Non-perturbative Interactions of Quarks and Gluons

AYSE KIZILERSU  
(CSSM, University of Adelaide)  
Australia

Collaborators:  
Jonivar Skullerud (Maynooth, Ireland)  
Andre Sternbeck (Jena, Germany)  
Orlando Oliveira (Coimbra, Portugal)  
Paulo J. Silva (Coimbra, Portugal)  
Anthony G. Williams (Adelaide)
Fundamental Components of QCD

- Ghost Propagator
- Quark Propagator
- Ghost-Gluon Vertex
- Quark-Gluon Vertex
- 4-Gluon Vertex
- 3-Gluon Vertex

Confinement
DCSB

Non-Perturbative

\[ \alpha_s(p^2) \]

Perturbative

\[ \Lambda_{\text{QCD}}^2 \]

Asymptotic Freedom

Quarks and gluons asymptotically free
Schwinger-Dyson Equations
The $O(a)$ improved quark propagator suffers from large tree-level lattice artefacts.

Need to reduce / remove lattice artefacts:

- tree level correction
- momentum cuts (cylinder cut)
  select momenta close to the diagonal in 4-dim momentum space, which have the smallest hypercubic artefacts.
Improved Wilson, Clover Fermion Action Rotated (Sheikholeslami-Wohlert) in Discrete

Simulation Parameters:

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\kappa$</th>
<th>$a$ [fm]</th>
<th>$V$</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_q$ [MeV]</th>
<th>$N_{\text{cfg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.20</td>
<td>0.13596</td>
<td>0.081</td>
<td>$32^3 \times 64$</td>
<td>280</td>
<td>6.2</td>
<td>900</td>
</tr>
<tr>
<td>5.29</td>
<td>0.13620</td>
<td>0.071</td>
<td>$32^3 \times 64$</td>
<td>422</td>
<td>17.0</td>
<td>900</td>
</tr>
<tr>
<td>5.29</td>
<td>0.13632</td>
<td>0.071</td>
<td>$32^3 \times 64$</td>
<td>295</td>
<td>8.0</td>
<td>908</td>
</tr>
<tr>
<td>5.29</td>
<td>0.1363</td>
<td>0.07</td>
<td>$64^3 \times 64$</td>
<td>290</td>
<td>150</td>
<td>750</td>
</tr>
<tr>
<td>5.40</td>
<td>0.13647</td>
<td>0.060</td>
<td>$32^3 \times 64$</td>
<td>426</td>
<td>18.4</td>
<td>900</td>
</tr>
</tbody>
</table>

The quark propagator using state-of-the-art gauge configurations with $N_f = 2$ flavors of $O(a)$ improved Wilson fermion:

- We thank the Regensburg Collaboration (RQCD) for allowing us to use their configurations in this project.


- The calculations (gauge fixing, propagators) were performed using the HLRN (Germany) for supercomputing.

https://www.hlrn.de/home/view/Service
Quark wave function renormalisation in Landau Gauge

\((m_\pi = 280 \text{MeV}; m_q = 6.2 \text{MeV})\)

\((m_\pi = 295 \text{MeV}; m_q = 8.0 \text{MeV})\)

\((m_\pi = 422 \text{MeV}; m_q = 17.0 \text{MeV})\)

\((m_\pi = 426 \text{MeV}; m_q = 18.4 \text{MeV})\)

All 4 ensembles on the \(32^3 \times 64\) volume, using the tree-level corrected \(Z(p)\)
Finite Volume Effects

Quark Wave-function for two lattice volumes at $\beta = 5.29$, $\kappa = 0.13632$
Mass function

All 4 ensembles on the $32^3 \times 64$ volume, using the hybrid correction scheme for $M(p)$.

$m_\pi = 422\ MeV\ ;\ m_q = 17.0\ MeV$

$m_\pi = 280\ MeV\ ;\ m_q = 6.2\ MeV$

$m_\pi = 295\ MeV\ ;\ m_q = 8.0\ MeV$

$m_\pi = 426\ MeV\ ;\ m_q = 18.4\ MeV$
Finite Volume Effects

Mass function for two lattice volumes at $\beta = 5.29$, $\kappa = 0.13632$
Quark Wave Function for the lighter quark masses

\[ \beta = 5.29, \ a=0.07 \]
Quark Mass Function for the lighter quark masses

\[ \beta = 5.29, \ a=0.07 \]

\[ M = 349.7 \pm 5.2 \text{ MeV} \ @ p= 136 \text{ MeV} \text{ for } m_\pi=290 \text{ MeV} \]

\[ M = 319 \pm 16 \text{ MeV} \ @ p= 136 \text{ MeV} \text{ for } m_\pi=150 \text{ MeV} \]
Summary on the Quark Propagator:

- \( M = 319 \pm 16 \text{ MeV} \) at \( p = 136 \text{ MeV} \) for \( m_\pi = 150 \text{ MeV} \)
- \( M = 349.7 \pm 5.2 \text{ MeV} \) at \( p = 136 \text{ MeV} \) for \( m_\pi = 290 \text{ MeV} \)

- Lattice artefacts under control for \( Z(p^2) \) but not for \( M(p^2) \)

- Quark wave function seems to be lattice space independent at UV but not in the IR and it has very weak volume dependence.

