

# Hyperon single-particle potentials from QCD on lattice

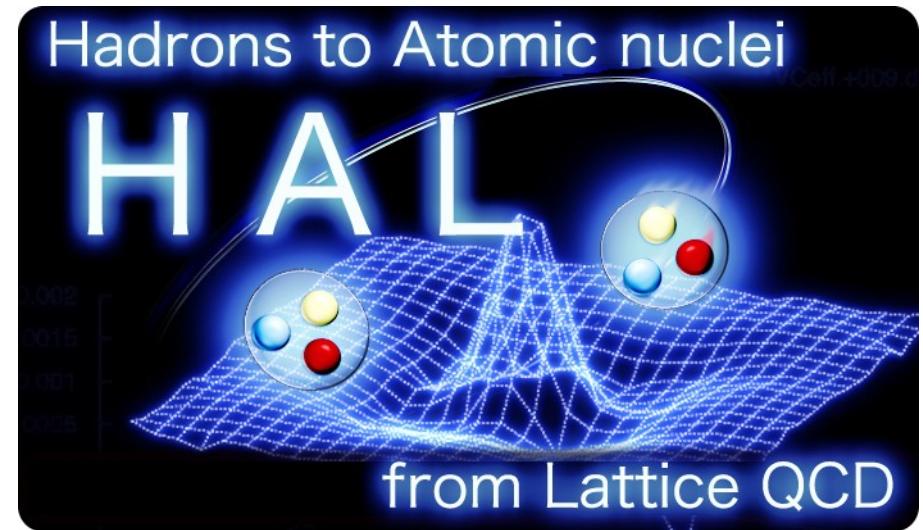
Takashi Inoue @Nihon Univ.

for

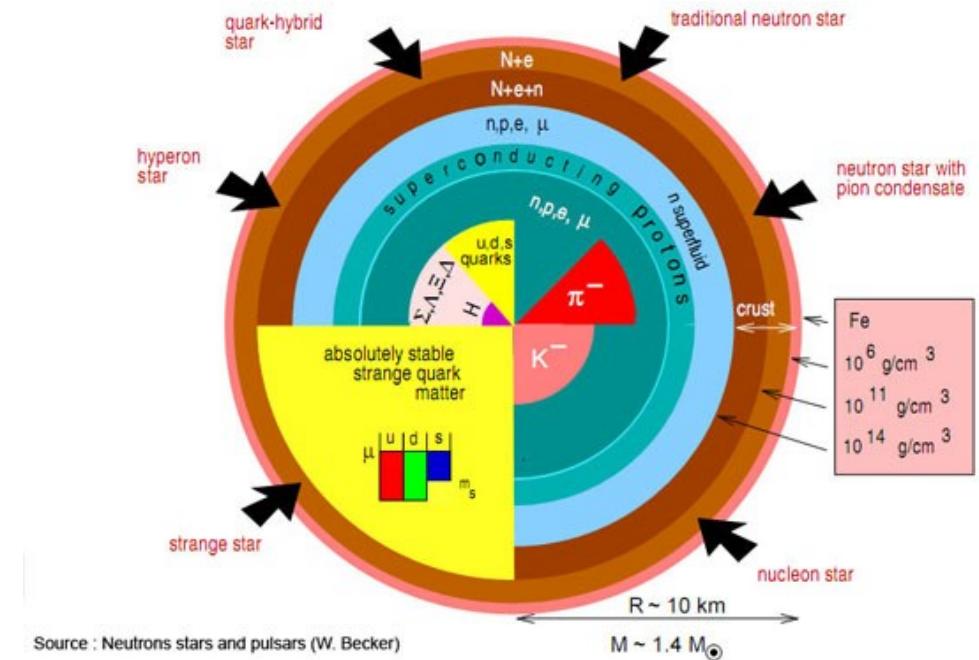
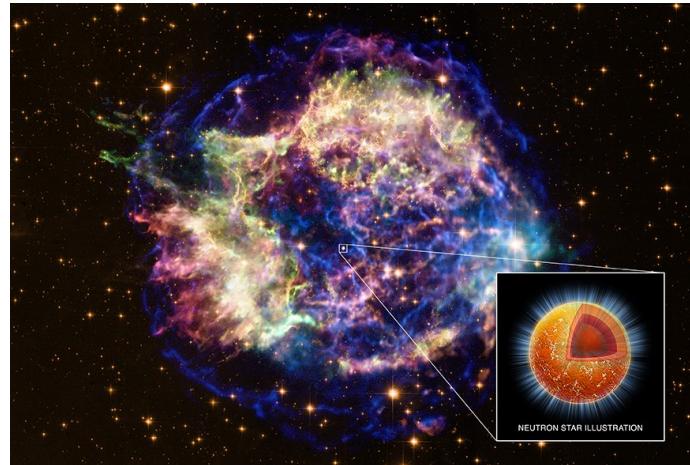
## HAL QCD Collaboration

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# Introduction



- ★ **Hyperon** is a serious subject in physics of NS.
  - Does hyperon appear inside neutron star core?
  - How EoS of NS mater can be so stiff with hyperon?  
cf. PSR J1614-2230  $1.97 \pm 0.04 M_{\odot}$
  
- ★ Tough problem due to **ambiguity** of hyperon forces
  - comes from difficulty of hyperon scattering experiment.

# Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from **QCD**.
  - measure h-h NBS w.f. in **lattice** QCD simulation. **HALQCD**
  - define & extract interaction “potential” from the w.f. **applapch**

# Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from **QCD**.
  - measure h-h NBS w.f. in **lattice** QCD simulation. HALQCD  
applapch
  - define & extract interaction “potential” from the w.f.
- Today, we study **hyperons in nuclear medium** by basing on YN YY interactions predicted from QCD.
  - We calculate hyperon **single-particle potential**  $U_Y(k;\rho)$
  - defined by  $e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$   $e_Y(k;\rho)$  : spectrum in medium
  - $U_Y$  is crucial for hyperon chemical potential.

# Introduction

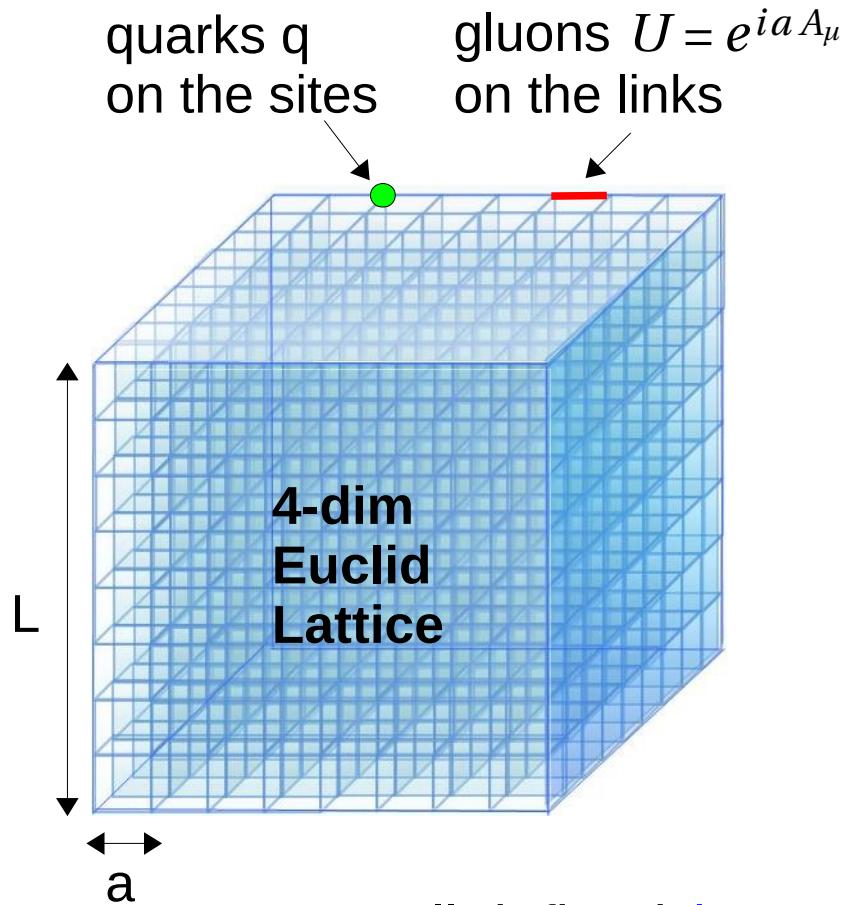
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  - $U_Y$  is crucial for hyperon chemical potential.
- Hypernuclear **experiment** suggest that  $\text{@ } \rho=0.17 [\text{fm}^3]$   
 $x=0.5$ 
  - $U_{\Lambda}^{\text{Exp}}(0) \simeq -30$ , attraction
  - $U_{\Xi}^{\text{Exp}}(0) \simeq -10$ , attraction small
  - $U_{\Sigma}^{\text{Exp}}(0) \simeq +10$  repulsion small[MeV]<sub>5</sub>

