# FLUCTUATIONS IN THE INHOMOGENEOUS CHIRAL TRANSITION

T.T., T-G. Lee, R. Yoshiike (Kyoto U.)

Refs: S. Karasawa, T.-G. Lee and T.T., PTEP (2016) 043D02.
T.T., T.-G. Lee and R. Yoshiike, in preparation

## I. Introduction

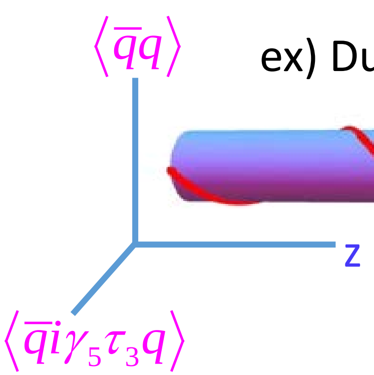
Chiral transition in the QCD phase diagram

→ Generalized order parameter

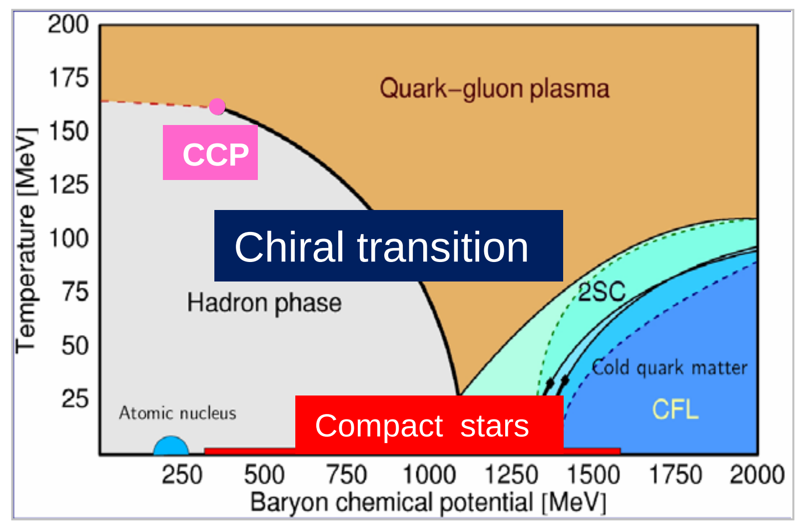
$$M \equiv \langle \bar{q}q \rangle + i \langle \bar{q} i \gamma_5 \tau_3 q \rangle = \Delta(\mathbf{r}) \exp(i\theta(\mathbf{r}))$$

*Inhomogeneous chiral phase* (iCP)

ex) Dual Chiral Density Wave (DCDW)




(T. T. and E. Nakano, hep-ph/0408294.
E. Nakano and T. T., PR**D 71** (2005) 114006.)




(B. Ruester)

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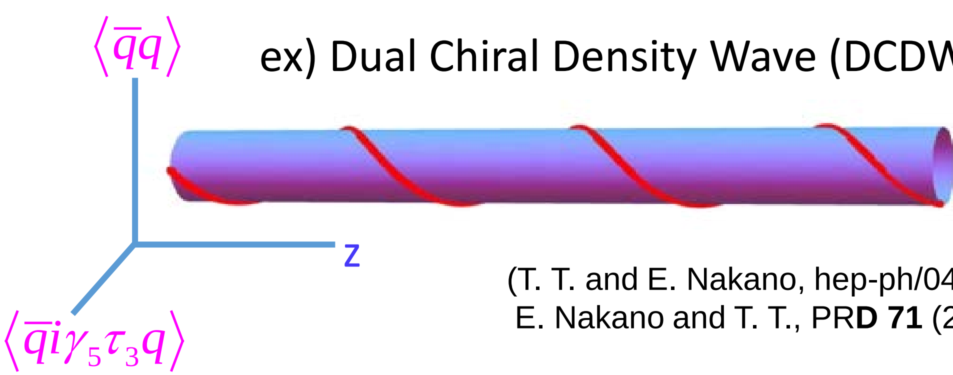


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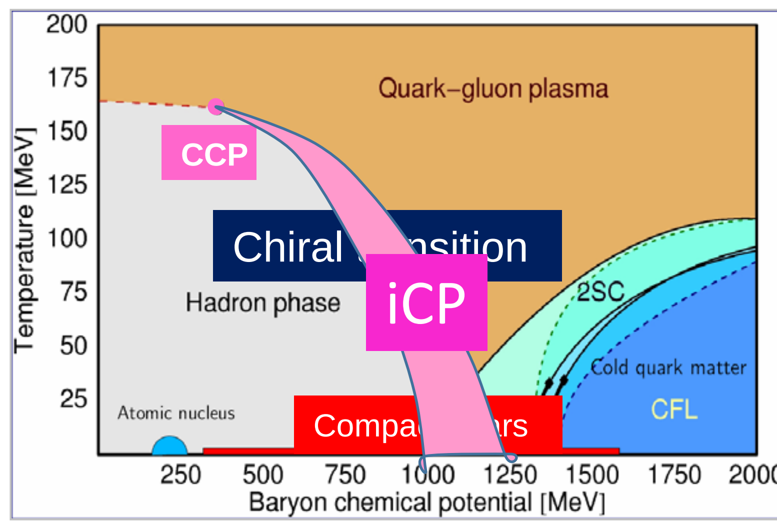
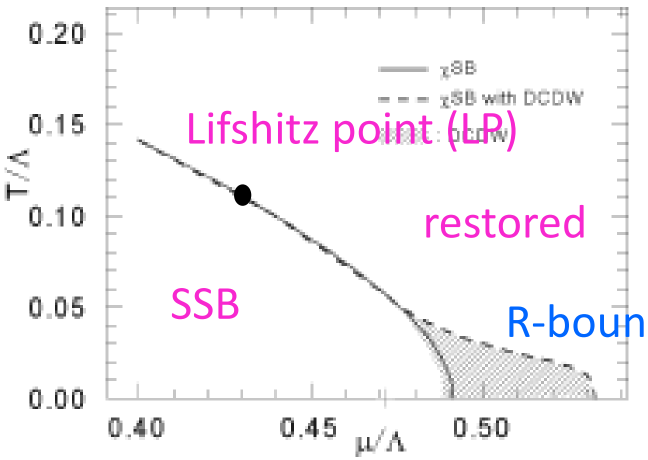


