ELECTROWEAK POLARISABILITIES USING FEYNMAN-HELLMANN

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Collaborators: Ross Young & James Zanotti

WEAK CHARGE OF PROTON

Q_W^P with radiative corrections

$$Q_W^P = [\rho_{NC} + \Delta_e]\left[1 - 4\sin^2\theta_W + \Delta_e'\right] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

Q_W^P with radiative corrections

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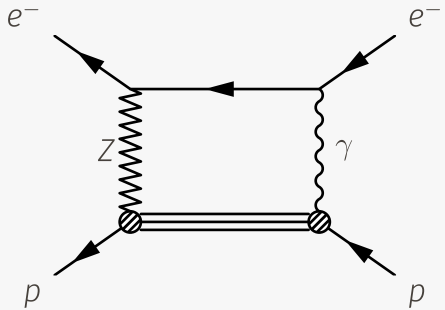

Lepton-Nucleon Hadron Tensor

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4x \, e^{iq\cdot x} \left\langle p', \epsilon' \,\middle|\, J^{\dagger}_{\mu}(x) J_{\nu}(0) \,\middle|\, p, \epsilon \right\rangle$$

Lepton-Nucleon Hadron Tensor

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$$\begin{aligned} W_{\mu\nu} = \; & \cdots + q_\mu q_\nu \frac{W_4}{m^2} + (P_\mu q_\nu + P_\nu q_\mu) \frac{W_5}{2m^2} + \cdots \\ & \cdots + i\epsilon_{\mu\nu\alpha\beta} q^\alpha P^\beta \frac{W_3}{2m^2} + \cdots \end{aligned}$$

CONTENTS

FEYNMAN-HELLMANN THEOREM

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→ Calculate matrix elements using two point methods

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Modify Action

$$S \to S + \lambda \int d^4x \, J(x)$$

FEYNMAN-HELLMANN THEOREM

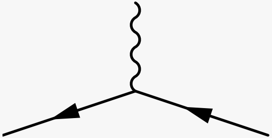
$\rightarrow$ Calculate matrix elements using two point methods

Modify Action

$$S \rightarrow S + \lambda \int d^4x \, J(x)$$

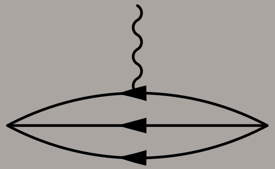
Feynman-Hellmann Theorem

$$\left.\frac{\partial E_{X,\mathbf{p}}}{\partial \lambda}\right|_{\lambda=0} \overset{\text{large } t}{=} \frac{1}{2E_{X,\mathbf{p}}} \langle X, \mathbf{p} \,|\, J(0) \,| X, \mathbf{p} \rangle$$



LATTICE CAVEAT

Connected Contribution

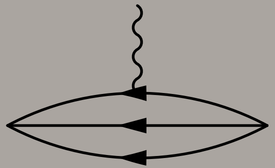


→ Modify quark propagator

→ High correlation for different λ

LATTICE CAVEAT

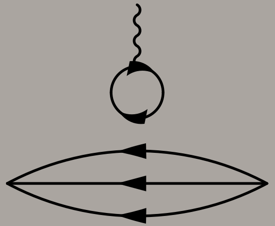
Connected Contribution



→ Modify quark propagator

→ High correlation for different λ

Disconnected Contribution



→ Modify weighting

→ No correlation for different λ

Modify Action

$$S \to S + \lambda \int d^4x \, J(x)$$

Modify Action

$$S \to S + \lambda \int d^4x\, J(x)$$

Second Order Feynman-Hellmann Theorem

$$\left.\frac{d^2 E_{X,\mathbf{p}}}{d\lambda^2}\right|_{\lambda=0} \overset{\text{large } t}{=} 2 \sum_{Y \neq X} \frac{\left|\langle X, \mathbf{p} \,|\, J(0) \,|\, Y, \mathbf{p}\rangle\right|^2}{4 E_{X,\mathbf{p}} E_{Y,\mathbf{p}} \left(E_{X,\mathbf{p}} - E_{Y,\mathbf{p}}\right)}$$