# Outline

1. Introduction
2. HALQCD method & simulation setup
3. Hyperon interactions from QCD
4. Hyperon s.-p. potentials from QCD
5. Summary

# Lattice QCD

$$L = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



Vacuum expectation value

$$\begin{aligned} & \langle O(\bar{q}, q, U) \rangle && \text{path integral} \\ &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) && \text{quark propagator} \\ & \{ U_i \} : \text{ensemble of gauge conf. } U && \\ & \text{generated w/ probability } \det D(U) e^{-S_U(U)} && \end{aligned}$$

- ★ Well defined (regularized)
- ★ Manifest gauge invariance

- ★ Fully non-perturbative
- ★ Highly predictive

# HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)  
 N. Ishii et al. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function  $\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x} + \vec{r}, t) B_j(\vec{x}, t) | B=2, \vec{k} \rangle$

Define a common potential  $U$  for all  $E$  eigenstates by a “Schrödinger” eq.

$$\left[ -\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3 \vec{r}' U(\vec{r}, \vec{r}') \phi_{\vec{k}}(\vec{r}') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})$$

Non-local but  
**energy independent**  
 below inelastic threshold

Measure 4-point function in LQCD

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x} + \vec{r}, t) B_j(\vec{x}, t) J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \dots$$

$$\left[ 2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r}, t) + \int d^3 \vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}', t) = -\frac{\partial}{\partial t} \psi(\vec{r}, t)$$

$\nabla$  expansion  
& truncation

$$U(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') V(\vec{r}, \nabla) = \delta(\vec{r} - \vec{r}') [V(\vec{r}) + \cancel{\nabla + \nabla^2 \dots}]$$

Therefor, in  
the **leading**

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2M_B$$

# Multi-hadron in LQCD

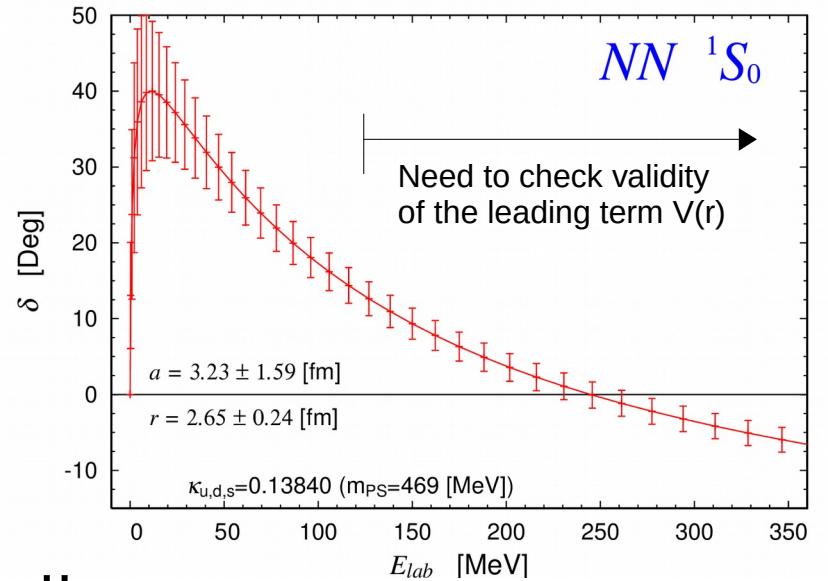
- Direct : utilize **energy eigenstates** (eigenvalues).
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL : utilize **spatial correlation** and “potential”  $V(\vec{r}) + \dots$

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2M_B$$

$\psi(\vec{r}, t)$  : 4-point function  
contains NBS w.f.

- Advantages
  - No need to separate E eigenstate.  
Just need to measure  $\psi(\vec{r}, t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume.  
No need to extrapolate to  $V=\infty$ .
  - Can output more observables.

★ We can attack **hyperon in matter** too!!



# Simulation setup

- $N_f = 2+1$  full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume  $96^4 \simeq (8 \text{ fm})^4$
  - $1/a = 2333 \text{ MeV}$ ,  $a = 0.0845 \text{ fm}$
  - $M_\pi \simeq 146$ ,  $M_K \simeq 525 \text{ MeV}$   
 $M_N \simeq 956$ ,  $M_\Lambda \simeq 1121$ ,  $M_\Sigma \simeq 1201$ ,  $M_\Xi \simeq 1328 \text{ MeV}$
  - Collaboration in HPCI Strategic Program Field 5 Project 1
- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - $\#stat = 414 \text{ confs} \times 4 \text{ rot} \times 28 \text{ src.}$

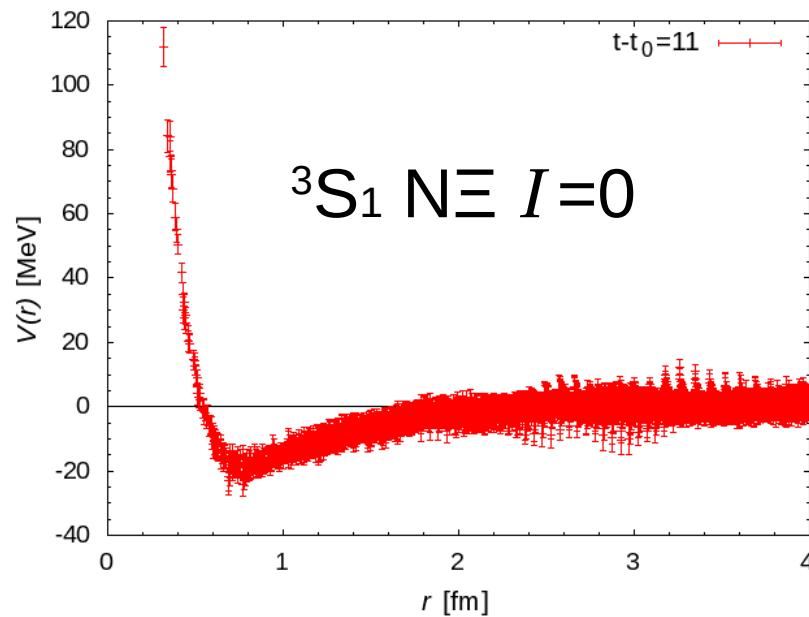
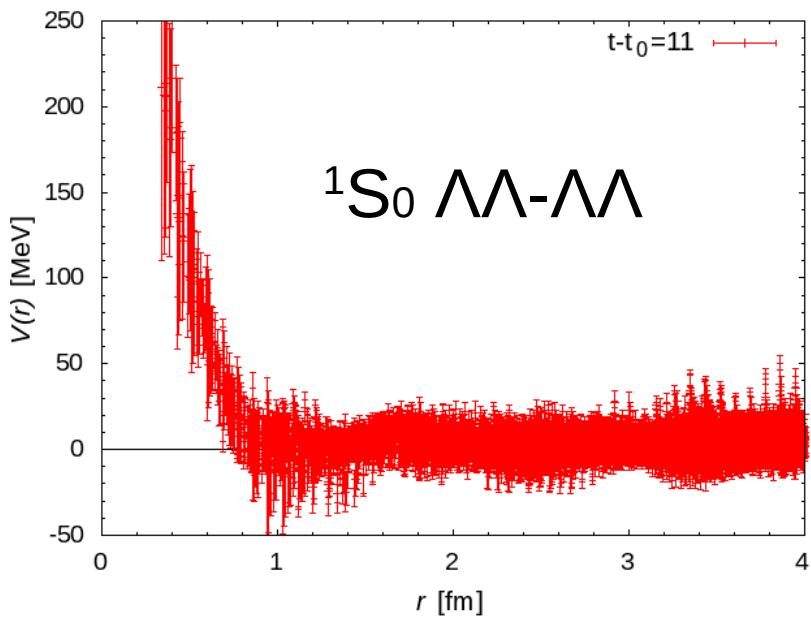
K-configuration

almost physical point

Not final. We are still increasing #stat.