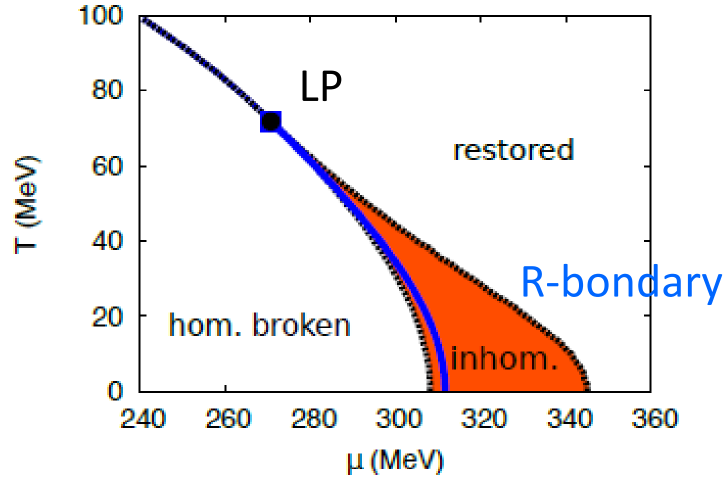

(B. Ruester)

# Phase diagram of iCP's within the mean-field approximation (MFA)

DCDW




E.Nakano,T.T. Phys.Rev. D **71** 114116.




Real kink crystal (RKC) $\Delta(z), \theta = 0$

(D.Nickel, PRL 103(2009) 072301; PR**D** 80(2009) 074025.)

There are few works about fluctuation effects: Quasi-Long-Range-Oder

Stability of the one-dimensional structure by the Nambu-Goldstone excitations

T.-G. Lee et al, PRD92,034024(2015)
Y. Hidaka et al., PRD92,(2015)

Here we discuss the fluctuation effects near the phase boundary.

Consider the right (R-) boundary, which is described by *the second order phase transition* within MFA.

# II Brazovskii and Dyugaev effect

Partition function Z within the NJL model:

$$Z=\int DqD\overline{q}e^{-S},$$

$$S=-\int_0^\beta d\tau\int d^3x\left[\overline{q}\left(-\gamma^0\frac{\partial}{\partial\tau}+i\boldsymbol{\gamma}\cdot\nabla+\mu\gamma^0\right)q+G\left[(\overline{q}q)^2+(\overline{q}i\gamma_5\tau q)^2\right]\right]$$

Introduce the auxiliary (collective) fields: $\phi_a=(-2G\overline{q}q,-2G\overline{q}i\gamma_5\tau q)$

After integrating out the quark degrees of freedom we have an effective action *S*eff in terms of these collective fields:

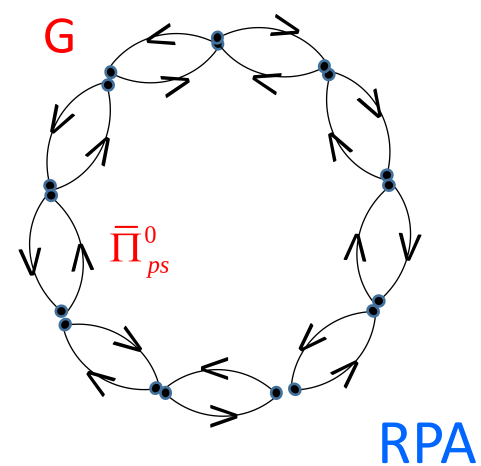
$$S_{\text{eff}}\sim\int d^4x\left[\frac{1}{2!}\Gamma^{(2)}\phi_a^2+\frac{1}{4!}\Gamma^{(4)}(\phi_a^2)^2+\ldots\right],$$

respecting $SU(2)_L\otimes SU(2)_R\simeq O(4)$ symmetry.

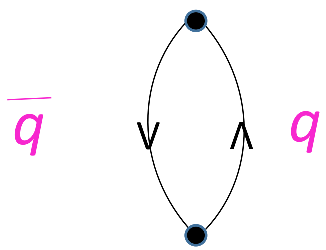
$$\phi_a=\left\langle\phi_{ps}\right\rangle+\xi_a\equiv\Phi(\Delta,q)+\xi_a$$

$$\Omega-\Omega_f=T\sum_q\sum_{\omega_n}\log\left(1-2G\overline{\Pi}^0_{ps}(q,\omega_n)\right)$$

$$+\int d^3x\left[\frac{1}{2!}\overline{\Gamma}^{(2)}|\Phi(\mathbf{x})|^2+\frac{1}{4!}\overline{\Gamma}^{(4)}|\Phi(\mathbf{x})|^4+.\frac{1}{6!}\overline{\Gamma}^{(6)}|\Phi(\mathbf{x})|^6..\right]$$



