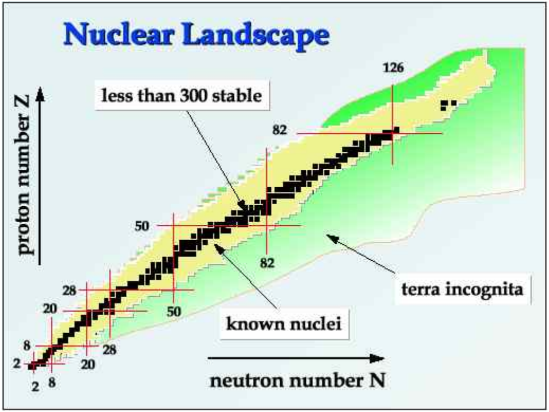
# Partial wave decomposition of finite-range effective interactions

A. Pastore[1], D. Davesne[2], P. Becker[2], J. Meyer[2] and J. Navarro[3]

[1]Department of Physics, University of York, Heslington, York, YO10 5DD, UK
[2]Université Lyon 1, 43 Bd. du 11 Novembre 1918, F-69622 Villeurbanne cedex, France
CNRS-IN2P3, UMR 5822, Institut de Physique Nucléaire de Lyon
[3]IFIC (CSIC-Universidad de Valencia), Apartado Postal 22085, E-46.071-Valencia, Spain

September 11, 2016

# Introduction




## Challenges in nuclear physics

How good our models? Can we explain systematic data?

# Several *possible* interactions

## M3Y: finite range → Yukawa form factor

$$v_N^C(\mathbf{r}_1,\mathbf{r}_2) = \sum_{i=1}^{3}\left[t_i^{(SE)}P_{SE} + t_i^{(TE)}P_{TE} + t_i^{(SO)}P_{SO} + t_i^{(TO)}P_{TO}\right]\frac{e^{-r\,\mu_i^C}}{r\,\mu_i^C},$$

## Gogny D1$x$: finite range → Gaussian form factor

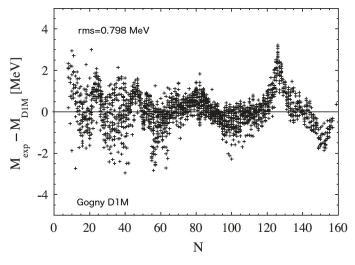
$$v_G(\mathbf{r}_1,\mathbf{r}_2) = \sum_{i=1}^{2}\left[W_i + B_iP_\sigma - H_iP_\tau - M_iP_\sigma P_\tau\right]e^{-(r/\mu_i^C)^2}$$

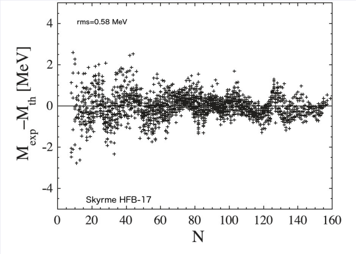
## Skyrme: zero range → Dirac delta

$$\begin{aligned} V_{\mathrm{N3LO}}^C &= t_0(1+x_0P_\sigma) + \frac{1}{2}t_1(1+x_1P_\sigma)(\mathbf{k}^2+\mathbf{k}'^2) + t_2(1+x_2P_\sigma)(\mathbf{k}\cdot\mathbf{k}') \\ &+\frac{1}{4}t_1^{(4)}(1+x_1^{(4)}P_\sigma)\left[(\mathbf{k}^2+\mathbf{k}'^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ &+t_2^{(4)}(1+x_2^{(4)}P_\sigma)(\mathbf{k}'\cdot\mathbf{k})(\mathbf{k}^2+\mathbf{k}'^2) + \ldots \end{aligned}$$

# Is it possible to assess the role of the range?

## Systematic performances in finite nuclei



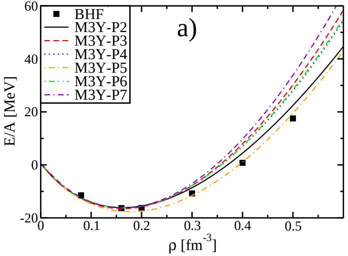



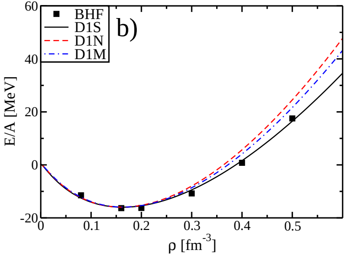

Similar average performances ⇒ Difficult to determine the role of the range

## Infinite Nuclear Matter

- Homogeneous system (no finite-size effects)
- Analytical calculations
- Comparison with other models ($ab-initio$)
- Applications to neutron-star core

