Effects of pairing correlation on the low-lying quasiparticle resonance in neutron drip-line nuclei

Prog. Theor. Exp. Phys. 2016, 013D01

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Back ground

- Most excitation modes become resonances in nuclei close to drip-line.
- Pair correlation influences strongly ground state and low-lying excitations.
- **Quasi-particle resonance** (Belyaev et al. 1987) has a chance to be observed, in place of single-particle potential resonance.

Purpose of present study

- We intend to disclose novel features of the quasi-particle resonance.
  Example: p-wave resonance in ^{47}Si =(^{46}Si + n)

Nucleon in continuum is influenced by pairing

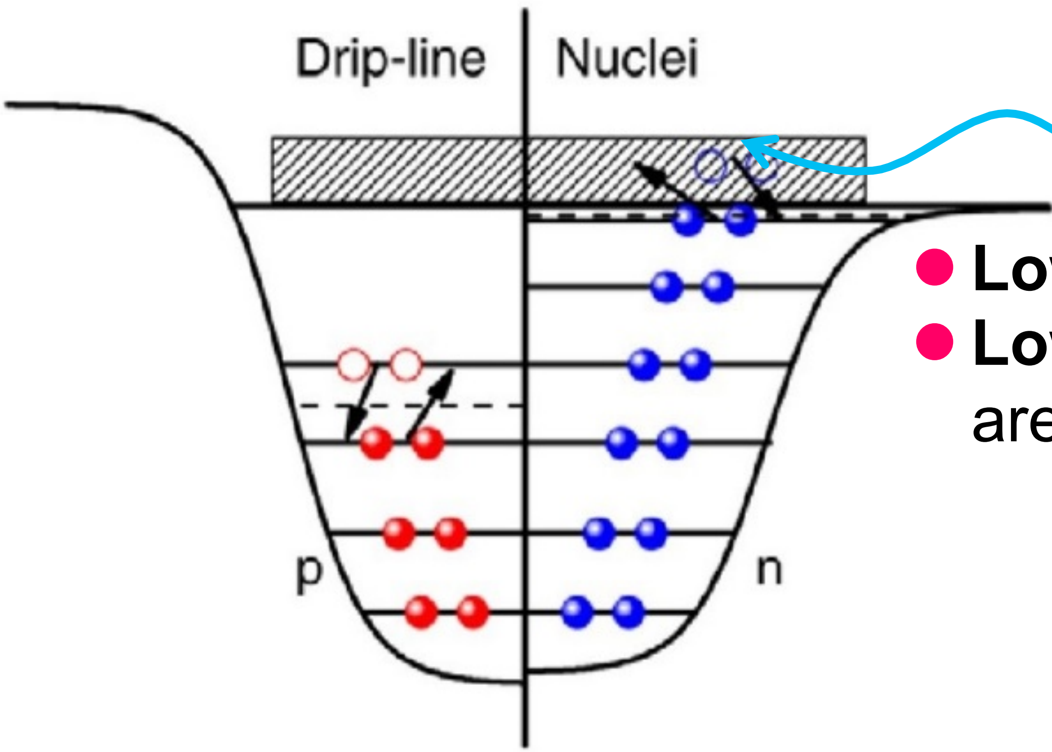
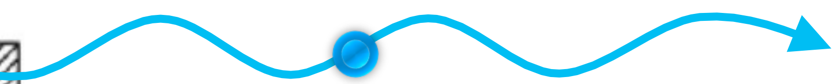
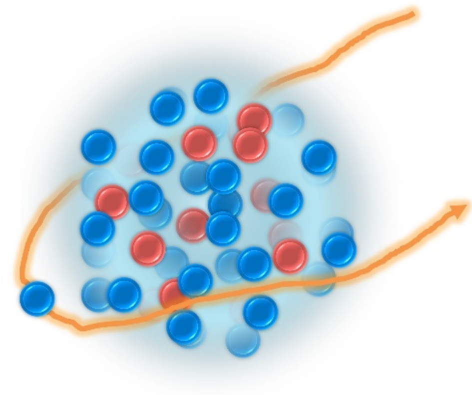


Fig. J. Meng, et al Prog. Part. Nucl. Phys. 57, 470 (2006)

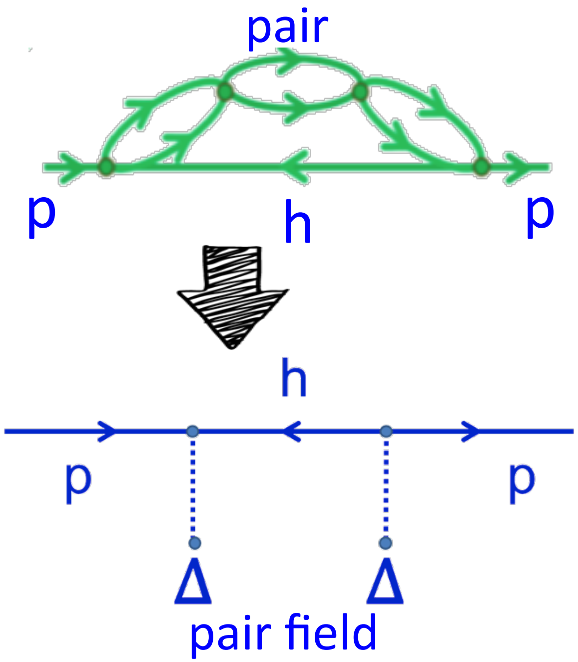


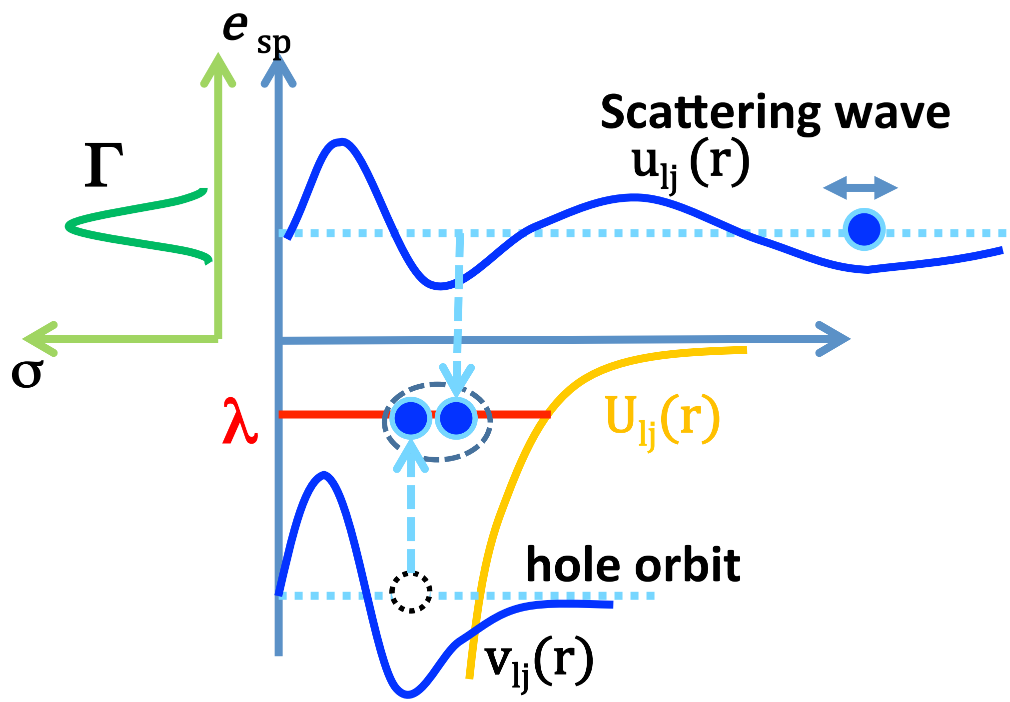
- **Low energy scattering particle**
- **Low-lying resonance**

  are influenced by pair correl.



Bogoliubov equation and quasi-particle resonance





- **<u>Bogoliubov equation for the coupled single-particle motion (hole & particle components)</u>**

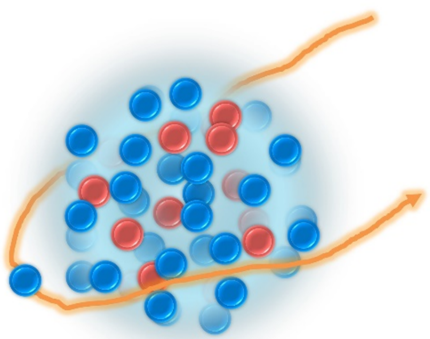
S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
A. Bulgac, arXiv:nucl-ph/9907088
J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

$$\begin{bmatrix} -\frac{\Delta}{2m} + U_{lj}(r) - \lambda & \Delta(r) \\ \Delta(r) & \frac{\Delta}{2m} - U_{lj}(r) + \lambda \end{bmatrix} \begin{bmatrix} u_{lj}(r) \\ v_{lj}(r) \end{bmatrix} = E_i \begin{bmatrix} u_{lj}(r) \\ v_{lj}(r) \end{bmatrix}$$

Bogoliubov quasi-particle in the continuum

- **Scattering boundary condition for the Bogoliubov's quasi-particle**

$$\frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = C\begin{pmatrix} \cos\delta_{lj}\, j_l(k_1 r) - \sin\delta_{lj}\, n_l(k_1 r) \\ D h_l^{(1)}(i\kappa_2 r) \end{pmatrix} \xrightarrow[r\to\infty]{} C\begin{pmatrix} \frac{\sin\left(k_1 r - \frac{l\pi}{2} + \delta_{lj}\right)}{k_1 r} \\ 0 \end{pmatrix}$$



- **phase shift, S-matrix**
- **elastic cross section**

for A+n scattering
resonances in (A+n) system

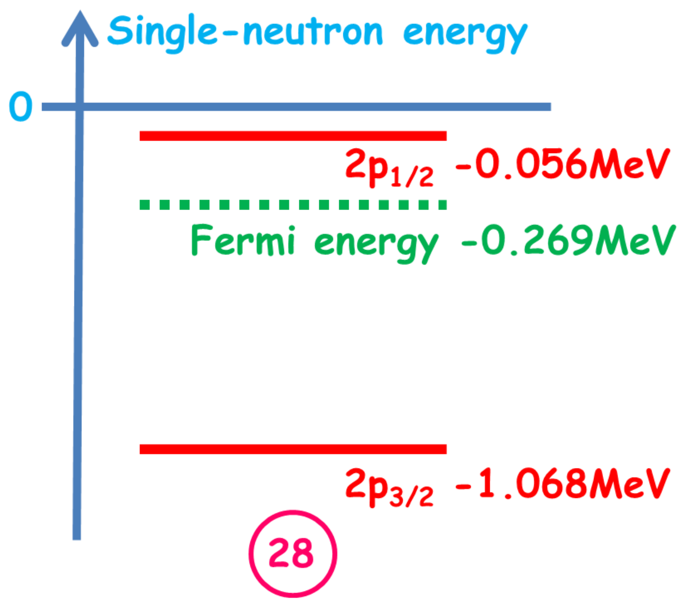
- **Numerical model**
  - **Mean-field U(r) : Woods-Saxon potential**
  - **Pair-field Δ(r): Woods-Saxon form**
    its strength (av. pair gap Δ) is varied

Neutron elastic scattering on ^{46}Si: (^{46}Si+n)*

- **Low-lying p wave quasi-particle resonance**

weakly bound p orbit
↕
low-lying p wave quasi-particle resonance

<u>single-particle energy in the WS potential</u>



☑ ^{46}Si is near the neutron drip-line

☑ deformation is small (spherical)

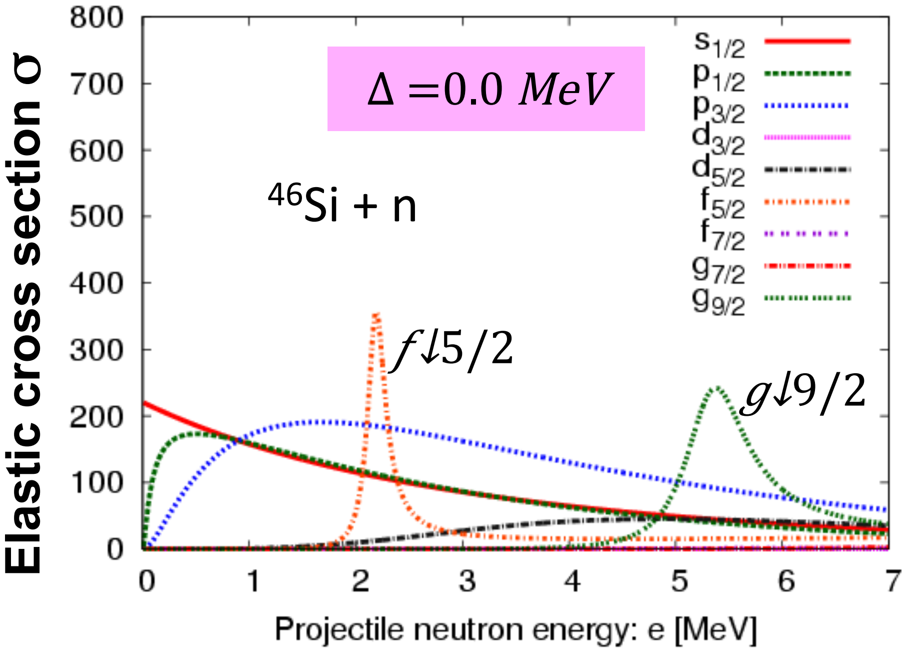
Ex) M. V. Stoitsov et al. PRC68. 054312 (2003)

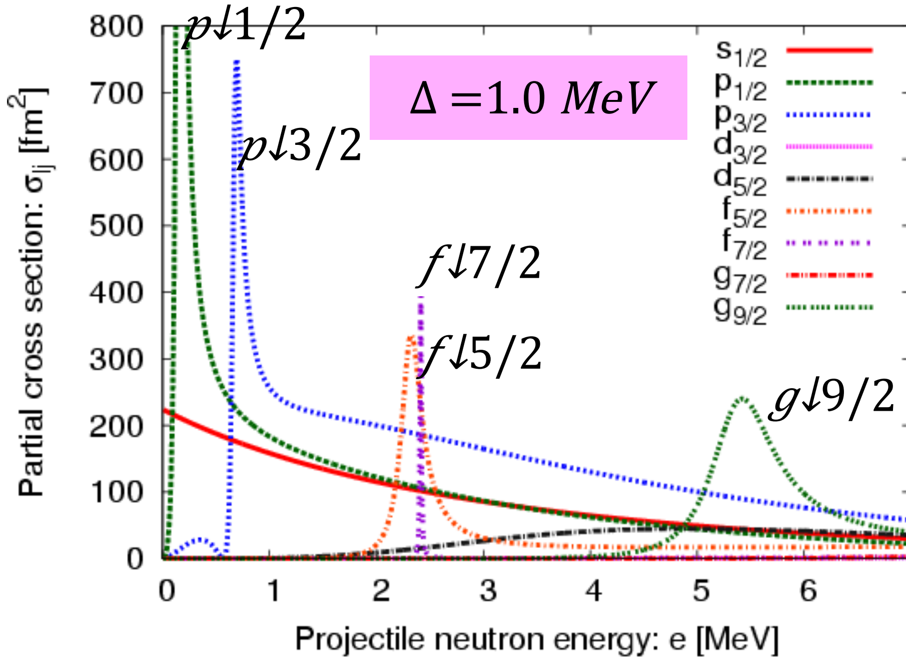
☑ Fermi energy : -0.269 MeV

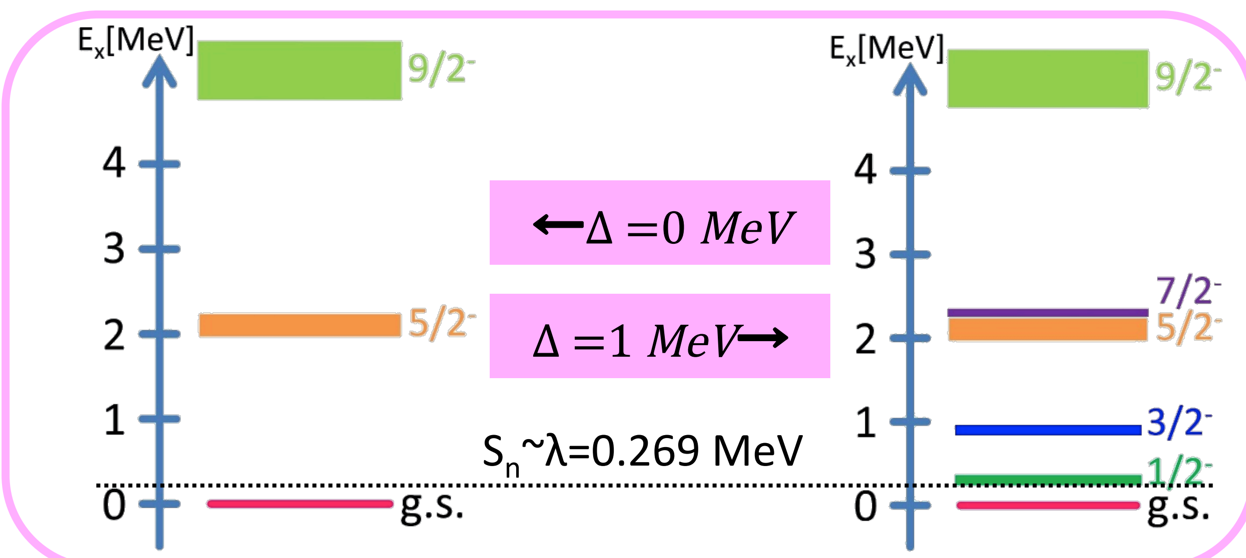
is estimated by the Woods-Saxon-Bogoliubov calculation

