QCD Thermodynamics on the Lattice from the Gradient Flow

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References:

FlowQCD coll. Phys.Rev. D90 (2014) 1, 011501

E.I., H.Suzuki, Y.Taniguchi, T.Umeda arXiv:1511.03009

and

Work in progress with S. Aoki, T.Hatsuda

INPC2016 @ International convention center, Aderaide, Australia 2016/09/16

thermodynamic quantity in finite-T QCD

pressure, entropy, traceanomaly…
shear / bulk viscosity

integration method
differential method

calculate free energy
classical thermodynamics
macroscopic picture

H.Meyer's plenary talk on Tuesday

energy-momentum tensor (EMT)

quantum field theory
microscopic picture

trace anomaly

$$\sum_{i=1}^{4} T_{ii} = \frac{\epsilon - 3P}{T^4}$$

entropy density

$$T_{44} - T_{11} = \frac{\epsilon + P}{T^4}$$

EMT on Lattice

Lattice regularization: a nonperturbative regularization

gauge invariant

discretize space-time coord.

- ✦ generator of general coord. transformation

EMT on Lattice

Lattice regularization: a nonperturbative regularization
gauge invariant
discretize space-time coord.

- ✦ generator of general coord. transformation
- ✦ same quantum number with the vac. (signal is noisy)

Basic Idea

Quantum field theory

(UV divergence)

perturbation with
dim. reg.
+YM gradient flow
(general covariance OK!)

lattice reg.
+Wilson flow
(with a->0 limit)

At finite flow time, UV finite!

Luescher and Weisz, JHEP 1102, 051(2011)

Firstly, we obtain the relation between them perturbatively.
Assume that it applies to the nonperturbative regime.

YM gradient flow

Flow equation

Luescher, JHEP 1008, 071 (2010)

Yang–Mills gradient flow (continuum theory)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) = \Delta B_\mu(t,x) + \cdots, \qquad B_\mu(t=0,x) = A_\mu(x)$$

Wilson flow (lattice theory)

$$\partial_t V(t,x,\mu) V(t,x,\mu)^{-1} = -g_0^2 \partial S_{\mathrm{Wilson}}, \qquad V(t=0,x,\mu) = U(x,\mu)$$

link variable: $U_\mu(x) = e^{ig_0 A_\mu(x)}$

t: fictitious time direction (flow-time)

$x = (\vec{x}, \tau)$

UV finiteness of the gradient flow

Flow equation (continuum)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x)$$ initial condition: $B_\mu(t=0,x) = A_\mu(x)$

perturbative solution in the leading order

$$B_\mu(t,x) = \int d^D y K_t(x-y) A_\mu(y)$$

$$K_t(z) = \int \frac{d^D p}{(2\pi)^D} e^{ipz} e^{-tp^2}$$

$p^2 > 1/t$ modes are suppressed (a smooth UV cutoff)

Smeared in the range $|x| < \sqrt{8t}$

signal becomes clear?

Finiteness is shown perturbatively in all order

Luescher and Weisz, JHEP 1102, 051(2011)

Energy-momentum tensor

$$T_{\mu\nu}(x) = \frac{1}{g_0^2}\left[F^a_{\mu\rho}(x)F^a_{\nu\rho}(x) - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x)\right]$$

Renormalized EMT within dim. reg.

$$\{T_{\mu\nu}\}_R(x) = T_{\mu\nu}(x) - \langle T_{\mu\nu}(x)\rangle$$

Dim=4 gauge invariant operator on Lattice

$$U_{\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$

$$E(t,x) \equiv \frac{1}{4}G^a_{\mu\nu}(t,x)G^a_{\mu\nu}(t,x)$$

Here, ops. are constructed by flowed field.

``Suzuki method"

- small flow-time expansion -

Suzuki, PTEP 2013, no8, 083B03, [Erratum: PTEP2015,079201(2015)],

relation…dim.=4 op on the lattice vs. renormalized EMT at small flow-time

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

coefficients…given by renormalized coupling and coeff. of beta fn.

$$\alpha_U(t)(g;\mu) = g^2\left\{1 + 2b_0\left[\ln(\sqrt{8t}\mu) + s_1\right]g^2 + O(g^4)\right\},$$

$$\alpha_E(t)(g;\mu) = \frac{1}{2b_0}\left\{1 + 2b_0 s_2 g^2 + O(g^4)\right\},$$

b_0 1-loop coeff. of beta fn. MSbar scheme $s_1 =$ -0.0863575

$s_2 =$ 0.05578512

cf.) Nonperturbative method: L.DelDebbio, A.Patella, A.Rogo, JHEP 1311,212(2013)

How to get EMT

for quenched QCD

Step 1

Generate gauge configuration at t=0 (usual process)

Step 2

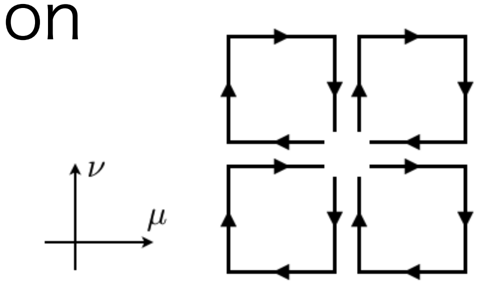
Solve the Wilson flow eq. and generate the gauge configuration at flow time (t)

$$a \ll \sqrt{8t} \ll \Lambda_{QCD}^{-1} \text{ or } T^{-1}$$

Step 3

Measure two dim=4 ops. using flowed gauge configuration

$$U_{\mu\nu}(t,x), E(t,x)$$



Step 4

Take the continuum limit. Then take t->0 limit.

