Multidimensional structure of chiral crystals in quark matter

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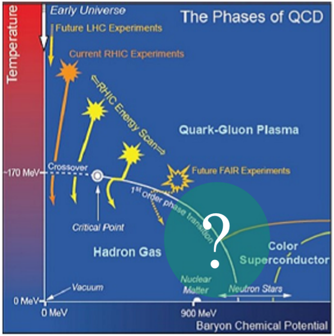
Nonvanishing baryon density

- Dense QCD phase diagram

Finite-density regime is still less well understood...



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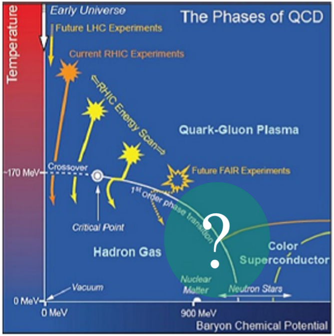

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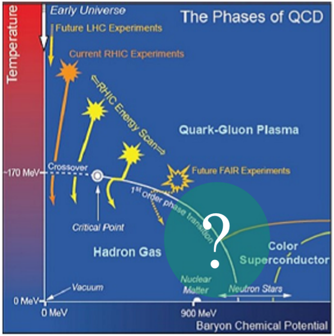

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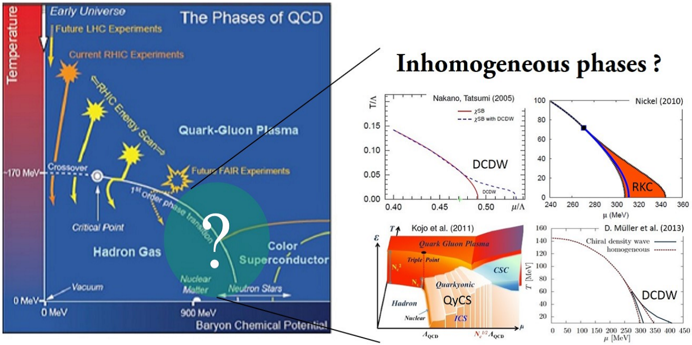
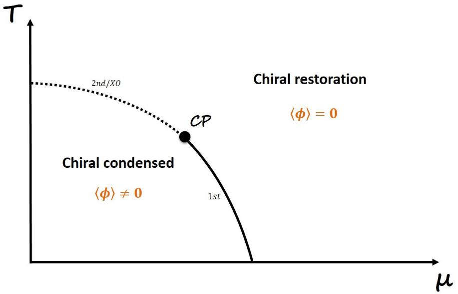
Recent theoretical studies predict inhomogeneous phases.



Inhomogeneous chiral phase

- Conventional picture (focused on the chiral phase transition)



$\Rightarrow$ chiral order parameter is constant in space (spatially homogeneously condensed)

$\Rightarrow$ what if one allows for the spatial dependence?

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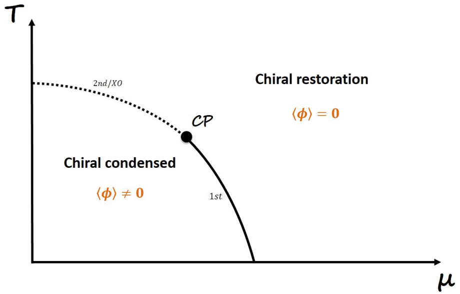

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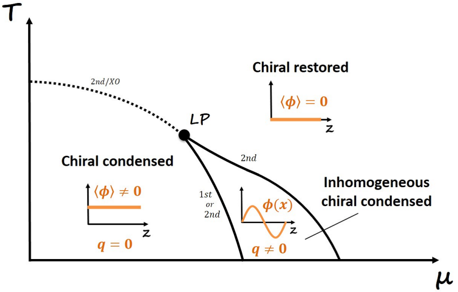
Inhomogeneous chiral phase

- Possible picture (focused on the chiral phase transition)



⇒ chiral-transition region is extended (chiral restoration is delayed)

⇒ restoration-picture may be changed (not the conventional 1st-order)

【*cf.* *Nakano-Tatsumi 2005; Nickel 2009; Müller et al. 2013, etc.*】

Outline

Basic features of 1D modulations

1D modulations (NJL results within MFA)

- NJL-model Lagrangian (chiral limit):