APPLICATION

SIMULATION

- → $N_f = 2+1$
- → $SU(3)$ symmetric point $m_q = \frac{1}{3}\left(m_u^{(\text{phys})} + m_d^{(phys)} + m_s^{(phys)}\right)$
- → Lattice spacing $a = 0.074\text{fm}$
- → $Z_A = 0.8728(33)$

$24^3 \times 48$	$32^3 \times 64$
→ 3000 trajectories with 2 sources each → $m_\pi \approx 480\text{MeV}$	→ 1600 trajectories with 2 sources each → $m_\pi \approx 470\text{MeV}$

FEYNMAN-HELLMANN

$\rightarrow \; S \rightarrow S + \lambda \int d^4x \, A_3^{(u-d)}$

- $S \to S + \lambda \int d^4x\, A_3^{(u-d)}$
- Do this for $\lambda = 0.0, 0.0125, 0.025, 0.0375$ and calculate corresponding two point functions $C_{\uparrow\downarrow}(\lambda, t)$

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- $\rightarrow$ Take advantage of correlation $\frac{C_\uparrow(\lambda,t)C_\downarrow(\lambda,t)}{C_\uparrow(0,t)C_\downarrow(0,t)} \propto e^{-2\Delta E_{even}(\lambda)t}$

FEYNMAN-HELLMANN

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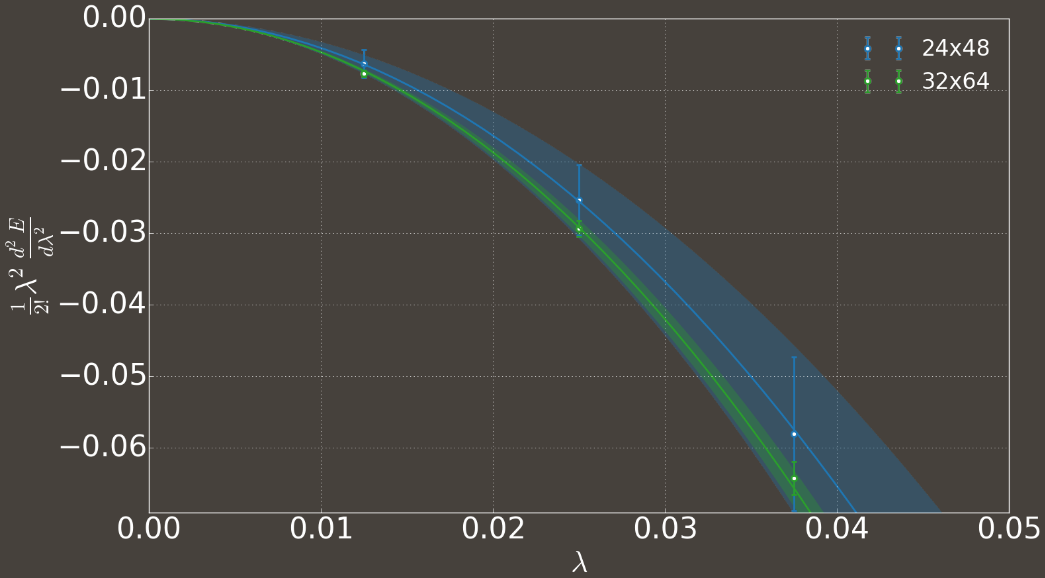
- → Take advantage of correlation $\frac{C_\uparrow(\lambda,t)C_\downarrow(\lambda,t)}{C_\uparrow(0,t)C_\downarrow(0,t)} \propto e^{-2\Delta E_{even}(\lambda)t}$
- → $\Delta E_{even} = \frac{1}{2!}\lambda^2 \frac{d^2E}{d\lambda^2} + \frac{1}{4!}\lambda^4 \frac{d^4E}{d\lambda^4} + \cdots$



