Sign of enhanced radius of Hoyle rotational state in $\alpha + {}^{12}C$ inelastic scattering

M. Tomita, Iwasaki, R. Otani, M. Ito

Department of Pure and Applied Physics, Kansai University

1. Background: Probe of 3α size through ${}^{12}C \Rightarrow 3\alpha$ inelastic scattering

   Present status and problem

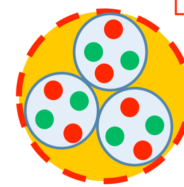
2. Our viewpoint and analysis: Comparison of 2_1^+ with 2_2^+

3. Results: $\alpha + {}^{12}C$ inelastic scattering and spatial size of the reaction

4. Discussion: Lower limit of the matter radius of Hoyle rotational 2_2^+

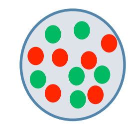
5. Summary and future studies

Background



There are several studies that try to get the signature of the enhanced 3α radius in the inelastic scattering of $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$.

A. Diffraction model



- p + ^{12}C: K. Iida et al., MPLA27 (2012)
- light-ion + ^{12}C: A. N. Danilov et al., PRC80 (2009)

An enhanced diffraction radius is obtained in the transition of $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$

B. Microscopic coupled-channel calculations

- α + ^{12}C inelastic scattering: S. Ohkubo et al., PRC70 (2004)
- Evolution of Airy structures are confirmed in the $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$ scattering

- Criticism by M. Takashina et al., PRC78 (2008), PRC74 (2006)

⇒ Inelastic scattering to Hoyle state does NOT reflect the size of the 3α matter radius

Angular distribution is mainly determined by the size of the coupling potential

Problem to get a sign of enhanced 3α radius in ^{12}C inelastic scattering

1. Comparison of 0_1^+ with 0_2^+

$0_2^+ \Rightarrow$ Inelastic scattering

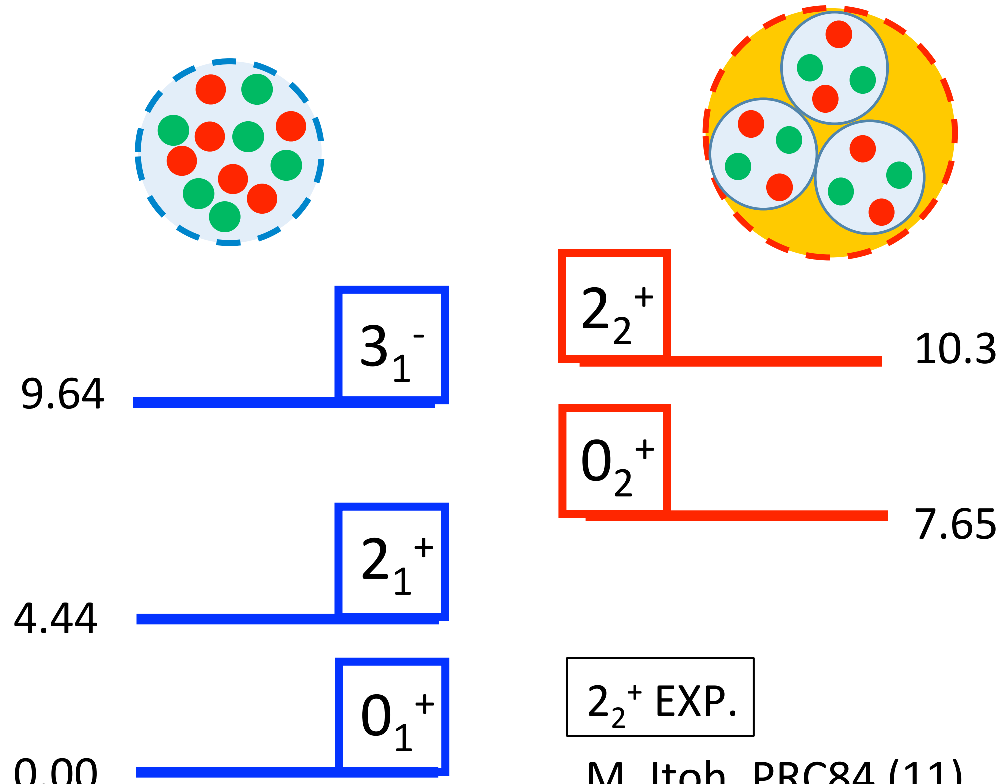
$0_1^+ \Rightarrow$ Elastic scattering

⇒Invalid comparison !

2. Comparison of 2_1^+, 3_1^- with 0_2^+

There are finite-spin effects in the 2_1^+ and 3_1^- channels

⇒Unfair comparison !



M. Itoh, PRC84 (11)

M. Freer, PRC80 (09)

Theory, Y. Funaki, EPJ24,321 (05)

Our viewpoint

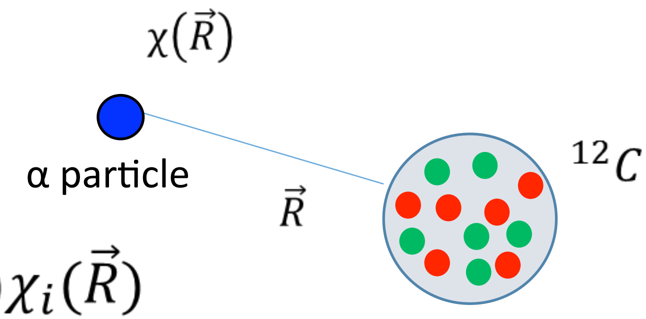
We consider comparisons of 2_1^+ with 2_2^+ channels (Fair comparison)

The difference of the 2_1^+ and 2_2^+ channels is just a size of their matter radius

Present report

We demonstrate that a sign of the enhanced radius of 2_2^+ appears in α + ^{12}C inelastic scattering by performing the microscopic coupled channels.

framework



Coupled-channel equation

$$\{\ T_f(\vec{R}) + V_{f,f}(\vec{R}) - E_f\ \}\chi_f(\vec{R}) = -\sum_{i\neq f} V_{f,i}(\vec{R})\chi_i(\vec{R})$$

Coupling potential

$$V_{f,i}(\vec{R}) = V_{f,i}^{CE}(\vec{R}) - iW(\vec{R})\delta_{f,i}$$

phenomenological WS potential

real potential

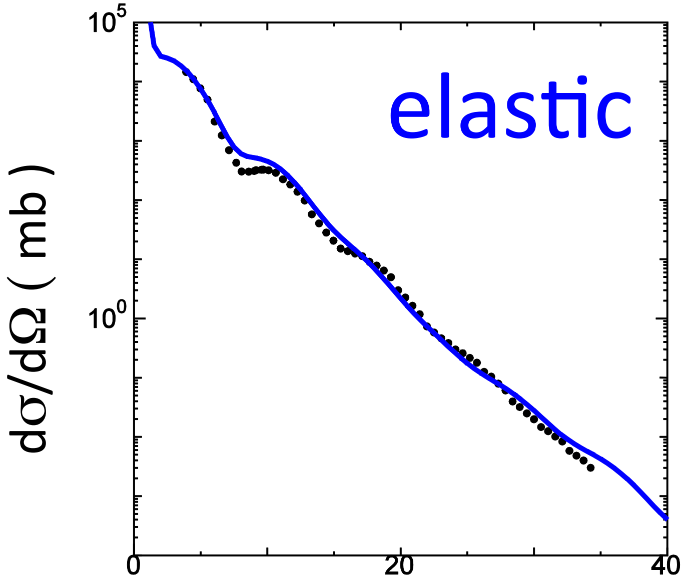
α+^{12}C : Folding pot. $$V_{f,i}(\vec{R}) = \iint \rho^{\alpha}(\vec{r}_1)\rho_{f,i}^{(^{12}C)}(\vec{r}_2)v_{NN}^{DDM3Y}(\vec{s})d\vec{r}_1 d\vec{r}_2$$

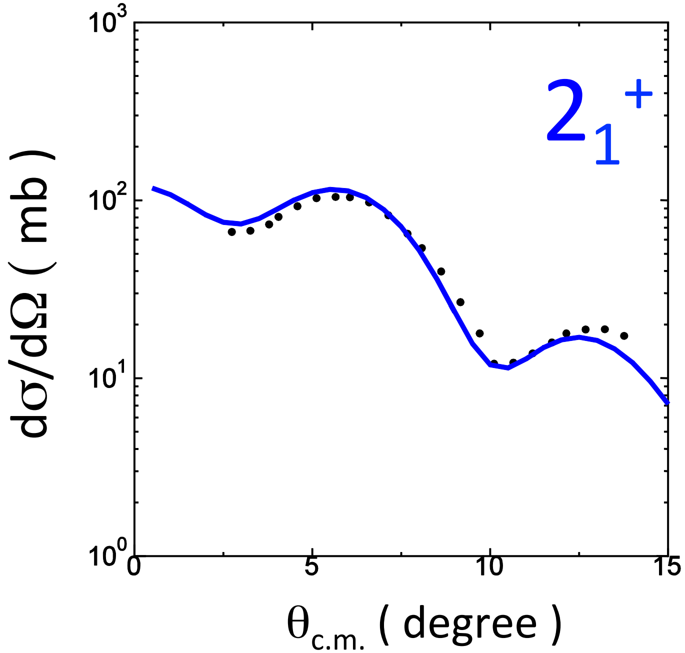
$\rho_{f,i}^{(^{12}C)}$: ^{12}C density is calculated by M. Kamimura 3α RGM

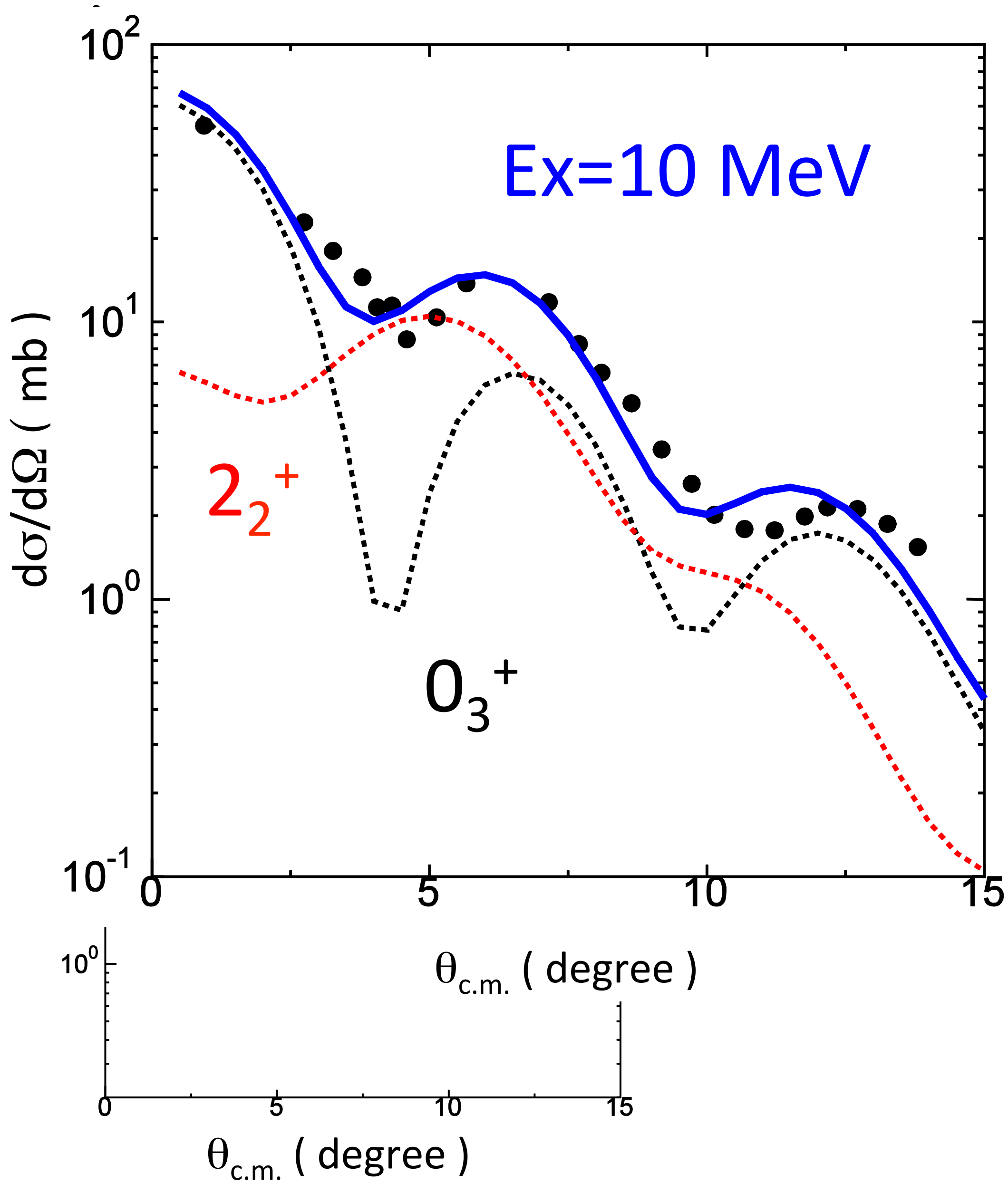
Channels: $0_1^+, 0_2^+, 0_3^+, 2_1^+, 2_2^+, 3_1^-$

