Lattice QCD and transport coefficients

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Cluster of Excellence

Institute for Nuclear Physics

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Plan

Strongly interacting matter at temperatures $T = 100 - 500\,\text{MeV}$

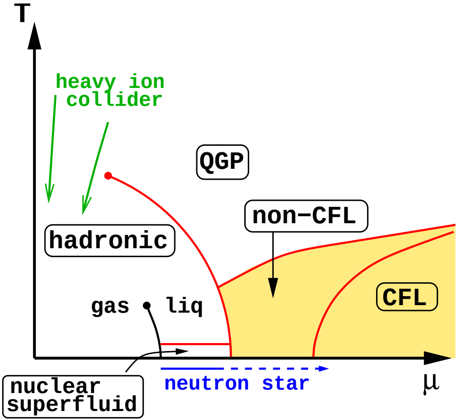
- probed in heavy-ion collisions: hadrons $\to$ quark-gluon plasma
- state of matter for the first microsecond after Big Bang

Thermal physics: $\beta = 1/(kT)$,

$$\langle A \rangle = \frac{1}{Z} \mathrm{Tr}\,\{e^{-\beta H} A\}, \qquad Z = \mathrm{Tr}\,\{e^{-\beta H}\}$$

- **Overview of equilibrium properties from lattice QCD**
- **Near-equilibrium (real-time) properties**
  - formalism
  - vector channel for light quarks: dilepton rate, diffusion coefficient
  - heavy-quark momentum diffusion coefficient
  - pion quasiparticle in the hadronic phase

QCD phase diagram



- at $\mu_B = 0$: $T_{\text{transition}} = 155 \pm 8\,\text{MeV}$ from lattice simulations (crossover)
- e.g. from chiral susceptibility $\int d^4x \langle\bar{\psi}(x)\psi(x)\ \bar{\psi}(0)\psi(0)\rangle$
  [see e.g. review Soltz et al. 1502.02296].

Fig. from Braun-Munzinger, Koch, Schäfer, Stachel, Phys.Rept. 621 (2016) 76

Thermodynamic potentials

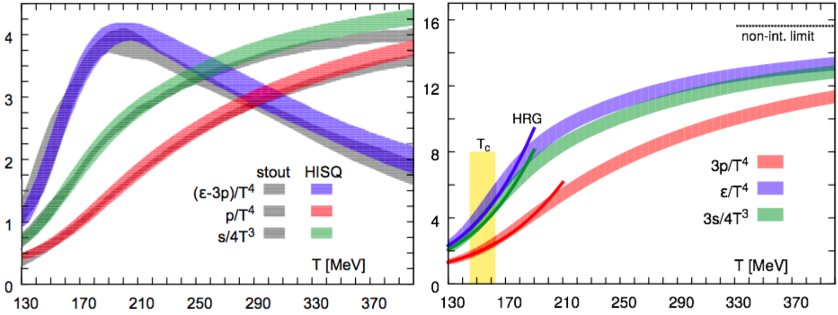
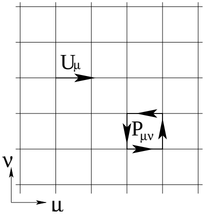


Fig. from review by Soltz et al. 1502.02296

- at $T = 260$MeV, $p \approx 1/2 p_{\rm SB}$: far from weakly interacting quarks and gluons;
- hadron resonance gas (HRG) model works well up to $T = 160\,$MeV;
- HRG also describes well the fluctuations of conserved charges, e.g. $\frac{1}{V}\times\,\langle Q^2\rangle$, $\langle B^2\rangle$ and $\langle S^2\rangle$.

Regularization of QCD on a lattice



Gluons: $U_\mu(x) = e^{iag_0 A_\mu(x)} \in SU(3)$
'link variables'

Quarks: $\psi(x)$ 'on site', Grassmann

Gauge-invariance exactly preserved; no gauge-fixing required.

Imaginary-time path-integral representation of QFT (Matsubara formalism):

Formally: | Lattice QCD = 4d statistical mechanics system |

Starting point for **Monte-Carlo simulations** using importance sampling

Near-equilibrium properties

Typical questions:

- What quasiparticles are there in the system?
- How fast does an external perturbation dissipate in the system?
  for long wavelength perturbations, the rate is parametrized
  by transport coefficients (shear/bulk viscosity, diffusion coefficients, ...)
  $\langle J_0(t,\boldsymbol{k})\rangle_{\theta(-t)\mu(\boldsymbol{k})} \overset{t \text{ large}}{\propto} e^{-Dk^2 t}$
- What is the production rate of photons or dileptons?

