Nuclear reaction path and inertial mass in the self-consistent collective coordinate method

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  - ASCC method
- Reaction path, potential, and inertial mass
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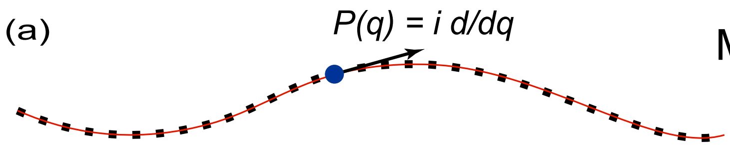
  - Symmetric reaction: $^{32}S \leftrightarrow {}^{16}O+{}^{16}O$
  - Asymmetric reaction: $^{20}Ne \leftrightarrow {}^{16}O+\alpha$

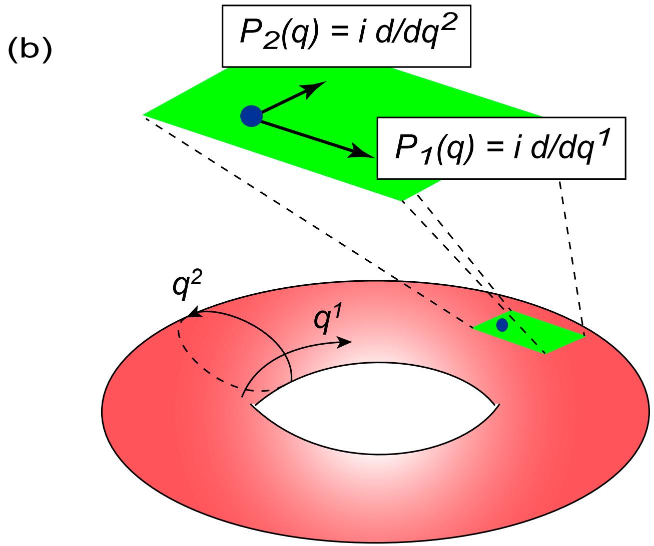
Microscopic determination of reaction path

- RGM
  - Assuming the “cluster configurations”
- GCM
  - Assuming the “generator coordinates”
- TDHF
  - An initial state produces a reaction path.
  - Not applicable to sub-barrier reaction.

Adiabatic Self-consistent Collective Coordinate (ASCC) method



Matsuo, Nakatsukasa, Matsuyanagi, PTP 103, 959 (2000)



Generators for canonical variables (q,p) are self-consistently constructed.

Collective submanifod spanned by (q,p) are determined.

ASCC method

Matsuo, Nakatsukasa, Matsuyanagi, PTP 103, 959 (2000)

"Moving mean-field equation"

(0th) $\delta\langle\Psi(q)|\hat{H}_M(q)|\Psi(q)\rangle = 0, \quad \hat{H}_M(q) \equiv \hat{H} - (\partial V/\partial q)\hat{Q}(q)$

(1st) $\delta\langle\Psi(q)|\left[\hat{H}_M(q), i\hat{Q}(q)\right] - B(q)\hat{P}(q)|\Psi(q)\rangle = 0$

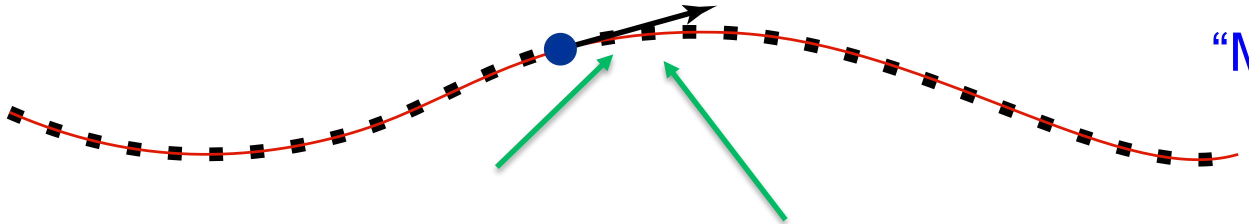
(2nd) $\delta\langle\Psi(q)|\left[\hat{H}_M(q), \hat{P}(q)/i\right] - C(q)\hat{Q}(q)$

$$- \frac{1}{2B(q)}\left[\left[\hat{H}_M(q), (\partial V/\partial q)\hat{Q}(q)\right], \hat{Q}(q)\right]|\Psi(q)\rangle = 0$$

"Moving RPA equation"

(a)

$P(q) = i\, d/dq$



Collective Hamiltonian

Identification of collective canonical variables; (q,p)

Determination of the optimal reaction path

Determination of collective mass

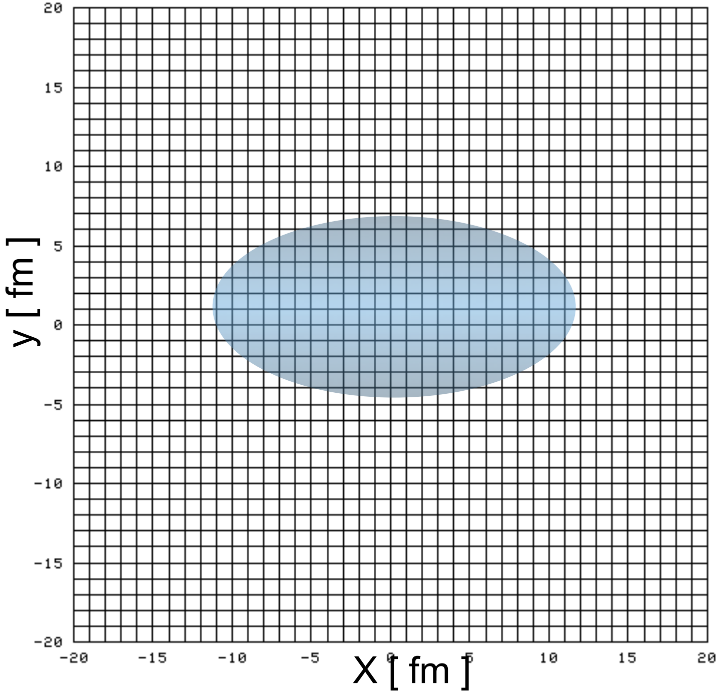
Construction of a collective Hamiltonian

$$H(q,p) = \langle \Psi(q,p) | \hat{H} | \Psi(q,p) \rangle \approx \frac{1}{2} B(q) p^2 + V(q)$$

$$V(q) = \langle \Psi(q) | \hat{H} | \Psi(q) \rangle, \quad B(q) = \langle \Psi(q) | \left[\left[\hat{H}, \hat{Q}(q) \right], \hat{Q}(q) \right] | \Psi(q) \rangle$$

Coordinate transformation; $(q,p) \rightarrow (R,P)$

3D real space representation



- 3D space discretized in lattice
- BKN functional
- Moving mean-field eq.: Imaginary-time method
- Moving RPA eq. : Finite amplitude method (PRC 76, 024318 (2007))

