QCD Mechanisms in the Parton Structure of Mesons

Glenelg

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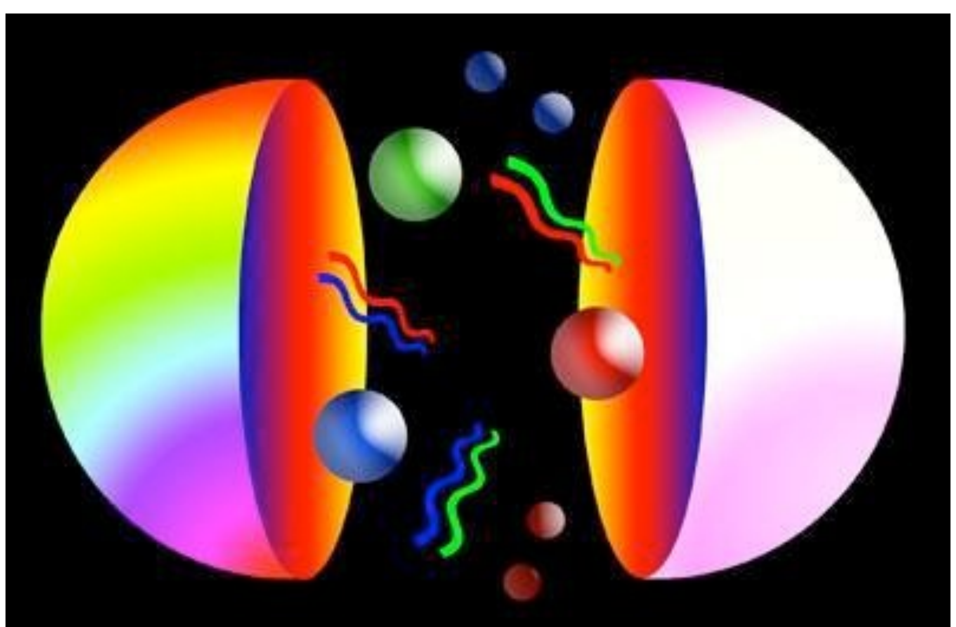
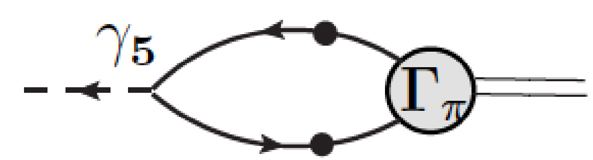
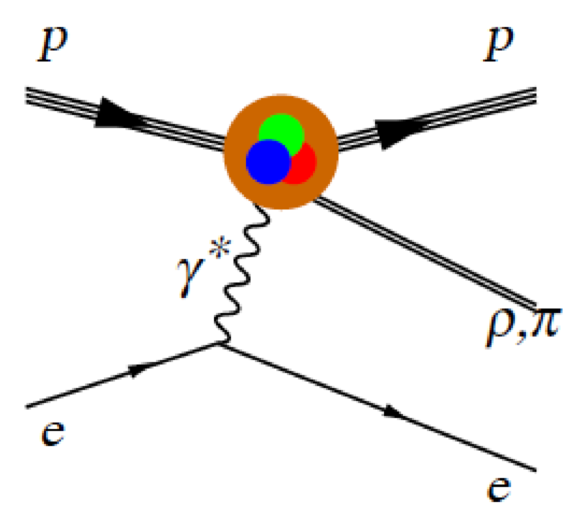






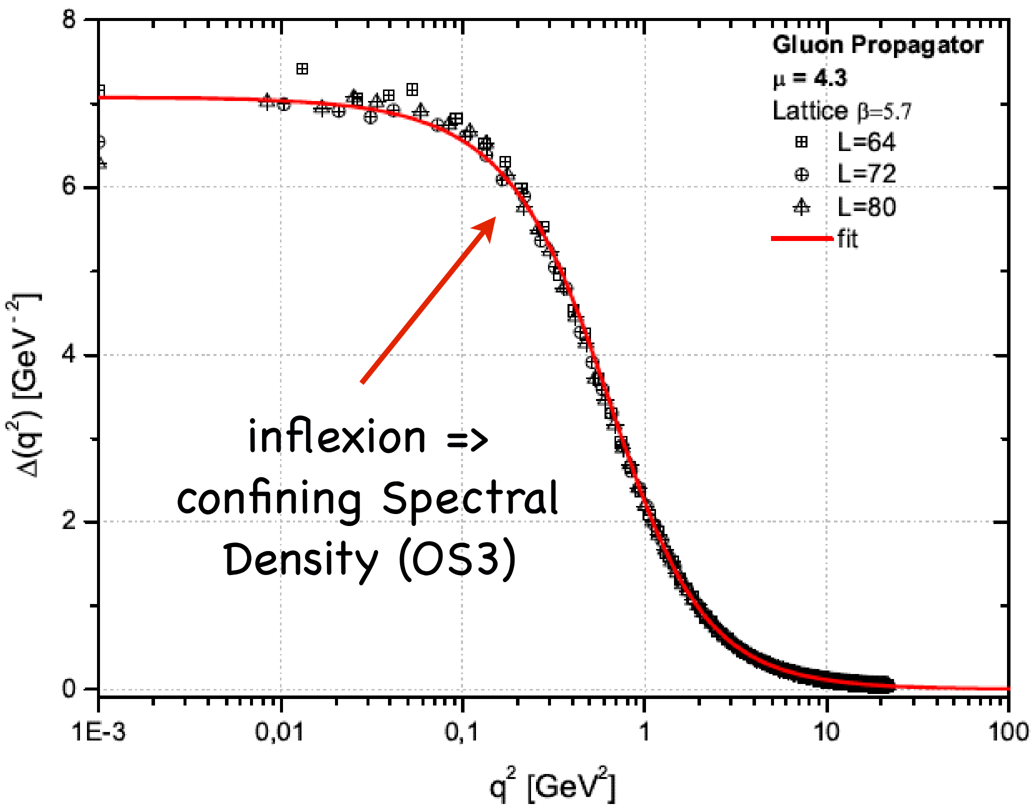
INPC 2016 Adelaide

DSE Modeling of Hadron Physics

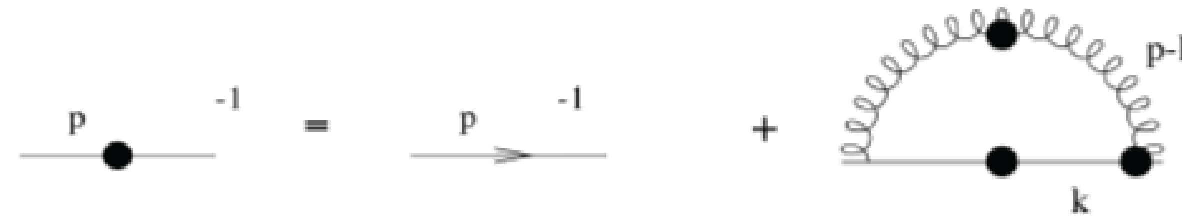
- Most common: Rainbow-ladder truncation of QCD's eqns of motion. Approximation to full BSE kernel now starting to produce results……
- Constrain modeling by preserving AV-Ward-Takahashi Id, V-WTI. [Color singlet] Naturally implements DCSB, conserved vector current, Goldstone Thm, PCAC…
- RL truncation only good for ground state vector & pseudoscalar mesons, q-qq descriptions of baryons with AV and S diquarks.
- At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant. Applicable to all scales, high Q^2 form factors, etc. Do not expect ab initio final-precision QCD results, except in special cases. [pion, kaon..]
- Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....

Modern Context for DSE Interaction Kernel

Landau gauge, lattice – QCD gluon propagator, I.L.Bogolubisky *et al.*, PosLAT2007, 290 (2007)

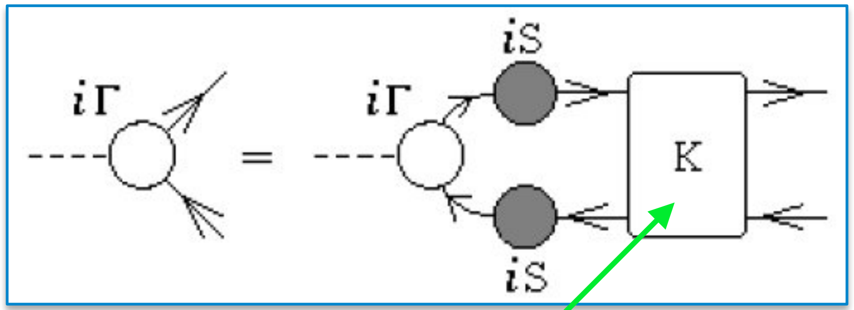


$\Rightarrow m_G(k^2)$ $m_G(0) \sim 0.38$ GeV



- $\alpha_{\rm eff}(q^2) \underset{IR}{\rightarrow} \langle \bar{q}q \rangle_{\mu=1\ \rm GeV} = -(240\rm MeV)^3$
- $\alpha_{\rm eff}(q^2) \underset{UV}{\rightarrow} \alpha_s^{1-\rm loop}(q^2)$
- P. Maris & P.C. Tandy, PRC60, 055214 (1999)

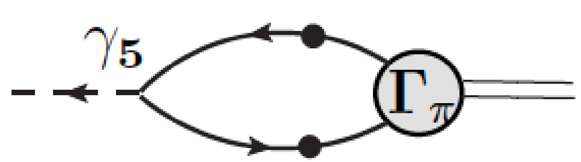
Bridging a gap between continuum-QCD and ab initio predictions of hadron observables
Daniele Binosi (ECT, Trento & Fond. Bruno Kessler, Trento), Lei Chang (Adelaide U., Sch. Chem. Phys.), Joannis Papavassiliou (Valencia U. & Valencia U., IFIC), Craig D. Roberts (Argonne, PHY). Dec 15, 2014. 6 pp.
Published in **Phys.Lett. B742 (2015) 183-188**



An Ansatz for the FULL BSE kernel: Chang, Roberts, PRL103, 081601 (2009), + S. Qin (2015).
Built on explicit qg vertex, incl 3g vertex contribution, + q act

Flavor Non-Singlet PS Mesons: Chiral Symmetry & PCAC Dominates over QCD “Details”

$$\mathbf{PCAC} \;\Rightarrow\; \langle\, \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{y})\; (\partial_\mu \mathbf{J}_{5\,\mu} = 2\mathbf{m_q}\,\mathbf{J_5})\,\rangle \;\Rightarrow \mathbf{AV - WTI}:$$

- $\mathbf{AV - WTI: m_q \to 0, P \to 0 \Rightarrow}$ $\boxed{\Gamma_{\pi q\bar{q}}(\mathbf{k}^2) = \mathbf{i}\gamma_5 \frac{\frac{1}{4}\mathbf{tr S_0^{-1}(k)}}{\mathbf{f_\pi^0}} + \mathcal{O}(\mathbf{P})}$ ie, Goldstone Thm
- $\mathbf{m_q \neq 0: \Rightarrow}$ $\boxed{\mathbf{f_\pi\; m_\pi^2 = 2\, m_q\, \rho_\pi(m_q)} \quad [\text{for all } \mathbf{m_q}, \text{ all ps mesons}]}$
- $\rho_{\rm ps}(\mu) = -\langle \mathbf{0}\,|\,\bar{\mathbf{q}}\,\gamma_5\,\mathbf{q}\,|\,\mathbf{ps}\rangle \;=$ 

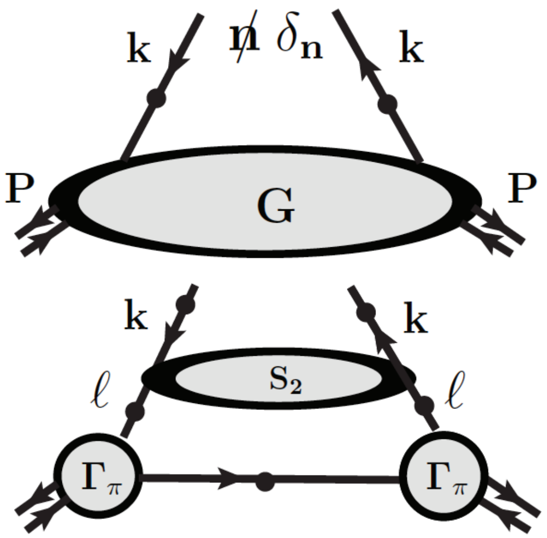
$\to \frac{|\langle \bar{\mathbf{q}}\mathbf{q}\rangle|}{\mathbf{f_\pi^0}} + \mathcal{O}(\mathbf{m_q})$ (GMOR)

Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) – –an exact result in QCD

The Leading Order PDF

$$\mathbf{q_f(x)} = \frac{1}{4\pi}\int d\lambda\ e^{-ixP\cdot n\lambda}\ \langle\pi(P)|\ \bar\psi_f(\lambda n)\ \not n\ \psi_f(0)\ |\pi(P)\rangle_c \qquad n = \hat{z}^{-}\ ,\ k\cdot n \to k^{+}$$

$$\mathbf{q_f(x)} = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ \delta(k\cdot n - xP\cdot n)\ \ \mathrm{tr}_{cd}[\,i\not n\ G(k,P)\,]$$