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau_a \psi)^2 \right]$$

- Mean-field approximation (space dependent condensates):

$$\sigma(\vec{x}) \equiv \langle \overline{\psi} \psi \rangle (\vec{x}), \quad \pi_a(\vec{x}) \equiv \langle \overline{\psi} i \gamma_5 \tau_a \psi \rangle (\vec{x}) \delta_{a3}$$

- Gap equations (minimizing thermodynamic potential $\mathcal{V}_{\mathrm{MF}}$):

$$\frac{\partial \mathcal{V}_{\mathrm{MF}}(T,\mu;\sigma,\pi_a)}{\partial \sigma(\vec{x})} = \frac{\partial \mathcal{V}_{\mathrm{MF}}(T,\mu;\sigma,\pi_a)}{\partial \pi_a(\vec{x})} = 0$$

- ▷ need to solve the Dirac eq. for arbitrary $\sigma(\vec{x}), \pi(\vec{x})$: $[i\partial\!\!\!/ + \sigma(\vec{x}) + i\gamma_5 \tau_3 \pi_a(\vec{x})]\psi = 0$
- ▷ assume the condensate shape based on known analytic solutions for 1+1D systems (use a possible ansätz for 1D modulations in 3+1D systems) 【*Başar-Dunne-Thies 2009; Nickel 2009*】
- ▷ obtain gap solutions by minimizing $\mathcal{V}_{\mathrm{MF}}$ w.r.t. variational parameters (Δ, q, ν)

1D modulations (typical examples)

General chiral order parameter: $\phi(x) \equiv \langle\bar{\psi}\psi\rangle(x) + i\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle(x) = \Delta(x)e^{i\theta(x)}$

- **DCDW modulation:** $\phi(z)=\Delta e^{iqz}$ 【*Nakano-Tatsumi 2005; Dautry-Nyman 1979; Flude-Ferrell 1964*】

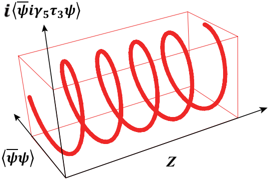
$$\langle\bar{\psi}\psi\rangle(z) = \Delta\cos(qz),\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle(z) = \Delta\sin(qz)$$

- **RKC modulation:** $\phi(z)=\Delta(z)$ 【*Nickel 2009; Thies 2006; Larkin-Ovchinnikov 1964*】

$$\langle\bar{\psi}\psi\rangle(z) = \frac{2\Delta\sqrt{\nu}}{1+\sqrt{\nu}}\mathrm{sn}\left(\frac{2\Delta z}{1+\sqrt{\nu}}|\nu\right),\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle(z) = 0$$

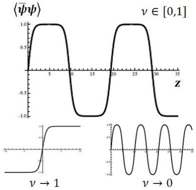
(Δ: amplitude, q: wavenumber, ν: elliptic modulus)

▷ Dual chiral density wave (DCDW)



(chiral spirals in 3+1D systems)

▷ Real kink crystal (RKC)



(solitonic domain walls in 3+1D systems)

1D modulations (NJL results within MFA)

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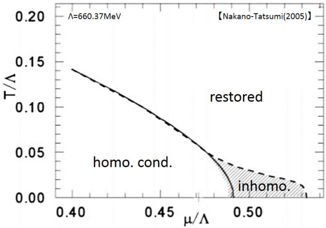
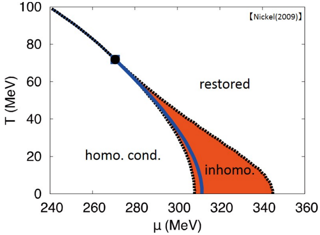
1D modulations (NJL results within MFA)

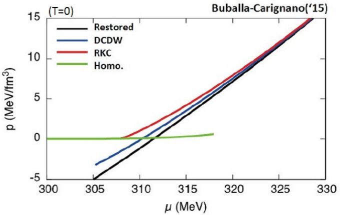
▶ DCDW: $\frac{\partial \mathcal{V}_{\text{MF}}(\phi_{\text{DCDW}})}{\partial \Delta, q} = 0$

▷ RKC: $\frac{\partial \mathcal{V}_{\text{MF}}(\phi_{\text{RKC}})}{\partial \Delta, \nu} = 0$





▷ free energies for DCDW and RKC condensates ($T = 0$)



⇒ RKC is energetically favored over DCDW within MFA in the chiral limit

⇒ but the situation is reversed at $\sqrt{eB} \neq 0$
【cf. Frolov et al. 2010, Tatsumi et al. 2014】

⇒ what if fluctuations are taken into account?

