Electromagnetic Form Factors of Nucleon Excitations from Lattice QCD

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- Existing parity projection techniques are vulnerable to opposite parity contaminations at nonzero momentum
- We propose the Parity Expanded Variational Analysis (PEVA) technique to resolve this issue

Baryon structure via variational analysis

Step 1: Construct correlation matrix

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Expect E^{eff}_α to approximately obey dispersion relation

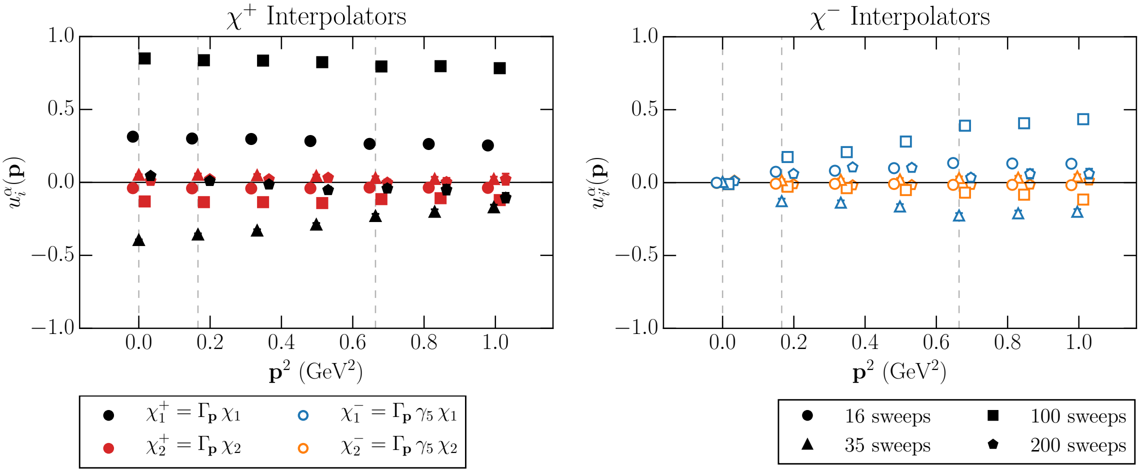
$$E^{\text{eff}}_\alpha(\mathbf{p}, t) \approx \sqrt{{m_\alpha}^2 + \mathbf{p}^2}$$

Second lightest PACS-CS $(2+1)$-flavour full-QCD ensemble

- $32^3 \times 64$ lattices
- $a = 0.0951(14)\,\mathrm{fm}$ by Sommer parameter
- $\kappa_{u,d} = 0.1377$, corresponding to $m_\pi = 280(5)\,\mathrm{MeV}$