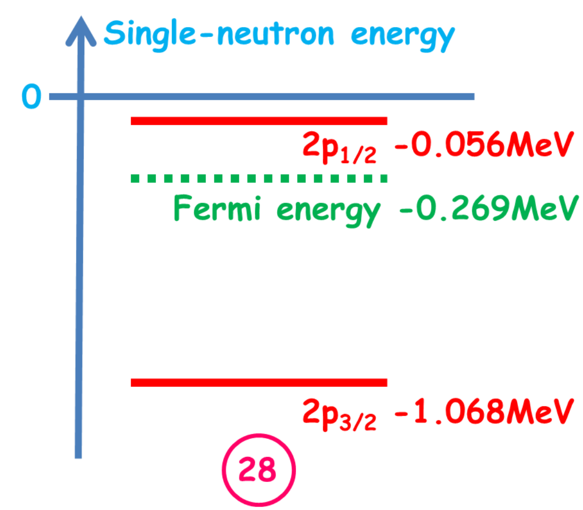
H. Oba and M. Matsuo, PRC80. 024301 (2009)

Weakly bound orbits emerge as resonances due to pairing







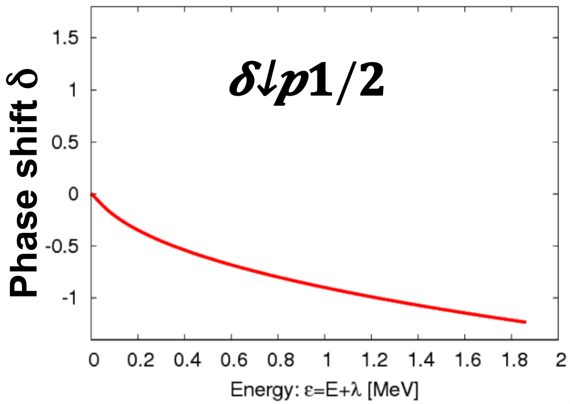


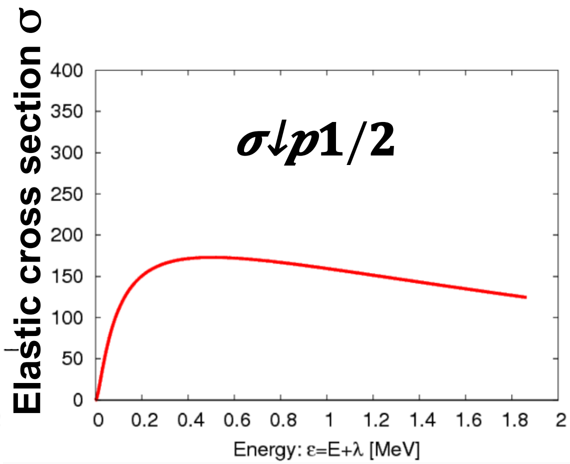
Quasi-particle resonance in S-matrix

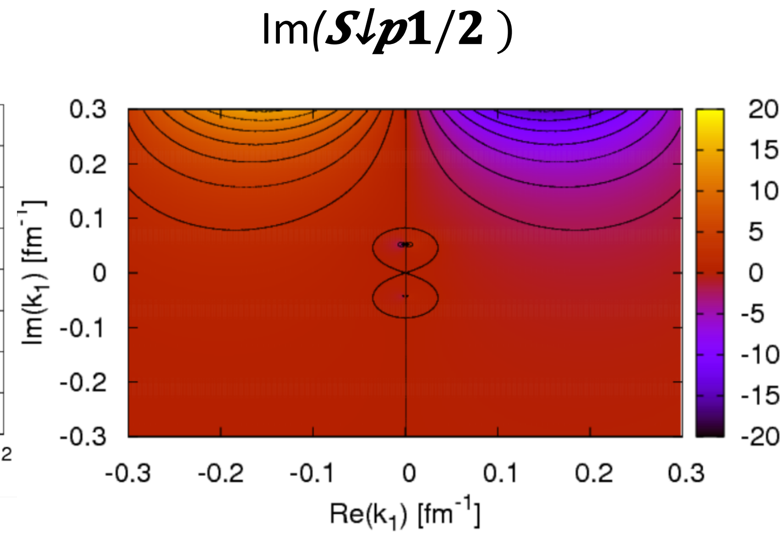
- The quasi-particle resonance appears as a pole of S-matrix in complex k plane.

$p_{1/2}$-wave

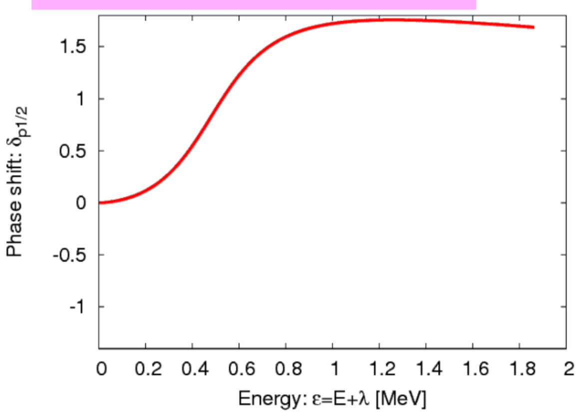
$\Delta = 0.0$ MeV

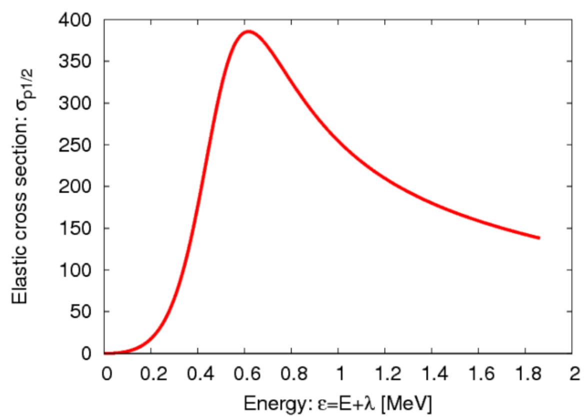


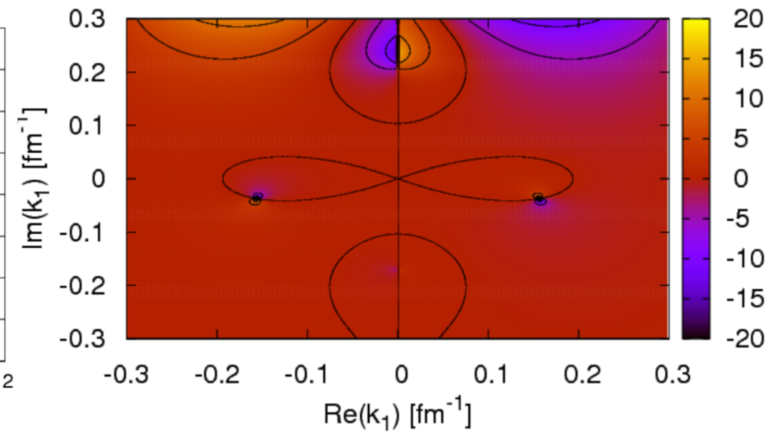




$\Delta = 2.0$ MeV

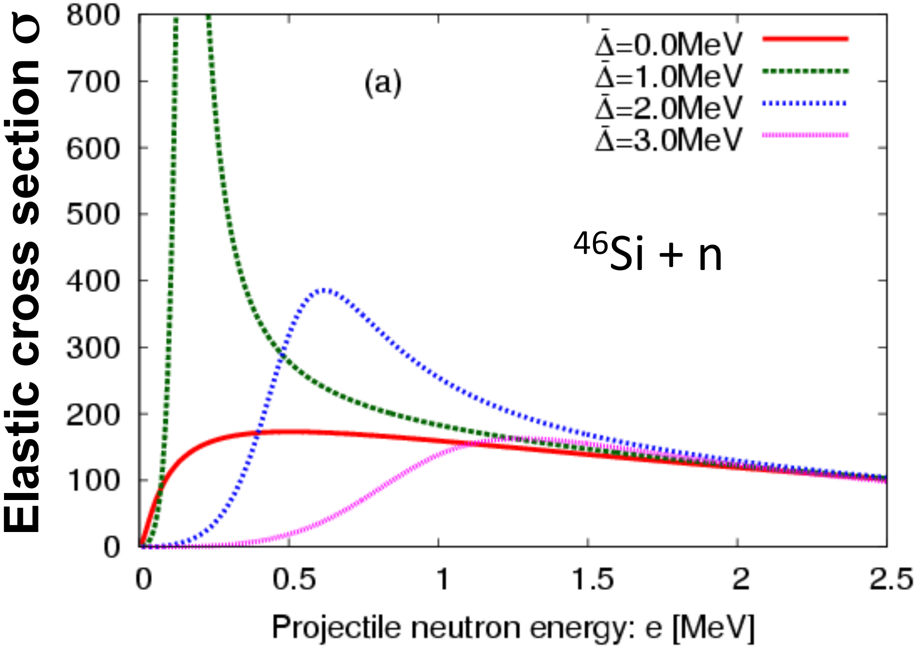


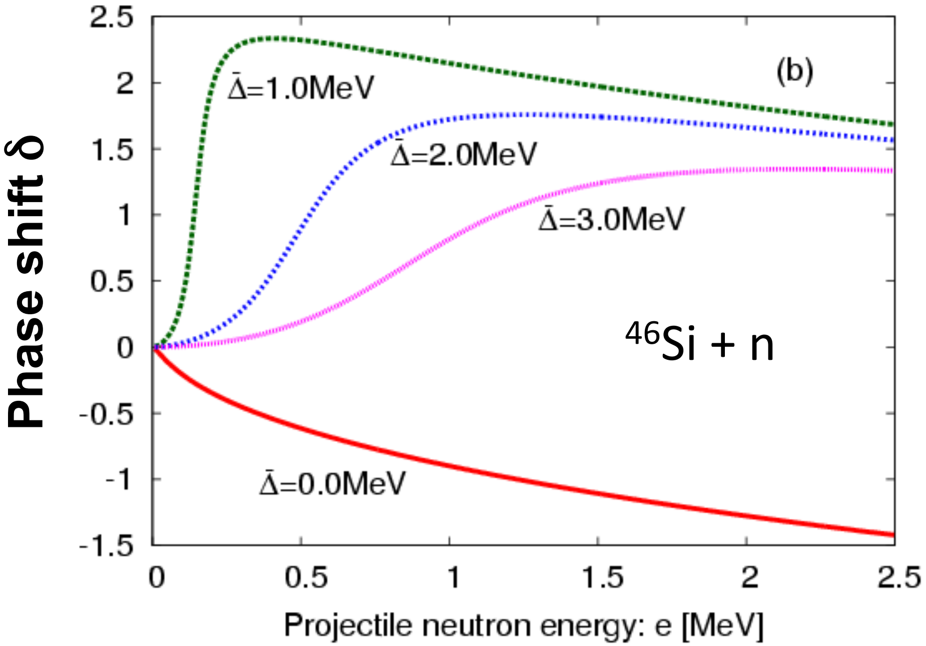




Dependence on pair correlation strength $\bar{\Delta}$

- **$p_{1/2}$-wave in (^{46}Si+n)**





- The pairing strength

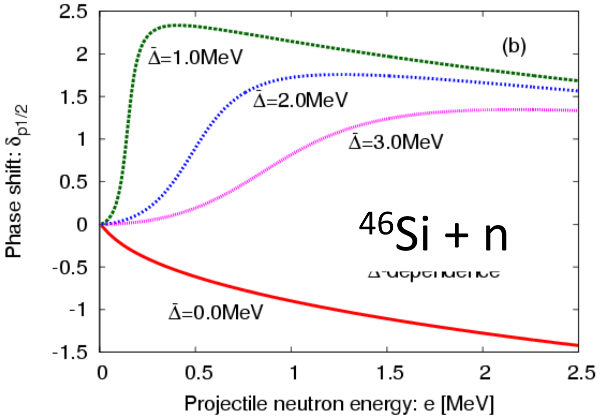
$\Delta = 0.0 - 3.0 MeV$

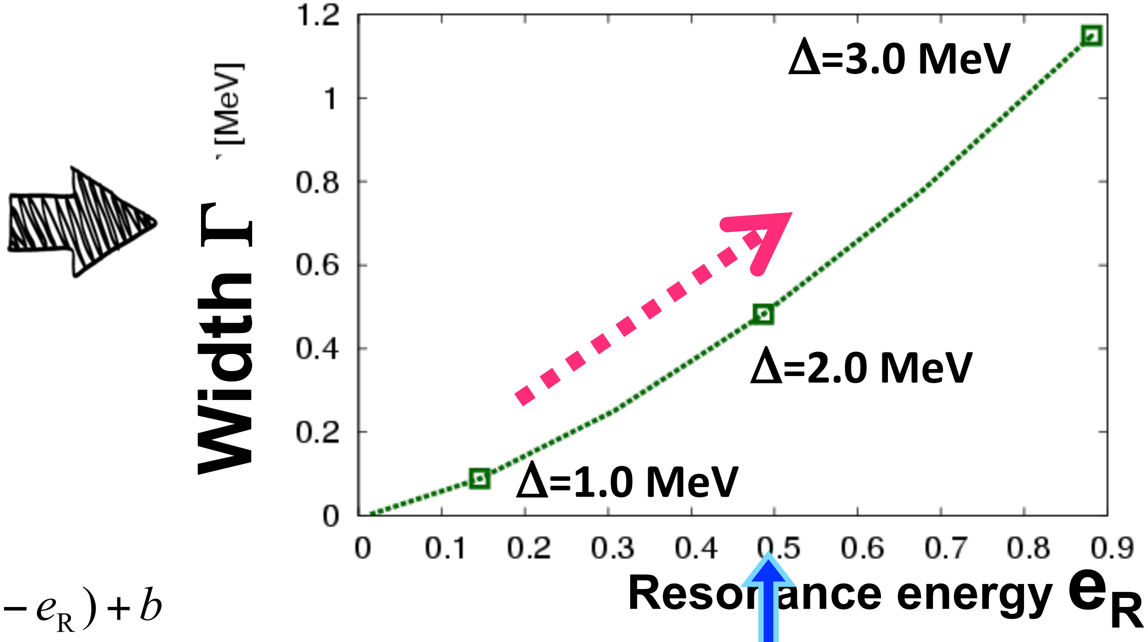
Cf. Typical value of pair gap

$\Delta = 12.0/\sqrt{A} \;\; MeV = 12.0/\sqrt{46} \; \cong 1.\mathbf{7} \; MeV$

Width Γ & Resonance energy e_R

Phase shift





e barriar =0.5 MeV

Fitting function

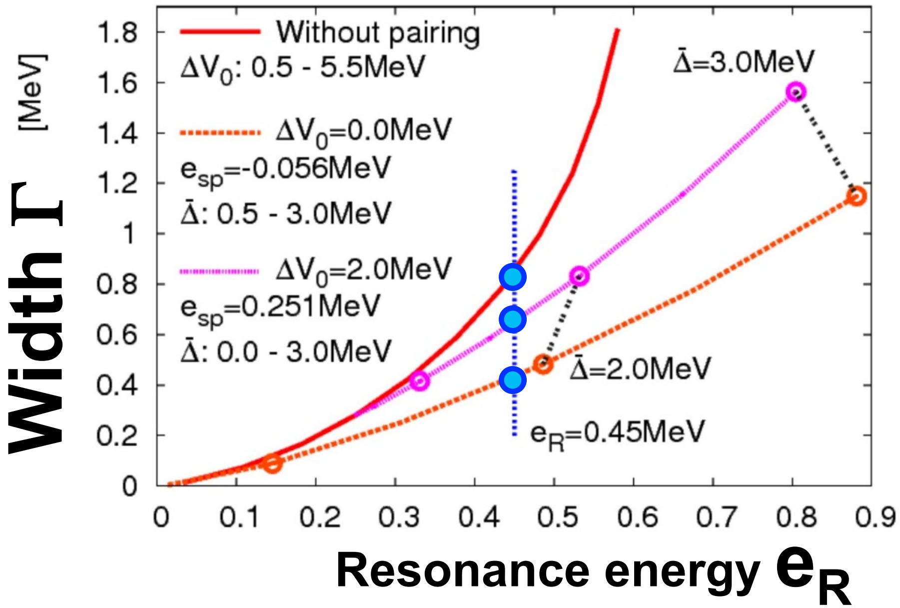
$$\delta(e) = \arctan\left(\frac{2(e-e_R)}{\Gamma}\right) + a(e-e_R) + b$$