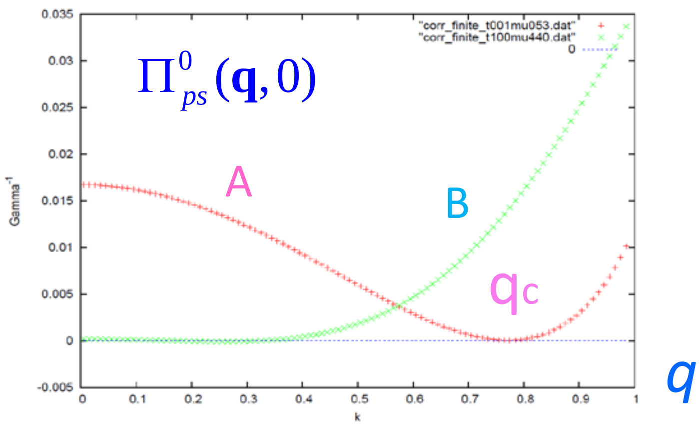
# Chiral pair fluctuations in the chiral-restored phase

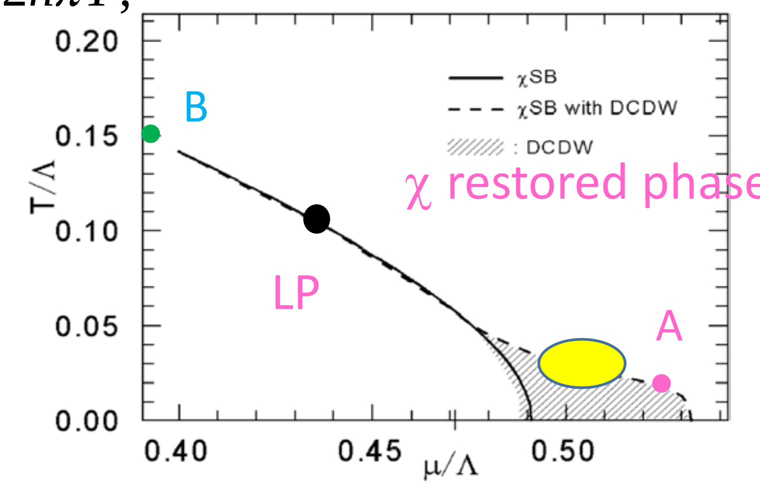


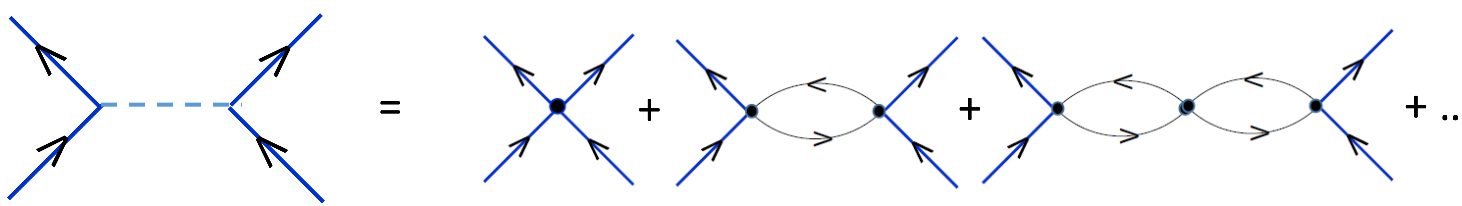

e.g. Pseudoscalar channel (Lindhard function)

$$\Pi^0_{ps}(\mathbf{q}, i\nu_n) = -T\sum_m \sum_{\mathbf{p}} \mathrm{tr}\left[i\gamma_5\tau_3 S_\beta(p+q) i\gamma_5\tau_3 S_\beta(p)\right]$$

$$p_0 = i(2m+1)\pi T + \mu, q_0 = i2n\pi T,$$



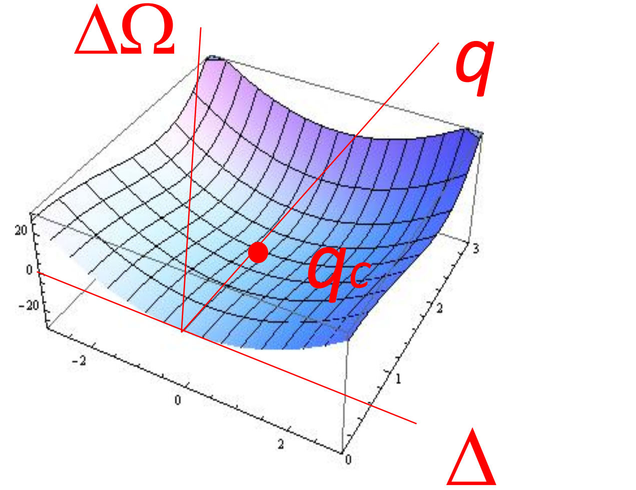




$$\phi \equiv -2G\bar{q}i\gamma_5\tau_3 q$$

$$G_{ps}(\mathbf{q},0) \sim \frac{1}{1-2G\Pi^0_{ps}(\mathbf{q},0)} = \frac{1}{\tau + \gamma(|\mathbf{q}|^2 - q_c^2)^2}$$

