Transport coefficients of hot and dense quark matter

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September 15, 2016

INPC 2016, Adelaide

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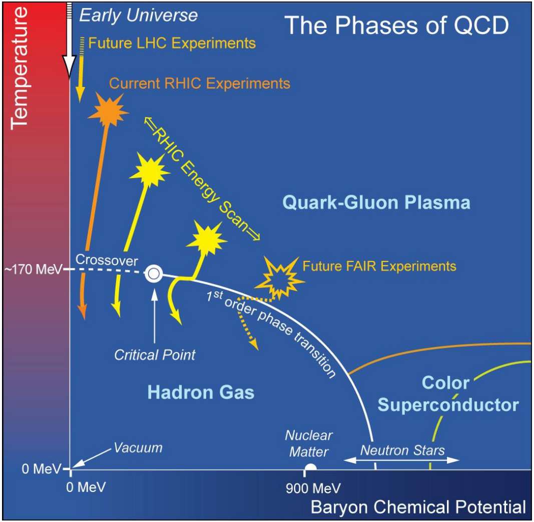
Outline

- Introduction
- Boltzmann equation and transport coefficients
- Nambu-JonaLasinio model, Thermodynamics and relaxation time estimation
- Results
- Summary and Outlook

Introduction

- Transport properties of hot/dense matter are important for heavy ion collision (HIC), cosmology and important for near equillibrium evolution of any thermodynamic system
- The most studied transport coefficient is perhaps shear viscosity η. In HIC spatial anisotrpy of colliding nuclei gets converted to momentum anisotropy throgh a hydro evoln. The equllibriation is decided by η. ($\frac{\eta}{s} \sim \frac{1}{4\pi}$, the KSS bound)
- The bulk viscosity ζ - thought earlier to be not important for HIC hydro evolution. Argument: $\zeta \sim (\epsilon - 3p)/T^4$ that vanishes for ideal gas. However, lattice simulation $\Rightarrow$ large $(\epsilon - 3p)/T^4$ near T_c. This,in turn, can give rise to different physical effects (Cavitation).
- The temperature and chemical potential dependence of transport coefficients may reveal the location of phase transition
- Most calculations are performed at zero baryon density ρ_B. Including finite density effects are relevent for upcoming HIC experiments, BES(Brookhaven), CBM at (GSI, Darmstadt), (NICA at Dubna).

QCD phase diagram and HIC



Boltzmann Equation

Boltzmann equation describes the evolution of particle distribution function

$$\frac{df_a}{dt} = \frac{\partial f^a}{\partial t} + \frac{p^i}{E_a}\frac{\partial f_a}{\partial x^i} - \frac{\partial E_a}{\partial x^i}\frac{\partial f_a}{\partial p^i} = C^a$$

The equllibrium distribution function

$$f_a^0 = \frac{1}{\exp\beta(u_\alpha p^\alpha \mp \mu) + 1}$$

To estimate viscosity coefficients, consider small departure from equllibrium

$$\frac{df_a}{dt} = \frac{p^\mu}{E_a}\frac{\partial f_a^0}{\partial x^\mu} - \frac{M}{E_a}\frac{\partial M}{\partial x^i}\frac{\partial f_a^0}{\partial p^i} = -\frac{\delta f_a}{\tau^a}$$

$$\partial_\mu f_0^a = -f_0^a(1 \mp f_0^a)\partial_\mu \left(\beta(E^a - \mu - \mathbf{p}\cdot\mathbf{u}\right)$$

Boltzmann Eq. relates non equllibrium part of distribution function to variation in fluid velocity and temperature and chemical potential

$T^{\mu\nu}$, J_μ and transport coefficients

Distribution function is related to the energy momentum tensor

$$T^{\mu\nu} = \sum_a \int d\Gamma_a p^\mu p^\nu f_a + g^{\mu\nu} U(\sigma); \quad d\Gamma_a = \nu_a \frac{d\mathbf{p}}{(2\pi)^3}$$

$$J^\mu = \sum_a t_a \int d\Gamma_a \frac{p^\mu}{E_a} f_a$$

Change in nonequllibrium part $\Rightarrow$

$$\delta T^{ij} = \sum_a \int d\Gamma^a \frac{p^i p^j}{T E_a} \tau_a f_a (1 - f_a) q_a(p, \beta, \mu)$$

$$\delta J^i = \sum_a t_a \int d\Gamma_a \frac{p^i}{E_a} \tau_a f_a (1 - f_a) \left(t_a - \frac{n E_a}{\epsilon + p} \right) p^j \partial_j \left(\frac{\mu}{T} \right)$$