# Hyperon interactions from QCD

# Hyperon int. potentials from LQCD

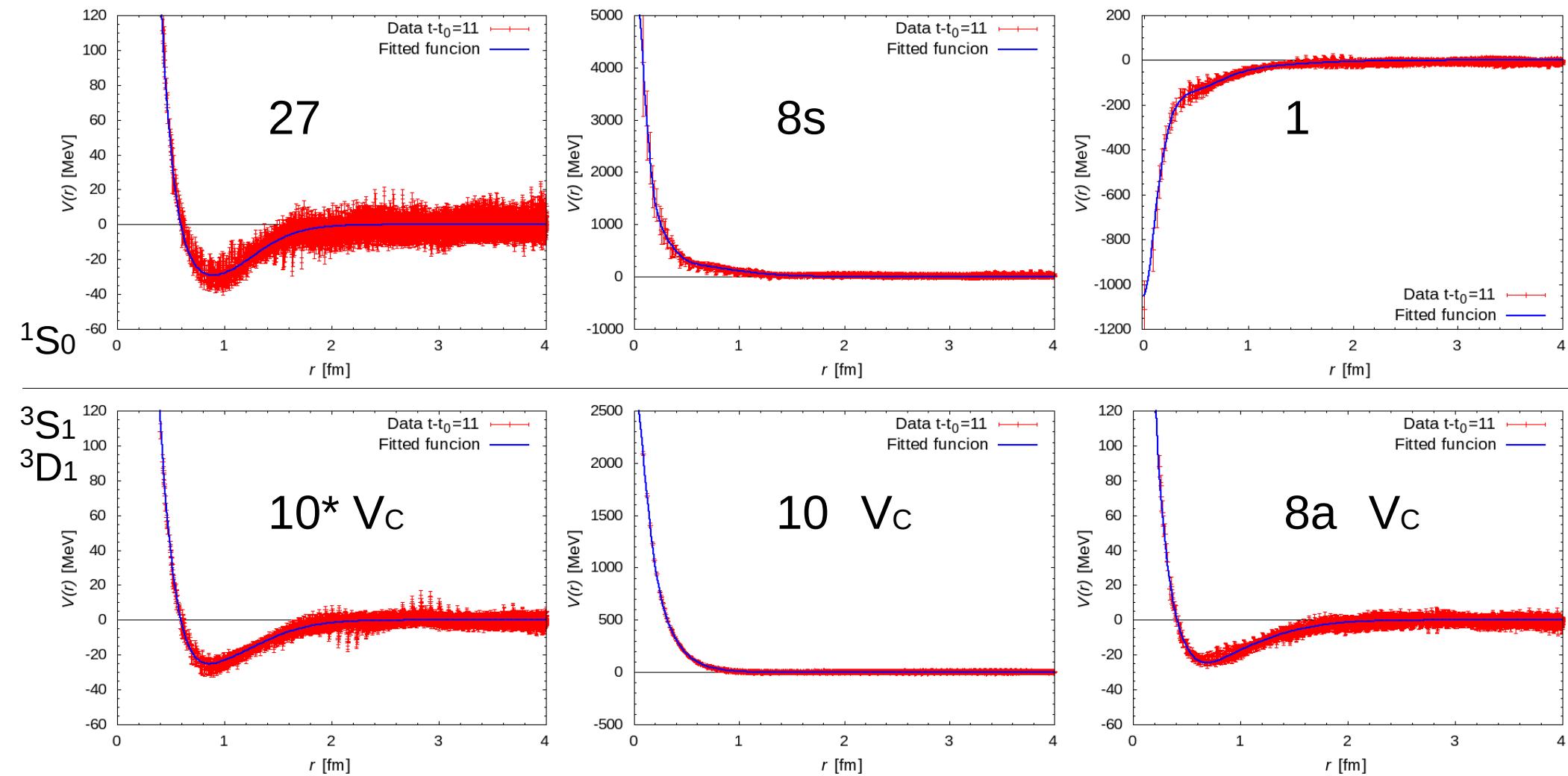


etc. for example

- There are **many** particle-base potentials. # $\approx 100$  in S-wave.
- For application, we need to **parameterize** potential data.
- It is **tough** to parameterize all needed potential data.
- So, today, for the moment, I use potential data **rotated** into the irreducible-representation base.

$$8 \times 8 = \underbrace{27 + 8s + 1 + 10^*}_{^1S_0} + \underbrace{10 + 8a}_{^3S_1, \ ^3D_1}$$

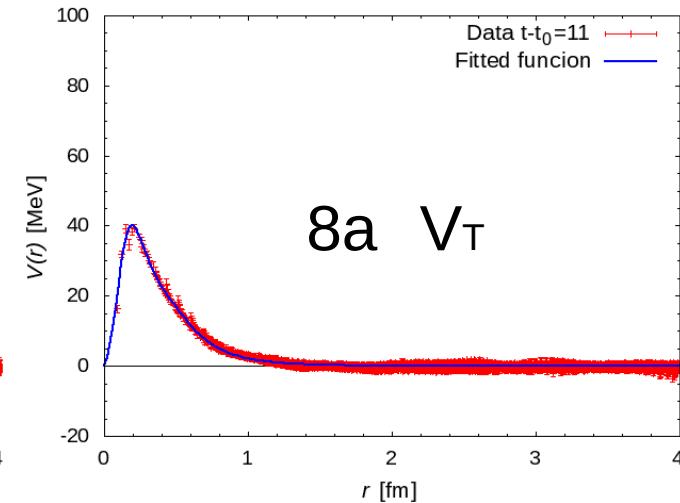
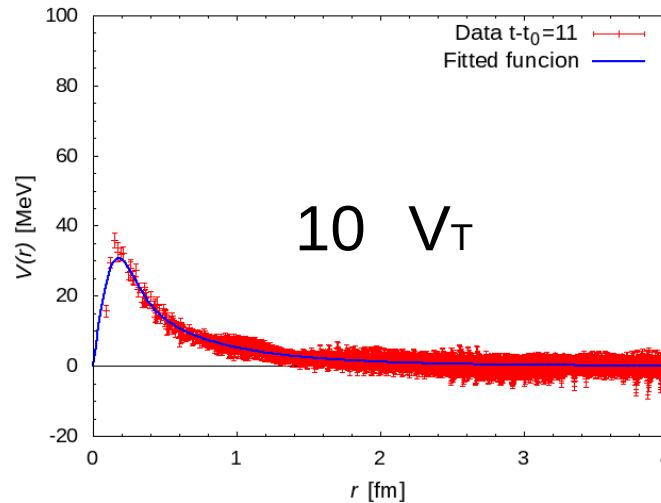
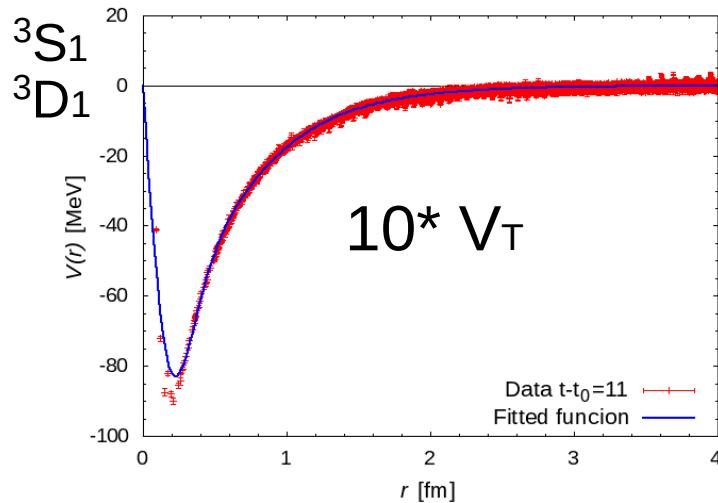
# Irre.-rep. base diagonal potentials



- Analytic function fitted to data

$$V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 \left[ \left( 1 - e^{-a_6 r^2} \right) \frac{e^{-a_7 r}}{r} \right]^2$$

# Irre.-rep. base diagonal potentials



- Analytic function fitted to data

$$V(r) = a_1 (1 - e^{-a_2 r^2})^2 \left( 1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2} \right) \frac{e^{-a_3 r}}{r} + a_4 (1 - e^{-a_5 r^2})^2 \left( 1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2} \right) \frac{e^{-a_6 r}}{r}$$

- Since  $SU(3)_F$  is **broken** at the physical point (K-conf.), there are irre.-rep. base **off-diagonal** potentials.
- But, I **omit** them and construct  $V_{YN}$ ,  $V_{YY}$  with these irre.-rep. diagonal potentials and the C.G. coefficient.