1D modulations (Beyond mean-field level)

▷ DCDW ground state: $\phi_0^T = (\Delta \cos qz, 0, 0, \Delta \sin qz)$

- introduce fluctuations around ϕ_0: 【*Lee-Nakano-Tsue-Tatsumi-Friman 2015*】

$$\phi(z) = U(\beta_i)\phi_0(z) = \begin{pmatrix} \Delta \cos(qz+\beta_3) \cos \beta_2 \cos \beta_1 \\ \Delta \cos(qz+\beta_3) \cos \beta_2 \sin \beta_1 \\ \Delta \cos(qz+\beta_3) \sin \beta_2 \\ \Delta \sin(qz+\beta_3) \end{pmatrix} \quad (= \phi_0 + \delta\phi)$$

- dispersion relation for gapless modes:

$$\omega^2 \sim a k_z^2 + b(\vec{k}^2)^2$$

⇒ spatially anisotropic (owing to the lack of $\vec{k}_t^2$-term)

⇒ symmetric under rotations about x-y (transverse) directions, as in smectic liquid crystals

- impacts of fluctuations:

$$\langle \phi(z) \rangle = \langle U(\beta_i)\phi_0(z) \rangle \simeq \begin{pmatrix} \Delta \cos(qz) e^{-\sum_i \langle \beta_i^2 \rangle /2} \\ 0 \\ 0 \\ \Delta \sin(qz) e^{-\langle \beta_3^2 \rangle /2} \end{pmatrix} \xrightarrow{\text{IR}} 0 \quad \text{(destroyed)}$$

where Gaussian fluctuations are logarithmically divergent at long-wavelength (IR) limit

$$\langle \beta_{i=1,2,3}^2 \rangle \simeq \frac{1}{2\Delta^2} \int \frac{d^3k}{(2\pi)^3} \frac{T}{\omega^2} \xrightarrow{\text{IR}} \infty \quad \text{(log div)}$$

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1D modulations (Beyond mean-field level)

▷ Landau-Peierls instability

- The DCDW phase is not expected to exist due to thermal fluctuations

$$\langle \phi(z) \rangle = 0$$

⇒ but exhibits quasi-long-range order with algebraically decaying correlation function with the nonzero power depending on T

$$\langle \phi(z\vec{e}_z)\cdot\phi^*(0)\rangle \sim \frac{1}{2}\Delta^2 \cos qz (z/z_0)^{-T/T_0} \qquad z_0 = 2q/\Lambda^2$$

$$\langle \phi(x_t\vec{e}_t)\cdot\phi^*(0)\rangle \sim \frac{1}{2}\Delta^2 (x_t/x_0)^{-2T/T_0} \qquad x_0 = 1/\Lambda,\ T_0 = 32\pi a_{6,1}\Delta^2 q$$

may be practically realized as a quasi-1D phase as in LCs 【*Lee-Nakano-Tsue-Tatsumi-Friman 2015*】

- The same applies to the RKC phase 【*Hidaka-Kamikado-Kanazawa-Noumi 2015*】
- There is no true LRO for 1D, but 2D/3D may be realized 【*cf. Landau-Lifshitz 1969*】

Beyond 1D modulations

Beyond 1D modulations (2D/3D modulations)

- ▷ no known analytic solutions for 2+1D or 3+1D systems (unlike purely 1+1D systems)
- ▷ assume possible ansätze for 2D/3D and compare their free energies with 1D

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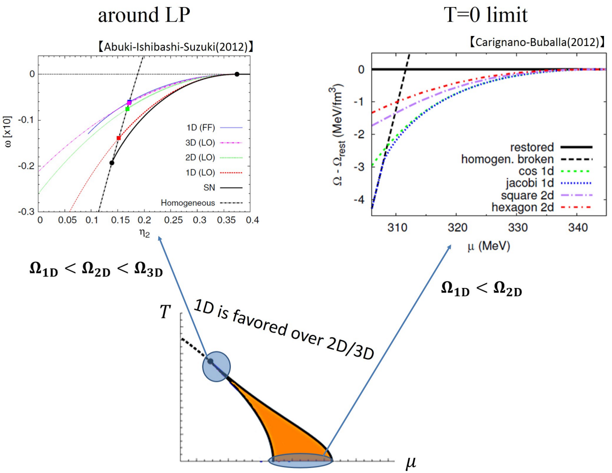
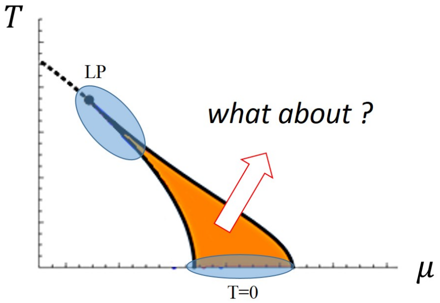
- ▷ assume possible ansätze for 2D/3D and compare their free energies with 1D
- ▷ there are a few studies for 2D/3D modulations 【*Abuki et al. 2012; Carignano et al. 2012*】