Results of the differential cross section in $\alpha + {}^{12}C$

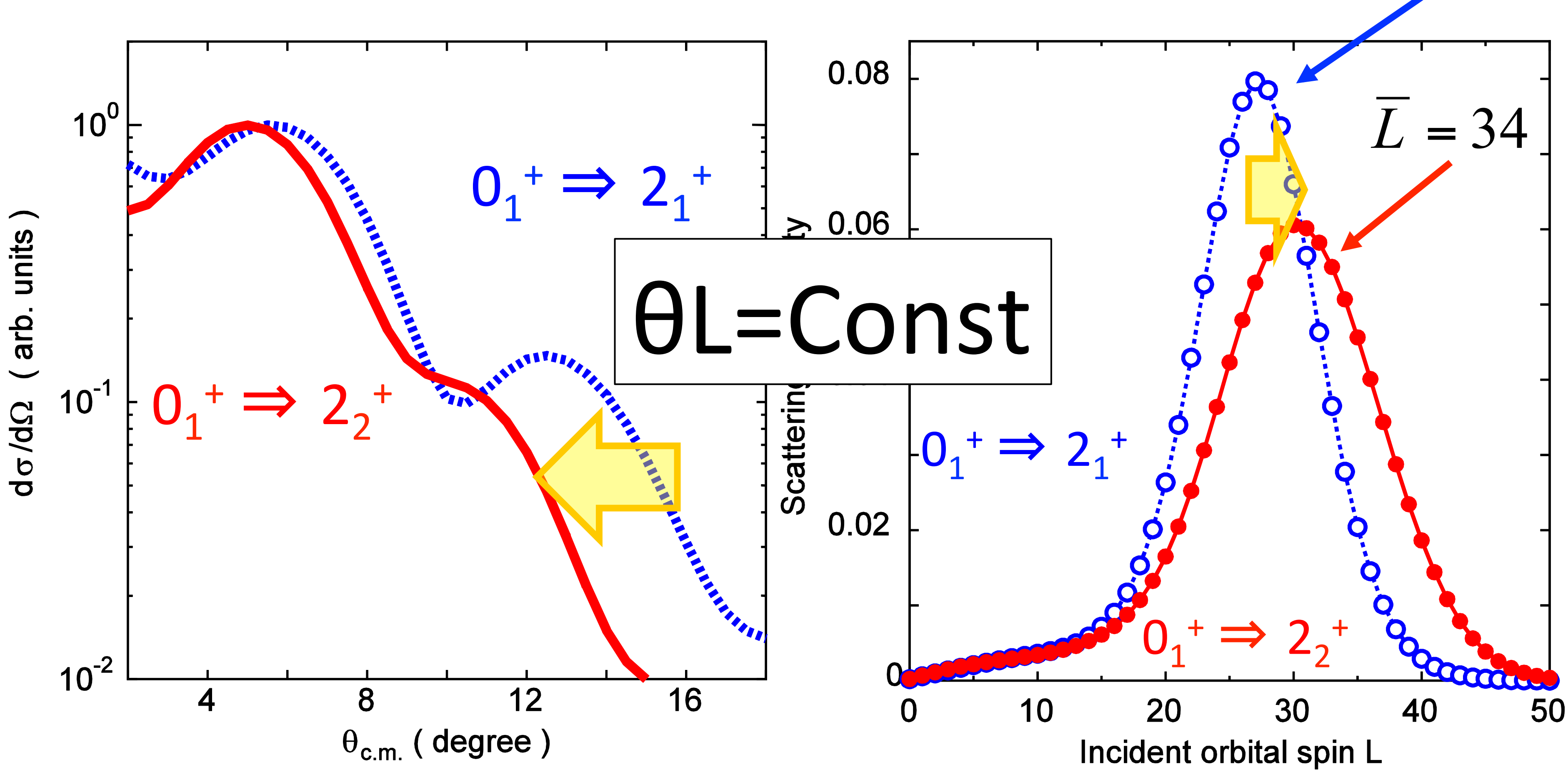
E_{lab} = 386 MeV ● : experimental data (M. Itoh, PRC84) Blue curves: Coupled-channels







Comparison of $\alpha + {}^{12}C(0_1^+) \Rightarrow \alpha + {}^{12}C(2_{1,2}^+)$



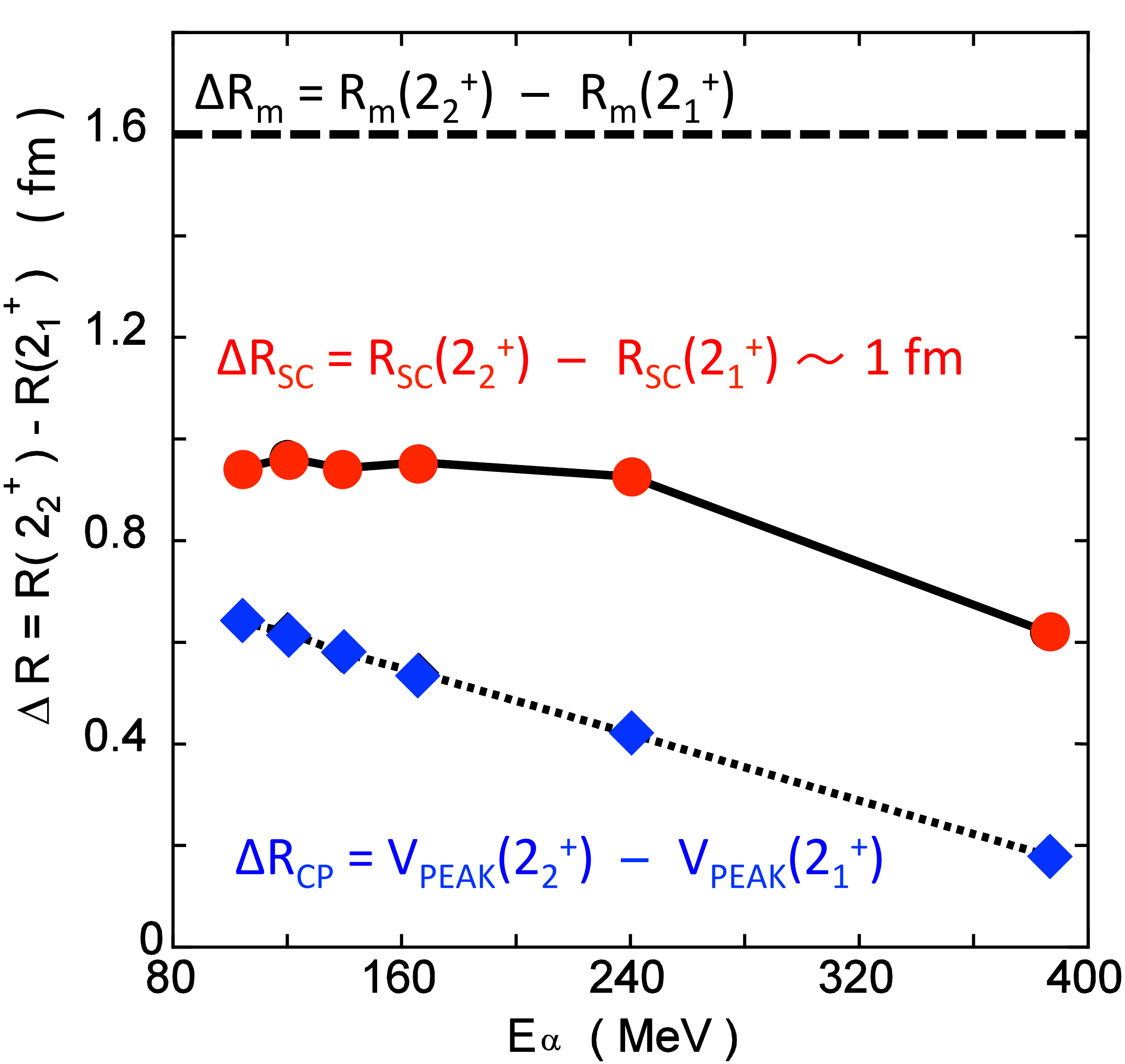
Distribution of 2_2^+ is Shrunk

Distribution of 2_2^+ is Extended

$\bar{L} = kR_{SC} \rightarrow$ $R_{SC}(2_2^+) = 5.2$ fm, $R_{SC}(2_1^+) = 4.6$ fm,

$\Delta R_{sc} = R_{SC}(2_2^+) - R_{SC}(2_1^+) = 0.6$ fm $\Rightarrow$ 3α structure in 2_2^+

Energy systematics of enhancement of $R_{sc}(2^+)$



Difference of matter radii (Theory of structure)

$R_m(2_2^+) \geqq 3.4$ fm

Difference of scattering radii

Size difference of coupling potential; $V_{PEAK}(0_1^+ \rightarrow 2^+)$

Summary

- A comparison of 2_1^+ and 2_2^+ exit channels is possible to probe an enhanced radius of Hoyle rotational 2_2^+ state.
- We introduce the scattering radius (R_{sc}) to characterize a size of reaction.
- Microscopic coupled-channels of α + ^{12}C are performed and scattering radii (R_{sc}) are derived for various exit channels.

Results

- Shrunk differential and extended partial cross sections are observed in 2_2^+
- R_{sc} of 2_2^+ is enhanced by about 1 fm in comparison to that of 2_1^+.

⇒ Calculation predicts that extension of Hoyle rotational 2_2^+ can be probed.

Future subject

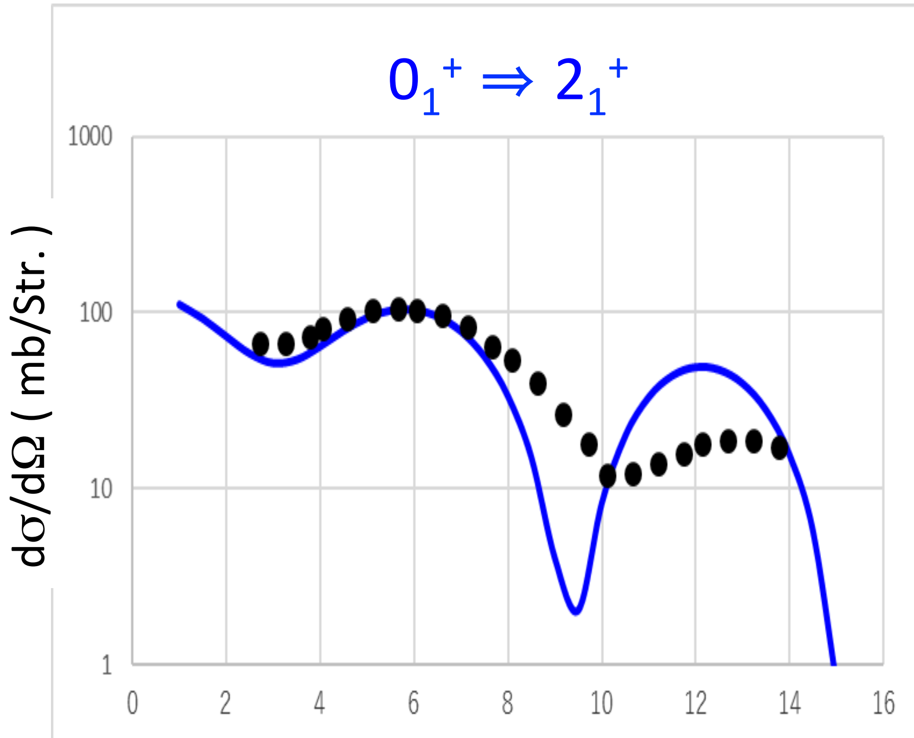
- Measurement of Ex=10MeV cross section over a wide energy and scattering angle is important.
- Careful MDA should be done for the differential cross section of Ex=10MeV state.

