The ‘Exotic Glue’ Structure Function

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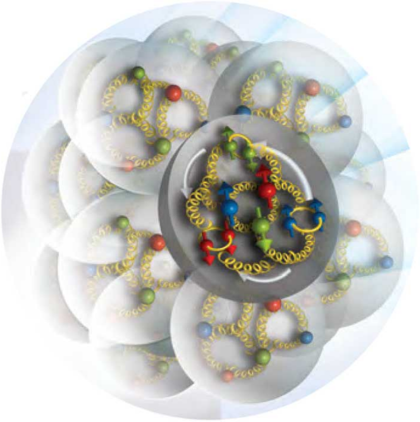
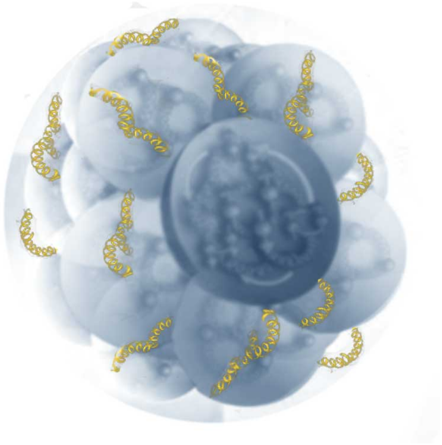
Motivation


Electron Ion Collider:
The Next QCD Frontier
Understanding the glue
that binds us all
BFKL
DGLAP
BK/JIMWLK
Geometric
Scaling

'Exotic' Glue in the Nucleus



‘Exotic’ Glue in the Nucleus



‘Exotic’ Glue

Contributions to gluon observables that are not from nucleon degrees of freedom.

Exotic glue operator:

operator in nucleon $= 0$

operator in nucleus $\neq 0$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

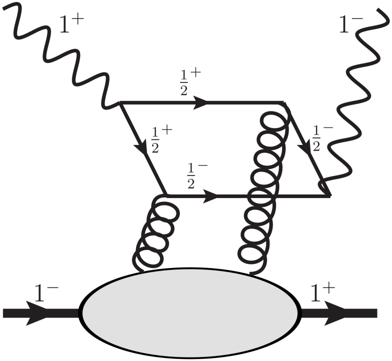
Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- In single hadrons: gluon transversity structure function
- Experimentally measurable (JLab LOI 2016)
- Moments are calculable using lattice QCD

First Lattice Study: arXiv:1606.04505 (PRD)

- First moment of $\Delta(x, Q^2)$ in spin-1 ϕ meson

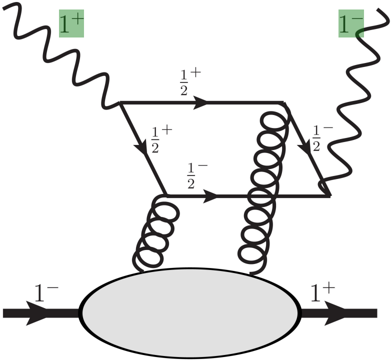
Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$



Double helicity flip amplitude:

$$\Delta(x, Q^2) = A_{+-,-+} = A_{-+,+-}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$



Double helicity flip amplitude:

Photon helicity

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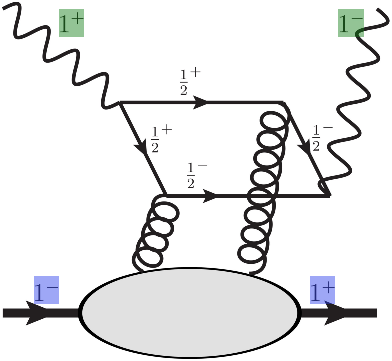

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Optical theorem, dispersion relation for hadronic forward scatt. amplitude, analytic continuation give **moments**:

Moment of Structure Function

Reduced Matrix Element

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \quad n = 2, 4, 6 \ldots,$$

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Operator Product Expansion to relate to matrix elements of operator

