Chiral Corrections to Electromagnetic Form Factors in the NJL Model

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Talk Outline

- Motivation
- Nucleon Electromagnetic Form Factors
- Calculation of Bare Form Factors
- Pion-Nucleon Effective Field Theory
- Results
- Further Work and Conclusion

Motivation

- Massless QCD is chirally symmetric.
- Important symmetry for models of QCD to replicate.
- In quark models, incorporate the pion field.
- Thomas and Krein:
  - *Chiral Corrections in Hadron Spectroscopy* (1999)
  - *Chiral Aspects of Hadron Structure* (2000)
- Two approaches: Parton Level, and Hadron Level
- Manohar and Georgi: *Chiral Quarks and the Non-Relativistic Quark Model* (1985)

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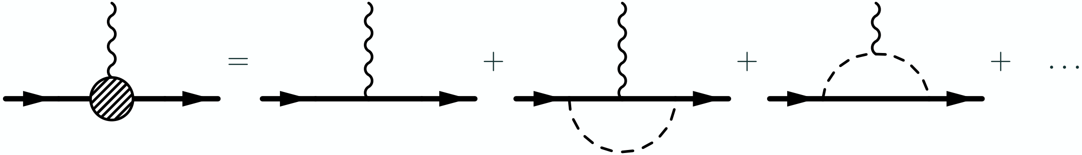
Proposal

- Cloët, Bentz and Thomas wrote paper - *Role of diquark correlations and the pion cloud in nucleon elastic form factors* (2014)
  - Pion loops calculated at parton level.
- Propose to calculate bare (pionless) form factors in NJL Model.
  - Perform pion loop corrections at hadron level (use nucleon-pion effective Lagrangian).
  - Similar to approach in Light Front Cloudy Bag Model (LFCBM), but several small differences.

Nucleon Electromagnetic Form Factors

- Form factors contain information about the structure of the nucleon.



$$\Gamma^{\mu}_{eff}(Q^2) = F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_N}$$

- Common to use the Sachs Parametrisation.

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

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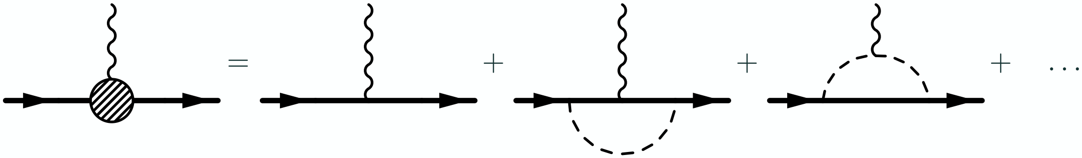

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The Nambu–Jona-Lasinio (NJL) Model: Bare Form Factors

- Bare form factors calculated in NJL Model.
  - Low energy approximation of QCD: 4 point contact force
- Confinement failure of basic model, but imposed via Proper Time Regularisation & infra-red cutoff.

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau X}$$

- Parameters are fitted to give nucleon mass.

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Performing Chiral Corrections to NJL Model at Hadron Level

The Self Energy

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{p - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops

$$= \frac{iZ}{p - m_N + i\epsilon}$$

$$m_N = m_N^{(0)} + \Sigma(p)\big|_{p=m_N}$$

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Pion-Nucleon Effective Field Theory

- Work with a pseudoscalar pion-nucleon interaction:

$$\mathcal{L}_{N\pi} = -g_0 \overline{\psi}_N i \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N$$

- After minimal substitution, one has three diagrams at first loop order.

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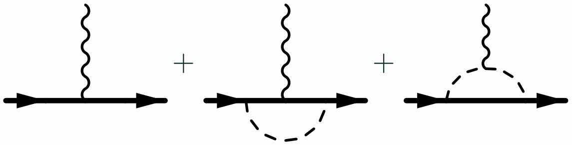
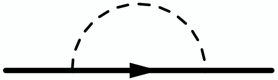
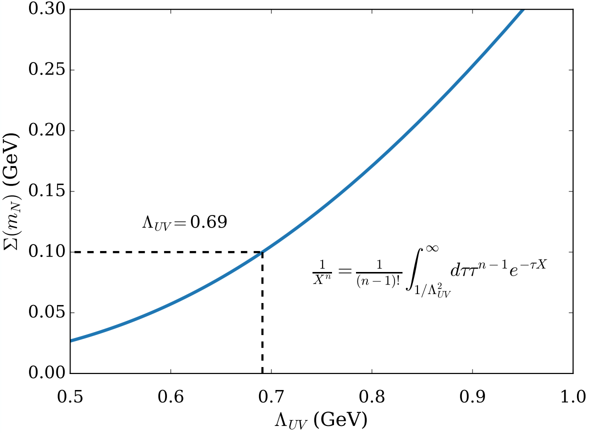