0.00
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$\frac{1}{2!}\lambda^2\frac{d^2E}{d\lambda^2}$
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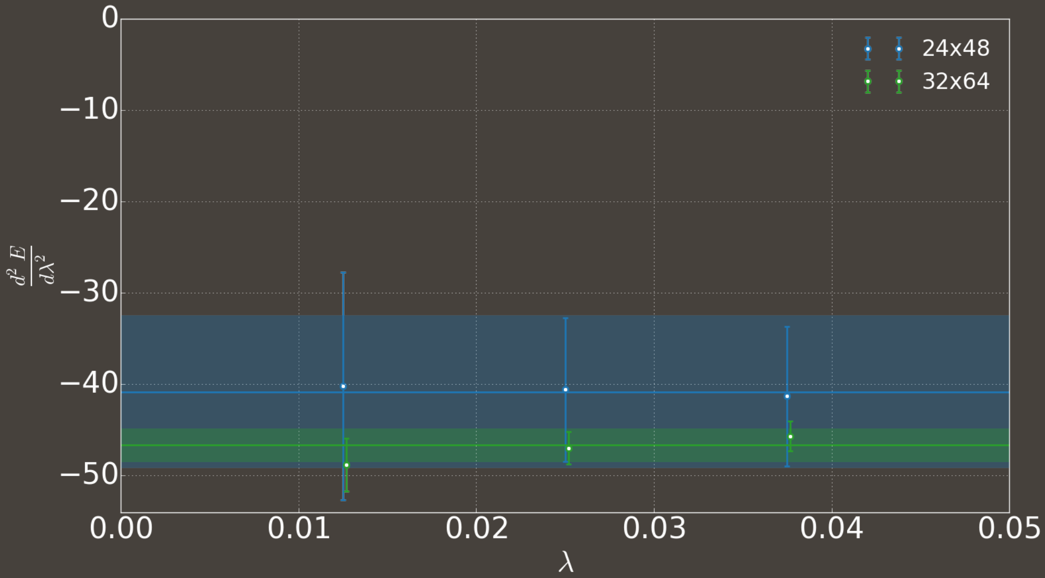
λ
24x48
32x64



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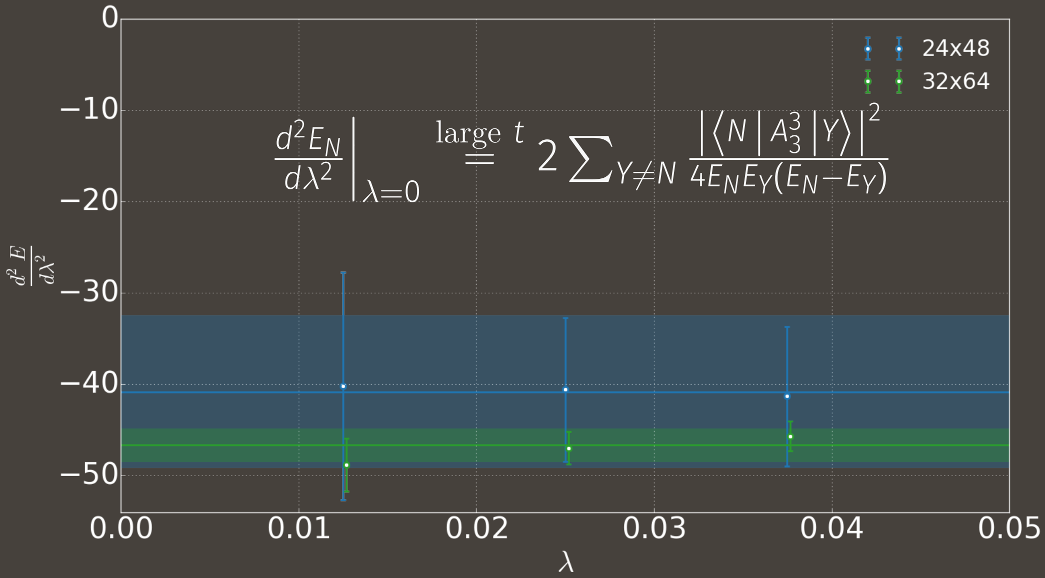
$\frac{d^2E}{d\lambda^2}$
λ
24x48
32x64