Beyond 1D modulations (2D/3D modulations)

- one knows only the area around the LP or at $T = 0$

  $\Rightarrow$ multidimensional modulation may be favored in different areas



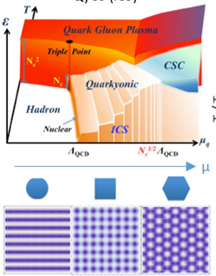
1D tends to be favored
around the LP or at T=0

BEYOND 1D MODULATIONS (2D/3D MODULATIONS)

- some remarkable results in different contexts

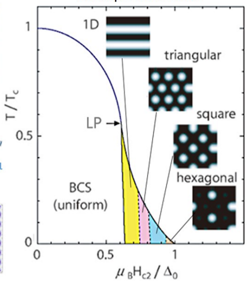
  ⇒ formation of multidimensional crystalline structures is predicted

QyCS (*ICS*)



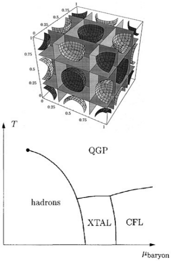
【*Kojo et al. 2012*】

2D superconductor



【*Shimahara 1998; Matsuda-Shimahara 2007*】

Crystalline CSC



【*Bowers-Rajagopal 2002*】

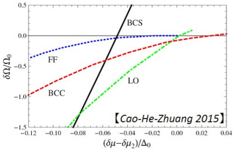
Pi cond.

№	Meson field	ξ
1	$\varphi_{\pi^0} = a\,(\cos kx + \cos ky + \cos kz)$	1
2	$\varphi_{\pi^0} = a\,(\cos kx + \cos ky)$	50/51
3	$\varphi_{\pi^0} = a \cos kx$	25/27
4	$\varphi_{\pi^0} = a \cos kx \ \cos ky \ \cos kz$	20/21

↑ favored

【*Migdal-Markin-Mishustin 1976*】

CSC (BCC)



【*Cao-He-Zhuang 2015*】

Beyond 1D modulations (2D/3D modulations)

- another possible way to find the multidimensional crystalline structures

  ⇒ Thomas-Fermi approximation

$$E_{TFA} = \sqrt{\cdots + \nabla\sigma(r) + \cdots}$$

effective derivative term

$$\frac{\delta E_{TFA}}{\delta\sigma(r)} = 0$$

for given σ(x);1D
σ(x,y);2D
σ(x,y,z);3D

⇒ using the expression for E_q obtained from the TFA

⇒ explore the lowest free energy for given 1D, 2D, 3D ansätze

⇒ need to effectively extract derivative terms of the condensate by scale trans.

⇒ but no self-consistency here

$$H_D\psi_\alpha = E\psi_\alpha$$

$$\langle\bar{\psi}\psi\rangle = \sigma(r)$$ condensate shape is fixed

Beyond 1D modulations (2D/3D modulations)

- self-consistent way to explore multidimensional structures w/o any ansätz

NJL model (w/RKC; $\Delta(r)$)

$$\mathcal{L}_{\rm MF} = \bar{\psi}\gamma^0 \left(i\partial_0 - H_D\right)\psi - \frac{\Delta(\mathbf{r})^2}{4G_s}, \quad \Delta(\mathbf{r}) \equiv -2G_s\langle\bar{\psi}\psi\rangle(\mathbf{r})$$

$$H_{D,\rm Weyl} = \begin{pmatrix} -i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla} & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r}) & \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix}$$

$$\Omega = -N_f N_c T \sum_{E_n} \ln\left(2\cosh\left(\frac{E_n-\mu}{2T}\right)\right)$$

using the finite-difference method

$$H_D\psi = E_n\psi, \quad \psi = \{f_n(z), g_n(z)\}$$

$$\begin{cases} E_n f_n + i f_n' - \Delta(z) g_n = 0 \\ -E_n g + i g_n' + \Delta(z) f_n = 0 \\ \Delta(z) = -2G_s \sum_{n<\mu}(|f_n|^2 - |g_n|^2) \end{cases} \longrightarrow E_i \text{ for } \{f_i, g_i\} \rightarrow \Delta(\mathrm{r})$$

both the discretized E and the corresponding $\{f, g\}$ can be simultaneously obtained