# Hyperon single-particle potentials

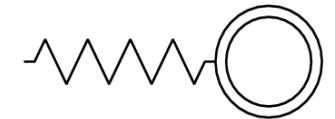
# Brueckner-Hartree-Fock

LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze,  
Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k k' | G_{(YN)(YN)}^{SLJ} (e_Y(k) + e_N(k')) | k k' \rangle$$



$$^{2S+1}L_J = {}^1S_0, {}^3S_1, {}^3D_1, \left| {}^1P_1, {}^3P_J \dots \right.$$

in our study

limitation

- YN G-matrix using  $V_{S=-1}^{\text{LQCD}}$ ,  $M_{N,Y}^{\text{Phys}}$ ,  $U_{n,p}^{\text{AV18,BHF}}$  and,  $U_Y^{\text{LQCD}}$

$Q=0$	$\begin{pmatrix} G_{(\Lambda n)(\Lambda n)}^{SLJ} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$	$Q=+1$	$\begin{pmatrix} G_{(\Lambda p)(\Lambda p)}^{SLJ} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$
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$Q=-1 \quad G_{(\Sigma^- n)(\Sigma^- n)}^{SLJ}$

$Q = +2 \quad G_{(\Sigma^+ p)(\Sigma^+ p)}^{SLJ}$

# Brueckner-Hartree-Fock

- Hyperon single-particle potential

$$U_{\Xi}(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k k' | G_{(\Xi N)(\Xi N)}^{SLJ} (e_{\Xi}(k) + e_N(k')) | k k' \rangle$$


- $\Xi N$  G-matrix using  $V_{S=-2}^{\text{LQCD}}$ ,  $M_{N,Y}^{\text{Phys}}$ ,  $U_{n,p}^{\text{AV18}}$ ,  $U_{\Lambda,\Sigma}^{\text{LQCD}}$  and,  $U_{\Xi}^{\text{LQCD}}$

Flavor symmetric  ${}^1S_0$  sectors

$Q=0$	$G_{(\Xi^0 n)(\Xi^0 n)}^{SLJ}$	$G_{(\Xi^0 n)(\Xi^- p)}$	$G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)}$	$G_{(\Xi^0 n)(\Sigma^0 \Sigma^0)}$	$G_{(\Xi^0 n)(\Sigma^0 \Lambda)}$	$G_{(\Xi^0 n)(\Lambda \Lambda)}$
	$G_{(\Xi^- p)(\Xi^0 n)}$	$G_{(\Xi^- p)(\Xi^- p)}$	$G_{(\Xi^- p)(\Sigma^+ \Sigma^-)}$	$G_{(\Xi^- p)(\Sigma^0 \Sigma^0)}$	$G_{(\Xi^- p)(\Sigma^0 \Lambda)}$	$G_{(\Xi^- p)(\Lambda \Lambda)}$
	$G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)}$	$G_{(\Sigma^+ \Sigma^-)(\Xi^- p)}$	$G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)}$	$G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Sigma^0)}$	$G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)}$	$G_{(\Sigma^+ \Sigma^-)(\Lambda \Lambda)}$
	$G_{(\Sigma^0 \Sigma^0)(\Xi^0 n)}$	$G_{(\Sigma^0 \Sigma^0)(\Xi^- p)}$	$G_{(\Sigma^0 \Sigma^0)(\Sigma^+ \Sigma^-)}$	$G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Sigma^0)}$	$G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Lambda)}$	$G_{(\Sigma^0 \Sigma^0)(\Lambda \Lambda)}$
	$G_{(\Sigma^0 \Lambda)(\Xi^0 n)}$	$G_{(\Sigma^0 \Lambda)(\Xi^- p)}$	$G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)}$	$G_{(\Sigma^0 \Lambda)(\Sigma^0 \Sigma^0)}$	$G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)}$	$G_{(\Sigma^0 \Lambda)(\Lambda \Lambda)}$
	$G_{(\Lambda \Lambda)(\Xi^0 n)}$	$G_{(\Lambda \Lambda)(\Xi^- p)}$	$G_{(\Lambda \Lambda)(\Sigma^+ \Sigma^-)}$	$G_{(\Lambda \Lambda)(\Sigma^0 \Sigma^0)}$	$G_{(\Lambda \Lambda)(\Sigma^0 \Lambda)}$	$G_{(\Lambda \Lambda)(\Lambda \Lambda)}$

$Q=+1$	$G_{(\Xi^0 p)(\Xi^0 p)}^{SLJ}$	$G_{(\Xi^0 p)(\Sigma^+ \Lambda)}$	$Q=-1$	$G_{(\Xi^- n)(\Xi^- n)}^{SLJ}$	$G_{(\Xi^- n)(\Sigma^- \Lambda)}$
	$G_{(\Sigma^+ \Lambda)(\Xi^0 p)}$	$G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)}$		$G_{(\Sigma^- \Lambda)(\Xi^- n)}$	$G_{(\Sigma^- \Lambda)(\Sigma^- \Lambda)}$

# Brueckner-Hartree-Fock

- $\Xi N$  G-matrix using  $V_{S=-2}^{\text{LQCD}}$ ,  $M_{N,Y}^{\text{Phys}}$ ,  $U_{n,p}^{\text{AV18}}$ ,  $U_{\Lambda,\Sigma}^{\text{LQCD}}$  and,  $U_{\Xi}^{\text{LQCD}}$   
 Flavor anti-symmetric  ${}^3S_1$ ,  ${}^3D_1$  sectors

$$Q=0 \quad \begin{pmatrix} G_{(\Xi^0 n)(\Xi^0 n)}^{SLJ} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0 \Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^- p)(\Sigma^0 \Lambda)} \\ G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+ \Sigma^-)(\Xi^- p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)} \\ G_{(\Sigma^0 \Lambda)(\Xi^0 n)} & G_{(\Sigma^0 \Lambda)(\Xi^- p)} & G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)} \end{pmatrix}$$

