Leptonic CP Violation and Mass Hierarchy in the Presence of a Light Sterile Neutrino

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Overview

- Neutrino oscillations are now well documented, culminating with the Nobel Prize for Arthur B. McDonald and Takaaki Kajita in 2015.
- Old problems, like the solar neutrino problem, are largely considered solved.
- There are now new areas of neutrino oscillation physics to be explored.

Motivation

Current experiments:

- Precision measurements of θ_{12}, θ_{13}, Δm^2_{21} and $|\Delta m^2_{31}|$.
- Mass Hierarchy-Charge-Parity (MH-CP) i.e. $\mathrm{sgn}(\Delta m^2_{31})$-$\delta_{13}$ degeneracy.
- Octant degeneracy ($\theta_{23} \approx 40^\circ$ or 50°).
- SBL anomaly.
- Sterile neutrinos.

Extending to Four Flavours

- 3+1 (three active, plus one sterile) model is introduced to account for the SBL anomaly.
- Adds one mass eigenstate, hence one new flavour which is assumed to be sterile.
- The new squared-mass splitting is of order $1\mathrm{eV}^2$, which is relatively large.
- This causes oscillations with high spatial frequency, which can account for the short range excesses.

PMNS Matrix (3ν)

- The usual three flavour parametrisation is: $U^{3\nu}_{\mathrm{PMNS}} = \Theta_{23}\tilde{\Theta}_{13}\Theta_{12}$.
- Where Θ_{ij} and $\tilde{\Theta}_{ij}$ are the $n \times n$ rotation matrices defined by the embedded 2×2 matrices:

$$\Theta_{ij} = \begin{pmatrix} \mathrm{c}_{ij} & \mathrm{s}_{ij} \\ -\mathrm{s}_{ij} & \mathrm{c}_{ij} \end{pmatrix},$$
$$\tilde{\Theta}_{ij} = \begin{pmatrix} \mathrm{c}_{ij} & \mathrm{s}_{ij}\mathrm{e}^{-i\delta_{ij}} \\ -\mathrm{s}_{ij}\mathrm{e}^{i\delta_{ij}} & \mathrm{c}_{ij} \end{pmatrix}.$$

- n is the number of neutrino flavours, $\mathrm{s}_{ij} = \sin\theta_{ij}$ and $\mathrm{c}_{ij} = \cos\theta_{ij}$.

3+1 Formalism

- We now need new parameters, these are: θ_{14}, θ_{24}, θ_{34}, δ_{14}, δ_{34} and Δm^2_{41}.
- A common and convenient parametrisation is: $U^{4\nu}_{\mathrm{PMNS}} = \tilde{\Theta}_{34}\Theta_{24}\tilde{\Theta}_{14}\Theta_{23}\tilde{\Theta}_{13}\Theta_{12}$, where the last three matrices make up the 3×3 PMNS matrix.
- This parametrisation is convenient because $P^{4\nu}_{\mu e}$ in vacuum does not depend on θ_{34} or δ_{34}.
- We assume that the new mixing angles are relatively small due to the near-unitarity of the three flavour case.

PMNS Matrix (4ν)

We now have a 4×4 PMNS matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

We can therefore write the flavour eigenstates as a linear combination of the mass eigenstates:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle,$$

$$(\text{for } \alpha = e, \mu, \tau, s \ \text{ and } i = 1, 2, 3, 4).$$

With sterile flavour eigenstate ν_s and new mass eigenstate ν_4.

Neutrino Mass Hierarchy

- We define the mass-squared differences as $\Delta m_{ij}^2 = m_i^2 - m_j^2$.
- We consider the case $|\Delta m_{21}^2| < |\Delta m_{31}^2| < |\Delta m_{41}^2|$.
- With $\Delta m_{21}^2, \Delta m_{41}^2 > 0$.
- The sign of Δm_{31}^2 is unknown.
- The best fits for the chosen independent mass-squared differences are:

$$\Delta m_{21}^2 \approx 7.5 \times 10^{-5}\,\mathrm{eV}^2$$
$$\Delta m_{31(\mathrm{NH})}^2 \approx 2.5 \times 10^{-3}\,\mathrm{eV}^2$$
$$\Delta m_{31(\mathrm{IH})}^2 \approx -2.4 \times 10^{-3}\,\mathrm{eV}^2$$
$$\Delta m_{41}^2 \approx 1\,\mathrm{eV}^2$$

- We do not consider other possibilities for the hierarchy involving different sign Δm_{41}^2 as LBL experiments cannot discriminate.

