Non-perturbative Interactions of Quarks and Gluons

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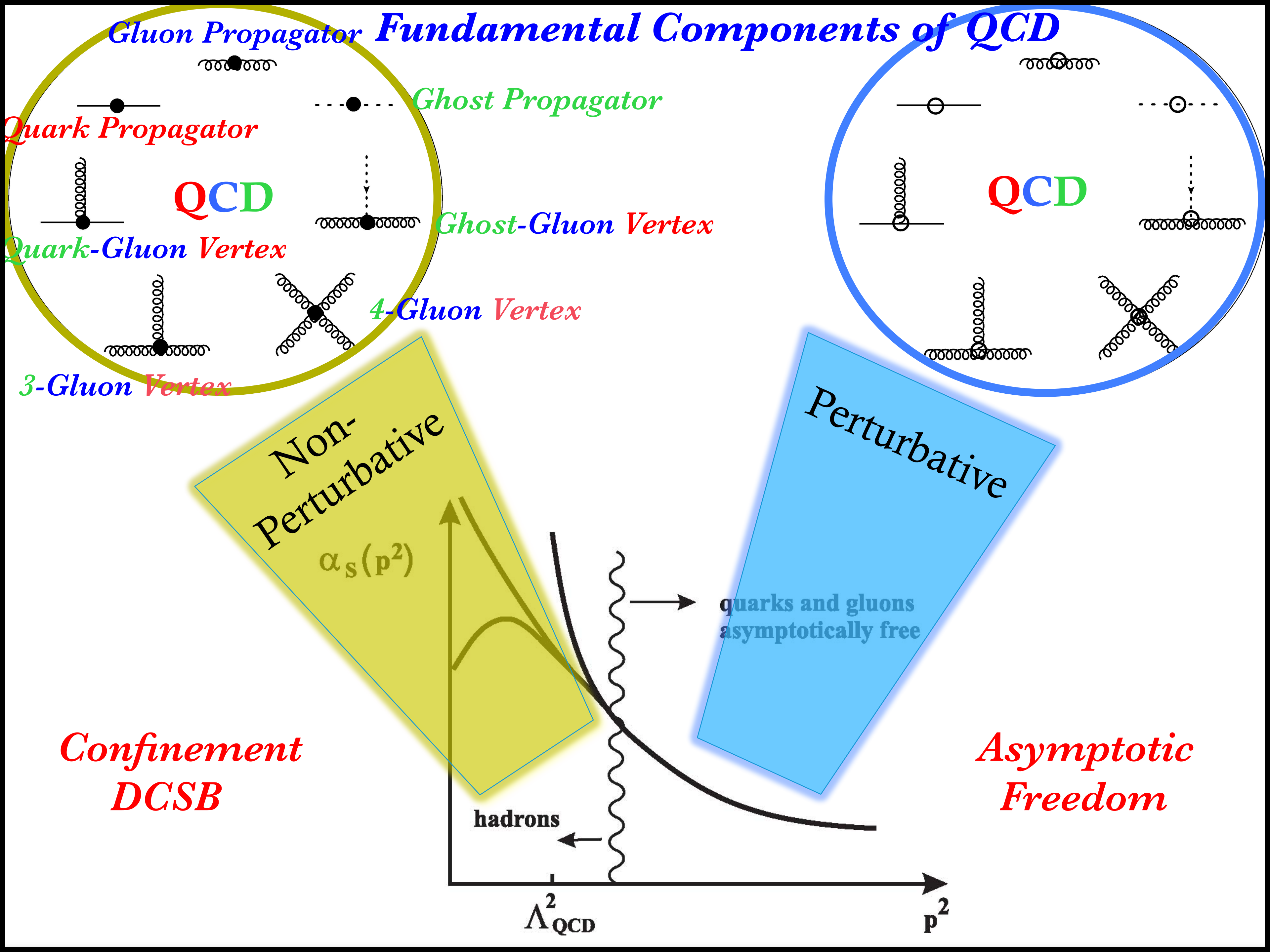
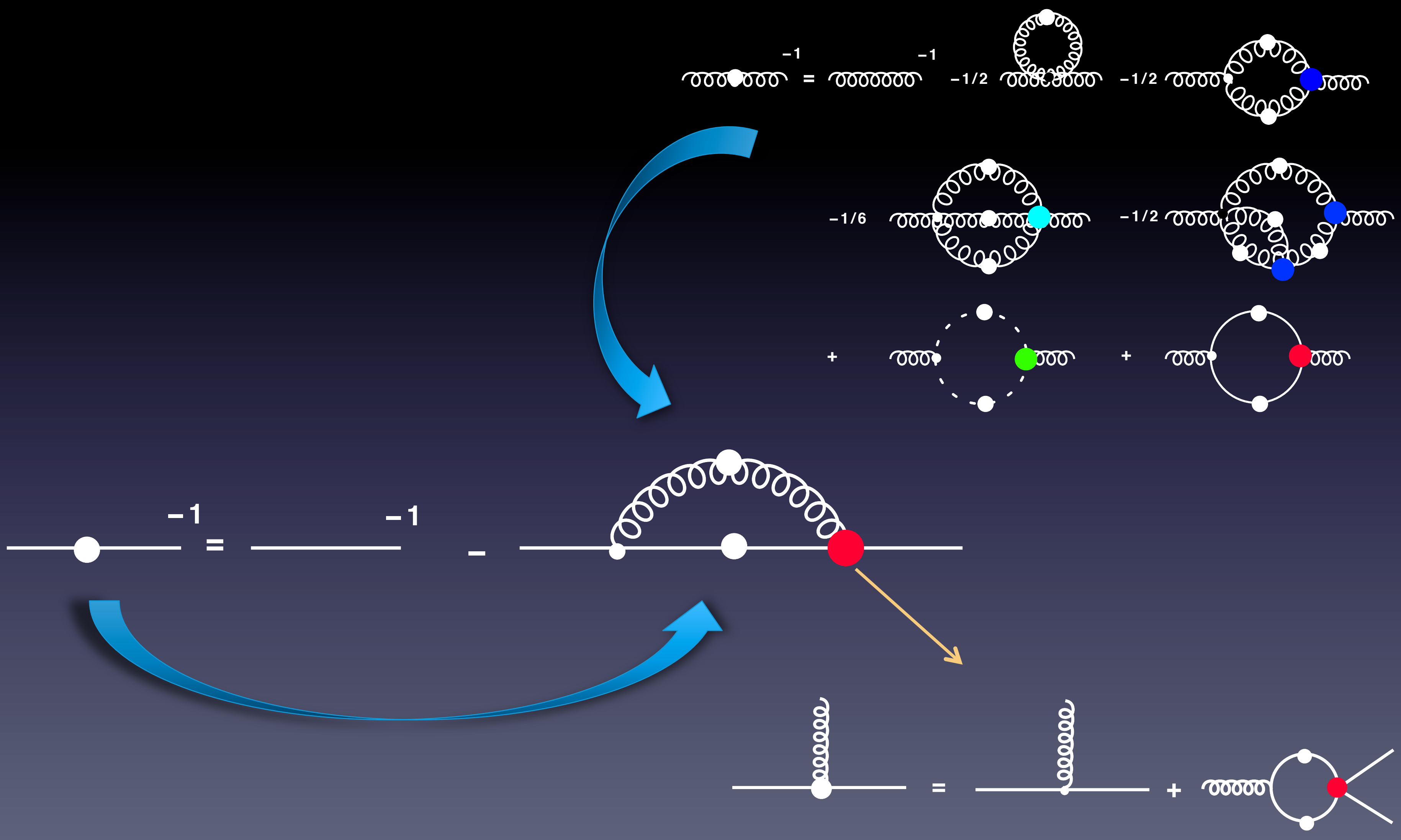
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER


Fundamental Components of QCD
Gluon Propagator
Ghost Propagator
Quark Propagator
QCD
Ghost-Gluon Vertex
Quark-Gluon Vertex
4-Gluon Vertex
3-Gluon Vertex
QCD
Non-Perturbative
Perturbative
αs(p²)
quarks and gluons
asymptotically free
hadrons
Confinement
DCSB
Asymptotic
Freedom
Λ²QCD
p²

Schwinger-Dyson Equations



Quark Propagator in Landau Gauge

Quark propagator:

In Continuum

$$S_F(p) = \frac{Z(p^2)}{\not p - M(p^2)} = \frac{1}{A(p^2)\ \not p - B(p^2)}$$

Quark propagator:

In Discrete

$$S^L(pa) = \frac{Z^L(pa)}{ia\,\not K(p) + aM^L(pa)}$$

$$K_\mu(p) \equiv \frac{1}{a} sin(p_\mu a)$$

$Z \equiv 1/A$ ***Quark (Fermion) Wave-Function Renormalisation*** Z^L

$M \equiv B/A$ ***Running Mass Function*** M^L

The O(a) improved quark propagator suffers from large tree-level lattice artefacts.

Need to reduce / remove lattice artefacts:

- *tree level correction*
- *momentum cuts (cylinder cut)*

  select momenta close to the diagonal in 4-dim momentum space, which have the smallest hypercubic artefacts.

Improved Wilson, Clover Fermion Action Rotated (Sheikholeslami-Wohlert) in Discrete

Simulation Parameters:

β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	N_{cfg}
5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900
5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900
5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908
5.29	0.1363	0.07	$64^3 \times 64$	290		750
	0.13640		64^3 x 64	150	2.1	400
5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900

The quark propagator using state-of-the art gauge configurations with $N_f = 2$ flavors of O(a) improved Wilson fermion

- *We thank the Regensburg Collaboration(RQCD) for allowing us to use their configurations in this project*