(Take care the feasible window of flow time)

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{ \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

One-point fn. of EMT in finite temperature quenched QCD

Asakawa, Hatsuda, E.I., Kitazawa, Suzuki (FlowQCD coll.)
Phys.Rev. D90 (2014) 1, 011501

Simulation setup

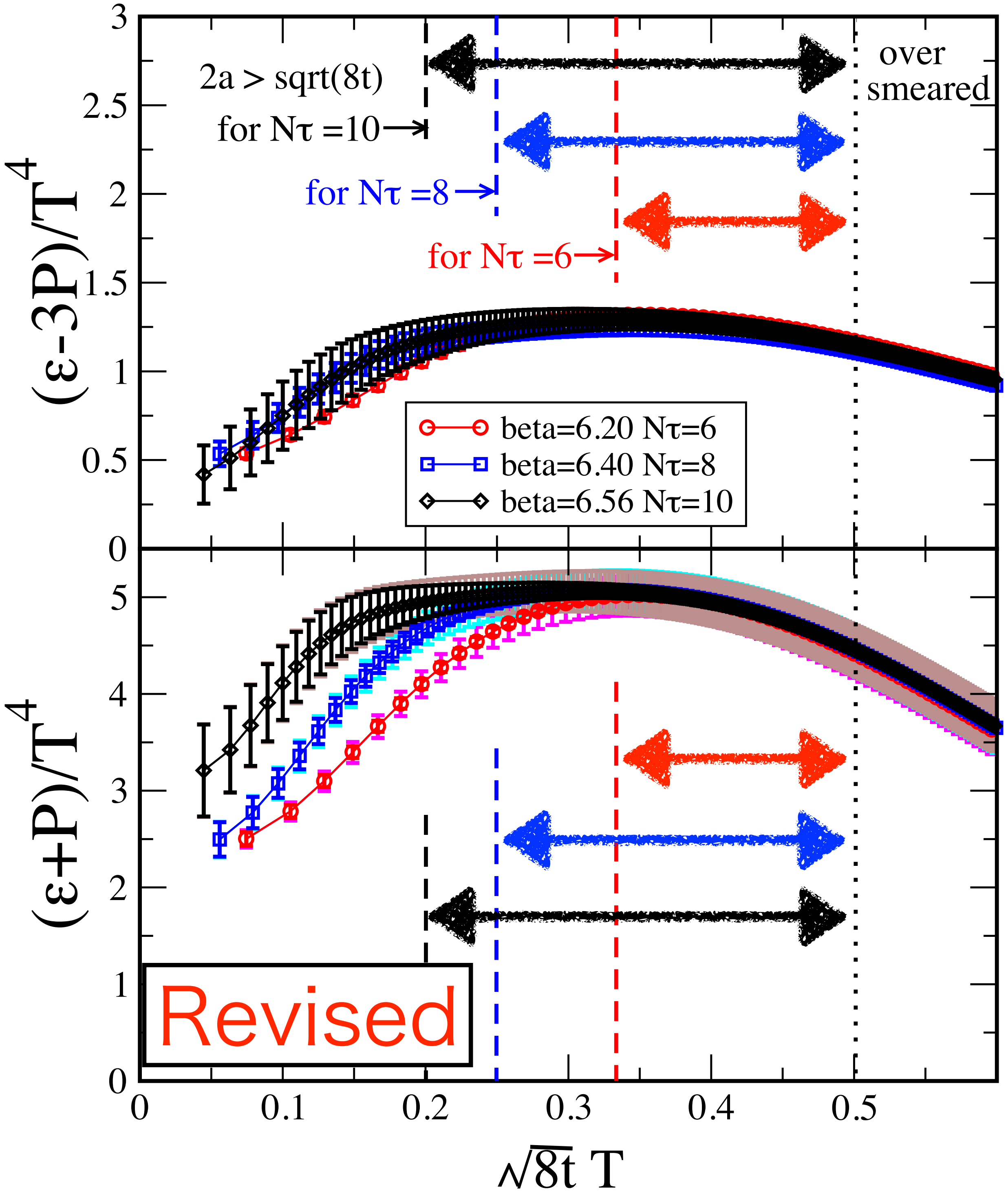
- Wilson plaquette gauge action
- lattice size (Ns=32, Nt=6,8,10,32)
- # of confs. is 100 - 300
- simulation parameters

N_τ	6	8	10	T/T_c
β	6.20	6.40	6.56	1.65
	6.02	6.20	6.36	1.24
	5.89	6.06	6.20	0.99

Temperature is determined by
Boyd et. al. NPB469,419 (1996)

Parametrization is given by
alpha collaboration NPB538,669 (1999)

flow time dependence
(T=1.65Tc)