Gluonic Operator

$$\langle pE' | S[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}] | pE \rangle$$
$$= (-2i)^{n-2} S\left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} A_n(Q^2) \ldots,$$

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Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

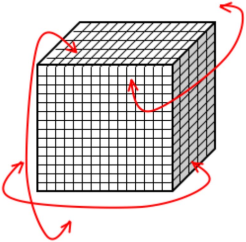
Reduced Matrix Element

Lattice QCD

Numerical first-principles approach

Discretise space-time (4D box)

Lattice spacing a, volume $L^3 \times T$
order $32^3 \times 64 \approx 2 \times 10^6$ lattice sites



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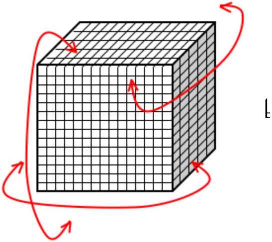

First moment of $\Delta(x, Q^2)$

⇕

Matrix elt. of $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S[G_{\mu\mu_1} G_{\nu\mu_2}]$

Lattice simulation in spin-1 ϕ meson

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1, 2, 3\}$ or $\{+, -, 0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units (1 unit $\sim$ 0.4GeV)
- Different discretisations of the operator (different irreps.)

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SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization (for now)

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Extraction of A_2

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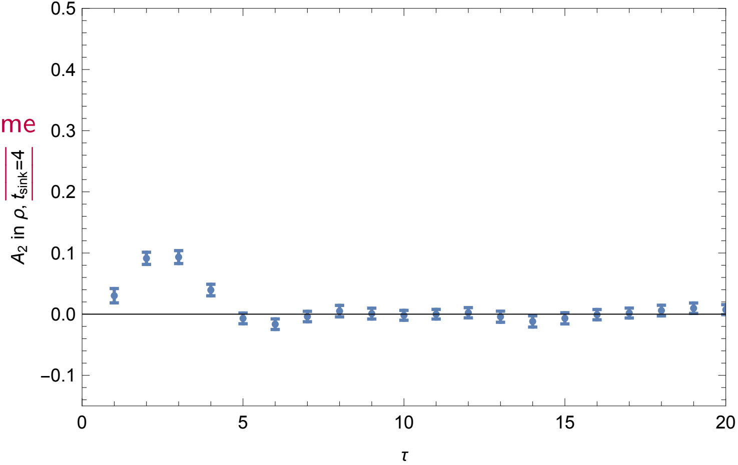
We calculate on the lattice:

factors of m and p

$$\left[\frac{C_{3\text{pt}}^{EE'}}{C_{2\text{pt}}^{EE'}}\right](t_{\text{sink}}, \tau) \propto A_2, \qquad 0 \ll \tau \ll t_{\text{sink}}$$

Extraction of A_2: 3pt/2pt ratio

sink time



operator insertion time

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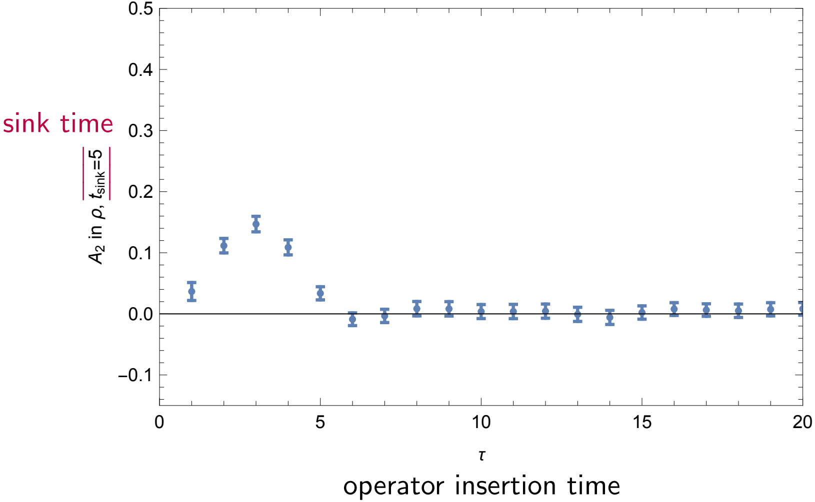