Oscillation Probability

- The general oscillation probability comes from $P_{\alpha\beta} = |\langle \nu_\beta(L)|\nu_\alpha(0)\rangle|^2$.
- Which becomes

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\mathrm{Re}\sum_{i>j}^{n} U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j} \sin^2(\Delta_{ij})$$
$$+ 2\mathrm{Im}\sum_{i>j}^{n} U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j} \sin(2\Delta_{ij}).$$

- Where $\Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E}$, is known as the oscillating factor for the ij^{th} mass-squared difference.

Oscillation Probability

- Evaluate $P_{\mu e}^{4\nu}$ using the 4ν PMNS matrix.
- Consider the small mixing parameters as small numbers, i.e. $s_{13}, s_{14}, s_{24} \approx \epsilon$, and keep terms up to 3^{rd} order in ϵ, this gives:

$$P_{\mu e}^{4\nu} \approx P^{\mathrm{ATM}} + P_{\mathrm{I}}^{\mathrm{INT}} + P_{\mathrm{II}}^{\mathrm{INT}},$$

where the individual terms are:

$$\begin{aligned}
P^{\mathrm{ATM}} &\approx 4 s_{23}^2 s_{13}^2 \sin^2 \Delta (1 - s_{14}^2 - s_{24}^2), \\
P_{\mathrm{I}}^{\mathrm{INT}} &\approx 8 s_{13} s_{12} c_{12} s_{23} c_{23} (\alpha\Delta) \sin\Delta \cos(\Delta + \delta_{13})(1 - s_{14}^2 - s_{24}^2), \\
P_{\mathrm{II}}^{\mathrm{INT}} &\approx 4 s_{14} s_{24} s_{13} s_{23} \sin\Delta \sin(\Delta + \delta_{13} - \delta_{14}),
\end{aligned}$$

and:

$$\alpha = \frac{\Delta_{21}}{\Delta_{31}}, \quad \Delta = \Delta_{31}.$$

Degeneracies

The probability equation allows MH-CP degenerate solutions, i.e.

$$P_{\mu e}(\Delta, \delta_{13}, \delta_{14}) \approx P'_{\mu e}(\Delta', \delta'_{13}, \delta'_{14})$$

where the false solutions are primed. For example:

$$\begin{array}{ccc} \text{True} & & \text{False} \\ \Delta = \Delta_{\mathrm{MH}} & \Longrightarrow & \Delta' \approx \Delta_{\mathrm{IH}} \\ \delta_{13} = 90^\circ & \Longrightarrow & \delta'_{13} = -90^\circ, \\ \delta_{14} = 90^\circ & \Longrightarrow & \delta_{14} = -90^\circ. \end{array}$$

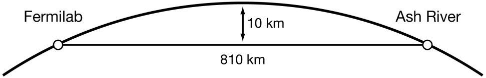
- Despite some terms flipping sign, both solutions are still close enough to look identical.
- Hence, sensitivity to these degeneracies depends almost entirely on baseline induced matter effects.

Experiments

- We focus on LBL (Long BaseLine) experiments which tend to have baselines on the order of 100's of km.
- In general these are the best for hierarchy determination due to the large MSW (matter) contribution to the probability.
- The MSW corrections have opposite signs for ν's and $\bar{\nu}$'s, so are an indirect source of CP violation.
- This allows combined ν and $\bar{\nu}$ running to potentially resolve the MH-CP degeneracy.
- This is important, as most simulations imply MH must be resolved before CP can be measured reliably.

NOνA


NOvA
Ash River
International
Falls
Duluth
MN
Minneapolis
WI
Fermilab
IL


Fermilab
10 km
Ash River
810 km

NOνA

- NOνA stands for NUMI Off-axis ν_e Appearance.
- Runs from the NUMI (NeUtrinos at Main Injector) proton accelerator at Fermilab in Illinois, through a near detector and on to the NOνA far detector facility in Ash River, Minnesota.
- Baseline is 810 km.
- Far detector is roughly 0.8 degrees off-axis.
- Current main goals are to measure θ_{13} and δ_{13}, and determine the mass hierarchy.
- Has greater sensitivity to the mass hierarchy than T2K due to the greater baseline.
- The far detector is the world's largest free-standing PVC structure.