each dark color shows statistical error

each light color includes systematic error

trace anomaly $\sum_{i=1}^{4} T_{ii} = \frac{\epsilon - 3P}{T^4}$

entropy density $T_{44} - T_{11} = \frac{\epsilon + P}{T^4}$

feasible flow time

longer than lattice cutoff

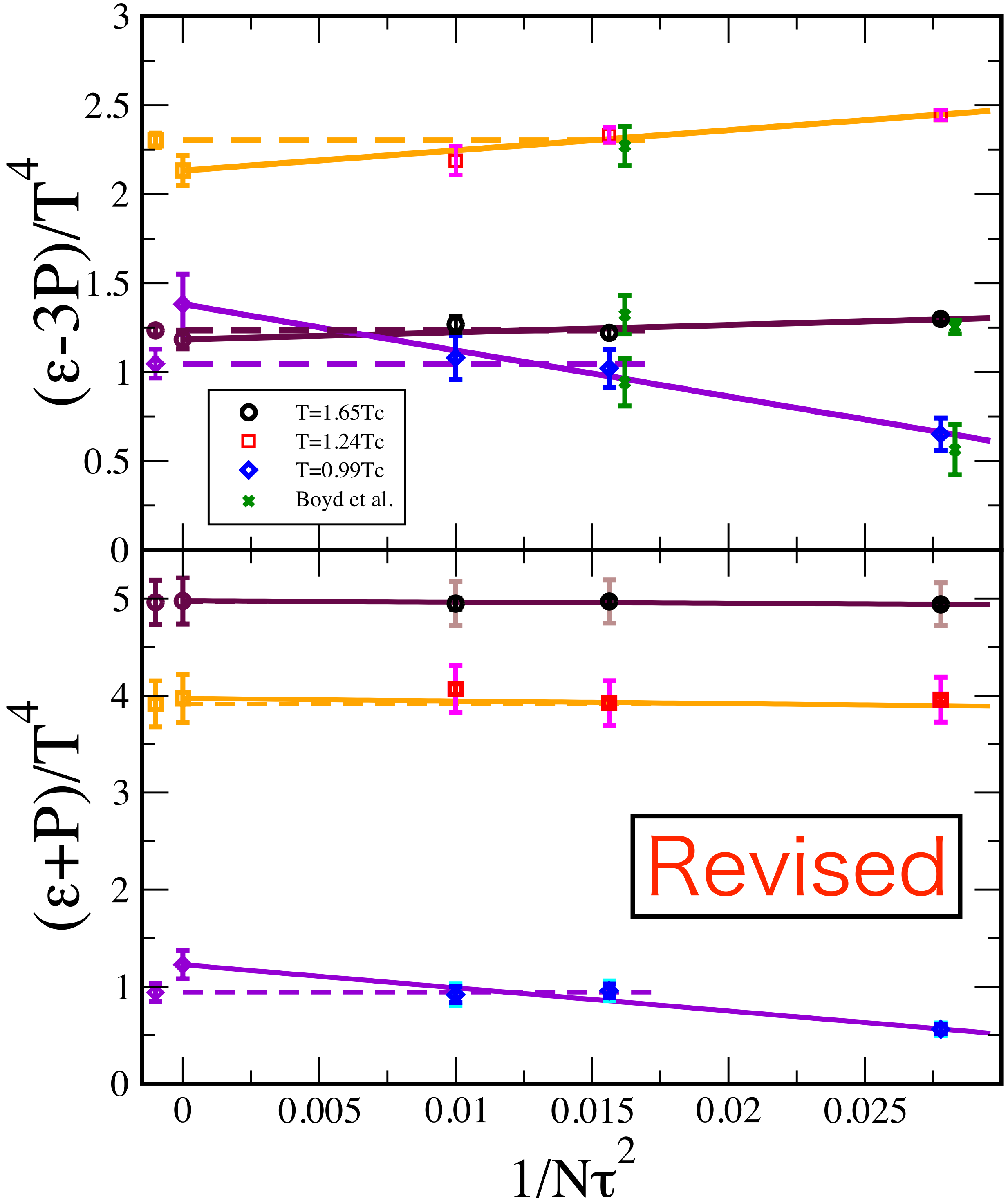
avoid an over-smeared regime

$$2a < \sqrt{8t} < N_\tau a/2$$

- show a plateau
  (small higher dimensional op.)
  Practically, no need t-> 0 limit
  **finer lattice simulation shows a slope

- systematic error coming from scale setting is dominated in entropy density

Continuum extrapolation



$\sqrt{8t}T = 0.40$

3point linear extrap.
(2pt. const. extrap.)

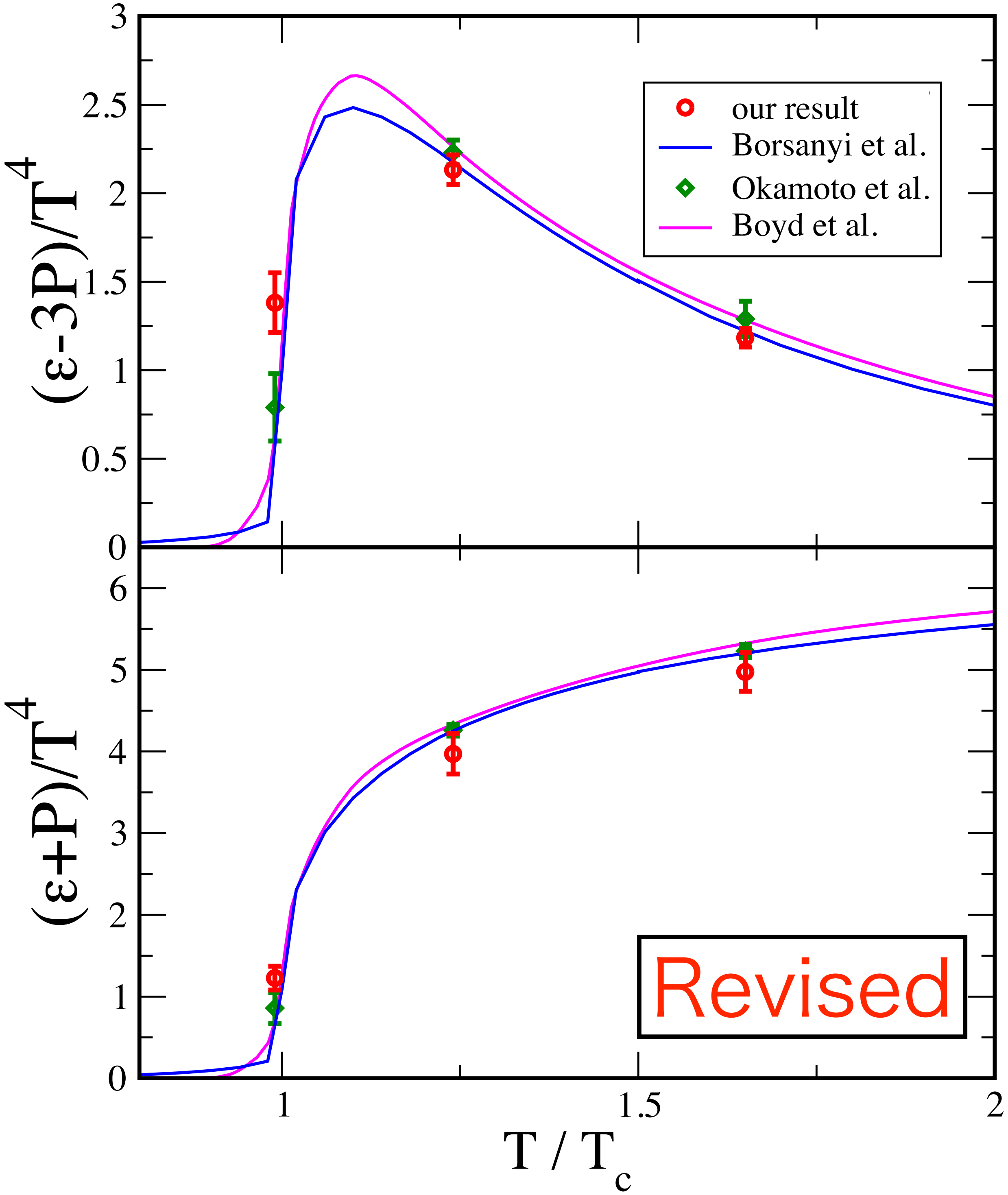
We also see the data at $\sqrt{8t}T = 0.35$