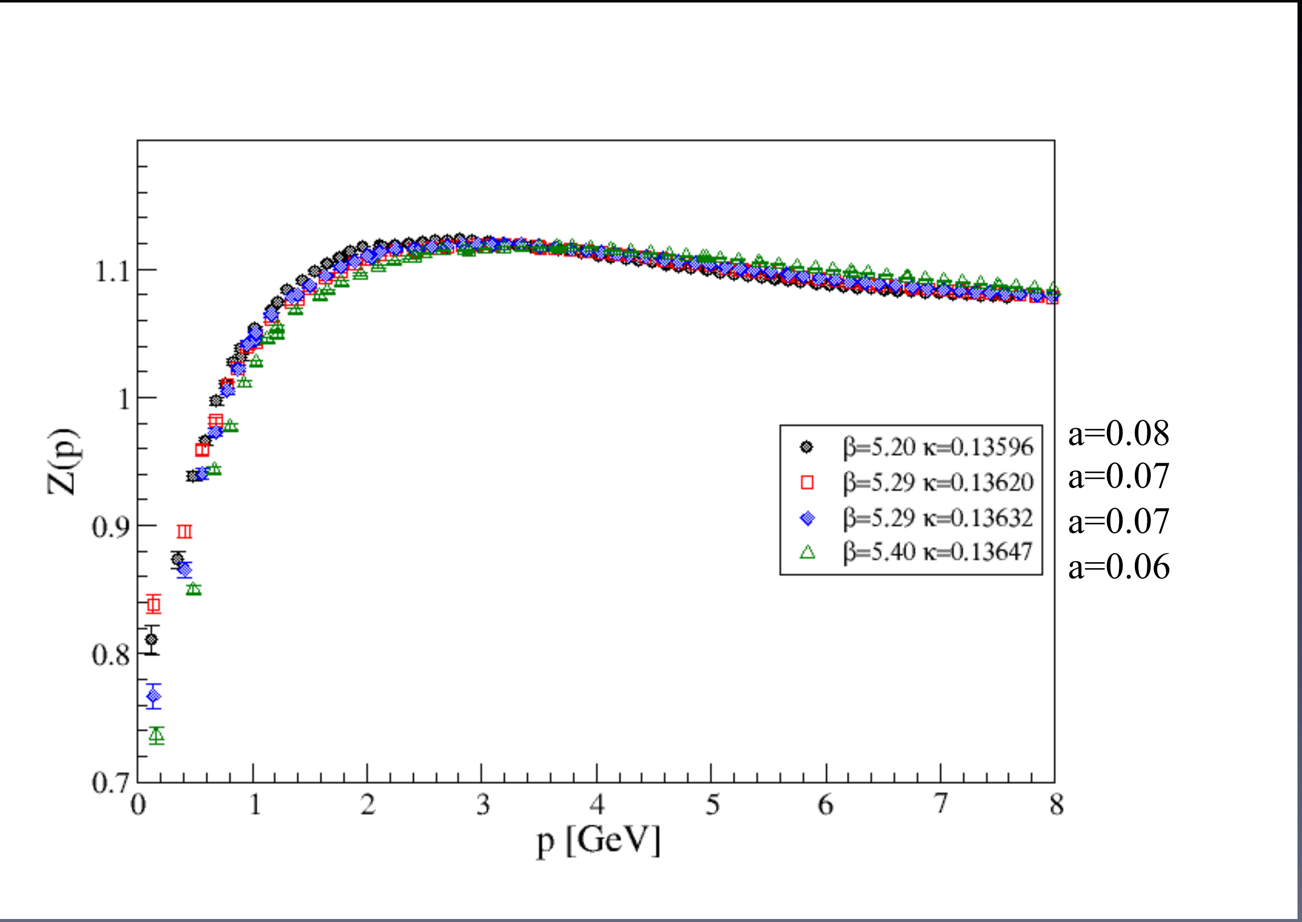
  G. S. Bali et all, Phys Rev D91, 054501 (2014) [arXiv:1412.7336]

- *The calculations (gauge fixing , propagators) where performed using the HLRN (Germany) for supercomputing*
  https://www.hlrn.de/home/view/Service

Quark wave function renormalisation in Landau Gauge

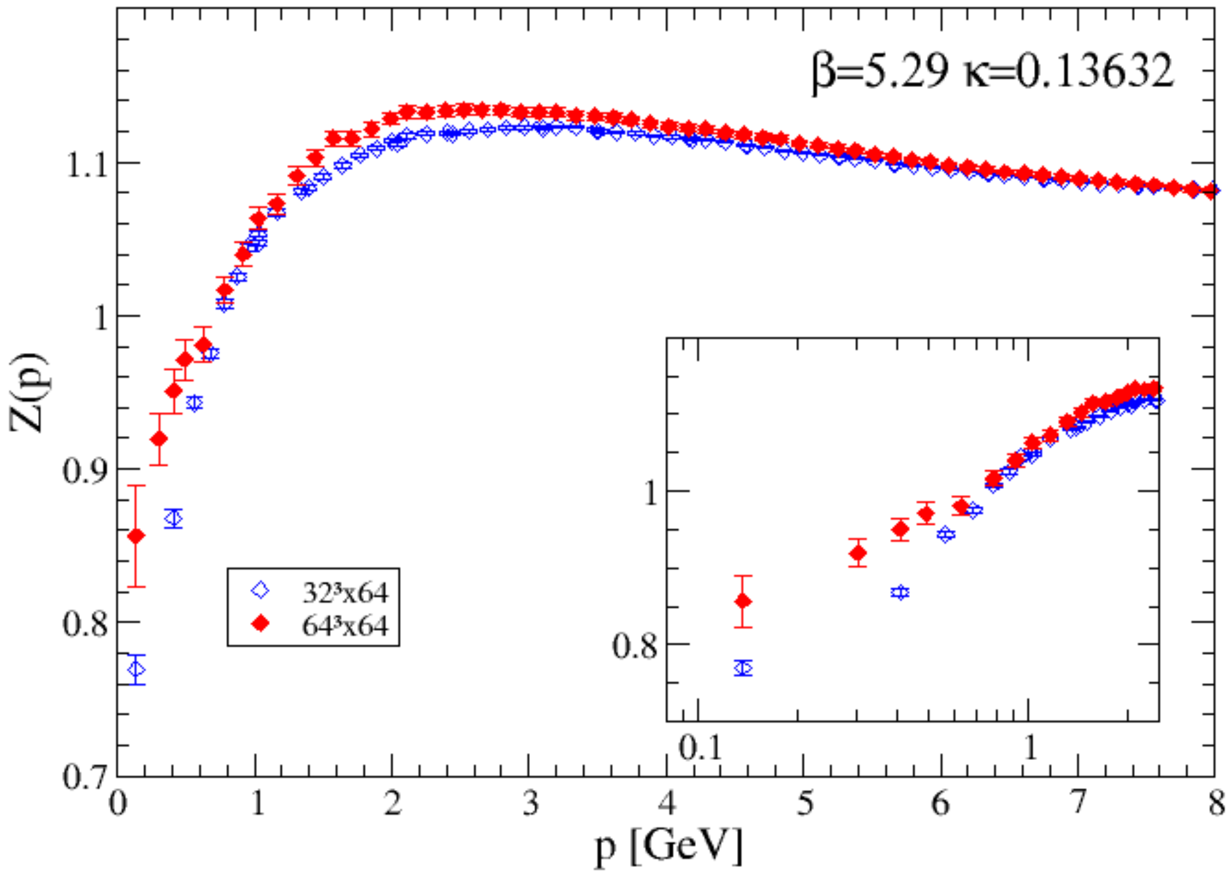
$$(m_\pi = 280MeV\,;\, m_q = 6.2MeV)$$
$$(m_\pi = 295MeV\,;\, m_q = 8.0MeV)$$
$$(m_\pi = 422MeV\,;\, m_q = 17.0MeV)$$
$$(m_\pi = 426MeV\,;\, m_q = 18.4MeV)$$

All 4 ensembles on the 32^3 x 64 volume, using the tree-level corrected Z(p)



Finite Volume Effects

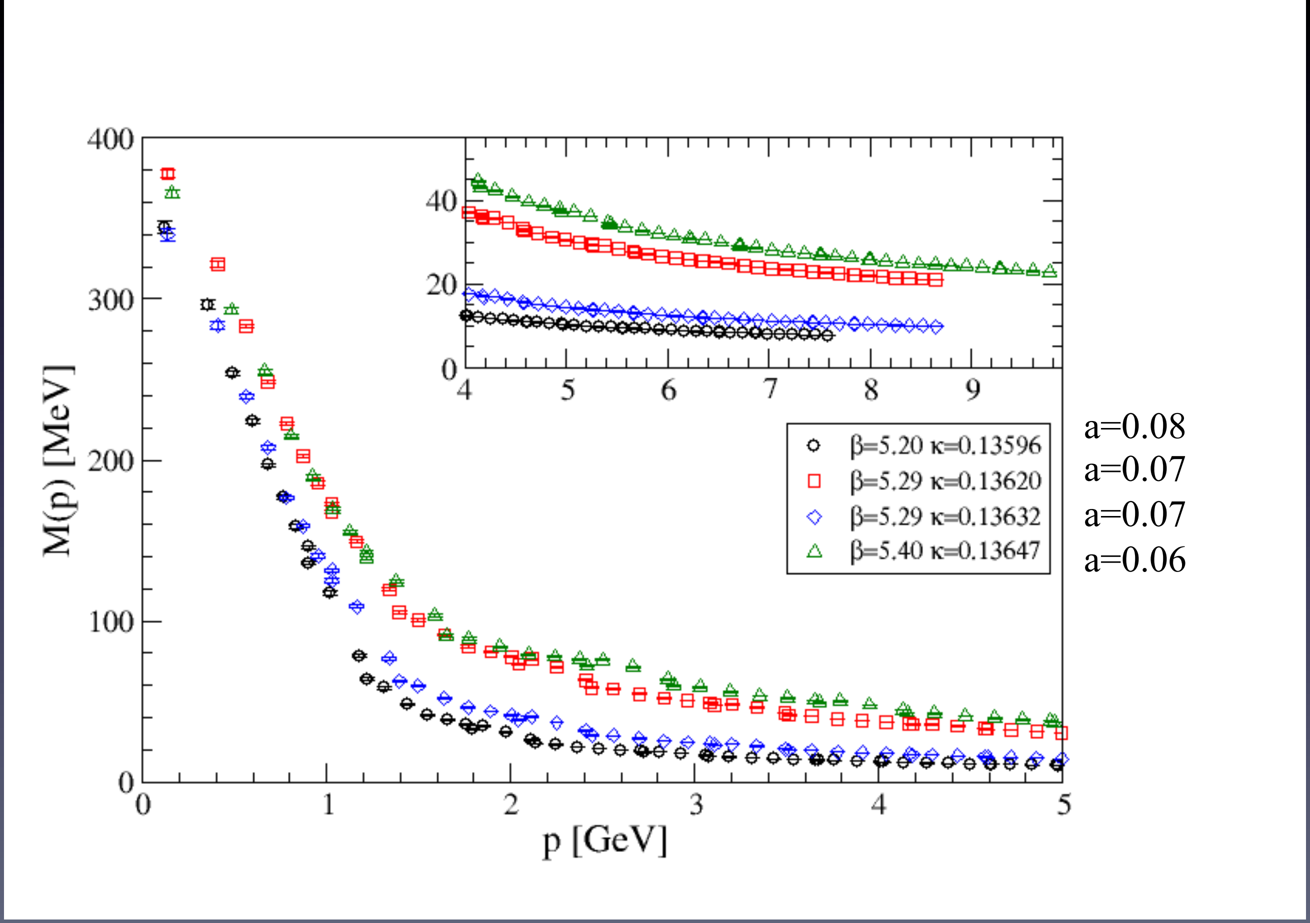
Quark Wave-function for two lattice volumes at β = 5.29, κ =0.13632