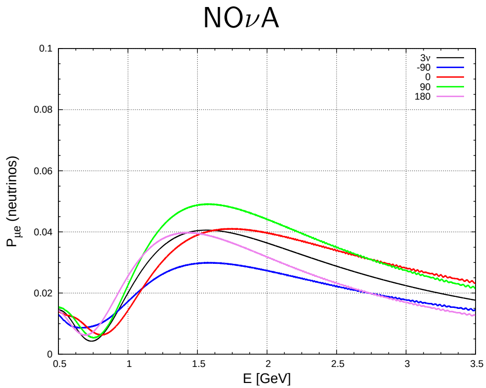
Simulation Overview

- Use the GLoBES package.
- Simulate the well known T2K and NOνA long baseline experiments with standard 3ν case.
- Construct χ^2 tests to see whether incorrect solutions can be rejected at a given confidence level (C.L.).
- See how sensitive these experiments should be to various degeneracies when analysed alone and in combination.
- Then if, given MH resolution, total CPV (CP Violation) can be detected.
- Adapt to simulate 4ν case with 1 light sterile neutrino.
- Predict whether the larger parameter space and new degeneracies reduce predictive power.

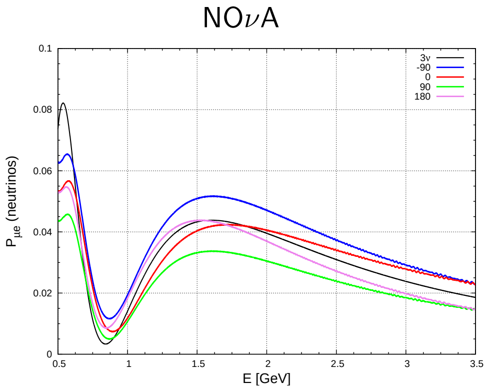
Predictions

- 3ν case:
  - T2K and NOνA can resolve the mass hierarchy for relatively large θ_{13}.
  - Given MH resolution, T2K and NOνA can potentially hint at δ_{13}.
  - If T2K and NOνA can't resolve MH, the next generation LBL detectors such as DUNE should be able to.
- 4ν case:
  - T2K and NOνA cannot resolve MH in the 3+1 case for all possible values of δ_{14}.
  - DUNE should be able to resolve MH but perhaps not total CPV.
  - Larger mixing angles $\theta_{14}, \theta_{24}, \theta_{34}$ and phases δ_{14}, δ_{34} can drastically reduce DUNE's sensitivity to MH and CPV.
  - Combined can these detectors resolve MH and measure CP phases?
  - Can multiple non-zero CP phases be resolved separately?