- \( Z(p^2) \) suppressed in IR and further suppressed as the quark mass and lattice spacing reduced but the suppression is weakened as the volume is increased.
Determining Quark-Gluon Vertex

1-Determining \(qqg\) vertex perturbatively

(Feynman gauge, 1-loop), (General covariant gauge, 1-loop),
(Massless, Landau Gauge, 2-loop)

2- Non-perturbatively in continuum by using SDE

\[ a- Either \text{ by solving DSE for the vertex itself...} \]

\[ b- By \text{ solving coupled SDE for the propagators self consistently by imposing constraints on Green’s functions ...} \]

\[ c- By \text{ using Ward-Green-Takahashi identities...} \]

3-Non-perturbatively in discreet space by using lattice gauge field theory

\[ \Gamma_F^\mu(p, P, q) = \sum_{i=1}^{4} \lambda^i L_i^\mu(p, P) + \sum_{i=1}^{8} \tau^i T_i^\mu(p, P) \]

Transverse Basis:

\[
T_{1\mu} = -i \left[ (pq)k_\mu - (kq)p_\mu \right] \\
T_{2\mu} = -P \left[ (pq)k_\mu - (kq)p_\mu \right] \\
T_{3\mu} = i q_\mu - q^2 \gamma_\mu \\
T_{4\mu} = -i \left[ q^2 \sigma_{\mu\nu} P_\nu + 2q_\mu \sigma_{\nu\lambda} p_\nu k_\lambda \right] \\
T_{5\mu} = -i \sigma_{\mu\nu} q_\nu \\
T_{6\mu} = (qP)\gamma_\mu - i P_\mu \\
T_{7\mu} = -\frac{i}{2} (qP)\sigma_{\mu\nu} P_\nu - i P_\mu \sigma_{\nu\lambda} p_\nu k_\lambda \\
T_{8\mu} = -\gamma_\mu \sigma_{\nu\lambda} p_\nu k_\lambda - \not{p} k_\mu + k p_\mu
\]

Longitudinal Basis:

\[
L_{1\mu} = \gamma_\mu \\
L_{2\mu} = -P P_\mu \\
L_{3\mu} = -i P_\mu \\
L_{4\mu} = -i \sigma_{\mu\nu} P_\nu
\]
Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\Gamma^T_{\mu}(p, k, q) = P^T_{\mu\nu}(q) \Gamma_\mu = \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Gamma_\mu(p, k, q)$$

We study transverse projected vertex:

$$\Lambda^{a, p}_{\mu}(p, q) \equiv P_{\mu\nu}\Lambda^{a, \text{lat.}}_{\nu}(p, q) = S(p)^{-1}V^a(p, q)S(p + q)^{-1}D(q^2)^{-1}$$

$$\Gamma^{T\mu} = -\frac{1}{q^2} \left[ \Lambda_1 \, T^\mu_3 + \Lambda_2 \, T^\mu_2 + \Lambda_3 \, T^\mu_1 + \Lambda_4 \, T^\mu_4 \right] + \tau_5 \, T^\mu_5 + \tau_6 \, T^\mu_6 + \tau_7 \, T^\mu_7 + \tau_8 \, T^\mu_8$$

$$\Lambda_1 = \lambda_1 - q^2 \tau_3 \quad \Lambda_2 = \lambda_2 - \frac{q^2}{2} \tau_2$$

$$\Lambda_3 = \lambda_3 - \frac{q^2}{2} \tau_1 \quad \Lambda_4 = \lambda_4 + q^2 \tau_4$$
Quark-Gluon Vertex in Special Kinematics

Soft Gluon Kinematics:

\( q_\mu = 0 \quad k_\mu = p_\mu = \frac{P_\mu}{2} \)

\[
(\Lambda^a_{\mu})^{ij}_{\alpha\beta} = -ig_0 t^a_{ij} (\Gamma^T_\mu)^{T}_{\alpha\beta}
\]

\[
(\Lambda^a_\mu)_{ij}^{\alpha\beta} = -ig_0 t^a_{ij} (\lambda_1^E \gamma_\mu - 4 \lambda_2^E p_\mu - 2i \lambda_3^E p_\mu - 2i \lambda_4^E \sigma_{\mu\nu} p_\nu)_{\alpha\beta}
\]

\[
Tr_4[I \Gamma^T_\mu] = -2i \lambda_3^E p_\mu
\]

\[
Tr_4[\gamma_\mu \Gamma^T_\mu] = \lambda_1^E \delta_{\alpha\mu} - 4 \lambda_2^E p_\alpha p_\mu
\]

\[
Tr_4[\sigma_{\alpha\beta} \Gamma^T_\mu] = -2i \lambda_4^E (p_\alpha \delta_{\beta\mu} - p_\beta \delta_{\alpha\mu})
\]

\[
\lambda_1^E, \lambda_2^E, \lambda_3^E, \lambda_4^E = 0
\]
Soft gluon Kinematics (Asymmetric)