- **Both Γ and e_R increase for larger pairing strength.**

- **Increase of Γ is modest**
- **moderate value of Γ even for e_R > barrier height**

Comparison with simple potential resonance

e_R-Γ relation



Potential resonance 2p1/2

$\Delta=0$, $e=0 \sim 0.6$ MeV

Quasi-particle resonance

$\Delta=0\sim3$ MeV, $e_{2p1/2}=0.251$ MeV

$\Delta=0\sim3$ MeV, $e_{2p1/2}=-0.056$ MeV

Pairing *reduces* the width

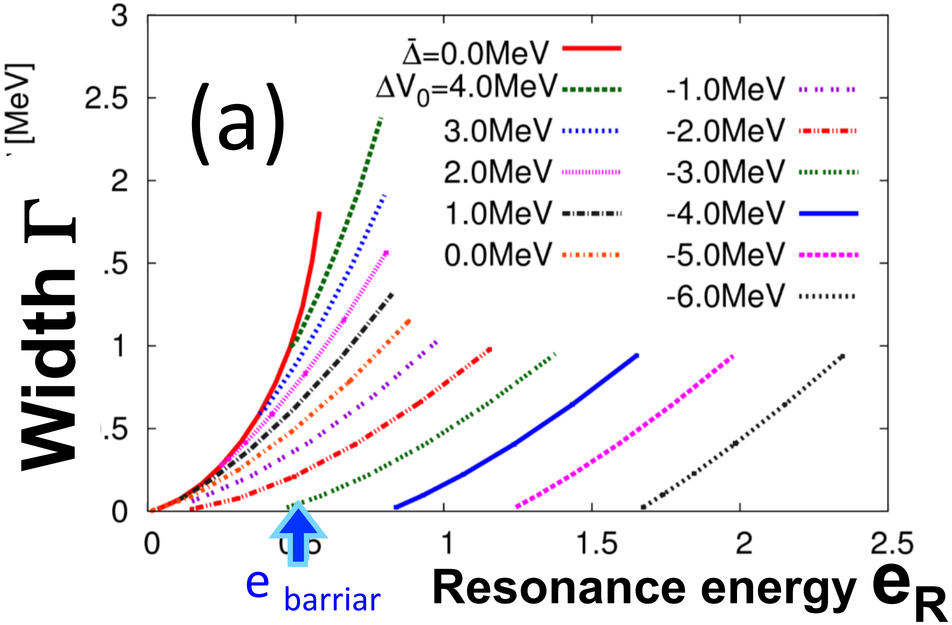
compared at the same resonance energy

NB. Opposite trend known previously for deep-hole quasi-particle resonance $\Gamma \propto \Delta^2$

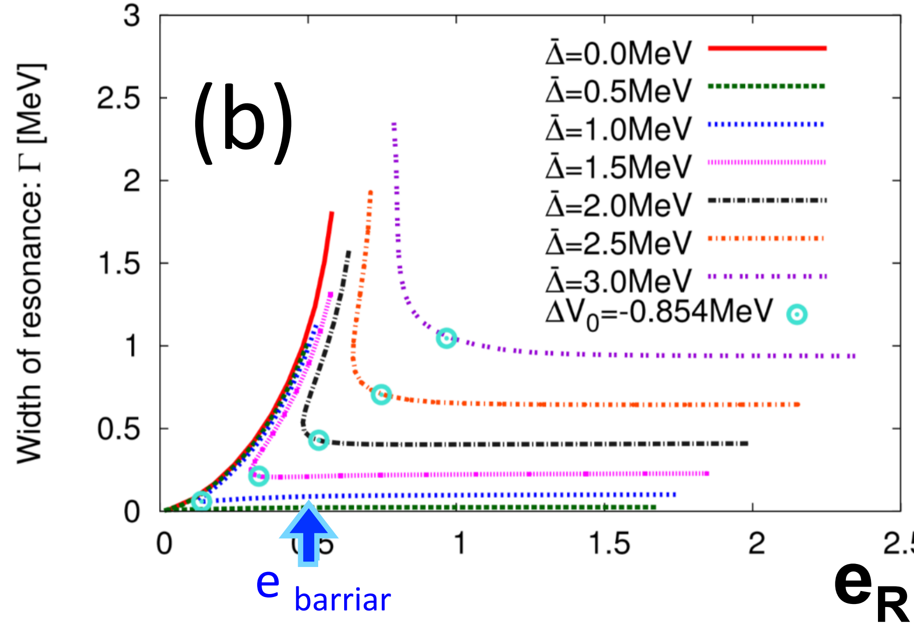
$e_R=0.45$ MeV			
Δ [MeV]	**0.0**	**1.634**	**1.897**
Γ [MeV]	**0.854**	**0.652**	**0.453**
ΔV[MeV]	3.677	2.0	0.0
e_{sp} [MeV]	0.450	0.250	-0.056

Systematics of resonance width Γ and resonance energy

Pairing strength Δ is varied

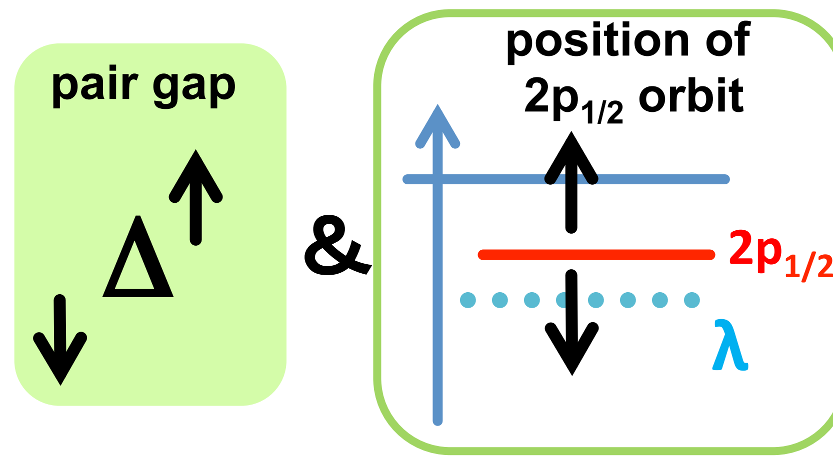


Position of $2p_{1/2}$ orbit is varied



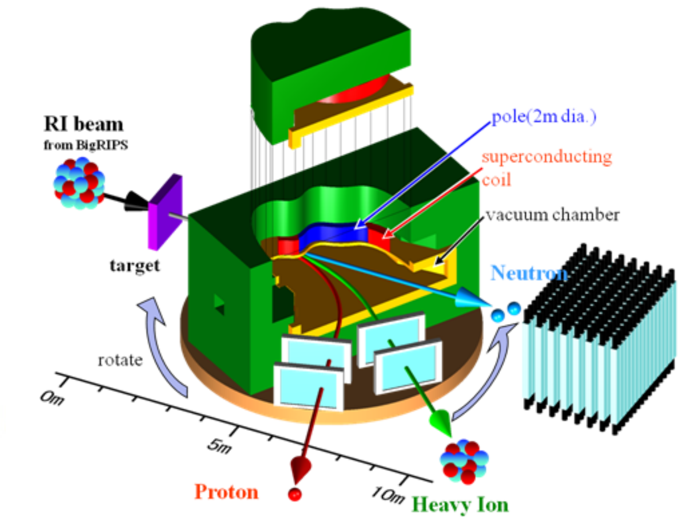
Novel features of quasi-particle resonance

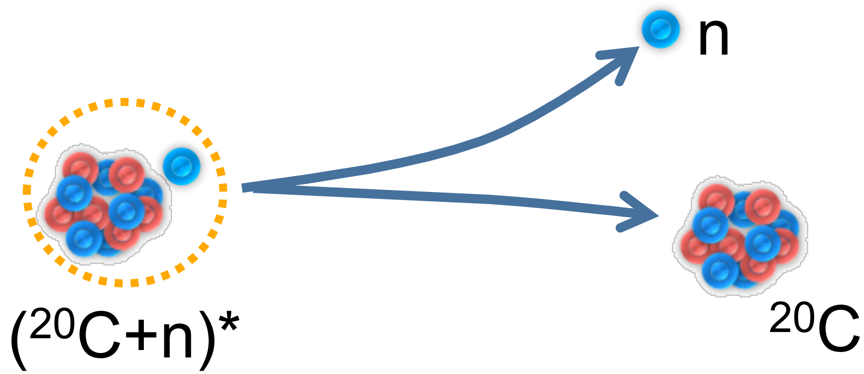
- The width has wide range of variation, not limited to the potential resonance
- It is allowed to exist above the barrier
- Resonance width is necessarily smaller than Γ(s.p. potential resonance)



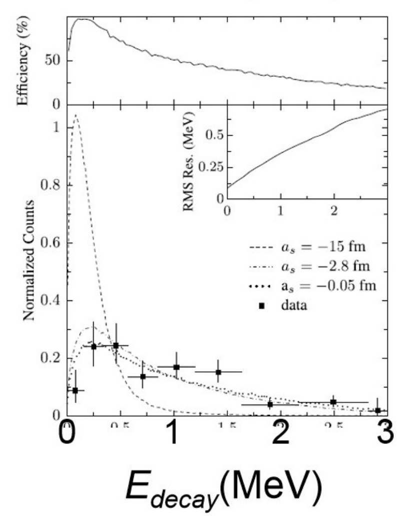
SAMURAI experiments for unbound nuclei

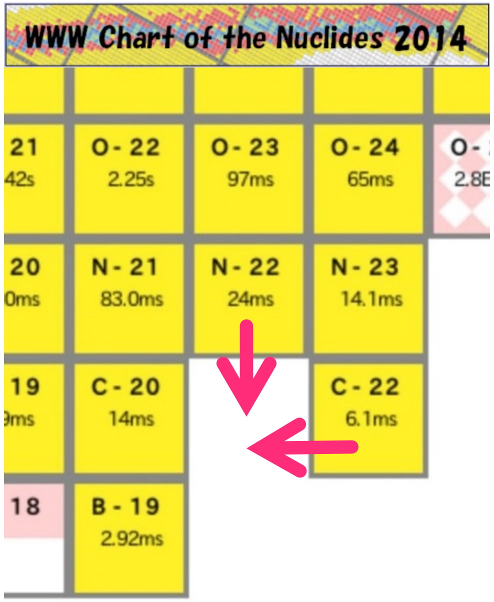
- SAMURAI experiments@RIBF（2012~）



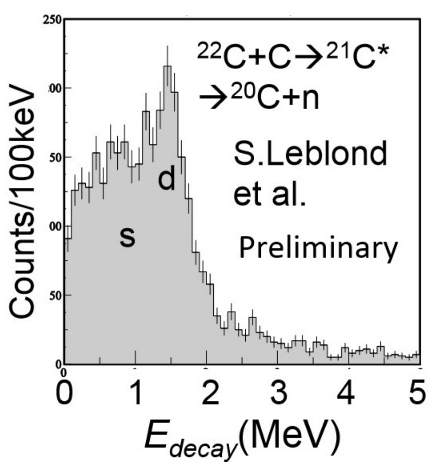


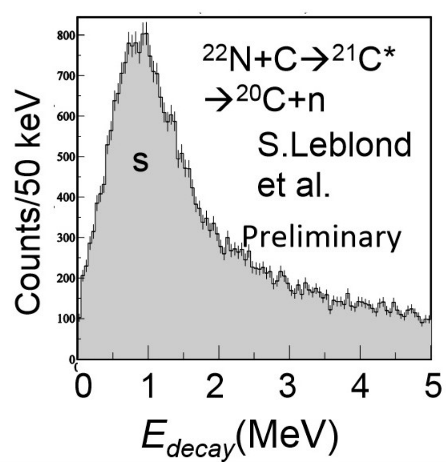
^{22}N→^{21}C*→^{20}C+n
S.Mosby et al.(MSU)
NPA909,69(2013).





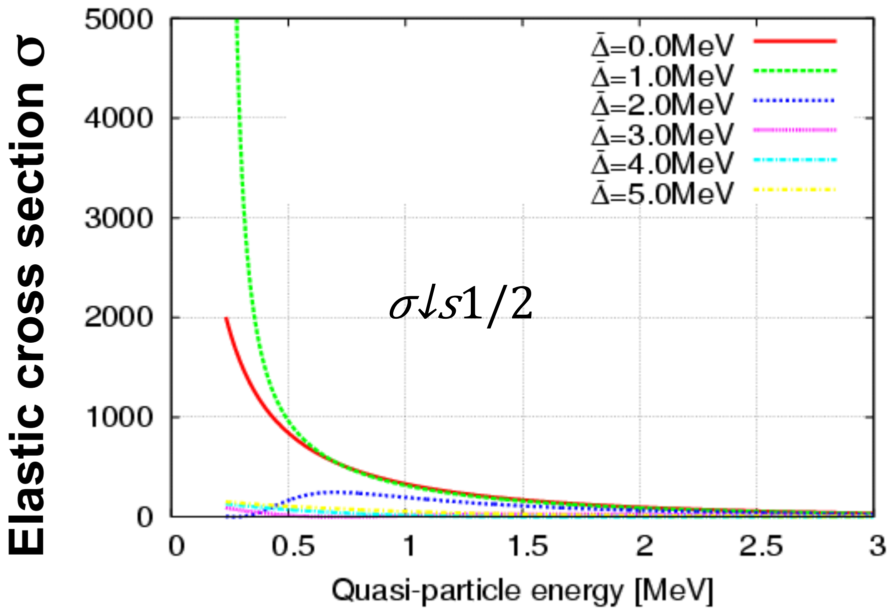
（↓）priv. comm. Takashi Nakamura

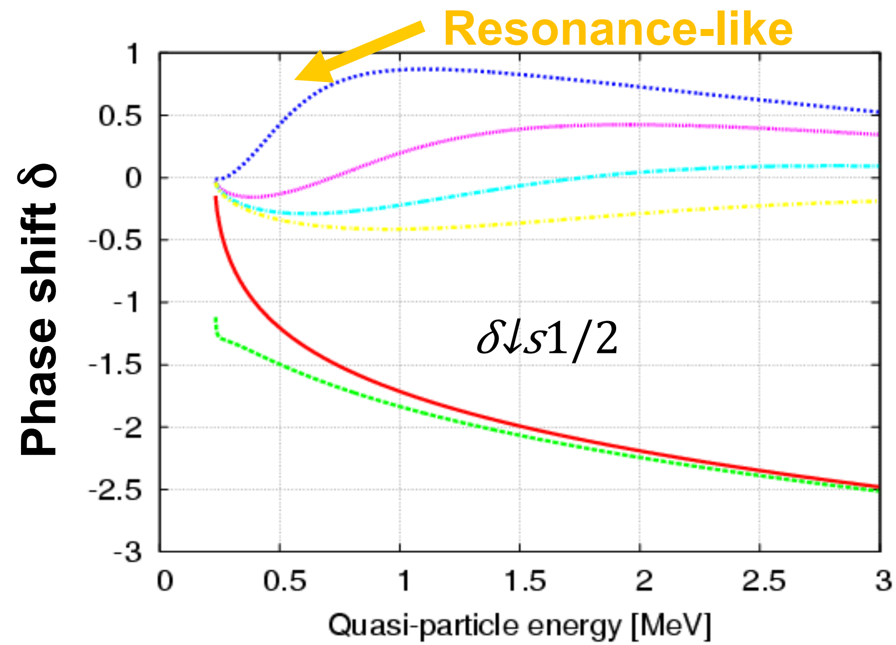




S-wave scattering in (^{20}C + n)

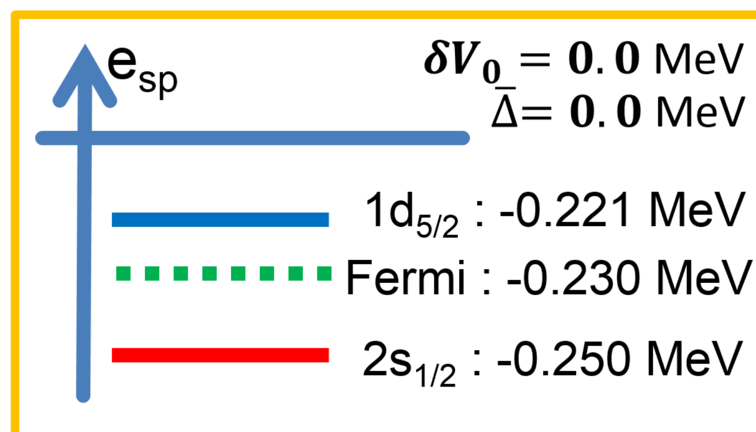
$\Delta = 0 \sim 5$ MeV varied





These are very different from the low-energy formula $k\cot\delta \cong -1/a + \frac{1}{2} k^2 r_{eff}$

Δ [MeV]	1/a [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.00825	-1.478
2.0	-0.9279	-109.617
3.0	0.3160	-69.521
4.0	0.3018	-14.192