Wen, Nakatsukasa, arXiv: 1608.02294
Wen, Washiyama, Ni, Nakatsukasa, Acta Phys. Pol. B Proc. Suppl. 8, 637 (2015)

^{8}Be: Canonical generators

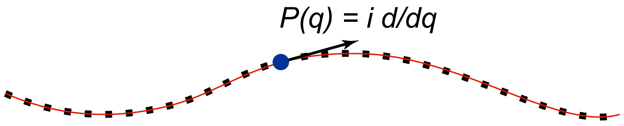
of the axial symmetry of the ground s
tion about the symmetry axis (z axis
pear. In Fig. 2 the calculation produces
modes of excitation around 2.8 MeV wi
tion matrix element of the $K = 1$ quadr
$\hat{Q}_{2\pm1} \equiv \int r^2 Y_{2\pm1}(\hat{r})\hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r})d\vec{r}$. The
these rotational modes comes from the
discretizing the space. Besides these fi
the lowest mode of excitation turns out t

$\rho(r)$

$\delta\rho(r)$

^{8}Be: Collective potential

- $V(q) = \Psi(q) H \Psi(q)$



⋮

distance $R \equiv \langle\psi(q)|R|\psi(q)\rangle$ with Eq.
shows the obtained potential energy a
collective path. As a reference, we also
Coulomb potential between two α parti
R, $4e^2/R + 2E_\alpha$, where E_α is the calculat
energy of the isolated α particle. Appar
totically approaches the pure Coulomb p
α's get closer, the potential starts to d
Coulomb potential at $R < 6$ fm and fin

^{8}Be: Collective inertial mass

$$k^2(R) = 2M(R) \left\{ E \right.$$

$$k_c^2(R) = 4m \left\{ E - \right.$$

where $k(R)$ and $k_c(R)$ a
radial motion with and w

Ground state

distance $R \equiv \langle\psi(q)|R|\psi(q)\rangle$ with Eq.
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totically approaches the pure Coulomb p
α's get closer, the potential starts to d
Coulomb potential at $R < 6$ fm and fin

Reduced mass

α + α scattering (phase shift)

Nuclear phase shift

$$k^2(R) = 2M(R)\left\{E - V(R) - \frac{(L}{4}\right.$$

$$k_c^2(R) = 4m\left\{E - \frac{4e^2}{R} - \frac{\left(L + \frac{1}{2}\right)}{4mR^2}\right.$$

where $k(R)$ and $k_c(R)$ are the wave
radial motion with and without the n

Dashed: Constant reduced mass

($M(R) \rightarrow 2m$)

^{8}Be: CHF+Cranking

larger than the ASCC inertia. The s
perturbative and perturbative cranking
are significantly different. For instance,
with $\hat{Q}_{20}$ constraint suggest prominent
in $M_{\rm cr}^{\rm NP(P)}(R)$. However, the peak po
different. It should be noted that the
should not be generalized to other energ
tionals, because the BKN interaction h
mean fields.