Q=+1

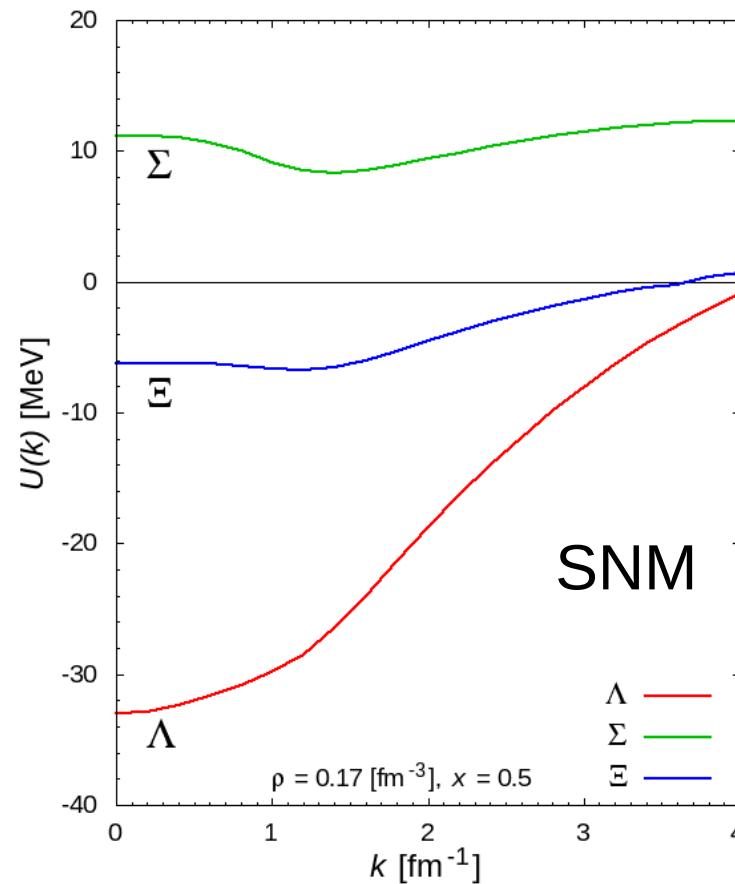
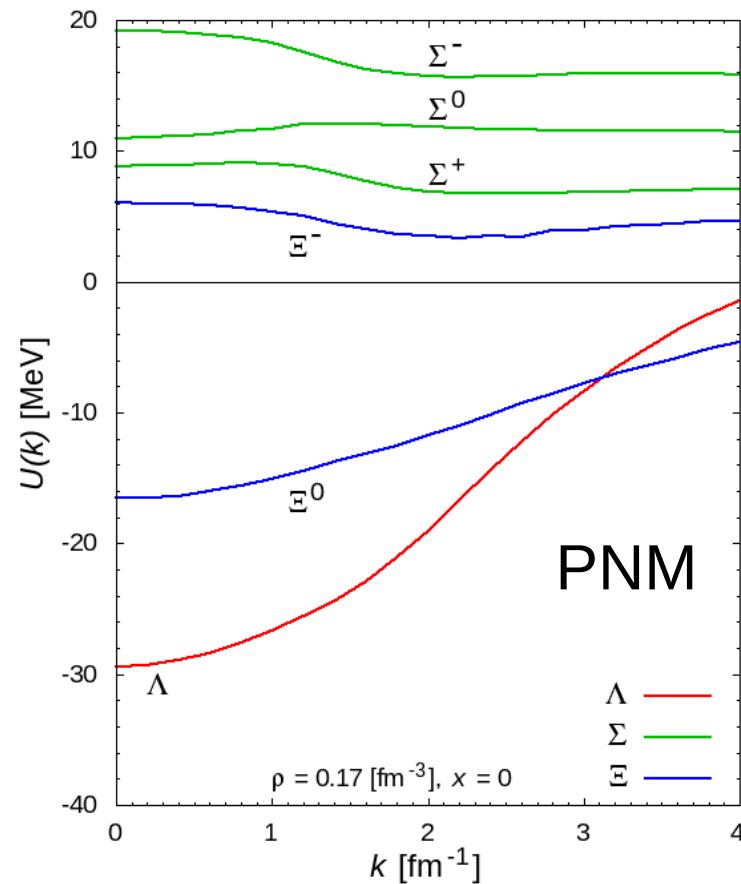
$$\begin{pmatrix} G_{(\Xi^0 p)(\Xi^0 p)}^{SLJ} & G_{(\Xi^0 p)(\Sigma^+ \Sigma^0)} & G_{(\Xi^0 p)(\Sigma^+ \Lambda)} \\ G_{(\Sigma^+ \Sigma^0)(\Xi^0 p)} & G_{(\Sigma^+ \Sigma^0)(\Sigma^+ \Sigma^0)} & G_{(\Sigma^+ \Sigma^0)(\Sigma^+ \Lambda)} \\ G_{(\Sigma^+ \Lambda)(\Xi^0 p)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Sigma^0)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)} \end{pmatrix}$$

Q=-1

$$\begin{pmatrix} G_{(\Xi^- n)(\Xi^- n)}^{SLJ} & G_{(\Xi^- n)(\Sigma^- \Sigma^0)} & G_{(\Xi^- n)(\Sigma^- \Lambda)} \\ G_{(\Sigma^- \Sigma^0)(\Xi^- n)} & G_{(\Sigma^- \Sigma^0)(\Sigma^- \Sigma^0)} & G_{(\Sigma^- \Sigma^0)(\Sigma^- \Lambda)} \\ G_{(\Sigma^- \Lambda)(\Xi^- n)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Sigma^0)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Lambda)} \end{pmatrix}$$

# Results

# Hyperon single-particle potentials

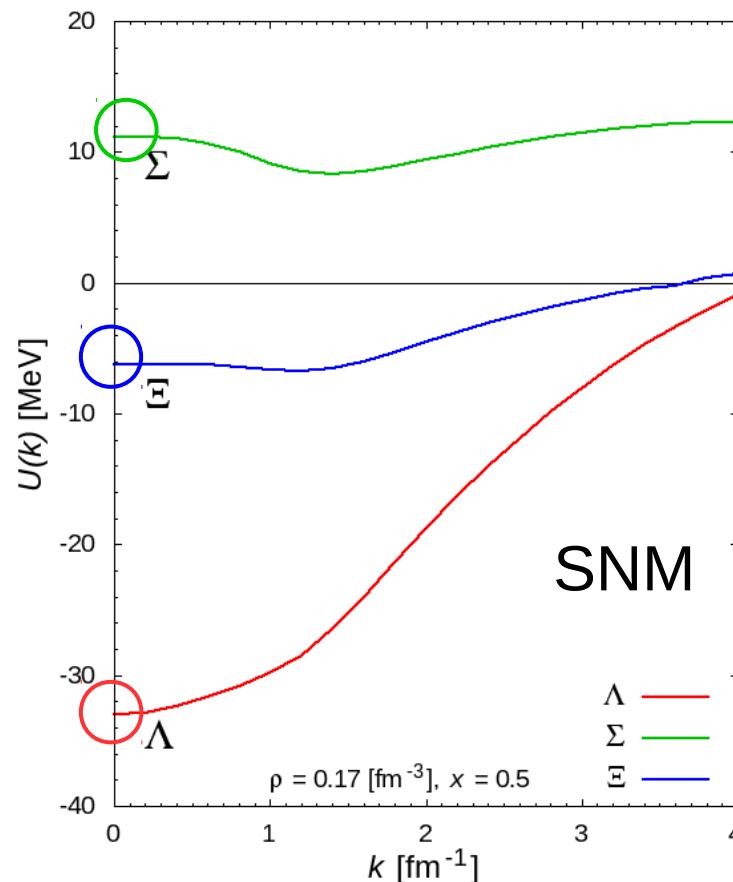
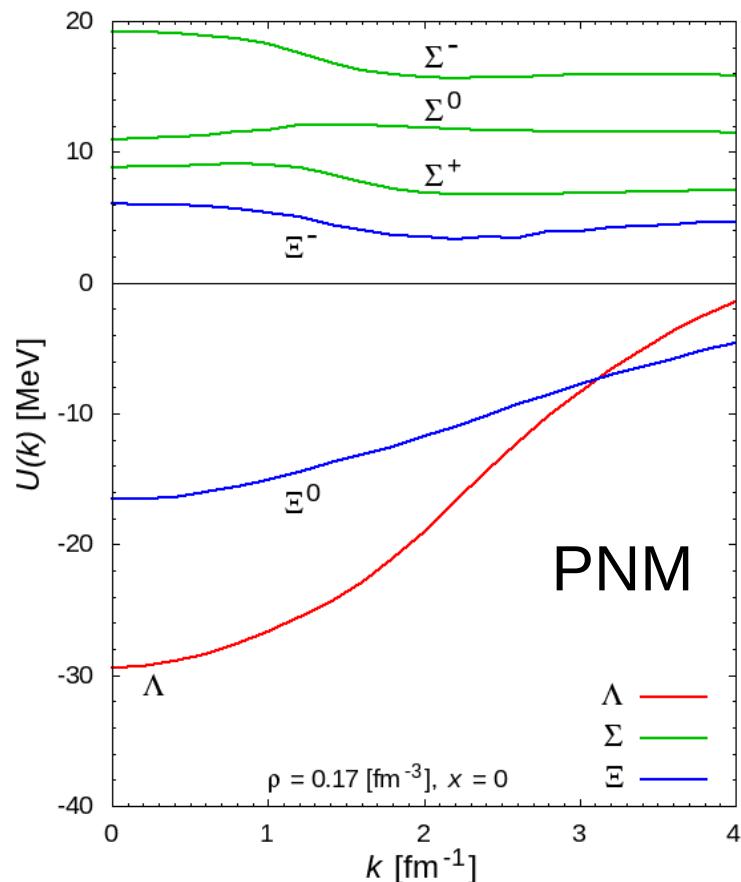


@ $\rho=0.17 [\text{fm}^3]$

Preliminary

- obtained by using YN,YY forces form QCD.