Formalism

Relation between the $\underbrace{\text{correlator}}_{\text{computed on the lattice}}$ and the $\underbrace{\text{spectral function}}_{\text{what we want to know}}$:

$$G(x_0,\mathbf{p}) \equiv \int d^3x\; e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle J(x)^\dagger J(0)\right\rangle \stackrel{\star}{=} \int_0^\infty \frac{d\omega}{2\pi}\,\rho(\omega,\mathbf{p})\,\frac{\cosh[\omega(\beta/2-x_0)]}{\sinh[\omega\beta/2]}.$$

- in the low-T phase, $J_i^{\rm em}$ can excite e.g. an ω-meson-like quasiparticle.
- for $J = J_i^{\rm em}$ electromagnetic current, $\rho(\omega,\mathbf{0}) \stackrel{\omega\to 0}{\sim} 6\chi_s D\omega$

  $\chi_s = \int d^4x \langle J_0^{\rm em}(x) J_0^{\rm em}(0)\rangle$ = static susceptibility of electric charge

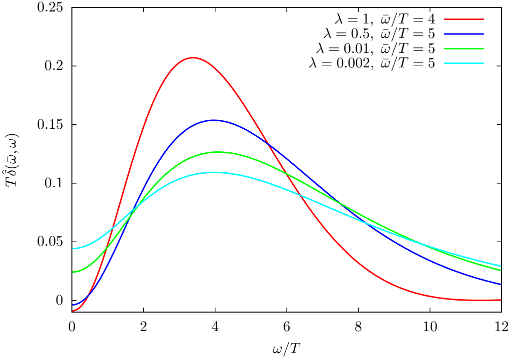
  D= diffusion coefficient
- photon rate: $\frac{d\Gamma}{d^3k} = \frac{e^2\sum_f Q_f^2}{2(2\pi)^3\,k}\,\frac{\rho(k,\boldsymbol{k})}{e^{\beta k}-1}$

$\star$ numerically ill-posed inverse problem for $\rho(\omega,\boldsymbol{p}) \stackrel{\omega>0}{\geq} 0$; $\quad 0 \le x_0 < \beta$.

The inverse problem has many faces: here is one of them

Linearity: $$\sum_{i=1}^{n} c_i(\bar{\omega})\, G(t_i) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho(\omega)\, \underbrace{\sum_{i=1}^{n} c_i(\bar{\omega})\, \frac{\cosh[\omega(\beta/2 - t_i)]}{\sinh[\omega\beta/2]}}_{\widehat{\delta}(\bar{\omega},\omega)}$$

- choose the coefficients $c_i(\bar{\omega})$ so that the 'resolution function' $\widehat{\delta}(\bar{\omega},\omega)$ is as narrowly peaked around a given frequency $\bar{\omega}$ as possible
  (idea behind the Backus-Gilbert method, [used in Robaina et al. PRD 92 (2015) 094510.])

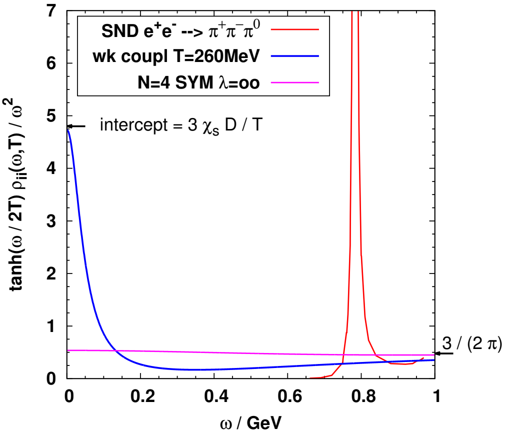


Resolution function at $\bar{\omega} = 4T$ for $N_t = 24$, $t_i/a = 5, \ldots 12$.

- Resolution only improves slowly with increasing n
- Large, sign-alternating coefficients $\Rightarrow$ need for ultra-precise input data.