In cont.lim. the result is consistent.

t->0 limit is not needed in this case

Comparison with the results given by integration method

Phys.Rev. D90 (2014) 1, 011501, arXiv:1312.7492v3[hep-lat]



Boyd et. al. NPB469,419 (1996)

Okamoto et. al. (CP-PACS) PRD60, 094510 (1999)

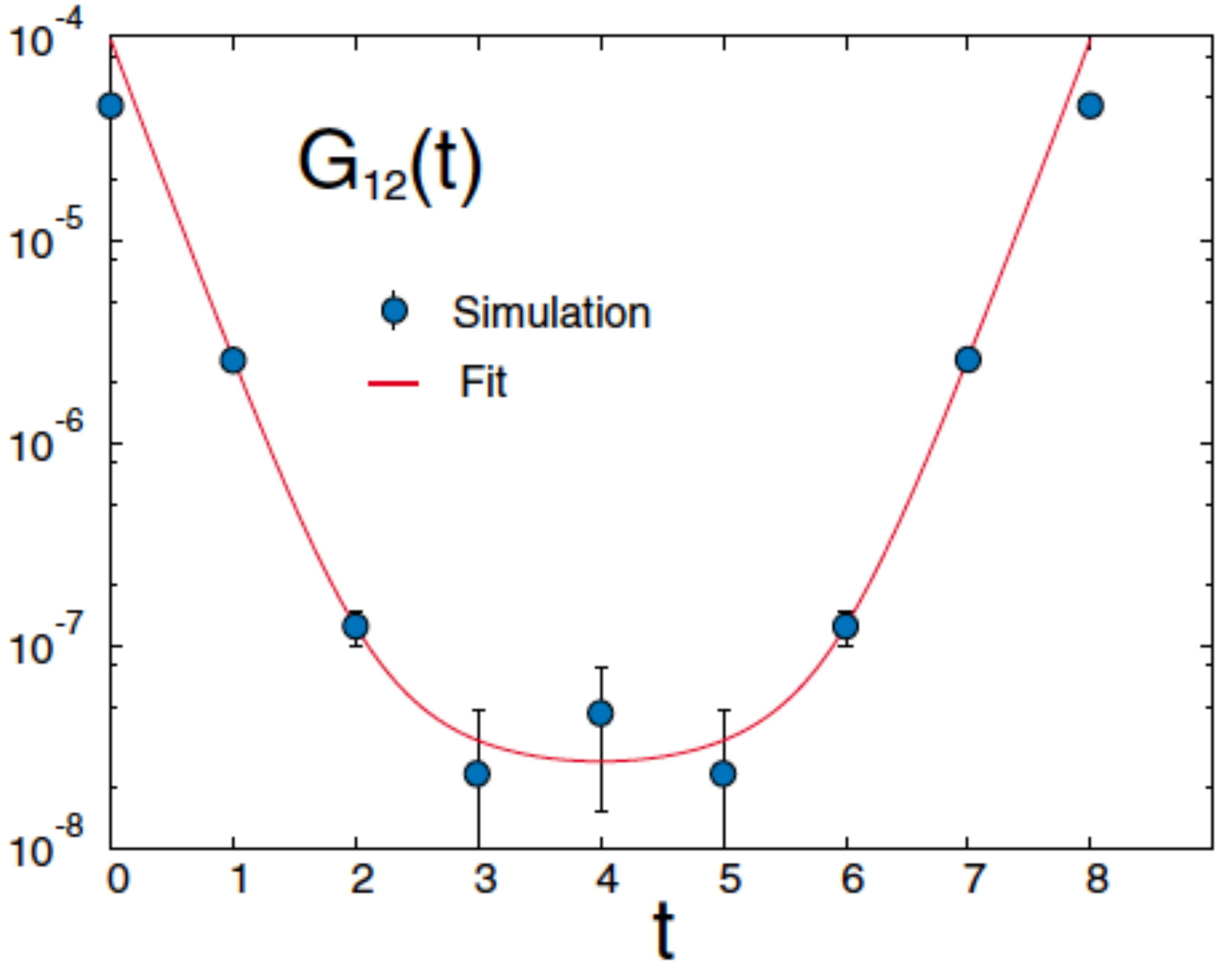
Borsanyi et. al. JHEP 1207, 056 (2012)

Integration method is based on (macroscopic) thermodynamics.