24x48
32x64
$\left.\frac{d^2 E_N}{d\lambda^2}\right|_{\lambda=0} \overset{\text{large } t}{=} 2\sum_{Y\neq N} \frac{\left|\langle N \,|\, A_3^3 \,|\, Y\rangle\right|^2}{4E_N E_Y (E_N - E_Y)}$
$\frac{d^2 E}{d\lambda^2}$
λ
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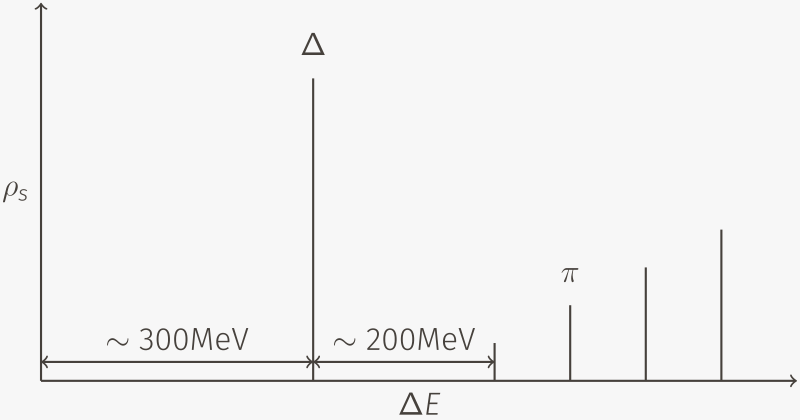
Δ^+ SATURATION



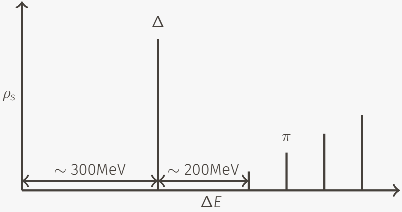
$$\left.\frac{d^2 E_N}{d\lambda^2}\right|_{\lambda=0} \overset{\text{large } t}{=} 2\sum_{Y\neq N} \frac{\left|\langle N \,|\, A_3^3 \,|Y\rangle\right|^2}{4E_N E_Y \left(E_N - E_Y\right)}$$

Δ^+ SATURATION

→ Proton to Δ transition form factor

→ In particular C_5^A

$$\left\langle P \middle| A_3^3 \middle| \Delta^+ \right\rangle \left\langle \Delta^+ \middle| A_3^3 \middle| P \right\rangle = \frac{16}{3} m_P \left[C_5^A\right]^2$$



Source		$C_5^A(0)$
Experiment	empirical νH/D scattering	1.15(23)
	low energy χPT pion scattering	0.93(10)

[1] S.L. Zhu, M.J. Ramsey-Musolf, Physical Review D 66, 076008 (2002).

ADLER FORM FACTOR

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Feynman-Hellmann	$24^3 \times 48$	0.84(11)
	$32^3 \times 64$	0.793(53)

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Feynman-Hellmann	$24^3 \times 48$	0.84(11)
	$32^3 \times 64$	0.793(53)
Quark Model[1]	physical masses	0.87
	$24^3 \times 48$	0.651(23)
	$32^3 \times 64$	0.646(17)

[1] S.L. Zhu, M.J. Ramsey-Musolf, Physical Review D 66, 076008 (2002).

SUMMARY

→ Second order extension to Feynman-Hellmann theorem as a technique to calculate polarisabilities

$$\left.\frac{d^2 E_{X,\mathbf{p}}}{d\lambda^2}\right|_{\lambda=0} \overset{\text{large } t}{=} 2\sum_{Y\neq X} \frac{\left|\langle X,\mathbf{p}\,|\,J(0)\,|Y,\mathbf{p}\rangle\right|^2}{4E_{X,\mathbf{p}}E_{Y,\mathbf{p}}\left(E_{X,\mathbf{p}}-E_{Y,\mathbf{p}}\right)}$$

→ Tested this with an isovector spatial axial current giving transition form factor $C_5^A\left(Q^2\approx 0\right)\sim 0.8 \implies$ higher energy intermediate states?