Expected thermal changes in spectral functions

Isoscalar vector channel: spectral fct. of $J_i = \frac{1}{\sqrt{2}}(\bar{u}\gamma_i\bar{u} + \bar{d}\gamma_i d)$



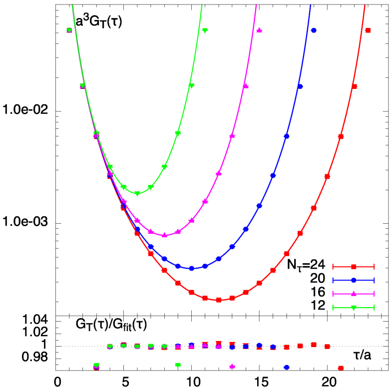
- presence of weakly coupled quasiparticles ⇒ transport peak at $\omega = 0$; is it really there at $T \approx 260$MeV ?

SND hep-ex/0305049

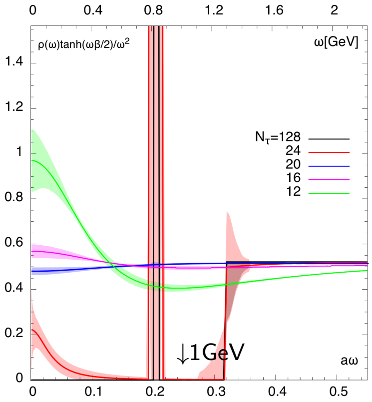
D = diffusion coefficient; χ_s = static susceptibility.

The isovector vector channel at $p = 0$

Lattice QCD correlators
$(T = 0.8, 1.0, 1.25, 1.67 \times T_c)$



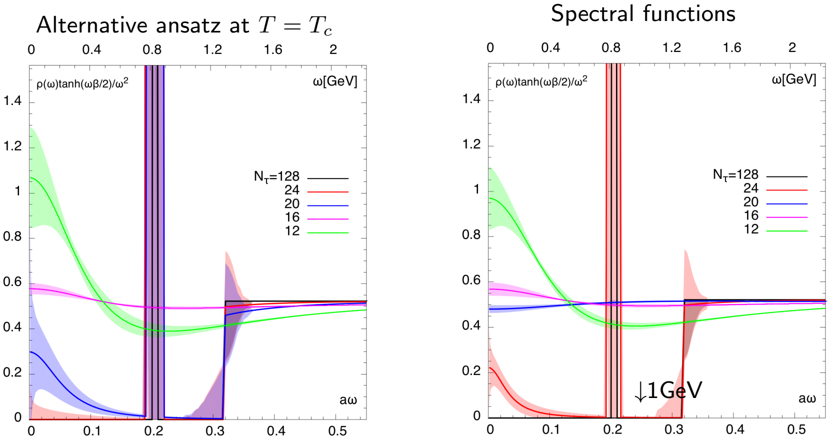
Spectral functions



- fit ansatz method: simultaneous fit to all temperatures, because $\rho(\omega) \overset{\omega\to\infty}{\sim} (\# + \# \frac{\alpha_s}{\pi} + \dots)\omega^2$ with T-independent coefficients;
- exact sum rule: $\int_0^\infty \frac{d\omega}{\omega}[\rho(\omega, T) - \rho(\omega, T')] = 0 \rightsquigarrow$ constraint;

Francis et al. ($N_f = 2$, $m_\pi|_{T=0} = 267$MeV, $T_c = 203$MeV), PRD93 (2016) 054510

Model-dependence of the spectral function



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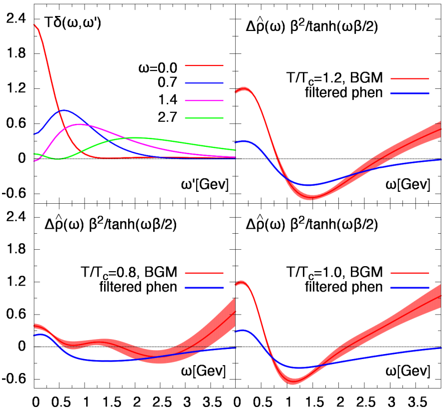
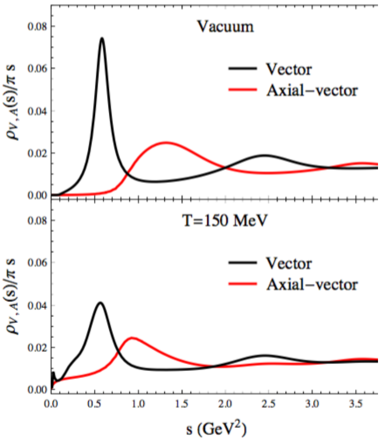
Lattice QCD vs. phenomenology

$\widehat{\Delta\rho}(\omega, T)$: lattice vs. pheno



Phenomenological model.

