Impact of pairing correlations on the chemical composition of the inner crust of a neutron star

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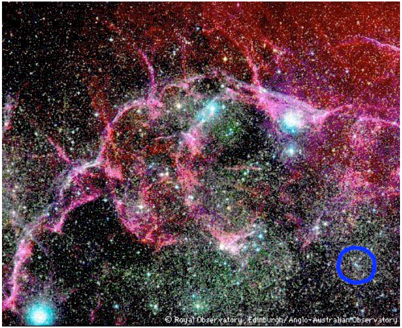
Neutron Stars

Several neutron stars detected in the universe

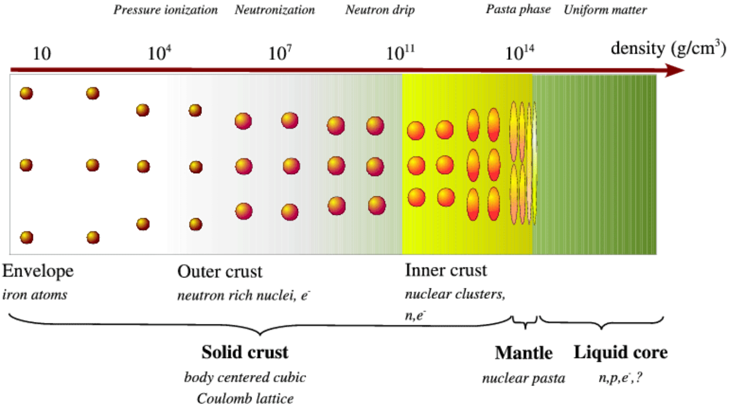


Vela Nebula

- 10^4 years ago explosion
- Rotating neutron star (pulsar)
- Radius ≈ 10 Km
- Density $\approx 5 - 10\rho_0$ ($\rho_0 \rightarrow$density of a nucleus)

The crust of a Neutron Star ($\approx 0.5 - 1$ Km)

The structure evolves with the density

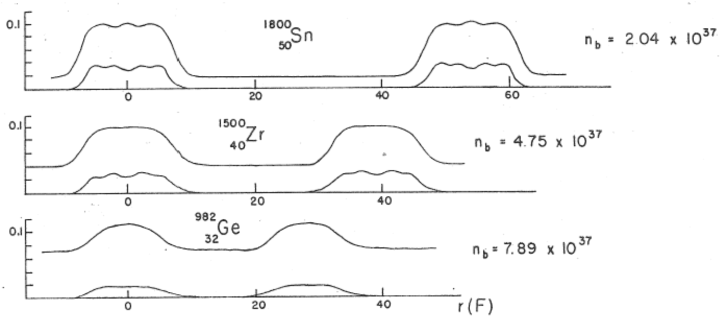


[N. Chamel, P. Haensel , http://www.livingreviews.org/lrr-2008-10]

- Neutron crust $\rho < \rho_0$
- Crystalline structure: isolated nuclei, nuclei+neutron gas

Inner crust of neutron star

Isolated nuclei in a crystalline structure surrounded by neutron gas



Interesting aspects

- Nucleus-gas interaction
- Neutron superfluidity
- Thermal evolution

How to determine the chemical composition?

Approximations

- Spherical cells $\rightarrow$ Wigner Seitz (WS) cells
- Non-interacting WS cells
- Uniform e distribution

We need to minimise the energy

$$E = Z(m_p + m_e) + (N - A)m_n + E_{Nuclear} + K_e + E_L$$

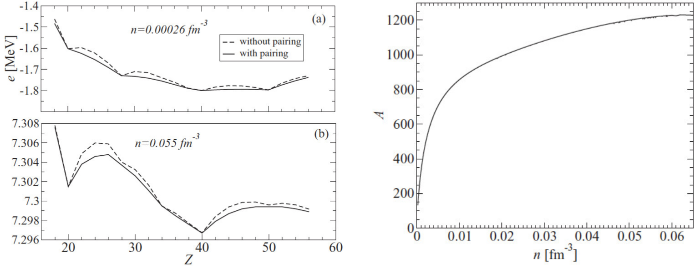
- $E_{nuclear}$: nuclear binding energy
- K_e electron kinetic energy (ultra-relativistic)
- E_L lattice energy

How to calculate $E_{nuclear}$

Nuclear physics input $\rightarrow$ calculate $E_{nuclear}$ for a wide range of densities, asymmetries and temperatures.