(cf. a similar way — finite-mode approach 【*Heinz et al. 2016*】)

Beyond 1D modulations (2D/3D modulations)

- **self-consistent way** to explore multidimensional structures w/o any ansätz

Before investigating arbitrary $\Delta(r)$

$\Rightarrow$ need to correctly reproduce the known result in $1+1D$ systems 【*Başar-Dunne-Thies 2009*】

1+1D Dirac eq.

$$H_D\psi = E\psi$$

$$\begin{pmatrix} -i\partial_x & \Delta \\ \Delta^* & i\partial_x \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix} = E\begin{pmatrix} f \\ g \end{pmatrix}$$

Δ(z): given

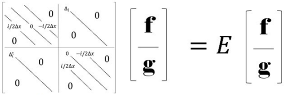
new Δ(z)

by searching for energy minimum

discretized Dirac eq.

$$-i\frac{f_{i+1}-f_{i-1}}{2\Delta x} + \Delta g_i = E_i f_i$$

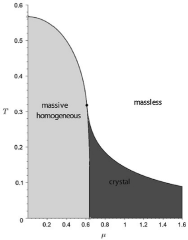
$$\Delta^* f_i + i\frac{g_{i+1}-g_{i-1}}{2\Delta x} = E_i g_i$$



eigenvalues for eigenvectors

thermodynamic pot.

(spatial discretization ~100 : need to solve a 200×200 Hermite matrix)



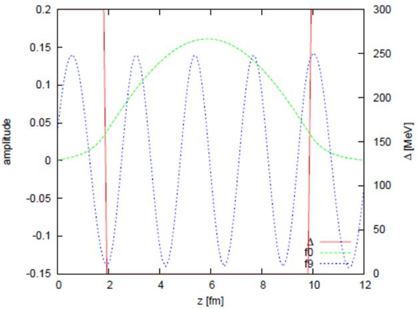
Beyond 1D modulations (2D/3D modulations)

- **self-consistent way** to explore multidimensional structures

$\Delta(z)$ for a well potential

analytic solution

$$E_n = \frac{(n+1)^2\pi^2\hbar^2}{8ma^2} \quad (n = 0, 1, 2, \cdots)$$

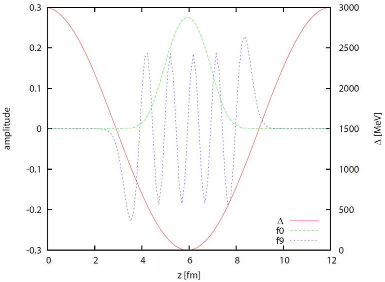


excited state	$\sqrt{E_n}$	$\sqrt{E_n/E_0}$
0	7.56645E+01	1.00000E+00
1	1.51288E+02	1.99946E+00
2	2.26831E+02	2.99785E+00
3	3.02252E+02	3.99464E+00
4	3.77511E+02	4.98927E+00
5	4.52567E+02	5.98123E+00
6	5.27380E+02	6.96998E+00
7	6.01910E+02	7.95499E+00
8	6.76117E+02	8.93573E+00
9	7.49961E+02	9.91167E+00
10	8.23403E+02	1.08823E+01
11	8.96402E+02	1.18471E+01
12	9.68920E+02	1.28055E+01
13	1.04092E+03	1.37570E+01
14	1.11236E+03	1.47012E+01
15	1.18320E+03	1.56374E+01
16	1.25340E+03	1.65653E+01
17	1.32294E+03	1.74842E+01

Beyond 1D modulations (2D/3D modulations)

