Relativistic approach to nuclear spin-isospin excitations including quasiparticle-vibration coupling

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The study of nuclear isospin-transfer excitations has many applications in

→ **Nuclear physics**: constraints on the (S,T) channels of the nuclear interaction...

→ **Particle physics**: nature of neutrinos (0νββ decay), ...

→ **Astrophysics**:

Fermi and Gamow-Teller transitions determine the rates of many weak processes occurring in stellar environments...

\[
\beta^+ \text{ decay: } (A, Z) \rightarrow (A, Z - 1) + e^+ + \nu_e
\]

\[
electron capture: (A, Z) + e^- \rightarrow (A, Z - 1) + \nu_e
\]

\[
\beta^- \text{ decay: } (A, Z) \rightarrow (A, Z + 1) + e^- + \bar{\nu}_e
\]

\[
\text{neutrino capture: } (A, Z) + \nu_e \rightarrow (A, Z + 1) + e^-
\]

Astrophysical modeling requires properties of nuclei far from stability not yet reached experimentally

→ *need precise and consistent information from theory*
Theoretical framework: spin-isospin response of nuclei with relativistic quasiparticle-vibration coupling

Applications: Gamow-Teller transitions in neutron-rich Nickel isotopes and β-decay half-lives

Conclusion
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Conclusion
Response function (particle-hole propagator) solution of the Bethe-Salpeter equation (BSE):

\[ \delta \rho_{k_1,k_2}(\omega) = \sum_{k_1 k_2 k_3 k_4} R_{k_1 k_4, k_2 k_3}(\omega) F_{k_3 k_4} \]

\[ R = GG - iGV R \]
Response of nuclei to a weak external field - linear response theory

Weak external field $F(t)$

Particle-hole excitations of nucleons

$$\delta \rho_{k_1,k_2}(\omega) = \sum_{k_1,k_2,k_3,k_4} R_{k_1,k_4,k_2,k_3}(\omega) F_{k_3,k_4}$$

**Response function (particle-hole propagator)**

Solution of the **Bethe-Salpeter equation (BSE):**

$$R = G G - i G G V R$$

Nucleon propagator

Self-energy
Response of nuclei to a weak external field - linear response theory

Weak external field $F(t)$

Simple quantum dots

Particle-hole excitations of nucleons

$$\delta \rho_{k_1,k_2}(\omega) = \sum_{k_1 k_2 k_3 k_4} R_{k_1 k_4,k_2 k_3}(\omega) F_{k_3 k_4}$$

Response function (particle-hole propagator)

Solution of the Bethe-Salpeter equation (BSE):

Nucleon propagator

Self-energy

$$V = i \frac{\delta \Sigma}{\delta G}$$

Effective in-medium interaction

$$R = GG - iGGVR$$

$$R = \sum \text{nucleon propagator} + \text{self-energy}$$
Expansion in term of the meson-exchange interaction:

\[ \sum \text{Fierz transformation} = \pi, \sigma, \omega, \rho, \gamma \]

Collective vibration (phonon)

Dynamic

Particle-vibration coupling (PVC)

\[ \infty \text{ serie} = \]

Relativistic Hartree(-Fock)

Static
Nucleonic self-energy

Expansion in term of the meson-exchange interaction:

\[ -\sum_{\pi, \sigma, \omega, \rho, \gamma} \]

\[ \text{Fierz transformation} \]

\[ = \left( \begin{array}{c} \text{static} \\ \text{relativistic Hartree(-Fock)} \\ \text{Bogoliubov} \end{array} \right) + \left( \begin{array}{c} \text{dynamic} \\ \text{collective vibration (phonon)} \end{array} \right) + \ldots \]