Cutoff Dependence

- Non-renormalisable theory, so must define regularisation prescription to fully define model.
- Choose to use Proper Time Regularisation.
- All observables become cutoff dependent.
  - Self energy cutoff dependent.
  - $m_N^{(0)} = m_N - \Sigma(m_N)$





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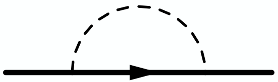

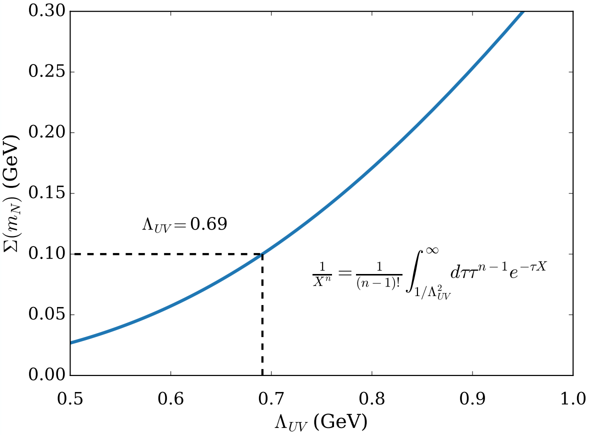

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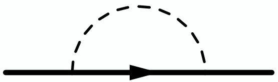
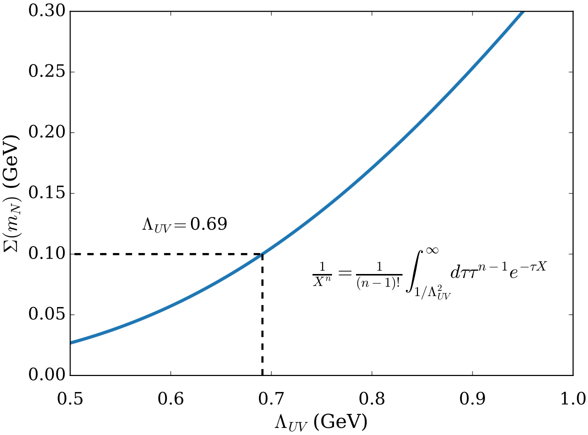
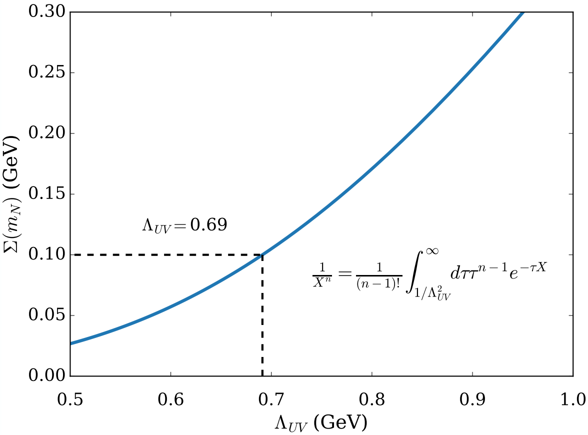




Results

- CBM predicts total pion self energy contribution of $\Sigma(m_N) = 0.2$ GeV.

  $\Longrightarrow \; m_N^{(0)} = m_N + 0.2 = 1.140$ GeV
  - Contains nucleon and delta contribution.
- CBM predicts $\approx$ half self energy contribution from nucleon alone.
  - Choose a UV cutoff $\Lambda_{UV} = 0.7$ GeV to obtain this self energy correction.