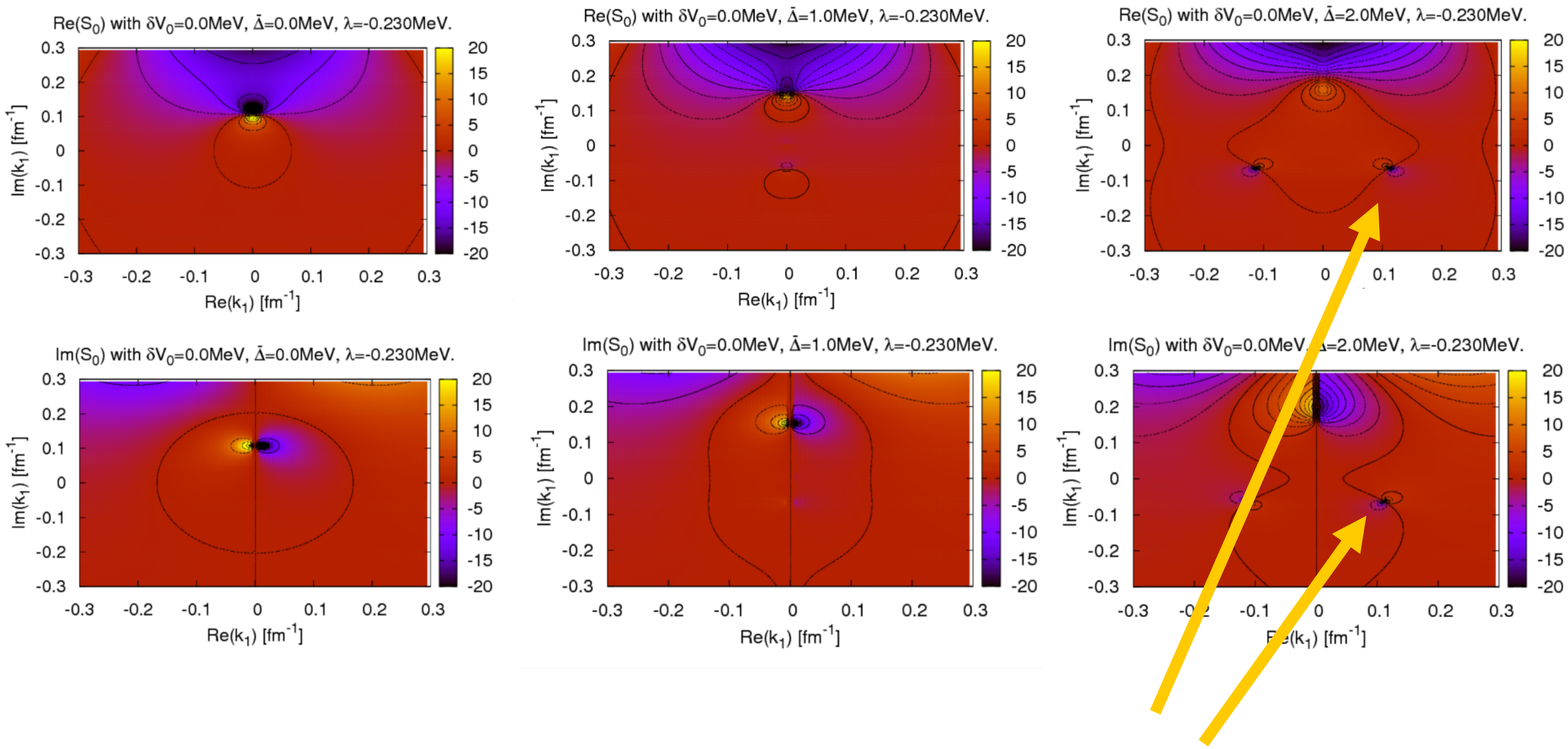


S-matrix behavior in (^{20}C + n) s-wave

$\mathbf{\Delta} = \mathbf{0.0}\,MeV$ $\mathbf{\Delta} = \mathbf{1.0}\,MeV$ $\mathbf{\Delta} = \mathbf{2.0}\,MeV$

$\mathrm{Re}(S_{s1/2})$

$\mathrm{Im}(S_{s1/2})$



A resonance pole emerges at low energy

Conclusion

We have investigated effects of the pair correlation on low-lying neutron resonance in n-rich drip-line nuclei

- P-wave resonance in $^{47}Si^*$ (^{46}Si + n scattering)

It exhibits novel behaviors, not seen in s.p. potential resonance

- Resonance is allowed to **exist above the barrier energy**
- The pairing effect on the resonance width has two faces:
  i) to increase the width (in case of hole origin $e_{s.p} <$ Fermi eng)
  ii) **to REDUCE the width** (in the other case $e_{s.p} >$ Fermi eng)
- Consequently,
  its width is necessarily smaller than the s.p. potential resonance

Outlook

- S-wave resonance /scattering is more dramatic ……………..under study
- Confrontation to experiments ……………. $^{45}Si^*$ $^{47}Si^*$ $^{21}C^*$ $^{10}Li^*$ ……

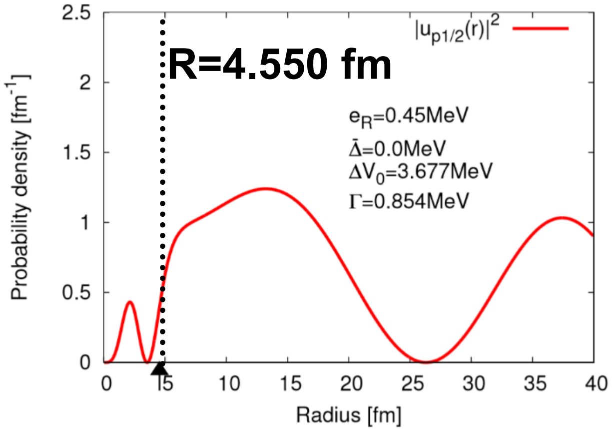
Backup

Analysis of the resonance wave functions

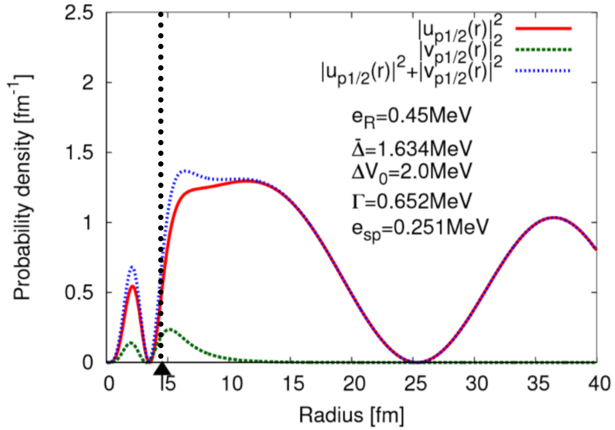
- Probability distribution of the resonances with $e_R = 0.45\ MeV$

$|u_{p1/2}(r)|^2 + |v_{p1/2}(r)|^2$: the sum of particle component and hole component

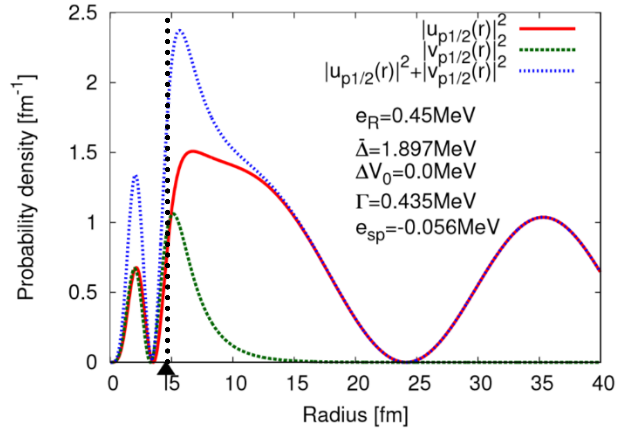
$\Delta = 0.0$ **MeV**



$\Delta = 1.63$ **MeV**



$\Delta = 1.897$ **MeV**



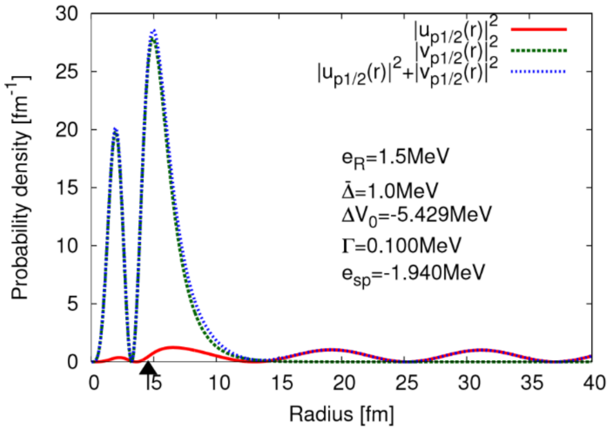
- The hole component which is localized inside the nucleus ($|v_{p1/2}(r)|^2$) becomes larger with increasing of Δ.

→ **The reducing of the width of particle-like quasi-particle resonance.**

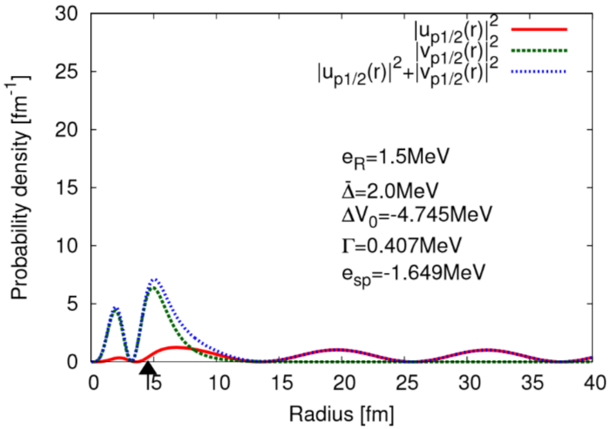
Analysis of the resonance wave functions

- Probability distribution of the hole-like quasi-particle resonances with $e_R = 1.50\ MeV$.

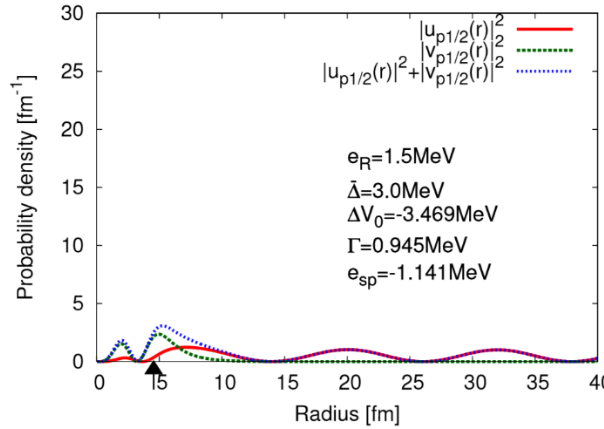
$\Delta = 1.0$**MeV**



$\Delta = 2.0$**MeV**



$\Delta = 3.0$**MeV**



- The hole component ($|v_{p1/2}(r)|^2$) decreases with increasing of Δ. (opposite behavior of particle-like case)

→ The width of hole-like quasi-particle resonance is increased by the pairing.

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
A. Bulgac, Preprint(1980); nucl-th/9907088

The Hartree-Fock-Bogoliubov equation

- The HFB equation in the coordinate spaced

J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

$$\begin{pmatrix} -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+U_{lj}(r)-\lambda & \Delta(r) \\ \Delta(r) & \frac{\hbar^2}{2m}\frac{d^2}{dr^2}-U_{lj}(r)+\lambda \end{pmatrix}\begin{pmatrix} u_{lj} \ldots$$

$$\begin{pmatrix} \varphi_i^{(1)}(x) \\ \varphi_i^{(2)}(x) \end{pmatrix} = \frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix}[Y_l(\theta,\varphi)\chi_{1/2}(\sigma)]_{jm}$$

$U_{lj}(r)$: HF potential with $l\cdot s$ interaction, $\Delta(r)$: Pair potential

- The pairing correlation is described by the pair potential.

- **Scattering boundary condition for the Bogoliubov's quasi-particle**

$$\frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = C\begin{pmatrix} \cos\delta_{lj}\, j_l(k_1 r) - \sin\delta_{lj}\, n_l(k_1 r) \\ D\, h_l^{(1)}(i\kappa_2 r) \end{pmatrix} \xrightarrow{r\to\infty} C\begin{pmatrix} \sin(k_1 r - l\pi/2 + \delta_{lj})/k_1 r \\ 0 \end{pmatrix}$$

$$k_1 = \sqrt{2m(\lambda+E)/\hbar^2}\ ,\quad \kappa_2 = \sqrt{-2m(\lambda-E)/\hbar^2}\qquad C=\sqrt{2mk_1/\hbar^2\pi}$$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
M. Grasso et al., Phys. Rev. C 64 064321 (2001)
I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

The BCS theory in nuclear physics

- Superconductors have been described by the BCS theory with the electron Cooper pairs (1957).

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

- **Bohr, Mottelson and Pines applied the BCS theory to the nuclear excitation spectra (1958).**

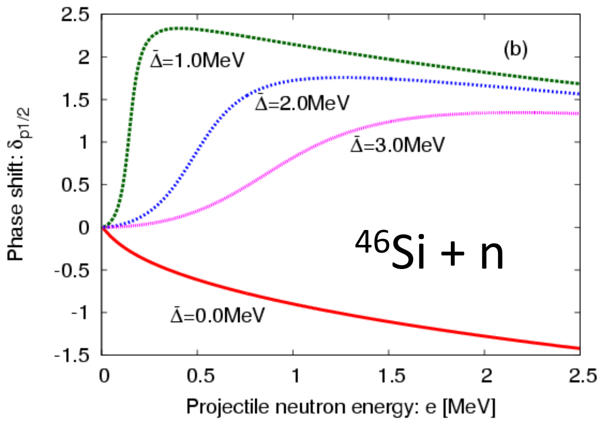
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

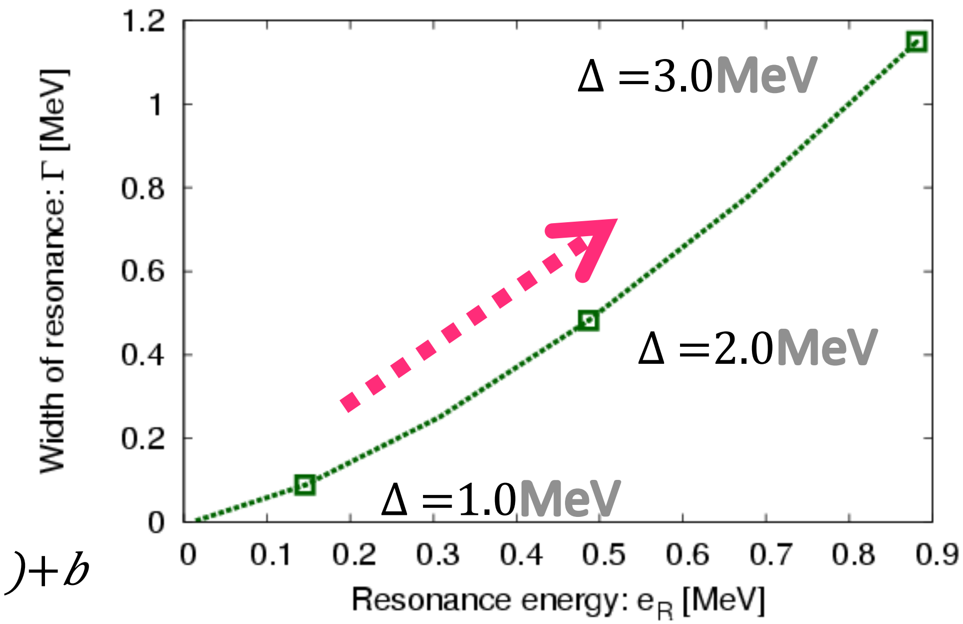
Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark
(Received January 7, 1958)

Width Γ vs. Resonance energy e_r

- The resonance width and energy are extracted from the calculated phase shifts for quantitative analysis.



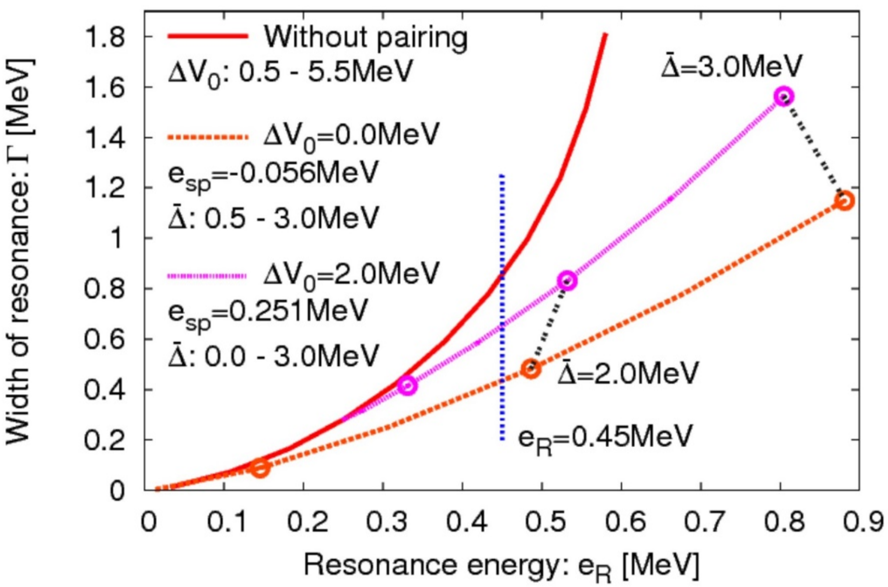


Fitting function

$$\delta(e)=\arctan(2(e-e_R)/\Gamma)+a(e-e_R)+b$$

Both the resonance width Γ and the resonance energy e_R increase as the strength of the pairing Δ increases.