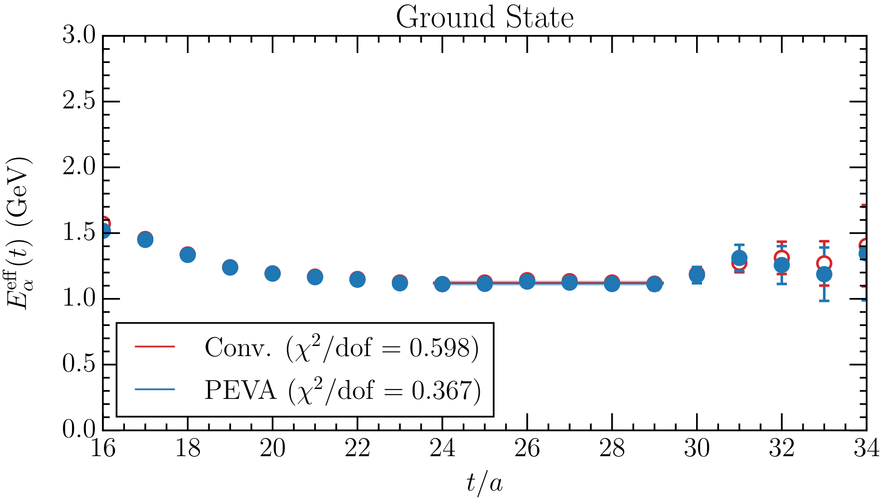
Eigenvector components

Ground state



Effective energy

Ground state - $\mathbf{p}^2 \simeq 0.166\,\mathrm{GeV}^2$



Effective energy
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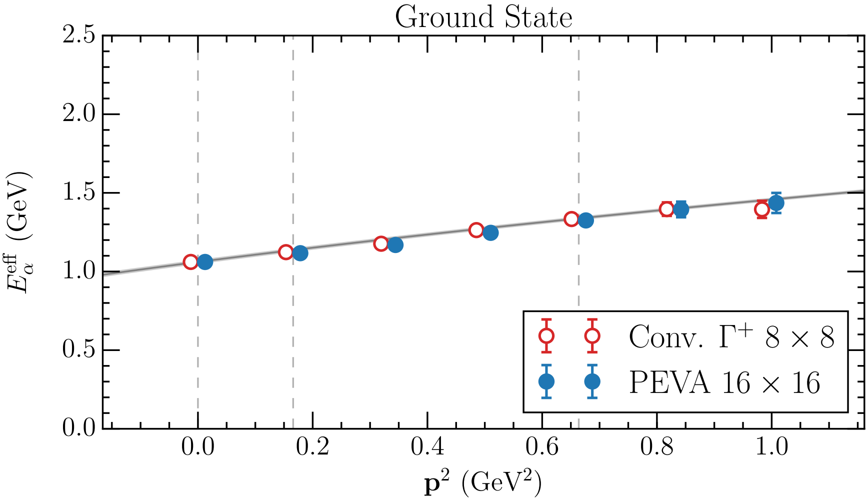
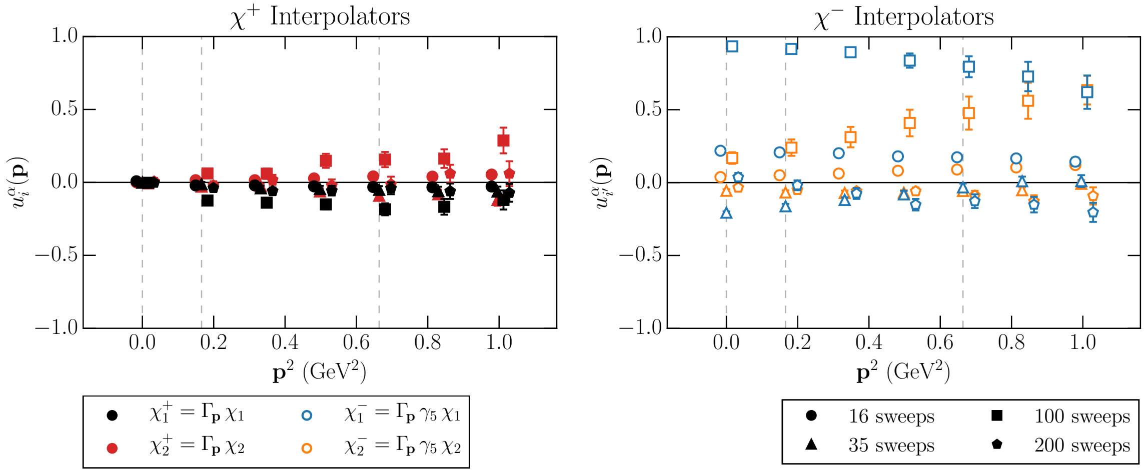


Eigenvector components

First negative parity excitation



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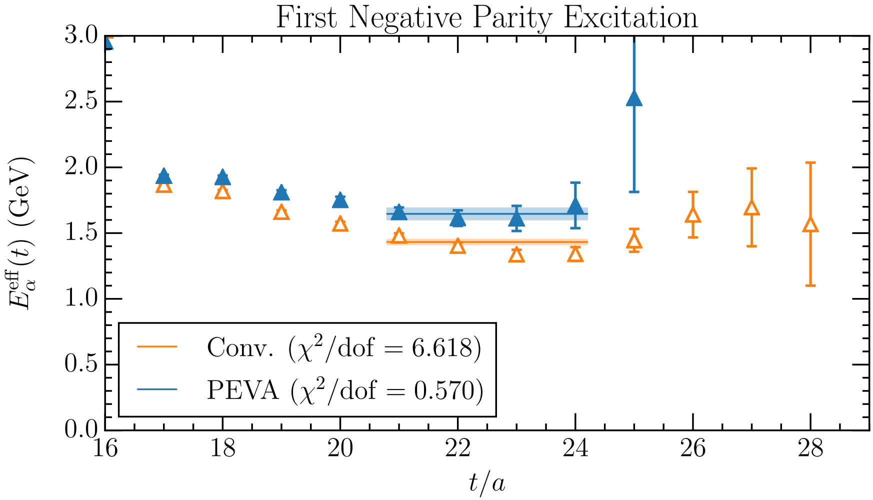

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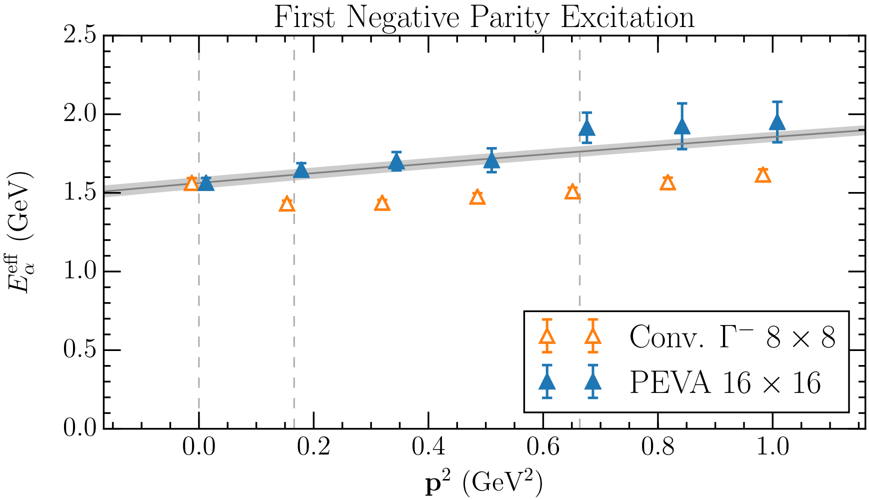