Mass function

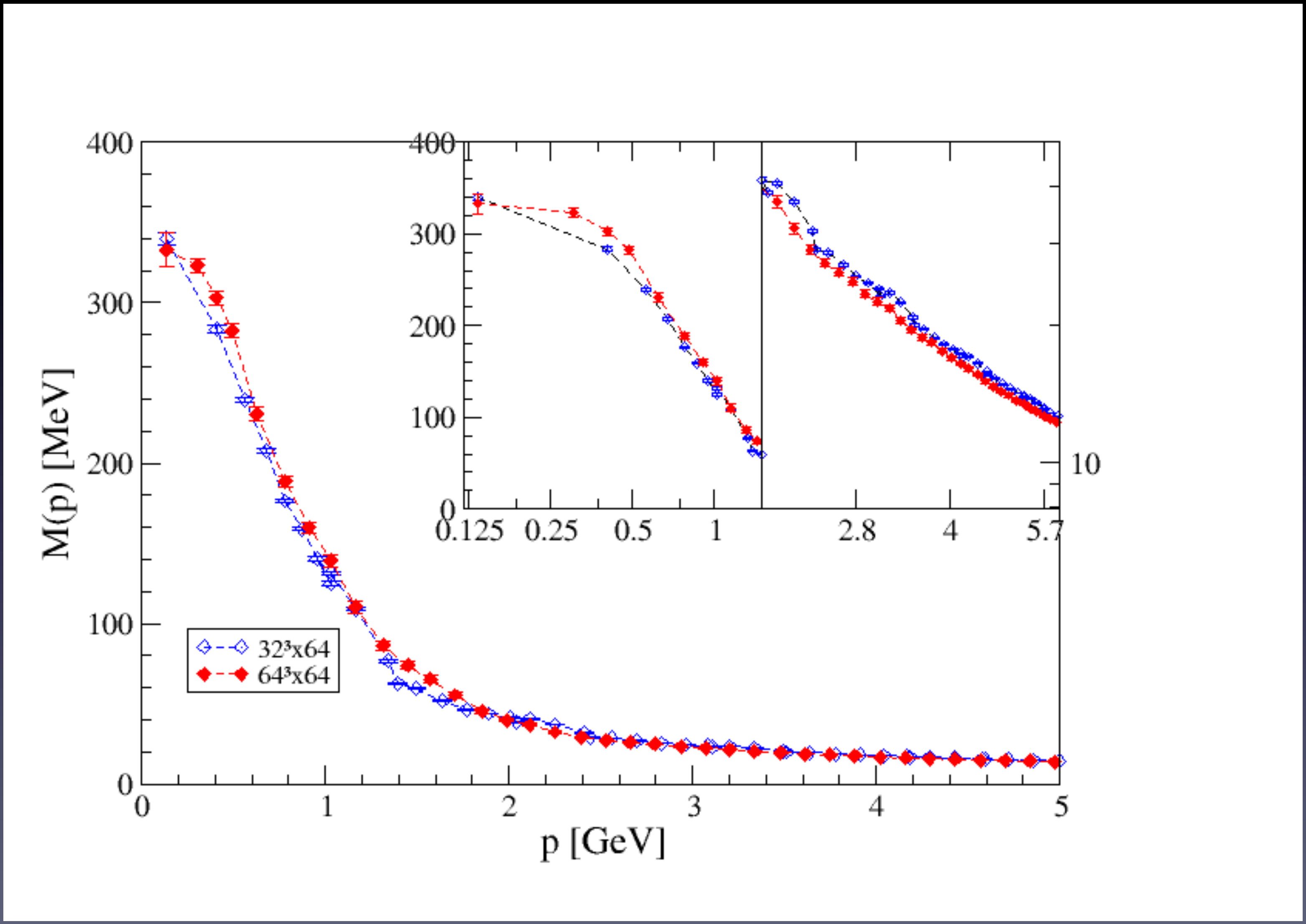
$$(m_\pi = 422 MeV\ ;\ m_q = 17.0 MeV)$$
$$(m_\pi = 280 MeV\ ;\ m_q = 6.2 MeV)$$
$$(m_\pi = 295 MeV\ ;\ m_q = 8.0 MeV)$$
$$(m_\pi = 426 MeV\ ;\ m_q = 18.4 MeV)$$

All 4 ensembles on the 32^3 x 64 volume, using the hybrid correction scheme for M(p).



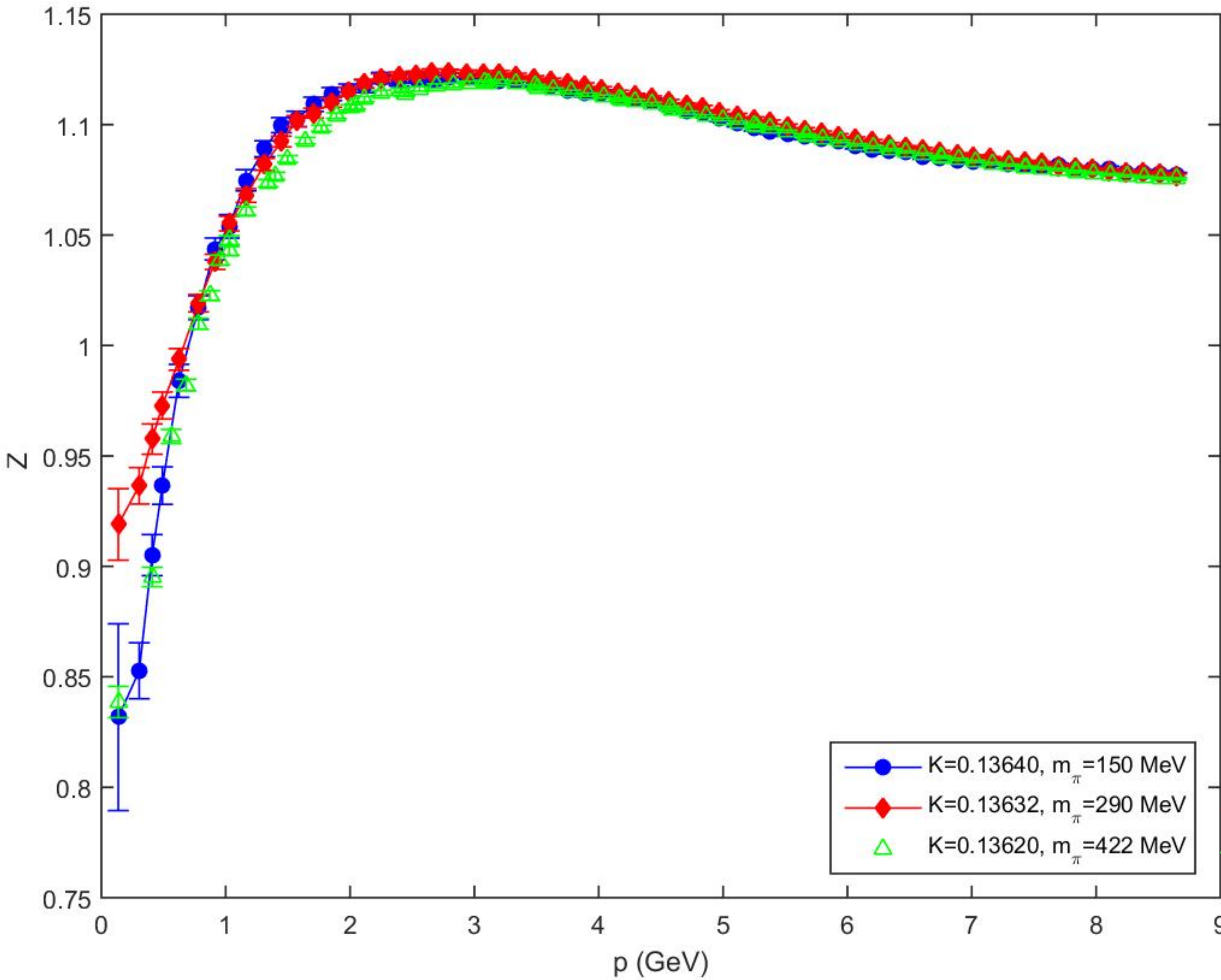
Finite Volume Effects

Mass function for two lattice volumes at β = 5.29, κ =0.13632



Quark Wave Function for the lighter quark masses

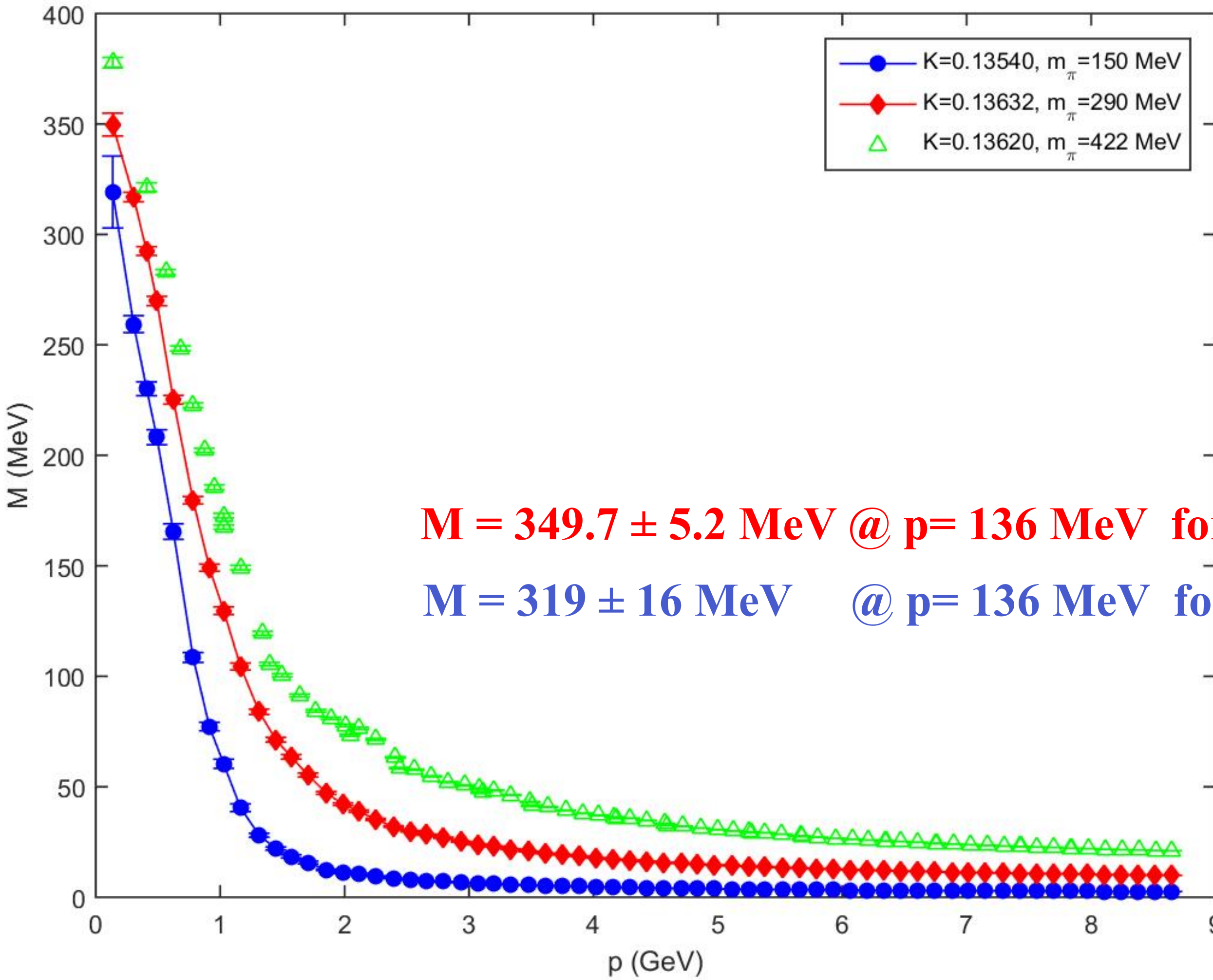
β = 5.29, a=0.07



64^3 x 64
64^3 x 64
32^3 x 64

Quark Mass Function for the lighter quark masses

β = 5.29, a=0.07



64^3 x 64
64^3 x 64
32^3 x 64

M = 349.7 ± 5.2 MeV @ p= 136 MeV for m_π=290 MeV

M = 319 ± 16 MeV @ p= 136 MeV for m_π=150 MeV

Summary on the Quark Propagator:

- M = 319 ± 16 MeV @ p= 136 MeV for m_π=150 MeV
  M = 349.7 ± 5.2 MeV @ p= 136 MeV for m_π=290 MeV

- *lattice artefacts under control for Z(p2) but not for for $M(p^2)$*

- *quark wave function seems to be lattice space independent at UV but not in the IR and it has very weak volume dependence.*

- *$Z(p^2)$ suppressed in IR and further suppressedas the quark mass and lattice spacing reduced but the suppresion is weakened as the volume is increased.*

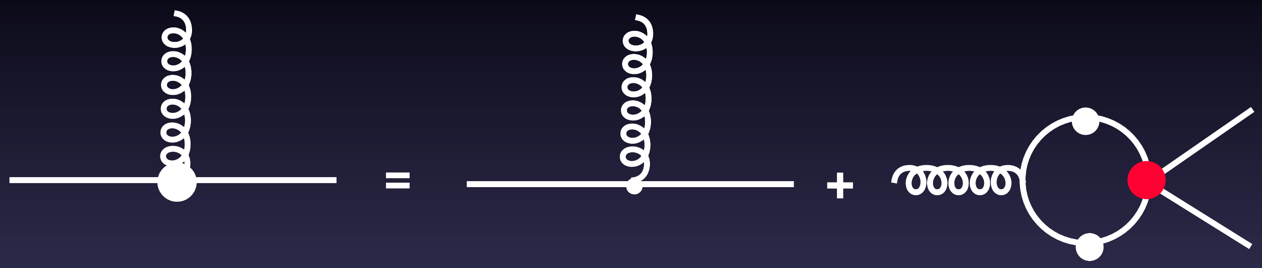
Determining Quark-Gluon Vertex

1-Determining qqg vertex perturbatively

(Feynman gauge, 1-loop), (General covariant gauge, 1-loop), (Massless, Landau Gauge, 2-loop)

2- Non-perturbatively in continuum by using SDE

a-Either by solving DSE for the vertex itself…



b-By solving coupled SDE for the propagators self consistently by imposing constraints on Green's functions …

c- By using Ward-Green-Takahashi identities…

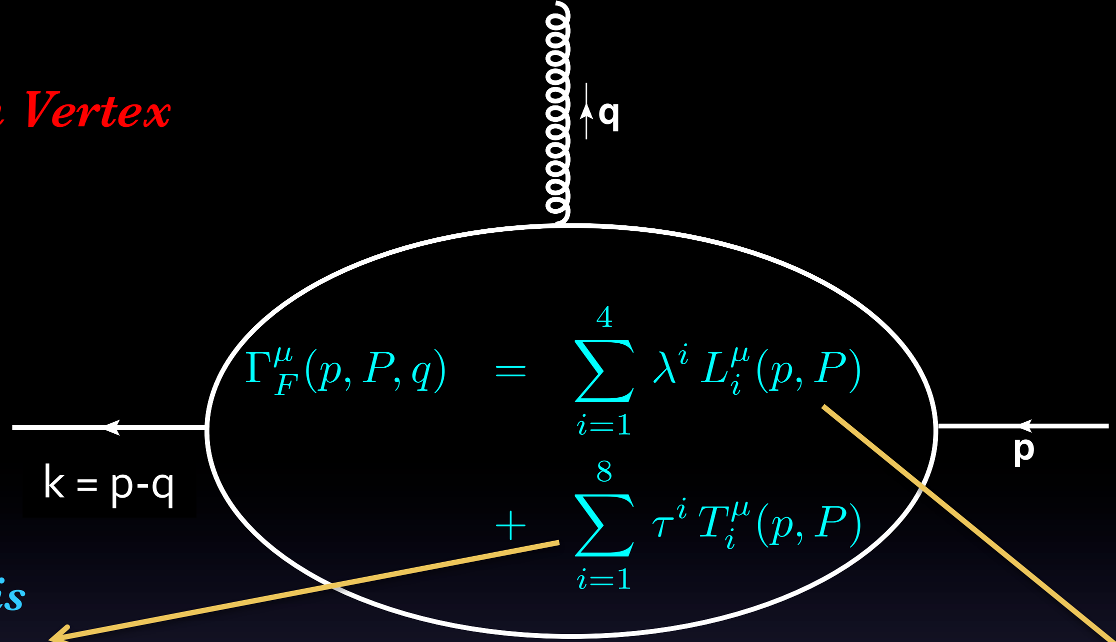
3-Non-perturbatively in discreet space by using lattice gauge field theory

J.I. Skullerud and A. Kızılersu, JHEP09,013 (2002).

J.I. Skullerud, P.O. Bowman, A. Kızılersu, D.B. Leinweber and A.G. Williams, JHEP04(2003)047

A. Kızılersu, J.I. Skullerud, D.B. Leinweber and A.G.Williams, Eur.Phys.J.C50(2007) 871

Quark Gluon Vertex



$$\Gamma^{\mu}_{F}(p,P,q) = \sum_{i=1}^{4} \lambda^{i} L^{\mu}_{i}(p,P) + \sum_{i=1}^{8} \tau^{i} T^{\mu}_{i}(p,P)$$

Transverse Basis

$$T_{1_\mu} = -i\,[(pq)k_\mu - (kq)p_\mu]$$

$$T_{2_\mu} = -\not{P}\,[(pq)k_\mu - (kq)p_\mu]$$

$$T_{3_\mu} = \not{q}\,q_\mu - q^2\gamma_\mu$$

$$T_{4_\mu} = -i\left[q^2\sigma_{\mu\nu}P_\nu + 2q_\mu\sigma_{\nu\lambda}p_\nu k_\lambda\right]$$

$$T_{5_\mu} = -i\,\sigma_{\mu\nu}q_\nu$$

$$T_{6_\mu} = (qP)\gamma_\mu - \not{q}\,P_\mu$$

$$T_{7_\mu} = -\frac{i}{2}(qP)\sigma_{\mu\nu}P_\nu - i\,P_\mu\,\sigma_{\nu\lambda}\,p_\nu\,k_\lambda$$

$$T_{8_\mu} = -\gamma_\mu\,\sigma_{\nu\lambda}\,p_\nu\,k_\lambda - \not{p}\,k_\mu + \not{k}\,p_\mu$$

Longitudinal Basis

$$L_{1_\mu} = \gamma_\mu$$

$$L_{2_\mu} = -\not{P}P_\mu$$

$$L_{3_\mu} = -i\,P_\mu$$

$$L_{4_\mu} = -i\,\sigma_{\mu\nu}P_\nu$$

Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\Gamma^T_\mu(p,k,q) = P^T_{\mu\nu}(q)\Gamma_\mu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Gamma_\mu(p,k,q)$$

We study transverse projected vertex:

$$\boxed{\mathbf{\Lambda^{a,P}_\mu(p,q) \equiv P_{\mu\nu}\Lambda^{a,lat.}_\nu(p,q) = S(p)^{-1}V^a_\nu(p,q)S(p+q)^{-1}D(q^2)^{-1}}}$$

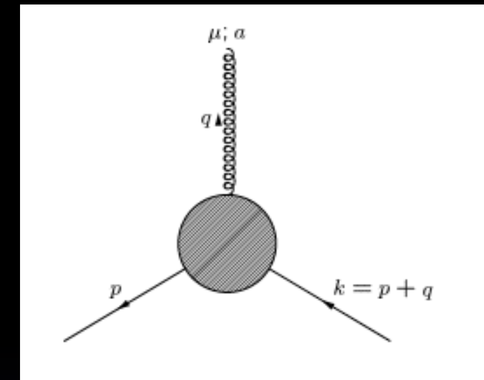
$$\Gamma^{T\,\mu} = -\frac{1}{q^2}\left[\Lambda_1\,T^\mu_3 + \Lambda_2\,T^\mu_2 + \Lambda_3\,T^\mu_1 + \Lambda_4\,T^\mu_4\right]$$
$$+\,\tau_5\,T^\mu_5 + \tau_6\,T^\mu_6 + \tau_7\,T^\mu_7 + \tau_8\,T^\mu_8$$

$$\Lambda_1 = \lambda_1 - q^2\,\tau_3 \qquad \Lambda_2 = \lambda_2 - \frac{q^2}{2}\,\tau_2$$

$$\Lambda_3 = \lambda_3 - \frac{q^2}{2}\,\tau_1 \qquad \Lambda_4 = \lambda_4 + q^2\,\tau_4$$

Quark-Gluon Vertex in Special Kinematics

- *Soft Gluon Kinematics :* $q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$



$$(\Lambda_\mu^{\mathbf{a}})_{\alpha\beta}^{\mathbf{ij}} = -\mathbf{i}\mathbf{g_0}\mathbf{t}_{\mathbf{ij}}^{\mathbf{a}}(\Gamma_\mu^{\mathbf{T}})_{\alpha\beta}$$

$$(\Lambda_\mu^{\mathbf{a}})_{\alpha\beta}^{\mathbf{ij}} = -\mathbf{i}\mathbf{g_0}\mathbf{t}_{\mathbf{ij}}^{\mathbf{a}}(\lambda_{\mathbf{1}}^{\mathbf{E}}\gamma_\mu - 4\,\lambda_{\mathbf{2}}^{\mathbf{E}}\,\not{\mathbf{p}}\mathbf{p}_\mu - \mathbf{2i}\lambda_{\mathbf{3}}^{\mathbf{E}}\,\mathbf{p}_\mu - \mathbf{2i}\lambda_{\mathbf{4}}^{\mathbf{E}}\,\sigma_{\mu\nu}\mathbf{p}_\nu)_{\alpha\beta}$$

$$Tr_4[\mathbf{I}\,\Gamma_\mu^T] = -2\,i\,\lambda_3^E\,p_\mu$$

$$Tr_4[\gamma_\mu\,\Gamma_\mu^T] = \lambda_1^E\delta_{\alpha\mu} - 4\,\lambda_2^E p_\alpha p_\mu$$

$$Tr_4[\sigma_{\alpha\beta}\Gamma_\mu^T] = -2i\lambda_4^E\,(p_\alpha\delta_{\beta\mu} - p_\beta\delta_{\alpha\mu})$$