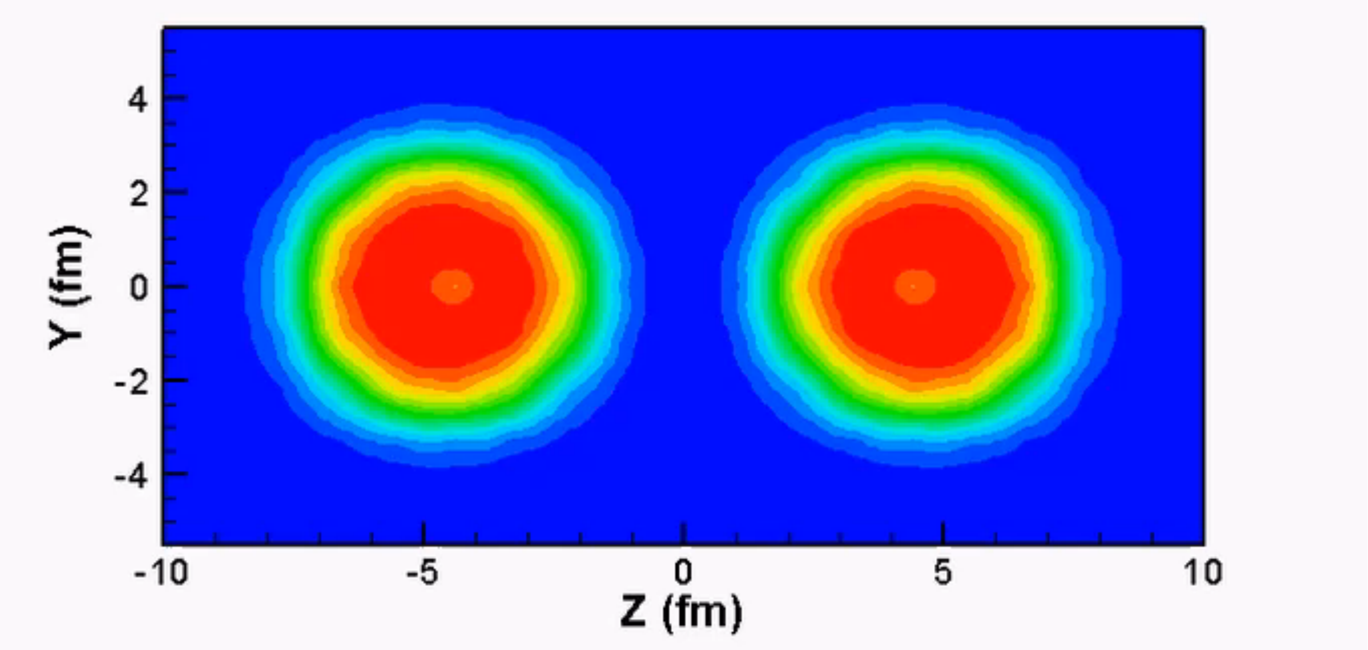
Perturbative cranking formula

Cranking formula for translational mass

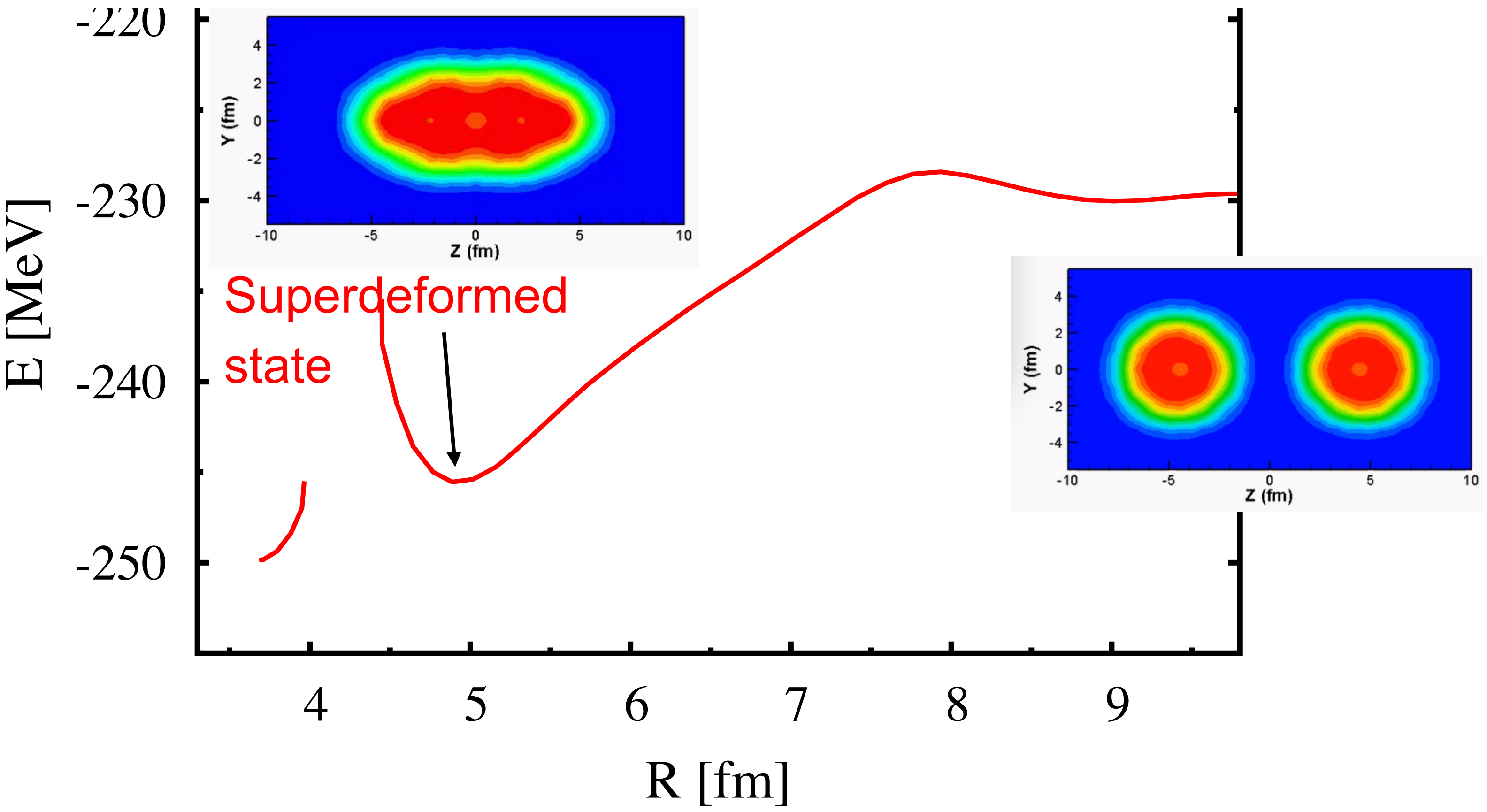
e.g.) $M_\alpha = 3.1\, m$ for $E_{BKN}[\rho] + B(\rho\tau - j^2)$ with $B = 50$ MeV fm^5