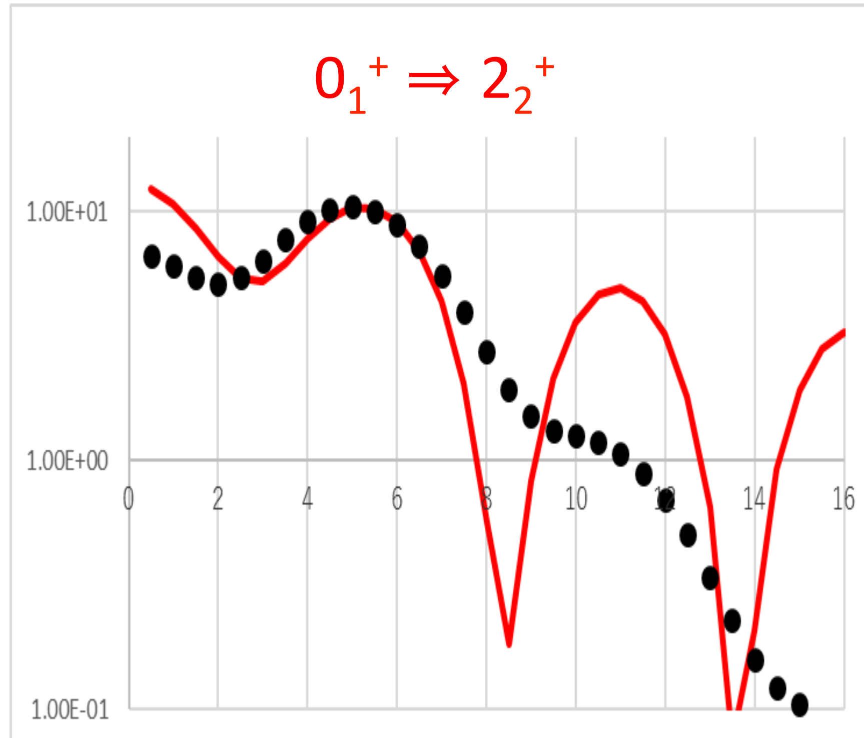
Application of the diffraction theory

K. Iida et al., MPLA 27 (2012)

J. S. Blair, Phys. Rev. 115 (1959)

$$\frac{d\sigma}{d\Omega}\left(0_1^+ \rightarrow 2^+\right) \propto J_0^2(x) + 3J_2^2(x) \qquad x = 2ka\sin\left(\frac{\theta_{c.m.}}{2}\right)$$





$\theta_{c.m.}$ (deg.) $\qquad\qquad$ $\theta_{c.m.}$ (deg.)

$a(2_1^+) = 4.95\ fm$ $\qquad\qquad$ $a(2_2^+) = 5.5\ fm$

Relation of scattering radius and 3α matter radius

In a naïve consideration, we can image the following relation,

$$R_{matter}(2_2^+) \geqq R_{matter}(2_1^+) + \Delta Rsc(2^+)$$

$$R_{matter}(2_1^+) \fallingdotseq R_{matter}(0_1^+) = 2.4 \text{ fm}$$

$$\Delta Rsc(2^+) = \sim 1 \text{ fm}$$

Prediction at
Elab ≦ 240 MeV

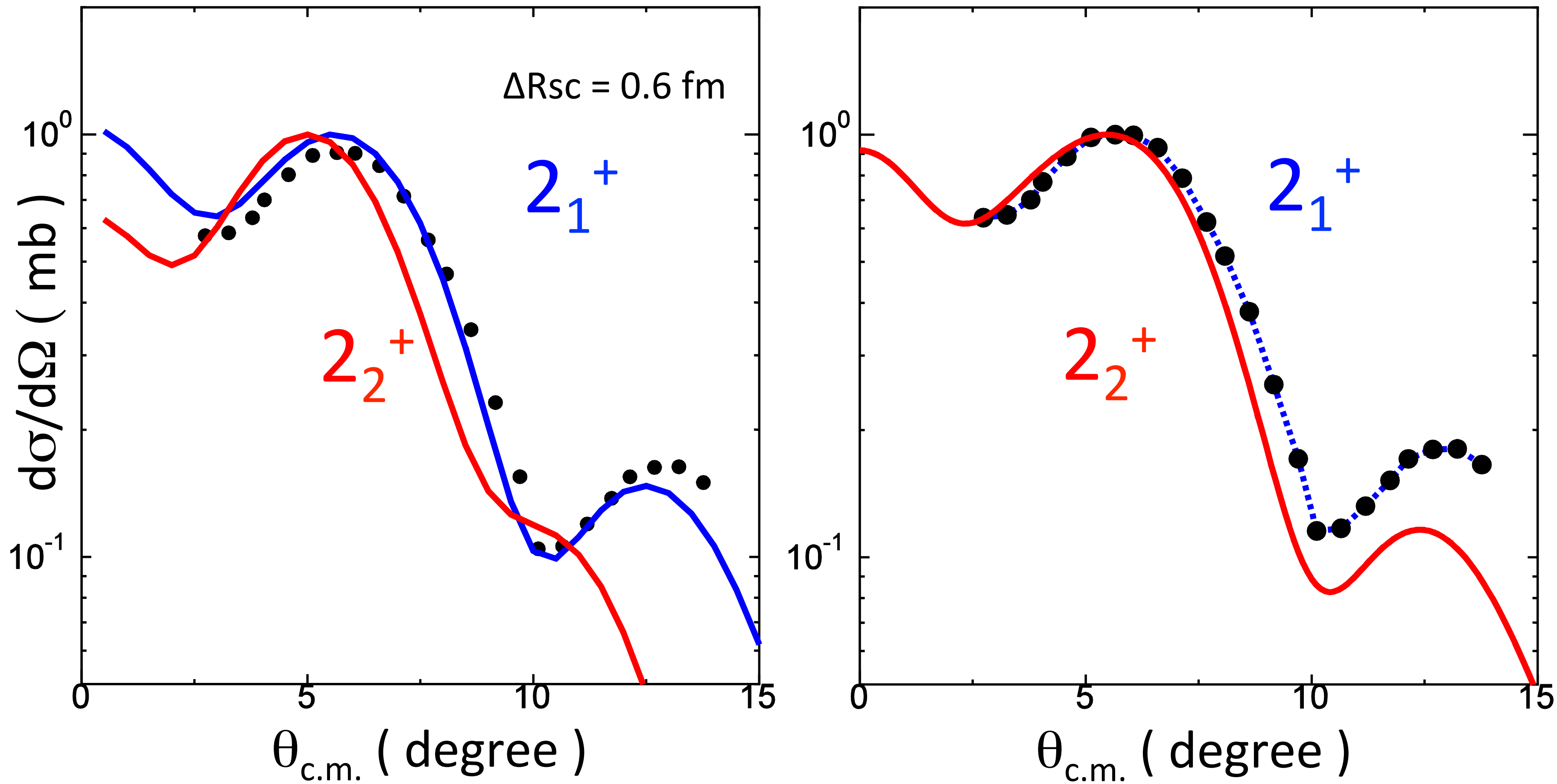
We can speculate the lower limit of the matter radius of 2_2^+

$$R_{matter}(2_2^+) \geqq 3.4 \text{ fm}$$

Comparison of experiment and Theory (Elab = 386 MeV)

Present calculations

Exp. MDA by M. Itoh et al., PRC84 (2011)



In the 2_2^+ distribution, shrinkage and rapid fall down can be clearly observed

ΔR_{SC} = Rsc(2_2^+) − R_{SC}(2_1^+) = 0.6 fm   (ΔR_{sc} ∼ 1 fm, Elab ≦ 240 MeV)

Scattering radius in α + ^{12}C (E_{lab} = 386 MeV)

$$\bar{L} = \sqrt{\frac{\sum_L \left[\sqrt{L(L+1)}\right]^4 \sigma(L)}{\sum_L \left[\sqrt{L(L+1)}\right]^2 \sigma(L)}} \qquad R_{SC} = \frac{\bar{L}}{k} \text{ : scattering radius}$$

Effective orbital spin ($\bar{L}$)

Incident orbital spin ($\sqrt{L(L+1)}$)

Partial cross section ($\sigma(L)$)

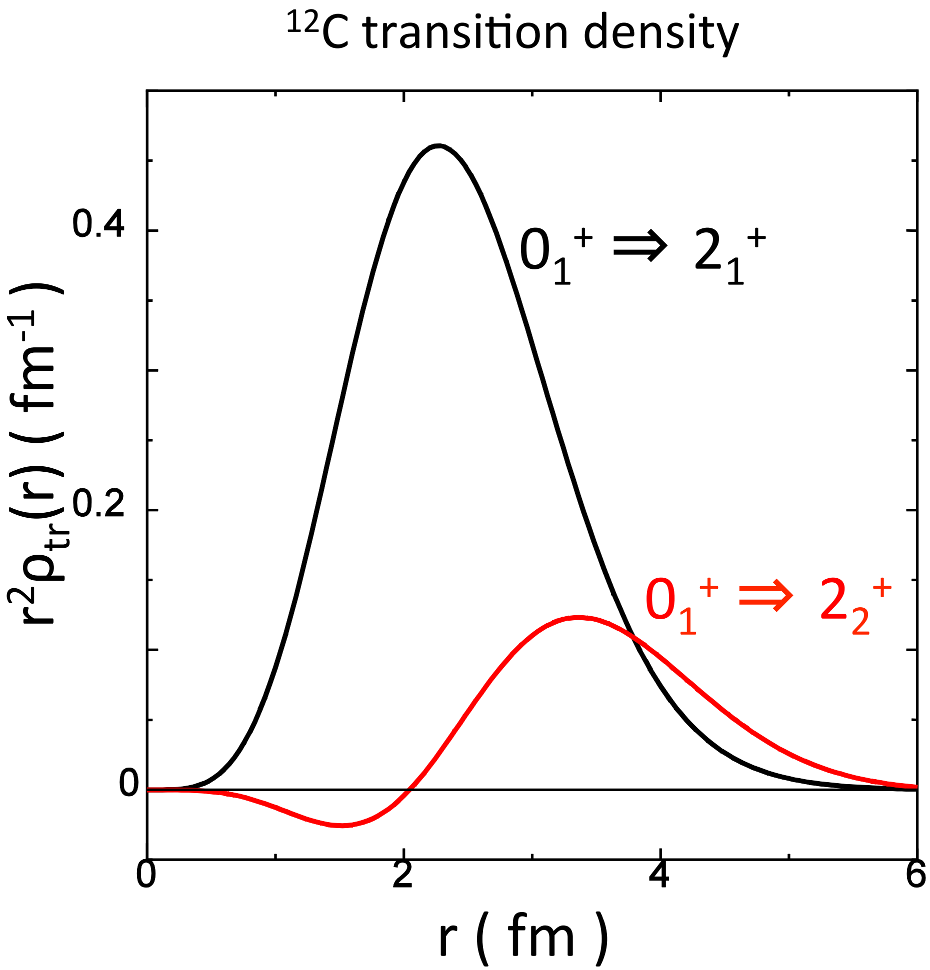
Ref.: M. Tomita et al., PRC89 (2014)