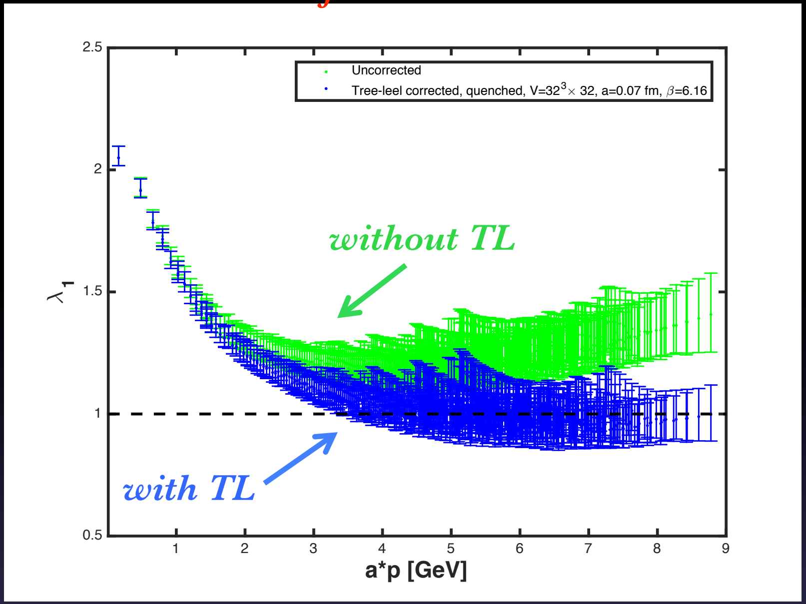
$\lambda_1^E, \lambda_2^E, \lambda_3^E$

$\lambda_4^E = 0$

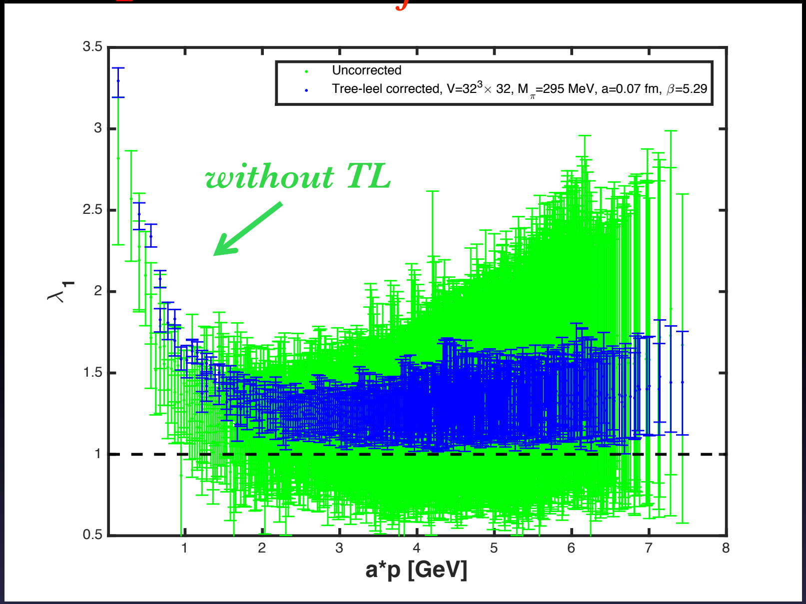
Soft gluon Kinematics (Asymmetric)

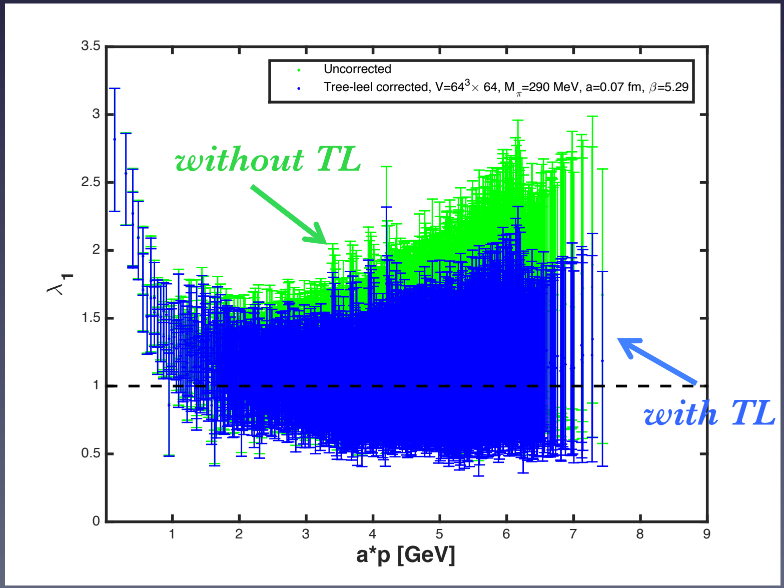
λ_1 with and without tree-level correction

Quenched, $N_f = 0$ 32^3x64



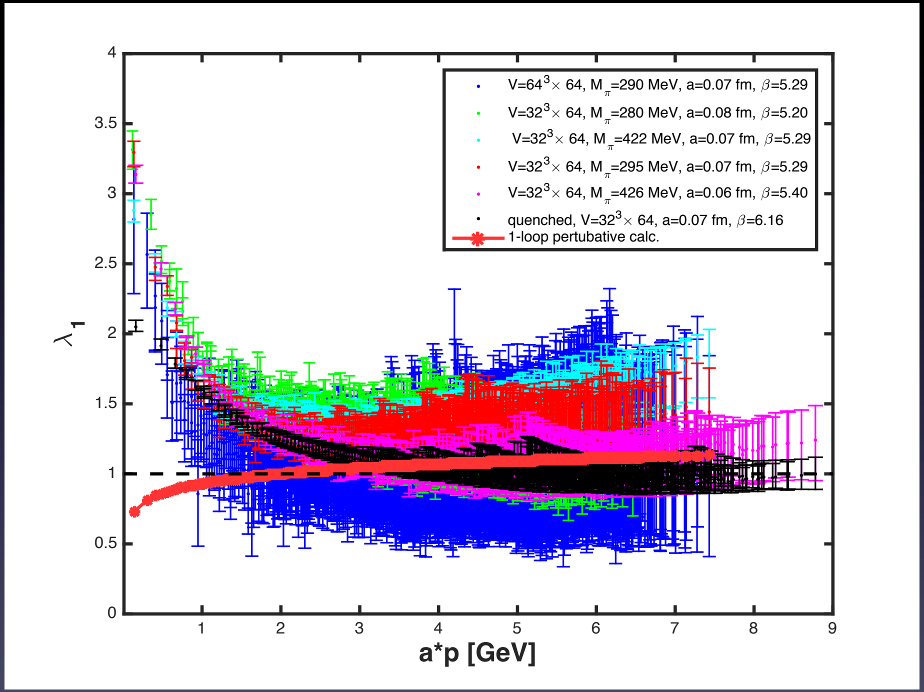
Unquenched, $N_f = 2$ 32^3x64

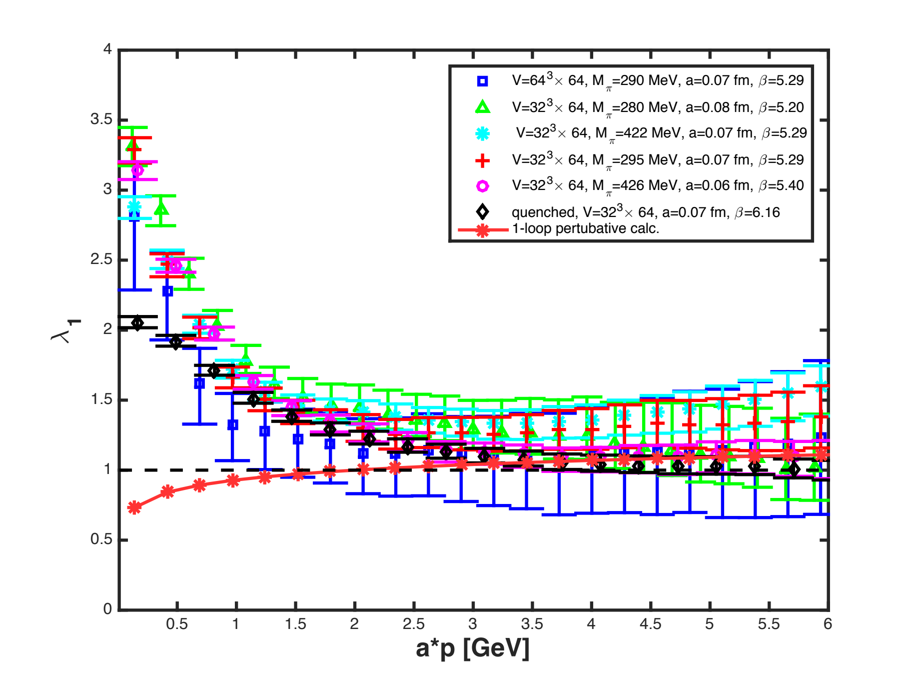




Unquenched, $N_f = 2$ 64^3x64

λ_1 *With and without Tree-level correction*

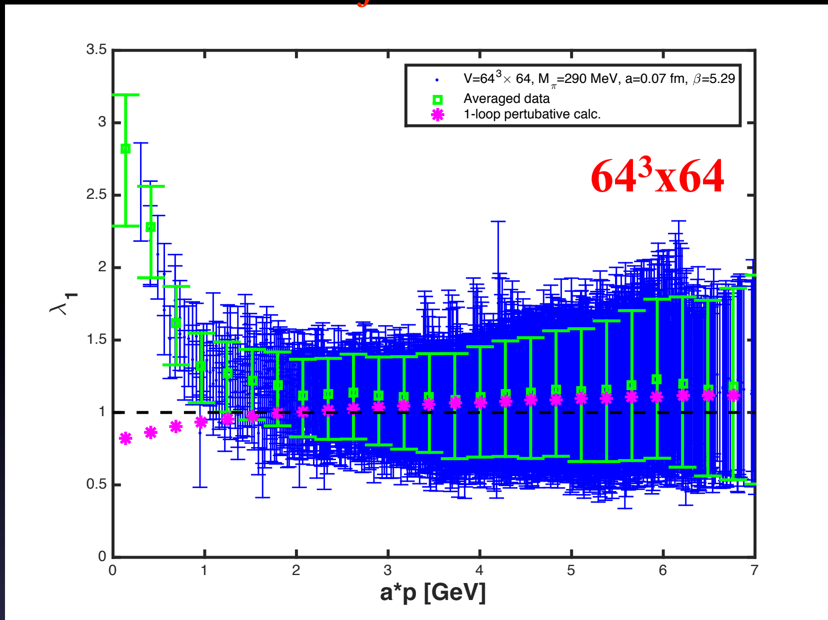




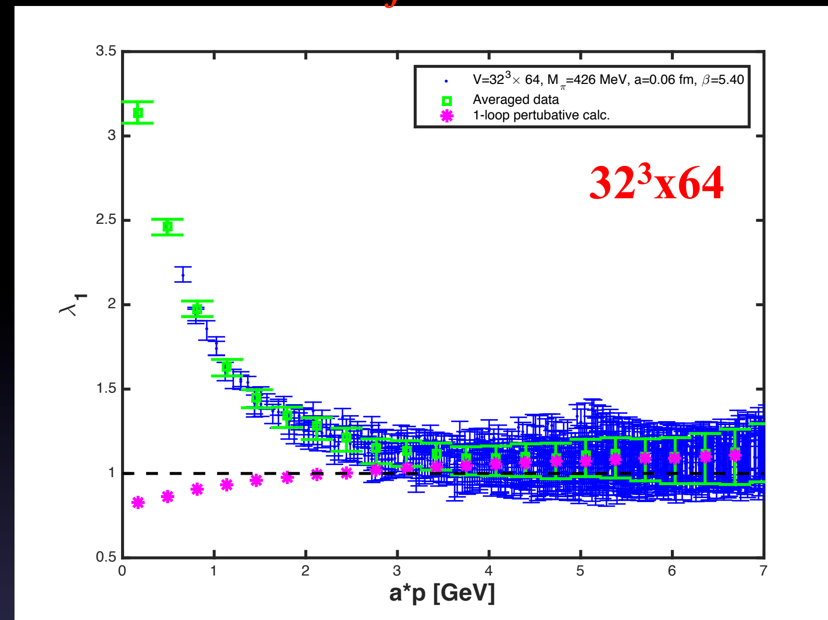
Soft gluon Kinematics (Asymmetric)