Our method is based on the (microscopic) quantum field theory.

two-point fn. of EMT

Shear viscosity in QGP phase



Matsubara-Green's function G12(t),
Nakamura-Sakai(2005)
800,000 conf.

shear viscosity: retarded Green's fn.

$$\eta = -\int \langle T_{12}(\vec{x},\tau)T_{12}(\vec{x}',0)\rangle_{\mathrm{ret.}}$$

obtained by the analytic continuation
of Matsubara Green's fn.

$$G_\beta(\vec{p},t) = \sum_n e^{i\omega_n t}\int d\omega\,\frac{\rho(\vec{p},\omega)}{i\omega_n-\omega}$$

Renormalization

$$\boxed{T_{\mu\nu}^{(R)}(g_0) = Z(g_0) T_{\mu\nu}^{(bare)}}$$

Meyer (2007)…1loop approximation

Fodor et al. (2013)…calculate Z-factor from entropy density

This work … Not necessary

(usage of Suzuki coefficient and MSbar coupling)

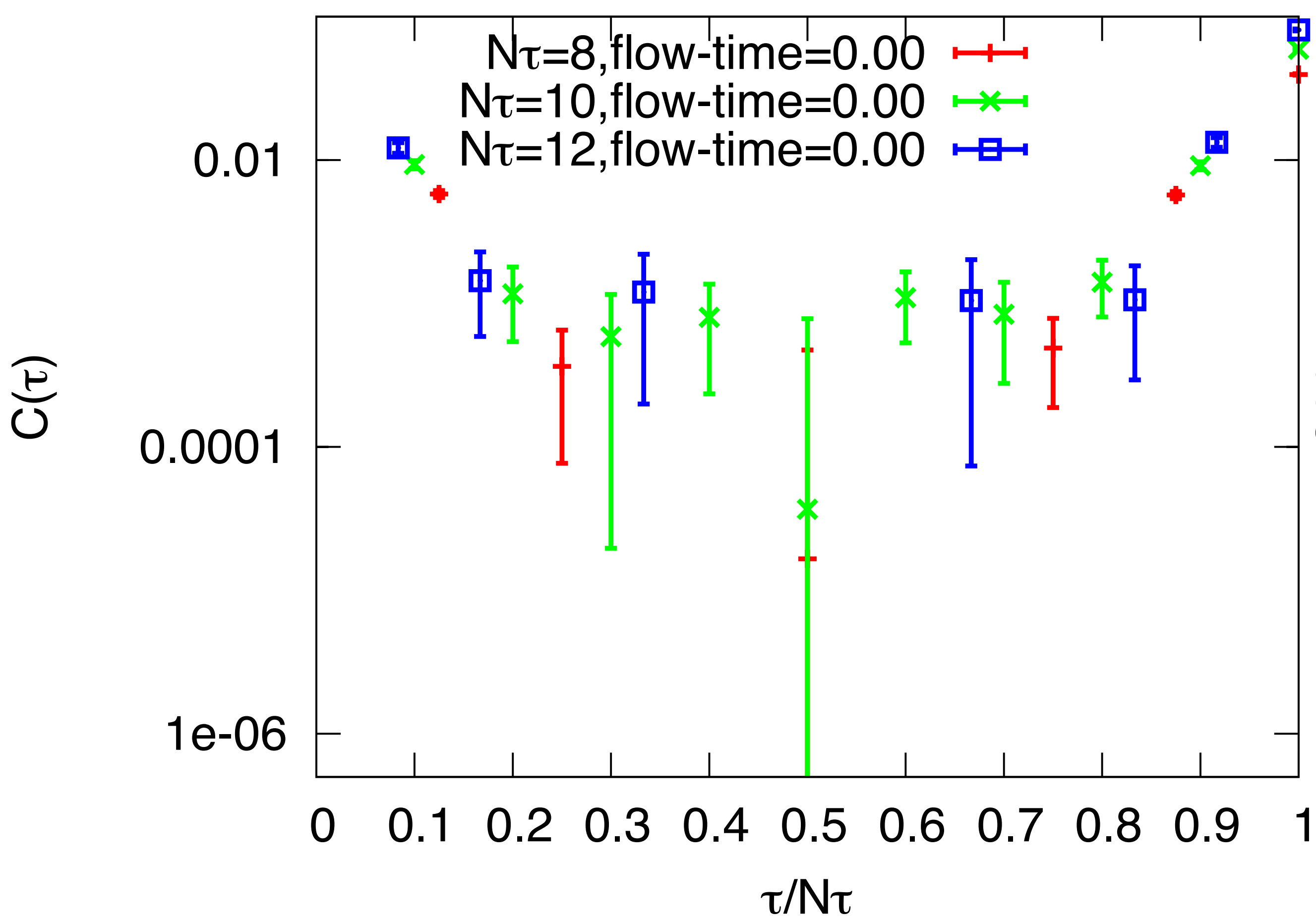
cf.) $\frac{sT}{4} = \langle T_{11}^{(R)} \rangle$

$$\langle T_{12} T_{12} \rangle = \frac{1}{4} \langle (T_{11} - T_{22})(T_{11} - T_{22}) \rangle$$