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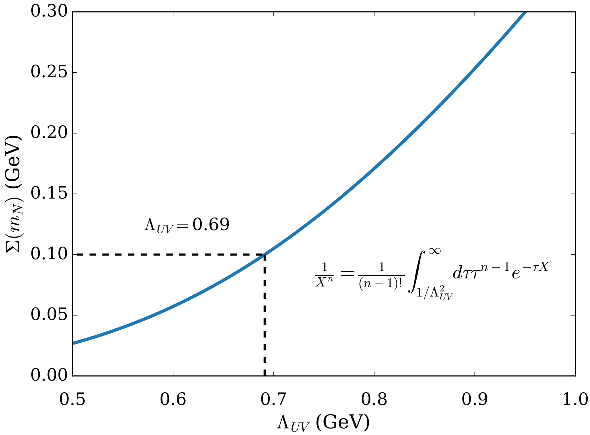

RESULTS

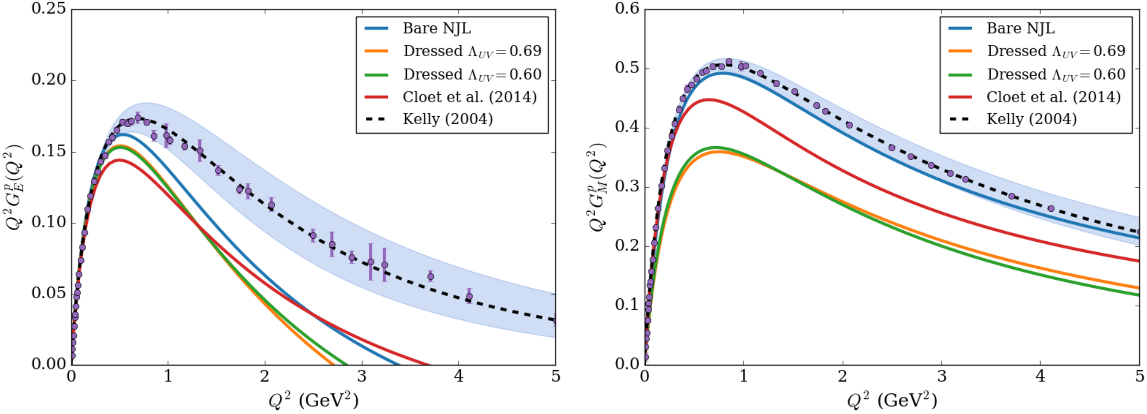

Figure 1: $Q^2 G_E^p$ and $Q^2 G_M^p$ (To be published). Experimental data from Arrington et al. (2007).

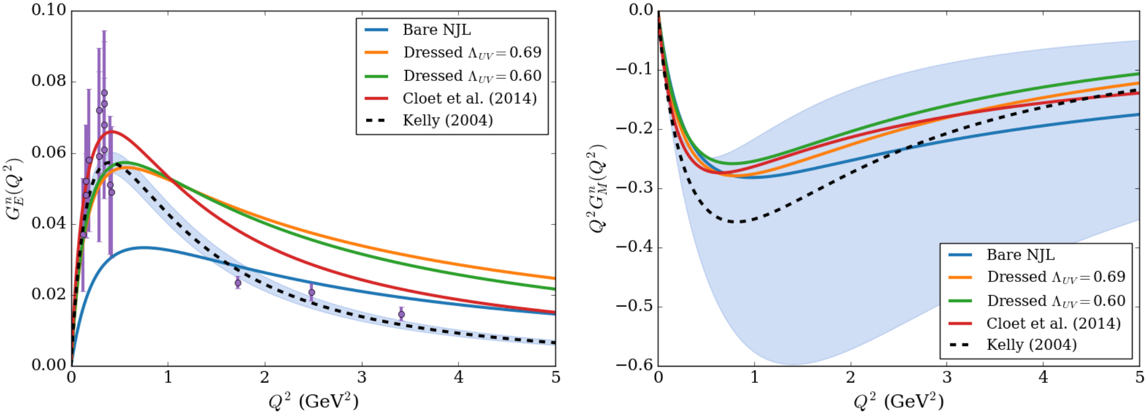

Figure 2: G_E^n and $Q^2 G_M^n$ (To be published).