# Equation of State (EoS)



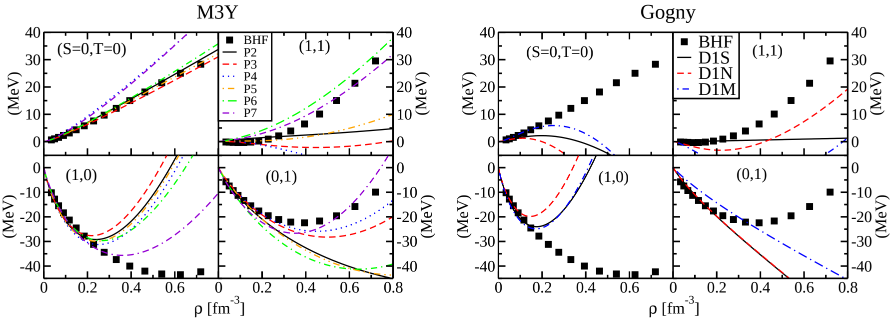

## Binding energy per particle (E/A) in symmetric matter (SNM)

$Ab-initio$ results as Brueckner-Hartree-Fock (BHF) are use to fix bulk properties

$$\frac{E}{A} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} + \mathcal{V}$$

# How flexible are finite-range interactions?

We investigare spin (S) isospin (T) decomposition of EoS in SNM




## Not *real* observables...

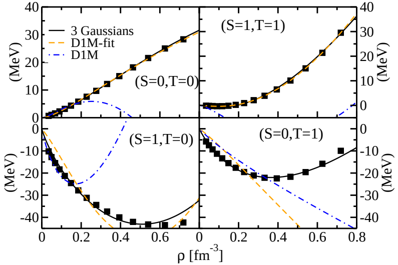
... included in the fit of both effective interactions!

# Shall we change the form of Gogny?

## 11 Gogny interaction

There are few Gogny interactions available on the market. Most of them have the *same* 2-Gaussian structure. D2 add a Gaussian on the $t_3$ term

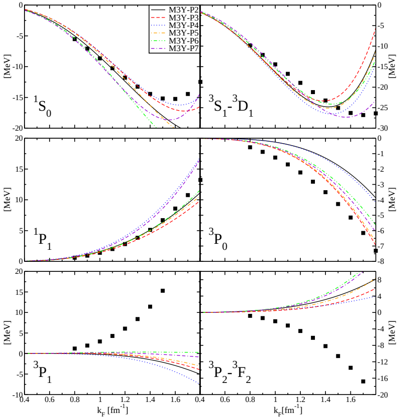
Why not adding a third Gaussian?




Possible only using 3 Gaussians $\mu_1 = 0.25fm, \mu_2 = 0.8fm, \mu_3 = 1.2fm$

# $JLS$-decomposition of EoS

We can decompose the potential energy in $JLS$ channels $\mathcal{V} = \sum_{JLS} \mathcal{V}(^{2S+1}L_J)$

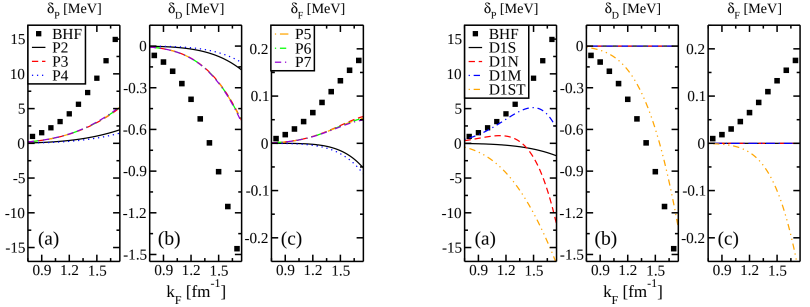



- $L$ degeneracy removed by spin-orbit and tensor
- $\mathcal{V}(^{2S+1}L_J)$ are calculate via $ab-initio$ methods
- Not observables, but in agreement (sign/magnitude) between different $ab-initio$ methods
- Gogny does not reproduce these quantities (no tensor)
- M3Y gives a fair description of the $JLS$ channels (not high order waves)

# $JLS$-decomposition to determine tensor terms

We *eliminate* the central contribution by considering

$$\left.\begin{aligned} \delta_P &= \mathcal{V}(^3P_1)/3 - \mathcal{V}(^3P_0) \\ \delta_D &= \mathcal{V}(^3D_2)/5 - \mathcal{V}(^3D_3)/7 \\ \delta_F &= \mathcal{V}(^3F_3)/7 - \mathcal{V}(^3F_4)/9 \end{aligned}\right\}$$