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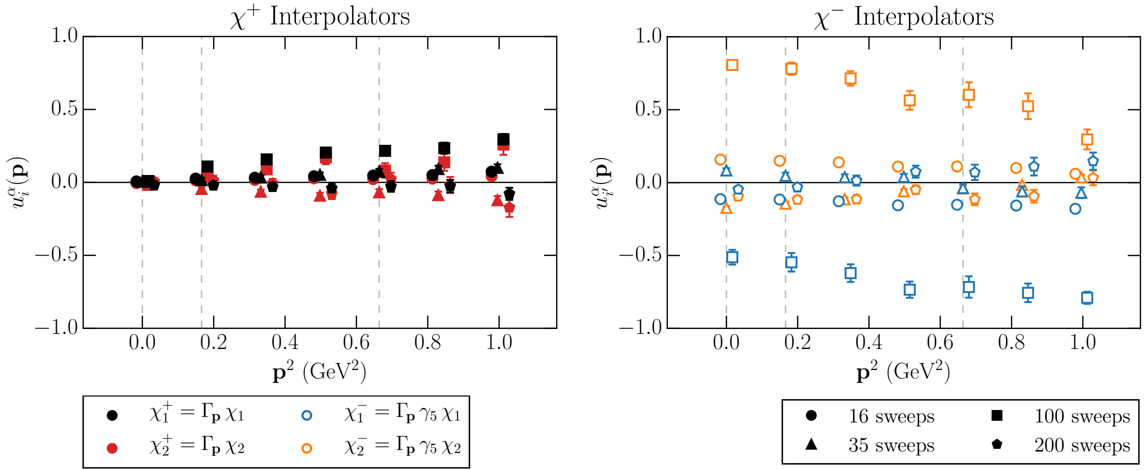

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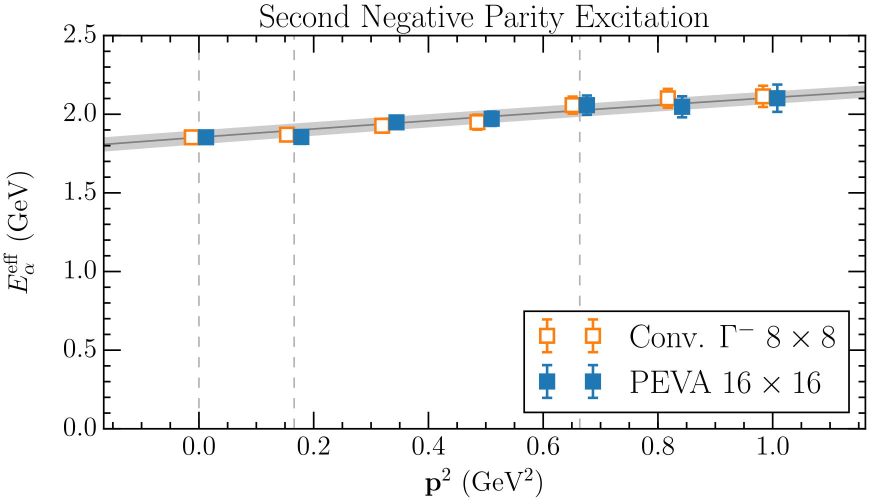