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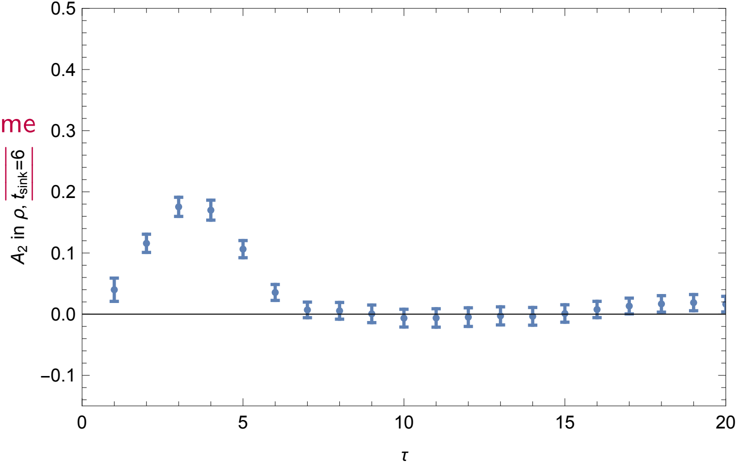

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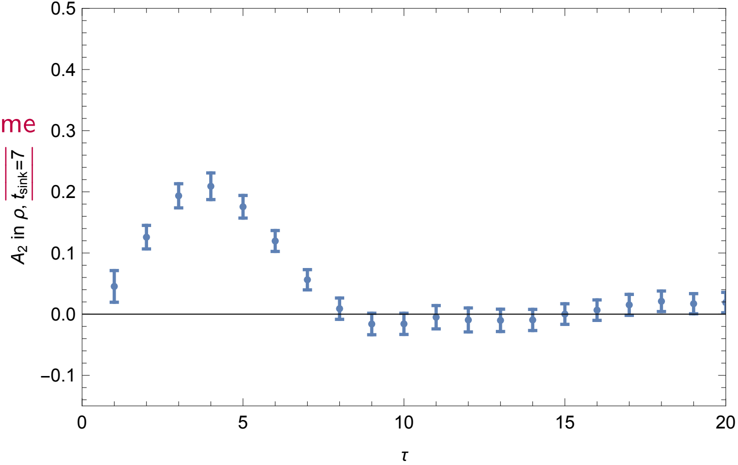

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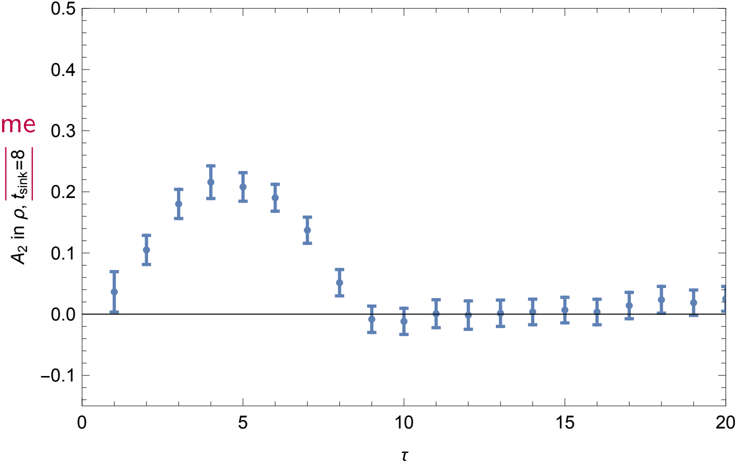