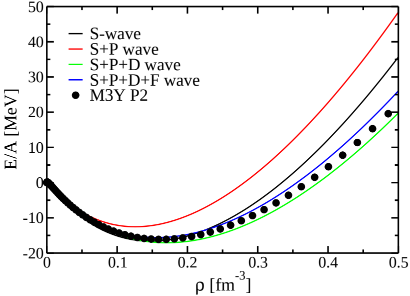


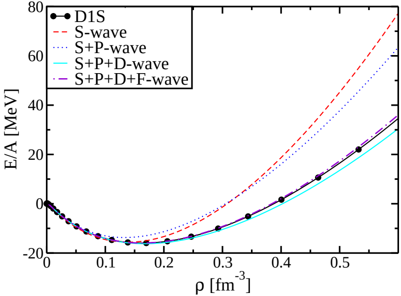

## Warning

This is *exact* only within the context of effective interactions (Gogny/M3Y).
$Ab-initio$ calculations are more complicated.

# Role of the range

The range includes contributions from $all$ partial waves... are they important?




**No free parameter!**

By including D-wave term (4th order) we have *almost* the complete physics at $\rho_0$

# Is there a relation between Skyrme and Gogny?

Fourier transform of the interaction

$$v_G^c(\mathbf{k},\mathbf{k}') = \sum_{n=1}^{2} \left[ W^n + B^{(n)} P_\sigma - H^{(n)} P_\tau - M^{(n)} P_\sigma P_\tau \right] \pi^{3/2} (\mu_n^{c,G})^3 e^{-(\mathbf{k}-\mathbf{k}')^2/4(\mu_n^{c,G})^2},$$

Taylor expansion of a scalar function $F(\mathbf{k}-\mathbf{k}')$ in $\mathbf{k}$-space

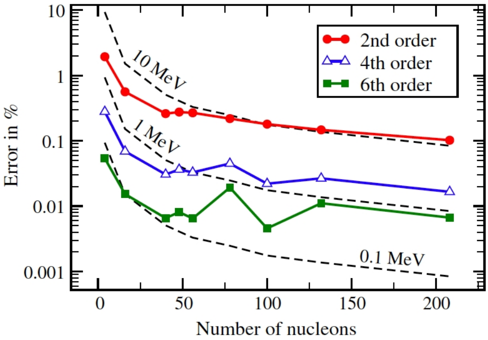
$$\begin{aligned} F(\mathbf{k}-\mathbf{k}') &= C_0 + C_2(\mathbf{k}-\mathbf{k}')^2 + C_4(\mathbf{k}-\mathbf{k}')^4 + \cdots \\ &= C_0 + C_2\left[\mathbf{k}^2 + \mathbf{k}'^2 - 2\mathbf{k}'\cdot\mathbf{k}\right] \\ &\quad + C_4\left[(\mathbf{k}^2 + \mathbf{k}'^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2 - 4(\mathbf{k}'\cdot\mathbf{k})(\mathbf{k}^2 + \mathbf{k}'^2)\right] + \cdots \end{aligned}$$

## Skyrme N2LO has *exactly* the same structure!

$$\begin{aligned} V^c_{\text{N3LO}} &= t_0(1 + x_0 P_\sigma) + \frac{1}{2} t_1(1 + x_1 P_\sigma)(\mathbf{k}^2 + \mathbf{k}'^2) + t_2(1 + x_2 P_\sigma)(\mathbf{k}\cdot\mathbf{k}') \\ &\quad + \frac{1}{4} t_1^{(4)}(1 + x_1^{(4)} P_\sigma)\left[(\mathbf{k}^2 + \mathbf{k}'^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ &\quad + t_2^{(4)}(1 + x_2^{(4)} P_\sigma)(\mathbf{k}'\cdot\mathbf{k})(\mathbf{k}^2 + \mathbf{k}'^2) + \ldots \end{aligned}$$

# Density Matrix Expansion (DME)

[G. Carlsson and J. Dobaczewski; Phys. Rev. Lett. (2013) ]



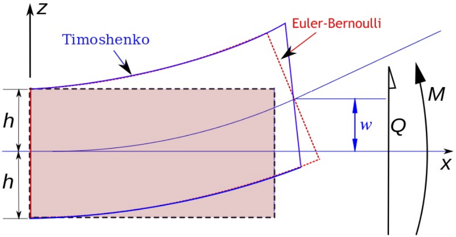

## Results

DME converges quite well at 4th order!!