$\tau = 0, |\mathbf{q}| = q_c$ :Thouless criterion n-th order renormalized vertex function:

$$\overline{\Gamma}^{(n)}(\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n) = (2\pi)^{3n} \left.\frac{\delta^n\Omega}{\delta\Phi(-\mathbf{q}_1)\delta\Phi(-\mathbf{q}_2)\ldots\delta\Phi(-\mathbf{q}_n)}\right|_{\Phi=0}$$



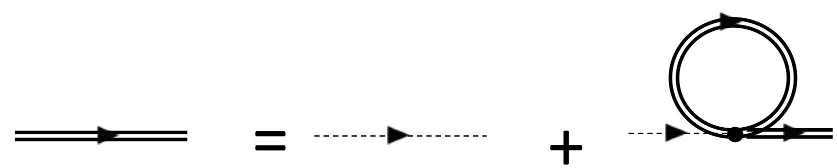

which includes the fluctuation effects given by $\xi_a$,

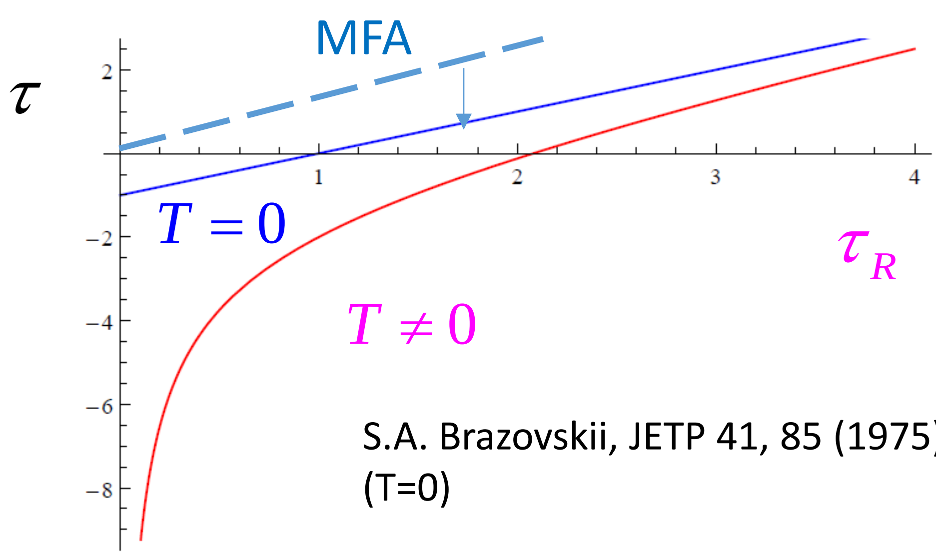
$\rightarrow \Gamma^{(n)}(\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n)$ within MFA.

$$\Delta\Omega = \int d^3x\left[\frac{1}{2!}\overline{\Gamma}^{(2)}|\Phi|^2 + \frac{1}{4!}\overline{\Gamma}^{(4)}|\Phi|^4 + \frac{1}{6!}\overline{\Gamma}^{(6)}|\Phi|^6 \ldots\right]$$

## Second-order vertex function

$$\overline{\Gamma}^{(2)}(\mathbf{q}_1,\mathbf{q}_2) \propto \delta(\mathbf{q}_1+\mathbf{q}_2)\left(G^R_{ps}(i\omega_n,\mathbf{q}_1)\right)^{-1},$$




S.A. Brazovskii, JETP 41, 85 (1975)
(T=0)

Dyson eq. gives

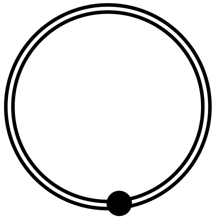
$$\tau = \tau_R - \frac{\lambda}{2}T\sum_{\omega_n}\int \frac{d^3p}{(2\pi)^3}G^R_{ps}(p,i\omega_n)$$

$$\simeq \tau_R - \frac{\lambda T q_c}{8\pi\gamma^{1/2}\tau_R^{1/2}}, \quad T\neq 0$$

$$\simeq \tau_R - \frac{\lambda\Lambda^3}{96\alpha\pi^{5/2}\gamma^{3/2}q_c^3}, \quad T=0$$

$$G^R_{ps}(\mathbf{p}, i\omega_n) = \frac{1}{\tau_R + \gamma(|\mathbf{p}|^2 - q_c^2)^2 + \alpha|\omega_n|}$$

$(p, i\omega_n)$



We can see how the difference is generated for quantum fluctuations and thermal fluctuations, Looking into the loop integral.

- All the Matsubara frequencies contribute to the integral at T=0

$$T\sum_{\omega_n}\int\frac{d^3p}{(2\pi)^3} \to \int\frac{d^4p}{(2\pi)^4} \qquad (\mathrm{Im}\,\overline{\Pi}^0_{ps} \neq 0 \text{ is important})$$

, and washed out the singularity in $G^R_{ps}$.