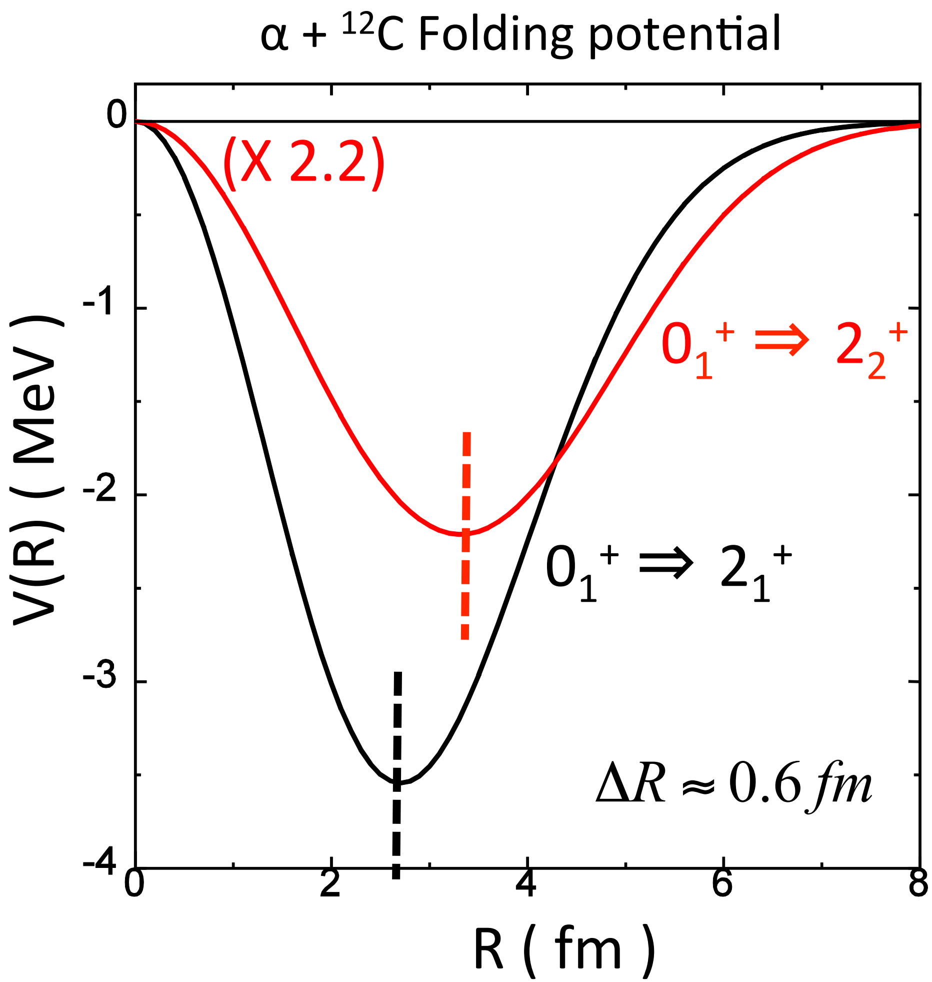
	0_1^+	2_1^+	0_2^+	2_2^+	3_1^-
effective orbital spin $\bar{L}$	22.87	29.67	33.83	33.69	31.76
scattering radius R_{SC}^{α} [fm]	3.53	4.58	5.21	5.20	4.90
matter radius $\bar{r}$[fm]※	2.40	2.38	3.47	4.00	2.76

※M. Kamimura, NPA351 (1981).

R_{SC}(2_2^+) is more enhanced by about 0.62 fm than R_{SC}(2_1^+) ⇒ 3α structure in 2_2^+

Transition densities and coupling potential: $0_1^+ \Rightarrow 2_1^+, 2_2^+$

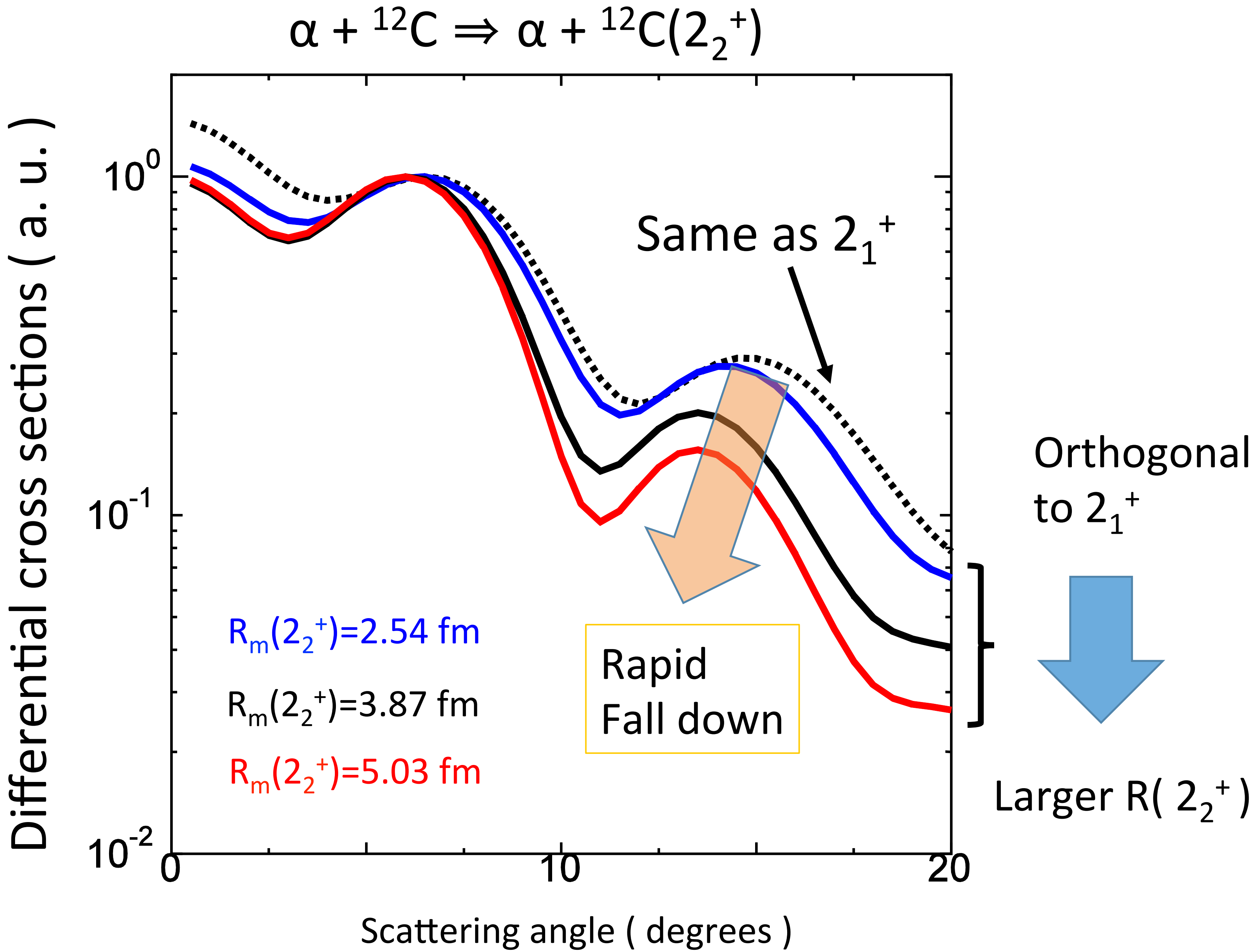




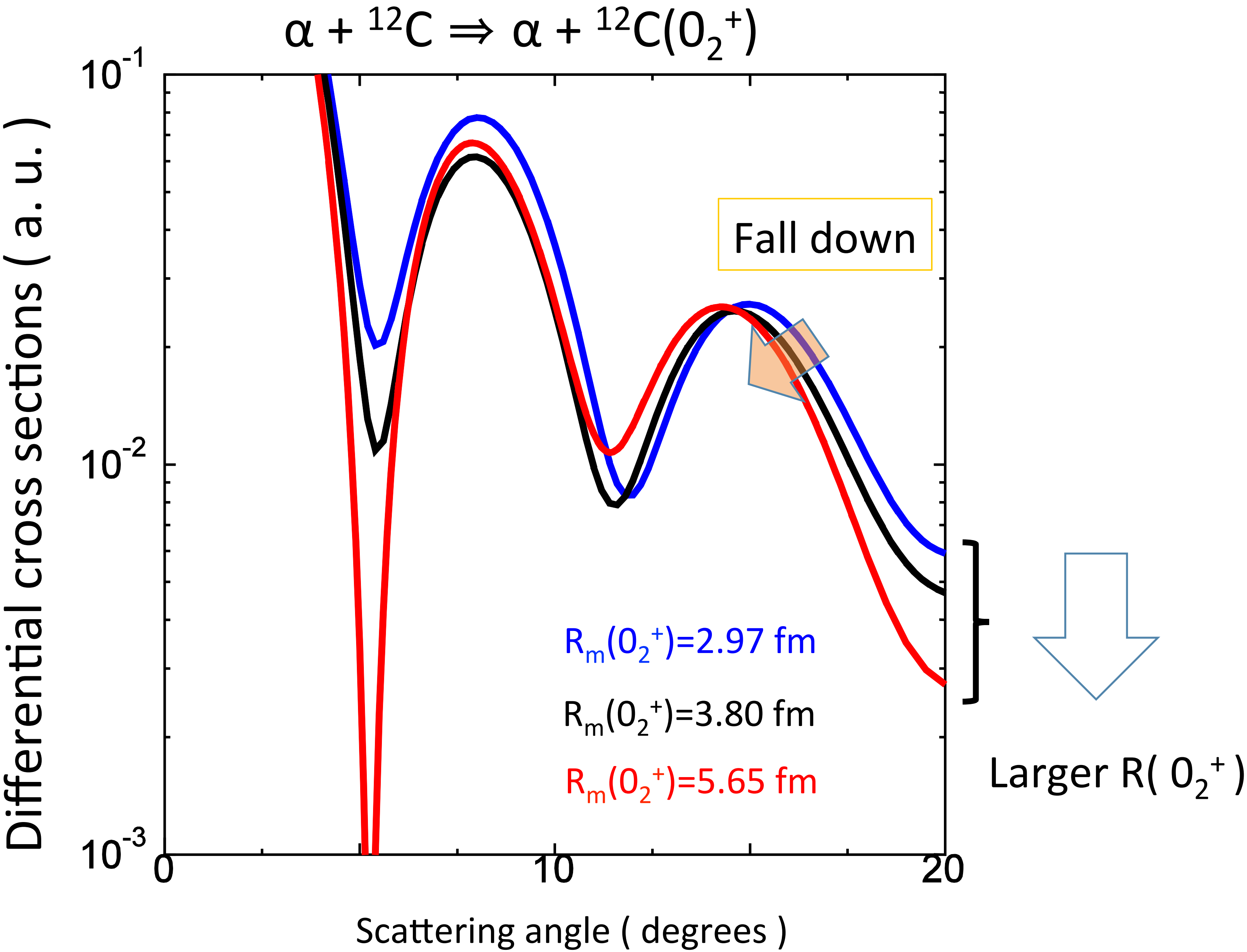
The coupling potential to 2_2^+ is extended by about 0.5 fm in comparison to 2_1^+