$P_{\mu e}$ Degeneracies



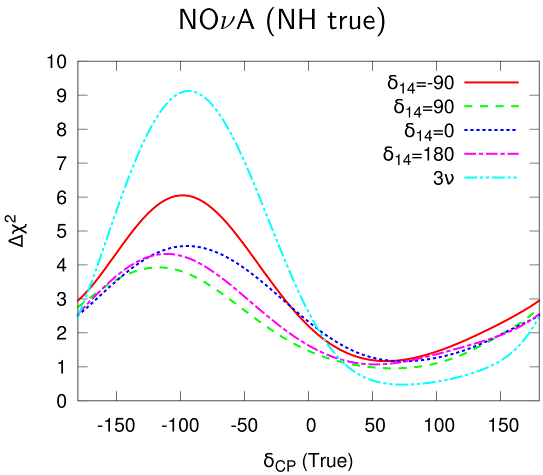
(a) NH, $\delta_{13} = 90^\circ$



(b) IH, $\delta_{13} = -90^\circ$

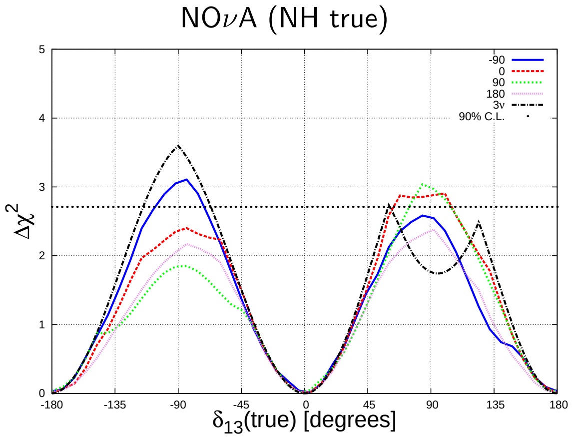
- These are the NH and IH unfavoured probabilities.
- The 3ν probabilities are (almost) degenerate.
- The NH, $\delta_{14} = \pm 90^\circ$ and IH, $\delta_{14} = \mp 90^\circ$ curves are also degenerate.

Wrong MH Exclusion



- For the favoured values the NOνA can exclude the wrong mass hierarchy solution at 90% C.L.
- It is predicted that for large θ_{13} in the 3ν case, NOνA and T2K combined can resolve MH for all δ_{13}.
- In the 4ν case, even for small sterile mixing angles this sensitivity can be noticeably degraded.

CPV Discovery



- For the favoured values the NOνA can reject CP conservation at 90% C.L.
- Without previously resolving the true MH the unfavoured solution cannot be resolved at 90% C.L.
- In the 4ν case the additional CP phases significantly lower the sensitivity.

Consequences of δ_{13}, $\delta_{14} \neq 0$

- "Direct" CP violation i.e. $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$, is allowed if at least one complex CP phase appears in the PMNS matrix.
- The CP violation comes from the imaginary part of the oscillation probability,

$$A_{\alpha\beta} \equiv P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}} = 4\mathrm{Im} \sum_{i>j}^{n} U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j} \sin(2\Delta_{ij})$$

- In the 3ν case we have:

$$A^{3\nu}_{\mu e} \propto -\sin \delta_{13}$$

  which vanishes if $\delta_{13} = 0, 180$.
- $A^{4\nu}_{\mu e}$ is more complicated, due to there being more CP phases.

Current Best Fits

- NOνA and T2K currently have roughly the same favoured values for MH and δ_{13}, they are MH=NH and $\delta_{13} \approx -90^\circ$ which are favoured for degeneracy resolution.
- These results are only initial measurements that should become more interesting once the NOνA anti-neutrino run has been started.
- NOνA also reports non-observation of sterile neutrino induced disappearance.
- The current limit on number of light neutrino flavours from PLANCK etc. is $n_{\mathrm{eff}} < 4$. This is clearly lower than we'd like!

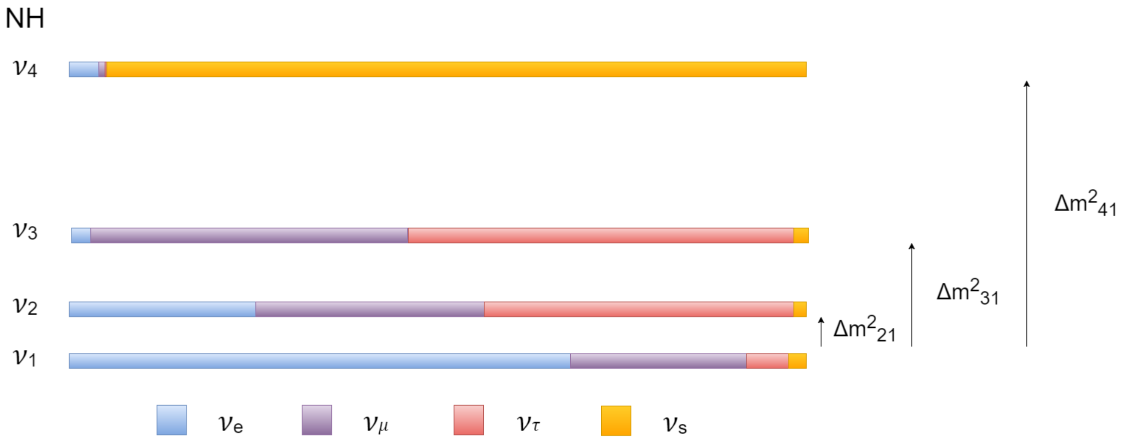
The Future

- Resolving the MH-CP degeneracy.
- Baryogenesis via leptogenesis.
- No sterile neutrinos?
- Octant degeneracy.
- More sterile flavours?