The pairing has the effect of *reducing* the width



The dependence of s.p. resonance width and energy ($\Delta = 0.0\ MeV$) on ΔV_0

The dependence of **q.p.** resonance width and energy ($e_{2p1/2} = 0.251\ MeV$) on Δ

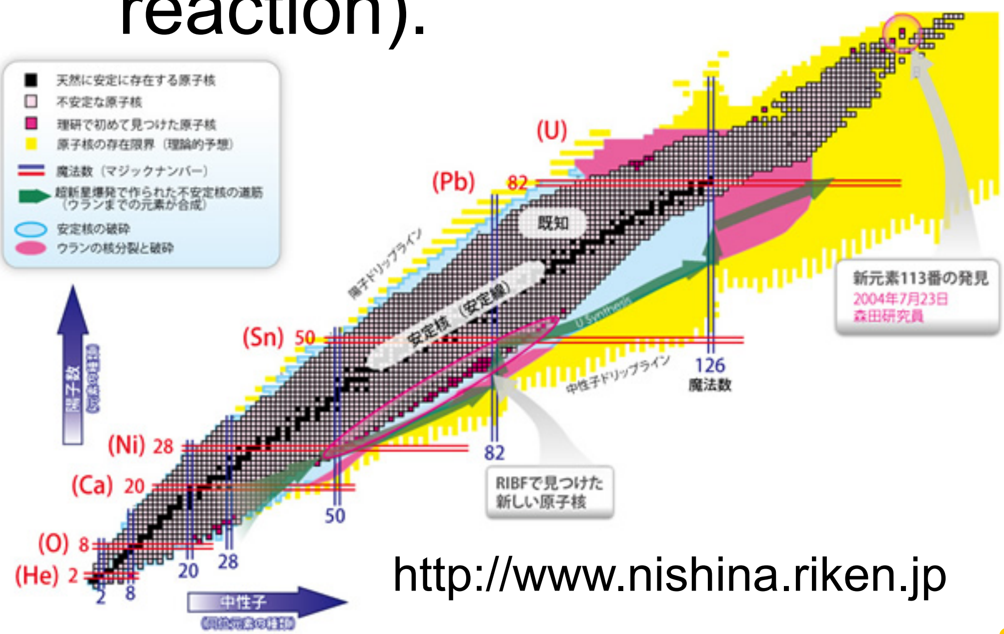
The dependence of **q.p.** resonance width and energy ($e_{2p1/2} = -0.056\ MeV$) on Δ

$e_R = 0.45$MeV			
Δ [MeV]	**0.0**	**1.634**	**1.897**
Γ [MeV]	**0.854**	**0.652**	**0.453**
ΔV_0 [MeV]	3.677	2.0	0.0
e_{sp} [MeV]	0.450	0.250	-0.056

- In order to extract the mixing effect by the pairing, we compare these three curves at the same resonance energy ($e_R = 0.45\ MeV$).

R-process and neutron-rich nuclei

- Our ultimate goal: we contribute the understand of **neutron capture phenomena** in the r-process using many-nucleon theory (nuclear structure and reaction).



http://www.nishina.riken.jp

- **R-process:**
  - Rapid neutron capture and β-decay in neutron-rich nuclei.
  - Site: supernova explosion and neutron star merger.

Energy scale: $E \lesssim 1\ MeV$

- **We need describe low-energy neutron capture phenomena in neutron-rich nuclei.**

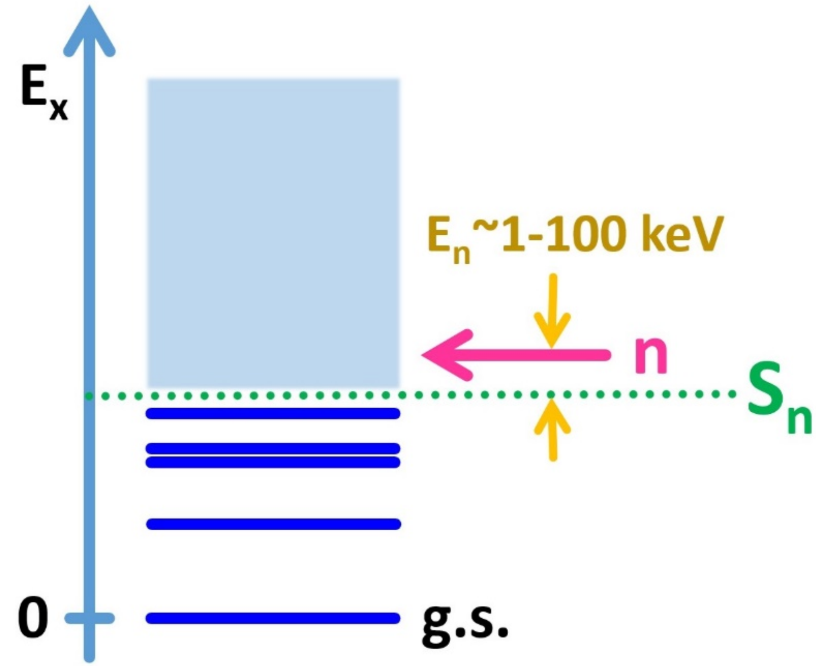
Low-energy neutron capture and continuum

- The temperature of supernova : $T \sim 10^7\ K - 10^9\ K$

  $E \sim 1 keV - 0.1 MeV$

- **The structure of continuum near neutron emission threshold in neutron-rich nuclei** is important for the r-process neutron capture phenomena.

- **We study low-lying single-particle resonance in neutron drip-line nuclei with the pairing correlation.**



The resonance width and the pairing correlation

Well bound nuclei $\lambda \cong -8.0$ **eV**

- Quasi-particle resonance associated with a deep-hole orbit can emerge.
- In analysis of the resonance width, the pairing effect is treated <u>in a perturbative way</u>.

$$(\epsilon_i - \lambda)^2 \gg \Delta^2$$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
A. Bulgac, Preprint(1980); nucl-th/9907088
J. Dobaczewski et al., Phys. Rev. C 53 2809 (1996)

- **The resonance width is evaluated on the basis of Fermi's golden rule.**

$$\Gamma_i = 2\pi \left| \int d^3 r \, \varphi_i(r) \Delta(r) \varphi_\epsilon(r) \right|^2 \propto |\Delta_{average}|^2$$

→ **The width is proportional to the square of the pair gap.**

The resonance width and the pairing correlation

Weakly bound nuclei $\lambda \cong 0.0 \sim -1.0$ **eV**

- The pairing correlation may cause strong configuration mixing between weakly bound orbits and low-lying continuum orbits. $(\epsilon_i - \lambda)^2 \lesssim \Delta^2$

- **The perturbative description may not be applicable.**

  ➡ **We expect an undisclosed relation between the quasi-particle resonance and the pairing.**

We analyze in detail how the width of the low-lying ($E \lesssim 1$MeV) quasi-particle resonance is governed by the pairing correlation in the neutron drip-line nuclei without perturbative way.

The Hartree-Fock-Bogoliubov theory

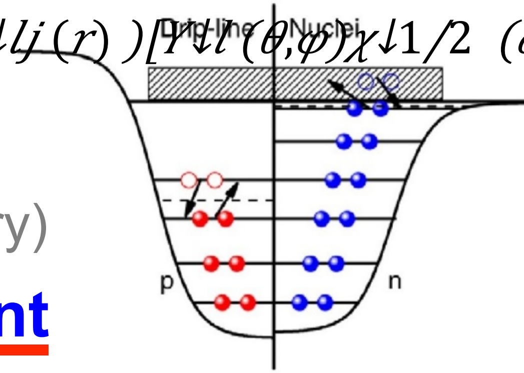
(which is equivalent to the Bogoliubov-de Gennes theory)

- The generalized Bogoliubov transformation

$$\psi(x)=\sum_{i=1}^{\infty}\varphi_i^{(1)}(x)\,\beta_i-\varphi_i^{(2)}(x)\,\beta_i^{\dagger} \qquad x=r,\sigma \qquad \beta_i\,|HFB\rangle=0$$

- Bogoliubov quasi-particle has the two components.

$$\varphi_i(x)=\begin{pmatrix}\varphi_i^{(1)}(x)\\ \varphi_i^{(2)}(x)\end{pmatrix}=\frac{1}{r}\begin{pmatrix}u_{lj}(r)\\ v_{lj}(r)\end{pmatrix}[Y_l(\theta,\varphi)\chi_{1/2}(\sigma$$



(with spherical symmetry)

Upper component: "particle" component

Lower component: "hole" component

The upper component could be scattering wave in weakly bound nuclei.

My notation is same as J. Dobaczewski, H. Flocard and J. Treiner, Nucl. Phys. A 422 103 (1984)
M. Matsuo, Nucl. Phys. A 696, 371 (2001)

The Hartree-Fock-Bogoliubov equation

- The density and the pairing density

$\rho(x)=\langle HFB|\psi^\dagger(x)\psi(x)|HFB\rangle=\sum_i |\varphi_i^{(2)}(x)|^2$ $\chi(x)=\langle HFB|\psi(x)\psi(x)|HFB\rangle=\sum_i \varphi_i^{(1)}(x)\varphi$

- **<u>The HFB equation in the coordinate space</u>**

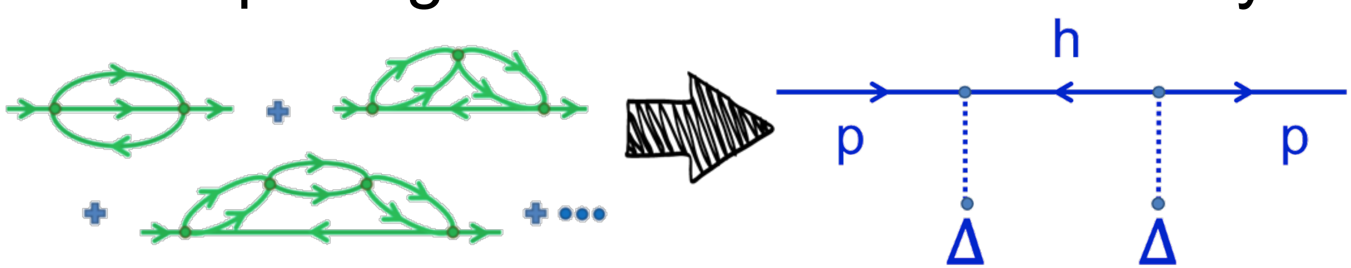
J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

$$\begin{pmatrix} -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+U_{lj}(r)-\lambda & \Delta(r) \\ \Delta(r) & \frac{\hbar^2}{2m}\frac{d^2}{dr^2}-U_{lj}(r)+\lambda \end{pmatrix}\begin{pmatrix} u_{lj}$$

$$\begin{pmatrix}\varphi_i^{(1)}(x) \\ \varphi_i^{(2)}(x)\end{pmatrix}=\frac{1}{r}\begin{pmatrix}u_{lj}(r) \\ v_{lj}(r)\end{pmatrix}[Y_l(\theta,\varphi)\chi_{1/2}(\sigma)]_{jm}$$

$U_{lj}(r)$: HF potential with $l\cdot s$ interaction, $\Delta(r)$: Pair potential

- The pairing correlation is described by the pair potential.



$\Delta(r)\leftrightarrow\boldsymbol{\rho}(\boldsymbol{r})$

Asymptotic form of quasi-particle in finite nuclei

- The nucleus has finite size.

For sufficiently large region ($r\to\infty$)

$$U_{lj}(r)|_{r\to\infty}=0,\ \Delta(r)|_{r\to\infty}=0$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u_{lj}(r)=(\lambda+E)u_{lj}(r)$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}v_{lj}(r)=(\lambda-E)v_{lj}(r)$$

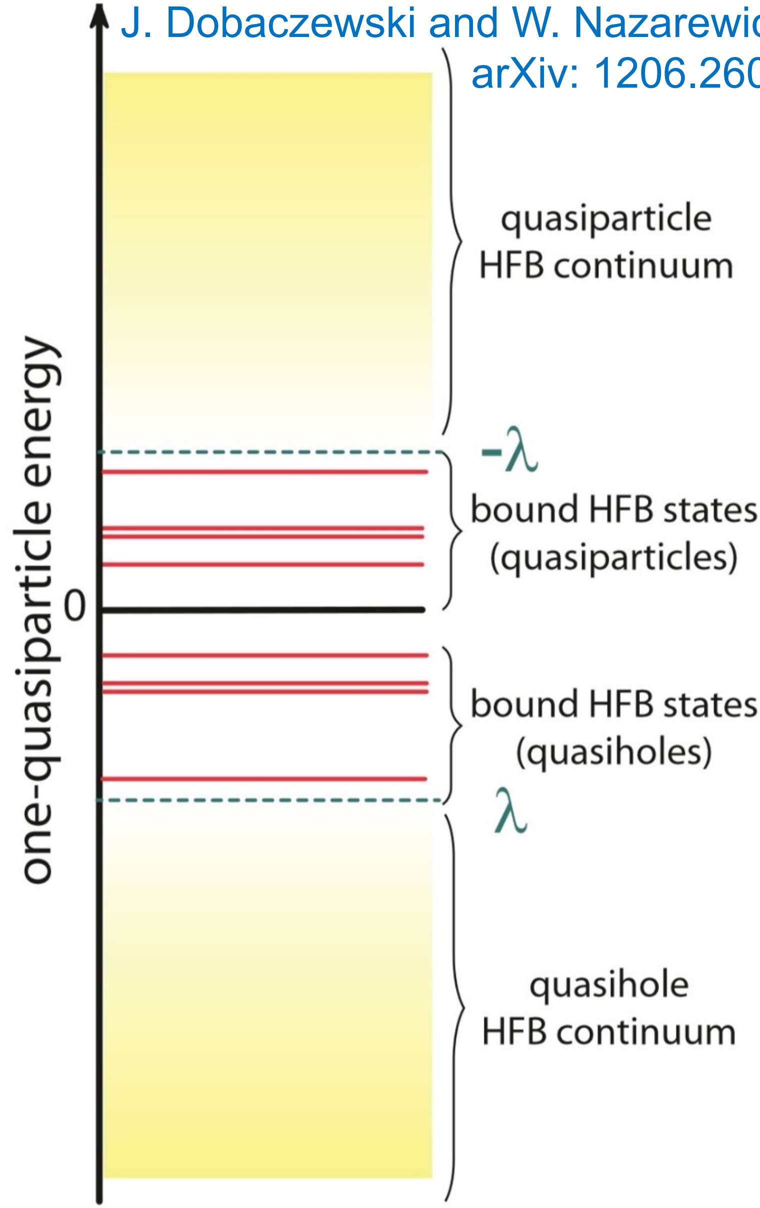
- The quasi-particle spectrum

J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

$E<|\lambda|$: **discrete states**

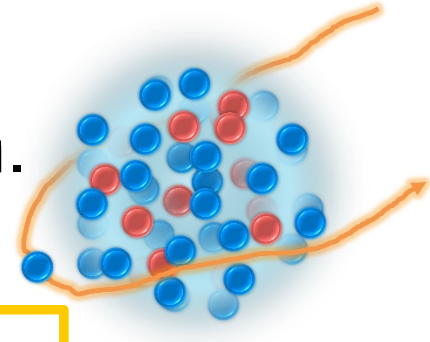
$E>|\lambda|$: **continuum states**

J. Dobaczewski and W. Nazarewicz arXiv: 1206.2600



Scattering boundary condition

- We consider a system consisting of a superfluid nucleus and impinging neutron.