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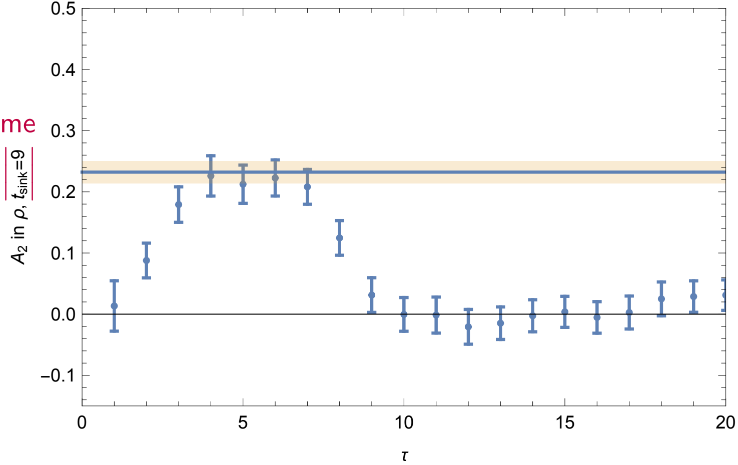

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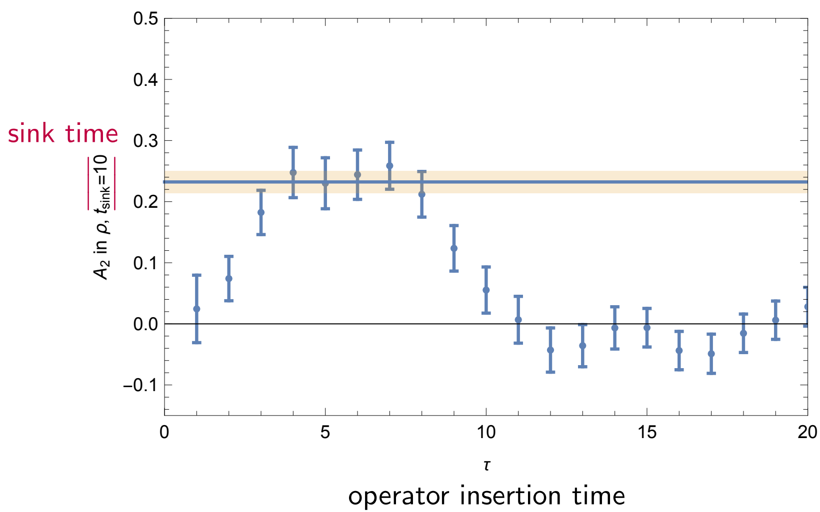

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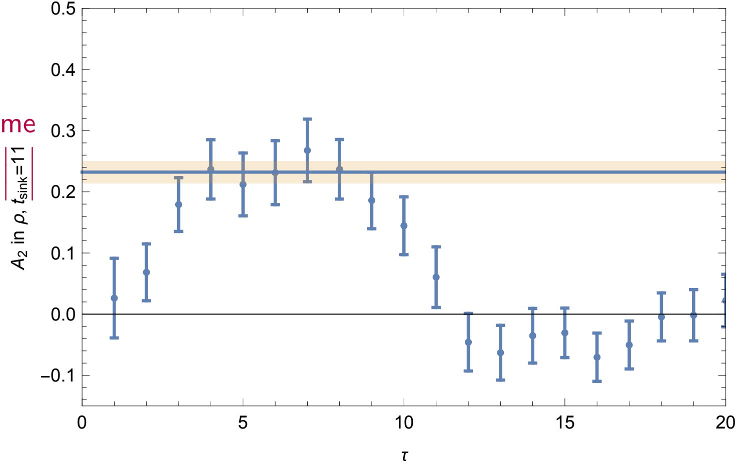