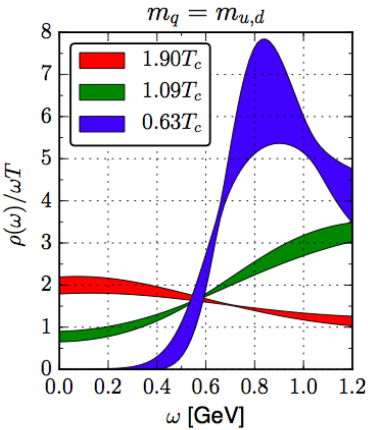


- compare $\frac{\widehat{\Delta\rho}(\omega,T)}{\tanh(\omega\beta/2)} \equiv \int_0^\infty d\omega'\ \delta(\omega,\omega')\frac{\Delta\rho(\omega',T)}{\tanh(\omega'\beta/2)}$: lattice result is model-independent.

- shift of spectral weight from the ρ to low frequency region as T increases.

Francis et al. PRD93 (2016) 054510; Rapp & Hohler, PLB731, 103 (2014).

Calculations of the diffusion coefficient

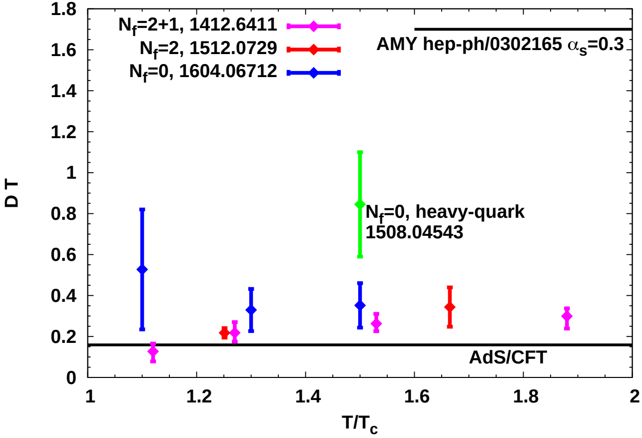


- inverse problem treated with the Maximum Entropy Method;
- $D \propto \rho(\omega)/(\chi_s\omega)|_{\omega=0}$ comes out very small;
- stability of the results tested under variations in the procedure.

$N_f = 2 + 1$ simulations, $m_\pi|_{T=0} = 384$MeV, Aarts et al. JHEP 1502 (2015) 186.

See also $N_f = 0$ continuum calculation using fit ansätze Ding, Kaczmarek, F. Meyer PRD94 (2016) 034504

Selected recent results for the light-quark diffusion coefficient D

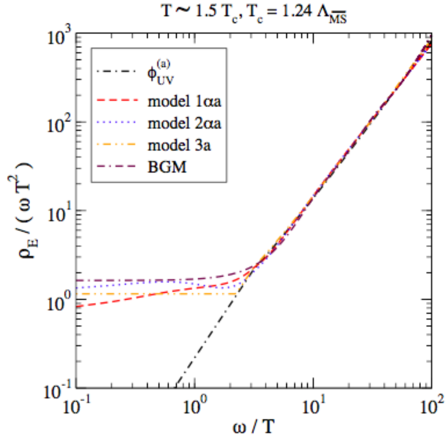


- lattice calculations yield very low values, $D \approx 1/(\pi T)$;
- however, all results assume that no narrow transport peak is present: these methods would fail at very high temperatures.
- Except green point!

$T > T_c$: Heavy-quark momentum diffusion coefficient κ

$$G(\tau) = \frac{\left\langle \mathrm{Re\,Tr}\left(U(\beta,\tau)gE_k(\tau,\mathbf{0})U(t,0)gE_k(0,\mathbf{0})\right)\right\rangle}{-3\,\langle \mathrm{Re\,Tr}\,U(\beta,0)\rangle} = \int_0^\infty \frac{d\omega}{2\pi}\,\rho(\omega)\,\frac{\cosh[\omega(\beta/2-\tau)]}{\sinh[\omega\beta/2]}$$

- color parallel transporters $U(t_2,t_1)$ are propagators of static quarks
- (Lorentz) force-force correlator on the worldline of the quark.