$\lambda_1$ with and without tree-level correction

Quenched, $N_f = 0$  $32^3 \times 64$

Unquenched, $N_f = 2$  $32^3 \times 64$

without TL

with TL

without TL

with TL

Unquenched, $N_f = 2$  $64^3 \times 64$
With and without Tree-level correction

$\lambda_1$

Soft gluon Kinematics (Asymmetric)
\[ \lambda_1 \quad \text{With tree-level correction} \]

**Unquenched, \( N_f = 2 \)**

64\(^3\)x64

**Unquenched, \( N_f = 2 \)**

32\(^3\)x64

**Quenched, \( N_f = 0 \)**

32\(^3\)x64

**Soft gluon Kinematics (Asymmetric)**
\( \lambda_2 \) without tree-level correction

Unquenched, \( N_f = 2 \)

64\(^3\)x64

32\(^3\)x64

Quenched, \( N_f = 0 \)
Soft gluon Kinematics (Asymmetric)

Mass dependence

$\lambda_2$ without tree-level correction

$M = 280$ MeV, $a = 0.08$ fm, $\beta = 5.20$

$M = 422$ MeV, $a = 0.07$ fm, $\beta = 5.29$

$M = 295$ MeV, $a = 0.07$ fm, $\beta = 5.29$

$M = 426$ MeV, $a = 0.06$ fm, $\beta = 5.40$

$32^3 \times 64$
\[ \lambda_3 \] without tree-level correction

Unquenched, \( N_f = 2 \)

- 64\(^3\)x64

Unquenched, \( N_f = 2 \)

- 32\(^3\)x64

Quenched, \( N_f = 2 \)

- 32\(^3\)x64
Soft gluon Kinematics (Asymmetric)

Mass dependence

$\lambda_3$ without tree-level correction

\[ (a^* p)^2 \ [\text{GeV}^2] \]

\[ 32^3 \times 64 \]
Unquenched versus Quenched

\( \lambda_3 \) With tree-level correction

\[ \text{Quenched } N_f = 0 \]

\[ \text{Unquenched, } N_f = 2 \]

32\( ^3 \times 64 \)

**Soft gluon Kinematics (Asymmetric)**
Soft gluon Kinematics (Asymmetric)

\[(\Lambda^a_{\mu})^{ij}_{\alpha\beta} = -ig_0t^a_{ij}(\lambda_1^E \gamma_\mu - 4 \lambda_2^E \not p p_\mu - 2i\lambda_3^E p_\mu - 2i\lambda_4^E \sigma_{\mu\nu} p_\nu)_{\alpha\beta}\]

Hirarchy of Form Factors

\[V=32^3 \times 64, \; M=280 \text{ MeV}, \; a=0.07 \text{ fm}, \; \beta=5.29\]

Unquenched, \(N_f = 2\)

\[V=32^3 \times 64, \; M=422 \text{ MeV}, \; a=0.07 \text{ fm}, \; \beta=5.29\]

Unquenched, \(N_f = 2\)

\[\Lambda^a_{\mu} \] \[= \]

\[\lambda_1^E \]

\[\lambda_2^E \]

\[\lambda_3^E \]

\[\lambda_4^E \]

Quenched, \(N_f = 0\)

\[V=32^3 \times 64, \; M=280 \text{ MeV}, \; a=0.07 \text{ fm}, \; \beta=6.16\]

Quenched, \(N_f = 0\)

\[\Lambda^a_{\mu} \] \[= \]

\[\lambda_1^E \]

\[\lambda_2^E \]

\[\lambda_3^E \]

\[\lambda_4^E \]
Conclusions: Quark-Gluon Vertex in Special Kinematics

- **Soft Gluon Kinematics**: \( q_\mu = 0 \quad k_\mu = p_\mu = \frac{P_\mu}{2} \)

\[ \lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0 \]

- **Hard Reflection Kinematics**: \( P_\mu = 0 \quad k_\mu = -p_\mu = \frac{q_\mu}{2} \)

\[ \Lambda_1, \tau_5 \]

- **Orthogonal Kinematics**: \( q \cdot P = 0 \quad k^2 = p^2 \)

\[ \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_7 \]

- **General Kinematics**: 

\[ \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_6, \tau_7, \tau_8 \]
Summary and Outlook

- high statistics quark propagator and quark-gluon vertex computation closer to the physical point

- $\lambda_1$ IR enhanced
- $\lambda_2$ IR strongly enhanced
- $\lambda_3$ IR suppressed
- Finite volume effects for $Z(p^2)$ are under control but for mass $M(p^2)$ needs to be better understood
- qualitative agreement with quark-gluon models
- explore further kinematics + form factors of the vertex
- need to have a better understanding of the lattice artefacts