- On the other hand, n=0 gives a leading contribution at finite temperature

$$T\sum_{\omega_n}\int\frac{d^3p}{(2\pi)^3} \to \int\frac{d^3p}{(2\pi)^3}$$

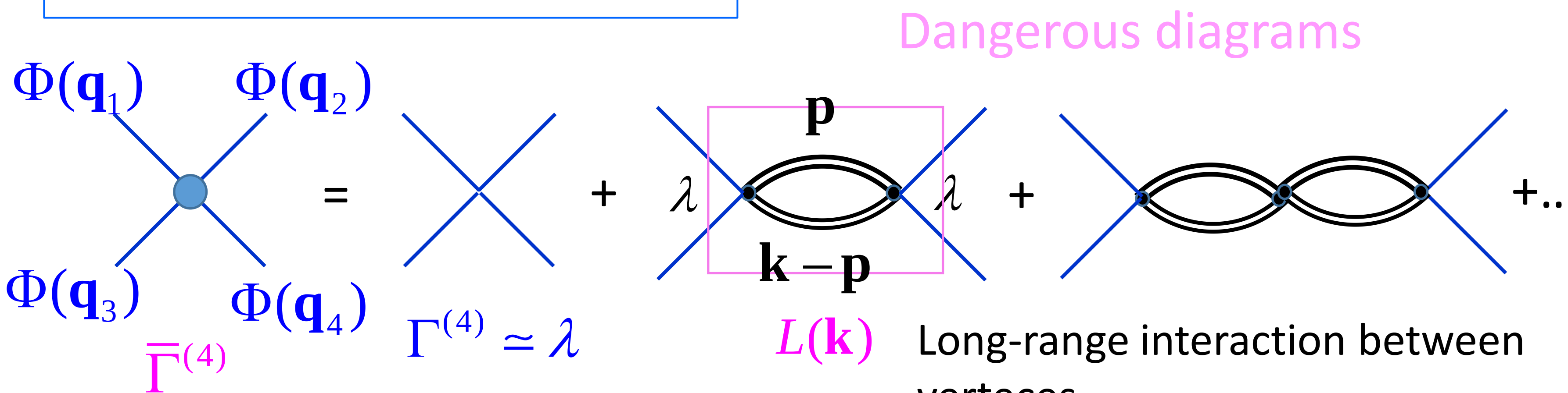
(dimensional reduction)

Momentum integral becomes more singular

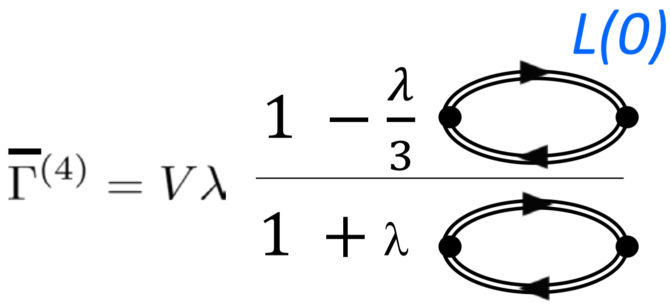
Fluctuation effect becomes more drastic.

(cf Coleman-Mermin-Wagner's theorem)

# Fourth-order vertex function




Long-range interaction between verteces




S.A. Brazovskii, Sov.Phys.JETP 41,85 (1975) (T=0).
A.M. Dyugaev, JETP Lett. 22, 83(1975) (T=0)

$$L(0) = \frac{\lambda}{2} T \lim_{k\to 0} \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} G^R_{ps}(\mathbf{p}, i\omega_n) G^R_{ps}(\mathbf{k}-\mathbf{p}, -i\omega_n)$$

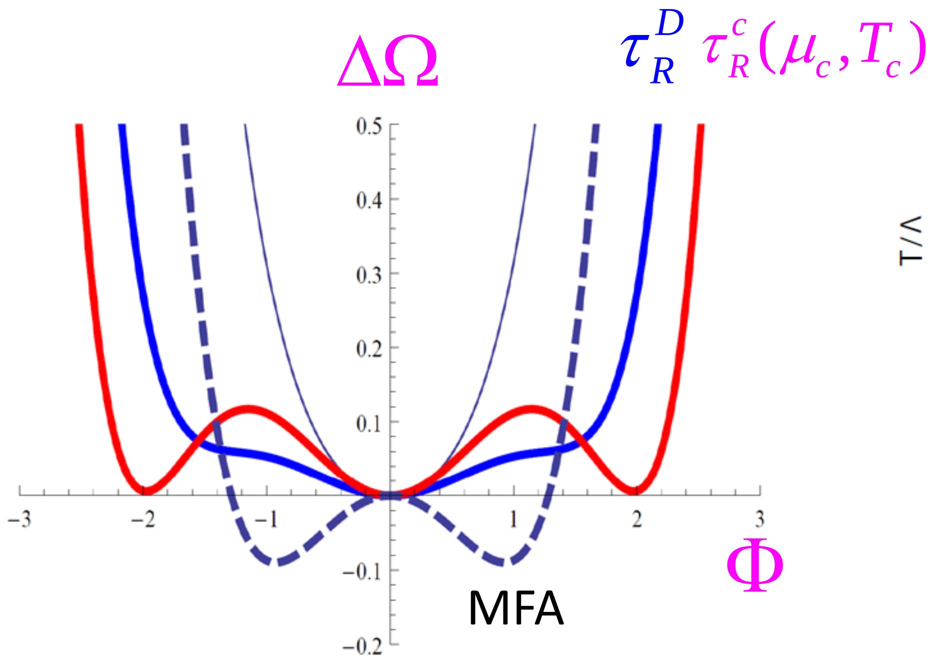
$$\simeq \frac{\lambda T q_c}{16\pi\gamma^{1/2}} \frac{1}{\tau_R^{3/2}}, \quad T \neq 0$$

$$\simeq \frac{\lambda q_c}{8 a_1 \pi^2 \gamma^{1/2}} \frac{1}{\tau_R^{1/2}}, \quad T = 0$$