RL DSE: **q(x) From Directly Obtained Moments**

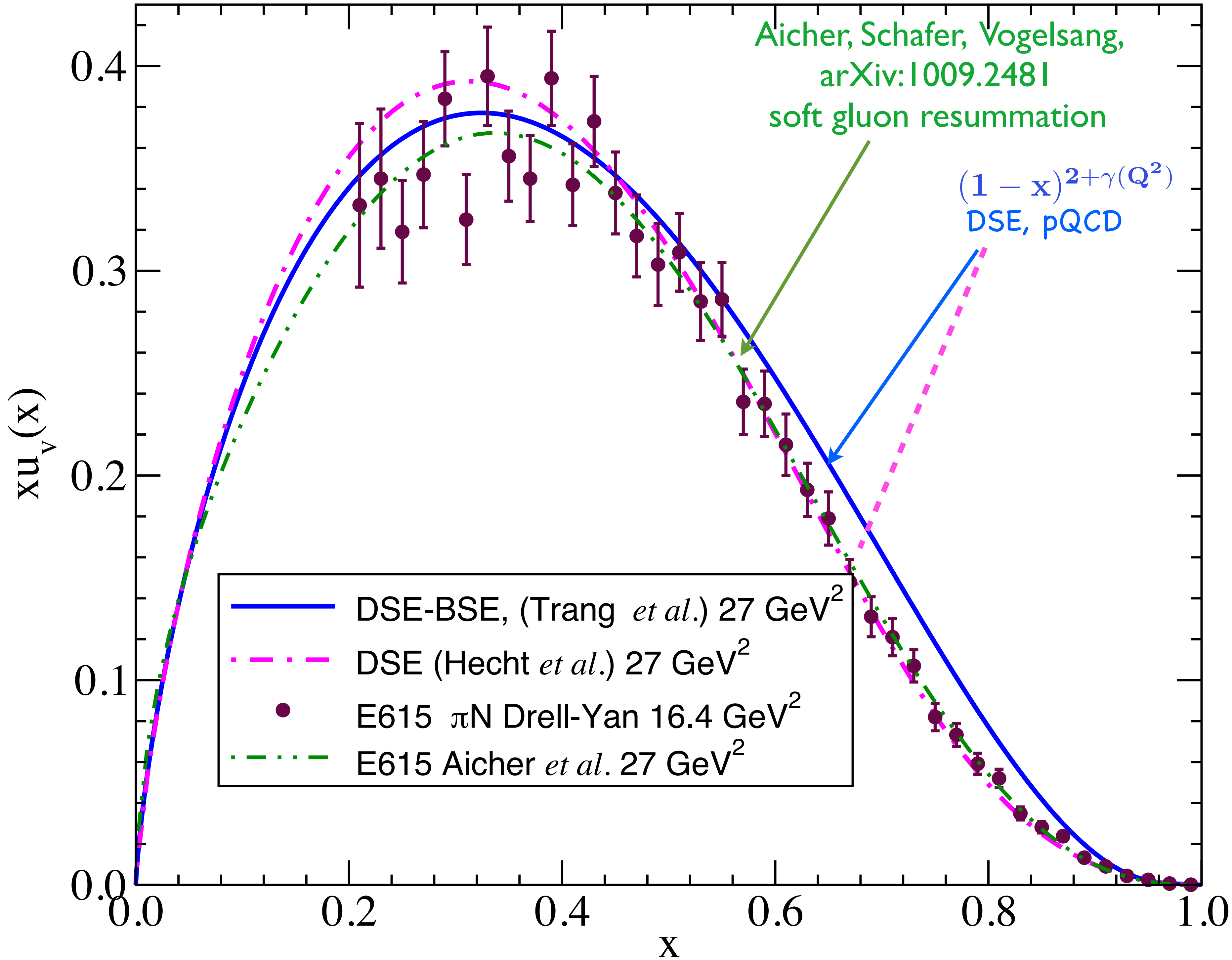
$$\langle x^m\rangle_v^{RL} = \frac{-N_c}{2P\cdot n}\mathrm{tr}\int_\ell \Gamma_\pi(\ell-\frac{P}{2})\ [(\frac{\ell\cdot n}{P\cdot n})^m\, n\cdot\partial_\ell S(\ell)]\ \Gamma_\pi(\ell-\frac{P}{2})\ S(\ell-P)$$

Method can easily exceed the Lattice – QCD practical limit : m = 3

$$\langle x^0\rangle = \ N_f^v = \int_0^1 dx\,[q_f(x) - q_{\bar f}(x)] = \frac{1}{2P^+}\,\langle\pi(P)|J^+(0)|\pi(P)\rangle_c = 1$$

Pion Valence PDF

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448



q_0

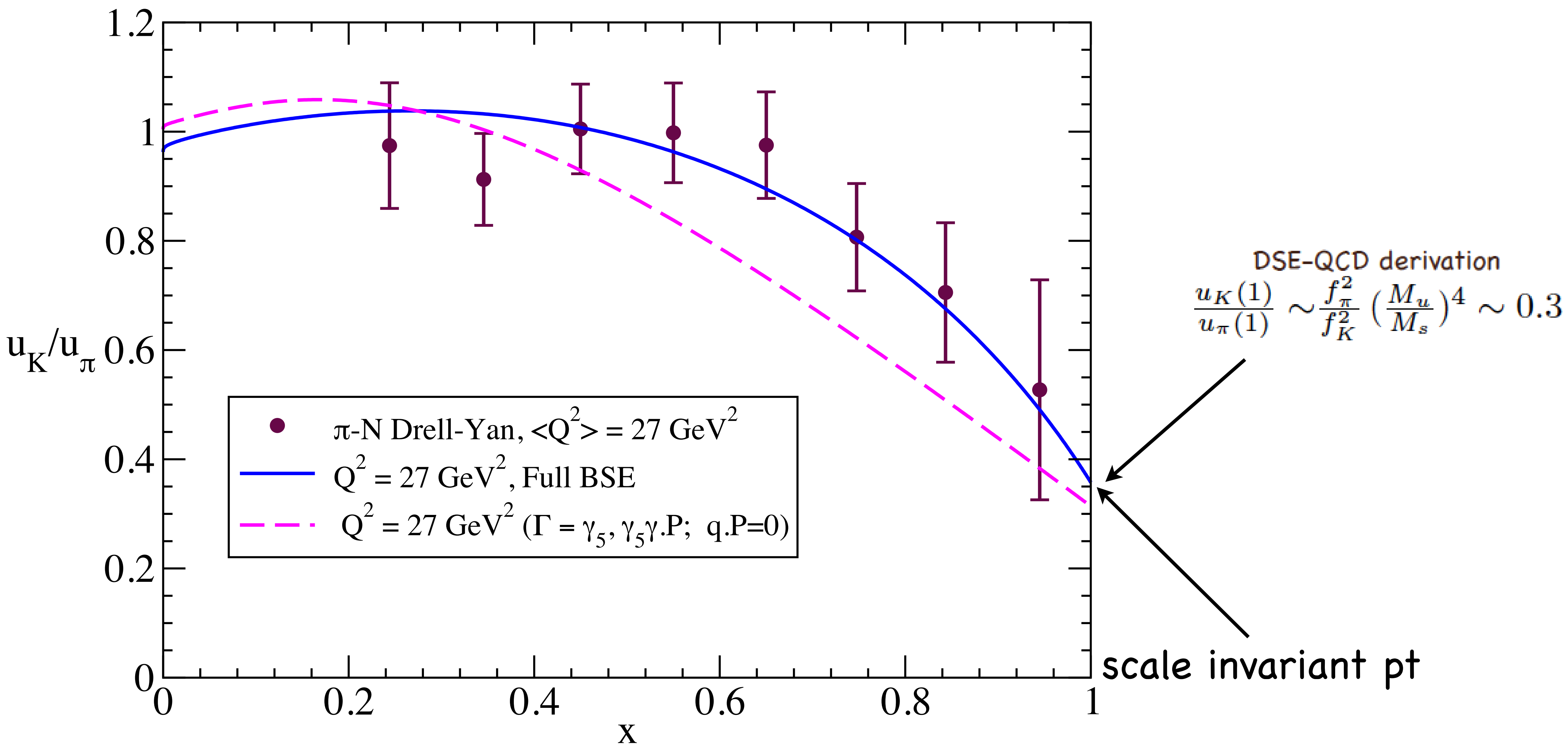
prev PDF expt parm $(1-x)^{1.5}$

CQM, duality.. $(1-x)^1$

NJL (pt π) : $(1-x)^0$

Environmental Dependence of Valence $u(x)$

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448



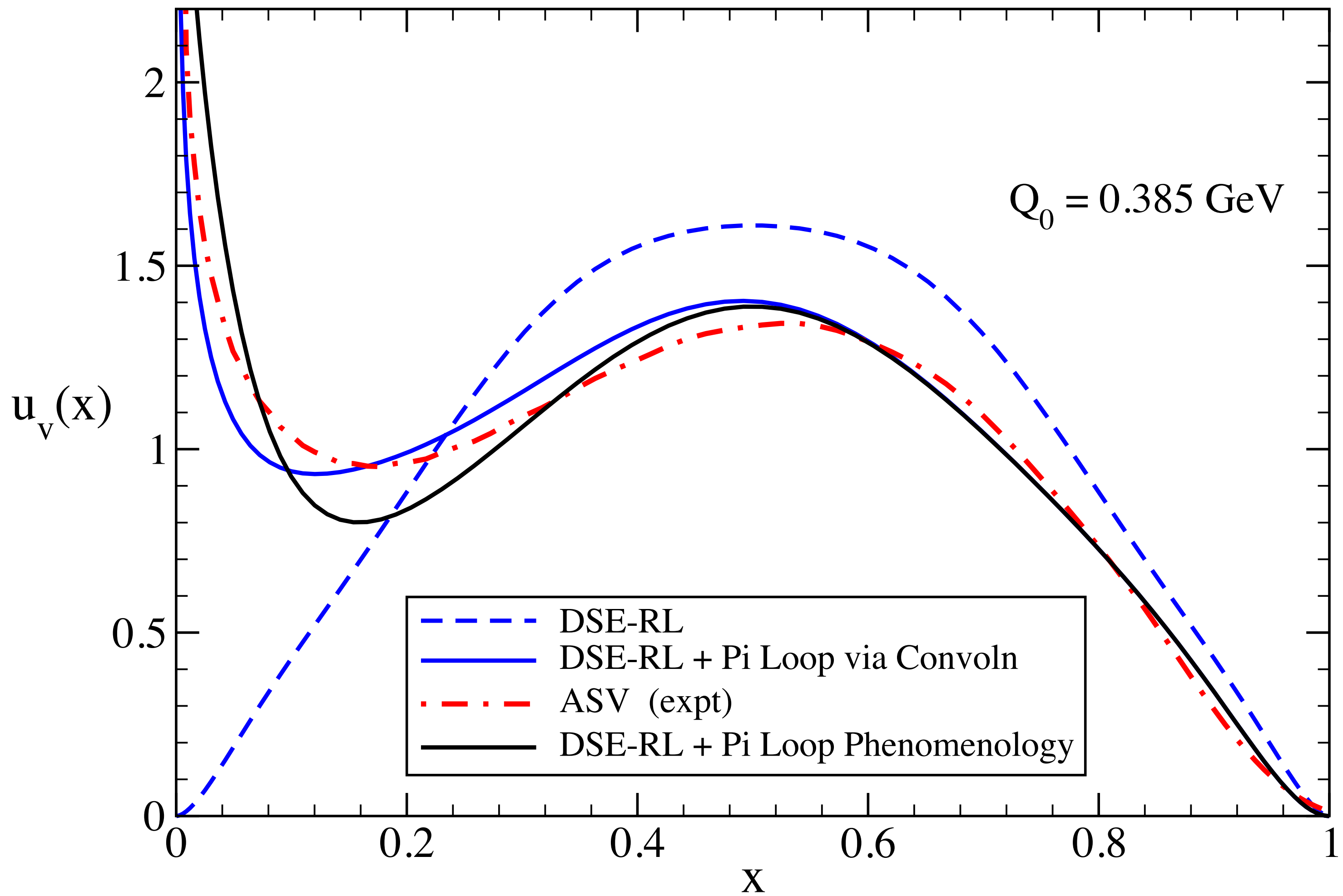
- CERN-SPS data: J. Badier et al, PLB **93**, 354 (1980)

DIVERSION———A full DSE calculation of the true pion valence PDF

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

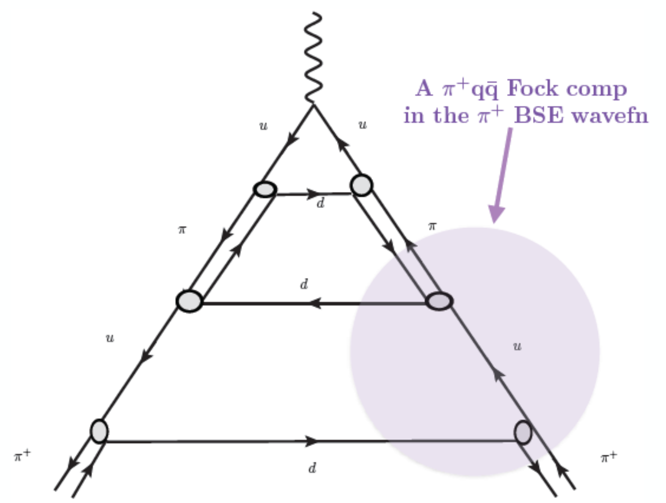
	$2\,q_{\rm val}^{\rm RL}$	$2\,q_{\rm val}^{\rm DSE}$	$4\,q_{\rm sea}^{\rm ASV}$	gluon	Total
$\langle x\rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)



$$q_v^{\pi-lp}(x) \sim \mathcal{P}_{q/T}(x) = \int_x^1 \frac{dy}{|y|}\; \mathcal{P}_{\pi/T}(y)\; \mathcal{P}_{q/\pi}\left(\frac{x}{y}\right),$$

$$T = \text{ target} = \pi \text{ here}$$



$$\Gamma_\pi = \sqrt{1-\alpha^2}\,\Gamma_{q\bar{q}}^{RL} + \alpha\,\Gamma_{\pi q\bar{q}}$$

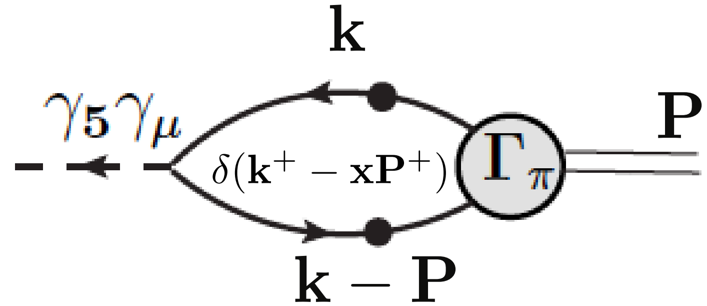
CPT: 18% effect

$$r_{ch}^2 = (1-\alpha^2)\,r_{RL}^2 + \alpha^2\,r_{\pi-lp}^2$$

DSE-RL: $r_{RL}^2 = r_{ch}^2 \Rightarrow \alpha^2 = 18\%$

Pion Distribution Amplitude (leading twist)