$^{16}O+^{16}O \rightarrow ^{32}S$: Reaction path

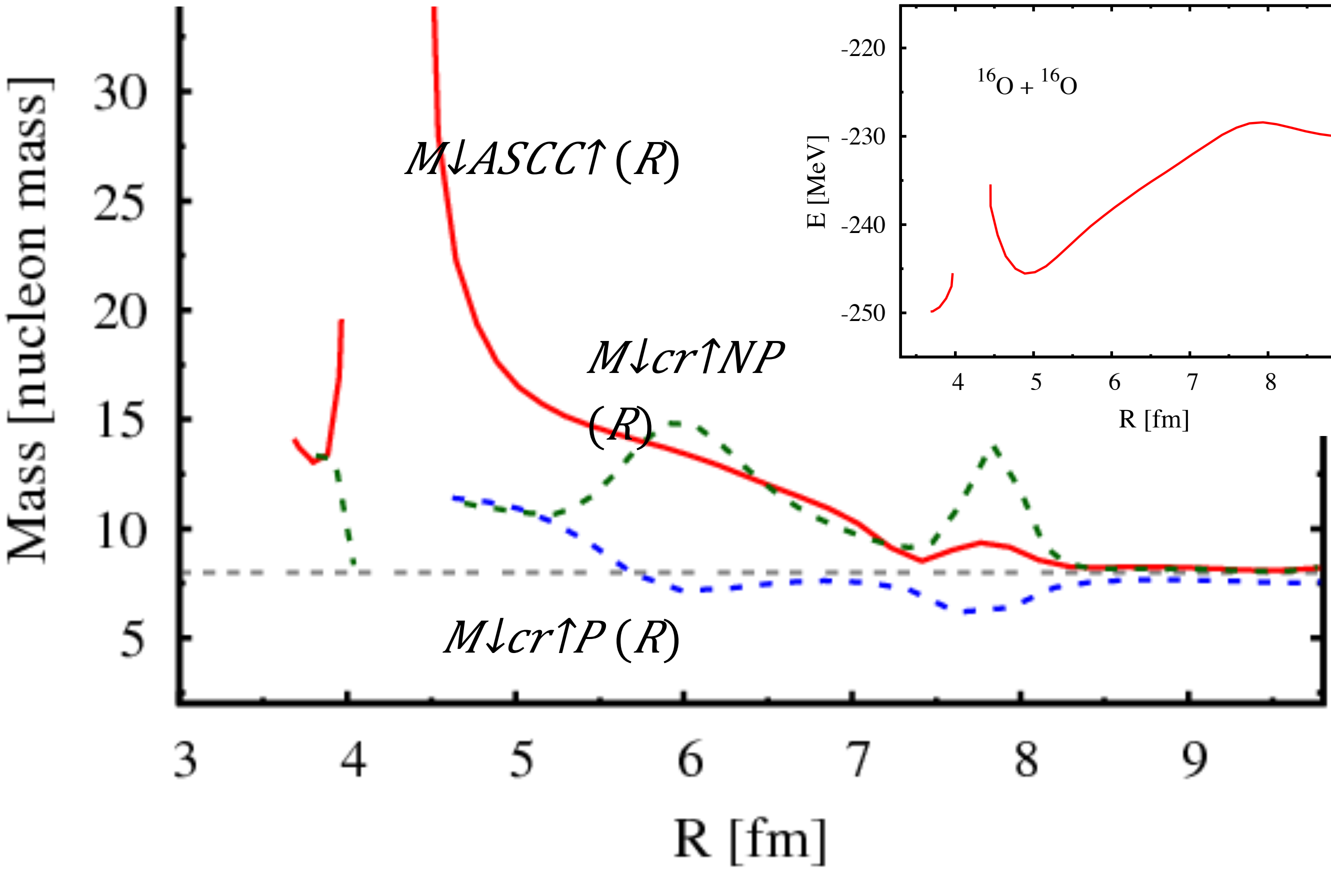
Starting from two ^{16}O configuration



$^{16}O+^{16}O \rightarrow ^{32}S$: Collective potential



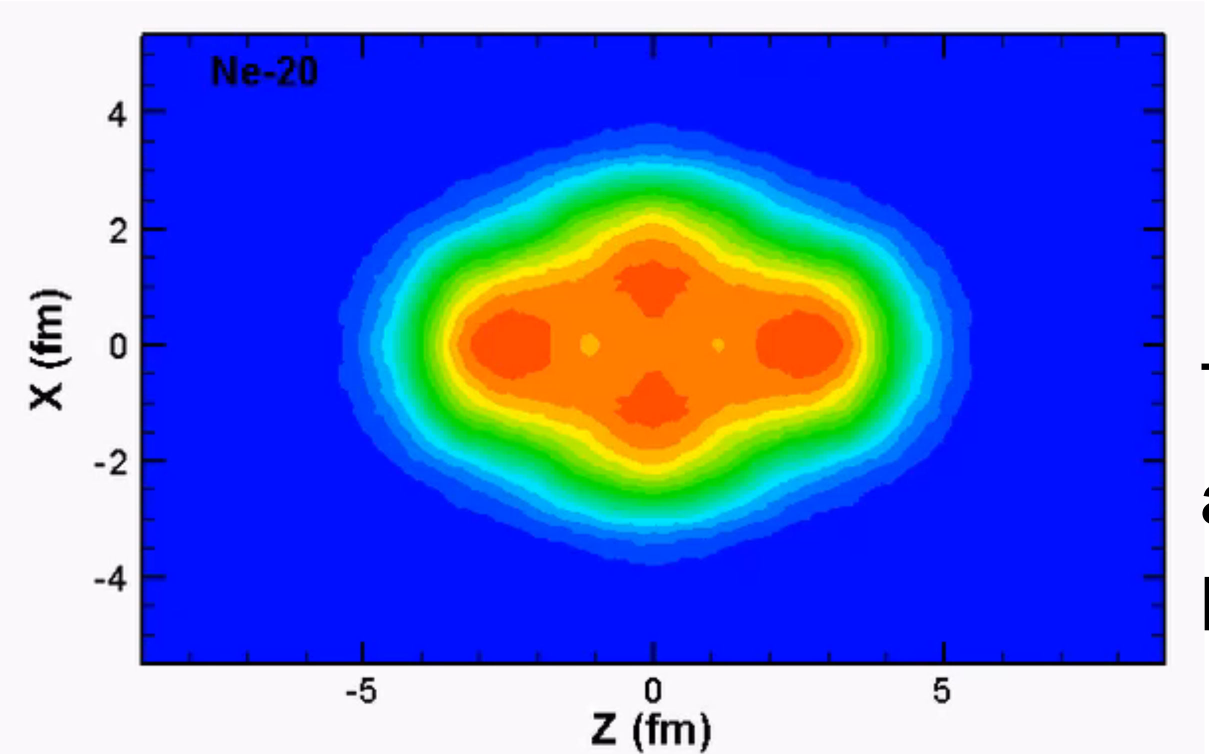
$^{16}O+^{16}O \rightarrow ^{32}S$: Collective mass