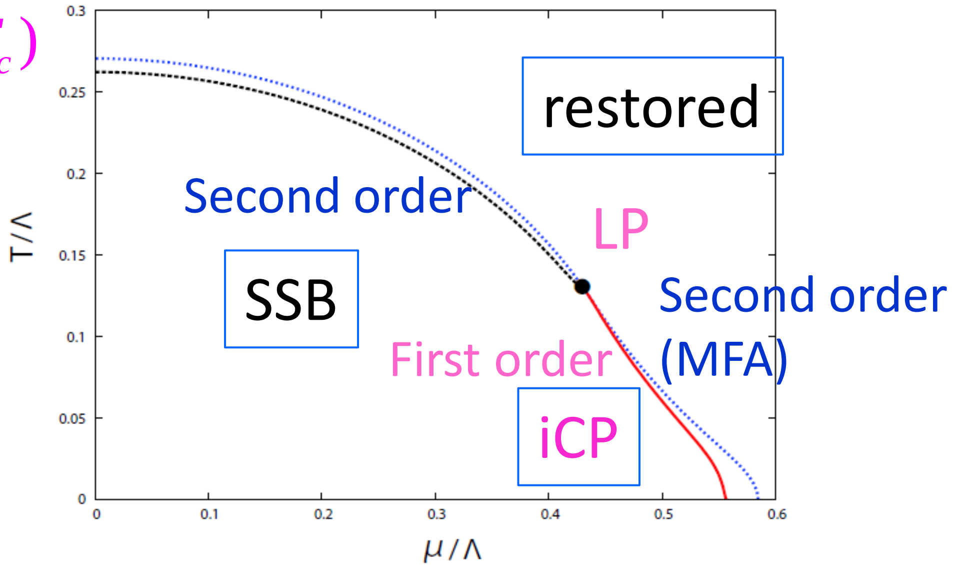
$\bar{\Gamma}^{(4)}$ changes the sign as $\tau_R \to 0$, which signals the *first order phase transition* (Brazovskii-Dyugaev effect).

To summarize

$\overline{\Gamma}^{(6)} > 0$ (positive definite)




Phase diagram




Fluctuation induced first order phase transition

Our results should be supported by
the renormalization group argument a la Shankar.

(P.C. Hohenberg and J.B. Swift, PRE 52, 1828 (1995))

# III. Anomalies of the thermodynamic quantities

Usual second-order phase transitions

Susceptibilities (second derivatives of $\Omega$)

Ex) Specific heat, number-susceptibility

$$C_v = -T\frac{\partial^2\Omega}{\partial T^2} \propto \left(T - T_c\right)^{-1/2}, \chi = -\frac{\partial^2\Omega}{\partial \mu^2}$$

How about the fluctuation-induced first-order phase transition?

Anomalies in the first derivatives

$$S = -\frac{\partial\Omega}{\partial T}, N = -\frac{\partial\Omega}{\partial \mu} \qquad \Omega - \Omega_f = T\sum_q \sum_{\omega_n} \log\left(1 - 2G\bar{\Pi}^0_{ps}(q, \omega_n)\right)$$

eg entropy density

$$s = s_f - \frac{q_c T}{2\pi\gamma^{1/2}}\tau_R^{-1/2} \longrightarrow$$

Implications for relativistic HI collisions?

$s_f$: entropy density for free quarks

# IV Summary and concluding remarks

- We have studied the fluctuation effects on the inhomogeneous chiral transition.

  Fluctuation induced first-order phase transition

- Peculiar dispersion of the fluctuations is very important.

  Singular behavior on the sphere $|\mathbf{q}| = q_c$

- The effects of quantum and thermal fluctuations are figured out;

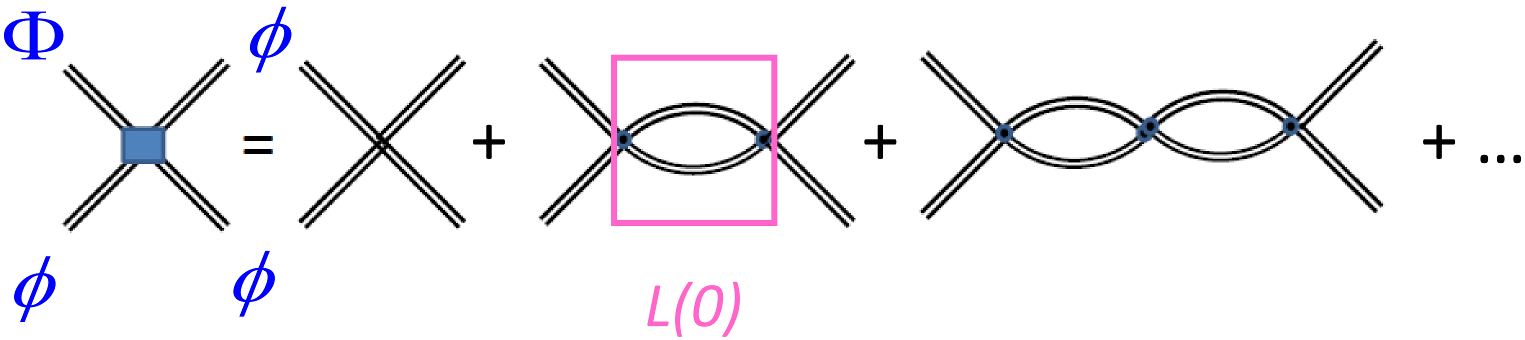
  Thermal fluctuations are more drastic than the quantum ones due to the dimensional reduction.

- Anomalous effect can be seen in the first derivatives of the thermodynamic potential, besides the discontinuous jump.

- Common features for inhomogeneous phase transitions.

  Application for the FFLO state should be interesting.