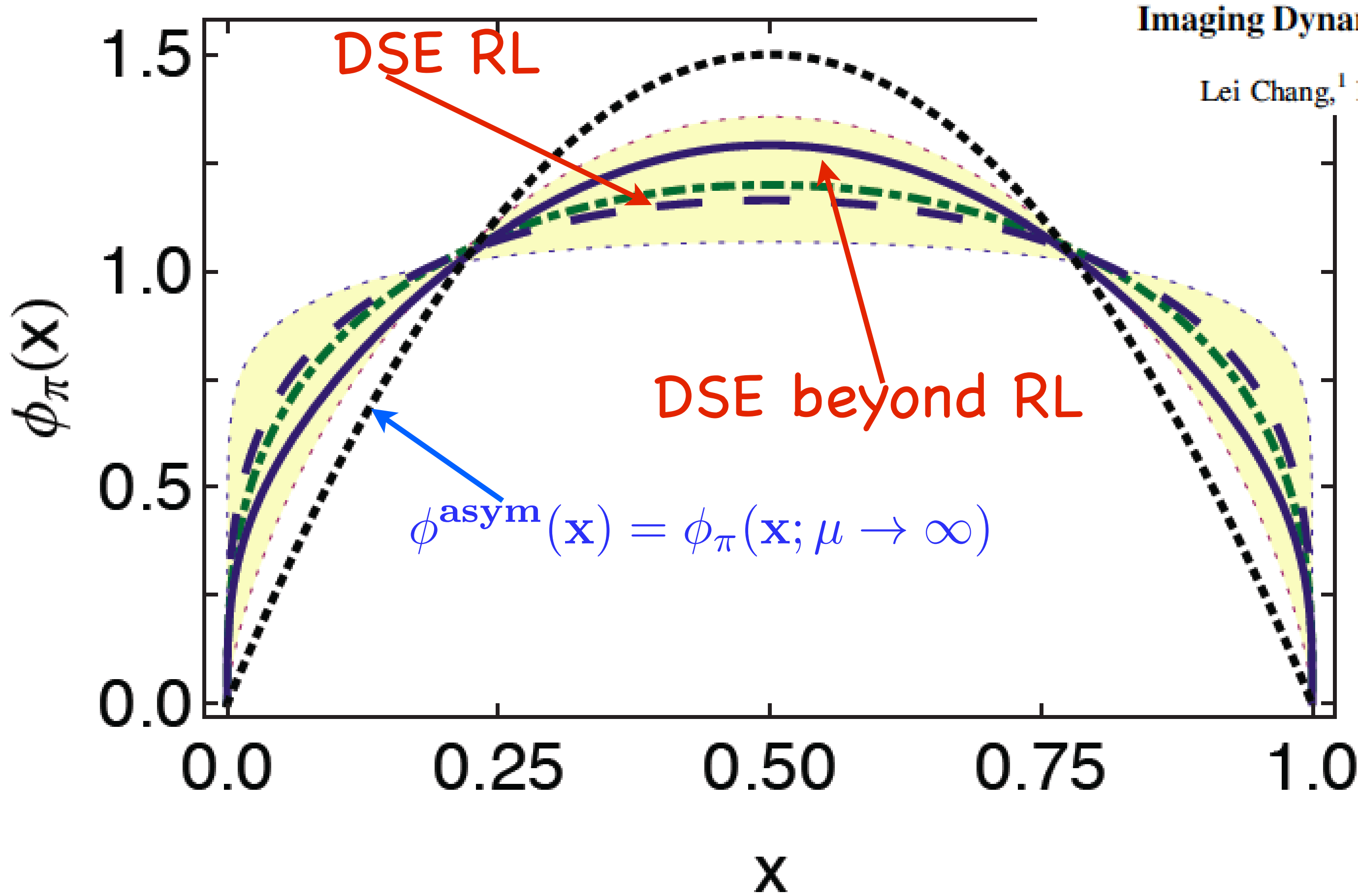
$$\mathbf{f_\pi\,\phi_\pi(x) = \int \frac{d\lambda}{2\pi}\, e^{-ixP.n\lambda}\, \langle 0|\, \bar{q}(0)\, \gamma_5 \not{n}\, q(\lambda n)\, |\pi(P)\rangle}$$



$$\mathbf{f_\pi\,\langle x^m\rangle_\phi = \frac{Z_2 N_c}{P\cdot n}\, tr \int_k \left(\frac{k\cdot n}{P\cdot n}\right)^m \gamma_5 \not{n}\, \left[S(k)\, \Gamma_\pi\left(k - \frac{P}{2}; P\right) S(k-P)\right]}$$

BS Wavefn

$\mu = 2$ GeV



PRL 110, 132001 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,[1] I. C. Cloët,[2,3] J. J. Cobos-Martinez,[4,5] C. D. Roberts,[3,6] S. M. Schmidt,[7] and P. C. Tandy[4]

Broadening of PDA is an expression of DCSB ---long sought after in LF QFT

Pion Distribution Amplitude

ERBL (~1980): $$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1} + \mathbf{\Sigma_{n=2,4\cdots}}\ \mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$$

$$\mathbf{a_n}(\mu) = \mathbf{a_n}(\mu_0)\left[\frac{\alpha_\mathbf{s}(\mu_0)}{\alpha_\mathbf{s}(\mu)}\right]^{\gamma_\mathbf{n}^{(0)}/\beta_0}$$

Evolution to higher scales is
EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit: $\mathbf{a_n}(\mu\to\infty) = \mathbf{0}$

Efficient representation of DSE results:

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{N}_\alpha\ \mathbf{x}^\alpha(\mathbf{1-x})^\alpha\ \{\,\mathbf{1} + \mathbf{\Sigma_{n=2}^\infty}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\,\mathbf{C_n^{\alpha+1/2}(2x-1)}\,\}$$

$$\phi_\mathbf{K}(\mathbf{x};\mu) = \mathbf{N}_\alpha\ \mathbf{x}^\alpha(\mathbf{1-x})^\alpha\ \{\,\mathbf{1} + \mathbf{\Sigma_{n=2,4\cdots}}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\ \mathbf{C_n^{\alpha+1/2}(2x-1)}\,\}$$

$$+\ \mathbf{N}_\beta\ \mathbf{x}^\beta(\mathbf{1-x})^\beta\ \{\,\mathbf{\Sigma_{n=1,3\cdots}}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\ \mathbf{C_n^{\beta+1/2}(2x-1)}\,\}$$

Low Order Truncation of ERBL-Gegenbauer Expn of PDA

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\left\{\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\ \mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\right\}$$

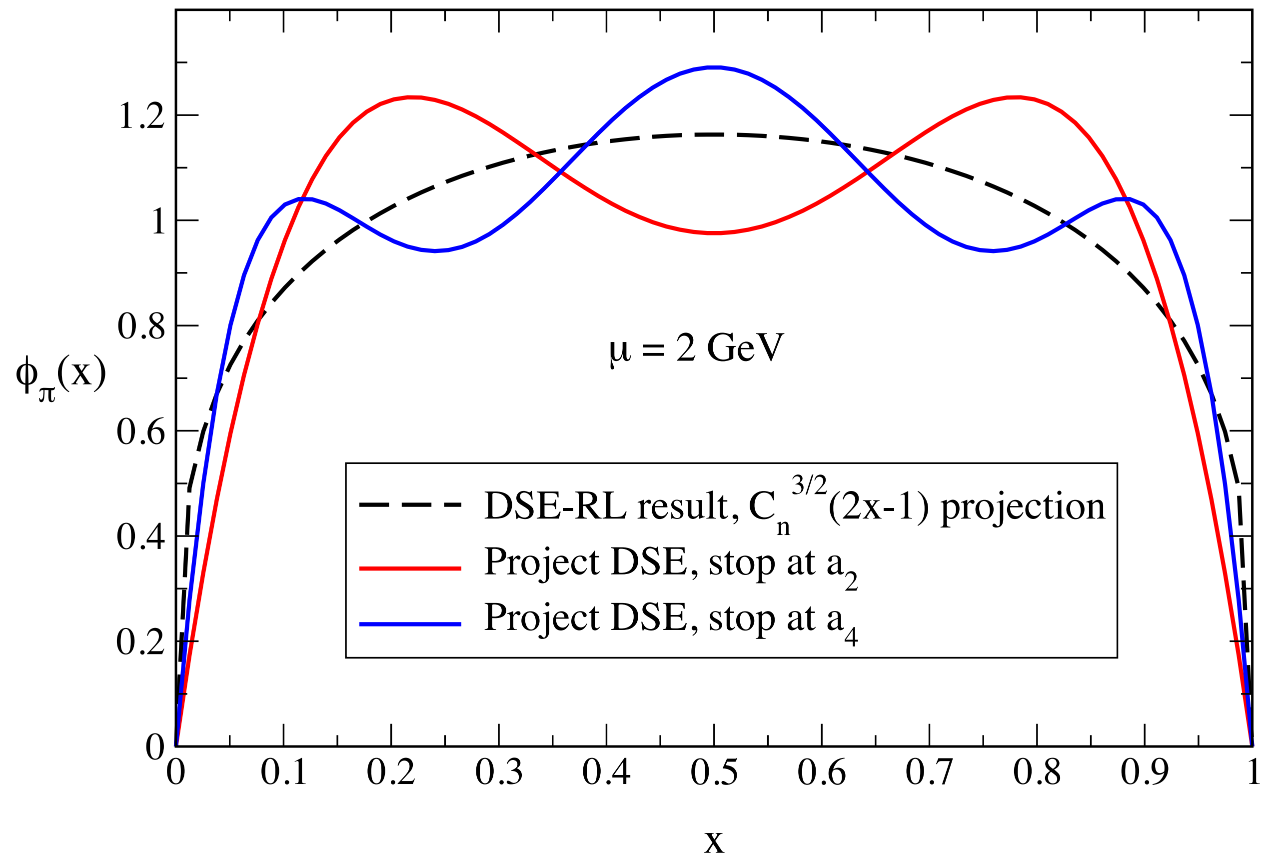
DSE soln

{{0, 1.}, {**2, 0.233104**}, {4,0.112135},

{6, 0.0683202}, {8, 0.0469145},
{10, 0.0346469}, {12, 0.0268732},
{**14, 0.0215933**}, {16, 0.0178199}, **10%**
{18, 0.0150159}, {20, 0.0128672},
{22, 0.0111788}, {24, 0.00982438},
{26, 0.00871886}, {28, 0.00780296},

{30, 0.00703438}, {32, 0.0063823},
{34, 0.00582279}, {36, 0.00534272},
{38, 0.00493277}, {**40, 0.00447911**}} **+.....**

2%



A double-humped PDA is almost ruled out by
V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)

$$\phi_\pi^{\mathbf{QCDSR}}(\mathbf{x = 1/2};\mu = \mathbf{2}) = \mathbf{1.2 \pm 0.3}$$

One Lattice-QCD Moment Almost Determines Pion DA

PRL 111, 092001 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Pion Distribution Amplitude from Lattice QCD

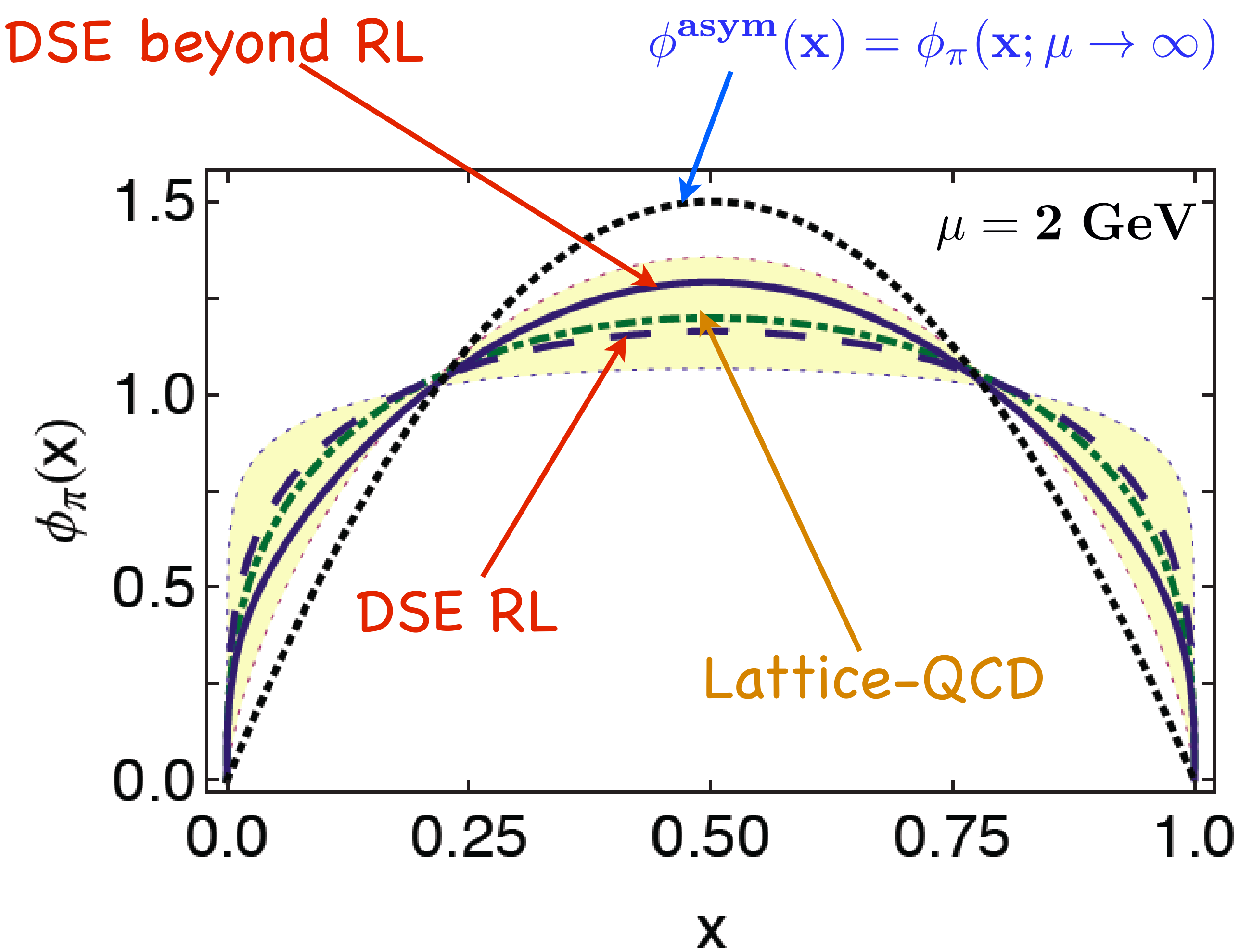
I. C. Cloët,[1] L. Chang,[2] C. D. Roberts,[1] S. M. Schmidt,[3] and P. C. Tandy[4]