# N2LO in finite nuclei

## A 4th order equation in spherical symmetry

$$A_4 R^{(4)}_{n\ell j} + A_3 R^{(3)}_{n\ell j} + A_{2R} R^{(2)}_{n\ell j} + A_{1R} R^{(1)}_{n\ell j} + A_{0R} R_{n\ell j} + \frac{\ell(\ell+1)}{r^2}\left[A_{2C} R^{(2)}_{n\ell j} + A_{1C} R^{(1)}_{n\ell j} + A_{0C} R_{n\ell j} + \frac{\ell(\ell+1)}{r^2} A_{0CC} R_{n\ell j}\right] = \epsilon_{n\ell j} R_{n\ell j}\,.$$




- We have derived an HFB code to solve N2LO equations (no tensor) Bessel basis and r-space
- Cross check with RPA code to avoid instabilities

# Summary and Conclusions

- We derive a new fitting protocol to determine N3LO Skyrme functional
  - Use of *meta-data* from *ab-initio* calculations
  - Use of RPA in IM to avoid instabilities
- N3LO and finite range interactions
- Interesting aspects on tensor properties

The next step...

- First applications to finite nuclei
- Generalisation of the fitting procedure to other functional forms

THANK YOU

# Example of Taylor expansion: M3Y-Pn

Same story with a tensor

$$v^t_{M3Y}(\mathbf{k},\mathbf{k}') = -\sum_n \left[t_n^{TNE}P_{TE} + t_n^{TNO}P_{TO}\right] \frac{32\pi}{\mu_n} \frac{1}{\left[\mu_n^2 + (\mathbf{k}-\mathbf{k}')^2\right]^3} \left[T_e(\mathbf{k},\mathbf{k}') - T_o(\mathbf{k},\mathbf{k}')\right]$$

where the tensor operators are defined as

$$\begin{aligned} T_e(\mathbf{k}',\mathbf{k}) &= 3(\sigma_1\cdot\mathbf{k}')(\sigma_2\cdot\mathbf{k}') + 3(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k}) - ({\mathbf{k}'}^2 + \mathbf{k}^2)(\sigma_1\sigma_2), \\ T_o(\mathbf{k}',\mathbf{k}) &= 3(\sigma_1\cdot\mathbf{k}')(\sigma_2\cdot\mathbf{k}) + 3(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k}') - 2(\mathbf{k}'\mathbf{k})(\sigma_1\sigma_2). \end{aligned}$$

now expanding in momentum space we obtain

$$\begin{aligned} v^t_{M3Y}(\mathbf{k},\mathbf{k}') \approx & -\sum_n \Big[(t_n^{TNE}+t_n^{TNO}) - (t_n^{TNE}-t_n^{TNO})P_\tau \\ + & (t_n^{TNE}+t_n^{TNO})P_\sigma - (t_n^{TNE}-t_n^{TNO})P_\sigma P_\tau\Big] \times \frac{8\pi}{\mu_n^7}\Big\{\Big(T_e(\mathbf{k},\mathbf{k}') - T_o(\mathbf{k},\mathbf{k}')\Big) \\ & -\frac{3}{\mu_n^2}\left[(\mathbf{k}^2+{\mathbf{k}'}^2)T_e(\mathbf{k},\mathbf{k}') + 2\mathbf{k}\cdot\mathbf{k}'T_o(\mathbf{k},\mathbf{k}')\right] + \frac{3}{\mu_n^2}\left[(\mathbf{k}^2+{\mathbf{k}'}^2)T_o(\mathbf{k},\mathbf{k}') + 2\mathbf{k}\cdot\mathbf{k}'T_e(\mathbf{k}, \right. \\ & +\frac{6}{\mu_n^4}\left[\left(\frac{1}{4}(\mathbf{k}^2+{\mathbf{k}'}^2)^2 + \frac{1}{8}(\mathbf{k}\cdot\mathbf{k}')^2\right)T_e(\mathbf{k},\mathbf{k}') + 2(\mathbf{k}\cdot\mathbf{k}')(\mathbf{k}^2+{\mathbf{k}'}^2)T_o(\mathbf{k},\mathbf{k}')\right] \\ & -\frac{6}{\mu_n^4}\left[\left(\frac{1}{4}(\mathbf{k}^2+{\mathbf{k}'}^2)^2 + \frac{1}{8}(\mathbf{k}\cdot\mathbf{k}')^2\right)T_o(\mathbf{k},\mathbf{k}') + 2(\mathbf{k}\cdot\mathbf{k}')(\mathbf{k}^2+{\mathbf{k}'}^2)T_e(\mathbf{k},\mathbf{k}')\right] + \dots\Big\} \end{aligned}$$