# Dangerous diagrams=long range interaction between $\phi^2$




$$L(0) = \frac{\lambda}{2} T \sum_{\nu_n} \int \frac{d^3 p}{(2\pi)^3} G^R_{ps}(\mathbf{p}, i\nu_n) G^R_{ps}(-\mathbf{p}, -i\nu_n)$$

$$= \frac{\lambda T}{4\pi^2} \int_0^\infty ds \left[ \frac{1}{2} \sqrt{\frac{\pi}{(4\gamma q_c^2)^3}} s^{-1/2} \tau_R^{-1/2} + \sqrt{\frac{\pi q_c^2}{4\gamma}} s^{1/2} \tau_R^{-3/2} \right] e^{-s} \coth\left( \frac{a_1 \pi T s}{\tau_R} \right)$$

$$\simeq \frac{\lambda T q_c}{16\pi \gamma^{1/2}} \frac{1}{\tau_R^{3/2}}, \quad T \neq 0$$

$$\simeq \frac{\lambda q_c}{8 a_1 \pi^2 \gamma^{1/2}} \frac{1}{\tau_R^{1/2}}, \quad T = 0$$

S.A. Brazovskii, Sov.Phys.JETP 41,85 (1975) (T=0).
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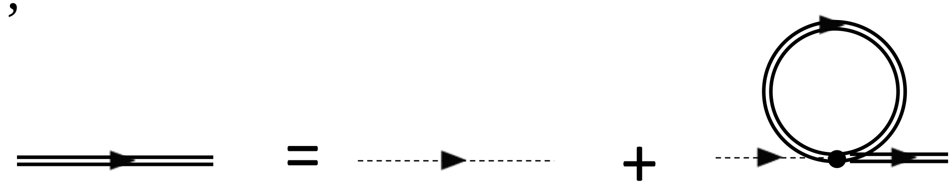
$$G_{ps}^{R}(p, i\nu_n) = \frac{1}{\tau_R + \gamma(|\mathbf{p}|^2 - q_c^2)^2 + a_1 |\nu_n|},$$

Dyson eq. gives



$$\tau = \tau_R - \frac{\lambda}{2} T \sum_{\nu_n} \int \frac{d^3 p}{(2\pi)^3} G_{ps}^{R}(p, i\nu_n)$$

$$= \tau_R - \frac{\lambda T}{4\pi^2} \int_{\tau_R/\Lambda^2}^{\infty} ds \left[ \frac{\tau_R^{1/2}}{2} \sqrt{\frac{\pi}{(4\gamma q_c^2 s)^3}} + \tau_R^{-1/2} \sqrt{\frac{\pi q_c^2}{4\gamma s}} \right] e^{-s} \coth\left( \frac{a_1 \pi T s}{\tau_R} \right)$$

$$\simeq \tau_R - \frac{\lambda T q_c}{8\pi \gamma^{1/2} \tau_R^{1/2}}, \quad T \neq 0$$

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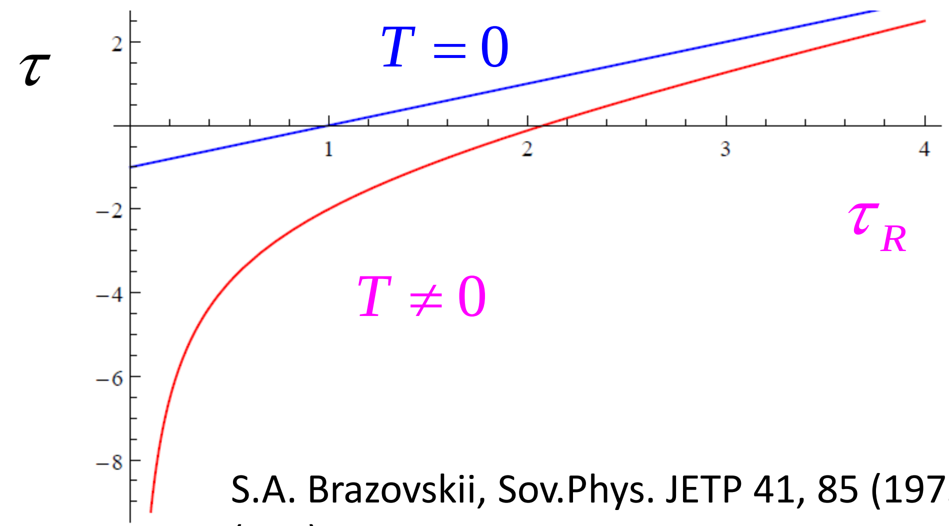


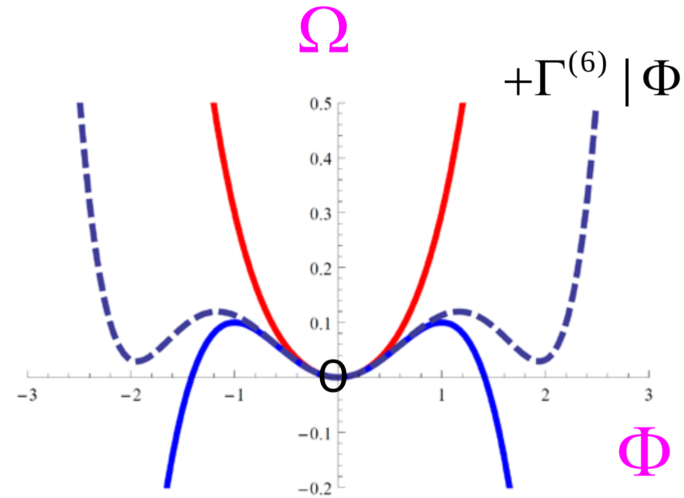

S.A. Brazovskii, Sov.Phys. JETP 41, 85 (1975)
(T=0)