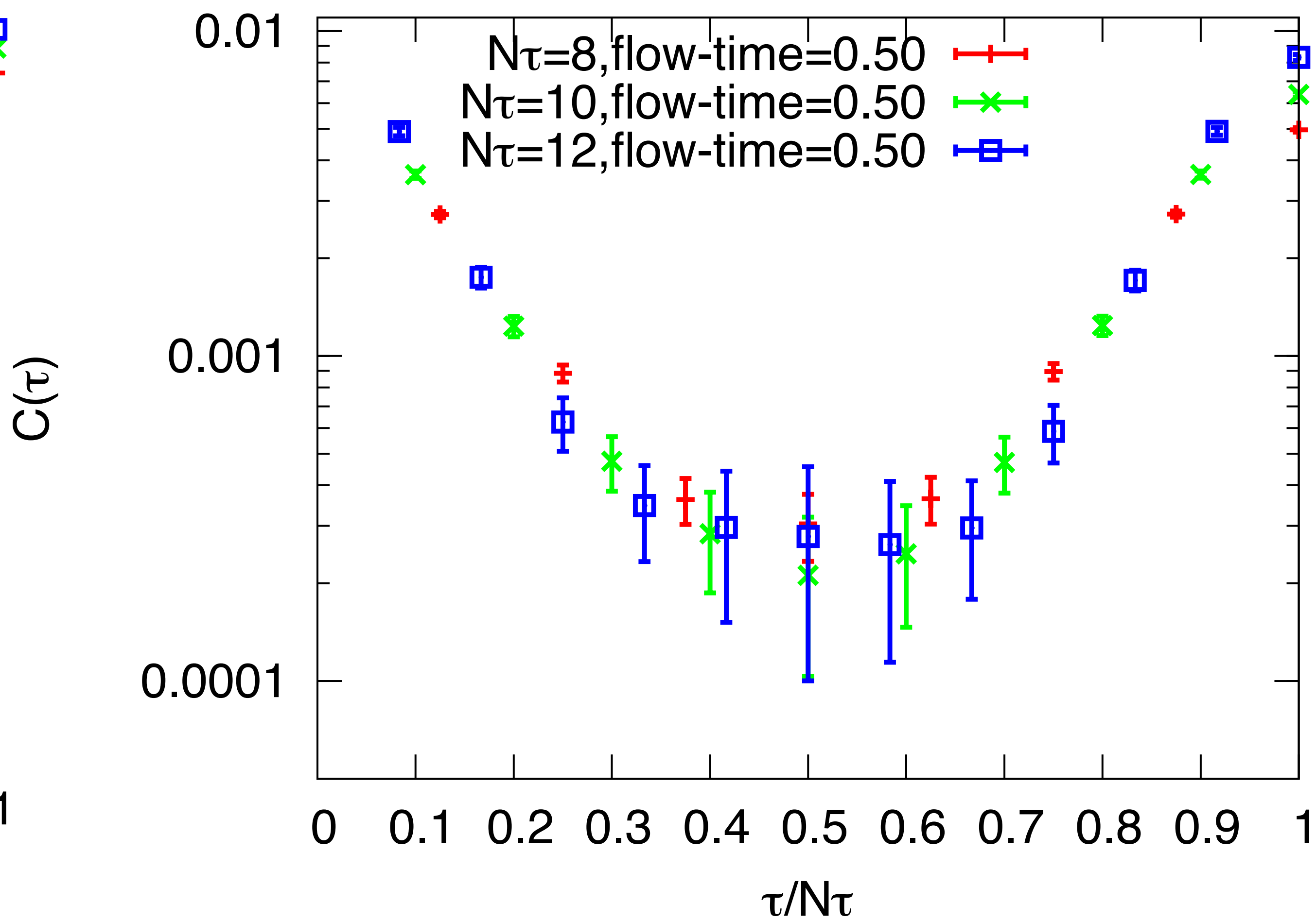
lattice raw data

$$C(\tau) = \langle \frac{1}{N_s^3} \sum_{\vec{x}} U_{12}(\vec{x},\tau) \frac{1}{N_s^3} \sum_{\vec{y}} U_{12}(\vec{y},0) \rangle$$

flow-time=0



flow-time t/a^2=0.50



fixed smeared length in lattice unit

beta=6.40,Nt=8, 2,000 conf.