FURTHER WORK

- → Investigate Axial Pion Production
- → Generalised Second Order Feynman-Hellmann Theorem

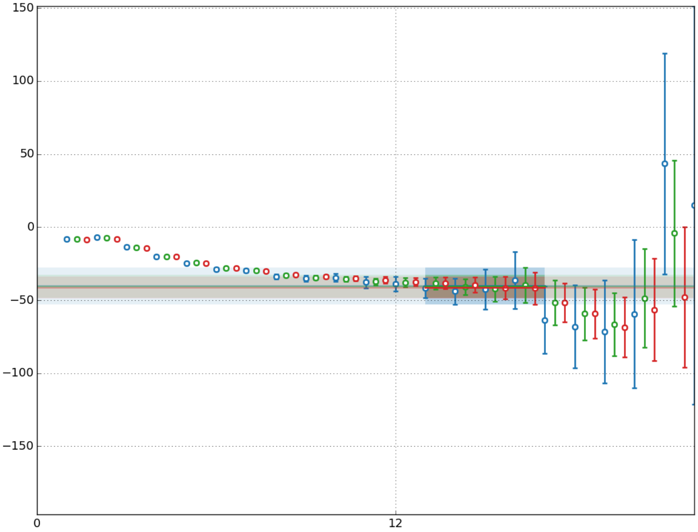
$$S \to S + \lambda_1 \int d^4x\, J_1(x) + \lambda_2 \int d^4x\, J_2(x)$$

$$\left.\frac{\partial^2 E_{X,\mathbf{p}}}{\partial\lambda_1\partial\lambda_2}\right|_{\boldsymbol{\lambda}=\mathbf{0}} = \sum_{Y\neq X} \frac{\langle X,\mathbf{p}|\,J_1(0)\,|Y,\mathbf{p}\rangle_0 \langle Y,\mathbf{p}|\,J_2(0)\,|X,\mathbf{p}\rangle_0 + \langle X,\mathbf{p}|\,J_2(0)\,|Y,\mathbf{p}\rangle_0 \langle Y,\mathbf{p}|\,J_1(0)\,|X,\mathbf{p}\rangle_0}{4E_{X,\mathbf{p}}E_{Y,\mathbf{p}}\left(E_{X,\mathbf{p}} - E_{Y,\mathbf{p}}\right)}$$

- → Momentum Transfer
- → Background E Field

Thank You

BACKUP SLIDES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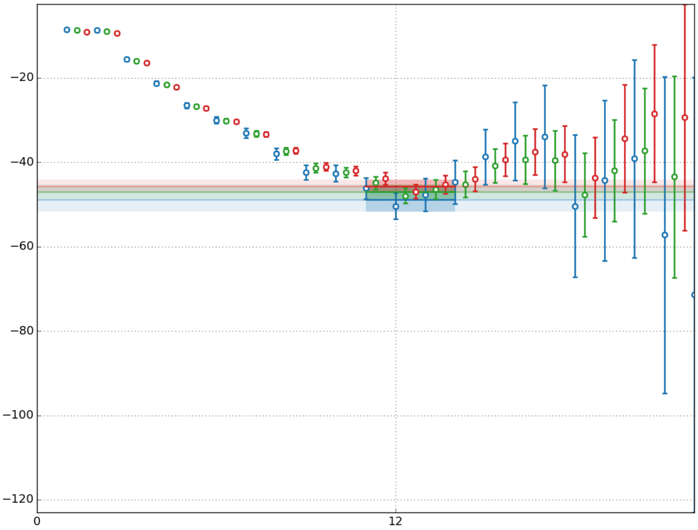

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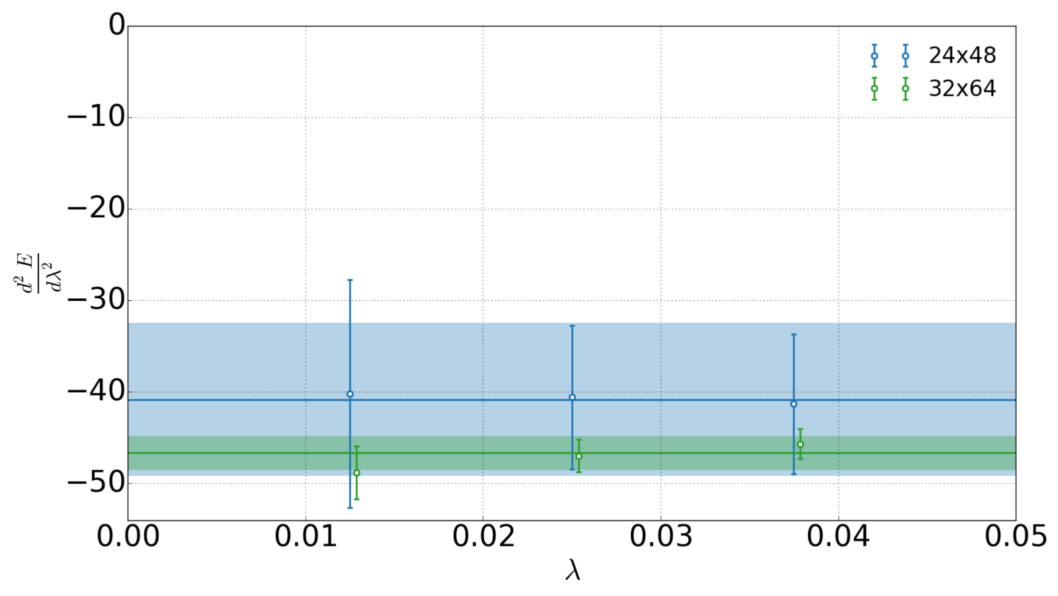