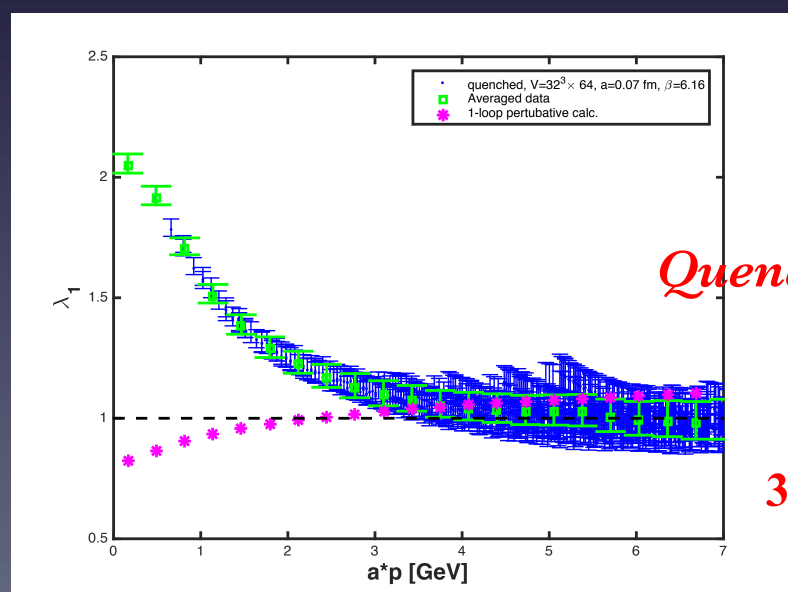
λ_1 *With tree-level correction*

Unquenched, $N_f = 2$



Unquenched, $N_f = 2$





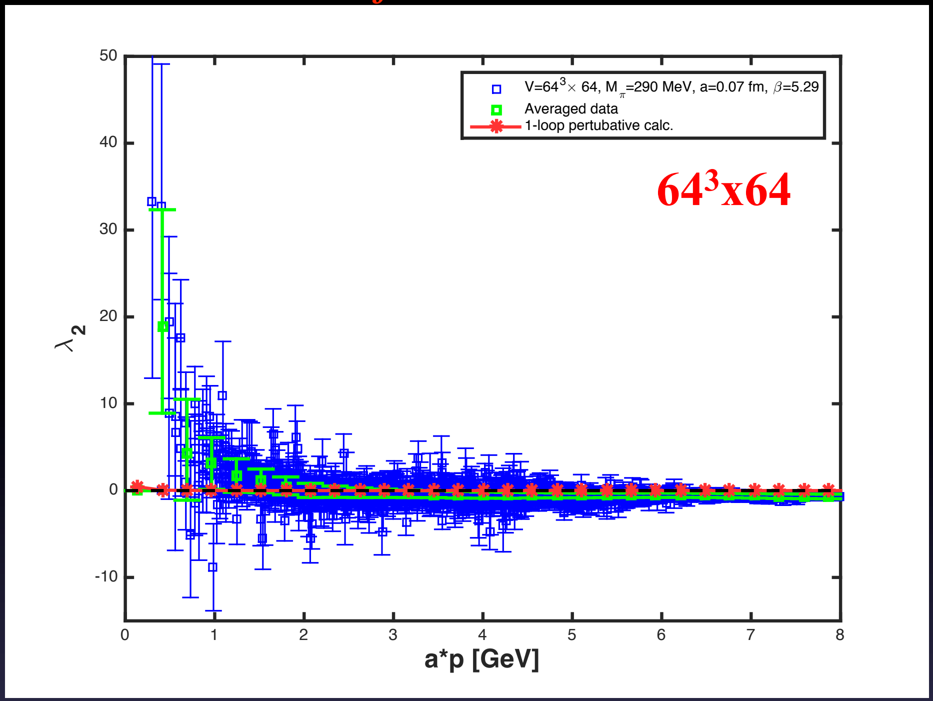
Quenched, $N_f = 0$

32^3x64

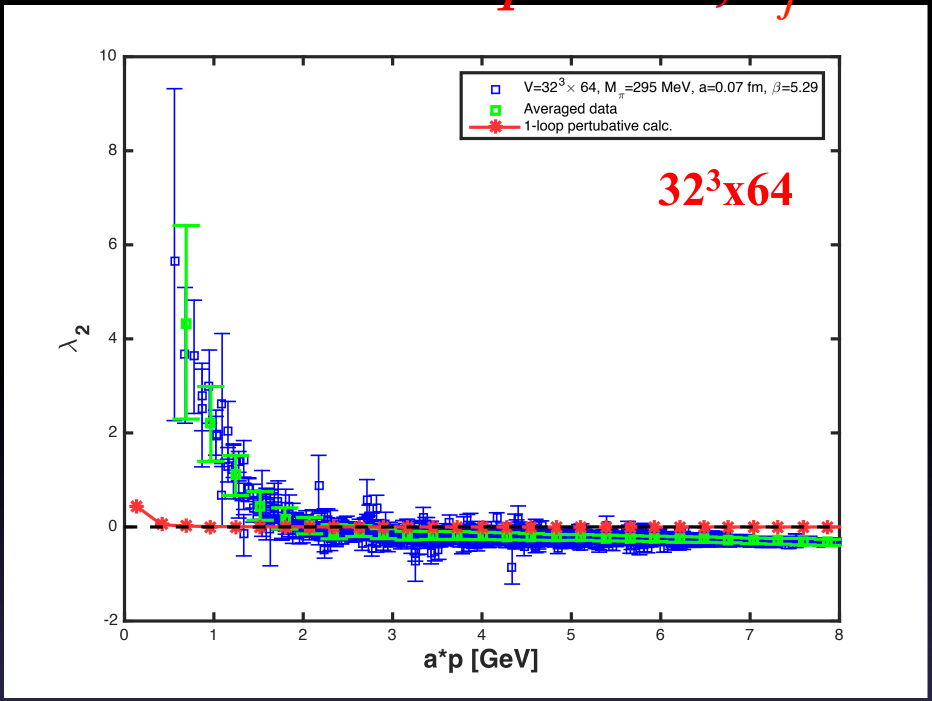
Soft gluon Kinematics (Asymmetric)

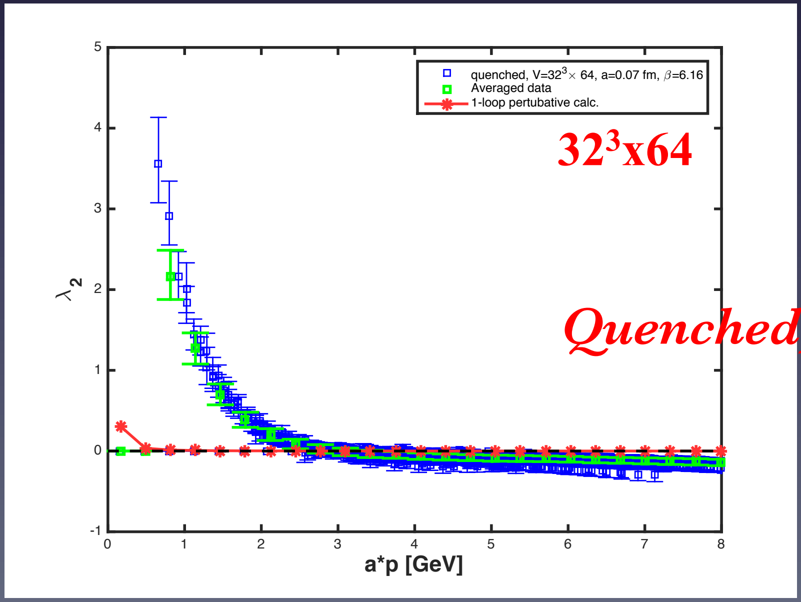
λ_2 without tree-level correction

Unquenched, $N_f = 2$



Unquenched, $N_f = 2$



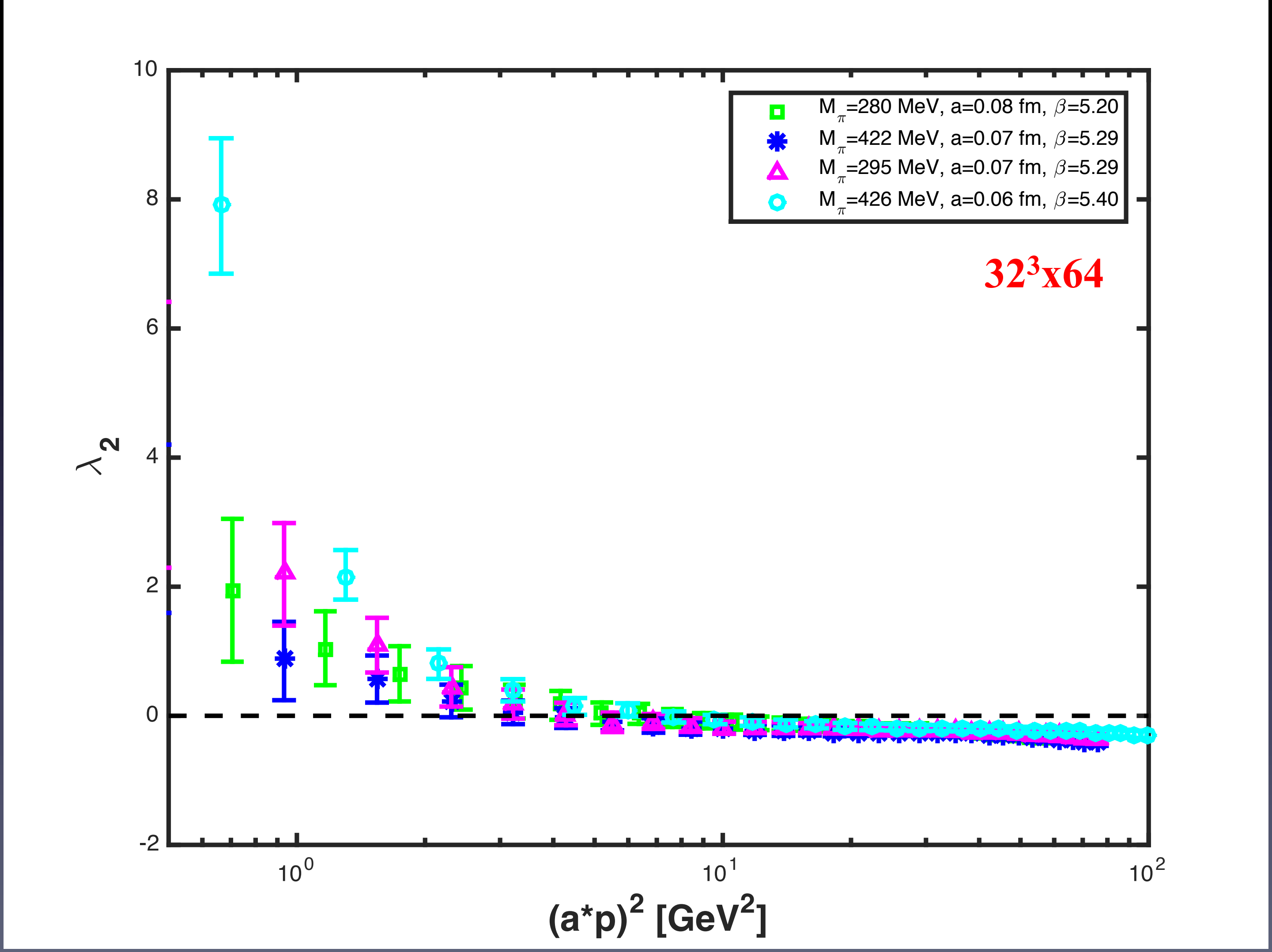


Quenched, $N_f = 0$

Soft gluon Kinematics (Asymmetric)