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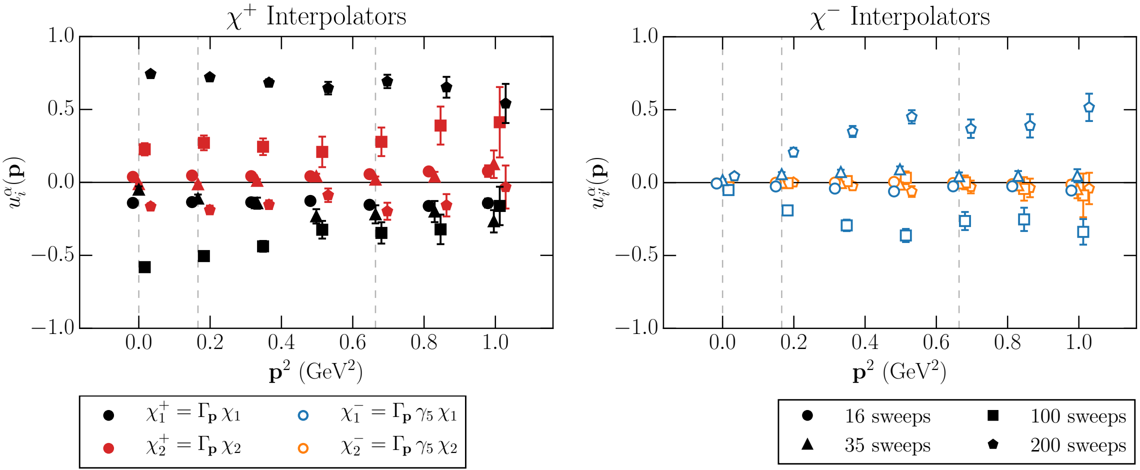
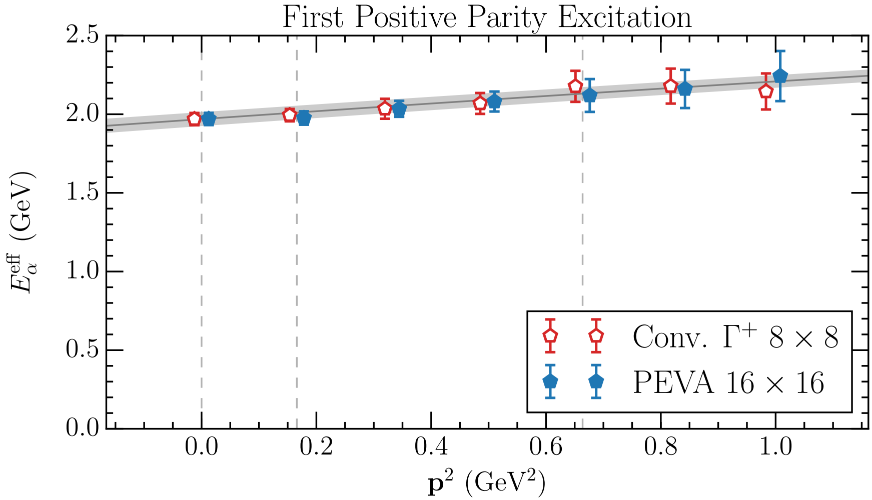
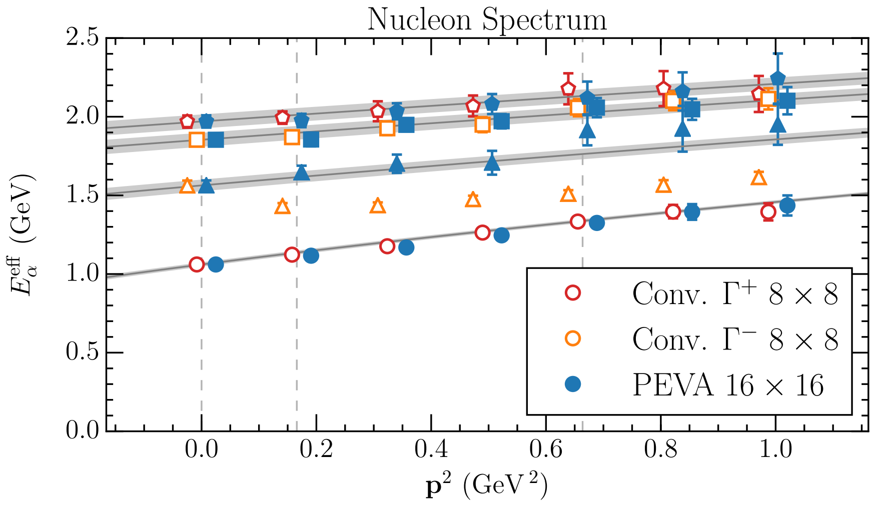



First Positive Parity Excitation
E_α^{eff} (GeV)
$\mathbf{p}^2$ (GeV^2)
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
Conv. Γ^+ 8×8
PEVA 16×16

Effective energy
Nucleon spectrum



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Eigenstate-projected three point correlation function

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- Define ratio

$$R^{\mu}_{\alpha\to\beta}(\mathbf{p}',\,\mathbf{p};\,\Gamma;\,t_1) := \sqrt{\frac{G^{\mu}_{\alpha\to\beta}(\mathbf{p}',\,\mathbf{p};\,\Gamma;\,t_2,\,t_1)\,G^{\mu}_{\beta\to\alpha}(\mathbf{p},\,\mathbf{p}';\,\Gamma;\,t_2,\,t_1)}{G_{\beta}(\mathbf{p}';\,t_2)\,G_{\alpha}(\mathbf{p};\,t_2)}}$$