- self-consistent way to explore multidimensional structures

  $\Delta(z)$ for a sine potential



Summary

Towards a multidimensional structure

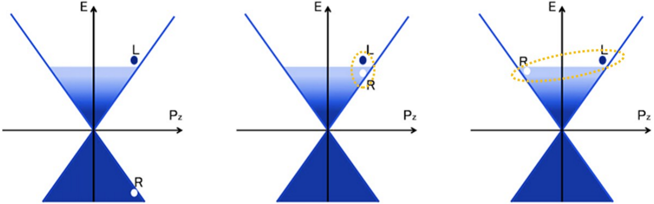
- inhomogeneous chiral phases with 1D modulations:
  - within MFA and beyond (fluctuations)
  - Landau-Peierls instability (no true LRO)
- inhomogeneous chiral phases with 2D/3D modulations:
  - disfavored around LP and at $T = 0$
  - nontrivial in different areas
  - TFA and self-consistent method
  - reproduce 1+1D $\rightarrow$ 2+1D
- additional consideration:
  - Coulomb interactions, β-equilibrium, charge neutrality, etc
  - may lead to the so-called "chiral pastas"?

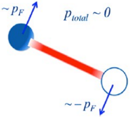
Thank you for your kind attention!

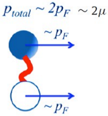
Back up

Inhomogeneous mechanism

Possible pairing pattern in quark matter







$q\bar{q}$ pairing in vac.

disfavored at large μ
(large energy cost)

qh pairing at intermediate μ

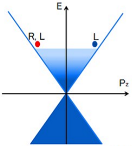
less favored than DW type
(large relative momentum disturbs
to form a bound state)

exciton type

qh pairing at intermediate μ

Favored
(small relative momentum
could form a bound state)

density wave type



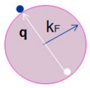
qq pairing at very high μ

【cf. *kojo et al. NPA 2010*】

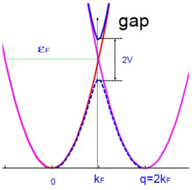
Inhomogeneous mechanism

Key mechanism: Nesting of the Fermi surface (Overhauser effect)

【cf. *Nakano-Tatsumi 2005*】



Particle-hole instability



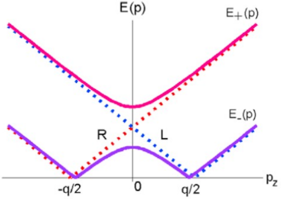
$$V = V_q\left(e^{iqx} + e^{-iqx}\right), \quad e^{ikx} \to e^{ikx} + e^{i(k\pm q)x}$$

Near the level crossing $\left(\varepsilon_k \approx \varepsilon_{k\pm q}\right)$:

$$H = \begin{pmatrix} \varepsilon_k & V_q \\ V_q & \varepsilon_{k\pm q} \end{pmatrix}$$

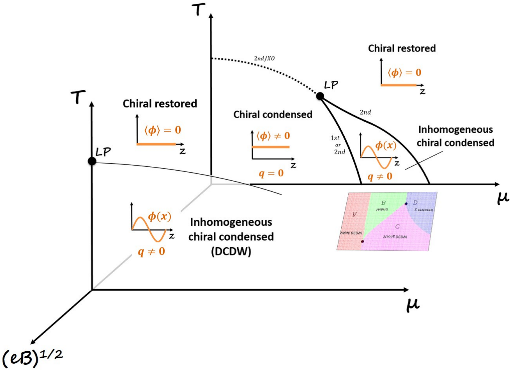
$$E_k^{\pm} = \frac{1}{2}\left[\varepsilon_k + \varepsilon_{k\pm q} \pm \sqrt{(\varepsilon_k - \varepsilon_{k\pm q})^2 + 4V_q^2}\right]$$

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_R\psi_L\rangle + \langle\bar{\psi}_L\psi_R\rangle \sim e^{iqx}$$



Inhomogeneous chiral phase

- In a background magnetic field



⇒ favored in an external magnetic field (due to nontrivial topology/anomaly)

【*cf. Frolov et al. 2010; Tatsumi et al. 2014; Nishiyama et al. 2015*】

Inhomogeneous chiral phase

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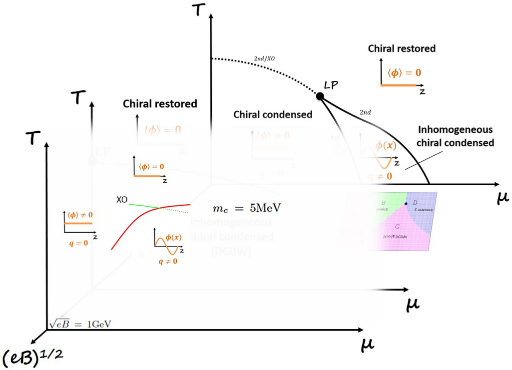