We adopt an approximation:

> **The unbound neutron is treated as an unbound quasi-particle state, governed by the HFB eq., built on a pair-correlated even-even nucleus.**

- **Scattering boundary condition on the Bogoliubov quasi-particle (with positive E).**

$$\frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = C\begin{pmatrix} \cos\delta_{lj}\, j_l(k_1 r) - \sin\delta_{lj}\, n_l(k_1 r) \\ D\, h_l^{(1)}(i\kappa_2 r) \end{pmatrix} \xrightarrow{r\to\infty} C\begin{pmatrix} \sin(k_1 r - l\pi/2 + \delta_{lj})/k_1 r \\ 0 \end{pmatrix}$$

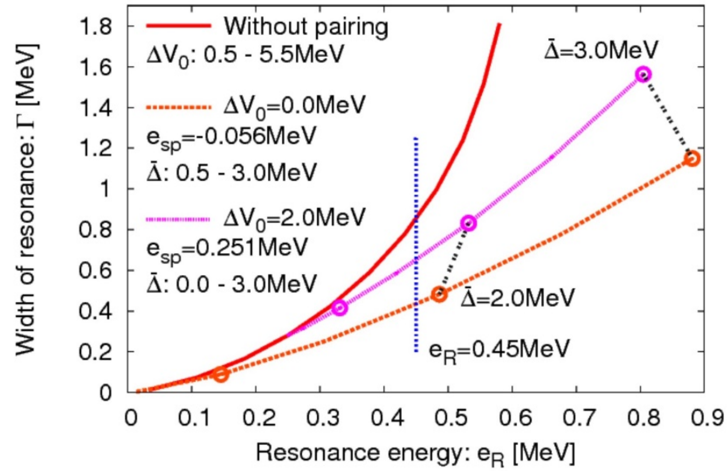
$k_1 = \sqrt{2m(\lambda+E)/\hbar^2}$, $\kappa_2 = \sqrt{-2m(\lambda-E)/\hbar^2}$, $C = \sqrt{2mk_1/\hbar^2\pi}$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
M. Grasso et al., Phys. Rev. C 64 064321 (2001)
I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

The possibility of observing the q.p. resonance

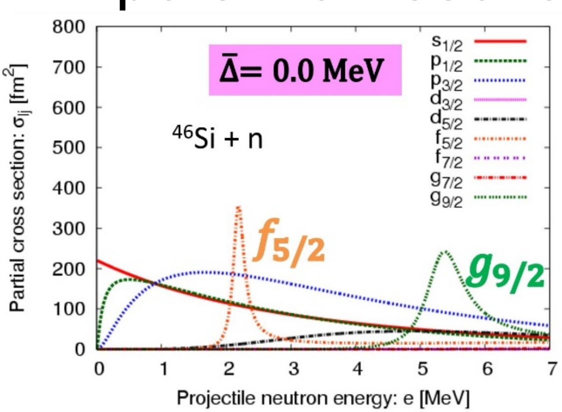
- The level density of low-lying region is low. Thus coupling to complex configuration are expected to be suppressed.

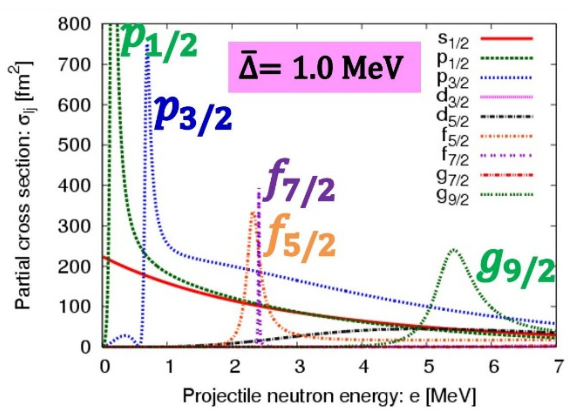
- At the same resonance energy, the width of particle-like q.p. resonance is narrower than that of s.p. potential resonance.

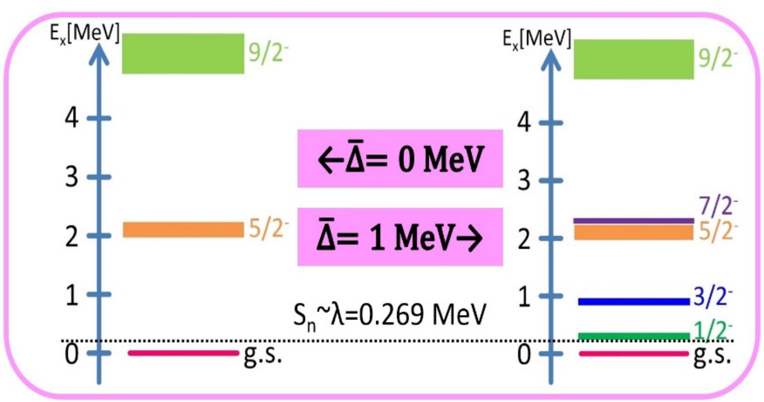


- The q.p. resonance can exist above the centrifugal barrier.

- The order of q.p. resonances is opposite to the order of s.p. potential resonances.



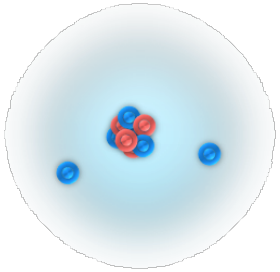




s wave and the pairing correlation in drip-line nuclei

- The neutron halo is a typical example of the pairing correlation effect on weakly bound s wave neutron.

Ex) The studies of s wave and the pairing correlation in drip-line nuclei

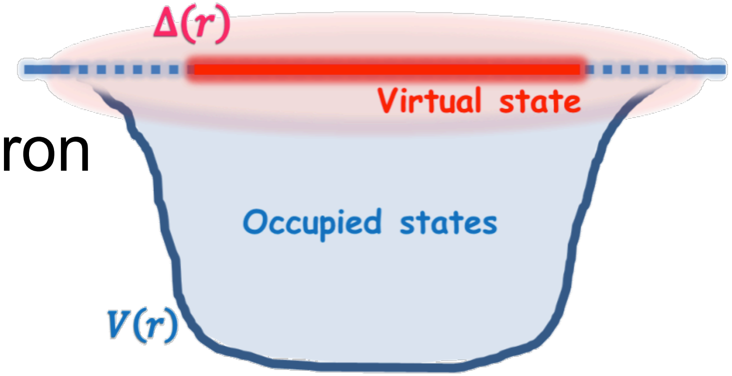


- ☐ Pairing anti-halo effect   K. Bennaceur et al., Phys. Lett. B 496 154 (2000)

  Diverging wave function is suppressed by the pairing

- ☐ Reduced effective pair gap   I. Hamamoto, B. R. Mottelson Phys. Rev. C 69 064302 (2004)

  Influence of the pairing on s wave is small



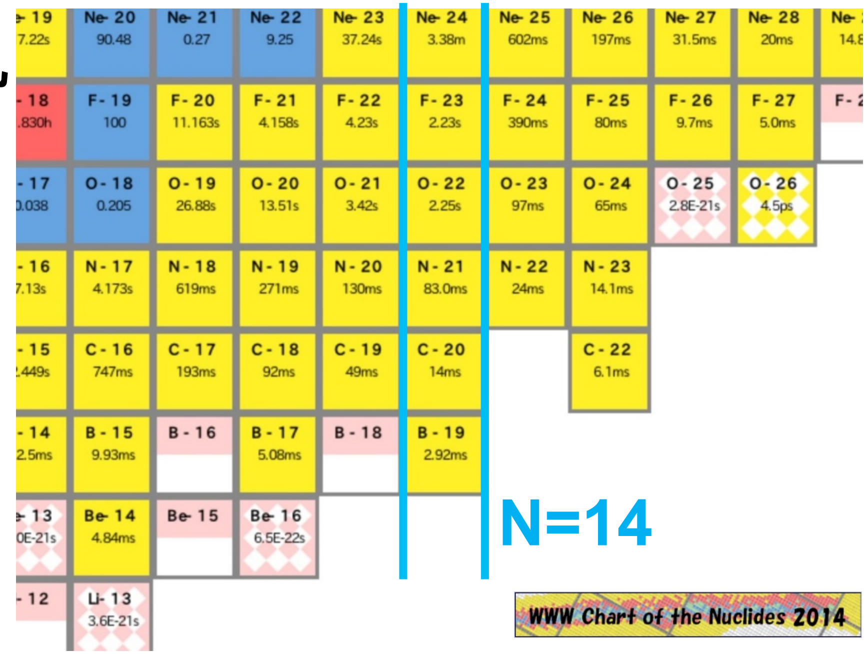
- Not only weakly bound s wave neutron but also…
  - **virtual state at 0 energy**
  - **Low-energy s wave scattering**

are influenced by the pairing.

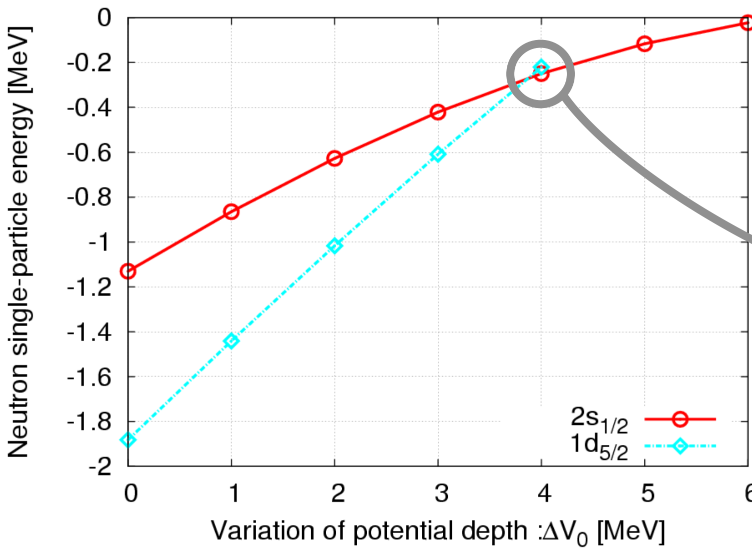
^{20}Cにおける一中性子弾性散乱：(^{20}C+n)*

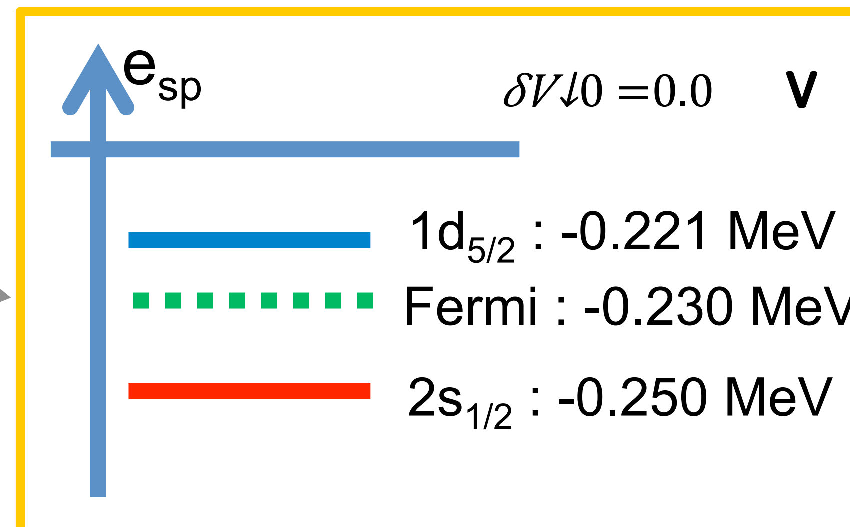
- 低エネルギーs波中性子散乱の分析を^{20}Cにおける一中性子弾性散乱を通して行う。

弱束縛s軌道 → virtual state

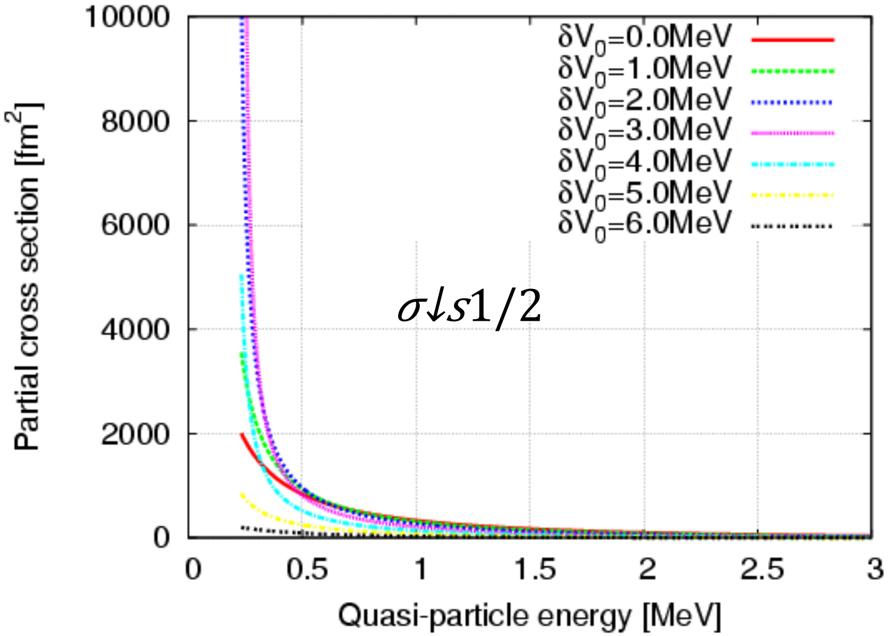


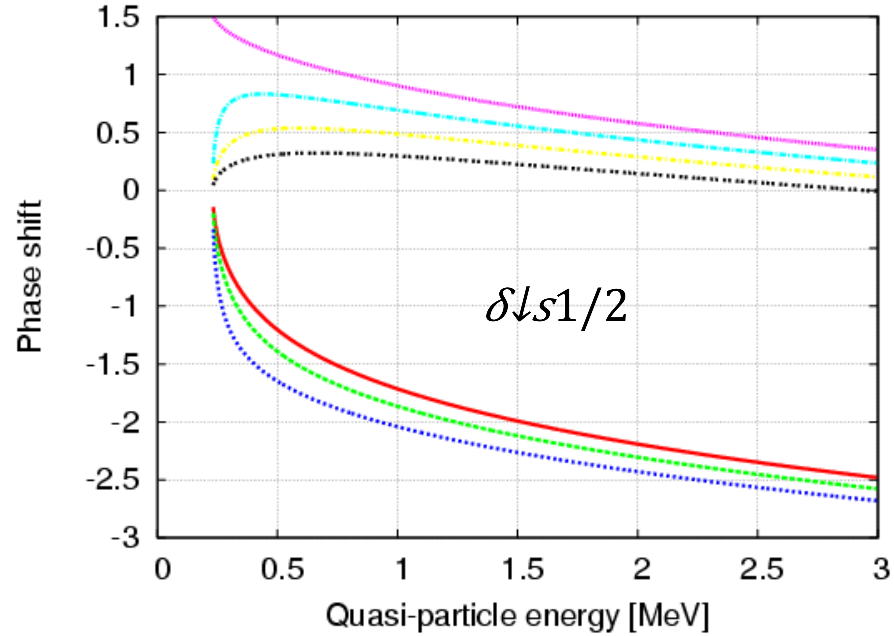
- Woods-Saxonポテンシャル中の弱束縛$2_{s1/2}$軌道を用意。



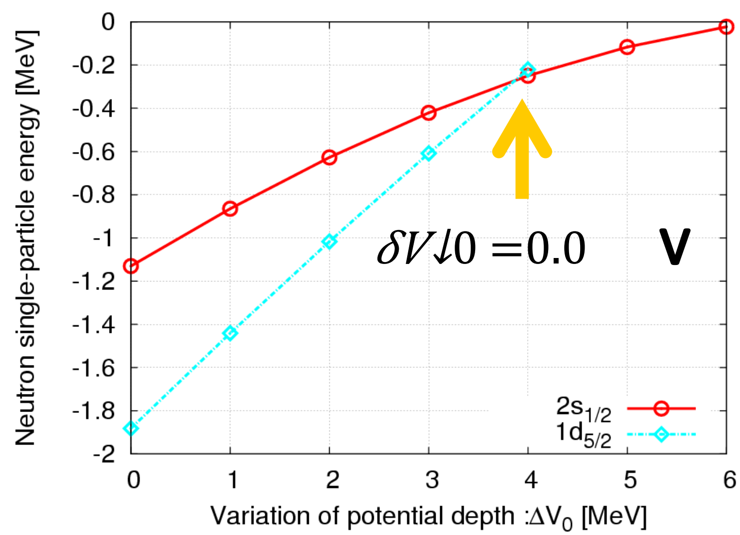


Virtual stateと一粒子軌道





- $\delta V\downarrow 0 = 3.0$ V のときに Virtual state となる。
- この振る舞いは、$2s_{1/2}$軌道の1粒子エネルギーの結果と矛盾しない。