Thank you for listening

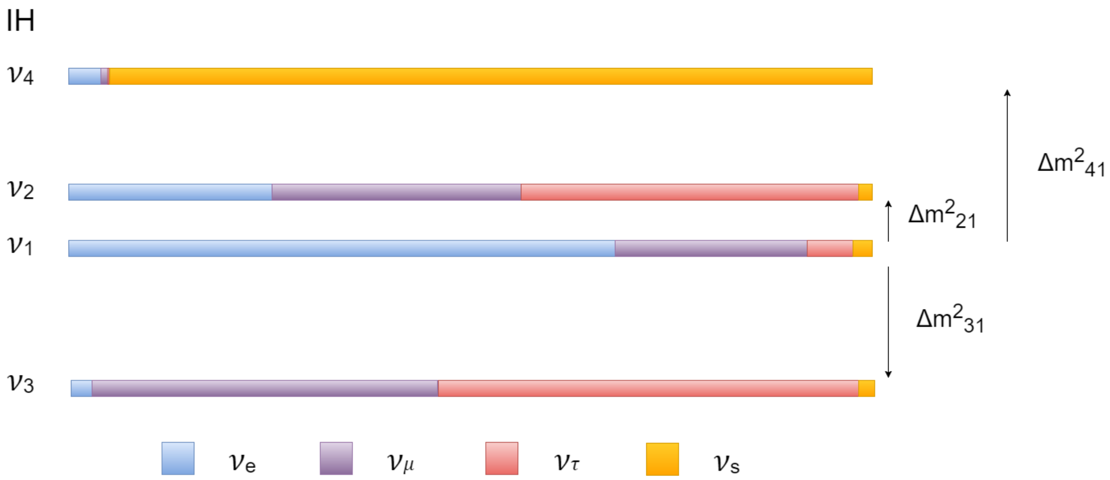
Backup

Normal Mass Hierarchy



- Normal Mass Hierarchy (NH) assumes Δm^2_{31} is positive.

Inverted Mass Hierarchy



- Inverted Mass Hierarchy (IH) assumes Δm^2_{31} is negative.

Oscillation Probability Expressions

- We are mostly dealing with muon neutrino beams, therefore we are most interested in $P_{\mu e}$ and $P_{\mu\mu}$, where $P_{ij} = P_{(\nu_i \longrightarrow \nu_j)}$
- These can be expanded perturbatively in the small mass difference $\Delta m^2_{21} \equiv \alpha$.

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \mathcal{O}(\alpha, \sin\theta_{13})$$

$$\begin{aligned} P_{\mu e} = {} & 4 \sin^2\theta_{13} \sin^2\theta_{23} \frac{\sin^2(A-1)\Delta}{(A-1)^2} + \alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2} \\ & + \alpha \sin\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) \frac{\sin(A-1)\Delta}{(A-1)} \frac{\sin A\Delta}{A} \end{aligned}$$

Where: $\Delta \equiv \frac{\Delta m^2_{31}}{4E}$, $A \equiv \frac{2EV}{\Delta m^2_{31}} = \frac{VL}{2\Delta}$, and $V = \sqrt{2} G_F n_e$ is the Wolfenstein matter term.

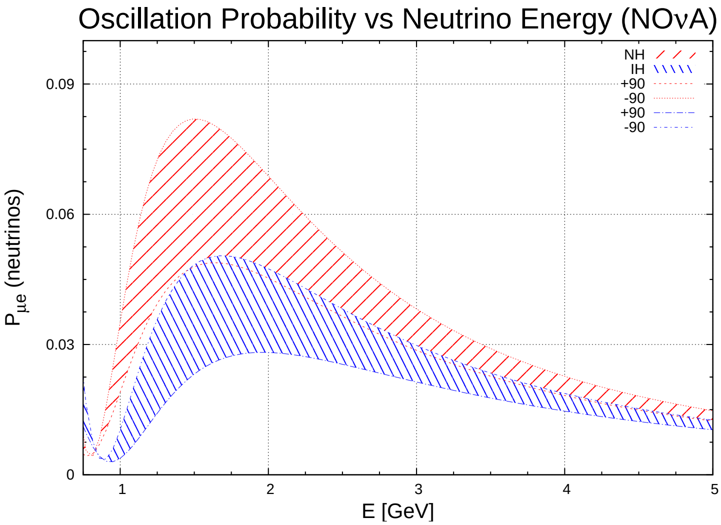
Parameter Degeneracies in Oscillation Probabilities