The extension of the coupling pot. of 2_2^+ is due to the developed 3α structure

Sensitivity to the size of the 2_2^+ state



Sensitivity to the size of the 0_2^+ state

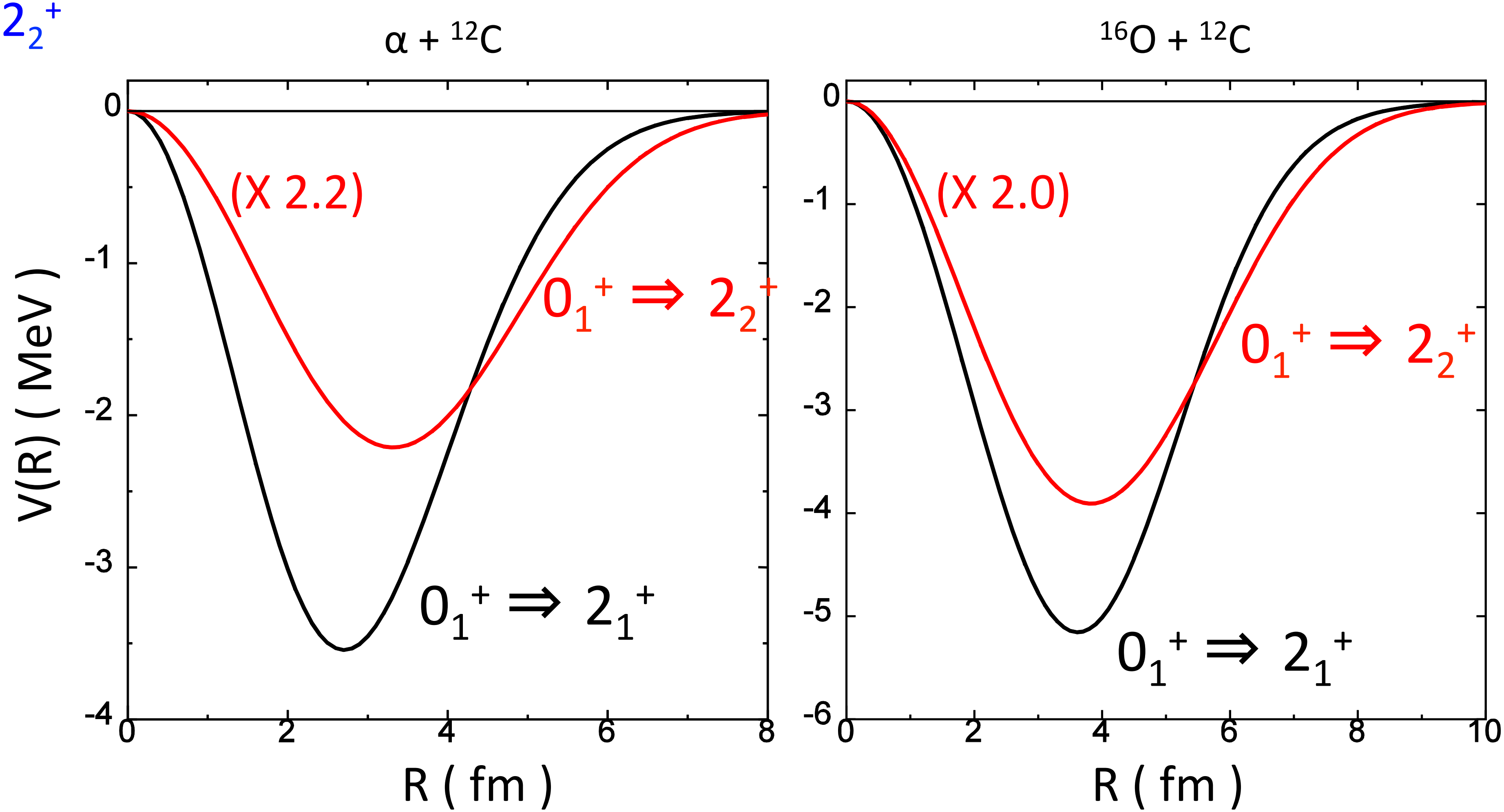


Final state distortion and CC effect

	2ch. w/o Distortion	2ch. With Mono-Dis.	2ch. With Full-Dis.	Full CC calculation
$Rsc(2_1^+)$	3.92	4.37	4.36	4.37
$Rsc(2_2^+)$	4.42	5.15	5.16	5.30
ΔRsc	0.50	0.78	0.80	0.93
ΔR(Vcp)	0.60	0.60	0.60	0.60

VCP:54%, Distortion:33%, CC-effect:13%

Systematics of the coupling potential: $0_1^+ \Rightarrow 2_1^+$, 2_2^+

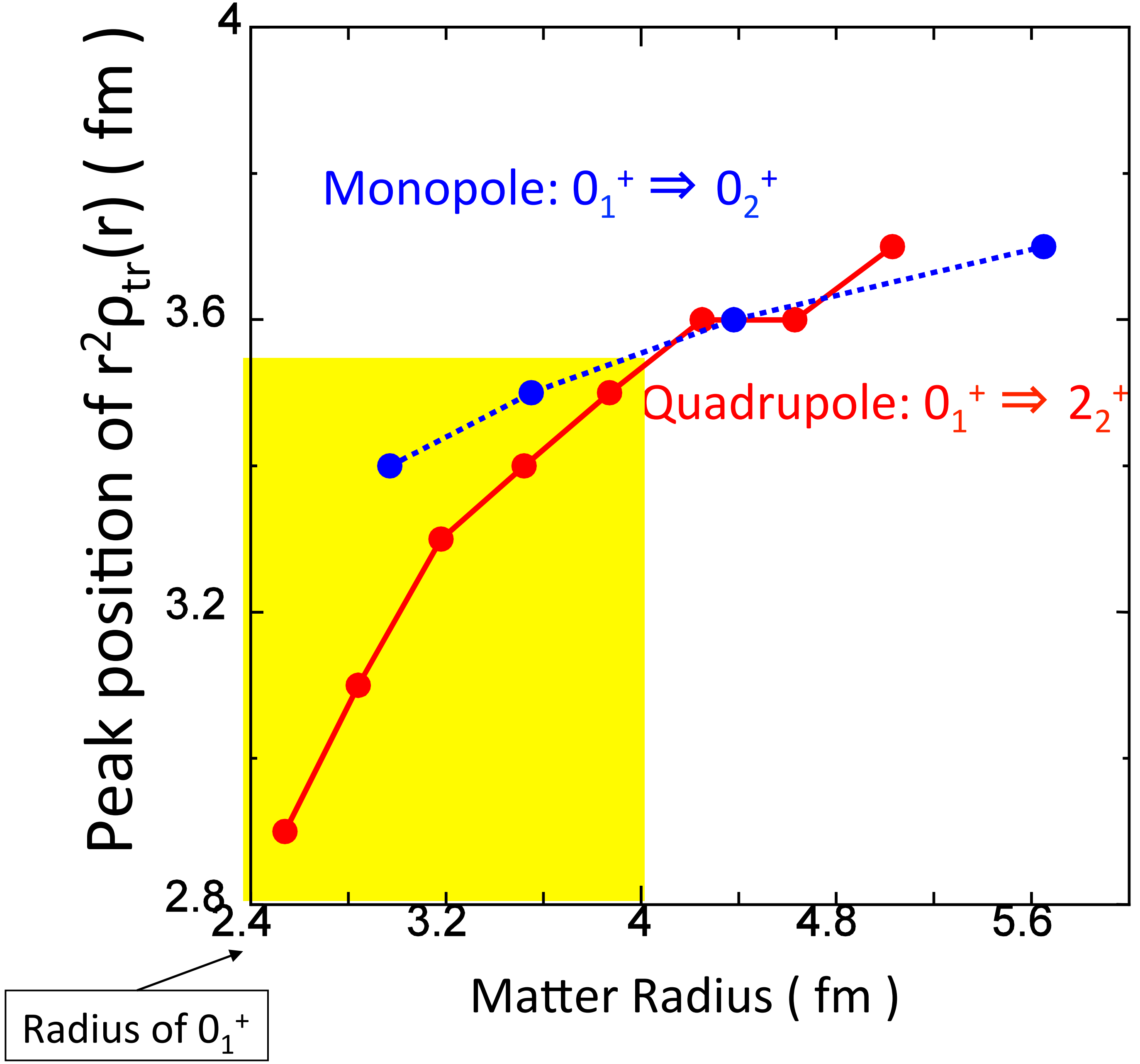


$$\overline{R}\left(0_1^+ \rightarrow 2_1^+\right) = 4.1\ fm \qquad \overline{R}\left(0_1^+ \rightarrow 2_1^+\right) = 5.1\ fm$$

$$\Delta R = 0.6\ fm \qquad \Delta R = 0.5\ fm$$

$$\overline{R}\left(0_1^+ \rightarrow 2_2^+\right) = 4.7\ fm \qquad \overline{R}\left(0_1^+ \rightarrow 2_2^+\right) = 5.6\ fm$$

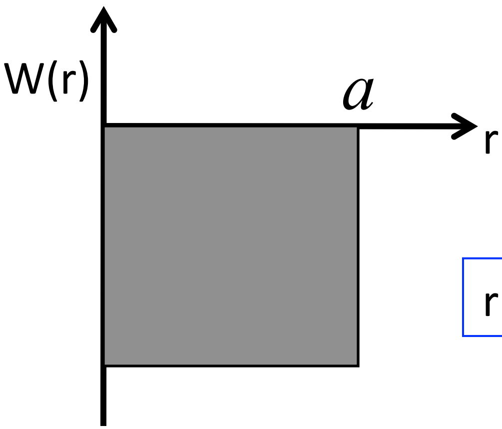
Sensitivity of the transition density to the matter radius



Black sphere limit of the scattering radius

We consider the high energy scattering by a black sphere potential

Black sphere potential



$$r_{BS} = \sqrt{\frac{3}{5}}\, a$$

r.m.s of Black sphere

Partial cross section of the BS scattering

$$L_{\max} = ka$$

$$\sigma(L) \propto \begin{cases} 2L+1 & \text{for } L \le L_{\max} \\ 0 & \text{for } L > L_{\max} \end{cases}$$

Effective orbital spin and the scattering radius

$$\bar{L} = \sqrt{\frac{\sum_{L=0}^{L_{\max}} L^4 \sigma(L)}{\sum_{L=0}^{L_{\max}} L^2 \sigma(L)}}$$

$$\bar{L} = \sqrt{\frac{2}{3}}\, ka = \sqrt{\frac{10}{9}}\, k\, r_{BS} \approx \sqrt{\frac{10}{9}}\, k\, r_{matter}$$

$$r_{BS} \approx r_{matter}$$

A. Kohama et al. PRC69 (2009)

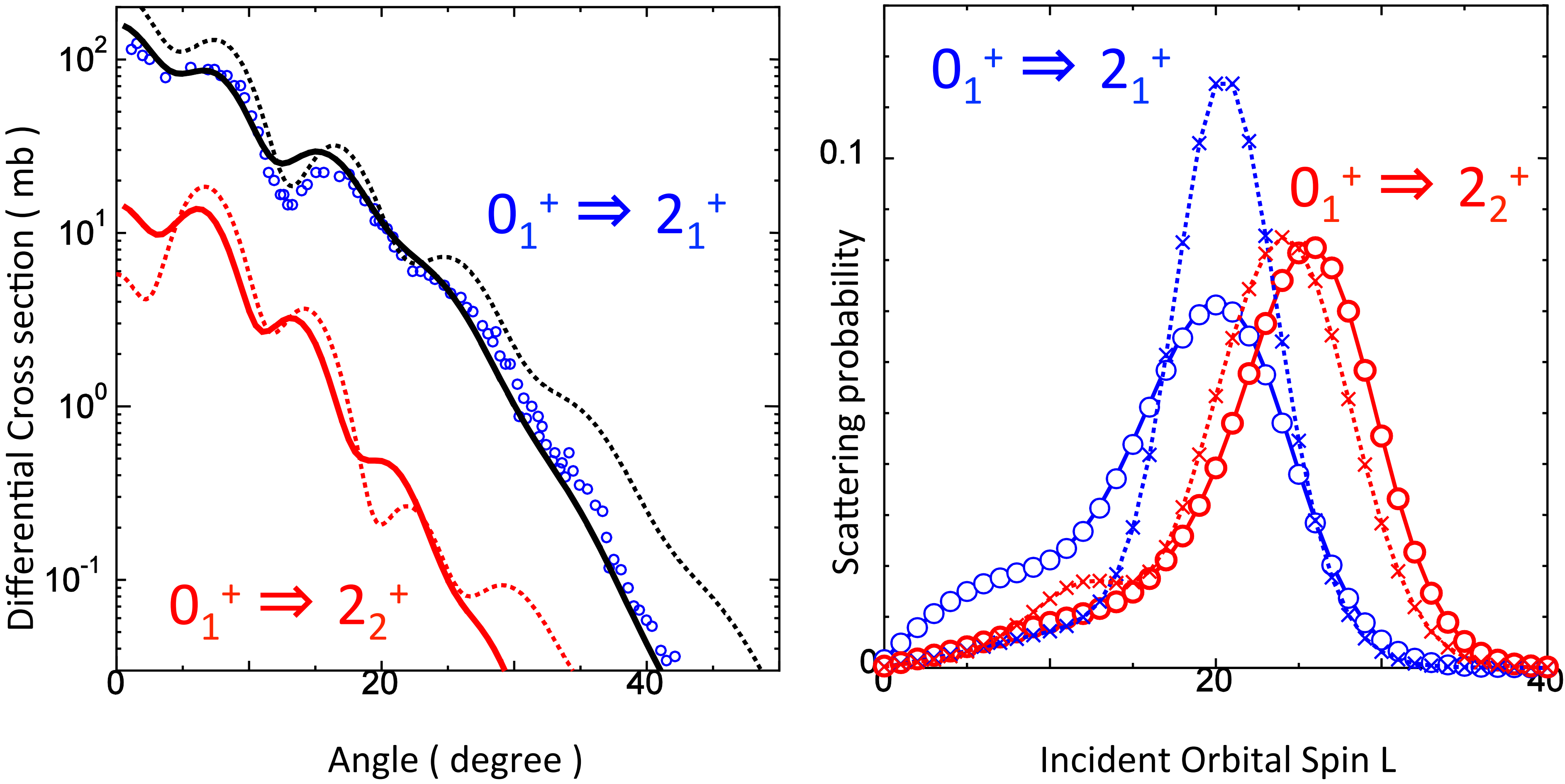
$$\bar{L}/k = R_{SC} \approx 1.05\, r_{matter}$$

(high Energy limit)

Comparison with the complex g-matrix NN int.