# Hyperon single-particle potentials



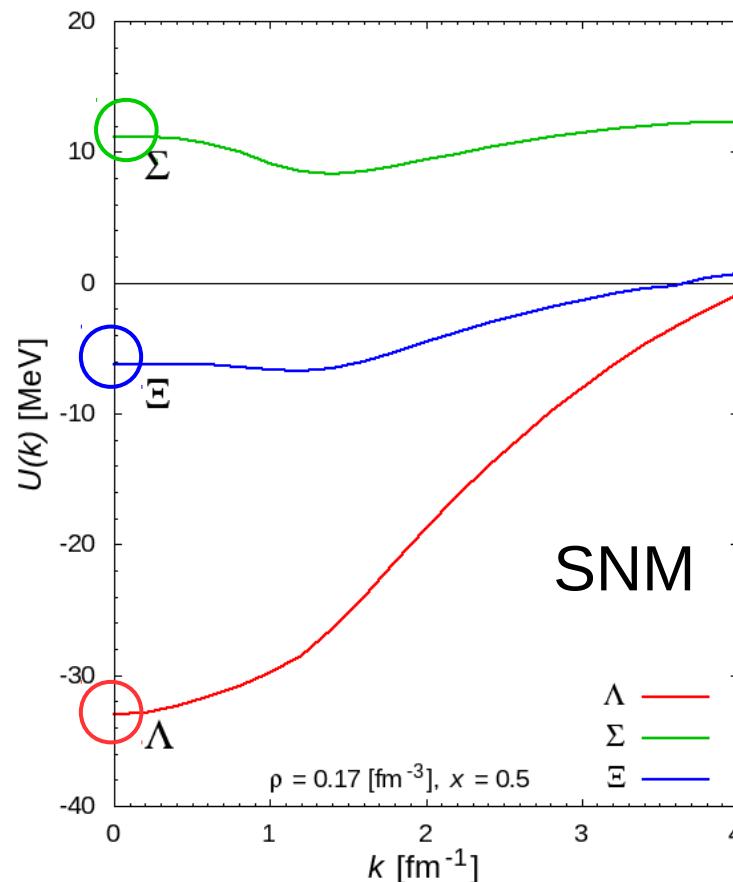
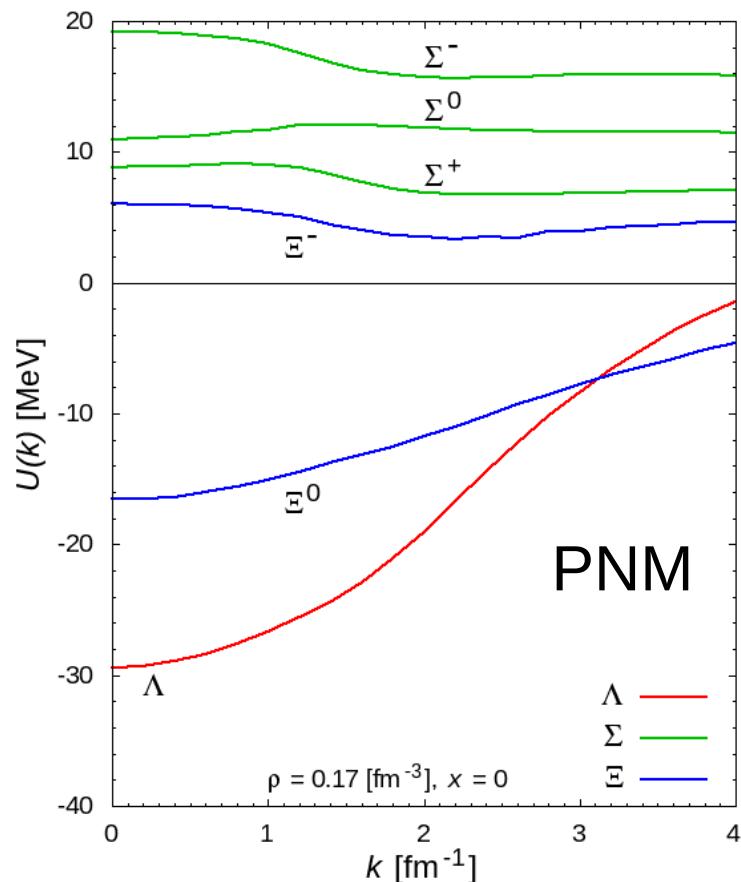
# Preliminary

- obtained by using YN,YY forces form QCD.
  - Results agree with experimental data!

$$U_\Lambda^{\text{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\text{Exp}} \simeq -10, \quad U_\Sigma^{\text{Exp}}(0) \simeq +10 \quad [\text{MeV}]$$

attraction                      attraction small                      repulsion small

# Hyperon single-particle potentials



# Preliminary

- obtained by using YN,YY forces form QCD.
  - Results agree with experimental data!

# Remarkable. Encouraging.

$$U_\Lambda^{\text{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\text{Exp}} \simeq -10, \quad U_\Sigma^{\text{Exp}}(0) \simeq +10 \quad [\text{MeV}]$$

attraction                      attraction small                      repulsion small

# Summary and Outlook

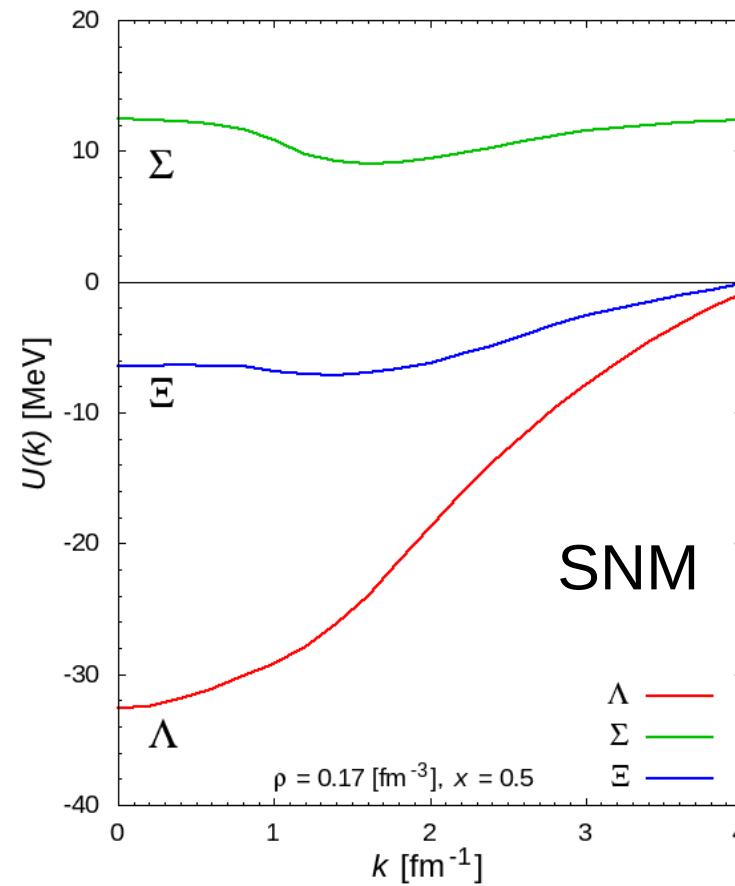
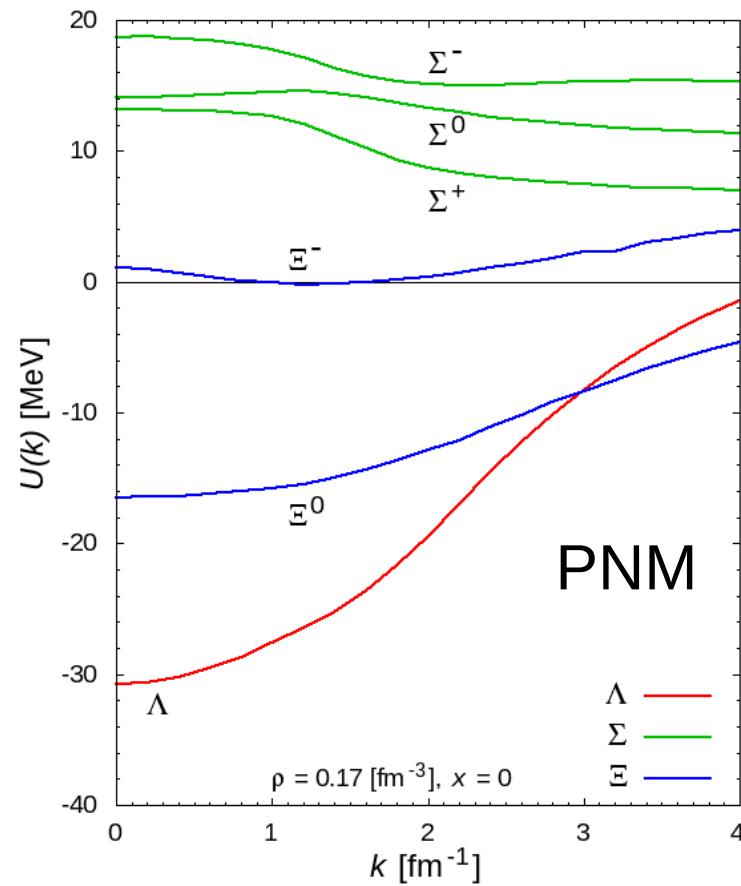
# Summary and outlook