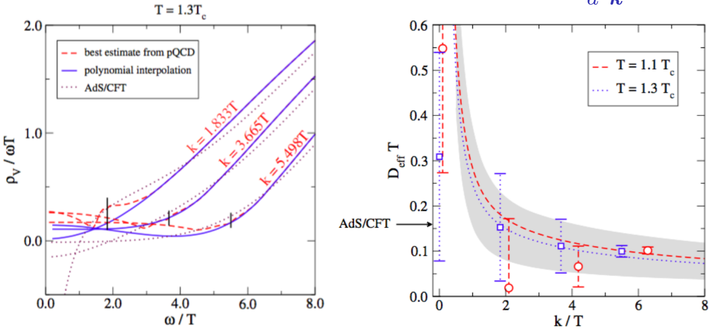
$$\kappa = \lim_{\omega\to 0} \frac{T}{\omega}\rho(\omega), \qquad D = 2T^2/\kappa.$$

NNLO calculation available:
$\rho(\omega) =$ smooth function $\overset{\omega\to\infty}{\sim} g^2\omega^3$.

Result: $2\pi T D = 3.7\ldots 6.9$

Francis, Kaczmarek, Laine, Neuhaus
PRD92 (2015) 116003

Spectral function on the light-cone $\rightsquigarrow$ photon rate $\frac{d\Gamma_\gamma}{d^3\boldsymbol{k}}$



$$D_{\rm eff}(k) = \begin{cases} \frac{\rho_V(k,\boldsymbol{k})}{2\chi_q k} & k > 0 \\ \lim_{\omega\to 0^+} \frac{\rho^{ii}(\omega,0)}{3\chi_q \omega} & k = 0 \end{cases} \quad ; \qquad \frac{d\Gamma_\gamma}{d^3\boldsymbol{k}} = \frac{2\alpha_{\rm em}\chi_q}{3\pi^2}\frac{D_{\rm eff}(k)}{e^{\beta k}-1}.$$

- at $k = 0$, a narrow transport peak cannot be excluded $\Rightarrow$ large uncertainty on result for D.
- for $k \approx 2T$, a more reliable result for $D_{\rm eff}(k)$ is possible: spectral function expected to be smooth;
- fit ansatz: polynom up to $\omega = \sqrt{k^2 + \pi^2 T^2}$, perturbation theory beyond.

$N_f = 0$, analysis in the continuum; Ghiglieri, Kaczmarek, Laine, F. Meyer PRD 94, 016005 (2016)

The pion quasiparticle in the low-temperature phase

- Chiral symmetry is spontaneously broken for $T < T_c$: $-\langle\bar{\psi}\psi\rangle > 0$.
- Goldstone theorem $\Rightarrow$ a divergent spatial correlation length m_π^{-1} exists in the limit $m_{u,d} \to 0$.
- also: a massless real-time excitation exists: the pion quasiparticle.
- dispersion relation: [Son and Stephanov, PRD 66, 076011 (2002)]

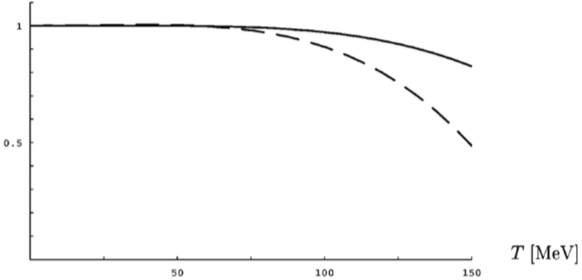
$$\omega_{\boldsymbol{p}} = u(T)\sqrt{m_\pi^2(T) + \boldsymbol{p}^2} + \dots$$

- $T \lesssim 100$MeV: Two-loop chiral perturbation theory prediction for the pion quasiparticle mass $u(T)m_\pi(T)$ [D. Toublan, PRD 56 5629 (1997)]

Quasiparticle mass $\frac{M_\pi^2(T)}{M_\pi^2}$

full: physical quark mass

dashed: massless case.