Semiclassical methods

Extended Thomas-Fermi+ BCS pairing (protons)



[N. Chamel, J.M. Pearson, A.P. and S. Goriely; Phys.Rev. C 91, 018801 (2015)]

- Proton pairing → small shift on energy minima, smaller differences between minima (more mixing?)
- No spurious *Box-effects*
- Shell effects → Strutinsky correction
- No neutron pairing

Semiclassical methods II

Force	$\bar{n}_{\rm drip}$ (fm^{-3})	Z	N	e (MeV)	P (MeV fm^{-3})
BSk19	2.63464×10^{-4}	40 (38)	96 (88)	-1.79426 (-1.87464)	5.072×10^{-4} (4.938×10^{-4})
BSk20	2.62873×10^{-4}	40 (38)	95 (88)	-1.79451 (-1.87305)	5.064×10^{-4} (4.923×10^{-4})
BSK21	2.57541×10^{-4}	40 (38)	94 (86)	-1.81718 (-1.90057)	4.984×10^{-4} (4.894×10^{-4})
SLy4	2.45897×10^{-4}	40 (38)	93 (82)	-1.78801 (-1.95898)	4.744×10^{-4} (4.807×10^{-4})

[J. M. Pearson et. ;Phys. Rev. C 85, 065803 (2012)]

Some problems at the drip-line....

- Inconsistent treatment at the drip-line: HFB vs Semi-classic (same interaction!)

HFB methods

DFT models

Solve HFB equations in a WS cell

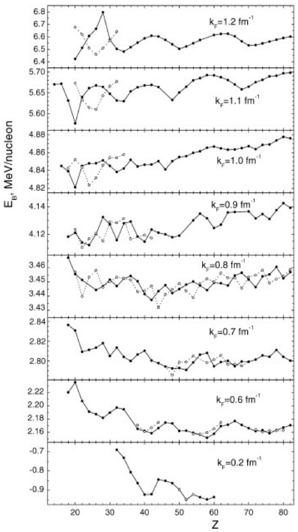
$$\sum_{n'}(h^q_{n'nlj} - \mu_{F,q})U^{i,q}_{n'lj} + \sum_{n'}\Delta^q_{nn'lj}V^{i,q}_{n'lj} = E^q_{ilj}U^{i,q}_{nlj}$$
$$\sum_{n'}\Delta^q_{nn'lj}U^{i,q}_{n'lj} - \sum_{n'}(h^q_{n'nlj} - \mu_{F,q})V^{i,q}_{n'lj} = E^q_{ilj}V^{i,q}_{nlj}$$

- Microscopic functionals + pairing (no approx.)
- Boundary conditions $\rightarrow$ continuum effects

$$\begin{array}{ll} u_{l\rightarrow\text{even}}(R_{Box}) = 0; & u'_{l\rightarrow\text{odd}}(R_{Box}) = 0 \\ u'_{l\rightarrow\text{even}}(R_{Box}) = 0; & u_{l\rightarrow\text{odd}}(R_{Box}) = 0 \end{array}$$

- No assumption on density shapes

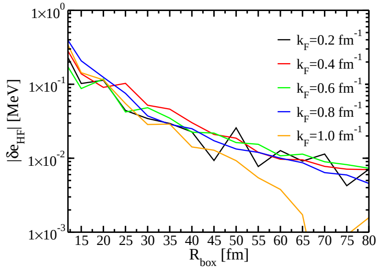
Boundary conditions I

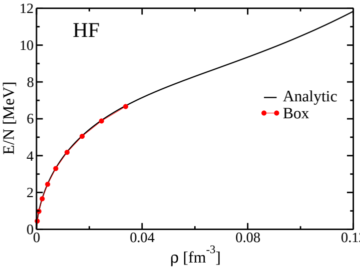


[M. Baldo et al. Nucl. Phys. A775, 235-244 (2006)]

Boundary conditions II

Compare E/A in pure neutron matter and in a box at same k_F





We need large boxes!

- We solve HFB equations in a large box $R_b = 80$ fm
- Approx. constant error 7keV/particle $\rightarrow$ constant shift
- Long CPU time....

Gaussian Process Emulators (GPEs)

- Emulate unknown outputs of a simulation
- Use Bayesian inference (i.e. not the same as basic interpolation)
- Probability of output being in certain region is also used by emulator
- Outputs of simulation are expected to vary smoothly with simulation inputs
- Outputs are modelled as a random Gaussian process in parameter space defined by simulation inputs
- In 1D, works by fitting set of polynomials to simulation output

Motivation for using GPEs and application to project

- Want to make the most of output expensive computer simulation
- Hopefully will produce same effect as decreasing grid size/ increasing size of basis set
- GPEs can be used for higher-dimensional problems, e.g. 2D potential energy surface for fission
- May be very useful if not working on equally-spaced grids (e.g. zeroes of Hermite polynomials)

Conclusions and Perspectives

Conclusions

- Pairing correlations impact chemical composition
- Strong shell effects $\rightarrow$ Need microscopic description
- Very difficult problem to solve

GPE methods

Advanced GPE methods $\rightarrow$ make the problem solvable with controlled approximations.

Perspectives

- Full HFB treatment (no *ad* $-$ *hoc* corrections)
- Very large boxes $\rightarrow$ better treatment of continuum
- Reduced CPU time $\rightarrow$ GPE