$$\phi_\pi^{\mathrm{LQCD}}(x; \mu = 2) = N\, x^\alpha (1 - x)^\alpha$$

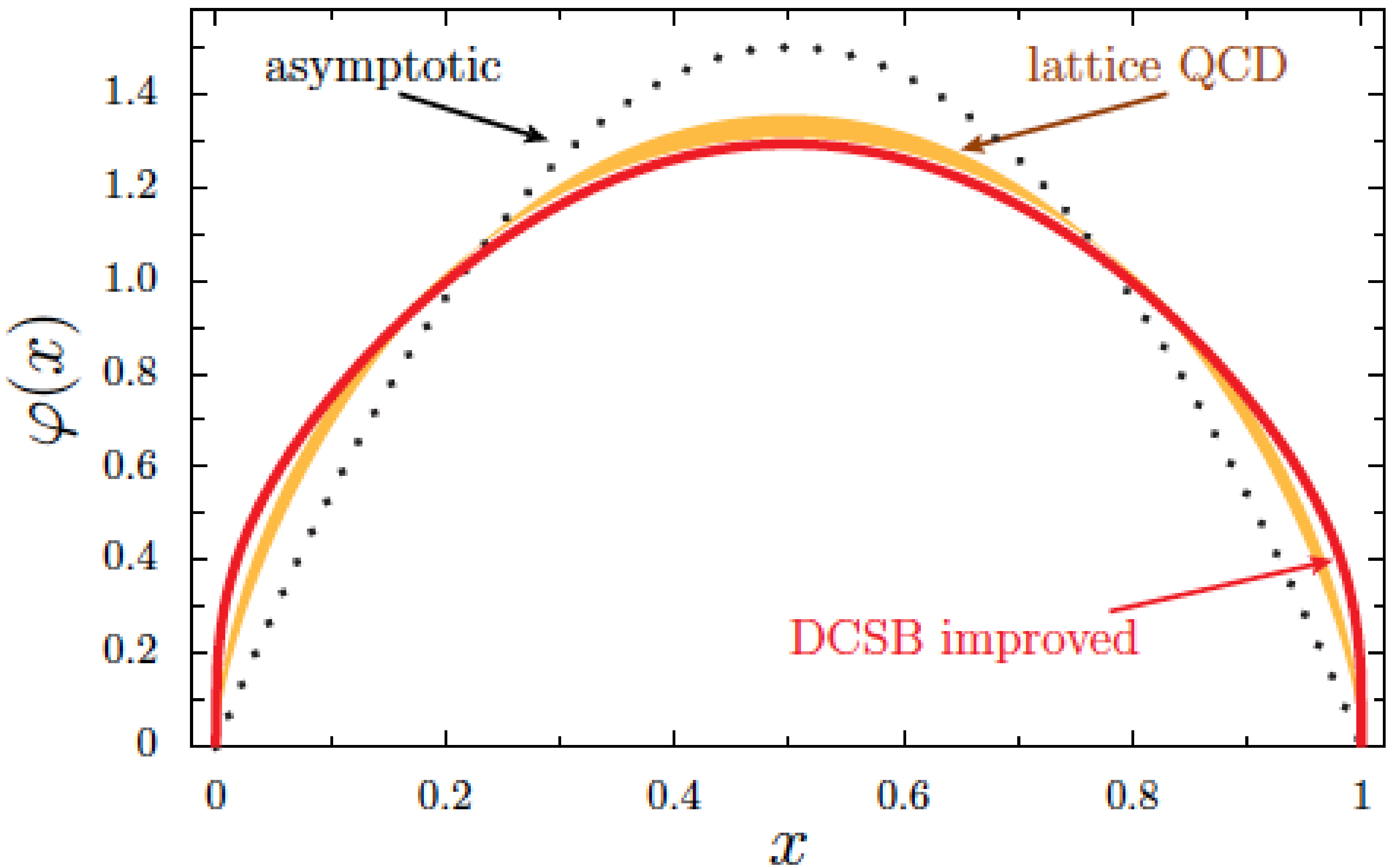
$$\alpha = 0.35 + 0.32 - 0.24$$

$$\langle (2x - 1)^2 \rangle^{\mathrm{LQCD}}_{\mu=2} = 0.27 \pm 0.04$$

V. Braun et al., PRD74, 074501 (2006)



Pion Distribution Amplitude



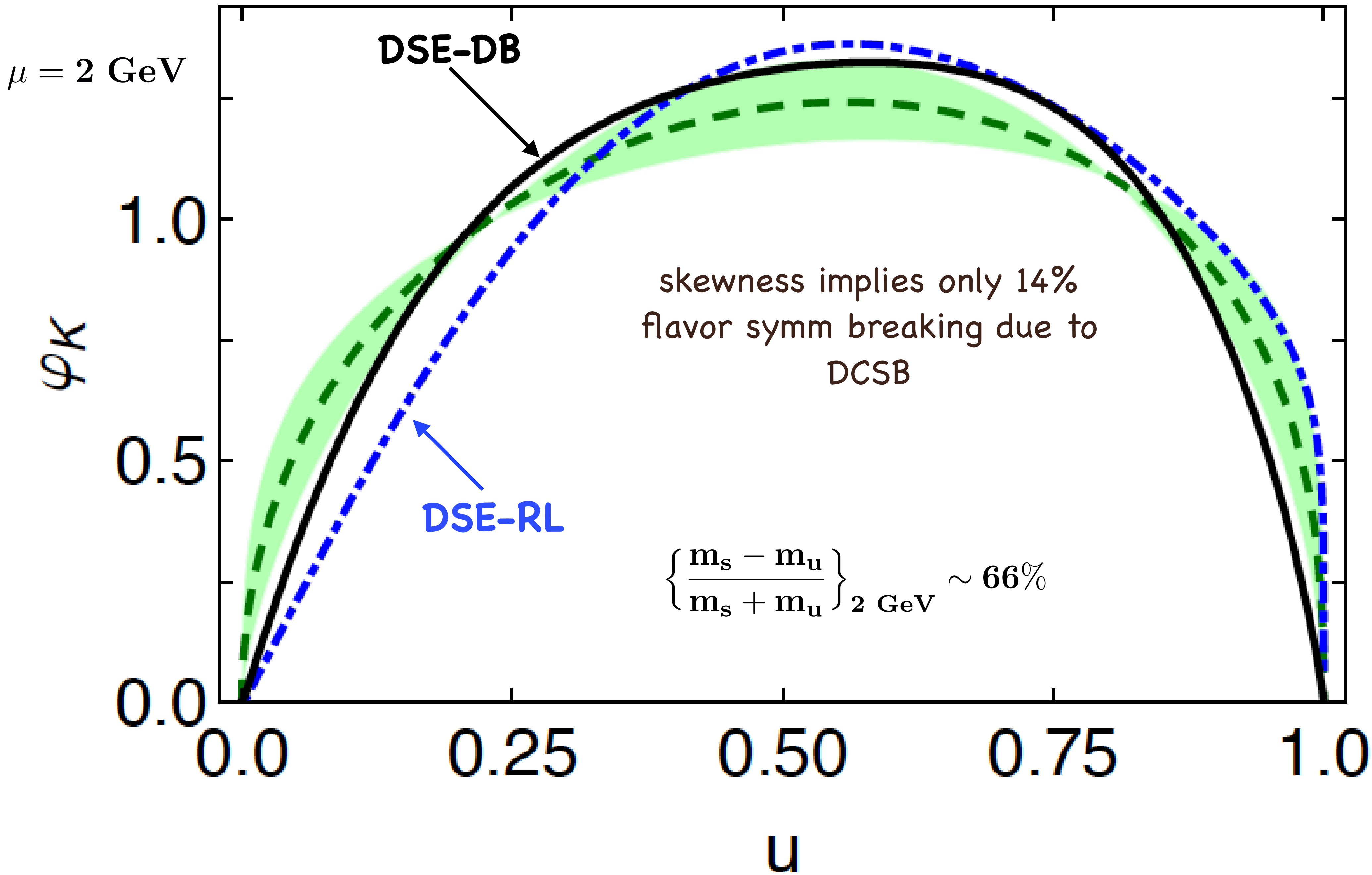
$$\langle (2x-1)^2 \rangle^{LQCD}_{\mu=2\ GeV} = 0.2361\,(41)\,(39)$$

V. Braun et al., arXiv:1503.03656 [hep=lat]

DSE prediction: 0.251

Kaon Distribution Amplitude

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)



R. Arthur, P. Boyle, D. Brommel, M. Donnellan, J. Flynn et al, PRD83, 074505 (2011)

Kaon DA Moments

$\mu = $ **2 GeV**

Table 1
Moments ($u_\Delta = 2u - 1$) of the K-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall *Ansatz* for the kaon's light-front wave function. We also list values obtained with $\varphi = \varphi^{asy}$, Eq. (14), and $\varphi = \varphi_{ms}$, Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

	$\langle u_\Delta^m \rangle$	$m = 1$	2	3	4	5	6
DSE-QCD:	RL	0.11	0.24	0.064	0.12	0.045	0.076
	DB	0.040	0.23	0.021	0.11	0.013	0.063
	[40]	0.027(2)	0.26(2)				
Lattice-QCD:	[41]	0.036(2)	0.26(2)				
	[10]	0.036(2)	0.26(2)	0.020(2)	0.13(2)	0.014(2)	0.085(15)
QCD Sum Rules:	[42–46]	0.035(8)					
	[47]	0.04(2)	0.24(1)				
	$\varphi = \varphi_{ms}$	0.33	0.33	0.2	0.2	0.14	0.14
	$\varphi = \varphi^{asy}$	0	0.2	0	0.086	0	0.048

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)

Other Pion & Kaon DAs

$\mu = 2$ GeV

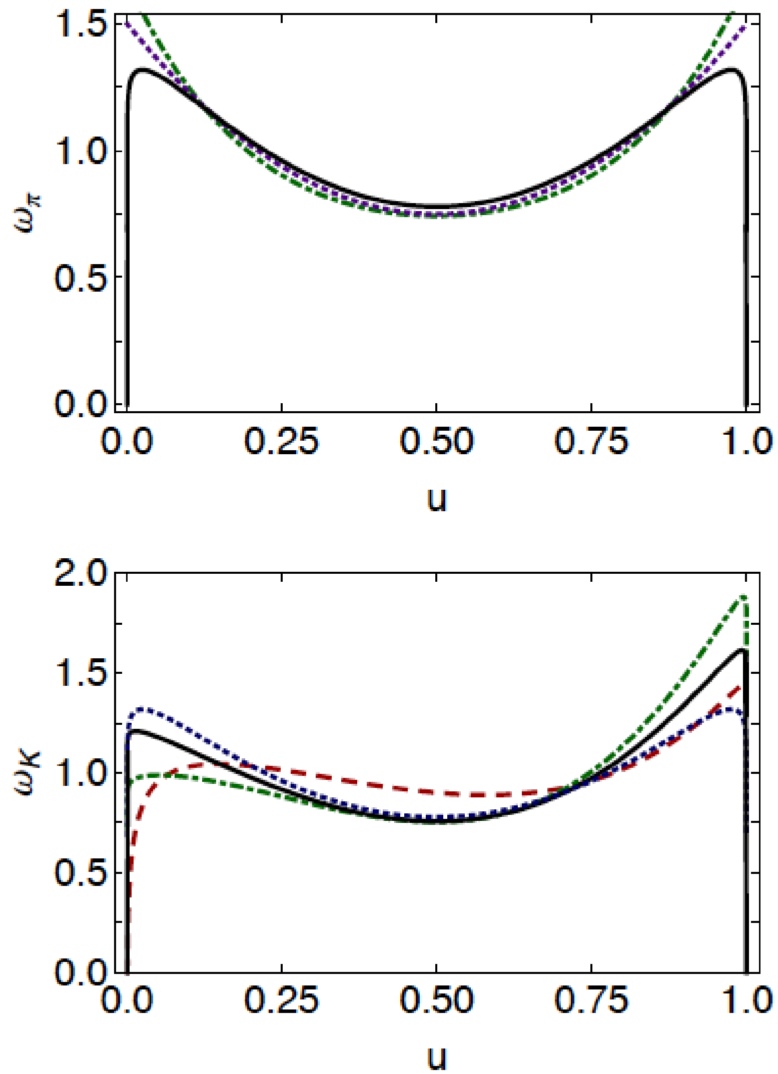
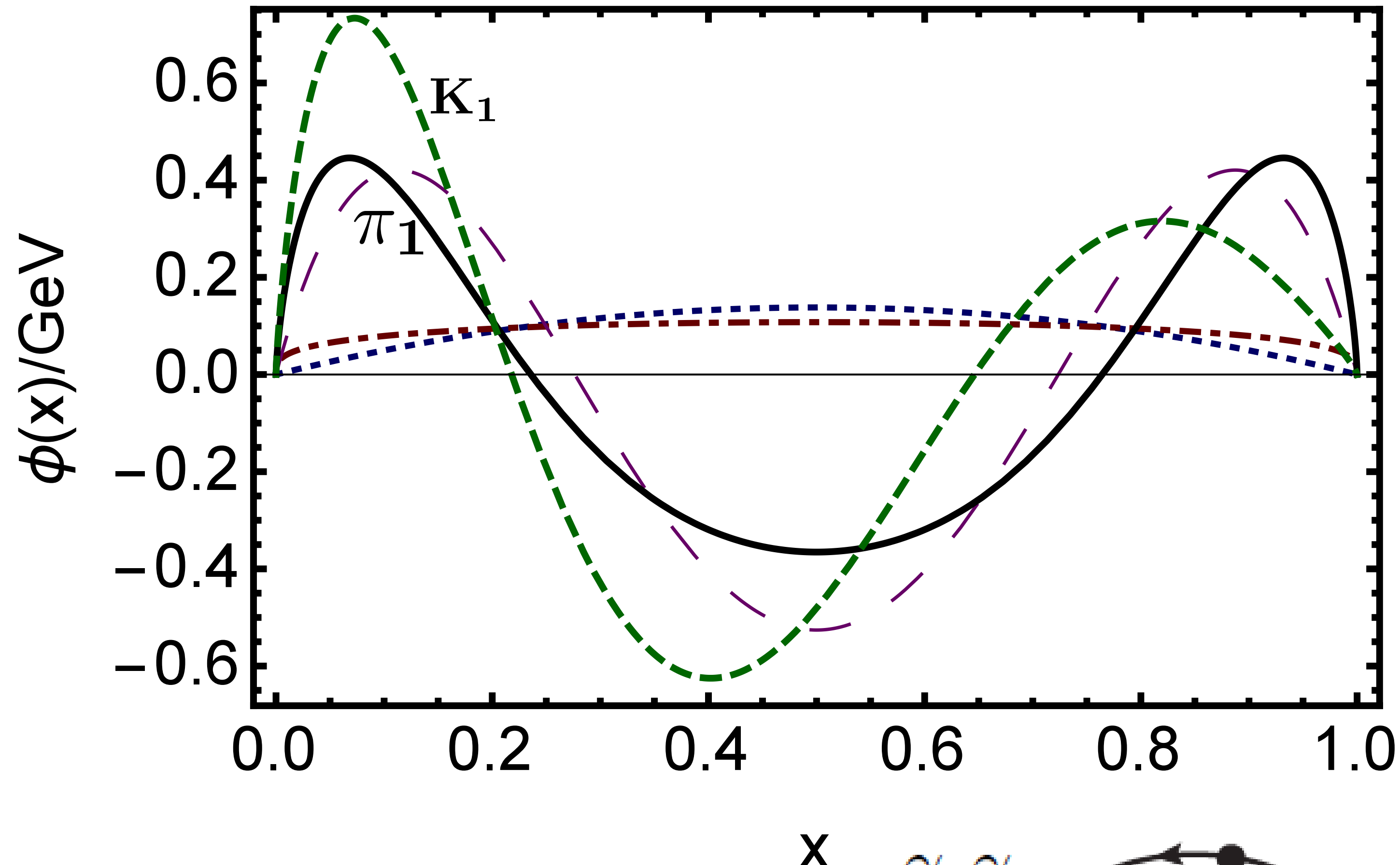
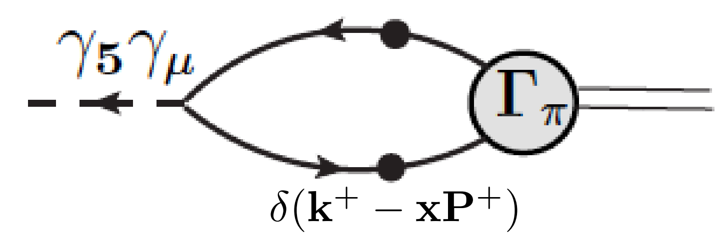
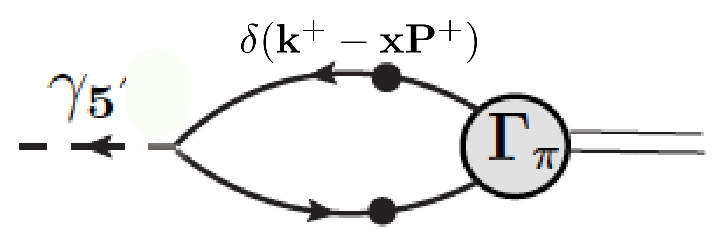