Reduced mass

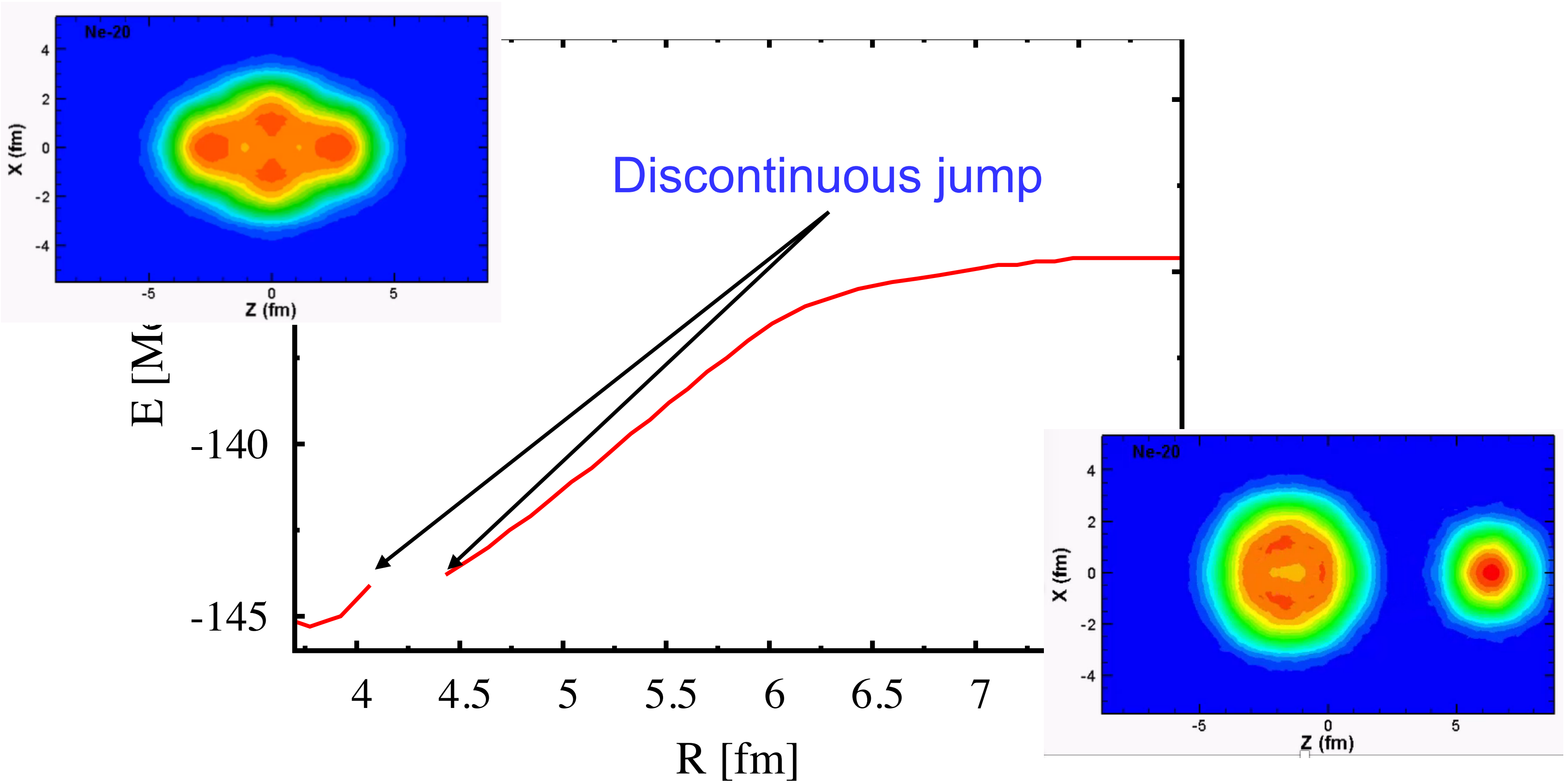
^{20}Ne: Reaction path

Starting from the ground state of ^{20}Ne

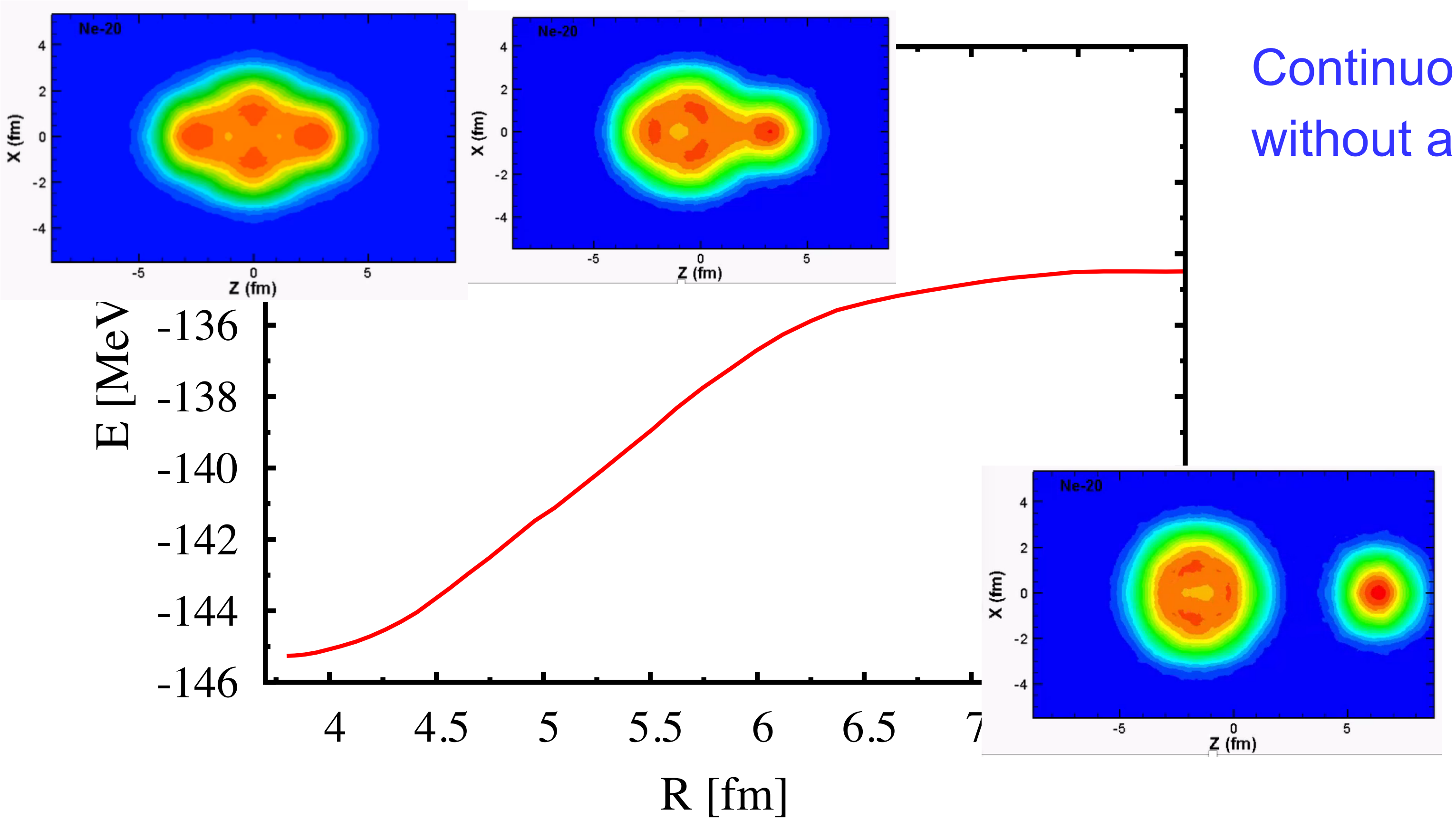


The final state automatically becomes ^{16}O + α

^{20}Ne: r^2Y_{20}-constrained cal.