ζ, η,λ contd.$\cdots$

The non equllibrium contribution related to the velocity gradients can be reorganised as

$$q^a = Q^a \partial_i u_i - \frac{p^i p^j}{2E_a} W_{ij}$$

;

$$W_{ij} = \partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k$$

Shear and bulk viscosities are defined through the dissipative part

$$\Delta T^{ij} = -\zeta \delta^{ij} \partial_k u_k - \eta W_{ij}$$

Thermal conductivity is defined through the dissipative part of the current

$$\Delta J_i = \lambda \left(\frac{nT}{w}\right)^2 \partial_i \left(\frac{\mu}{T}\right)$$

ζ, η,λ contd.$\cdots$

$$\eta = \frac{1}{15T}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a}\left(\tau_a f_a^0(1-f_a^0) + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\right)$$

$$\zeta = -\frac{1}{3T}\sum_a \int d\Gamma_a \frac{\mathbf{p}_a^2}{E_a}\left(\tau_a f_a^0(1-f_a^0)Q_a + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\bar{Q}_a\right)$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a t_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} f_a(1-f_a)\tau_a \left(t^a - \frac{nE_a}{w}\right)$$

In the bulk viscosity coefficient, the coefficient Q^a depends upon the equation of state

$$Q_a = -\left[\frac{\mathbf{p}_a^2}{3E_a} + \left(\frac{\partial P}{\partial n}\right)_\epsilon \left(\frac{\partial E}{\partial \mu} - 1\right) - \left(\frac{\partial P}{\partial \epsilon}\right)_n \left(E_a - T\frac{\partial E_a}{\partial T} - \mu \frac{\partial E_a}{\partial \mu}\right).\right]$$

ζ contd.

However, Q_a has to be supplemented by the conditions $u_\mu \delta J^\mu = 0$ and $u_\mu \delta T^{\mu\nu} u_\nu = 0$ corresponding to baryon number and energy momentum conservation. Within the relaxation time approximation, these Landau-Lifshitz conditions reduce to

$$\sum_a t_a \langle \tau_a Q_a \rangle = 0, \quad \sum_a \langle \tau_a E_a Q_a \rangle = 0$$

$$\langle \phi_a(p) \rangle = \int d\Gamma_a [\phi_a(p) f_a^0 (1 - f_a^0)]$$

If Landau Lifshitz conditions are not satisfied, replace

$$\tau_a Q_a \to \tau_a Q_a + \alpha t_a + \beta E_a$$

The unknown coefficients to be determined from the baryon number and energy momentum conservation equation. The expression for bulk viscosity consistent with the Landau Lifshitz condition is then given as

$$\zeta = -\frac{1}{T} \sum_a \langle (\tau_a Q_a + \alpha t_a + \beta E_a) \frac{\mathbf{p}^2}{3E_a} \rangle$$

η,ζ,λ contd.

The expressions for the transport coefficients become simpler when one realises that for ideal hydrodynamics the entropy per baryon (σ) is constant.

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1-f_a^0)$$

$$\zeta = \frac{1}{9T}\sum_a \int d\Gamma_a \frac{\tau_a f_a^0(1-f_a^0)}{E_a^2}\left[\mathbf{p}^2 + 3v_n^2 T^2 E_a \frac{\partial}{\partial T}\left(\frac{E_a-\mu_a}{T}\right)_\sigma\right]^2$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2}\tau_a f_a(1-f_a)\left(t_a - \frac{nE_a}{w}\right)^2$$

- Transport coefficients are nonnegative as they must be.
- It is important to include the Landa-Liftshitz conditions to obtain the above results.

Knowing the equation of state and other thermodynamic quantities like velocity of sound etc. and the relaxation time one can estimate the viscosity coefficient.

This thermodynamics and estimation of relaxation time is done within the Nambu Jonalasinio model.

Nambu JonaLasinio model : Thermodynamics

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi - G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 \mathbf{t}^a \psi)^2\right)$$

The thermodynamic potential (negative of pressure):

$$\Omega(\beta,\mu) = -\frac{\gamma}{(2\pi)^3}\int E(\mathbf{k})d\mathbf{k} - \frac{\gamma}{(2\pi)^3\beta}\int d\mathbf{k}\left(\ln(1+\exp(-\beta(E-\mu)) + \mu \to -\mu\right) + \frac{(M-m_0)^2}{4G}$$

$\gamma = 2N_c N_f$ (degeneracy); $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + M^2}$,$M$: Constituent quark mass get determined self consistently solving the mass gap equation

$$M = m_0 - 2G\langle\bar{\psi}\psi\rangle\rho_s = m_0 + \frac{\gamma}{(2\pi)^3}\int \frac{M}{E(\mathbf{k})}\left(1 - f_-(\mathbf{k},\beta,\mu) - f_+(\mathbf{k},\beta,\mu)\right)d\mathbf{k}$$