- We deduced hyperon forces from QCD on lattice.
- We calculated hyperon s.p. potentials using them.
  - This time, I used rotated data diagonal in the irre.-rep. base.
- Obtained  $U_Y$  are compatible with experiment!
  - In SNM,  $\Lambda$  and  $\Xi$  feel attraction, while  $\Sigma$  feels repulsion.
  - I need to confirm this agreement since I used simplifications.
- This is remarkable success, at least encouraging.
  - Recall that we've never used any experimental data about hyperon forces, but we used only QCD.
- We continue to improve this study.
  - We have to estimate statistical error & systematic uncertainty.
  - We try to reduce them as possible.
- In near future, we will be able to reproduce hypernuclei from QCD and reveal hyperons in NS from QCD.

Thank you !!

# Back up

# Hyperon single-particle potentials



@  $\rho = 0.17 \text{ [fm}^3\text{]}$

Preliminary

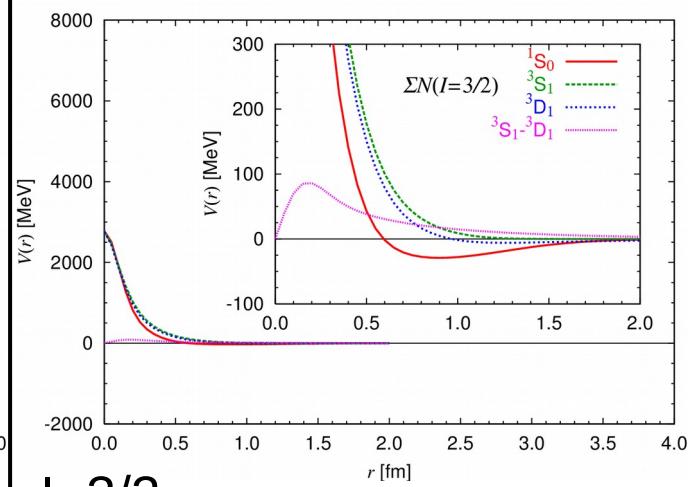
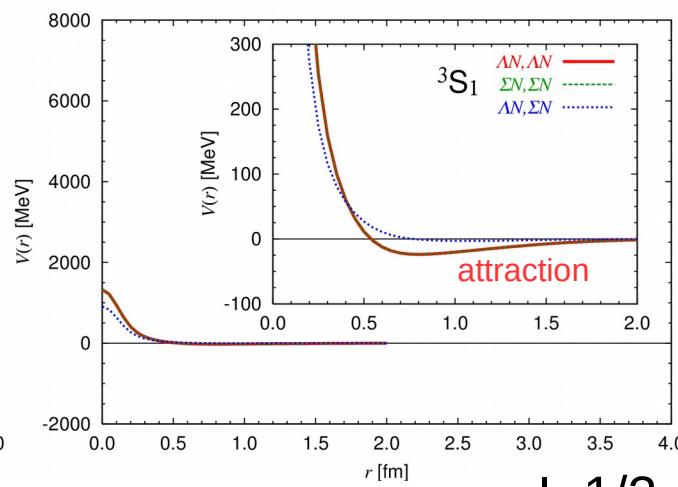
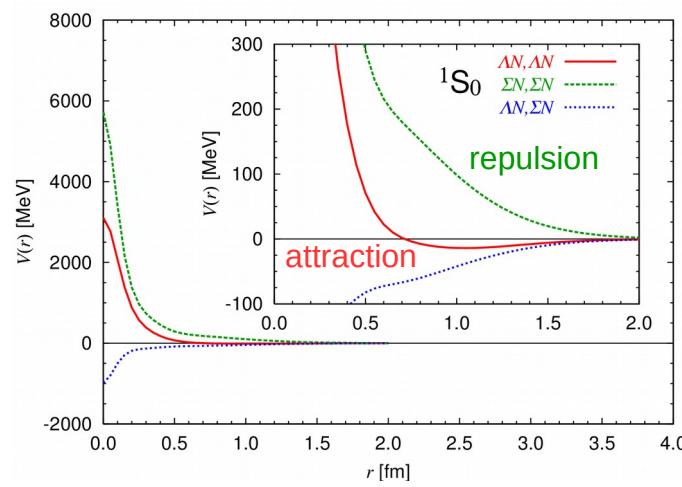
- obtained with LQCD YN,YY pot. +  $M_{N,Y}^{\text{LQCD}}$  +  $U_{n,p}^{\text{LQCD,BHF}}$

	N	$\Lambda$	$\Sigma$	$\Xi$
$M_B$ [MeV]	956	1121	1201	1328

- YN,YY pot. are essential.  $M_B$  and  $U_{n,p}$  have minor effect.

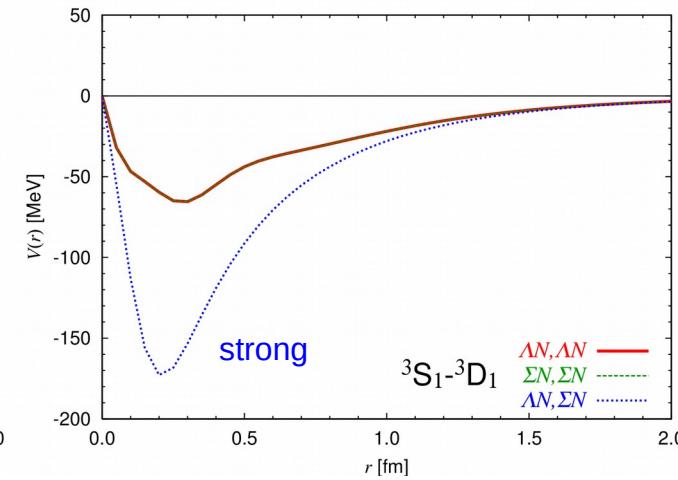
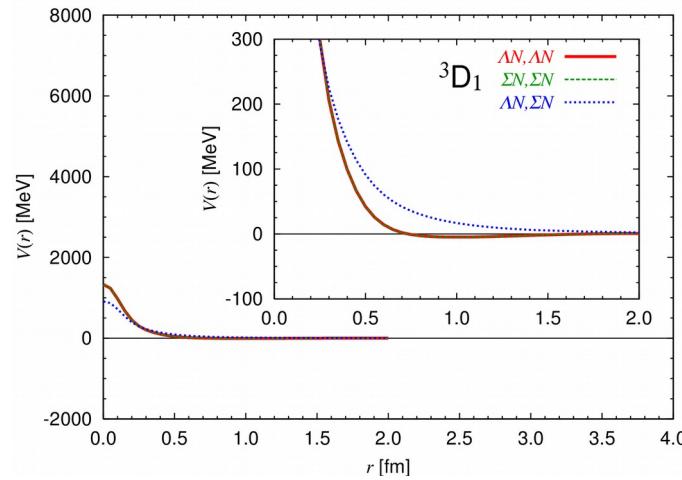
# LQCD $\Lambda N$ - $\Sigma N$