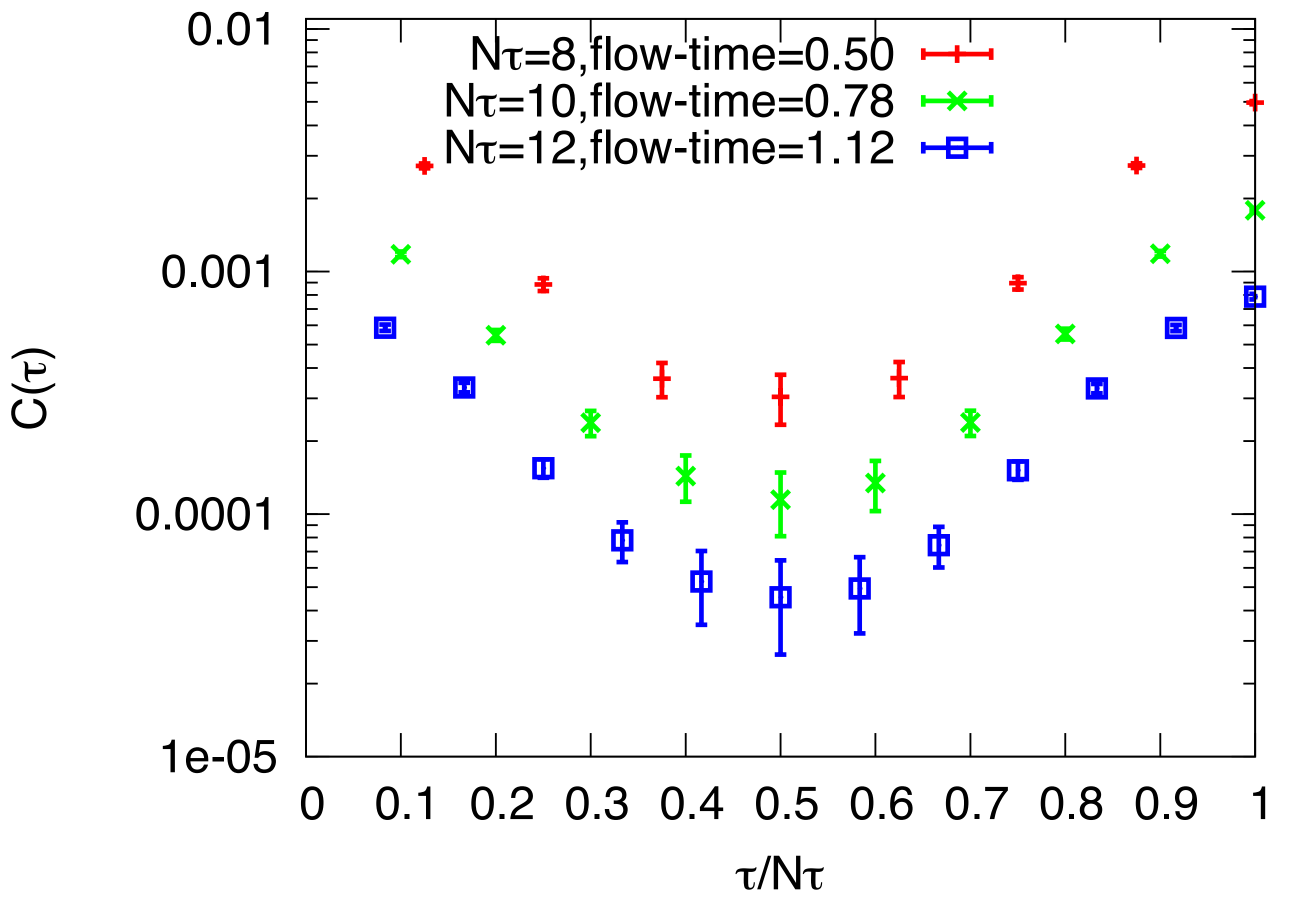
beta=6.57,Nt=10, 1,100 conf.

beta=6.72,Nt=12, 650 conf.

EMT correlator

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

$$C(\tau) = \langle \frac{1}{N_s^3}\sum_{\vec{x}} U_{12}(\vec{x},\tau)\frac{1}{N_s^3}\sum_{\vec{y}} U_{12}(\vec{y},0)\rangle$$



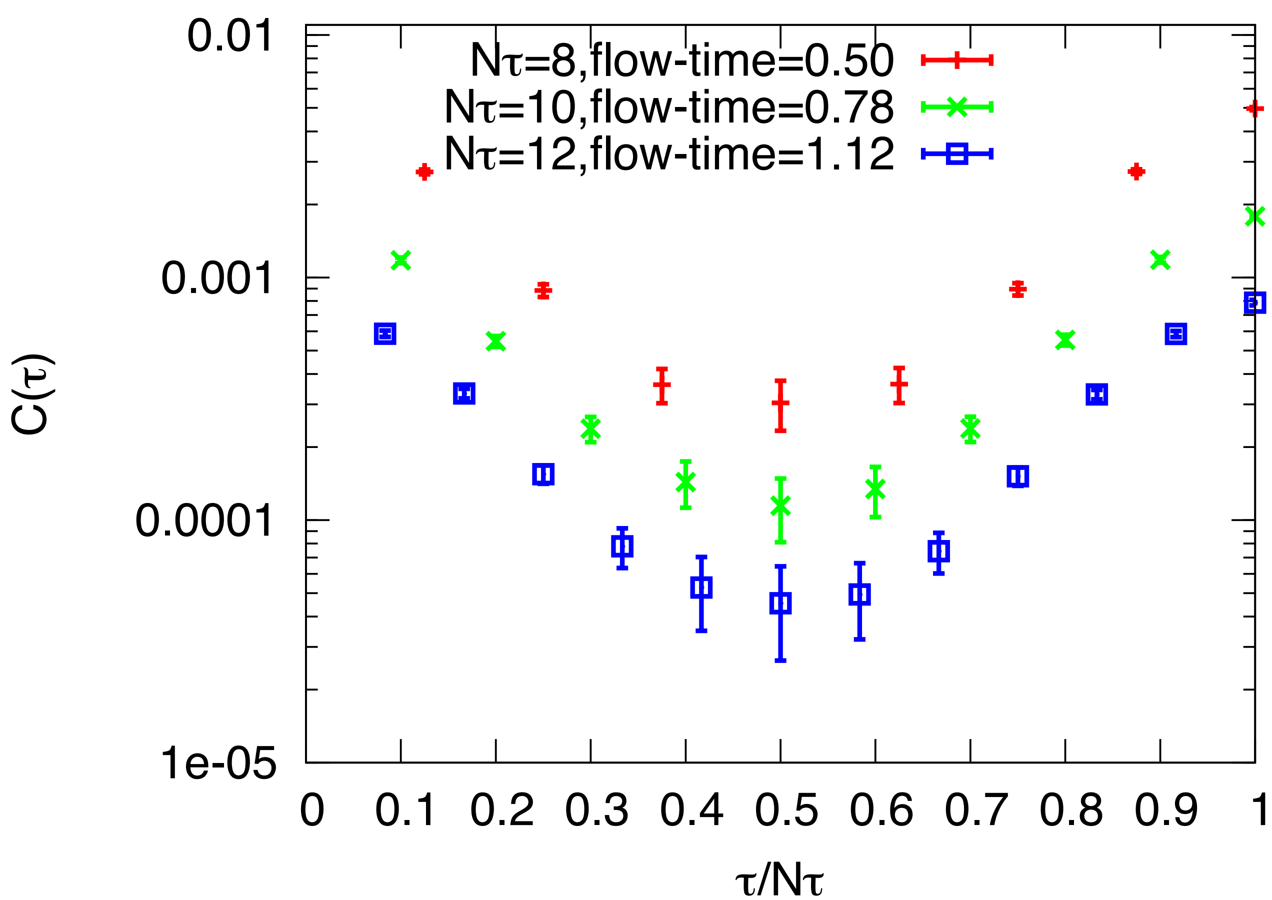
fixed smeared length in physical unit $\sqrt{8t}T = 0.25$