- For a given probability, there may be several solutions for the oscillation parameters.
- These can be summarised:

$$
\begin{aligned}
P_{\mu\mu}(\Delta m^2_{31}) &= P_{\mu\mu}(-\Delta m^2_{31}) \\
P_{\mu\mu}(\theta_{23}) &= P_{\mu\mu}(\pi/2 - \theta_{23}) \\
P_{\mu e}(\theta_{13}, \delta_{CP}) &= P_{\mu e}(\theta'_{13}, \delta'_{CP}) \\
P_{\mu e}(\Delta m^2_{31}, \delta_{CP}) &= P_{\mu e}(-\Delta m^2_{31}, \delta'_{CP})
\end{aligned}
$$

- Where: Δm^2_{31} corresponds to the "Normal Hierarchy" and $-\Delta m^2_{31}$ (assuming $-\Delta m^2_{31} + \Delta m^2_{21} \approx -\Delta m^2_{31}$) corresponds to the "Inverted Hierarchy". Primed values are false values.

Degeneracies (NOνA)



More Than One Sterile Neutrino?

- Some models add more than one sterile neutrino.
- 3+2 or 3+3 models are common.
- A large mass splitting allows the seesaw mechanism to produce the light active neutrino masses.
- A mass matrix:

$$\begin{pmatrix} 0 & M_D \\ M_D & M_M \end{pmatrix}$$

where M_D and M_M are Dirac and Majorana masses respectively, with $M_D << M_M$, has two disproportionate eigenvalues:

$$\lambda_L \approx \frac{M_L^2}{M_R}$$

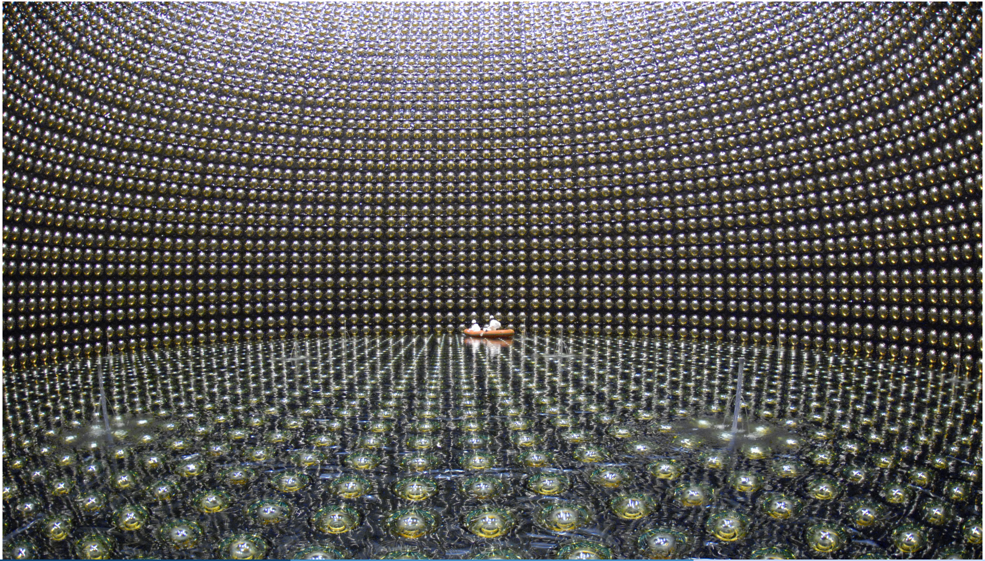
$$\lambda_R \approx M_R$$

where, similarly $\lambda_L << \lambda_R$. These represent the physical left and right handed neutrino masses.

GLoBES

- GLoBES stands for General Long Baseline Experiment Simulator.
- Uses experiment files and cross sections defined in Abstract Experiment Definition Language (AEDL).
- Contains functions for calculating event rates and χ^2 values between test and true probabilities.
- Good for LBL/reactor simulation, solar and atmospheric results are accounted for using Gaussian priors.
- Has it's own χ^2 minimiser.
- Can accept modified PMNS matrices and oscillation parameters.