FIG. 3. Pseudoscalar two-particle, twist-three PDAs, computed at ζ_2. *Upper panel* – pion: solid curve (black), prediction in Eq. (35); dot-dashed curve (dark green), QCD sum rules estimate [29]; and dotted curve (indigo), ω_P^{asy} in Eq. (26). *Lower panel* – kaon: solid curve (black), DB result; dot-

Bo-Lin, L. Chang, F. Gao, C.D.Roberts, S.M.Schmidt, H-S., Zong, Phys. Rev. D93 114033 (2016).







Chao Shi, C. Chen, L. Chang, C.D.Roberts, S.M.Schmidt, H-S., Zong, Phys. Rev. D92 014035 (2015).

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$\mathbf{F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy\, \phi^\star_\pi(x;Q)\, [T_H(x,y;Q^2)]\, \phi_\pi(y;Q)}$$ + NLO/higher twist.....

---LFQCD, Brodsky, LePage PRD (1980)

$$\mathbf{Q^2 >> \Lambda^2_{QCD} :\ Q^2 F_\pi(Q^2) \to 16\,\pi\, f^2_\pi\, \alpha_s(Q^2)\, \omega^2_\phi(Q^2) + \mathcal{O}(1/Q^2)}$$

$$\mathbf{\omega_\phi(Q^2) = \frac{1}{3}\int_0^1 dx\, \frac{\phi_\pi(x;Q)}{x}}$$

$\mathbf{\to 1 ,\quad Q^2 \to \infty}$ but deathly slow

at $\mathbf{Q^2 \sim 3 - 4\ GeV^2, \Rightarrow 0.1}$

JLab expt, Theory $\Rightarrow$ 0.45

But, recent DSE theory $\Rightarrow \phi_\pi(x;\mu = 2\ GeV) \Rightarrow \omega^2_\phi = 3.3$

PRL **111,** 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

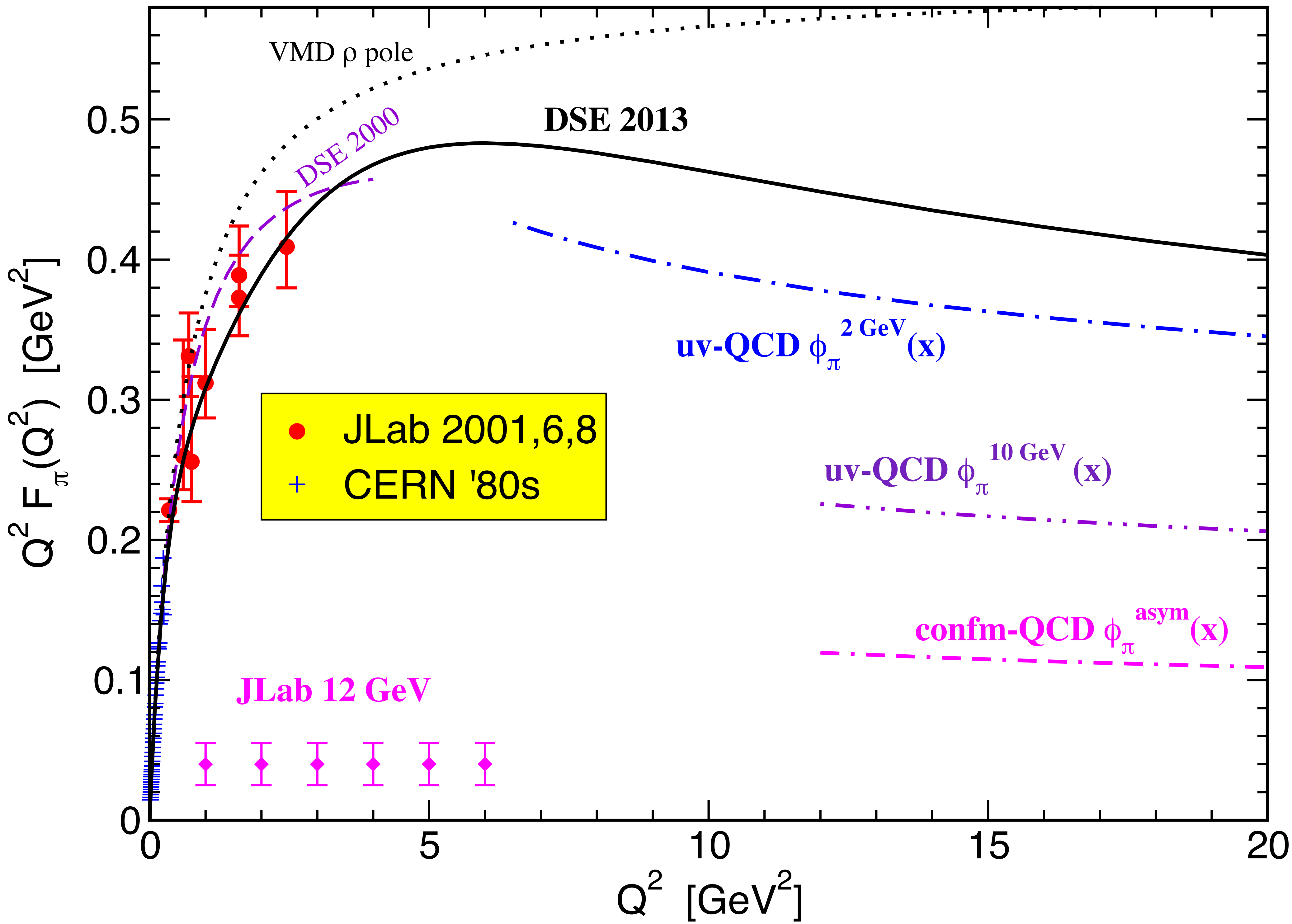
Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I. C. Cloët,[2] C. D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

PRL 111, 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I.C. Cloët,[2] C.D. Roberts,[2] S.M. Schmidt,[3] and P.C. Tandy[4]



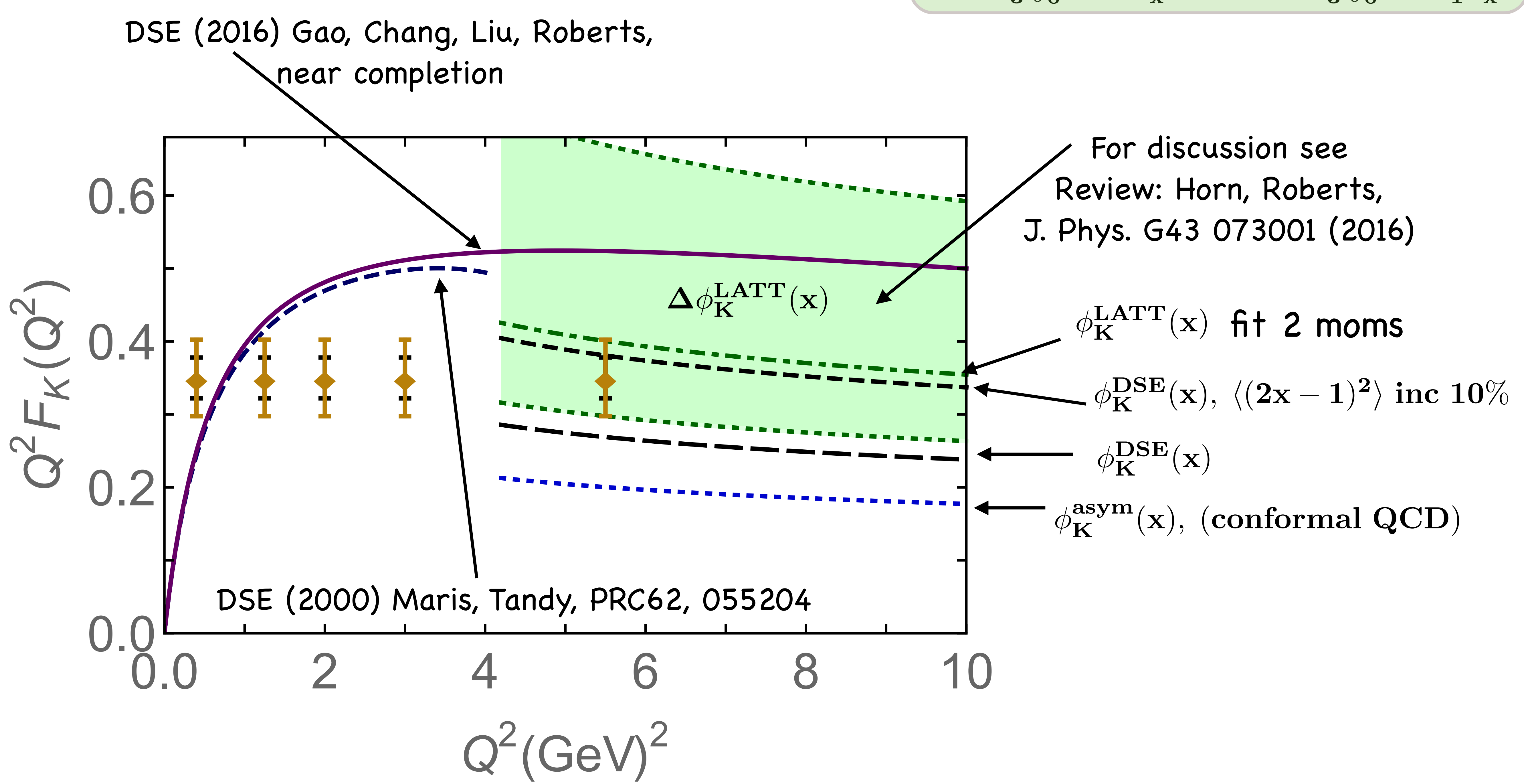
Jab data: G. Huber et al., PRC78, 045203 (2008)

Kaon Elastic Form Factor

$$\text{HS/UV}: \quad Q^2 F_K(Q^2) \to 16\,\pi\,\alpha_s(Q^2)\, f_K^2\, \omega_K^2(Q^2)$$

$$\omega_K^2 = e_u\,\omega_u^2 + e_{\bar s}\,\omega_{\bar s}^2 \;\to 1,\, Q^2 \to \infty$$

$$\omega_u = \frac{1}{3}\int_0^1 dx\, \frac{\phi_K(x)}{x} \neq \omega_{\bar s} = \frac{1}{3}\int_0^1 dx\, \frac{\phi_K(x)}{1-x}$$



Hard Scattering Kaon Elastic Form Factor

$$\mathbf{HS/UV:}\quad Q^2F_K(Q^2) \to 16\,\pi\,\alpha_s(Q^2)\, f_K^2\, \omega_K^2(Q^2) \qquad \omega_K^2 = e_u\,\omega_u^2 + e_{\bar{s}}\,\omega_{\bar{s}}^2 \quad \to 1,\, Q^2 \to \infty$$

$$\omega_u = \frac{1}{3}\int_0^1 dx\, \frac{\phi_K(x)}{x} \neq \omega_{\bar{s}} = \frac{1}{3}\int_0^1 dx\, \frac{\phi_K(x)}{1-x}$$

$$\text{Expt } (s_U = 17.4 \text{ GeV}^2 \text{ timelike}): \quad \frac{F_K(s_U)}{F_\pi(s_U)} = 0.92(5)$$