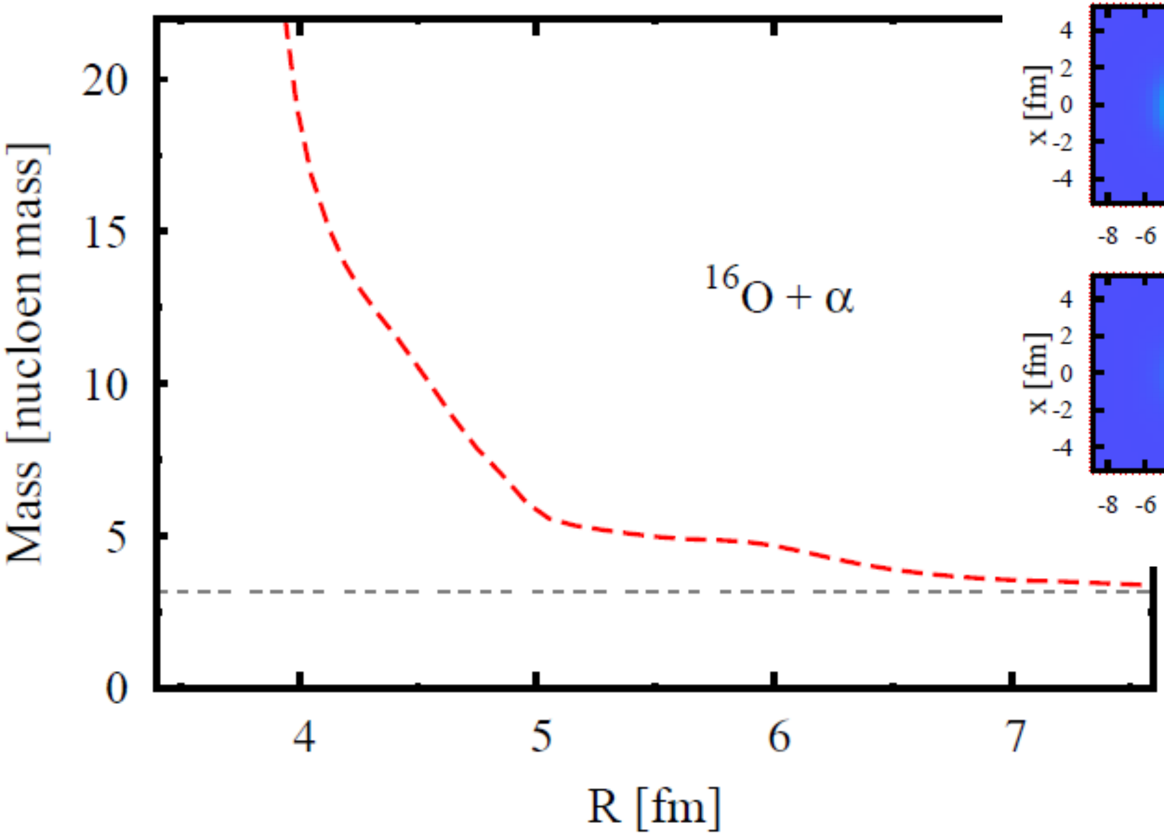


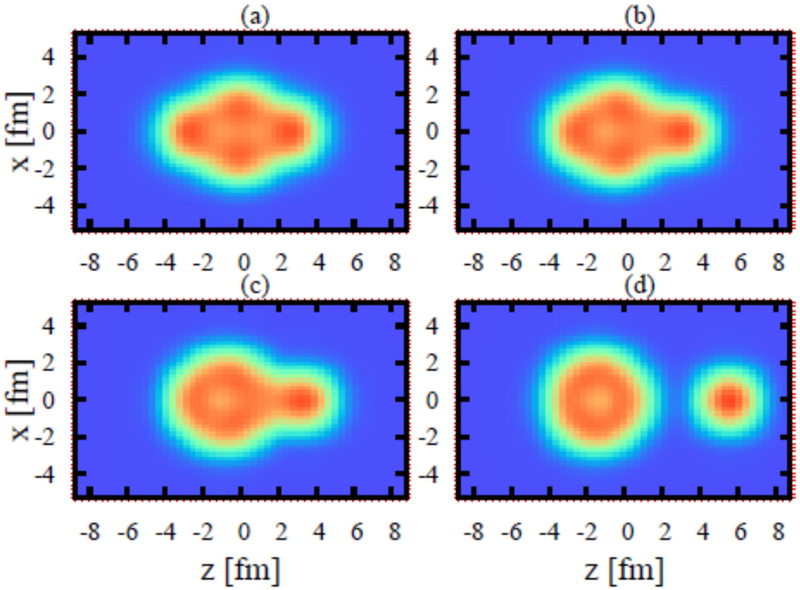
^{20}Ne: Collective potential



Continuous curve without a jump

^{20}Ne: Collective inertial mass



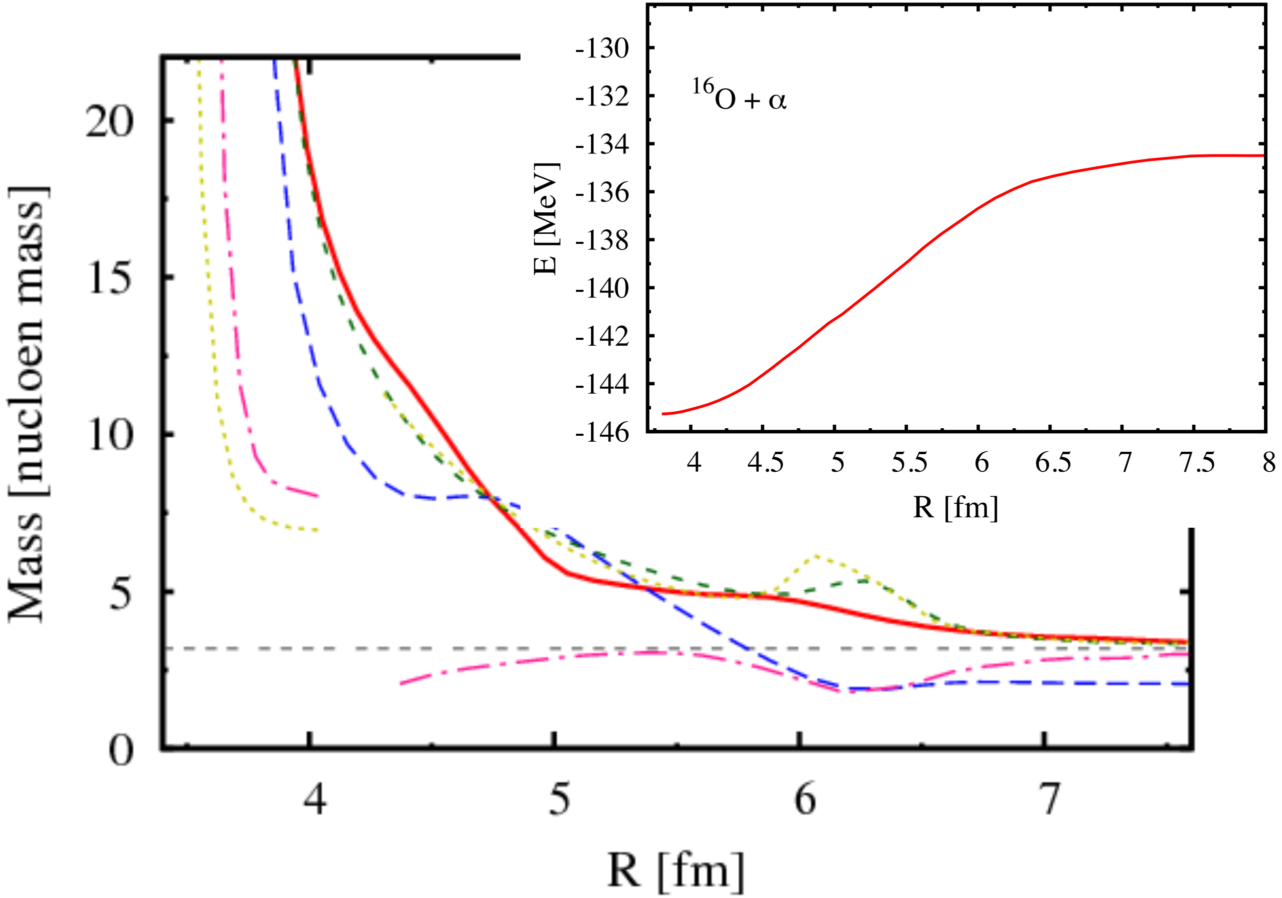


Reduced mass for ^{16}O + α

Summary

- Reaction path and inertial mass
  - Scattering/fusion/fission reaction
  - Determination of the reaction path
  - Inertial mass with proper account of time-odd effects (different from the cranking/GOA inertia)
- Applications
  - Symmetric mode: α+α ←→ ^{8}Be
    - Reaction path: Quadrupole vib. into rel. motion between two α's
  - Symmetric mode: ^{16}O + ^{16}O ←→ ^{32}S(SD)
    - Reaction path: Two ^{16}O's into the superdeformed ^{32}S
  - Asymmetric mode: ^{20}Ne ←→ ^{16}O+α
    - Reaction path: Octupole vib. into ^{16}O + α

^{20}Ne: Collective inertial mass



Constrained Hartree-Fock + cranking inertial mass

larger than the ASCC inertia. The se
perturbative and perturbative cranking
are significantly different. For instance,
with $\hat{Q}_{20}$ constraint suggest prominent
in $M_{\rm cr}^{\rm NP(P)}(R)$. However, the peak po
different. It should be noted that the
should not be generalized to other energ
tionals, because the BKN interaction h
mean fields.