MASSES ; NJL MODEL CONTD.···

Within RPA Meson propagators:

$$D = \frac{2iG}{1 - 2G\Pi_{\sigma/\pi}}$$

Mass of the meson determined by pole position of the Real part of meson propagator:

$$1 - 2GRe\Pi_M(m_M, \mathbf{0}) = 0,$$

For $m_M < 2M$, Π_M is real while for $m_M > 2M$, Π_M has imaginary part: Decay width of resonance $\Gamma_M = Im\Pi_M(m_M, 0)$
This affects the quark scatterings through meson exchange and hence on the relaxation time.

MASSES ···

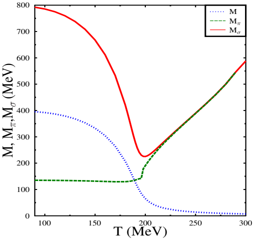


Fig. 1-a

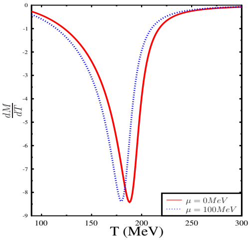


Fig. 1-b

Figure: Temperature dependence of the masses of constituent quark (M), and pions (M_π) and sigma mesons (M_σ) for $\mu = 0$ (Fig1-a) and temperature derivative of the constituent quark mass for $\mu = 0$MeV and $\mu = 100$MeV (Fig. 1-b).

$$T_\chi = 188\text{MeV};\ m_\pi(T_{Mott}) = 2M(T_{Mott}); \quad T_{Mott} \simeq 197\text{MeV}.$$

Estimating the average relaxation time

Avg. relaxation time

$$\tau_a^{-1} = \sum_b n_b \bar{W}_{ab}$$

Thermally averaged transition rate

$$\bar{W}_{a,b} = \frac{1}{n_a n_b} \int f_a f_b W_{ab} d\pi_a d\pi_b$$

$$W_{ab}(s) = \frac{2\sqrt{s(s-4m^2)}}{1+\delta_{ab}} \int_{t_{min}}^{0} dt(\frac{d\sigma}{dt})(1 - f_c(\frac{\sqrt{s}}{2}, \mu))(1 - f_d(\frac{\sqrt{s}}{2}, \mu))$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s} \frac{1}{p_{ab}} |M|^2$$

Estimating the average relaxation time

- For two flavors we consider the following possible scatterings.

$$u\bar{u} \to u\bar{u}, \quad u\bar{d} \to u\bar{d}, \quad u\bar{u} \to d\bar{d},$$

$$uu \to uu, \quad ud \to ud, \quad \bar{u}\bar{u} \to \bar{u}\bar{u},$$

$$\bar{u}\bar{d} \to \bar{u}\bar{d}, \quad d\bar{d} \to d\bar{d}, \quad d\bar{d} \to u\bar{u},$$

$$d\bar{u} \to d\bar{u}, \quad dd \to dd, \quad \bar{d}\bar{d} \to \bar{d}\bar{d},$$

- Using i-spin symmetry, charge conjugation symmetry as well as the crossing symmetry to relate the matrix element square for the above 12 processes reduce to evaluating only two independent matrix elements $u\bar{u} \to u\bar{u}$ and $u\bar{d} \to u\bar{d}$
- Dominant contribution comes from propagation of pion and sigma mode in the s-channel.
- The temperature dependence of π and σ modes play an important role in these cross section evaluation.

Zhuang etal Phys Rev D51,3728, 1995

Relaxation time; η/s: T behavior

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1-f_a^0)$$

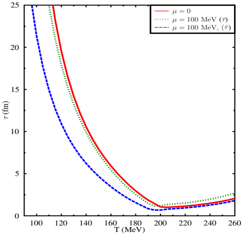


Fig. 2-a

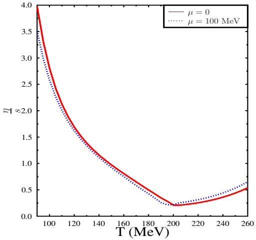


Fig. 2-b

Figure: Relaxation time as a function of temperature for $\mu = 0$ MeV and for $\mu = 100$MeV (Fig 5-a). In Fig (5-b), shear viscosity to entropy density ratio is shown for $\mu = 0$ MeV and $\mu = 100$ MeV.

η/s: Contd.