Further Work and Conclusion

Further Work:

- Use pseudovector, rather than pseudoscalar interaction.
- Incorporate delta fluctuation.

In Conclusion...

- Discussed the two ways one could incorporate chiral loop corrections.
- Earlier work has shown the correct way is at the hadron level.
- Calculated chiral loop corrections to the NJL model at the Hadron Level.

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Thanks for listening!

Scaling the Form Factors

- The calculation at non-physical mass requires one to modify the equations for the loop corrections.
- The outputs of NJL Model are $F_1^{(0)}(Q^2)$, and $F_2^{(0)}(Q^2)$, and come from separating the gamma function Γ^μ

$$\Gamma^\mu = F_1^{(0)}(Q^2)\gamma^\mu + F_2^{(0)}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N^{(0)}}$$

- When inserting this into the chiral correction equations, one must rescale $F_2^{(0)}(Q^2)$, as

$$f$$

Results

Λ_{UV}	Z	$\langle r^2 \rangle_p^{1/2}$	$\langle r^2 \rangle_n^{1/2}$	μ_p	μ_n
0.69	0.67	0.83	0.32	1.92	-1.46
0.60	0.76	0.83	0.32	1.99	-1.39

THE STORY SO FAR...

PARTON LEVEL

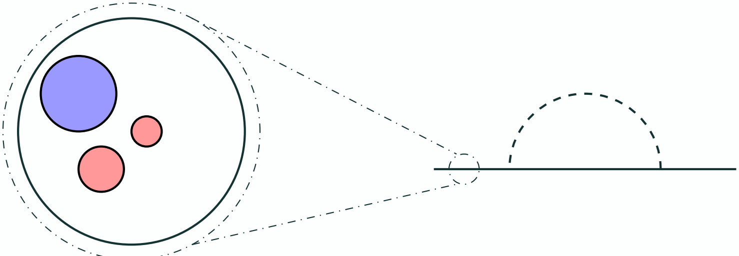
Caculate bare (pionless) form factors in NJL Model

→

HADRON LEVEL

Calculate chiral loops

Distance Scale →



Equations

$$F^p_{1,b} = \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} d\tau \Bigg([4x(x+y)m_N^2 - 2(x+y)m_N^2 + \frac{1}{2}(x+y)q^2 - \frac{1}{\tau}]$$

$$\times (F^n_{1,a} + \frac{1}{2}F^p_{1,a}) + [x(x+y)q^2](F^n_{2,a} + \frac{1}{2}F^p_{2,a})\Bigg) e^{-\tau\Delta}$$

$$+ \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \frac{d\tau}{\tau}\frac{1}{1-x}(F^n_{1,a} + \frac{1}{2}F^p_{1,a})e^{-\tau\Delta'}$$

$$F^p_{2,b} = \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} d\tau \Bigg(- [4x(x+y)m_N^2](F^n_{1,a} + \frac{1}{2}F^p_{1,a})$$

$$+ [\frac{1}{2}(x+y)q^2 - 2(x+y)m_N^2 - \frac{2}{\tau} - 2xyq^2](F^n_{2,a} + \frac{1}{2}F^p_{2,a})\Bigg) e^{-\tau\Delta}$$

$$+ \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \frac{d\tau}{\tau}\frac{1}{1-x}(F^n_{2,a} + \frac{1}{2}F^p_{2,a})e^{-\tau\Delta'}$$

Equations

$$F^{p}_{1,c} = \frac{Zg_0^2}{(4\pi)^2} F_\pi(q^2) \int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \left[\frac{1}{\tau} - 2m_N^2 z^2\right] e^{-\tau\Delta}$$

$$F^{p}_{2,c} = \frac{Zg_0^2}{(4\pi)^2} F_\pi(q^2) \int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \left[2m_N^2 z^2\right] e^{-\tau\Delta}$$

References

[1] A. Thomas and G. Krein, "Chiral corrections in hadron spectroscopy," *Physics Letters B*, vol. 456, pp. 5–8, jun 1999.

[2] A. Thomas and G. Krein, "Chiral aspects of hadron structure," *Physics Letters B*, vol. 481, pp. 21–25, may 2000.