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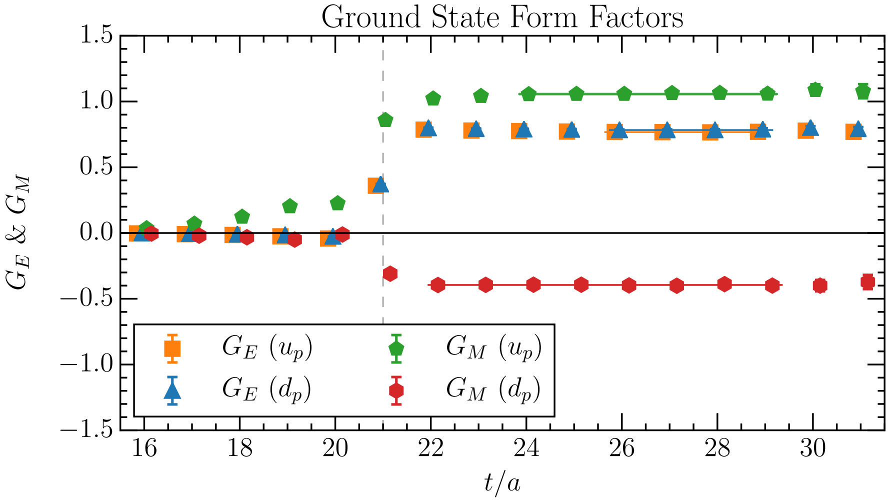
- Extract $G_E(Q^2)$ and $G_M(Q^2)$ from ratios

Step 4: Extract form factors

Fits to ground state form factors



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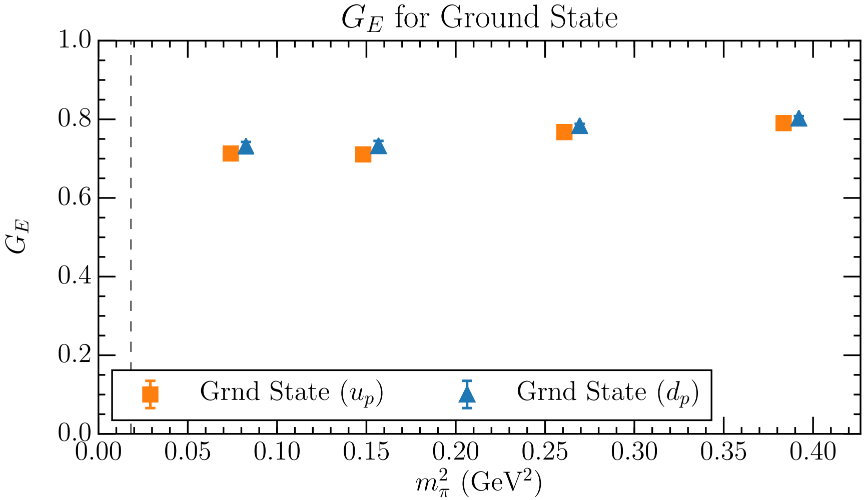
$G_E(Q^2 = 0.15(1)\,\mathrm{GeV}^2)$ for ground state



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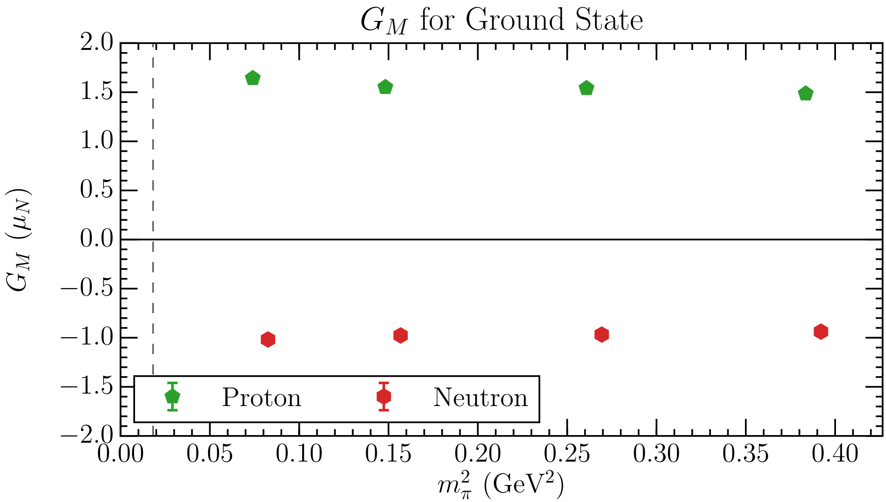