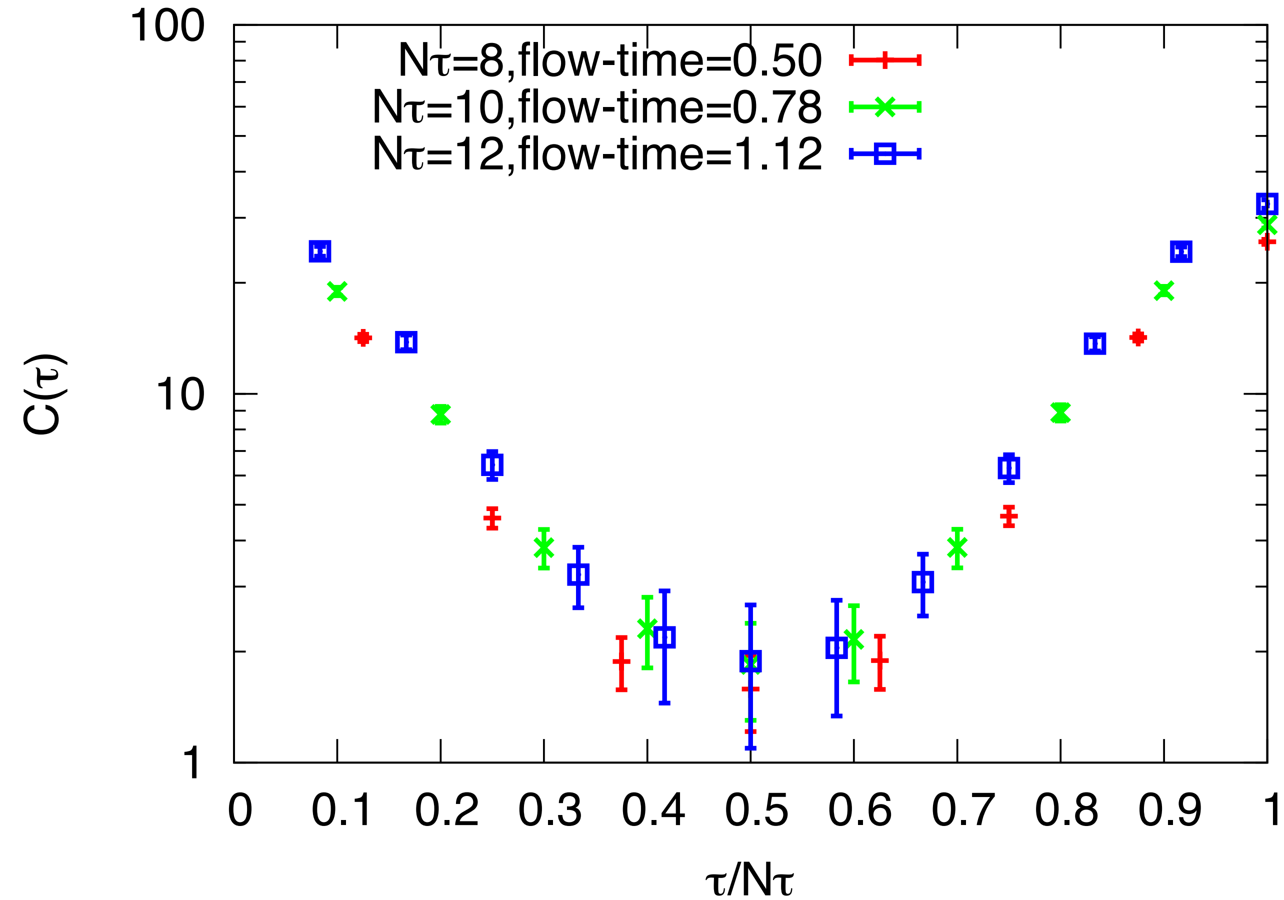
EMT correlator

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

$$C(\tau) = \langle \frac{1}{N_s^3}\sum_{\vec{x}} U_{12}(\vec{x},\tau)\frac{1}{N_s^3}\sum_{\vec{y}} U_{12}(\vec{y},0)\rangle$$

$$C(\tau) = \frac{1}{T^5}\langle \sum_{\vec{x}} T_{12}(\vec{x},\tau)\sum_{\vec{y}} T_{12}(\vec{y},0)\rangle$$





fixed smeared length in physical unit $\sqrt{8tT} = 0.25$

conclusion

- ✦ Novel method to obtain EMT using the lattice simulation
- ✦ quenched results (1pt.fn) show that the small flow time expansion is promising
- ✦ clear statistical signal, small systematic error
- ✦ Z-factor of the bosonic ops. are not needed
- ✦ 2pt. fn. and full QCD simulation are also doable!!

Nf=2+1 QCD
E.I. et al.; arXiv:1511.03009
WHOT coll, arXiv:1609.01417

future directions

- ✦ two-point function of EMT (shear and bulk viscosity, heat capacity)
- ✦ application to the other theories
- ✦ conformal field theory (central charge, dilation physics)
- ✦ nonlinear sigma model