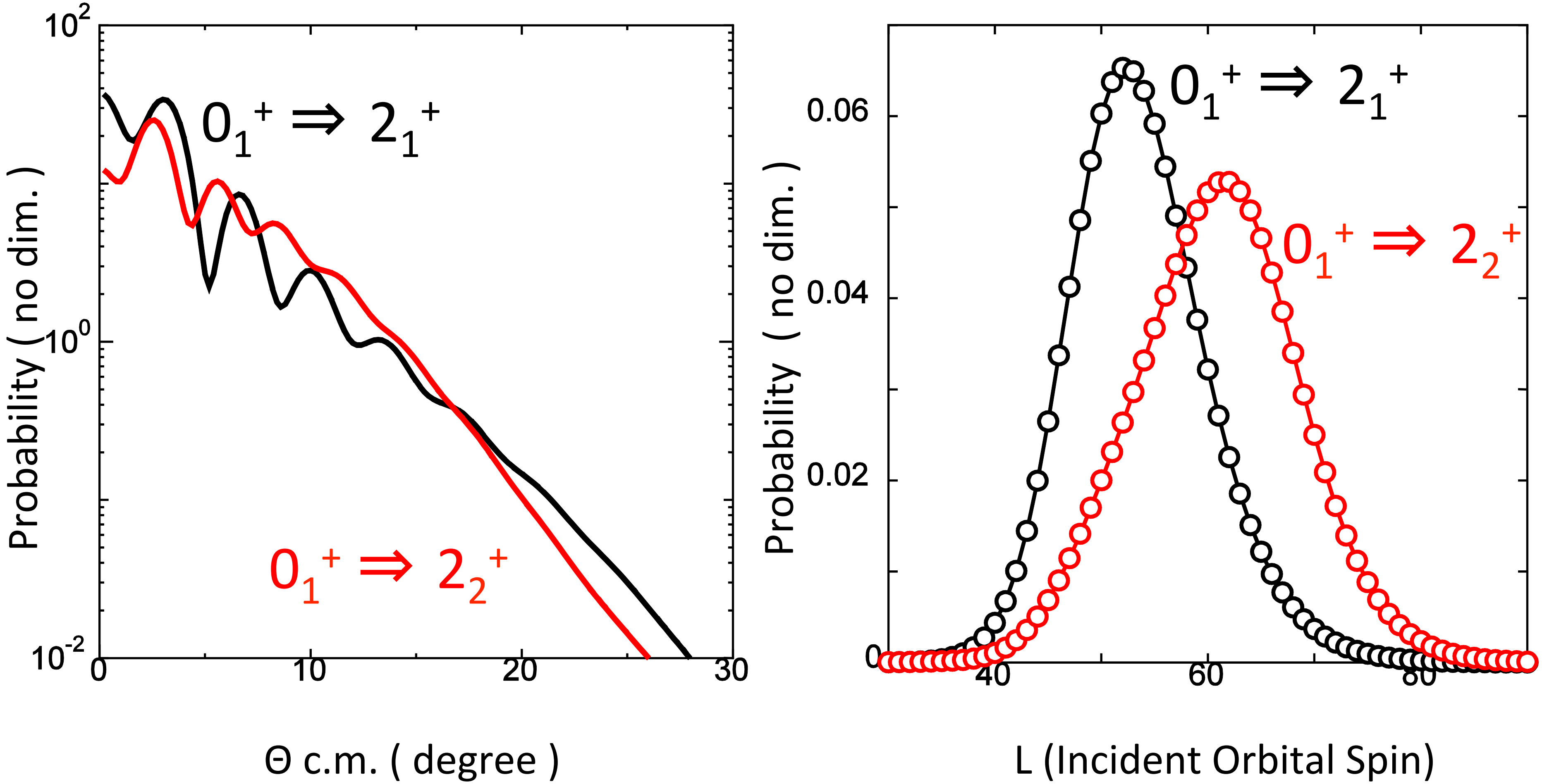
Dotted curve: complex g-matrix NN int.

Solid curve: DDM3Y NN int.