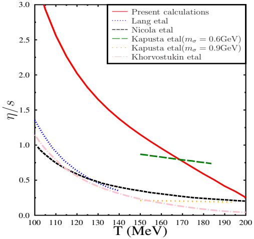


Fig. 3-a

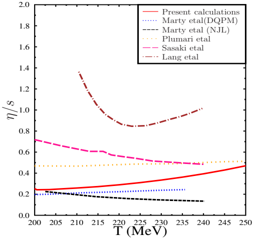


Fig. 3-b

Figure: Shear viscosity to entropy ratio for $\mu = 0$ for low temperatures (Fig 3-a). The present calculations is shown by the solid lines. The other results correspond to Lang etal (2012) of interacting pion gas, Fernandez-Fraile and Nicola (2009). The two curves by Kapusta etal (2011) correspond to different masses for the sigma mesons. The green dashed curve is for $m_\sigma = 600$ MeV while the orange dotted curve is with $m_\sigma = 0.9$ GeV. The pink dot dashed curve is for the SHMC model by Khvorostukhin etal(2010). In Fig. 3-b is shown the ratio for higher temperatures. Present calculations is shown by solid red line, the two curves of Marty etal(2013)' , correspond to dynamical quasi particle model (DQPM) and the 3 flavor NJL model, the orange dotted curve by Plumari etal(2012) , the pink dashed curve by Sasaki etal(2010) is for two flavor NJL model while the brown dot dashed curve is from Lang etal (2014).

Bulk viscosity: T behavior

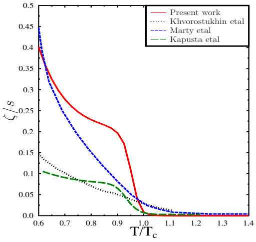


Fig. 4-a

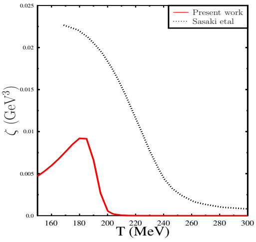


Fig. 4-b

Figure: Bulk viscosity to entropy ratio as a function of temperature in units of T_c for zero baryon density (4-a). Also shown results from different models, the SHMC model of Khvosthukin etal (2010), Kapustaand Chakraborty (2009), the three flavor NJL results of Marty etal. Bulk viscosity in units of GeV^3 as a function of temperature is shown in Fig. 4-b. Solid red curve corresponds to the present calculations while dotted curve correspond to the results by Sasaki and Redlich (2010)

Bulk viscosity: T behavior

For zero chemical potential

$$\zeta = \frac{1}{9T}\sum_a d\Gamma^a \frac{\tau_a}{E_a^2}\left[\mathbf{p}^2(1-3v^2) - 3v^2(M^2 - TM\frac{dM}{dT})\right]^2$$

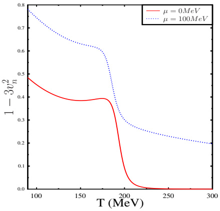


Fig. 5-a

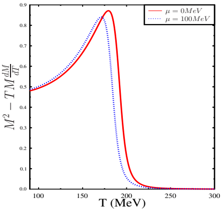


Fig. 5-b

Figure: The violation of conformality measure $C_1 = 1 - 3v_n^2$ (Fig 5a) and $C_2 = M^2 - TM\frac{dM}{dT}$ (Fig 5b) as a function of temperature for $\mu = 0$ MeV and for $\mu = 100$ MeV

$$v_n^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_n = \frac{s\chi_{\mu\mu} - n\chi_{\mu T}}{T(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)}$$

$\lambda(T)$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} \tau_a f_a(1 - f_a) \left(t_a - \frac{nE_a}{w}\right)^2$$

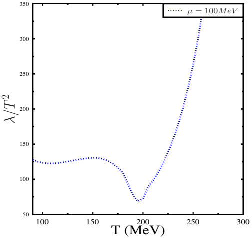


Figure: Thermal conductivity(λ) in units of T^2 for $\mu = 100$ MeV.

Summary,conclusions and outlook

- We tried to derive the viscosity coefficients using Boltzmann kinetic equation withing relaxation time approximation within NJL model.
- While η depends only on the behaviour of relaxation time and the medium dependent masses, ζ depends on other thermodynamic quantities and the equation of state.
- The deviation from equllibrium should be consistent the Landau Lifshitz conditions.
- The thermodynamics of hot and dense matter is estimated within NJL model.
- The transport coefficients are non negative in the relaxation time approximation which is a consequence of Landau-Liftshitz conditions of fit.
- Relaxation times are estimated using quark scatterring through meson exchange.
- Medium dependence of meson masses and widths affect the relaxation time and hence the transport coefficients.

Thank you