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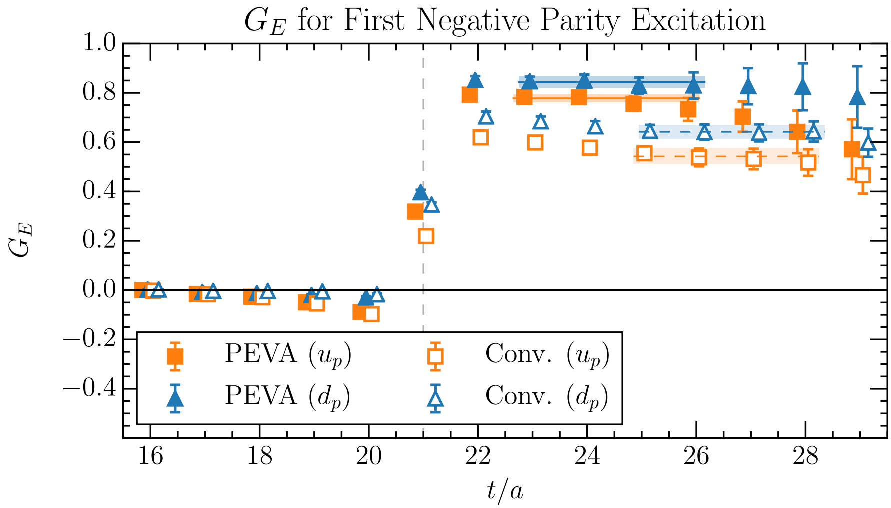
Fits to G_E for first negative parity excitation



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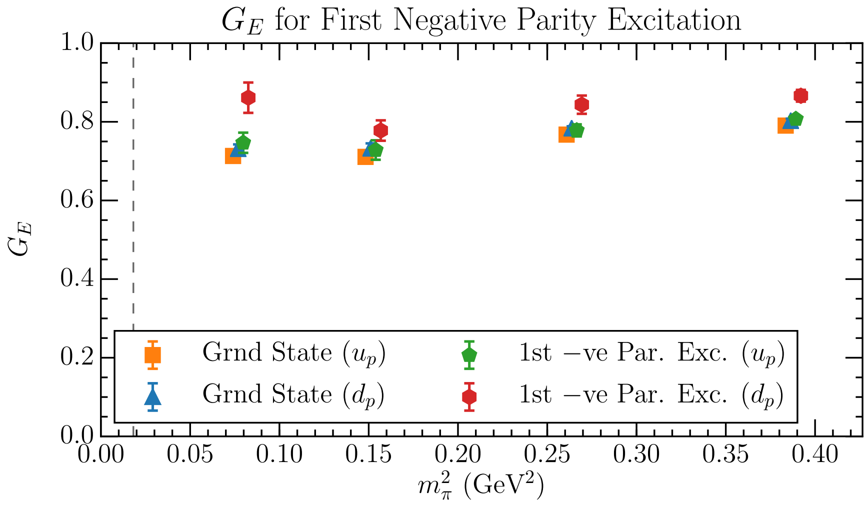

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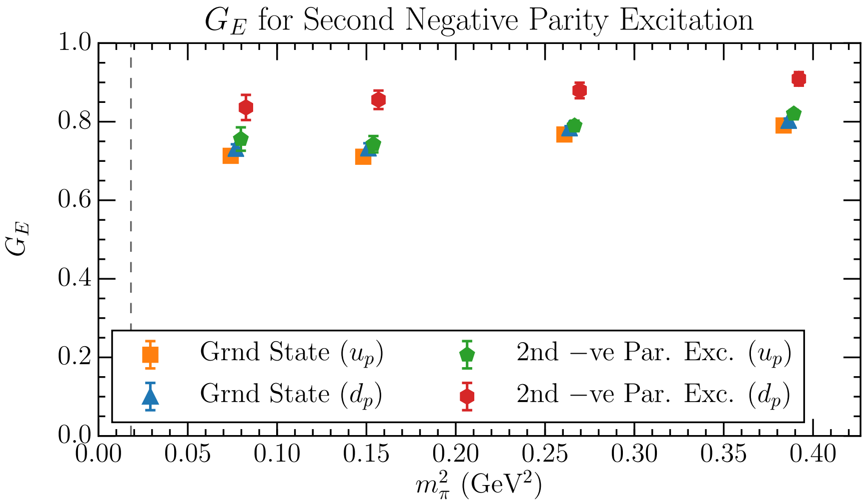

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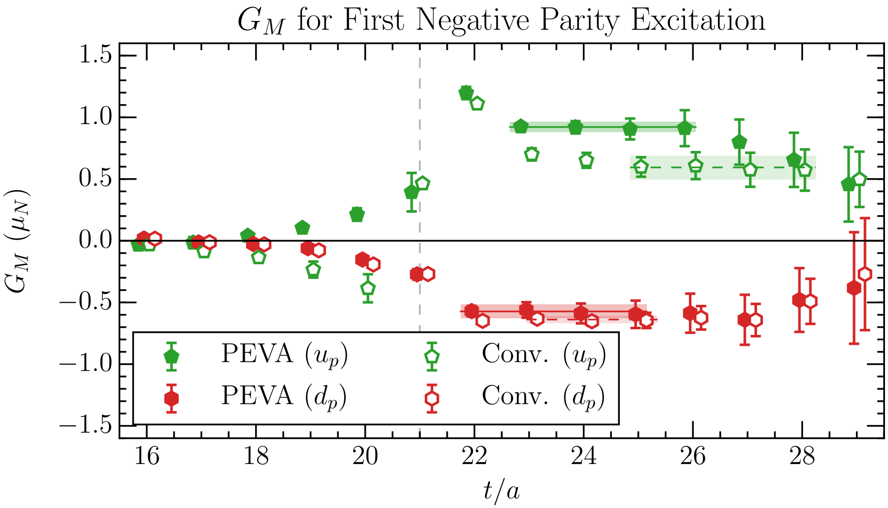