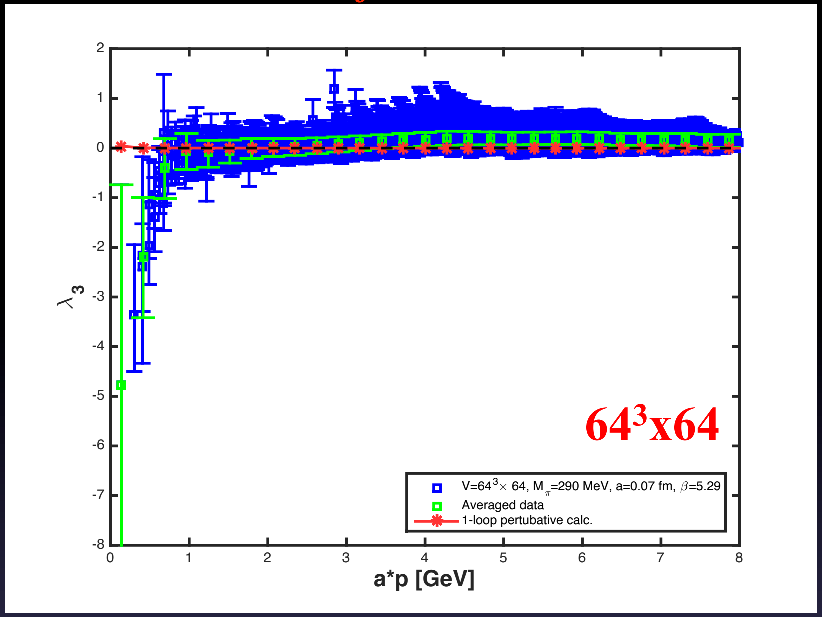
Mass dependence

λ_2 *without tree-level correction*

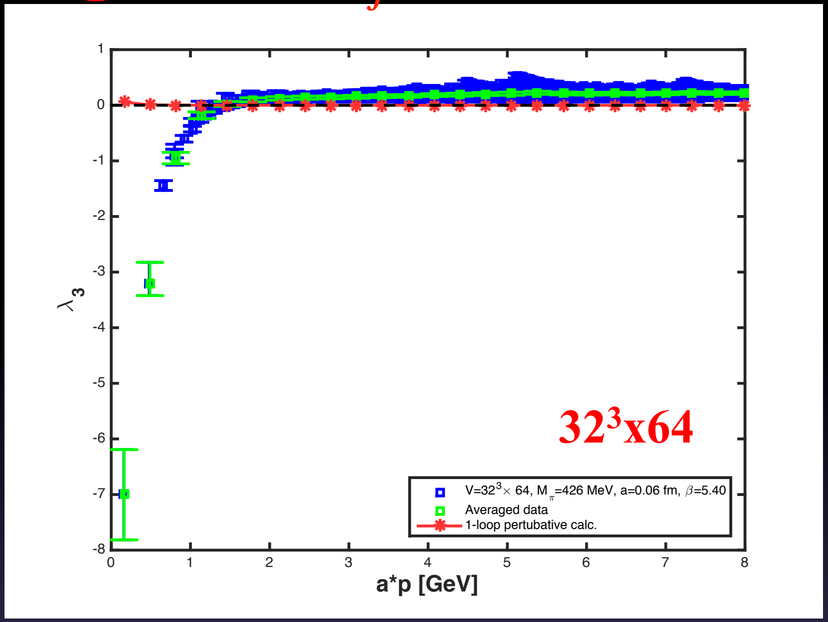


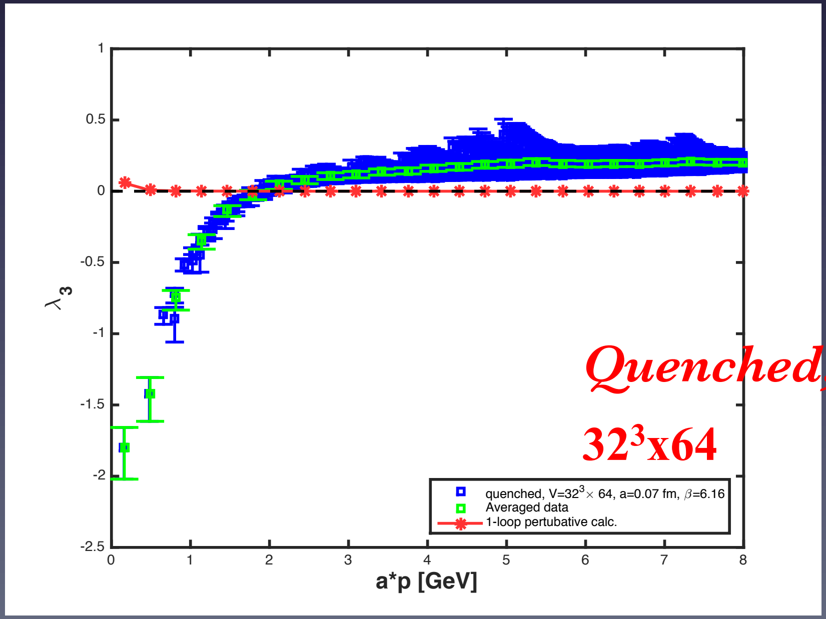
λ_3 *without tree-level correction*

Unquenched, $N_f = 2$



Unquenched, $N_f = 2$





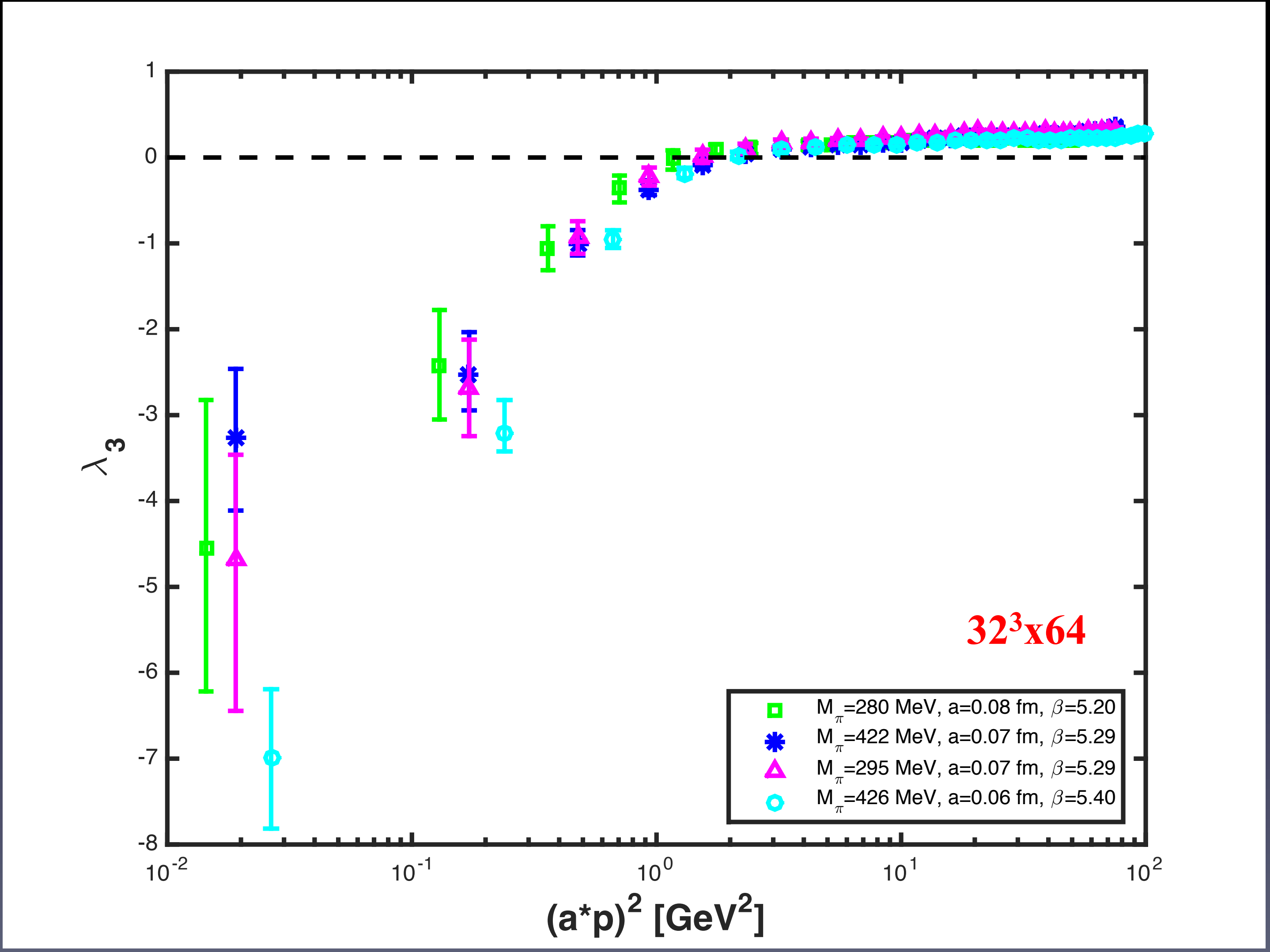
Quenched, $N_f = 2$

32^3x64

Soft gluon Kinematics (Asymmetric)