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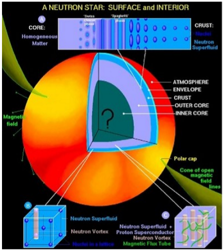
Inhomogeneous chiral phase

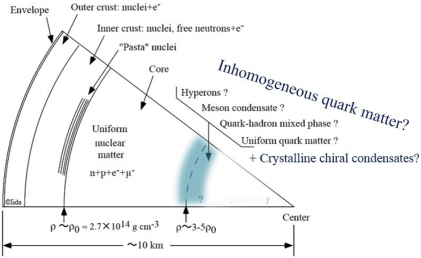
- Compact stars

A dense cold matter is relevant for compact stars.

Inhomogeneous chiral condensates may be realized in the core region.

This may also have astronomical implications (e.g., cooling, starquake, etc.)





TRUE LRO

▷ Possibilities of long-range order

- $T = 0$ limit (or sufficiently low temperatures)

  $\langle\phi\rangle = \langle\phi_0 + \delta\phi\rangle \neq 0$ (LRO)

  $\Rightarrow$ stable against quantum fluctuations (not diverge: $\langle\beta^2\rangle \propto \int d^3k\omega^{-1} \neq 0$)

- External magnetic fields

  $\omega^2 \sim ak_z^2 + b\vec{k}^2 + \mathcal{O}((\vec{k}^2)^2)$ for $B \neq 0$ $(b \propto B)$ (cf. $\omega^2 \sim \tilde{a}k_z^2 + \mathcal{O}((\vec{k}^2)^2)$ for $B = 0$)

  $\Rightarrow$ modified dispersion (explicit rotational symmetry breaking: k_t^2-term)

  $\Rightarrow$ could be stabilized (improved: $\langle\beta^2\rangle \propto T(\Lambda_{\text{uv}} + \mathcal{O}(\Lambda_{\text{IR}})) \neq 0$)

- Finite-size effects

  long wave-length fluctuations are cutoff by the system size (effectively stabilized)

  IR cutoff as system size: $\Lambda_{\text{IR}} = L^{-1}$ (no log div: $\langle\beta^2\rangle \propto T\ln(\mathcal{O}(1/\Lambda_{\text{IR}})) \sim T\ln(\mathcal{O}(L)) \neq 0$)

  $\Rightarrow$ QLRO can effectively mimic a true LRO (depending on L or experimental resolutions)
  【*cf. Als-Nielsen et al. 1980; Baym-Friman-Grinstein 1982*】

- Two- and three-dimensional modulations (inferred from Landau-Peierls theorem)

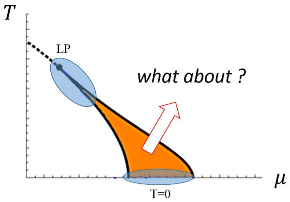
  similar suppression of IR div can be expected

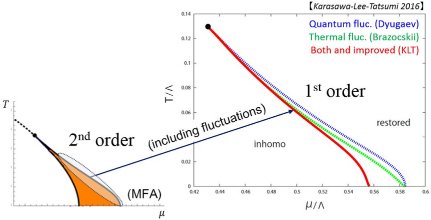
  $\Rightarrow$ stabilization could occur

MULTIDIMENSIONAL STRUCTURE

- one knows only the areas around a LP and at $T = 0$

  $\Rightarrow$ multidimensional chiral crystals may be realized in different areas





Incidentally, fluctuation effects give rise to a weakly 1st-order p.t. (Brazovskii-Dyugaev effect)

【*Brazovskii 1975; Dyugaev 1975; Hohenberg-Swift 1995; Ohashi 2002; Karasawa-Lee-Tatsumi 2016; Tatsumi-Yoshiike-Lee*】

$\Rightarrow$ may cause multidimensional structures?

(as in mesoscopic structures associated with 1st-order trans.)