- key point: pion dominates parametrically the Euclidean two-point function of the axial charge density ($\int d^3x\, e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\, \bar{\psi}\gamma_0\gamma_5\frac{\tau^a}{2}\psi$) and its second derivative at $x_0 = \beta/2 \approx 0.6$fm and $|\boldsymbol{p}| \lesssim 300$MeV
- inverse problem can be solved via the ansatz

$$\rho_A(\omega, \boldsymbol{p}, T) = f_\pi^2(T)\,(m_\pi^2(T) + \boldsymbol{p}^2)\,\delta(\omega^2 - u^2(T)(m_\pi^2(T) + \boldsymbol{p}^2))$$

- here $m_\pi(T)$ and $f_\pi(T)$ are determined from screening (=static) correlation functions; from time-dependent correlator: $u = 0.75(2)$ and

$$T = 0: \qquad \text{pion mass} = 267(2)\,\text{MeV}$$
$$\swarrow \qquad \searrow$$
$$T = 169\text{MeV}: \qquad \text{quasiparticle mass} = 223(4)\text{MeV} \qquad \text{screening mass} = 303(4)\text{MeV}.$$

- Simulation details: $N_f = 2$ (no strange quark); 24×64^3 lattice;
- Transition temperature $T_c \simeq 203$MeV.

▲ How does this fit in with the success of the hadron-resonance gas model?

Robaina et al. PRD 90 (2014) 054509; PRD 92 (2015) 094510.

Conclusion

Significant progress in lattice QCD on near-equilibrium quantities:

- few-permille precision on correlation functions at small lattice spacings, even continuum in 'quenched' approximation
- advanced weak-coupling calculations, effective field theories, exact sum rules, ... provide crucial prior information on spectral function.
- many channels not discussed here: fate of quarkonium in the quark-gluon plasma, open-charm spectral functions, shear/sound channels, ...

Backup slides

Thermal fluctuations and correlations

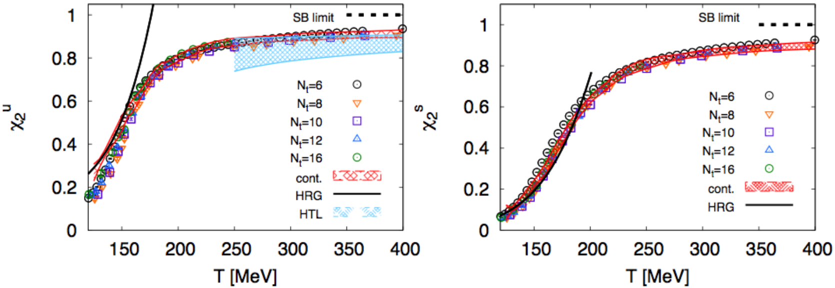
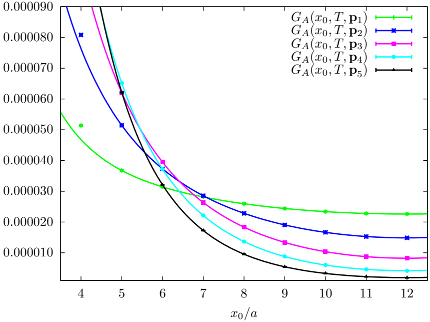
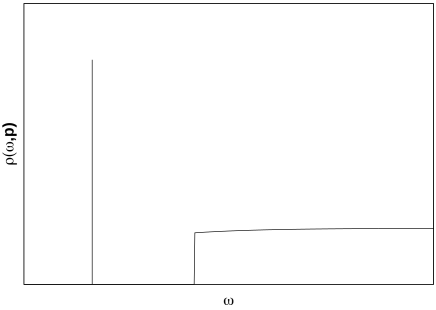


Fig. from S. Borsanyi et al. 1112.4416

- Light-quark number susceptibility: suggests that deconfinement occurs practically at the same temperature as chiral restoration.
- Successful predictions of the HRG.

Pion channel, continued: description of the lattice data





$$\frac{1}{3}\int d^3x\; e^{i\mathbf{p}\cdot\mathbf{x}}\,\langle A_0^a(x)A_0^a(0)\rangle = \int_0^\infty \frac{d\omega}{2\pi}\;\rho^A(\omega,\mathbf{p})\frac{\cosh[\omega(\beta/2-x_0)]}{\sinh[\omega\beta/2]}.$$

$$\text{Ansatz}:\ \rho^A(\omega,\mathbf{p}) = a_1(\mathbf{p})\delta(\omega-\omega_{\mathbf{p}}) + a_2(\mathbf{p})(1-e^{-\omega\beta})\theta(\omega-c).$$