Differential and Partial cross sections: $^{16}O + ^{12}C$

$$^{16}O + ^{12}C(0_1^+) \Rightarrow ^{16}O + ^{12}C(2^+)$$



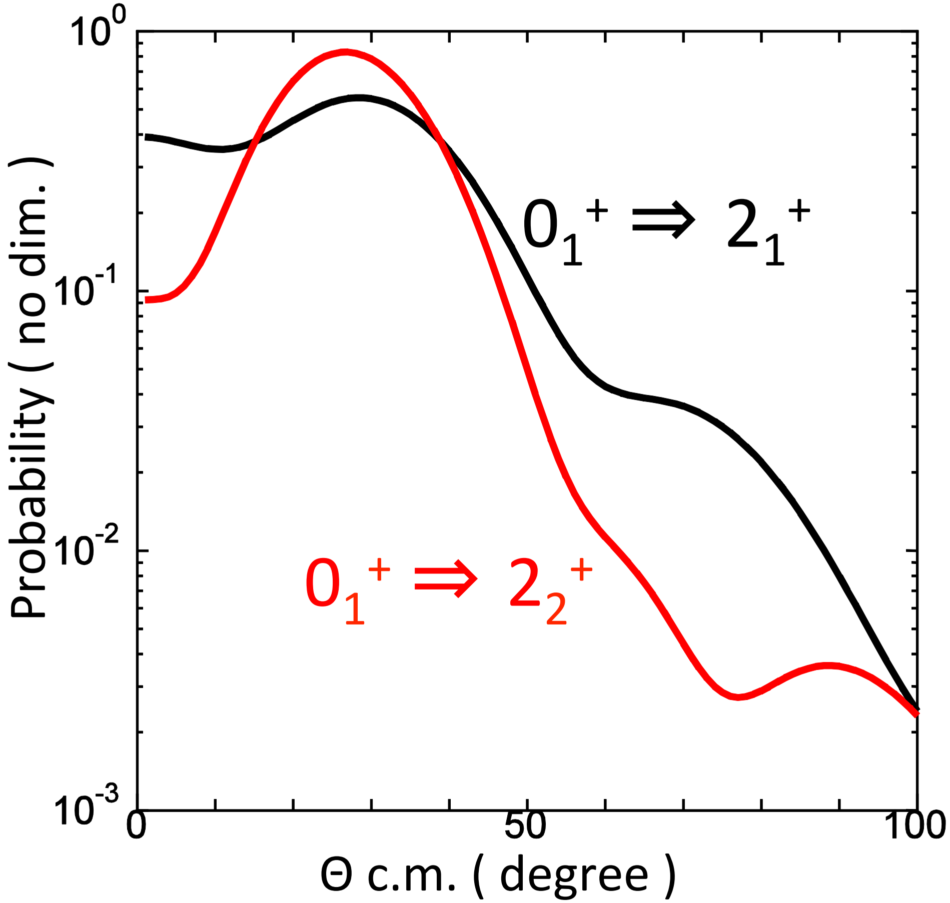
Distribution of 2_2^+ is shrunk

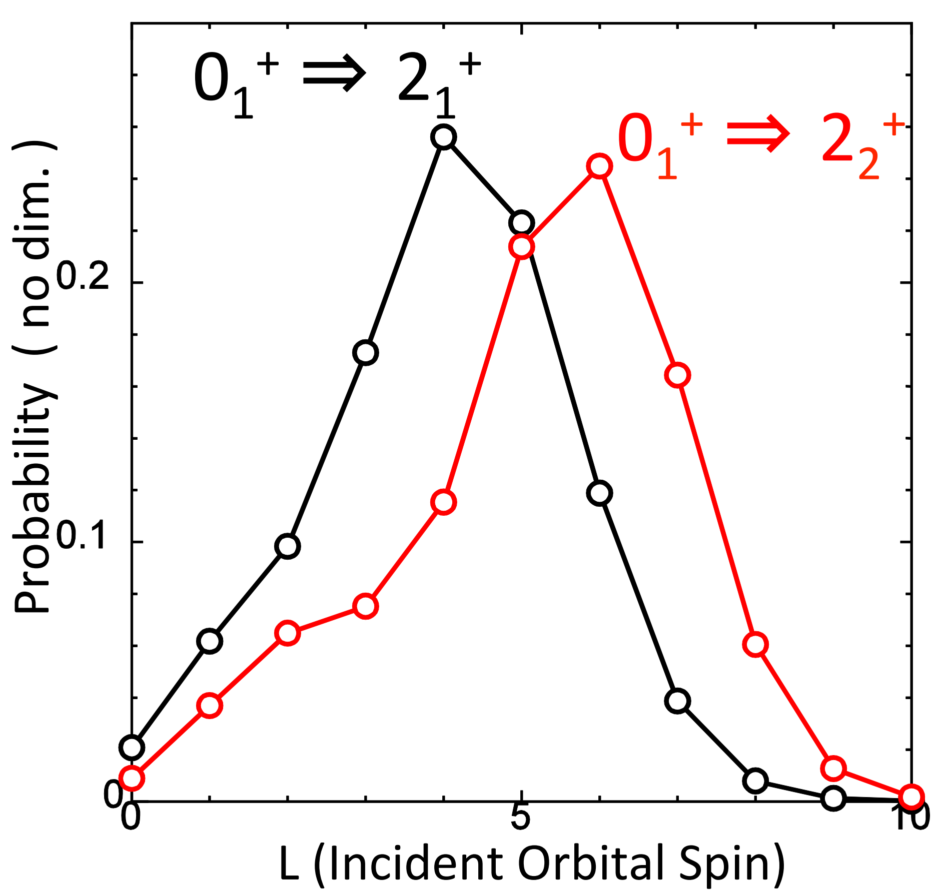
Distribution of 2_2^+ is Extended

Uncertainty Relation: $L\theta \approx \hbar$

Differential and Partial cross sections: 1H + 12C

$$p + {}^{12}C(0_1^+) \Rightarrow p + {}^{12}C(2^+)$$



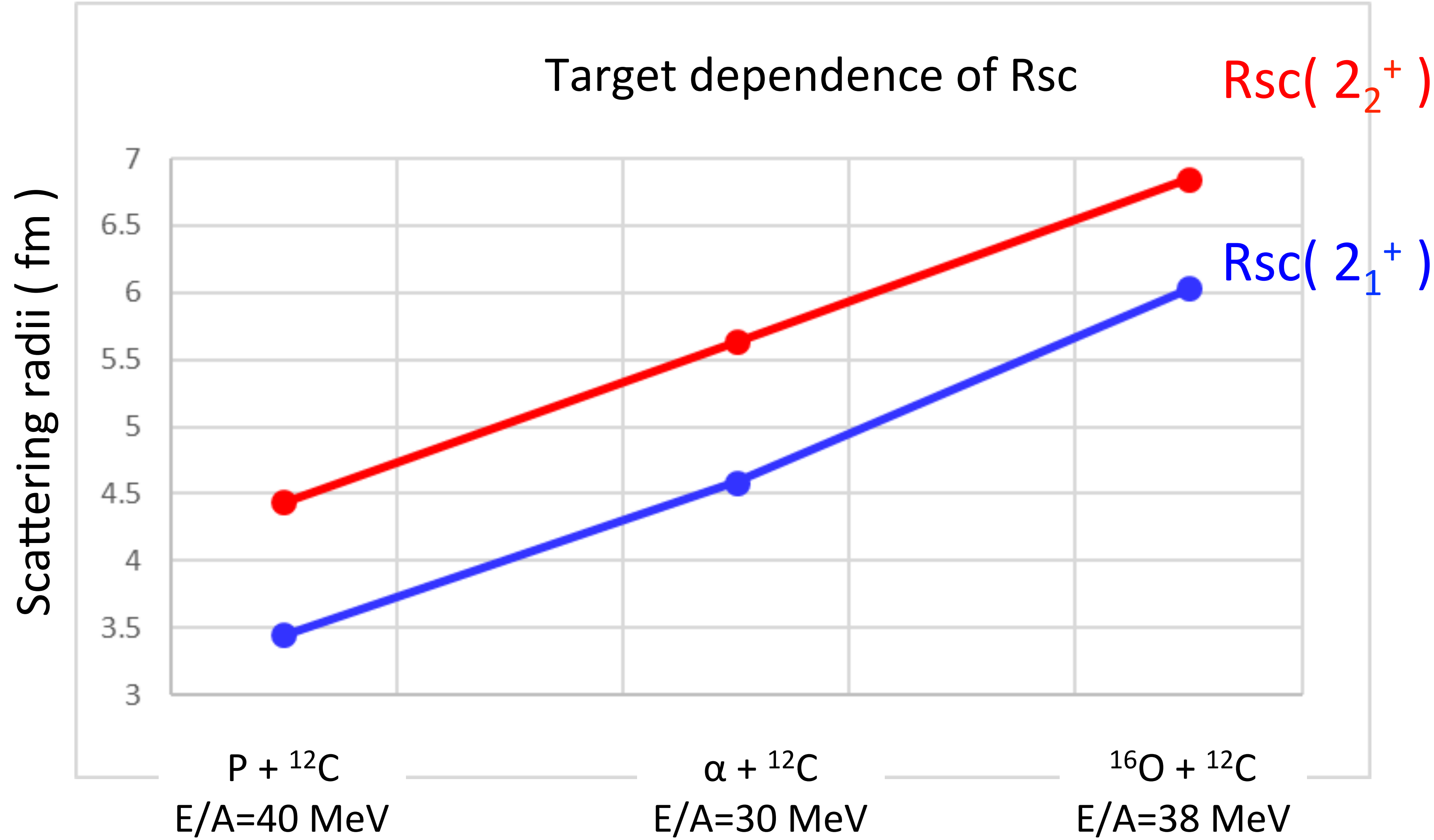


Distribution of 2_2^+ is shrunk

Distribution of 2_2^+ is Extended

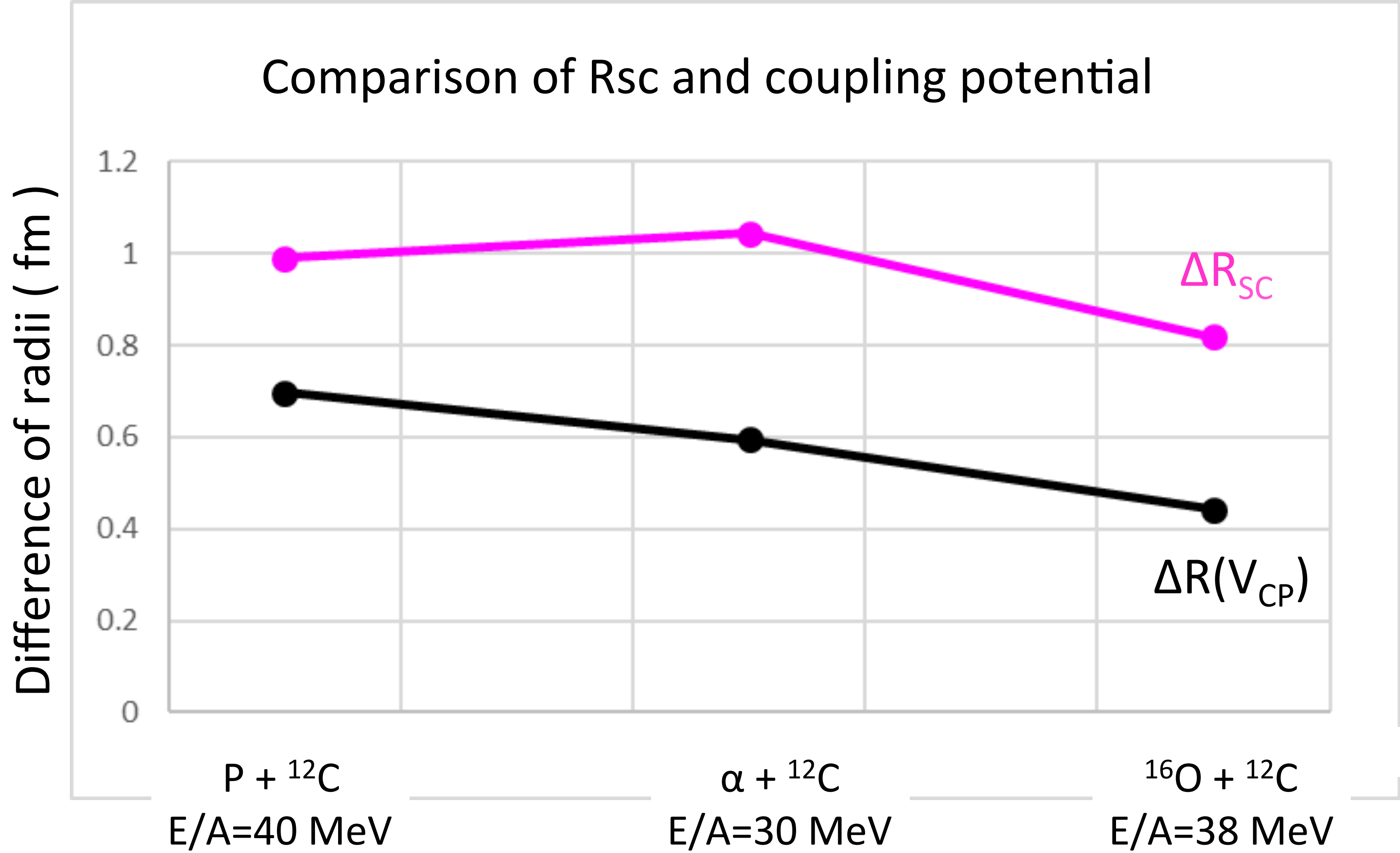
Uncertainty Relation: $L\theta \approx \hbar$

Systematics of the scattering radii (1)



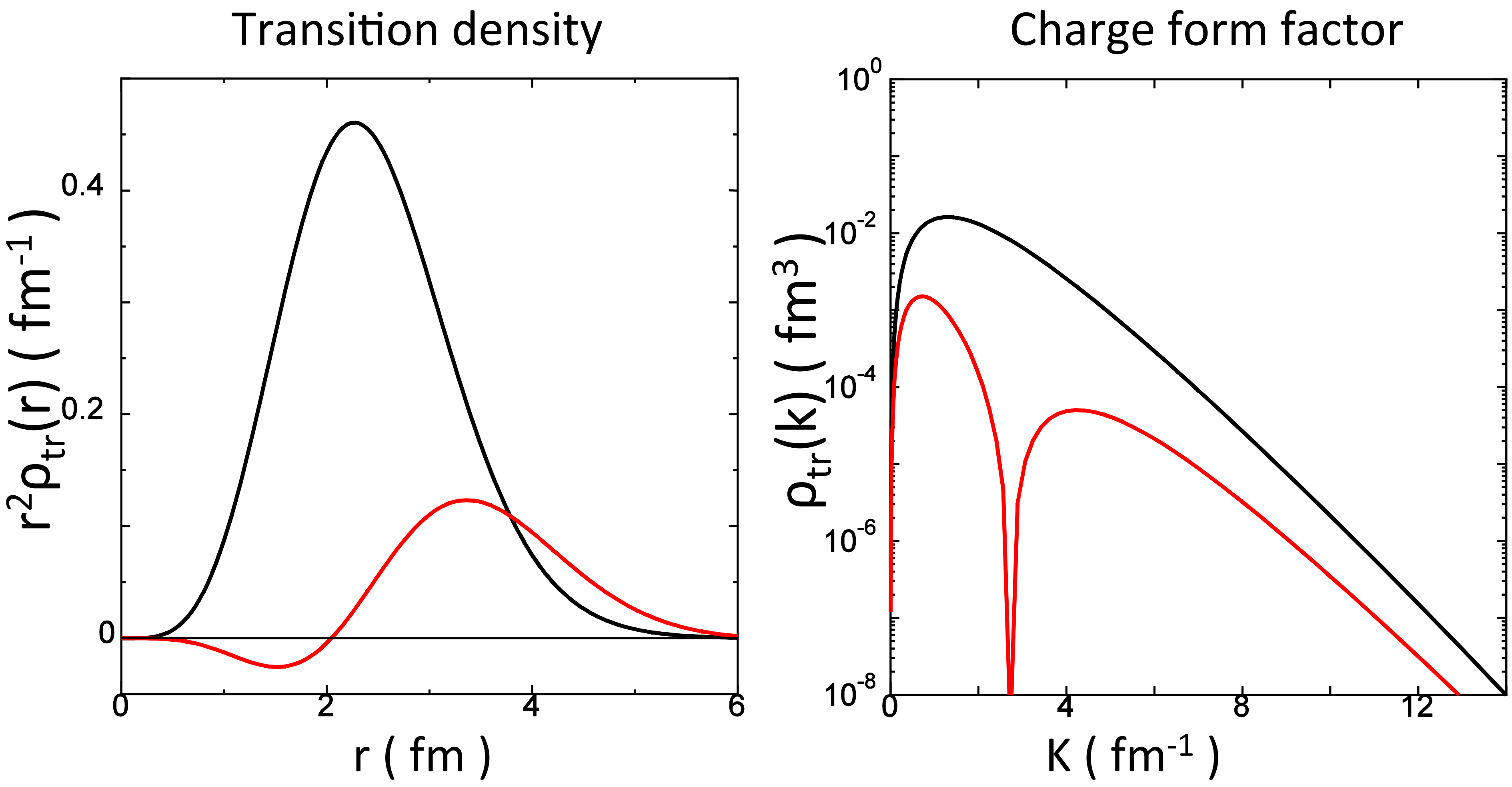
ΔR_{SC} = Rsc(2_2^+) − R_{SC}(2_1^+) = 1.0 fm in average

Systematics of the scattering radii (2)



ΔR_{SC} is about 1.3～2 times larger than $\Delta R(V_{CP})$

Transition Densities: $0_1^+ \Rightarrow 2_1^+, 2_2^+$



Red: $0_1^+ \Rightarrow 2_2^+$

Black: $0_1^+ \Rightarrow 2_1^+$

Previous works (1): Diffraction model（p+^{12}C）

K. Iida, A. Oyamatsu, A. Kohama, MPLA27 (2012)

In $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$, diffraction radius is enhanced in the 3α final channel.