K. Seth et al., PRL110, 022002 (2013)

$$\frac{f_K^2}{f_\pi^2} = 1.43$$

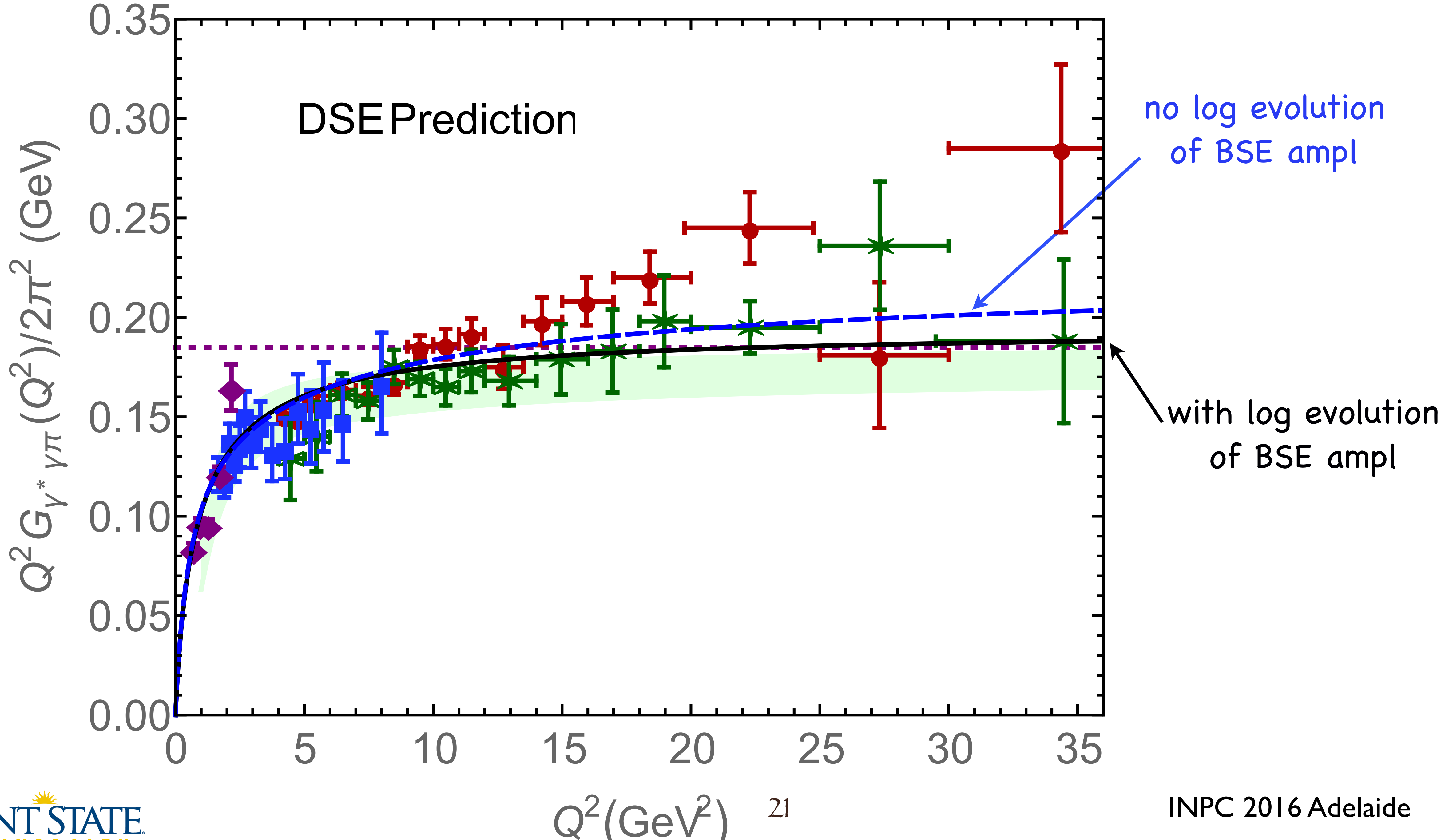
$$\frac{F_K(s_U)}{F_\pi(s_U)} = 1.16\,(\phi^{DSE});\quad 1.16 + 50\%\,(\phi^{LQCD})$$

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)

Pion Transition Form Factor

K. Raya, L. Chang, A. Bashir, J.J.Cobos-Martinez, L.X. Gutierez-Guerrero, C.D.Roberts, P.C.Tandy,
PRD93, 074017 (2016)

From unified treatment of DA, elastic FF, and transition FF



Summary

- DSE approach works extremely well for pion & kaon due to symmetry dominance. Time to declare we understand the pion and kaon in QCD ?

- Parton DAs (pion, kaon) & PDFs. DSE approach shows good contact with available LQCD moments. Flavor sum breaking & DCSB evident and quantitative in the shapes. Empirical PDF data fits reproduced by DSE-RL q-qbar + empirical pion loop analysis. Connection with QCD ultraviolet /hard scattering form factors reconciled.

- QCD mechanisms being revealed: DCSB, PDF ~ (1-x)^2+, HS behavior of Fpi/K(Q^2) & ERBL evolution, constit. to partonic evolution in FFs, explicit gq, 3g couplings…

The End

Collaborators

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Sixue Qin, Argonne National Lab, USA
- Hong-shi Zong, Nanjing Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico

To help lattice-QCD be more applicable to hadron PDFs and GPDs than just the first 3 moments ?

PRL 110, 262002 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Parton Physics on a Euclidean Lattice

Xiangdong Ji[1,2]

Standard light-cone correlator, leading twist: $x = k \cdot n / P \cdot n = k^+/P^+ \;\; \epsilon \;\; [0,1]$

$$q_f(x) = \frac{1}{4\pi} \int d\lambda \; e^{-ix\, P\cdot n\, \lambda} \, \langle \pi(P) | \, \bar{\psi}_f(\lambda n) \; \not{n} \; \psi_f(0) \, | \pi(P) \rangle_c$$

$$n^2 = 0 \;\; ; \;\; z^- = \lambda\, n \;\; ; \;\; z^+ = 0 = z_\perp$$

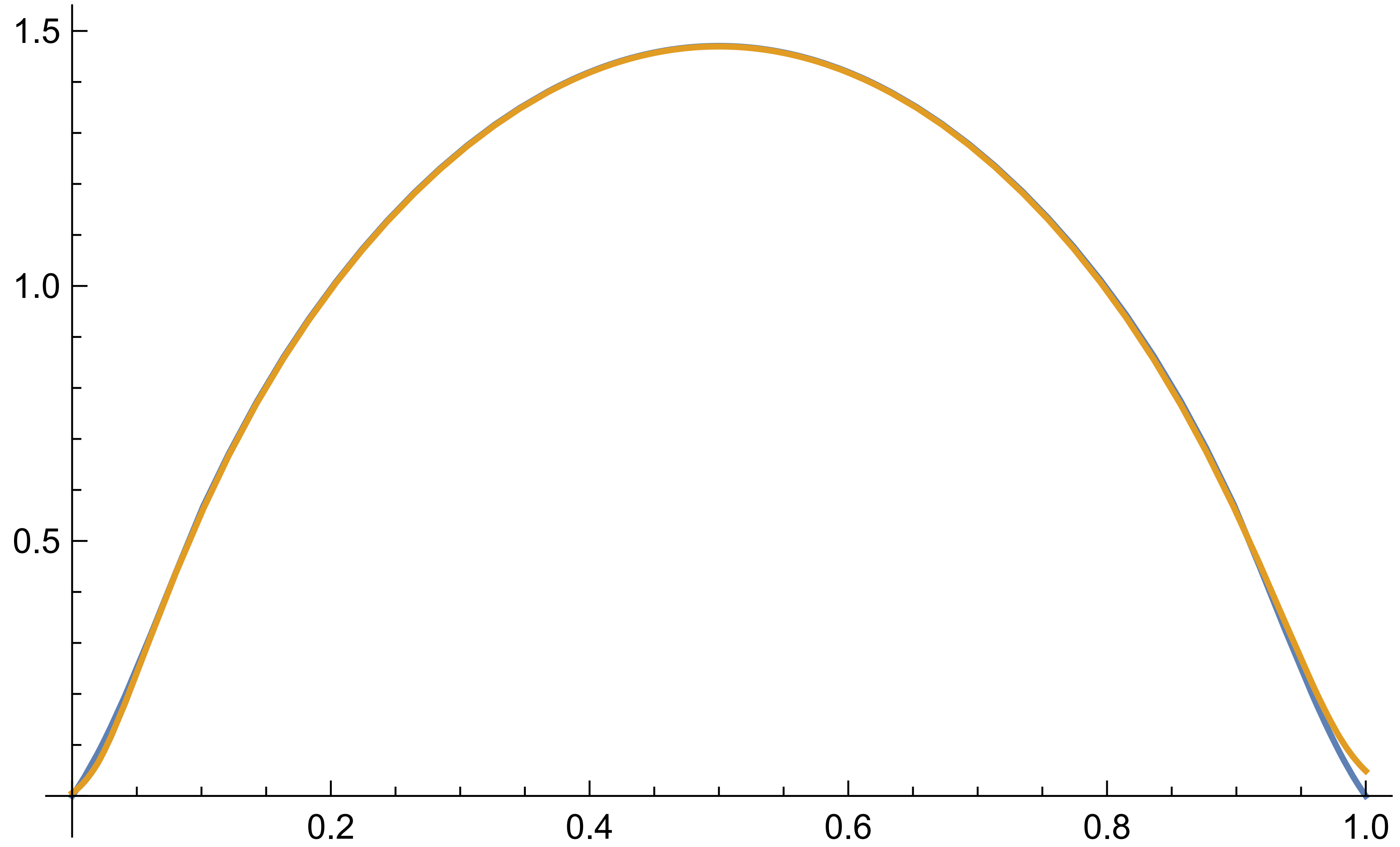
Ji: Take large Pz limit of frame-dependent equal-time correlator: $x = kz/Pz \;\; \epsilon \;\; [-\infty, +\infty]$

$$\tilde{q}_f(x; Pz) = \frac{1}{4\pi} \int dz \; e^{-ix\, P_z\, z} \, \langle \pi(P) | \, \bar{\psi}_f(z) \; \gamma_z \; \psi_f(0) \, | \pi(P) \rangle_c$$

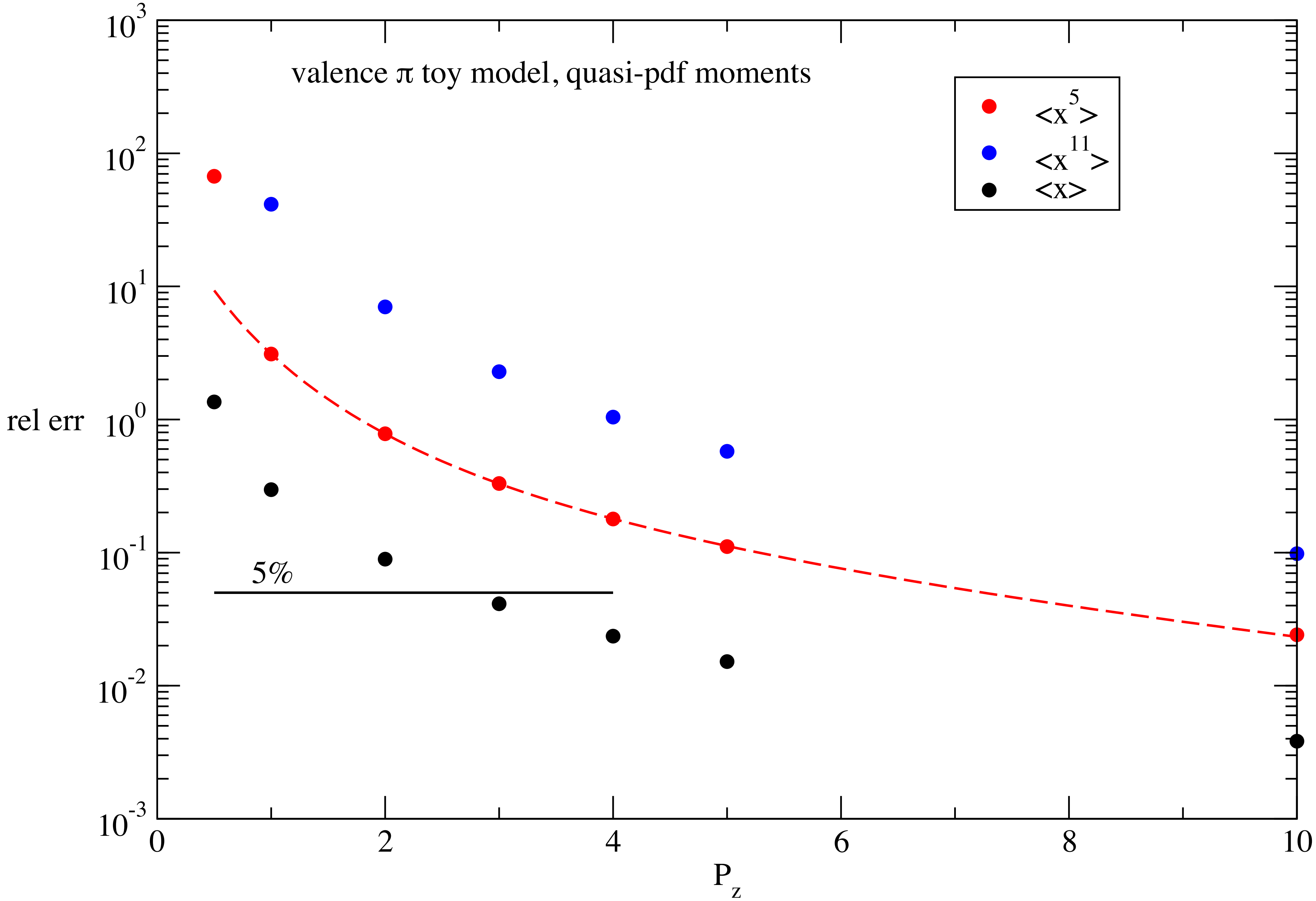
$\to q_f(x)$ as $Pz \to \infty$ **How fast?**