ATDHF method

(Goeke, Gruemmer, Reinhard 1983)

inertial mass increases drastically. This
from the result of the former calculatio
reason of which is currently under investi
encounter a difficulty to obtain the collec
asymptotic region at large R. A larger r
finer mesh size seems to be needed to obta
in the asymptotic region and to reprodu
mass $2m$. We should also mention that t
with $dV/dR = 0$ is extremely difficult to

Translational motion of α particle

Dipole

Quadrupole

Monopole

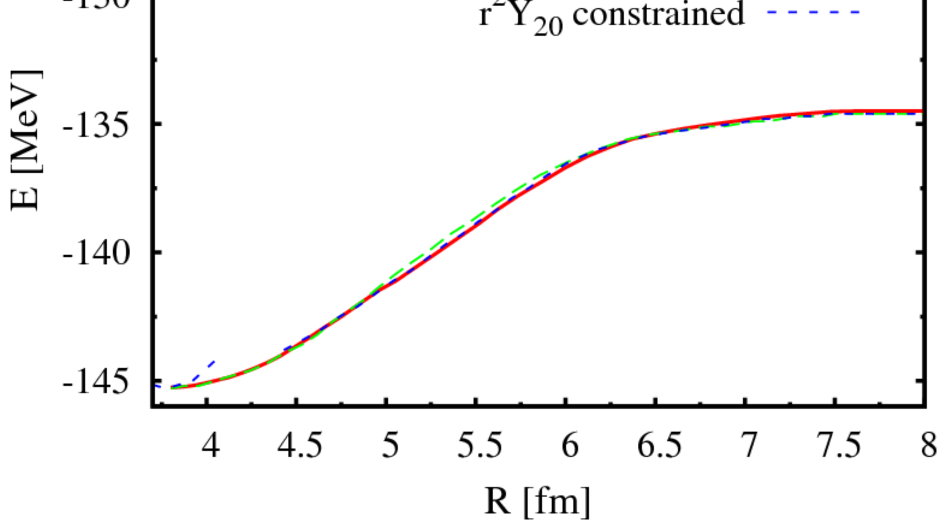
which is degenerated with the othe
modes along the x and y axis at abou
culation of Fig. (1) the space is discre
equal to 0.8 fm. With finer mesh size
translational motion approaches to (
pact nature of alpha particle, the n
is 20 MeV higher than these translat

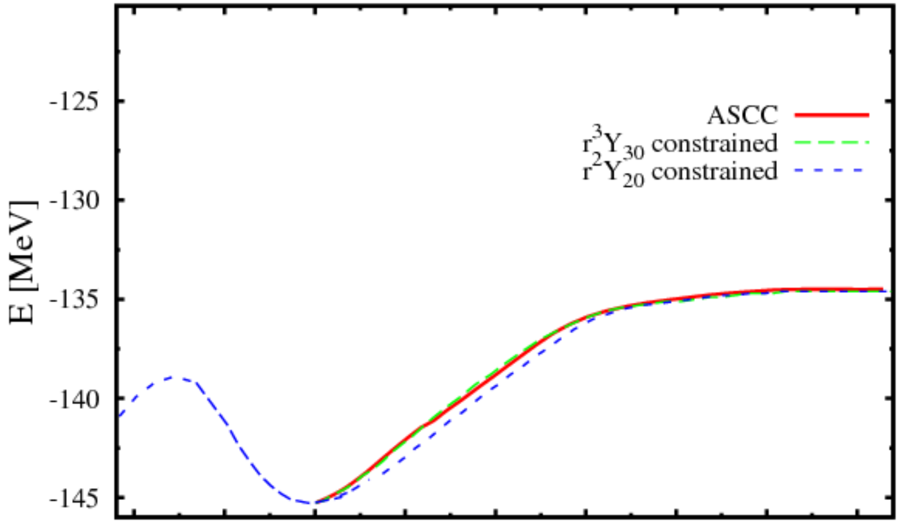
CoM

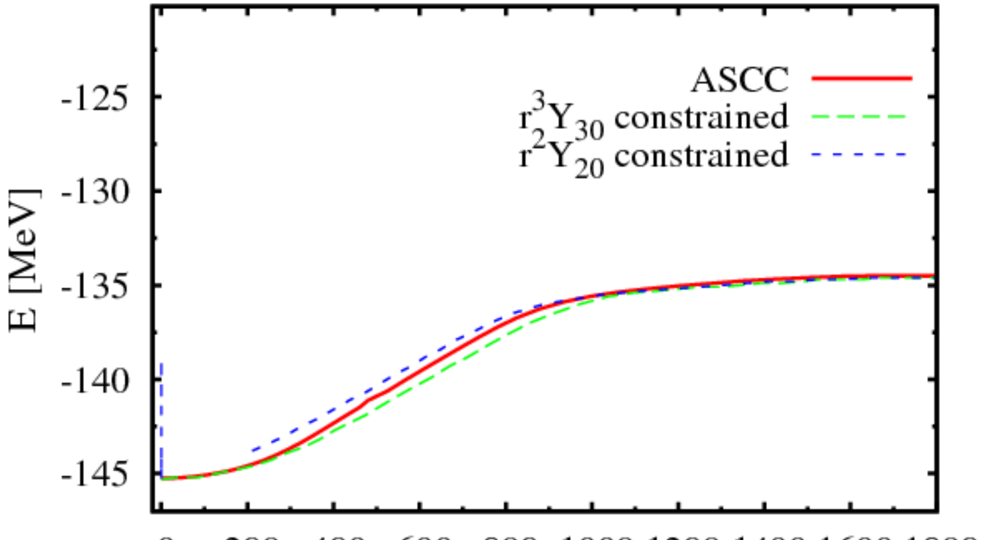
can be picked out with non-zero valu
which is degenerated with the othe
modes along the x and y axis at abou
culation of Fig. (1) the space is discre
equal to 0.8 fm. With finer mesh size
translational motion approaches to (
pact nature of alpha particle, the n
is 20 MeV higher than these translat