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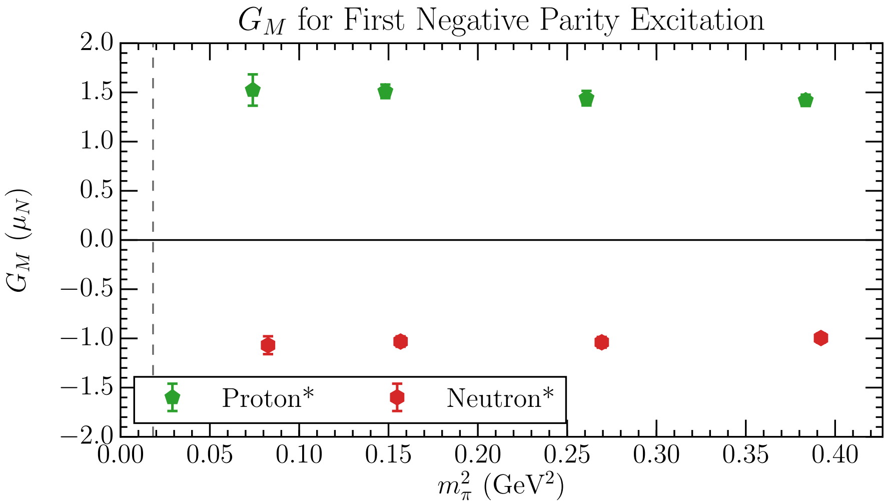

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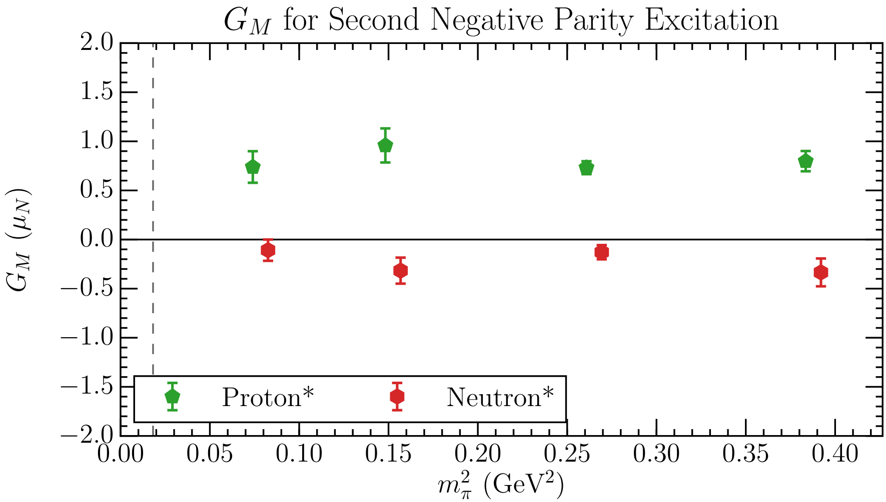