Back to: Spacelike Correlator Approximation for PDFs

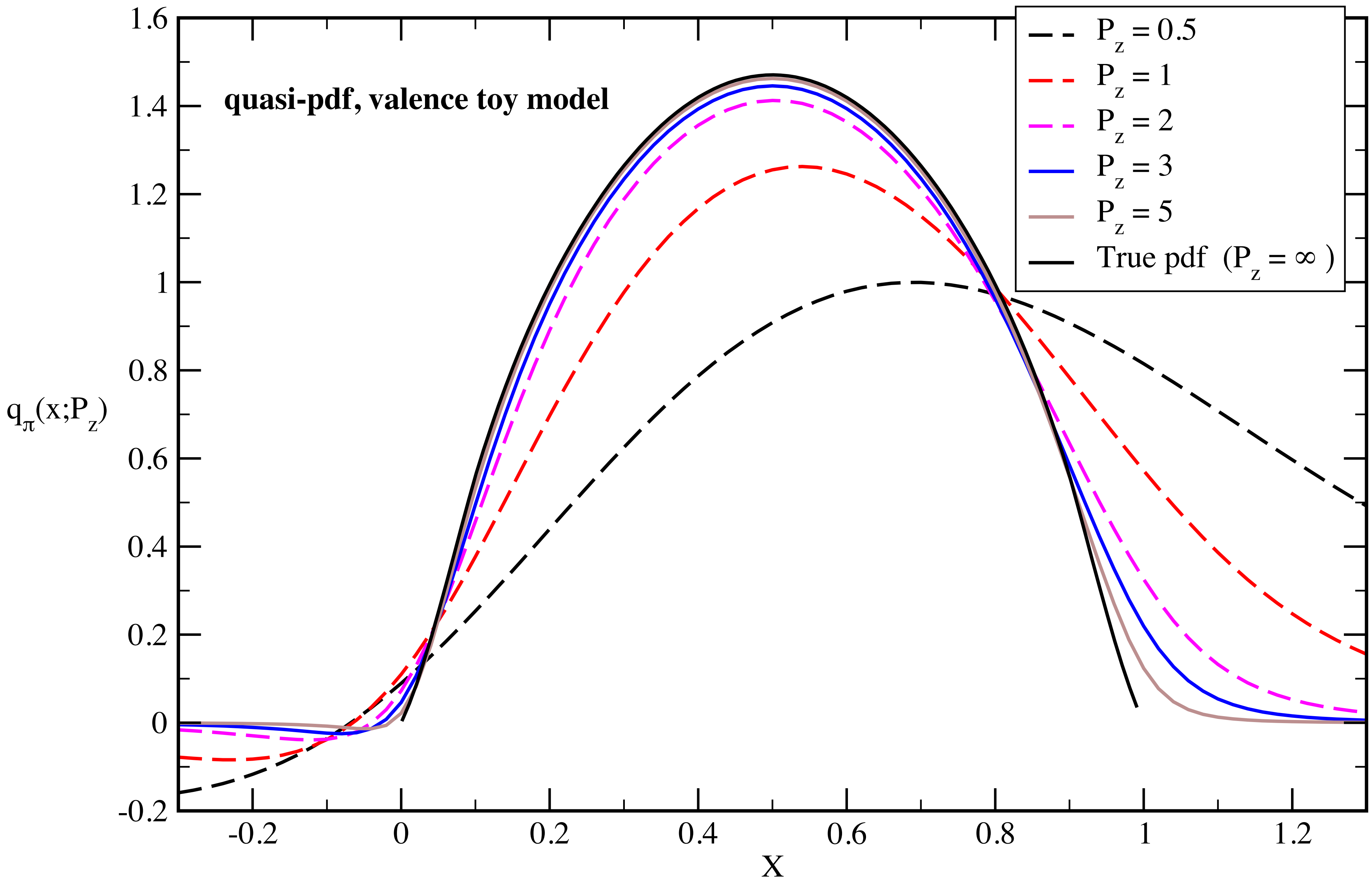
Model-exact PDF & Quasi-PDF @ Pz=10 GeV



Pz Dependence of quasi-pdf of valence model pion



Pz Dependence of quasi-pdf of u-ubar "pion"



---I.Cloet, Lei Chang, PCT, in progress (2015).......

KENT STATE UNIVERSITY

Fit numerical DSE-BSE solns to PTIRs (Nakanishi)

EG: $\Gamma_\pi(\mathbf{q^2}, \mathbf{q}\cdot\mathbf{P}) = \gamma_5 \left\{\mathbf{E}_\pi(\mathbf{q^2}, \mathbf{q}\cdot\mathbf{P}) + \not{P}\, \mathbf{F}_\pi(..) + \not{q}\, \mathbf{q}\cdot\mathbf{P}\ \mathbf{G}_\pi(..) + \sigma : \mathbf{qP}\ \mathbf{H}_\pi(..)\right\}$

Use Nakanishi Repn (or PTIR) (1965) :- $\mathcal{F} = \mathbf{E},\ \mathbf{F},\ \mathbf{G},\ \text{or } \mathbf{H}$

$$\mathcal{F}(\mathbf{q^2}; \mathbf{q}\cdot\mathbf{P}) = \int_{-1}^{1} \mathbf{d}\alpha \int_{0}^{\infty} \mathbf{d}\Lambda\ \left\{\frac{\rho_{\rm IR}(\alpha; \Lambda)}{(\mathbf{q^2} + \alpha\mathbf{q}\cdot\mathbf{P} + \Lambda^2)^{\mathbf{m+n}}} + \frac{\rho_{\rm UV}(\alpha; \Lambda)}{(\mathbf{q^2} + \alpha\mathbf{q}\cdot\mathbf{P} + \Lambda^2)^{\mathbf{n}}}\right\}$$

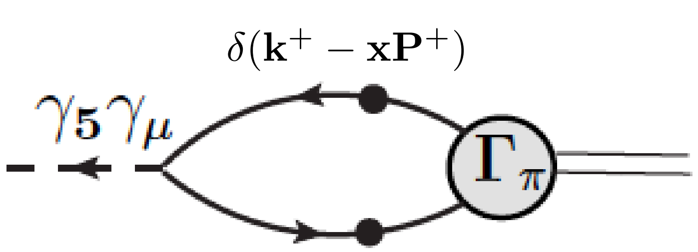
npQCD info is in the variables and constants that are not momenta
---Wick rotation is trivial as in pert thy.

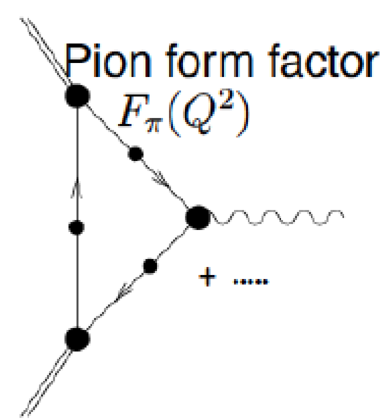
$$\rho_{\rm IR}(\alpha; \Lambda) \to \rho_1(\alpha)\ \delta(\Lambda - \Lambda_{{\rm IR}_1})\ +\ \cdots 3$$

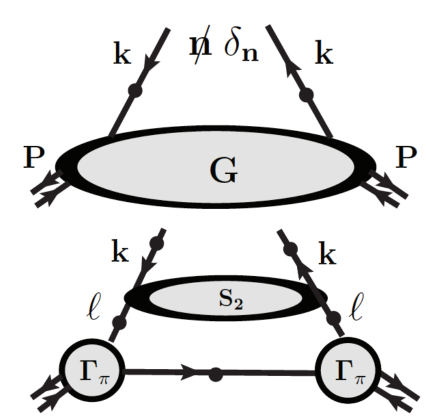
$$S(q) = \sum_{k=1}^{3} \left(\frac{z_k}{i\ \not{q} + m_k} + \frac{z_k^*}{i\ \not{q} + m_k^*}\right)$$

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.

N. Souchlas, PhD thesis KSU, (2009), J. Phys. G37, 115001 (2010)







Topics

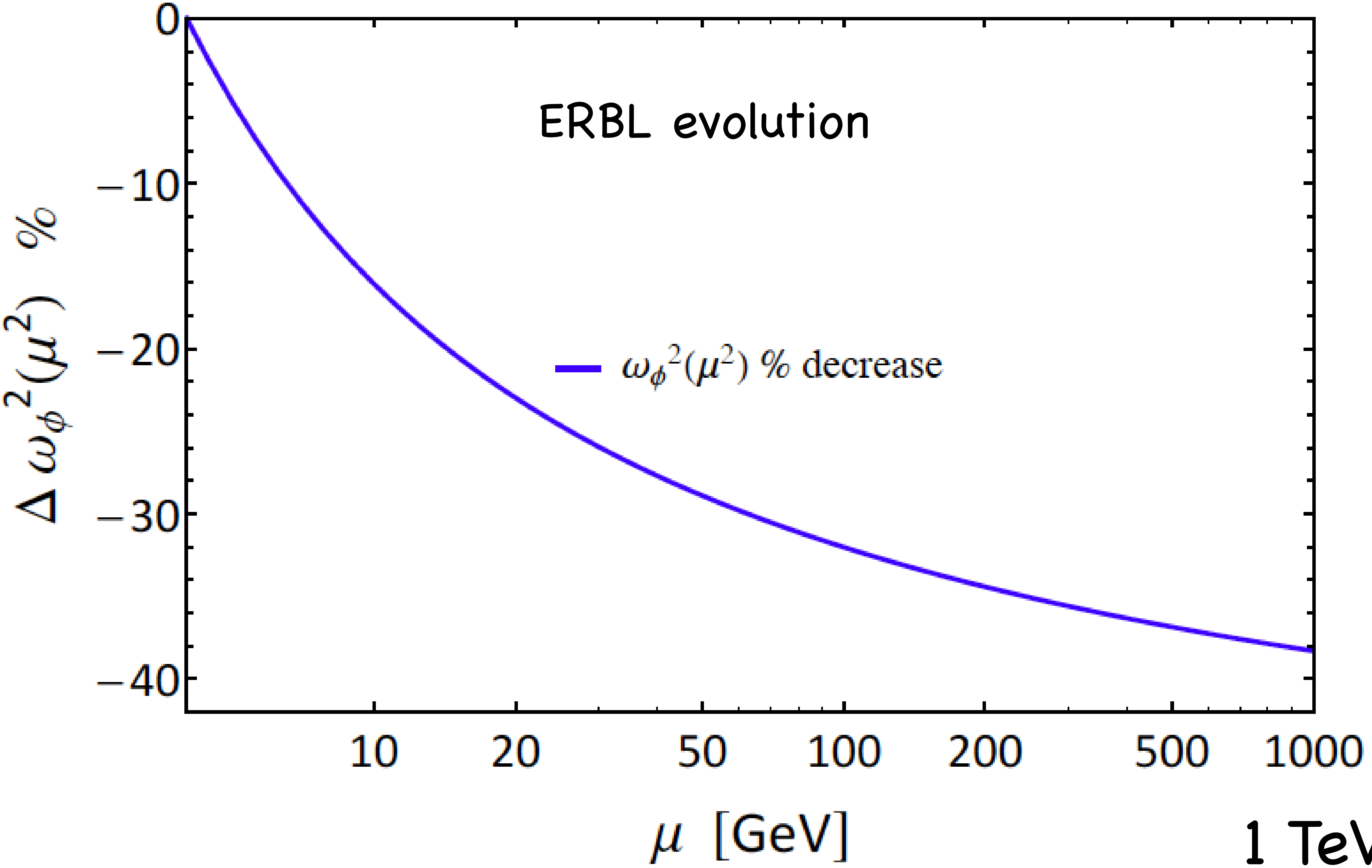
- Parton distribution amplitudes and PDFs— mesons as an example. DSE-model calculations with direct connection to QCD. Comparison to LQCD.

- Some applications to uv physics (Form Factors, HS behavior of exclusive processes)

- PDFs including X. Ji's space-like correlator approximation for LQCD—a model investigation.

UV-QCD is not Asymptotic QCD

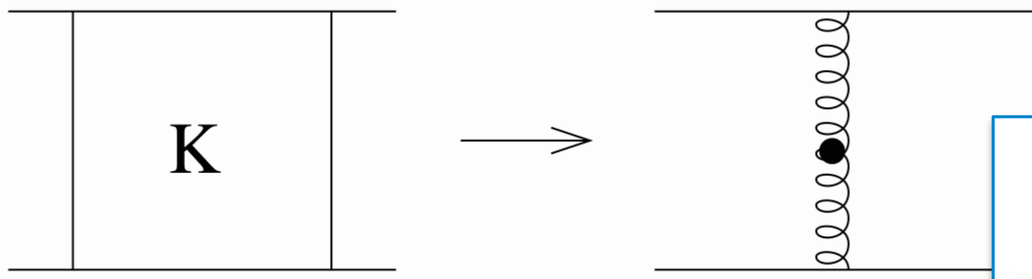
$$\mathbf{Q^2} >> \mathbf{\Lambda^2_{\rm QCD}} : \ \mathbf{Q^2 F_\pi(Q^2)} \to \mathbf{16\,\pi\,f_\pi^2\,\alpha_s(Q^2)}\ \omega_\phi^2(\mathbf{Q^2})\ +\ \mathcal{O}(\mathbf{1/Q^2})$$