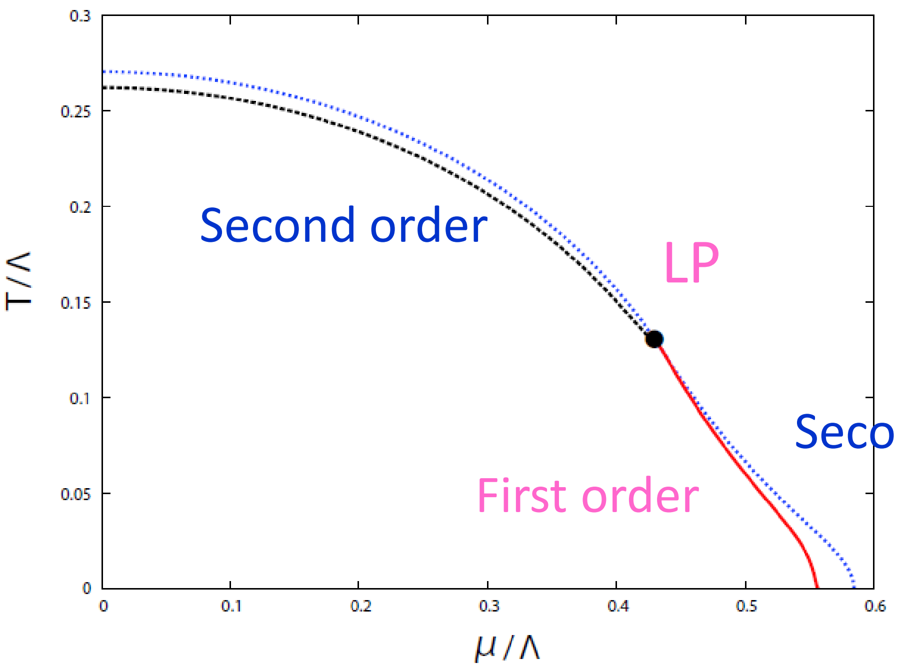
We can prove similar behavior in the FFLO case.

$$\bar{\Gamma}^{(4)} = V\lambda \frac{1 - \frac{\lambda}{2}}{1 + \frac{\lambda}{2}}$$

Change of the sign of $\bar{\Gamma}^{(4)}$ as $\tau_R(\tau) \to 0$.

$+\Gamma^{(6)} |\Phi|^6$ ?



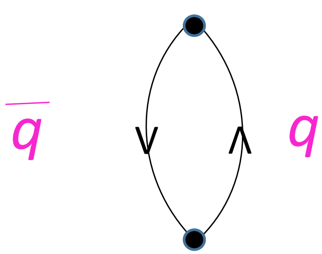

Fluctuation induced first order p.t.

# II Brazovskii and Dyugaev effect

We discuss the properties of the phase transition to the inhomogeneous phases.

Pion condensation, liquid crystal,
FFLO state of superconductivity, diblock copolymer,…

Chiral pair fluctuations in the chiral-restored phase




e.g. Pseudoscalar channel

(Lindhard function)

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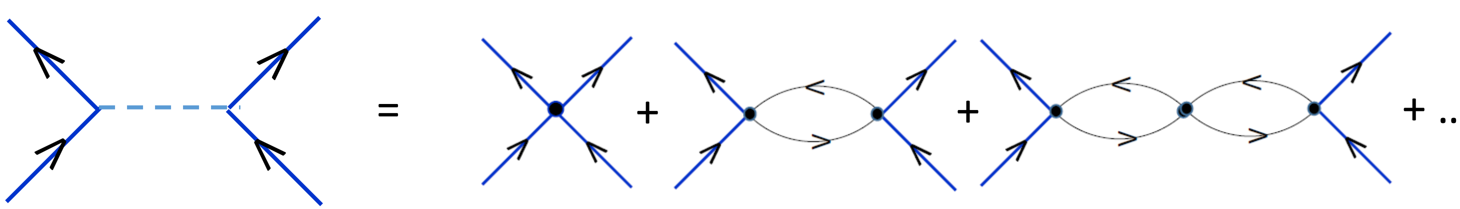
$$p_0 = i(2m+1)\pi T + \mu, q_0 = i2n\pi T,$$

$G_{ps}(q, i\omega_n)$




$\phi \equiv -2G\bar{q}i\gamma_5\tau_3 q$

# Fluctuation effects:

Stability of the one-dimensional structure
by the Nambu-Goldstone excitations ϕ

T.-G. Lee et al, PRD92,034024(2015)
Y. Hidaka et al., PRD92,(2015)

Anisotropic dispersion

$$\omega^2 = ak_z^2 + bk_\perp^4$$

Correlation function at large distance:

$$\boxed{\begin{aligned} &\left\langle \phi(z)\phi^*(0) \right\rangle \to \frac{1}{2}\Delta^2 \cos(qz)\left(\frac{z}{z_0}\right)^{-T/T_0}, z_0 = 2q/\Lambda^2 \\ &\left\langle \phi(\mathbf{r}_\perp)\phi^*(0) \right\rangle \to \frac{1}{2}\Delta^2 \left(\frac{r_\perp}{r_0}\right)^{-2T/T_0}, r_0 = \Lambda^{-1} \end{aligned}}$$

Quasi-long-range order (QLRO)

Note that LRO exists at T=0