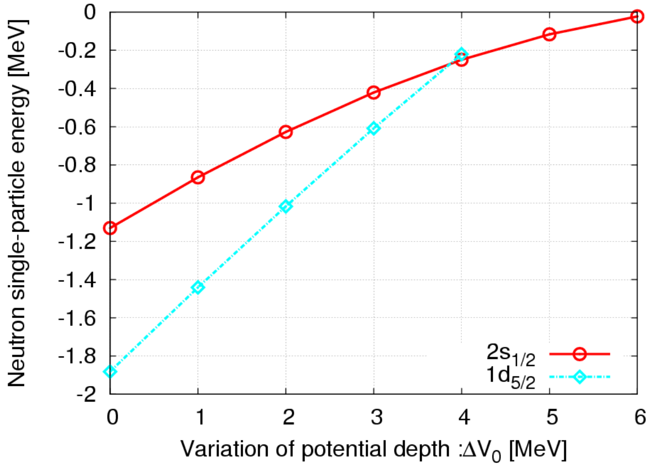


低エネルギー公式でvirtual stateを記述する

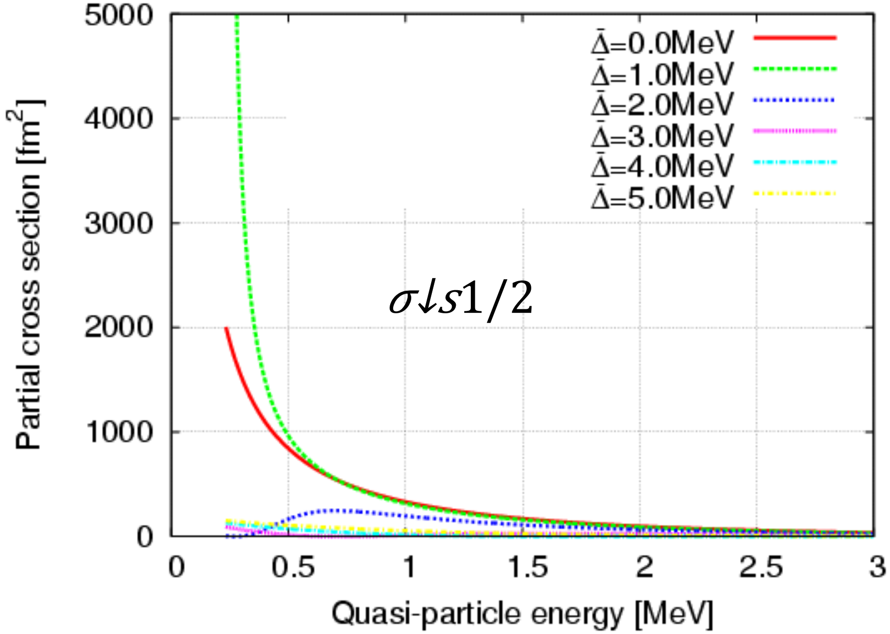
- 低0エネルギー極限の低エネルギー公式を用いたFittingにより、散乱長(a)と有効距離(r_{eff})を位相のずれから抽出する。 $k\cot\delta \cong -1/a + 1/2\, k^2 r_{eff}$
- 低エネルギー公式が有効だと考えられる範囲でFittingを行う。(0<e_{sp}<0.3 MeV) → $k r_{eff} \ll 1.0$

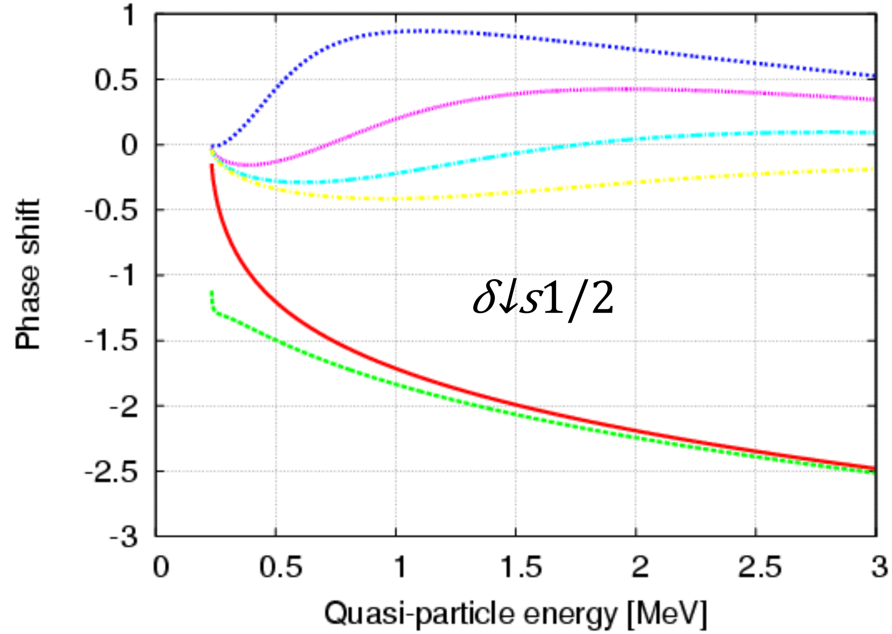
δV_0 [MeV]	1/a [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.0590	5.839
2.0	0.0335	6.401
3.0	-0.000363	7.275
4.0	-0.0475	8.762
5.0	-0.1981	11.296
6.0	-0.2466	16.637

1/aが非常に小さい値。(Virtual state)



Virtual stateは対相関によっても生ずることがある



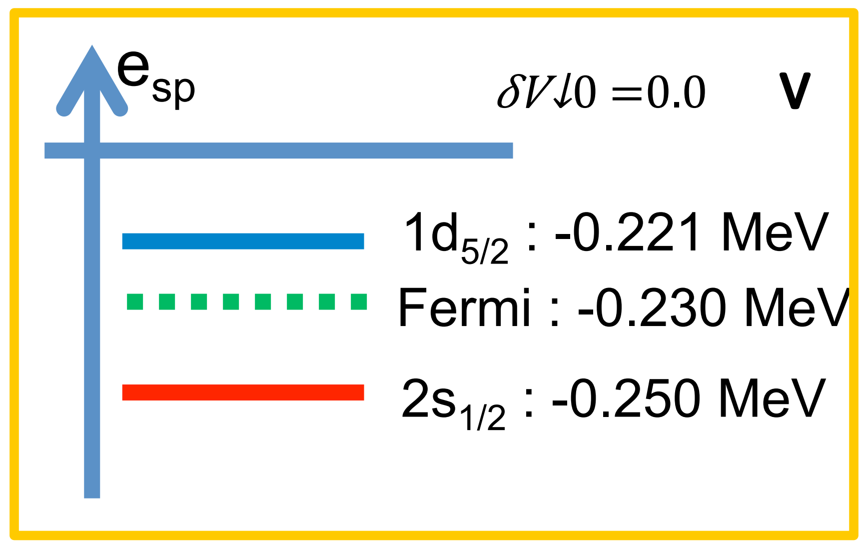


- Δ　の変化によっても影響を受けていることが分かる。

- 位相のずれの振る舞いより…

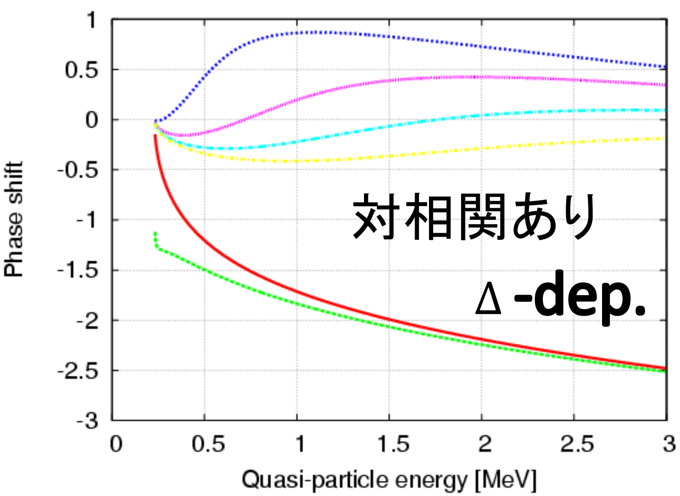
  $\Delta = 1.0$**MeV**　:ギリギリ束縛

  $\Delta = 2.0$**MeV**　:連続状態中



対相関効果は低エネルギー公式では記述できない

- 対相関なしのときと同様に、低エネルギー公式から散乱長と有効距離を抽出する。 $k_1 = \sqrt{2m(\lambda+E)/\hbar^2}$
- 波数はparticle成分波動関数のもの。



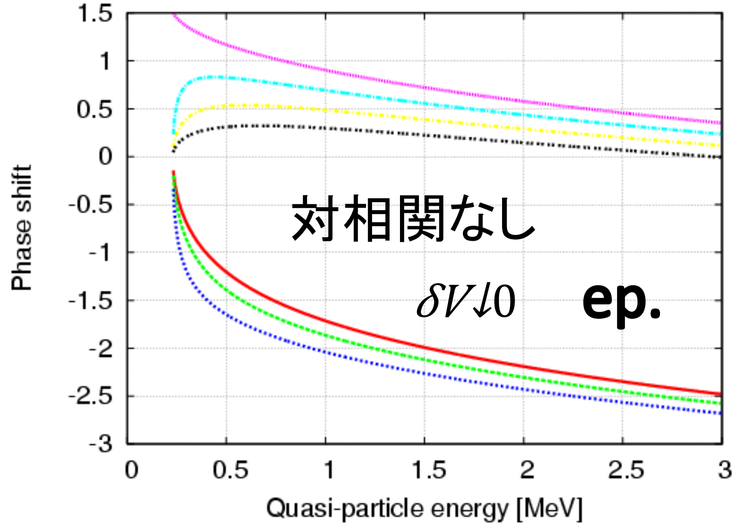
Δ [MeV]	1/a [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.00825	-1.478
2.0	-0.9279	-109.617
3.0	0.3160	-69.521
4.0	0.3018	-14.192
5.0	0.2862	-5.711

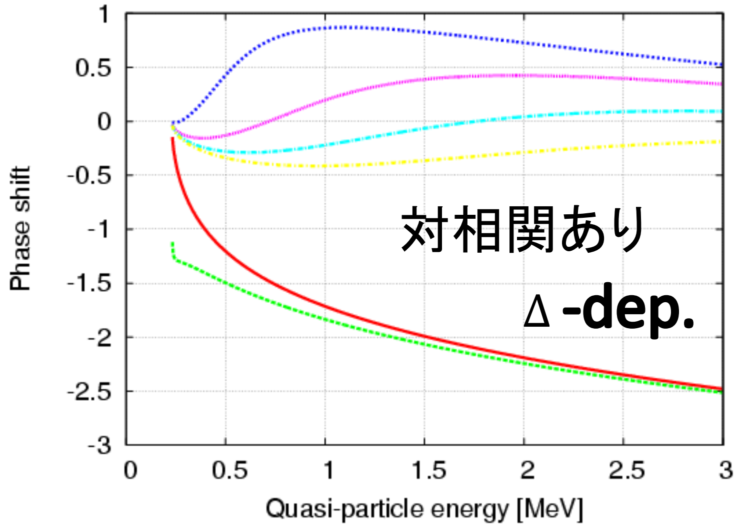
- 散乱長の符号がポテンシャル散乱とは異なる。
- 有効距離が負の値になる。

→ <u>低エネルギー公式を越える振る舞い。</u>

結論：virtual state（s波散乱）に対する対相関効果

- 対相関の効果によってもVirtual stateが生ずる。
- 位相のずれや弾性散乱断面積は、ポテンシャル散乱のときとは異なる振る舞いをする。 $k\cot\delta \cong -1/a + 1/2\, k^2 r_{eff}$
- 低エネルギー公式を超える振る舞い。





課題：平方井戸型ポテンシャルを用いた解析的分析
実験データ（SAMURAI）との比較

2016/08/29（Mon.）
The 15th CNS Summer School@Nishina hall, RIKEN

Effects of pairing correlation on the s-wave scattering in neutron-rich nuclei

Yoshihiko Koayashi, Masayuki Matsuo
（***Niigata University, Japan***）

- Pairing correlation and continuum coupling in weakly bound nuclei (neutron-rich nuclei)
- The Hartree-Fock-Bogoliubov theory in the coordinate space (Bogoliubov-de Gennes theory)
- Numerical results for (^{20}C+n)*: $\sigma_{s1/2}$, $\delta_{s1/2}$, a, and r_{eff}
- Conclusion and perspective

Pairing correlation influences the continuum

- Many nuclei with open-shell configuration have **superfluidity** generated by **the pairing correlation**.

Pair gap in nuclei

$\Delta \sim 12.0/\sqrt{A} \sim O(1\ MeV)$

A. Bohr and B. R. Mottelson
Nuclear Structure

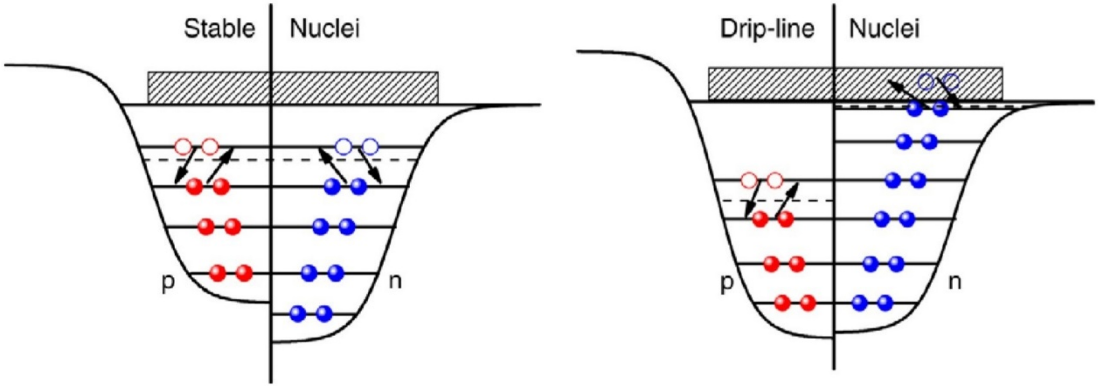


Figure is taken from J. Meng et. al., Prog. Part. Nucl. Phys. 57, 470 (2006)

- The pairing correlation causes configuration mixing
  - among bound orbits in well bound nuclei.
    $\lambda \sim 8\ MeV$
  - involving both **bound and unbound (continuum) orbits** in weakly bound nuclei.
    $\lambda \sim 0\ MeV$

Scattering particle is influenced by the pairing

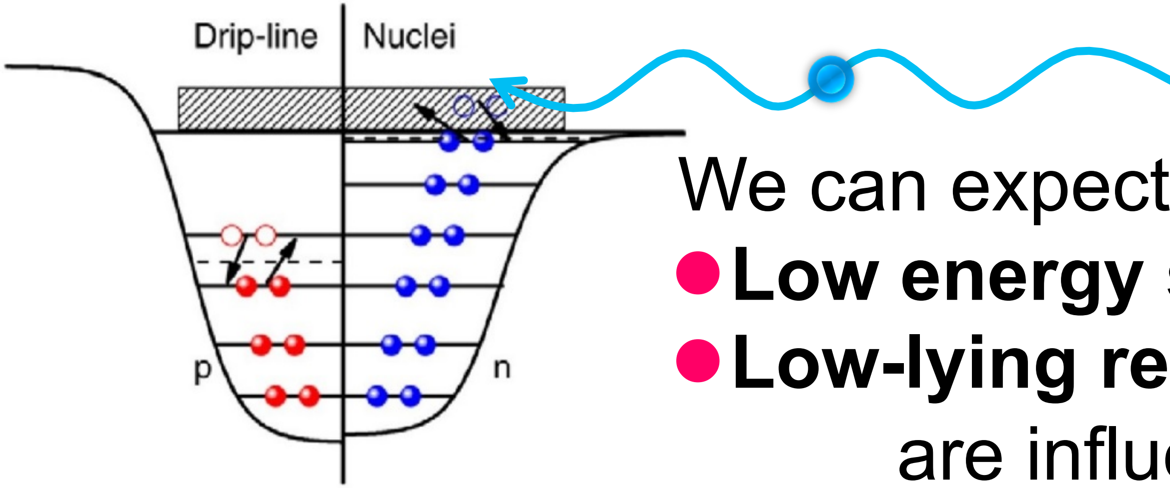


Figure is taken from
J. Meng et. al., Prog. Part. Nucl. Phys. 57, 470 (2006)

We can expect that…

- **Low energy scattering particle**
- **Low-lying resonance**

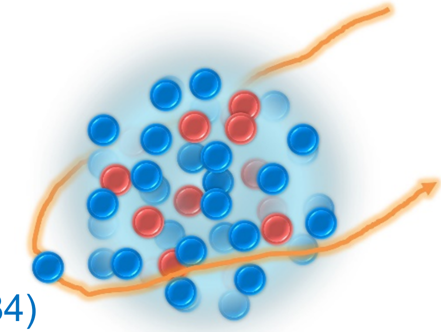
are influenced by the pairing.

- **In present study, we analyze properties of <u>low energy s-wave scattering and virtual state</u> on neutron-rich nuclei with the pairing correlation.**

- Low angular momentum wave (s and p) can approach nuclei easily due to no or small centrifugal barriers.

Pairing theory in the coordinate space is needed

- **The Hartree-Fock-Bogoliubov theory** can describe both the pairing correlation and scattering waves.



J. Dobaczewski, H. Flocard and J. Treiner, Nucl. Phys. A 422 103 (1984)

※This is called the Bogoliubov-de Gennes theory in solid state physics.

Hole component (bound w.f.)

Generalized Bogoliubov transformation

ψ(x)=∑i=1↑▒φ↓i↑(1) (x) β↓i + φ↓i↑(2) (x) β↓i↑†

Particle component (can be scattering w.f.)