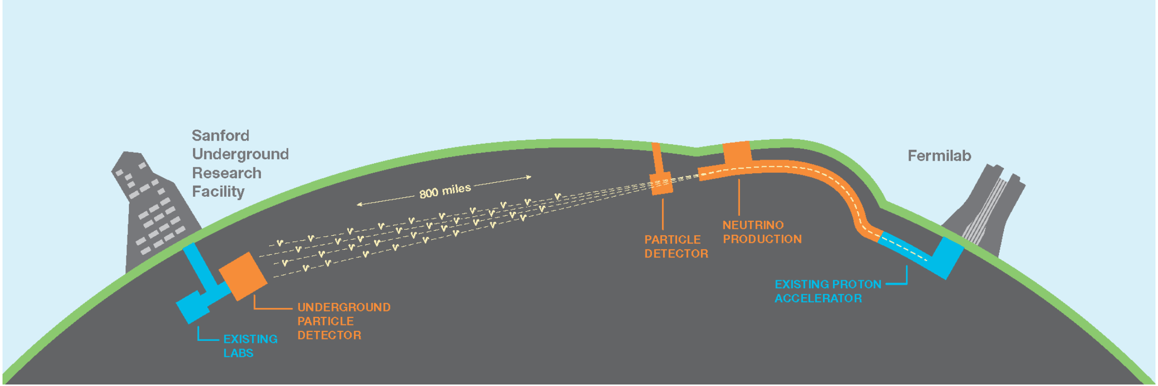
T2K



T2K

- T2K stands for Tokai to Kamioka.
- Runs from the J-PARC proton accelerator in Tokai to the Kamiokande Detector at Kamioka Observatory.
- Baseline is 295km.
- Far detector is roughly 2 to 3 degrees off-axis.
- Announced definitive evidence for muon to electron flavour oscillation in July 19 2013.
- Combined with other experiments may help with resolution of several degeneracies.

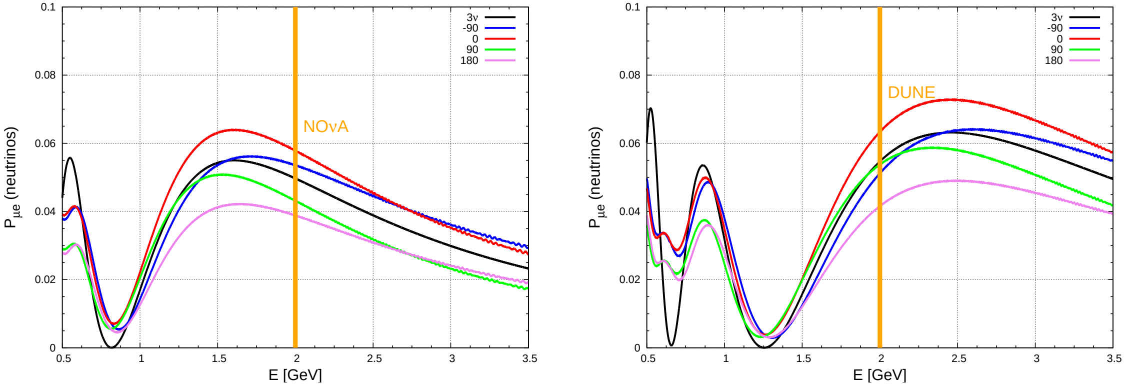
DUNE


Sanford
Underground
Research
Facility
800 miles
Fermilab
PARTICLE
DETECTOR
NEUTRINO
PRODUCTION
EXISTING PROTON
ACCELERATOR
UNDERGROUND
PARTICLE
DETECTOR
EXISTING
LABS

DUNE

- DUNE stands for Deep Underground Neutrino Experiment.
- Construction proposed to begin in 2018 to be operational by 2025.
- Will have a very long baseline of 1300km.
- Will have near and far detectors.
- Assuming the 3ν case should be able to resolve the MH and allow precision measurements of δ_{13} to occur (if this hasn't occurred yet).
- May also be able to observe observe supernova neutrinos.
- Detector would be sensitive to proton decay if it occurs.

4ν Probability



$P_{\mu e}$ for various δ_{14} with $\delta_{13} = 0$, averaged over fast Δ_{41} induced oscillations (approximate mean beam energy marked).