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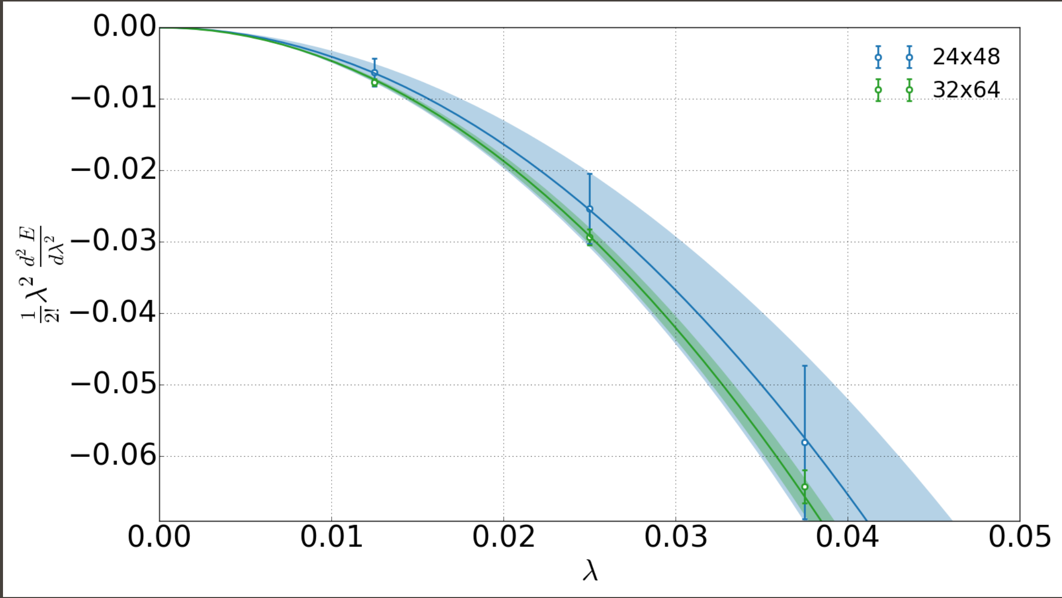

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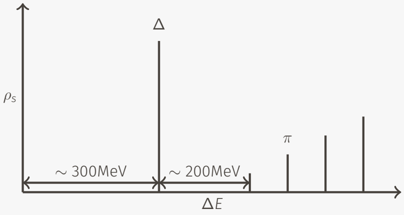
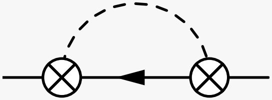
AXIAL PION PRODUCTION (PRELIMINARY)





Use Nucleon-Pion EFT

$$\frac{1}{V}\sum_{\mathbf{k}} \frac{\left|\left\langle P(0)\,\middle|\,A_3^3\,\middle|N(-\mathbf{k})\pi^+(\mathbf{k})\right\rangle\right|^2}{4E_P E_{N\pi^+}\left(E_P - E_{N\pi^+}\right)} = \frac{1}{V}\sum_{\mathbf{k}} \frac{2}{f_\pi^2}\frac{E_N - m}{E_{N\pi^+}\left(m - E_{N\pi^+}\right)}$$