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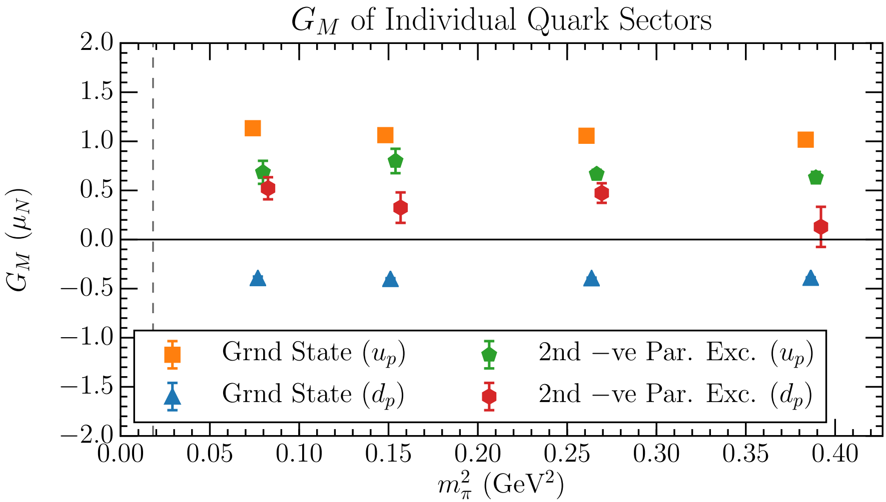
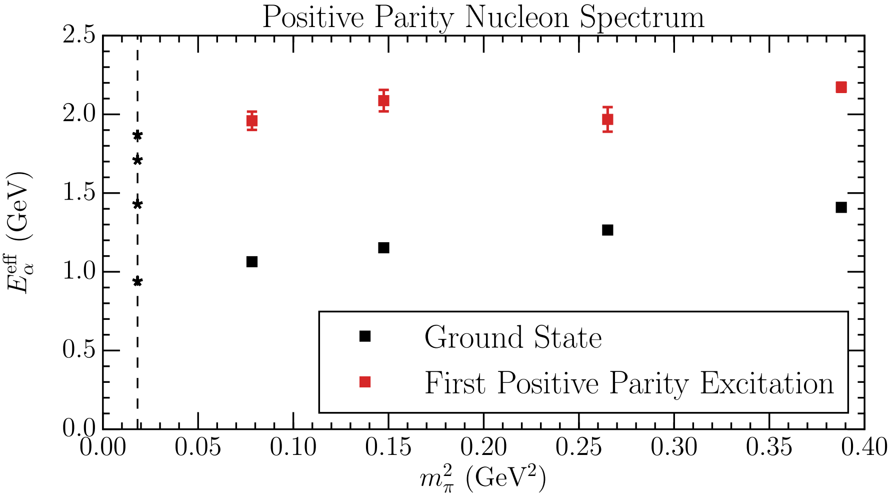


Nucleon mass spectrum
Positive Parity



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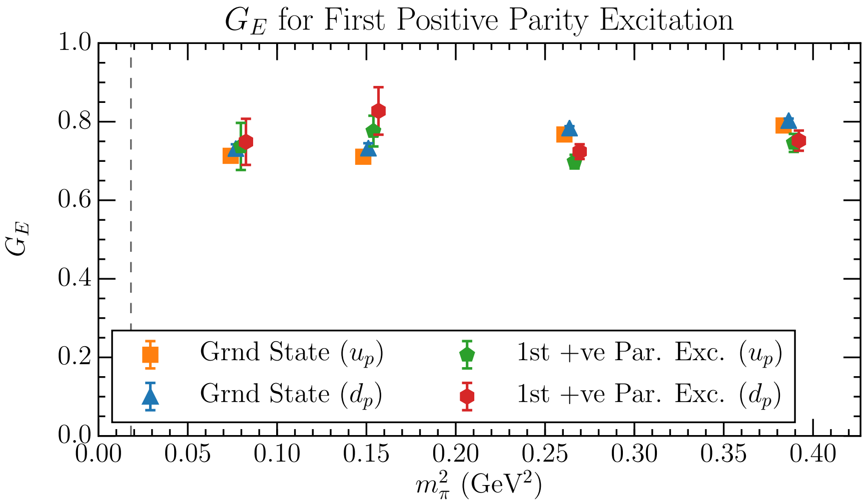

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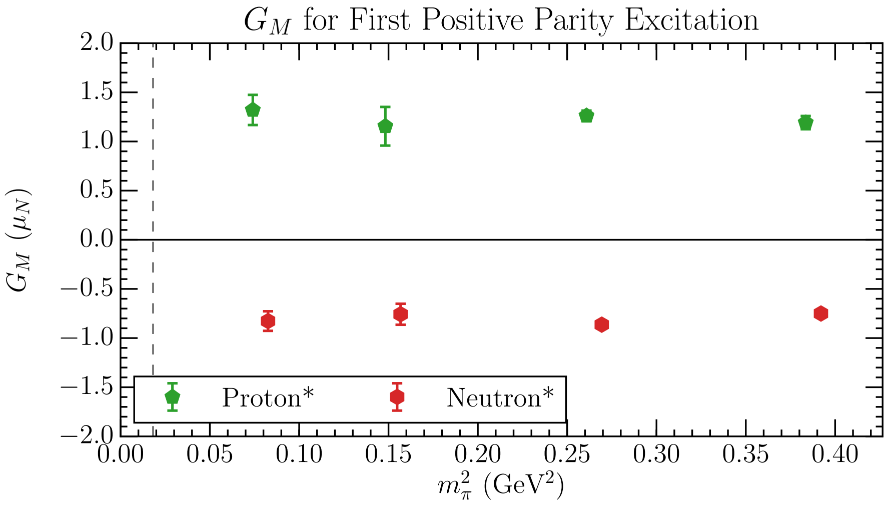

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- Clear effect on two point function for lowest lying negative parity excitation
- Has significant effects on three point functions for excited states
- Calculations of finite volume baryon transition moments are almost complete
- This is an important step towards making contact with experiment

More information

“Parity-expanded variational analysis for nonzero momentum”
F. M. Stokes, W. Kamleh, D. B. Leinweber, M. S. Mahbub, B. J. Menadue, B. J. Owen
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