In open-shell nuclei:

\[ = \left( \begin{array}{c} \text{quasiparticle} \\ \text{in single-particle space} \end{array} \right) \]

\[ \text{Gorkov propagator} \]
Nuclear response with dynamic self-energy

Bethe-Salpeter equation in the proton-neutron channel:

\[
R(\omega) = \tilde{G}\tilde{G} - i\tilde{G}\tilde{G}(\tilde{V} + \Phi(\omega)) R(\omega)
\]
Nuclear response with dynamic self-energy

\[ R(\omega) = \tilde{G}\tilde{G} - i\tilde{G}\tilde{G}(\tilde{V} + \Phi(\omega))R(\omega) \]

Isovector static meson exchange
\[ \tilde{V} = V_\pi + V_\rho + V_{\delta\pi} \]

Landau-Migdal contact term
Nuclear response with dynamic self-energy

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energy-dependent interaction:

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Bethe-Salpeter equation in the proton-neutron channel:

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RQRPA state accounts for 2qp configurations (on correlated ground state)

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Energy-dependent interaction:
Bethe-Salpeter equation in the proton-neutron channel:

\[
R(\omega) = \tilde{G}\tilde{G} - i\tilde{G}\tilde{G}(\bar{V} + \Phi(\omega))R(\omega)
\]

- **RQRPA** accounts for 2qp configurations (on correlated ground state)
- **QPVC** energy-dependent interaction:
- Isovector static meson exchange
  \[\bar{V} = V_\pi + V_\rho + V_\delta\pi\]
- Landau-Migdal contact term

\[\Rightarrow\] fragmentation of many-body states & possible shift
Theoretical framework: spin-isospin response of nuclei with relativistic quasiparticle-vibration coupling

Applications: Gamow-Teller transitions in neutron-rich Nickel isotopes and β-decay half-lives

Conclusion
Gamow-Teller resonance in Nickel (Ni → Cu)

1st application:
coupling of quasiparticles to neutral (non isospin-flip) phonons
($2^+, 3^-, 4^+, 5^-, 6^+$ up to 30 MeV)

\[ S(E) = -\frac{1}{\pi} \lim_{\Delta \to 0^+} \text{Im} \Pi(E + i\Delta) \]
\[ \Pi(\omega) = \langle F^{\dagger} R(\omega) F \rangle \]

Strength function:

\[ F_{GT^-} = \sum_{i} \sigma^{(i)} \tau^{(i)} \]

(Smearing $\Delta = 200$ keV)

QPVC brings fragmentation of the strength and distribution over a larger energy range

Gamow-Teller resonance in Nickel (Ni \rightarrow Cu)

"Quenching problem":
The observed GT strength (~up to the GR region) in nuclei is \sim 40\% less than the Ikeda sum rule

\[ + \text{ transitions from the Fermi sea to the Dirac sea (~10\%)} \]
[N. Paar et al., PRC 69, 054303]

Up to GR region
\sim 86\% of the total GT \rightarrow strength

\rightarrow \text{RQRPA strength naturally "quenched" due to complex configurations}
Beta-decay half-lives

\[ \frac{1}{T_{1/2}} = \frac{g_a^2}{D} \int_{\Delta B}^{\Delta nH} f(Z, \Delta_{np} - E) S(E) dE \]

\( g_a = 1 \) effective weak axial coupling constant

→ big improvement due to QuasiParticle Vibration Coupling!

exp data from nndc.bnl.gov


68Ni: appearance of strength in the \( Q_\beta \) window due to QPVC → finite lifetime

78Ni: more strength with RQRPA but located at higher energies → smaller lifetime with QPVC due to phase space factor
New developments: coupling to charge-exchange phonons

Existence of low-energy isospin-flip modes which can couple to single-nucleon degrees of freedom → additional terms in the effective interaction:

\[
\begin{align*}
\phi & = p p' + p' p + p p'' + p' p'' + n'' n' + n n'' + n' n
\end{align*}
\]

(No extra phonon-exchange term because of charge conservation)

Effect on the nuclear response:

- Preliminary results for \(^{78}\text{Ni}\) (coupling to \(0^\pm \rightarrow 6^\pm\)):

\[
\begin{align*}
S_{\text{GT}}(E^*) \text{ (MeV)} &
\end{align*}
\]

\[
\begin{align*}
\Sigma B(\text{GT}) &
\end{align*}
\]

\[
\begin{align*}
T_{1/2} (s) \text{ \(^{78}\text{Ni}\)} &
\end{align*}
\]

\(~81\% \text{ vs } 86\% \text{ of the total GT strength}\)

Additional decrease of the half-life

- pnRQRPA
- pnRQTBA neutral phonons only
- pnRQTBA neutral+ charge-ex. phonons

\(~81\% \text{ vs } 86\% \text{ of the total GT strength}\)
Outline

**Theoretical framework:** spin-isospin response of nuclei with relativistic quasiparticle-vibration coupling

**Applications:** Gamow-Teller transitions in neutron-rich Nickel isotopes and β-decay half-lives

**Conclusion**
The pn-RQTBA (pn-RQRPA + QPVC) has potential to describe both

- the details of the low-lying transition strength → important for beta-decay half-lives and other rates of weak processes, which are very sensitive to the coupling between single-particle and collective degrees of freedom
- the overall strength distribution up to high excitation energy → important to reproduce (part of) the observed “quenching” of the GT strength without any artificial factor

Perspectives:

- Inclusion of the coupling to pn pairing phonons in doubly magic (N=Z) nuclei
- Inclusion of ground-state correlations (backward-going diagrams) beyond the RQRPA ones
- Application to double-beta decay
- Together with RQTBA in neutral channel, this framework provides a high-quality and consistent description of both phases of the r-process nucleosynthesis, (n,γ) and β-decay ⇒ implementation in astrophysical modeling

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Thank you!
Backup
**Problem:** Integration over all intermediate times $\Rightarrow$ complicated BSE (integrations do not separate), appearance of $NpNh$ configurations:

$R$, $\ldots$

**Solution:** *Time-Blocking Approximation* 
[V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)]

$1p1h \otimes 1$ phonon $(2p2h)$ $\Rightarrow$ spreading

$\rightarrow$ allowed configurations:

$\rightarrow$ blocked configurations: $3p3h$, $4p4h$...

... but can be included in a next step (under development)
Response function in the proton-neutron channel:

\[ V = V_{\pi} + V_{\rho} + V_{\delta\pi} \]

Note: simple-pole structure due to TBA → ensures locality and unitarity of the response → strength is positive-definite.
Numerical scheme

Relativistic mean-field with pairing
(RH+BCS, NL3, monopole pairing force)

Solve RQRPA
⇒ neutral (non-isospin flip) phonons with natural parities
$2^+, 3^-, 4^+, 5^-, 6^+$ up to 30 MeV and their coupling vertices

Solve BSE with RQTBA for the proton-neutron response function
(Gamow-Teller: $J^\pi = 1^+$)

Nuclear Polarizability:
\[
\Pi(\omega) = F^\dagger R(\omega)F
\]

Strength function:
\[
S(E) = -\frac{1}{\pi} \lim_{\Delta \to 0^+} \text{Im} \, \Pi(E + i\Delta)
\]

Strength function:
\[
F_{GT^-} = \sum_i \sigma^{(i)} \tau^{(i)}
\]
Gamow-Teller resonance in Nickel

Convergence of the strength according to the phonon spectrum (neutral phonons):

$^{68}\text{Ni}$

C. Robin and E. Litvinova EPJA 52, 205 (2016).
Gamow-Teller resonance in Nickel

Effect of pairing correlations on the strength distribution:

Pairing can bring fragmentation at the QRPA level (Landau damping)

QPVC brings spreading effects
Gamow-Teller resonance in Nickel

Graphs showing the Gamow-Teller strength function $S_{GT}(E^*)$ for $^{68}\text{Ni}$, $^{70}\text{Ni}$, and $^{72}\text{Ni}$, with calculated results from pnRQRPA, pnRQTBA(10), and pnRQTBA(30).