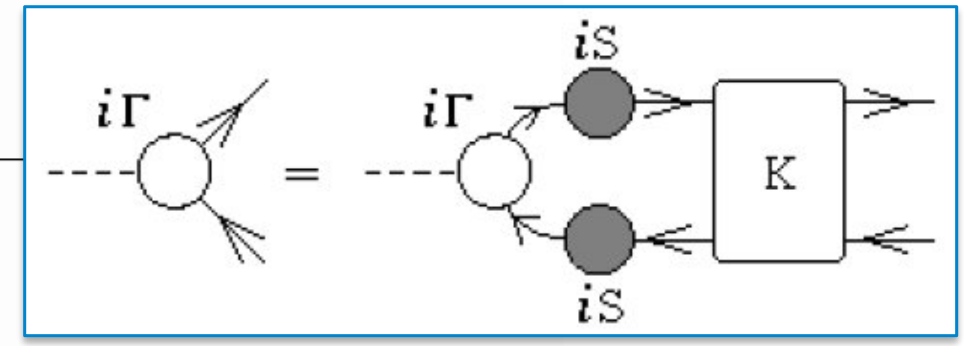


1 TeV

Ladder-Rainbow Model

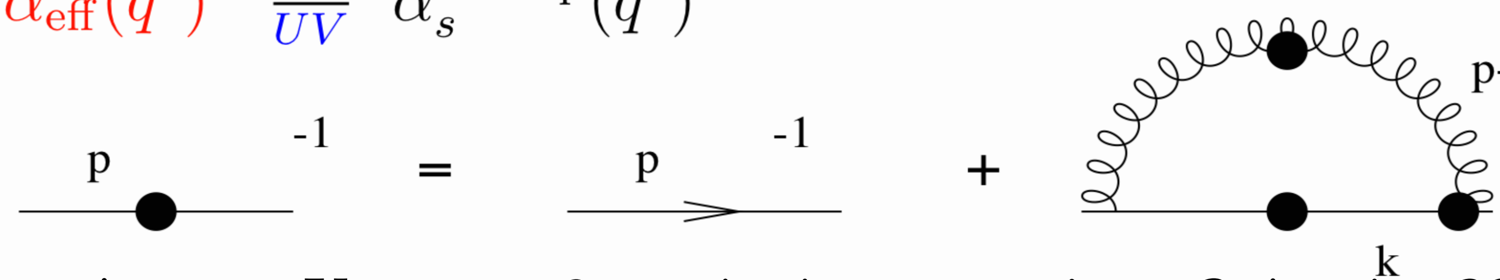
Landau gauge only





1 true phen parameter

- $K_{\rm BSE} \to -\gamma_\mu \frac{\lambda^a}{2} \; 4\pi\alpha_{\rm eff}(q^2) \; D^{\rm free}_{\mu\nu}(q) \; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \;\; \overrightarrow{IR} \;\; \langle \bar{q}q\rangle_{\mu=1\ {\rm GeV}} = -(240{\rm MeV})^3$, incl vertex dressing
- $\alpha_{\rm eff}(q^2) \;\; \overrightarrow{UV} \;\; \alpha_s^{1-{\rm loop}}(q^2)$



modern π, K qDSE-BSE strategy: Maris & Roberts, PRC56, 3369 (1997)

- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10% $[\mathbf{fit:m_\pi, m_K, f_\pi}], \mathbf{f_K}(2\%)$

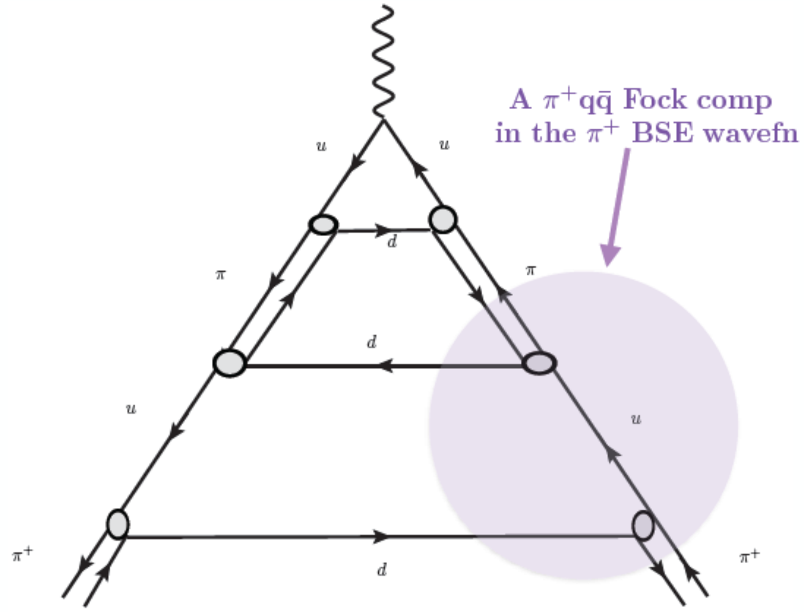
An Ansatz for the FULL QCD kernel: L. Chang, C.D. Roberts, PRL103, 081601 (2009), + S. Qin (2015).

A more modern RL kernel: S. Qin, L. Chang, C.D. Roberts, D.J. Wilson, PRC84, 042202 (2011).

Convolution Model for q(x) from virtual pi loop

$$\mathbf{q_v^{\pi-lp}(x)} \sim \mathcal{P}_{\mathbf{q/T}}(\mathbf{x}) = \int_{\mathbf{x}}^{\mathbf{1}} \frac{\mathbf{dy}}{|\mathbf{y}|}\ \mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})\ \mathcal{P}_{\mathbf{q}/\pi}\left(\frac{\mathbf{x}}{\mathbf{y}}\right),$$

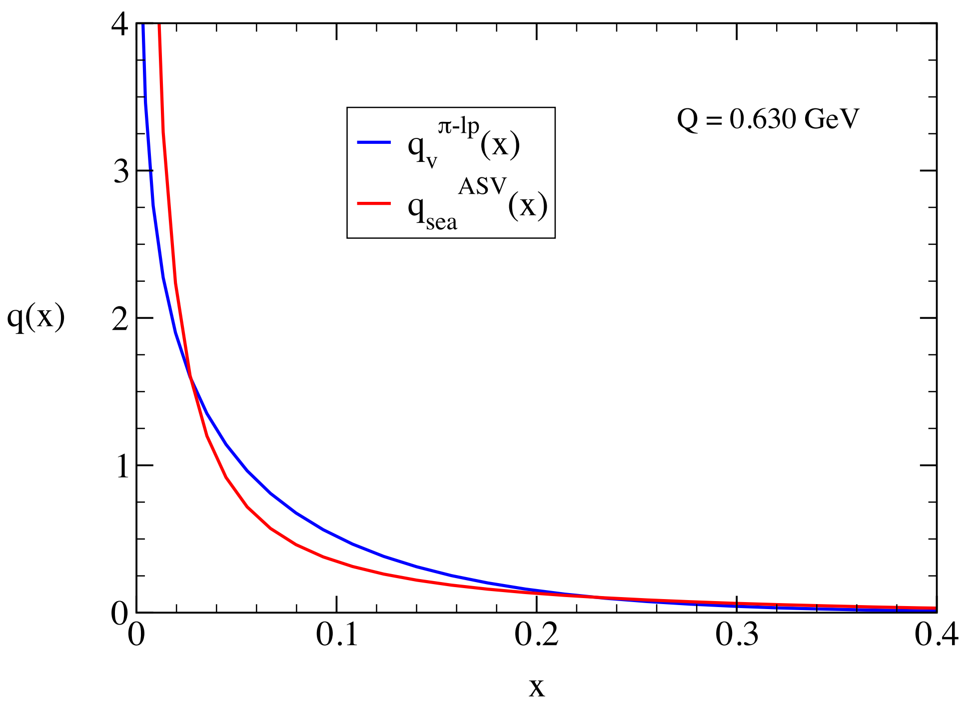
$$\mathbf{T} = \ \mathbf{target} = \pi\ \mathbf{here}$$



$\mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})$ **should strongly favor** $\mathbf{y} \leq \frac{\mathbf{m}_\pi}{\mathbf{2M_q+m_\pi}} \approx \mathbf{0.2}$,

$\mathcal{P}_{\mathbf{q}/\pi}(\frac{\mathbf{x}}{\mathbf{y}})$ **is self – consistently determined**

Result is strongly constrained



Estimate 1-Pion Loop Contribution to Pion PDF

$$\pi^+ \; : \; \langle x^1 \rangle_\mu = \int_0^1 dx\, x\, \{u + \bar{u}_{sea} + \bar{d} + d_{sea} + g(x)\} \;\; \approx 2\langle x\, q_v(x)\rangle + 4\langle x\, q_{sea}(x)\rangle + \langle x\, g(x)\rangle = 1$$

$$u = u_v + u_{sea}\;, \quad \bar{d} = \bar{d}_v + \bar{d}_{sea}$$

Empirical GRS/ASV $\Rightarrow$ universal $q_v(x)$, $q_{sea}(x)$ at $\mu = 0.630$ GeV

$$\Gamma_\pi = \sqrt{1-\alpha^2}\, \Gamma^{RL}_{q\bar{q}} + \alpha\, \Gamma_{\pi q\bar{q}}$$

CPT: 18% effect

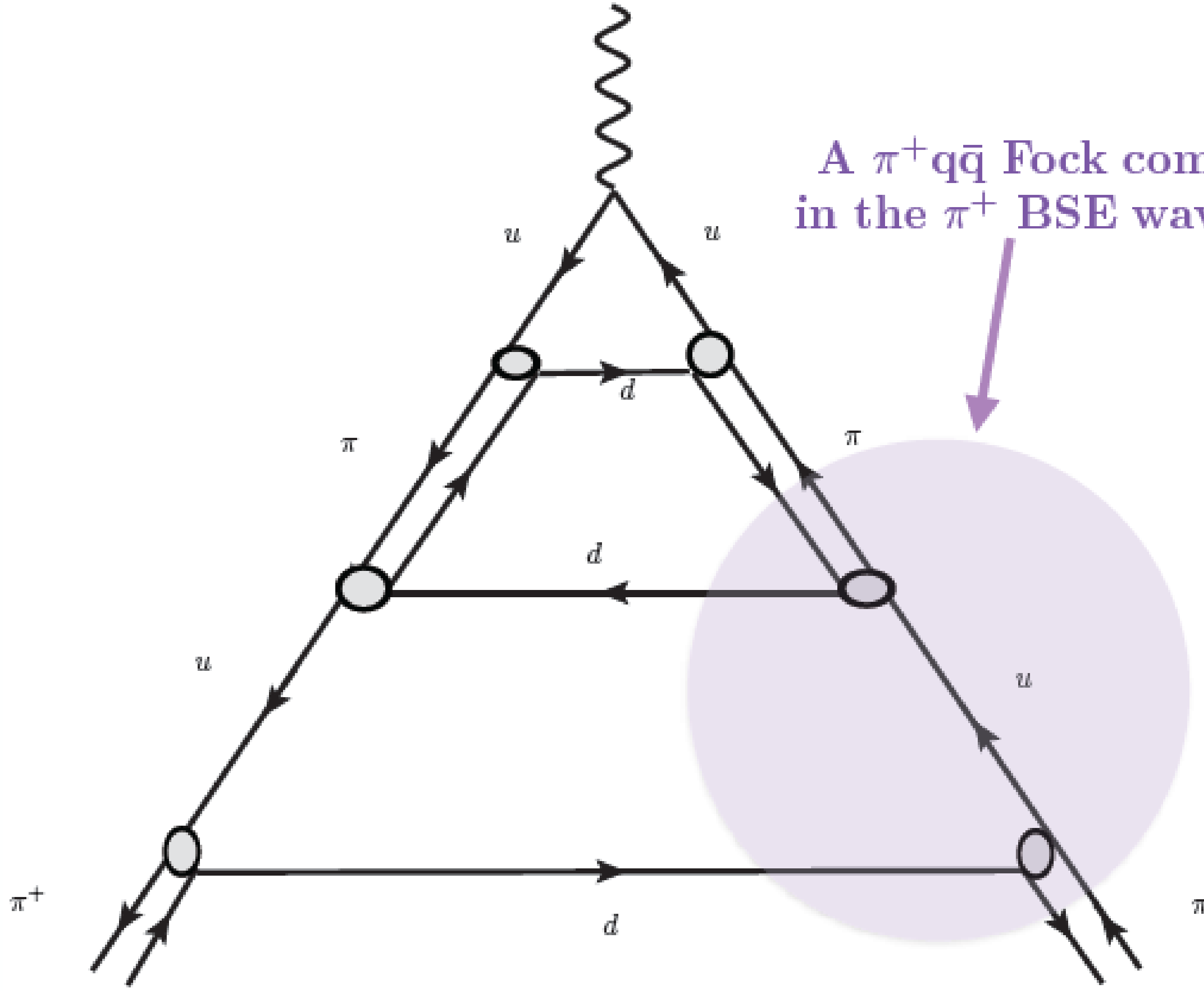
$$r^2_{ch} = (1-\alpha^2)\, r^2_{RL} + \alpha^2\, r^2_{\pi-lp}$$

DSE-RL: $r^2_{RL} = r^2_{ch} \Rightarrow \alpha^2 = 18\%$

PDF Consequence:

$$q_v(x) = (1-\alpha^2)\, q^{RL}(x) + q_v^{\pi-lp}(x)$$

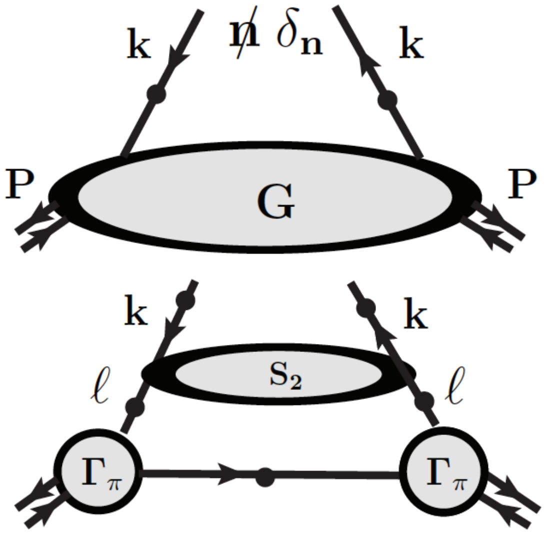
with $\langle q_v^{\pi-lp}(x)\rangle = \alpha^2 = 0.18$



Simple model for pion PDF & Quasi-PDF

$$\mathbf{S(k) = 1/(i\not{k} + M)\ ,\quad M = 0.4\ GeV}$$

$$\mathbf{\Gamma_\pi(q,P) = \gamma_5 N_\pi \int_{-1}^{1} d\alpha\, \frac{\rho(\alpha)}{q^2 + \alpha q\cdot P + \Lambda^2}\ ,\quad \rho(\alpha) = even}$$



Euclidean to Minkowski:-

Evaluate q(x) directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} dk^-$

$$\mathbf{q_A(x) = i\,N_c\,tr \int \frac{dk^+\,dk^-\,d^2k_\perp}{(2\pi)^4}\ \delta(k^+ - x\,P^+)\ tr[\Gamma_\pi\ S\ (i\gamma^+)\ S\ \Gamma_\pi\ S]}$$

Evaluate $\tilde{q}(x;P_z)$ **directly using Cauchy Residue Thm for** $\int_{-\infty}^{\infty} dk^0$

$$\mathbf{\tilde{q}_A(x) = i\,N_c\,tr \int \frac{dk^0\,dk_z\,d^2k_\perp}{(2\pi)^4}\ \delta(k_z - x\,P_z)\ tr[\Gamma_\pi\ S\ (i\gamma^z)\ S\ \Gamma_\pi\ S]}$$

<x^m> for toy model pion at Pz = 3 GeV