# Example of Taylor expansion: M3Y-Pn

We can make a Taylor expansion of a finite range interaction

$$v^c_{M3Y}(\mathbf{k},\mathbf{k}') = \sum_{n=1}^{3} \left[ t_n^{(SE)} P_{SE} + t_n^{(TE)} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO} \right] \frac{1}{\mu_n (\mathbf{k}-\mathbf{k}')^2 + \mu_n^3}$$

now applying exactly the idea of Skyrme we have

$$\begin{aligned} v^c_{M3Y}(\mathbf{k},\mathbf{k}') \quad &\approx \quad \sum_{n=1}^{3} \frac{C_n}{\mu_n^3} - \frac{C_n}{\mu_n^5} [\mathbf{k}'^2 + \mathbf{k}^2 + 2\mathbf{k}' \cdot \mathbf{k}] \\ &+ \quad \frac{C_n}{\mu_n^7} \left[ (\mathbf{k}^2 + \mathbf{k}'^2)^2 + 4(\mathbf{k}' \cdot \mathbf{k})^2 - 4(\mathbf{k}' \cdot \mathbf{k})(\mathbf{k}^2 + \mathbf{k}'^2) \right] \\ &- \quad \frac{C_n}{\mu_n^9} \left\{ (\mathbf{k}'^2 + \mathbf{k}^2) \left[ (\mathbf{k}'^2 + \mathbf{k}^2)^2 + 12(\mathbf{k}' \cdot \mathbf{k})^2 \right] - 2(\mathbf{k}' \cdot \mathbf{k}) \left[ 3(\mathbf{k}'^2 + \mathbf{k}^2)^2 + 4(\mathbf{k}' \cdot \mathbf{k})^2 \right] + \dots \right. \end{aligned}$$

where we have used

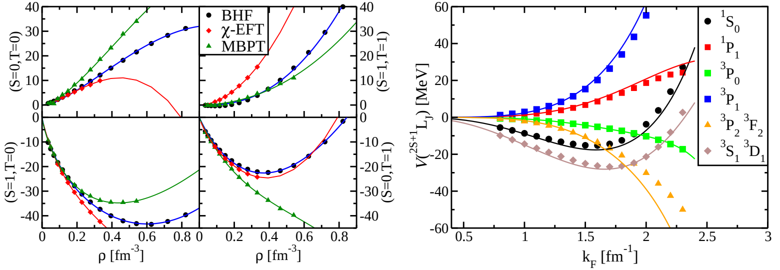
$$\begin{aligned} 4C_n \quad &= \quad (t_n^{(SE)} + t_n^{(SO)} + t_n^{(TE)} + t_n^{(TO)}) + (t_n^{(SE)} - t_n^{(SO)} - t_n^{(TE)} + t_n^{(TO)}) P_\tau \\ &+ \quad (-t_n^{(SE)} - t_n^{(SO)} + t_n^{(TE)} + t_n^{(TO)}) P_\sigma + (-t_n^{(SE)} + t_n^{(SO)} - t_n^{(TE)} + t_n^{(TO)}) P_\sigma P_\tau \end{aligned}$$

# Test N3LO in infinite matter

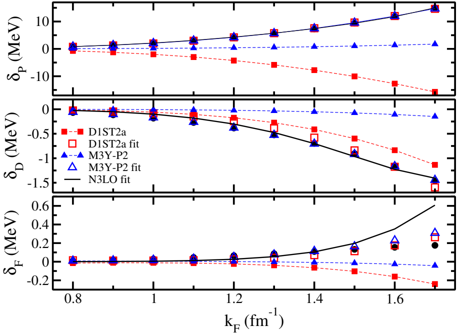
## Fitting protocol of LYVA1 (6th order gradient)

- EoS of PNM based on BBG results
- Spin (S)/ Isospin(T) decomposition of the EoS BBG

[D. Davesne, A. Pastore and J. Navarro, Astronomy and Astrophysics, 585, A83 (2015)]



# Are tensor terms compatible with $ab-initio$ methods?




Black circles are the BHF results while filled red squares and blue triangles have been calculated with interactions D1ST2a and M3Y-P2, respectively. Open symbols and solid line are the result of a fit of the tensor parameters, except for $\delta_F$.