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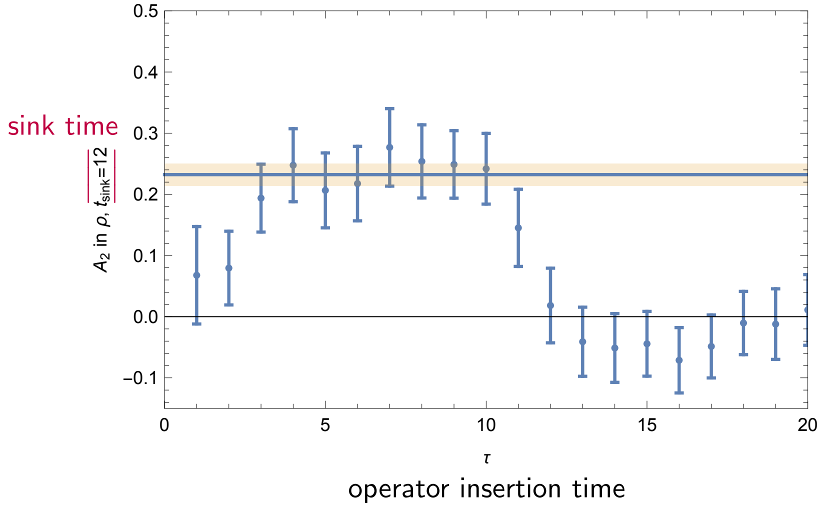

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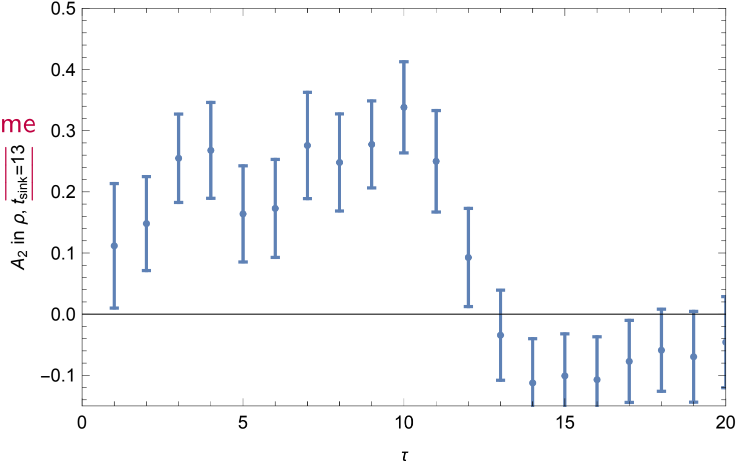
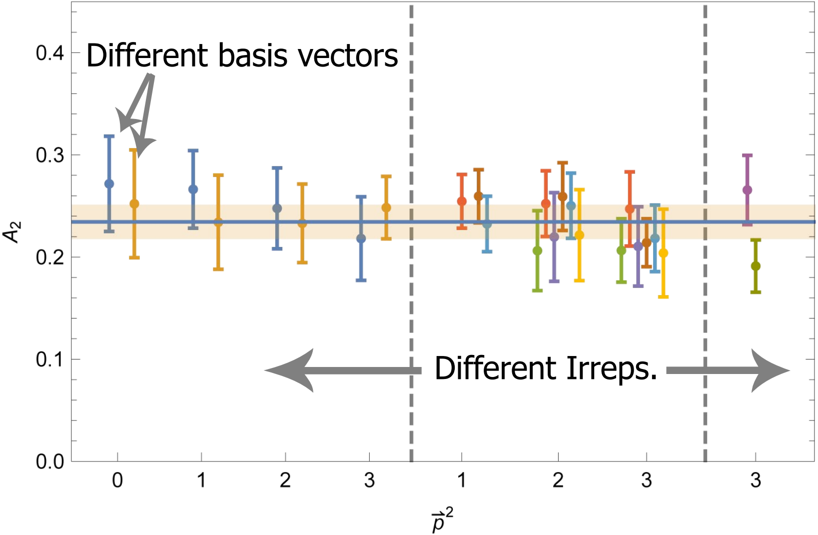