$^{20}Ne \leftarrow\rightarrow {}^{16}O + \alpha$

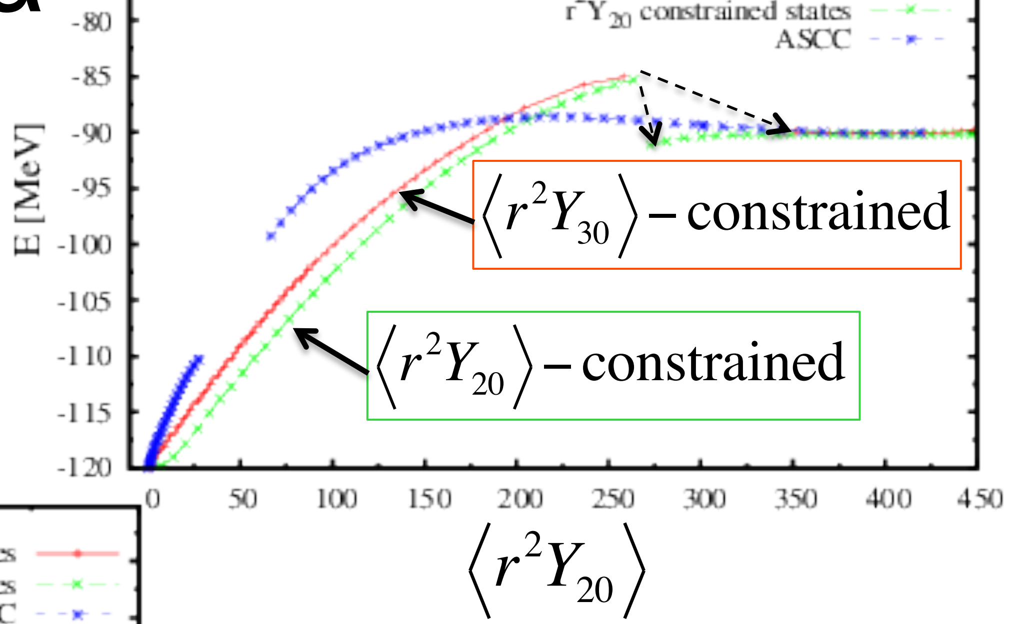
Preliminary result

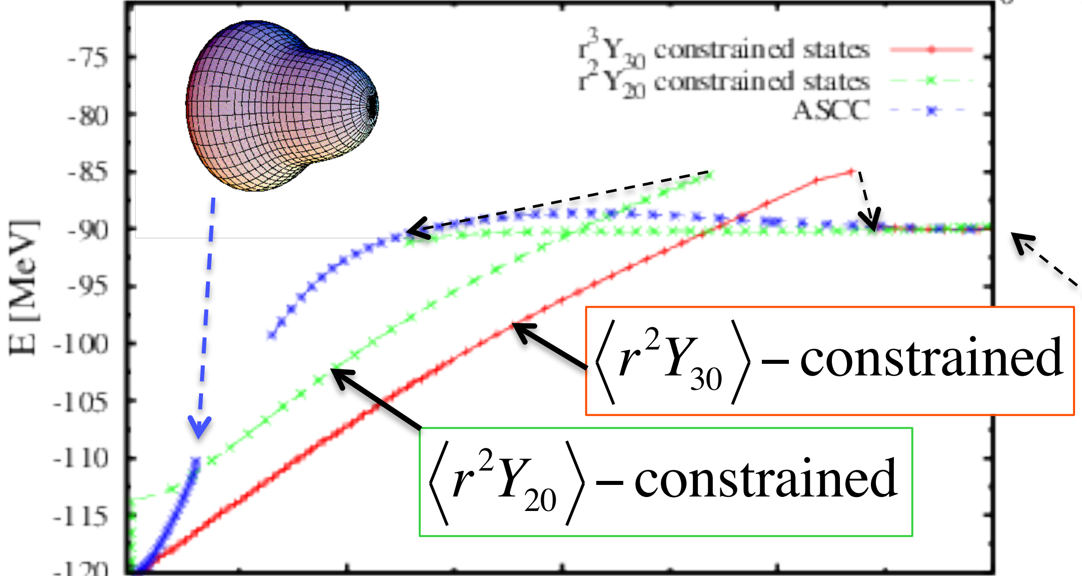


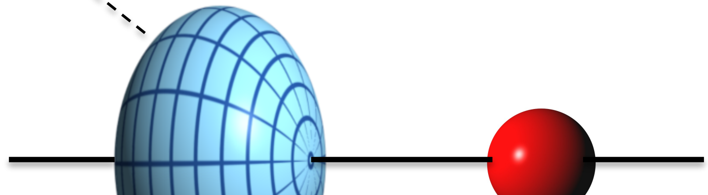




- Constraint operator
  - LHE generator
  - IS Quadrupole op.
  - IS Octupole op.
- Different path

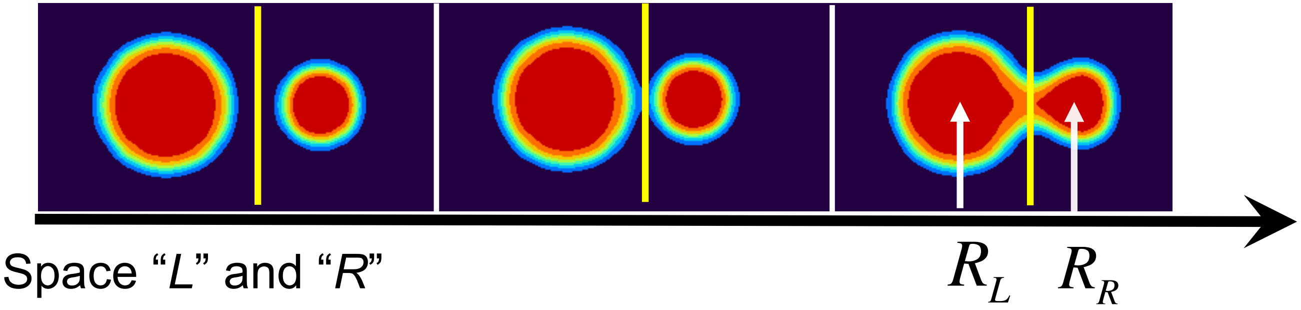






Choice of variables (*R*,*P*)

- *R* is defined by $R = |R_R - R_L|$



- *P* is calculated from current in *L* & *R*
- This definition is questionable after two nuclei touch each other.
- *Need reliable definition of canonical*

Inertial mass & coordinate transf.

Finding decoupled canonical variables

$$\left(\xi^\alpha,\pi_\alpha\right)\rightarrow(q,p;\bar{q}^i,\bar{p}_i)$$

$$H\left(\xi,\pi\right)=\frac{1}{2}B^{\alpha\beta}(\xi)\pi_\alpha\pi_\beta+V(\xi)$$

$$\approx\frac{1}{2}B(q)p^2+V(q)+H(\bar{q},\bar{p})$$

(map from q to R)

$$\Rightarrow\frac{1}{2}B(R)P_R^2+V(q(R))$$

$$B(R)=\left(\frac{\partial R}{\partial q}\right)^2 B(q)$$

Assuming the collective variables

$$R=R\left(\xi\right)$$

R is chosen by hand

$$H\left(\xi,\pi\right)=\frac{1}{2}B^{\alpha\beta}(\xi)\pi_\alpha\pi_\beta+V(\xi)$$

$$\Rightarrow\frac{1}{2}\tilde{B}(R)P_R^2+V(R)$$

$$\tilde{B}(R)=\frac{\partial R}{\partial\xi^\alpha}\frac{\partial R}{\partial\xi^\beta}B^{\alpha\beta}(\xi)$$

$$=B(R)+\frac{\partial R}{\partial\bar{q}^i}\frac{\partial R}{\partial\bar{q}^j}B^{ij}(\bar{q})$$

$$B(R)\neq\tilde{B}(R)$$

Reaction path