Hartree-Fock potential

Hartree-Fock-Bogoliubov equation

(■−ℏ↑2 /2m d↑2 /dr↑2 +U↓lj (r)−λ&Δ(r)@Δ(r)&ℏ↑2 /2m d↑2 /

Pair potential

※ (■φ↓i↑(1) (x)@φ↓i↑(2) (x))= 1/r (■u↓lj (r)@v↓lj (r))[Y↓l (θ,φ)χ↓1/2 (σ)]↓jm

Numerical calc.: Boundary condition and potentials

- Scattering boundary condition（$E>-\lambda$）

$$\frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = C\begin{pmatrix} \cos\delta_{lj}\, j_l(k_1 r) - \sin\delta_{lj}\, n_l(k_1 r) \\ D\, h_l^{(1)}(i\kappa_2 r) \end{pmatrix} \xrightarrow{r\to\infty} C\begin{pmatrix} \sin(k_1 r - l\pi/2 + \delta_{lj})/k_1 r \\ 0 \end{pmatrix}$$

$k_1 = \sqrt{2m(\lambda+E)/\hbar^2}$, $\kappa_2 = \sqrt{-2m(\lambda-E)/\hbar^2}$, $C = \sqrt{2mk_1/\hbar^2\pi}$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
M. Grasso et al., Phys. Rev. C 64 064321 (2001)
I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

- HF potential and pair potential ← **Woods-Saxon shape**

$$U_{lj}(r) = \left[V_0 + (\boldsymbol{l}\cdot\boldsymbol{s})V_{SO}\frac{r_0^2}{r}\frac{d}{dr}\right]f_{WS}(r) \qquad f_{WS}(r) = \left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

$$\Delta(r) = \Delta_0 f_{WS}(r)$$

We can control the shapes easily through the parameters.

- Δ_0 is controlled by the average pair gap $\bar{\Delta}$.

$$\bar{\Delta} = \int dr\, r^2 \Delta(r) f_{WS}(r) \Big/ \int dr\, r^2 f_{WS}(r)$$

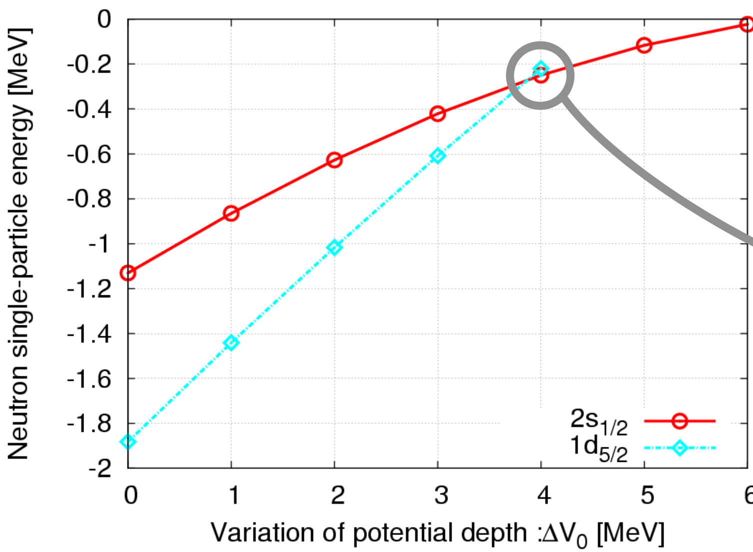
I. Hamamoto, B. R. Mottelson, Phys. Rev. C 68 034312 (2003)

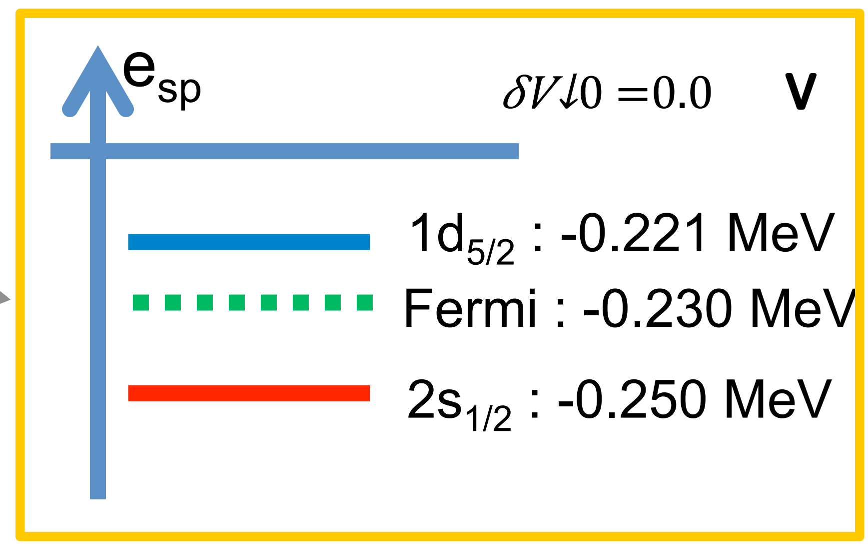
Neutron elastic scattering on ^{20}C : (^{20}C+n)*

- Low energy s-wave scattering on ^{20}C

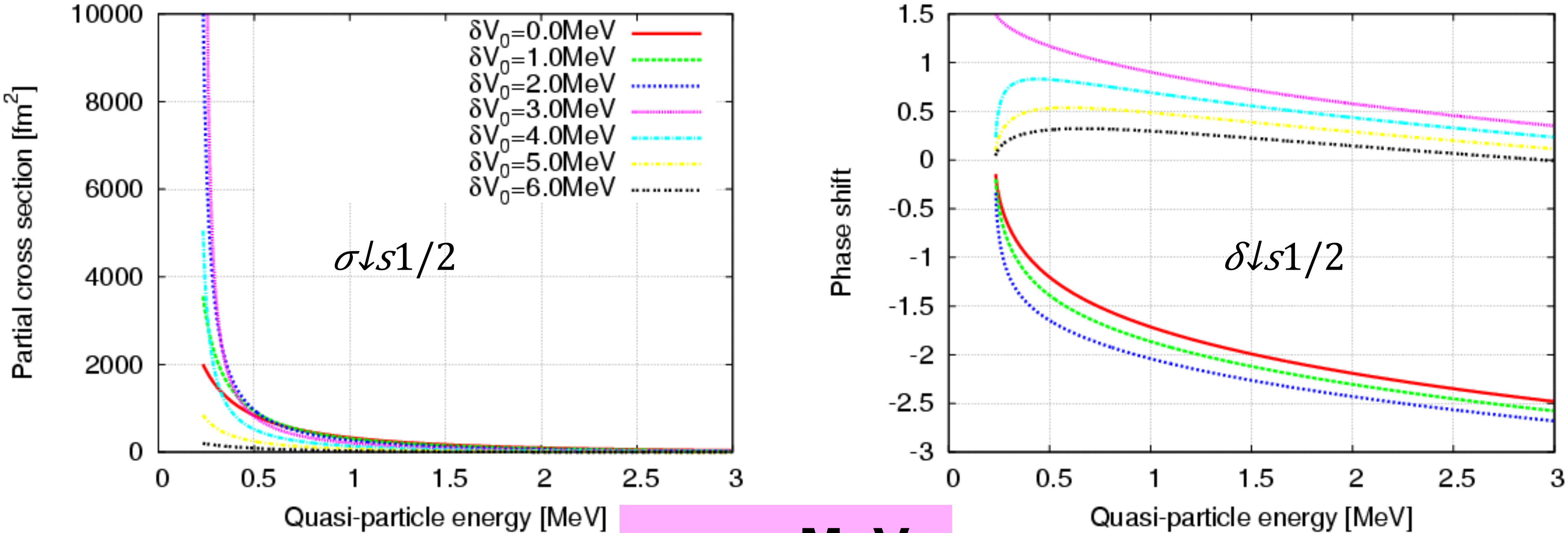
$2s_{1/2}$ orbit is located around the continuum threshold → virtual state

- Weakly bound $2s_{1/2}$ orbit in Woods-Saxon potential



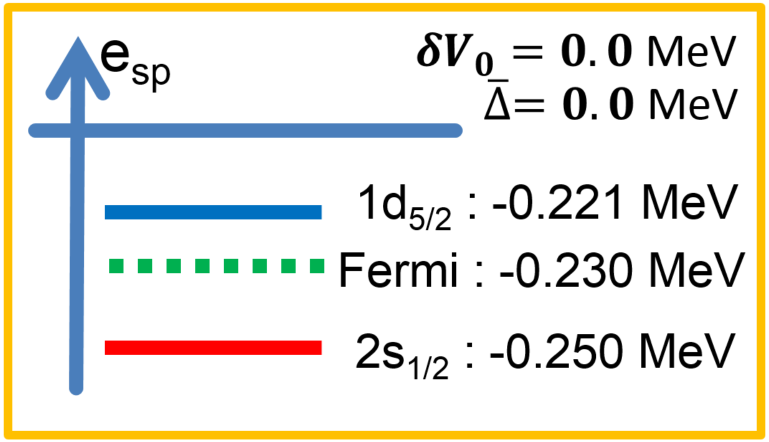


(^{20}C+n)*: elastic cross sections and phase shifts



$\Delta = 0.0$ MeV

- Calculation is performed for various values of the potential depth (δV_0).
- $\delta V_0 = 3.0$ MeV case corresponds to a virtual state.



$(^{20}C+n)^*$: scattering length and effective range

- The scattering length (a) and the effective range (r_{eff}) are extracted from the calculated phase shift with low-energy effective range formula.

$$k\cot\delta \cong -\frac{1}{a} + \frac{1}{2}k^2 r_{eff}$$

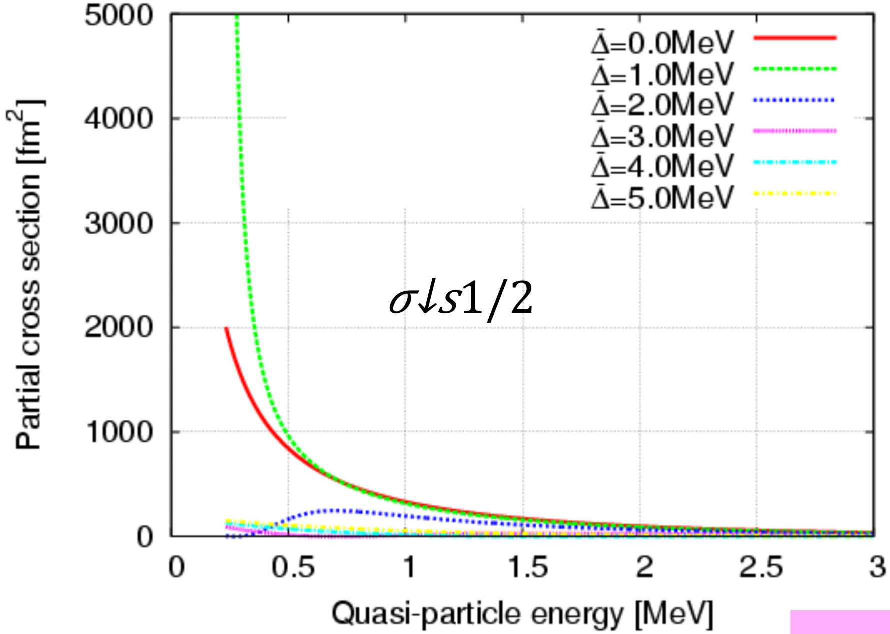
- Fitting is done in $0 < e_{sp} < 0.3$ MeV.

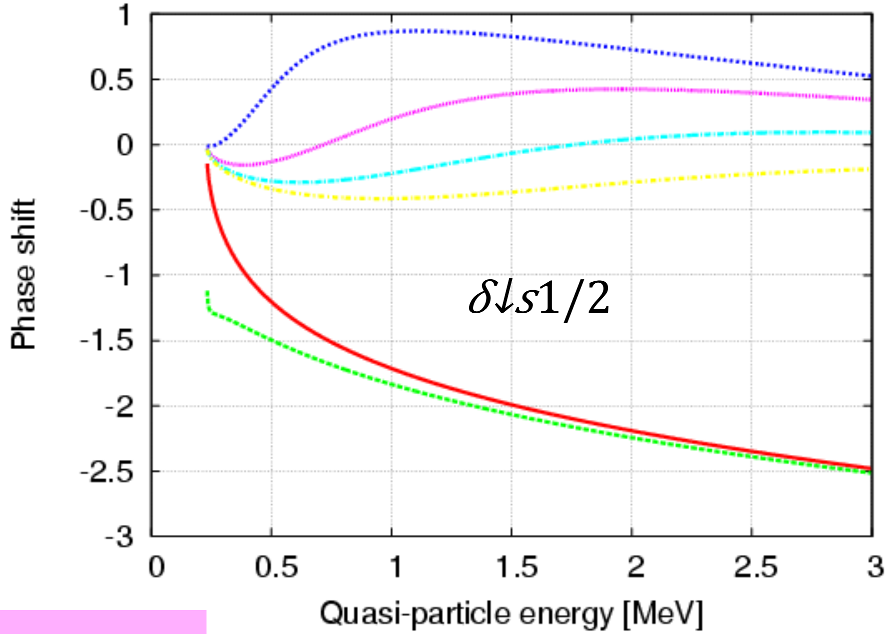
※applicable region of the effective range formula: $k\, r_{eff} \ll 1.0$

δV_0 [MeV]	1/a [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.0590	5.839
2.0	0.0335	6.401
3.0	-0.000363	7.275
4.0	-0.0475	8.762
5.0	-0.1981	11.296
6.0	-0.2466	16.637

Virtual state ($1/a$ is very small value.)

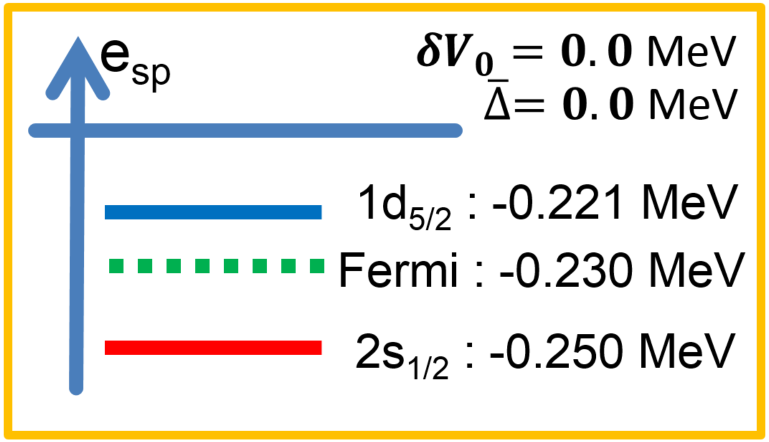
Elastic cross section and phase shift with pairing





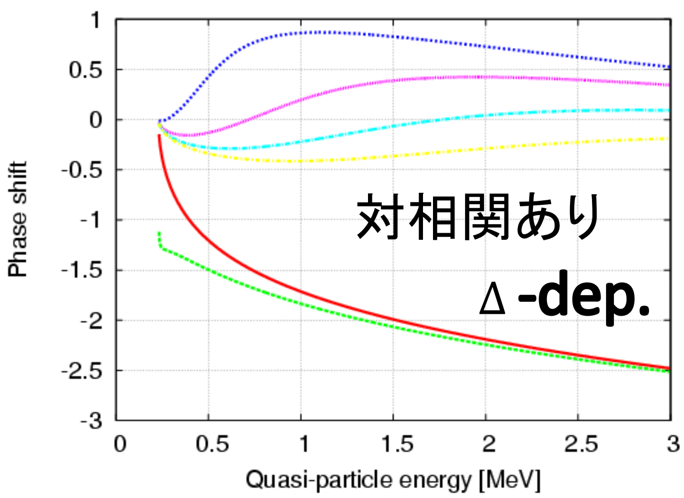
$\Delta \neq 0.0$ **MeV**

- Calculation is performed for various values of the pairing strength (Δ).
- $\boldsymbol{\sigma_{s1/2}}$ and $\boldsymbol{\delta_{s1/2}}$ are influenced by the pairing.



Pairing effect cannot be described by effective range formula

- The scattering length and the effective are extracted from the calculated phase shift. $k_1 = \sqrt{2m(\lambda+E)/\hbar^2}$
- k_1 is k of the particle component.



Δ [MeV]	1/a [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.00825	-1.478
2.0	-0.9279	-109.617
3.0	0.3160	-69.521
4.0	0.3018	-14.192
5.0	0.2862	-5.711

- **The nature of extracted results are very different from $\Delta = 0.0$ MeV case.**

→ **beyond the effective range formula**

Conclusion and perspective

- Elastic cross section σ_{lj} and phase shift δ_{lj} are influenced by the pairing strongly.
- The effect of pairing correlation cannot be described by the effective range formula.
- In progress…: **analysis of the S-matrix pole with Δ.**

$\Delta = 0.0 MeV$ | $\Delta = 1.0 MeV$ | $\Delta = 2.0 MeV$

Re($S_{s1/2}$)

Im($S_{s1/2}$)