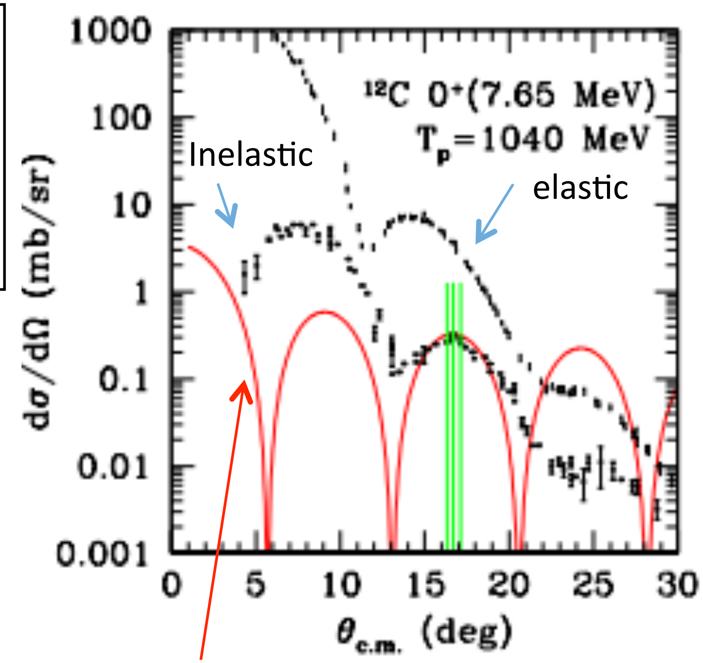
Formula of the inelastic Fraunhofer diffraction

$$\frac{d\sigma}{d\Omega}(0 \to l) \propto \sum_{m=-l,-l+2,\ldots}^{l} \frac{(l-m)!(l+m)!}{[(l-m)!!(l+m)!!]^2} \alpha_{lm}^2 J_{|m|}^2(2pa_l \sin(\theta_{\rm c.m.}/2))$$

Diff. radius

Target	Final state	$E_{\rm ex}$ (MeV)	T_p (MeV)	a or a_l (fm)
^{12}C	g.s.	0.00	1040	2.75±0.06
	2^+	4.44	1040	2.70±0.06
	0^+	7.65	1040	3.20±0.07

Enhanced radius in 3α



Inelastic Fraunhofer diff. pattern

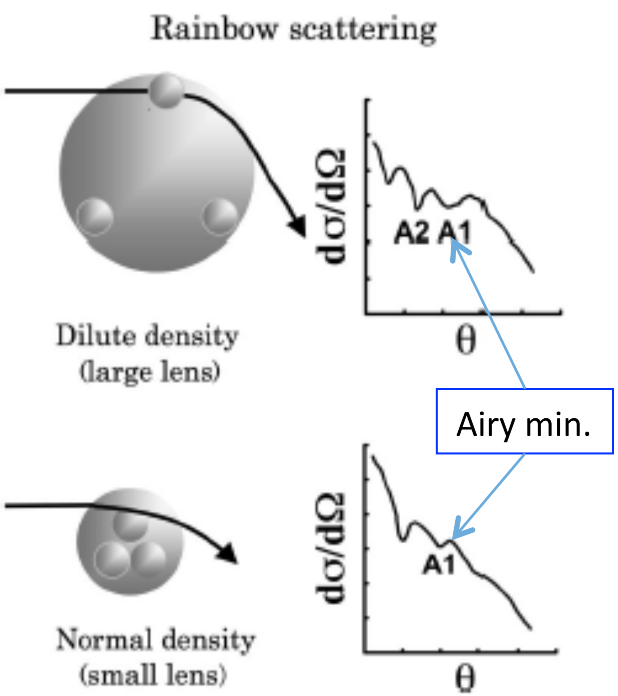
The enhancement of the diffraction radius is clearly confirmed,
but the relation of the 3α wave function and diffraction radius still remains unclear.
Diff. radius ⇒ the size of the coupling potential ?? (Pointed out by Takashia).

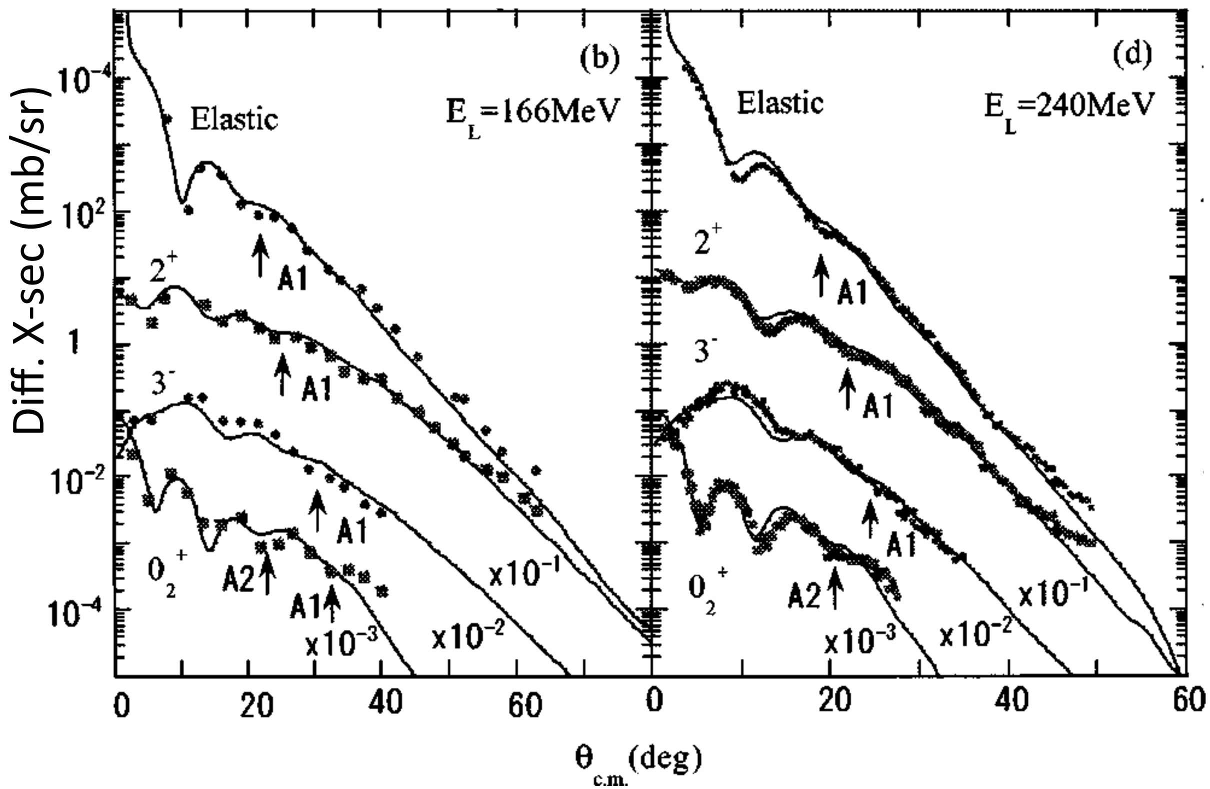
Previous works (2): Microscopic coupled-channel (α+^{12}C)

S. Ohkubo and Y. Hirabayashi, Phys. Rev. C70, 041602(R) (2004)

The microscopic coupled-channel calculation is performed for an α scattering by ^{12}C

(Folding model with 3a RGM w.f. + DDM3Y NN int.)





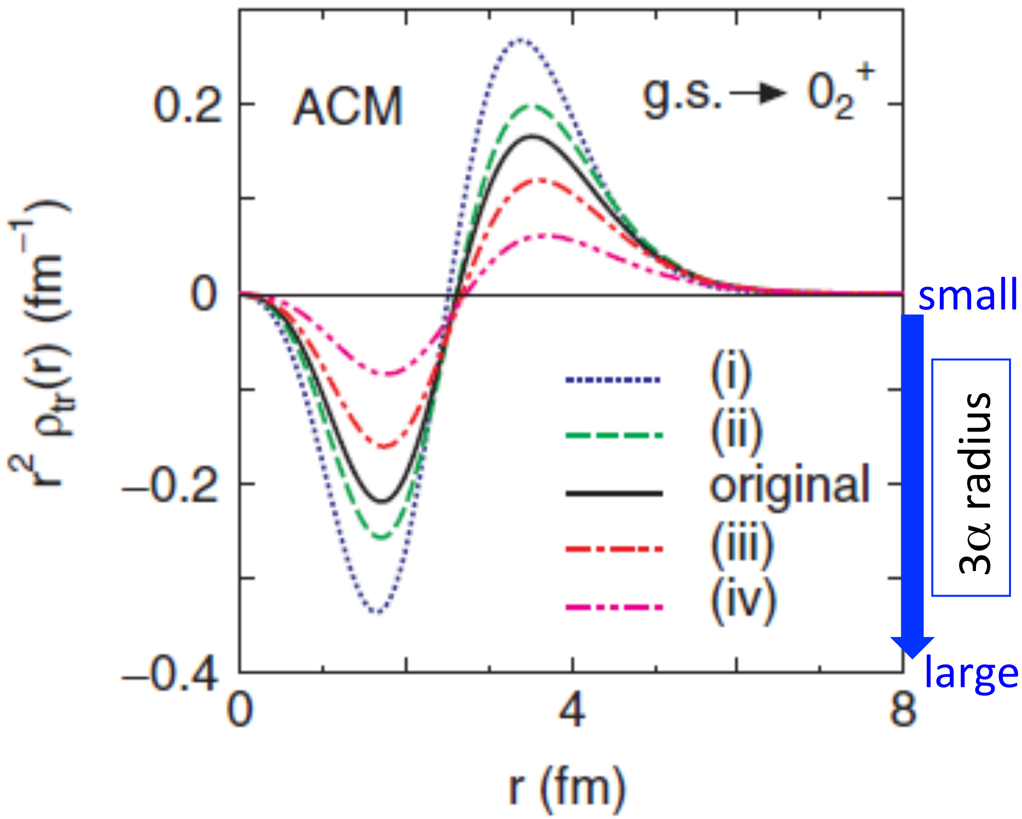
In $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$, Evolution of the Airy minima is observed.

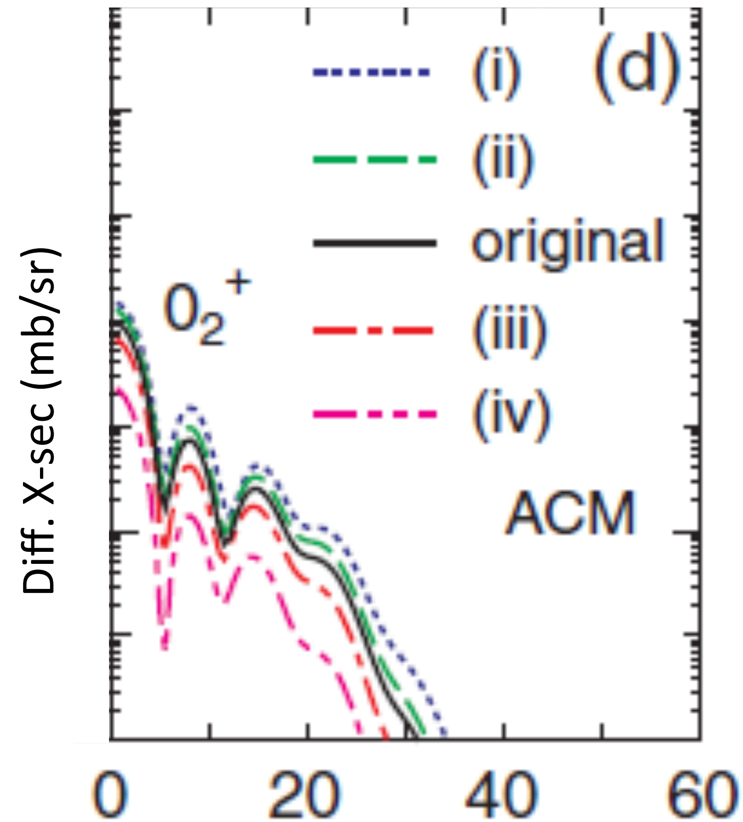
⇒ Evolution is a sign of the enhanced attraction at the surface region of the 3a state !?

Previous works (3): Analysis of the MCC ($^{4}He+^{12}C$)

M. Takashina and Y. Sakuragi, Phys. Rev. C74, 054606 (2006)

In the microscopic coupled-channel calculation, the sensitivity of the angular distribution to the size of the 3α state is investigated.





Oscillating pattern is quite INSENSITIVE to the radius of the final 3α state

Oscillation is quite SENSITIVE to a size of the transition potential of $^{12}C_{g.s.} \rightarrow {}^{12}C(0_2^+)$