operator insertion time

UNRENORMALISED reduced matrix element: ϕ meson



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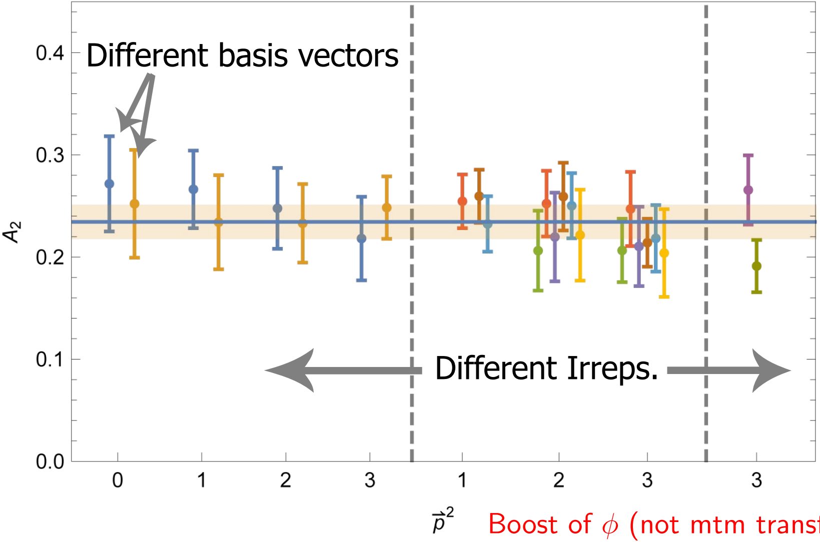

Boost of ϕ (not mtm transfer)

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity — Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Spin-dependent

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Direct analogue for leading moments of gluon distributions:

$$|A_2| \leq \frac{1}{2} B_2$$

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Direct analogue for leading moments of gluon distributions:

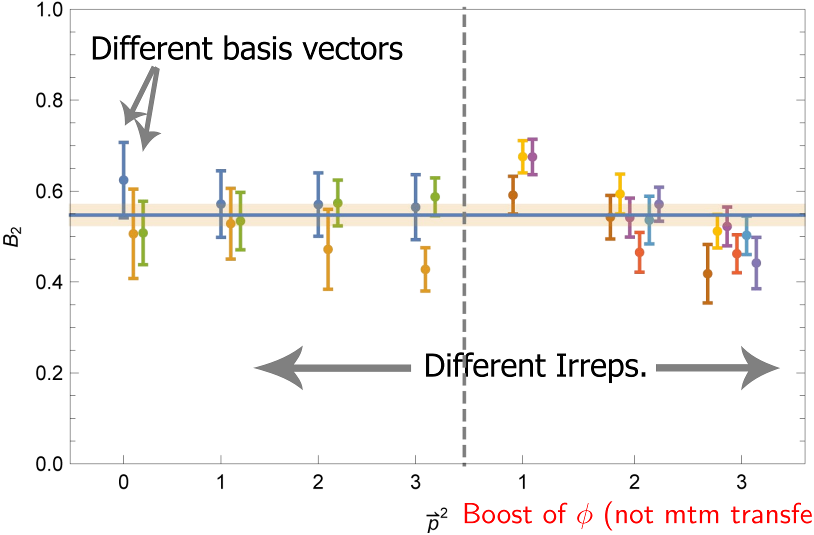
$G_{\mu\mu_1} G_{\nu\mu_2}$

$G_{\mu_1\alpha} G_{\mu_2}{}^{\alpha}$

$$|A_2| \leq \frac{1}{2} B_2$$

$\widetilde{G}_{\mu_1\alpha} G_{\mu_2}{}^{\alpha} \to 0$

UNRENORMALISED reduced matrix element: ϕ meson



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If we assume approx. the same renormalisation for A_2 and B_2:

$$\underbrace{|A_2|}_{G_{\mu\mu_1}G_{\nu\mu_2}} \leq \frac{1}{2}\underbrace{B_2}_{G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}}$$

$$0.25 \leq \frac{1}{2}\,0.6$$

First two moments of quark distributions: Soffer bound saturated to 80% (lattice QCD, Diehl *et al.* 2005)

Summary

ROBUST NON-ZERO signal
for 'exotic glue' operator in the ϕ meson

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Explore gluon structure of hadrons more generally
e.g., Soffer bound analogue

BUT: SYSTEMATICS IGNORED
$\Rightarrow$ no physically meaningful number (yet)