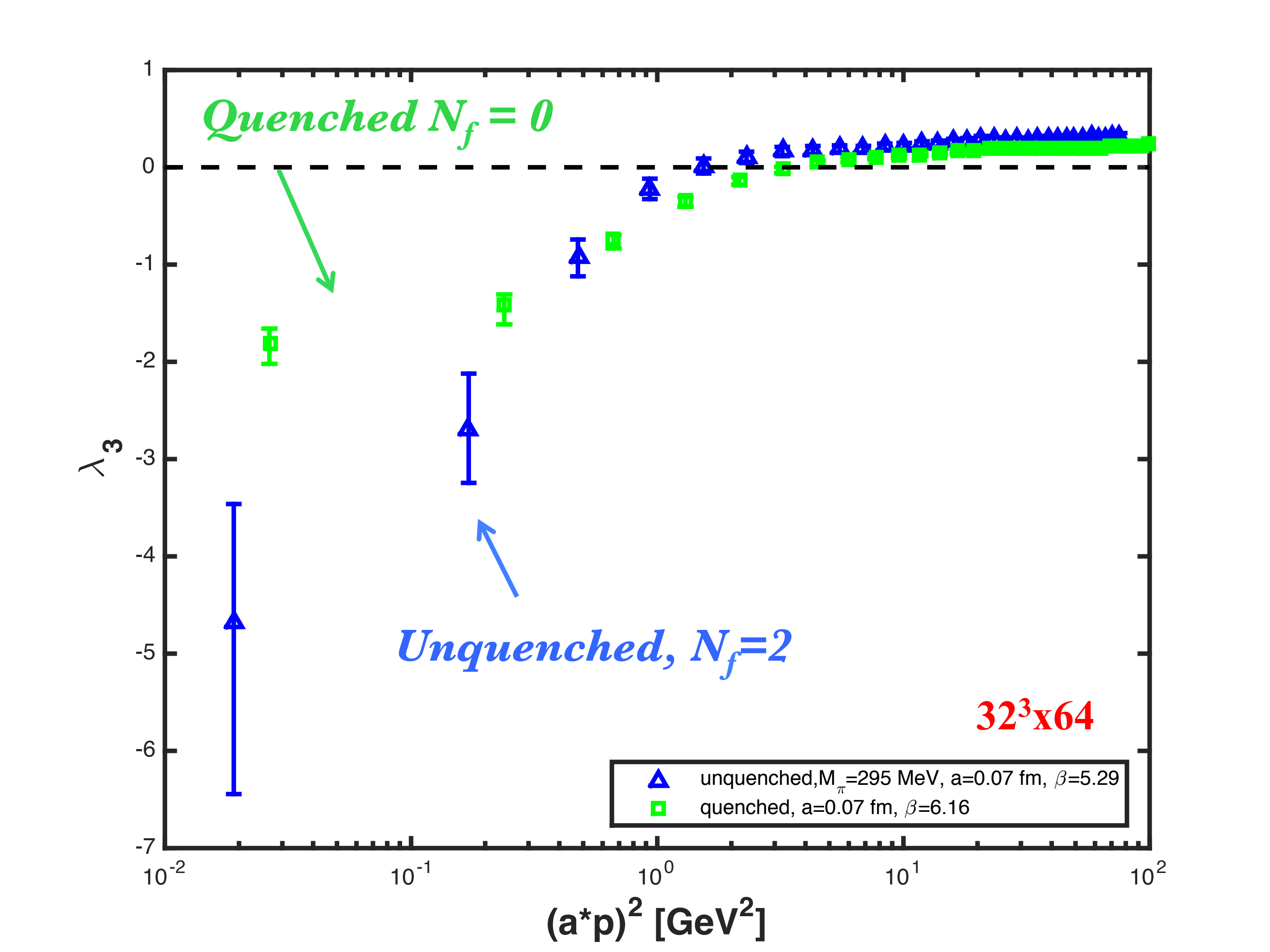
Mass dependence

λ_3 *without tree-level correction*



Unquenched versus Quenched

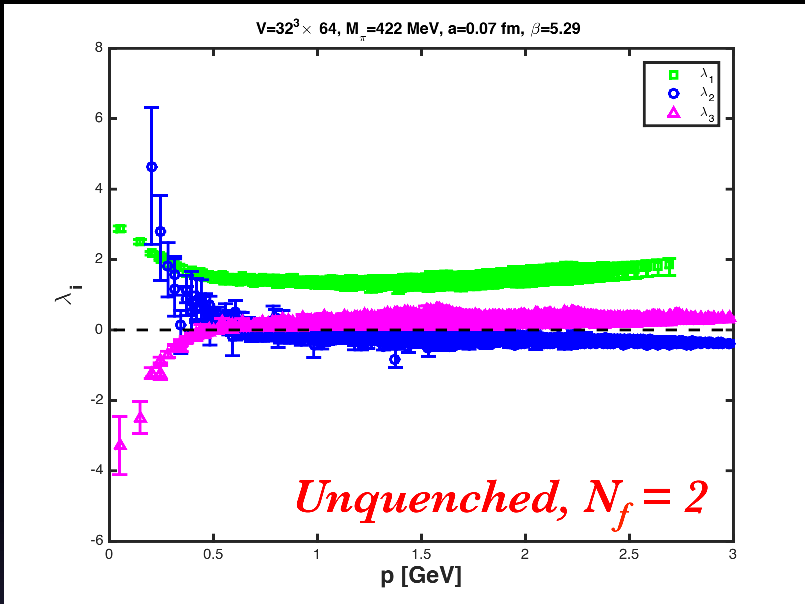
λ_3 With tree-level correction

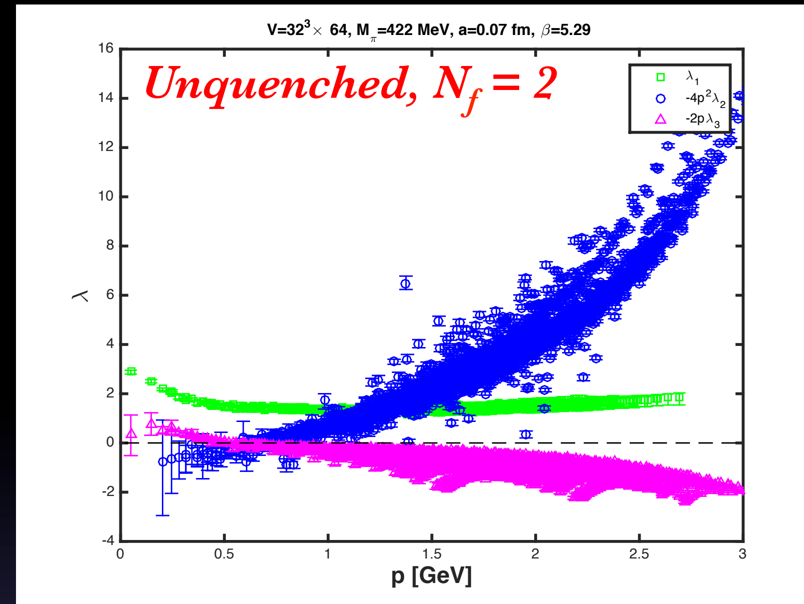


Soft gluon Kinematics (Asymmetric)

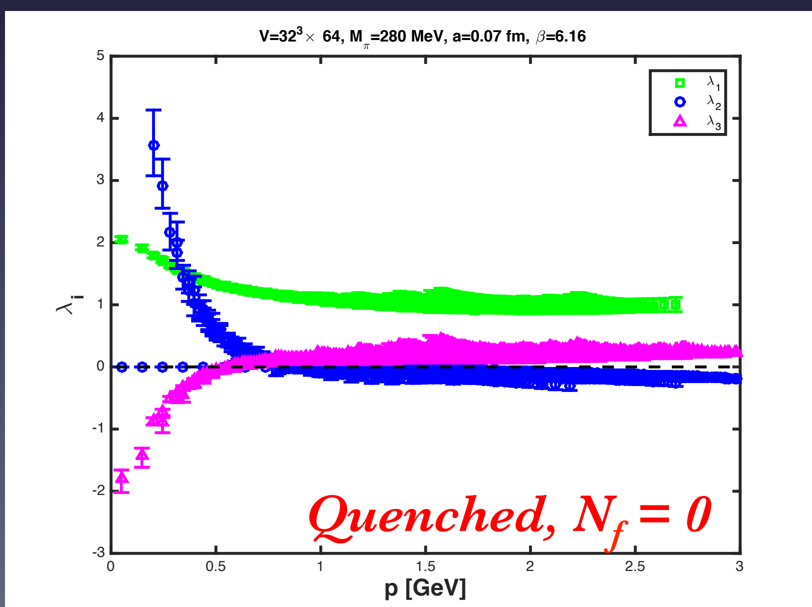
Soft gluon Kinematics (Asymmetric)

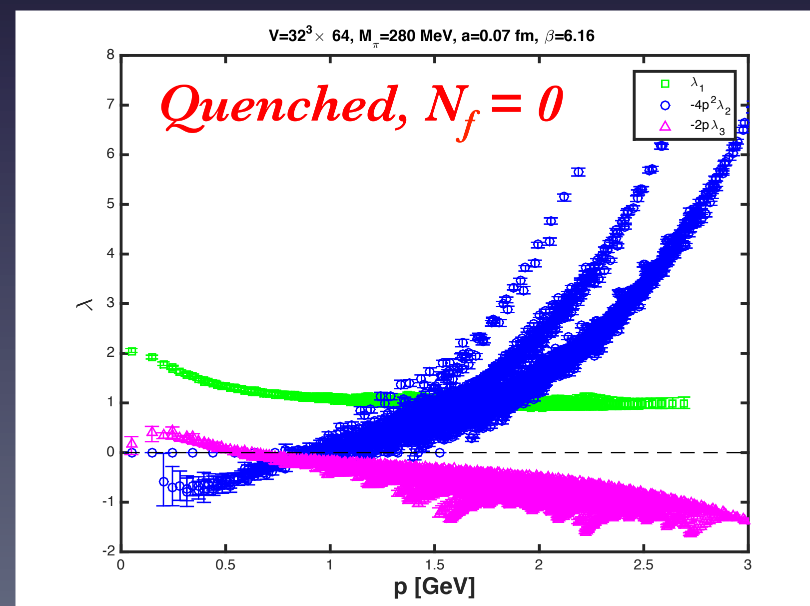
Hirarchy of Form Factors





$$(\Lambda^a_\mu)^{ij}_{\alpha\beta} = -ig_0 t^a_{ij}(\lambda^E_1 \gamma_\mu - 4\lambda^E_2 \not{p} p_\mu - 2i\lambda^E_3 p_\mu - 2i\lambda^E_4 \sigma_{\mu\nu} p_\nu)_{\alpha\beta}$$





Conclusion: *Quark-Gluon Vertex in Special Kinematics*

- ***Soft Gluon Kinematics :*** $q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0$$

- ***Hard Reflection Kinematics :*** $P_\mu = 0 \qquad k_\mu = -p_\mu = \frac{q_\mu}{2}$

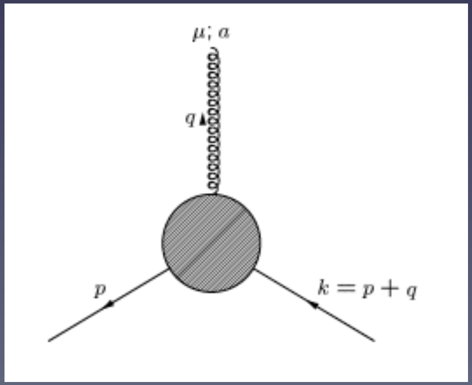
$$\Lambda_1, \tau_5$$

- ***Orthogonal Kinematics :*** $q \cdot P = 0 \qquad k^2 = p^2$

$$\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_7$$

- ***General Kinematics :***

$$\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_6, \tau_7, \tau_8$$



Summary and Outlook

- *high statistics quark propagator and quark-gluon vertex computation closer to the physical point*

- λ_1 *IR enhanced*
- λ_2 *IR strongly enhanced*
- λ_3 *IR suppressed*
- *Finite volume effects for Z(p2) are under control but for mass M(p2) needs to be better understood*
- *qualitative agreement with quark-gluon models*
- *explore further kinematics + form factors of the vertex*
- *need to have a better understanding of the lattice artefacts*