PARAMETRIZATION

▷ General fluctuations on the DCDW phase with $\phi_0 = \Delta\,(\cos qz, 0, 0, \sin qz)^T$

$$\phi \;=\; (\Delta+\delta)\begin{pmatrix} \cos(qz+\beta_3)\cos\beta_2\cos\beta_1 \\ \cos(qz+\beta_3)\cos\beta_2\sin\beta_1 \\ \cos(qz+\beta_3)\sin\beta_2 \\ \sin(qz+\beta_3) \end{pmatrix} \;=\; (\Delta+\delta)U(\vec{\beta})\begin{pmatrix} \cos(qz) \\ 0 \\ 0 \\ \sin(qz) \end{pmatrix}$$

δ: amplitude fluctuation, $\vec{\beta} = \{\beta_1, \beta_2, \beta_3\}$: NG modes (parameters: 4D-sphere of chiral circle)

$U(\vec{\beta}) := e^{\vec{\beta}\cdot\vec{L}}$ with axial isospin generators $\vec{L}$,

- "z-direction displacement" = "β_3-rotation" under the above parametrization

$$\phi = (\Delta+\delta)\begin{pmatrix} \cos qz \\ 0 \\ 0 \\ \sin qz \end{pmatrix} + \Delta\begin{pmatrix} -\beta_3\sin qz \\ \beta_1\cos qz \\ \beta_2\cos qz \\ \beta_3\cos qz \end{pmatrix} + \mathcal{O}(\beta_i^2, \beta_i\delta, \delta^2)$$

$\Rightarrow$ we can identify linear fluctuations as pions in background inhomogeneous phase

Plugging this parametrization ϕ into $\mathcal{L}$,

$\Rightarrow$ we can obtain **a low-energy effective theory** for fluctuation fields $\delta(x)$ and $\vec{\beta}(x)$.

Low energy collective excitations

▷ Dispersion relations

- for mixing δ-β_3

$$\omega_+^2 \simeq M^2 + a_{6,1}\left[u_{z+}^2 k_z^2 + \left(\vec{k}^2\right)^2\right] + a_{6,4}\Delta^2\vec{k}^2 + A\vec{k}^2 k_z^2 + Bk_z^4$$
$$\omega_-^2 \simeq a_{6,1}\left[u_{z-}^2 k_z^2 + \left(\vec{k}^2\right)^2\right] - A\vec{k}^2 k_z^2 - Bk_z^4$$

- for $\vec{\beta}_T(=\vec{\beta}_{1,2})$

$$\omega_k^2 = a_{6,1}\left[4q^2k_z^2 + \left(\vec{k}^2\right)^2\right] + \mathcal{O}(k^6)$$

where $A \equiv 4q^2\Delta^2 a_{6.2}(4M^2a_{6.1} - \Delta^4 a_{6.2}a_{6.4})/M^4$, $B \equiv -(2q\Delta^2 a_{6.2})^4/M^6$
$u_{z\pm}^2 = 4q^2[1 \pm a_{6.2}^2\Delta^4/a_{6.1}M^2]\ (> 0)$

Impacts of low-energy fluctuations

- ▷ consider low energy fluctuations on the order parameter

$$\langle(\Delta+\delta)U(\beta_i)\phi_0\rangle = \Delta\langle U(\beta_i)\phi_0\rangle + \langle\delta U(\beta_i)\phi_0\rangle$$

where

$$\langle U(\beta_i)\phi_0\rangle \simeq \begin{pmatrix} \cos(qz)\exp(-\sum_i\langle\beta_i^2\rangle/2) \\ 0 \\ 0 \\ \sin(qz)\exp(-\langle\beta_3^2\rangle/2) \end{pmatrix}$$

$$\langle\delta U(\beta_i)\phi_0\rangle \simeq \begin{pmatrix} -\sin(qz)\langle\delta\beta_3\rangle\exp(-\sum_i\langle\beta_i^2\rangle/2) \\ 0 \\ 0 \\ \cos(qz)\langle\delta\beta_3\rangle\exp(-\langle\beta_3^2\rangle/2) \end{pmatrix}$$

Here, 2nd-order fluctuations

$$\langle\delta\beta_3\rangle = 0, \quad \langle\beta_{1,2}^2\rangle \simeq \frac{1}{2\Delta^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{T}{\omega_k^2}, \quad \langle\beta_3^2\rangle \simeq \frac{1}{2\Delta^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{T}{\omega_-^2}$$

$\Rightarrow \langle\beta_{1,2,3}^2\rangle \propto \int(T/\omega^2)d^3k$: log div (IR).

[fluctuations are all longitudinally divergent due to soft modes in transverse direction]