.-G. Huang et al., PRD 81(2010) 045015.

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P_\perp \Xi^{\mu\nu} + P_\parallel b^\mu b^\nu, \quad P_\perp = P - MB, P_\parallel = P$$

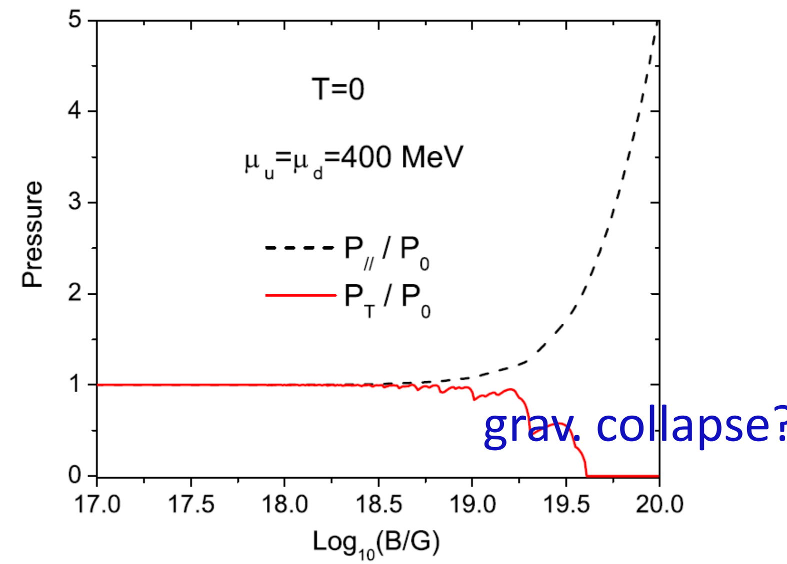
(thermodynamic pressure)

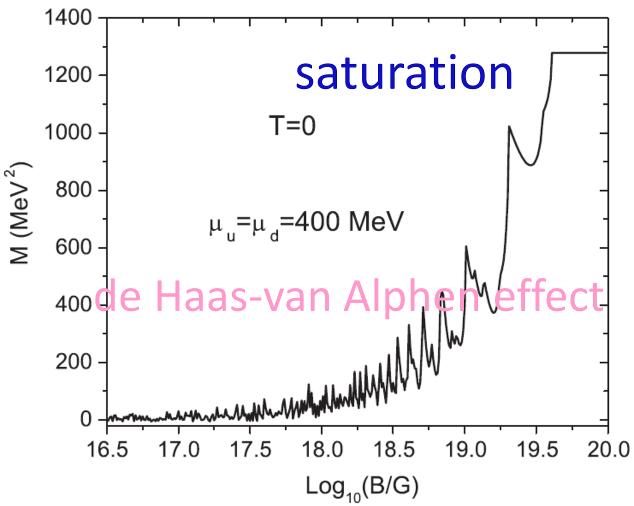
$$T_{EM}^{\mu\nu} = \frac{1}{2} B^2 \left(u^\mu u^\nu - \Xi^{\mu\nu} - b^\mu b^\nu \right),$$

$$\Xi^{\mu\nu} = \Delta^{\mu\nu} + b^\mu b^\nu,$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

(MHD)





Magnetization

Anisotropic pressure or
Deformation of star is still controversial

Pressure is isotropic due to
a back reaction from matter?

(A.Y. Pothekhin and D.G. Yakovlev, PRC 85, 039801 (2012))

We assume here an isotropic pressure.

tiny quark core

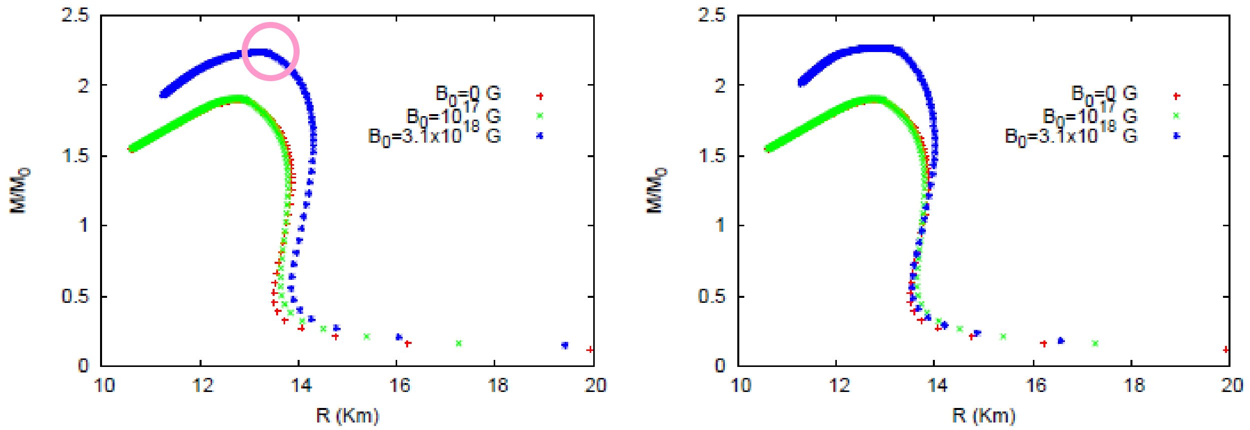


FIG. 7. Mass-radius curves for hybrid stars without mixed phase built with the GM1 and SU(3) HK NJL parametrizations, SLOW (left panel) and FAST (right panel) cases.

(R. Casali et al., PRC 89, 015805 (2014))

Hybrid star

RMF+NJL
Maxwell construction

Virial theorem

cf D. Lai and S.L. Shapiro, Ap.J. 383, 745 (1991)

$$2T + W + 3\Pi + M = 0,$$

$$T > 0, \Pi > 0, W < 0$$

$$\frac{4\pi R^3}{3}\frac{B_{\max}^2}{8\pi} \sim \frac{GM^2}{R}(+\mathrm{GR}\,?)$$

$$B_{\max} \sim 2\times 10^8 (M/M_\odot)(R/R_\odot)^{-2} G$$

	B[G]	R[cm]
Sun	10^3	10^{10} (7X10^10cm)
NS	10^{11}	10^6
Magnetar	10^{15}	10^4

White Dwarf（MHD）:

$$B_{surf} \sim 10^6 - 10^8 G$$

$$B_{in} \sim 10^{12} G$$

$$B_{\max} \sim 10^{18} G(\text{neutron star})$$

$$\int d^3r \frac{B^2(r)}{8\pi} = \frac{4\pi R^3}{3}\left\langle \frac{B^2(r)}{8\pi}\right\rangle_{\mathrm{av}}$$