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



$|l|=1/2$

$|l|=3/2$

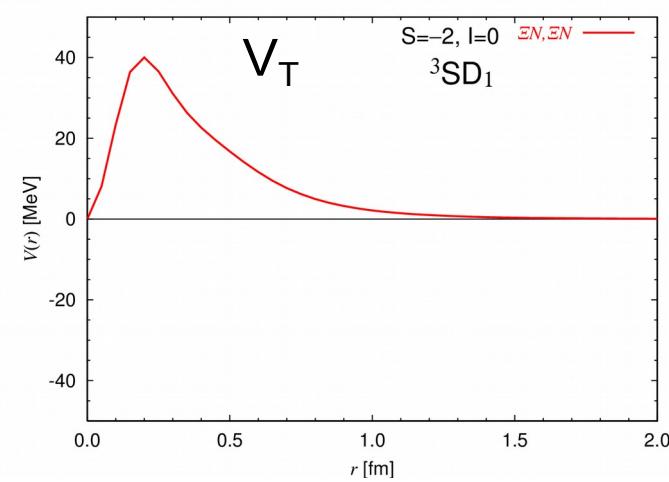
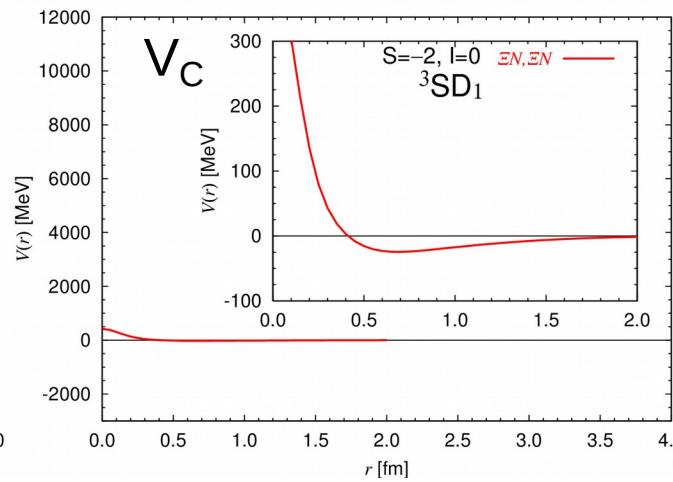
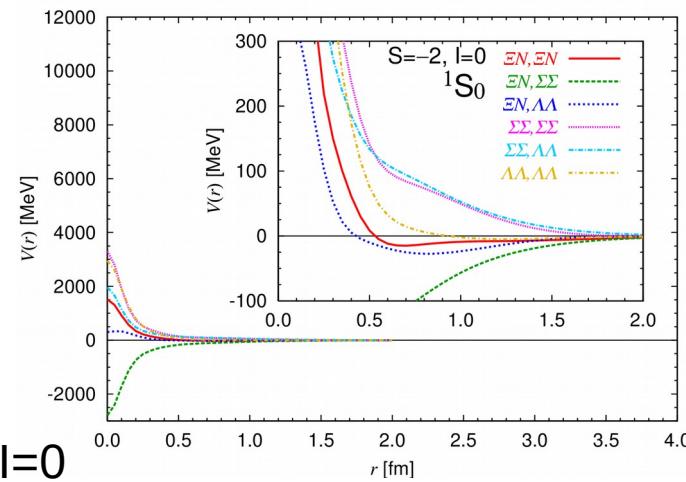


- In  $|l|=1/2$ ,  $^1S_0$  channel,  $\Lambda N$  has an **attraction**, while  $\Sigma N$  is **repulsive**.
- In  $|l|=1/2$ ,  $^3S_1$  channel, both  $\Lambda N$  and  $\Sigma N$  have an **attraction**.
- In  $|l|=1/2$ , **strong** tensor coupling in flavor off-diagonal.

No attraction  
in Nijmegen

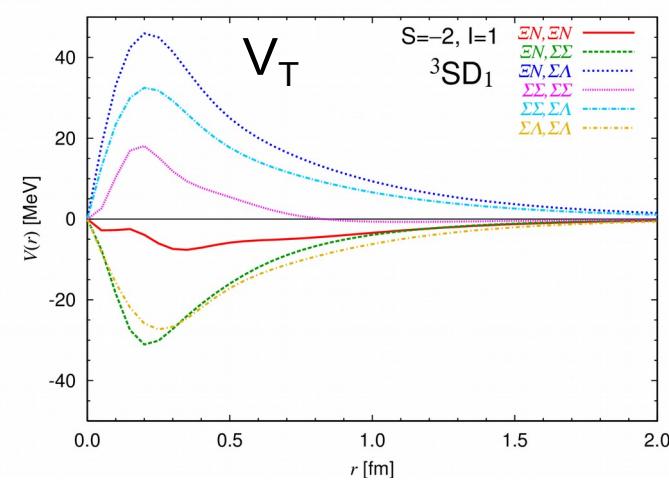
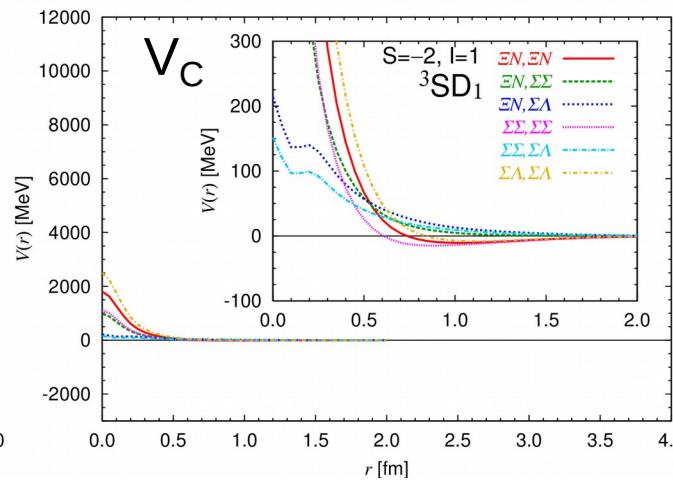
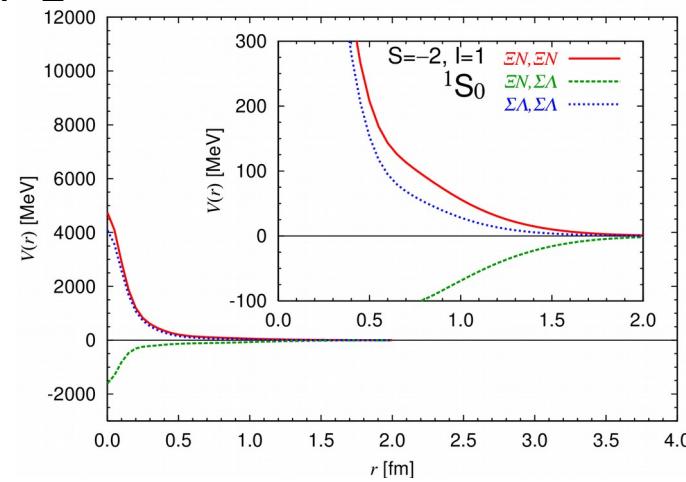
# LQCD $\Xi N - \bar{Y} Y$

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



$|l=0$

$|l=1$



- Many experimentally **unknown** coupled-channel potentials.
- One can see **predictive** power of the HALQCD method.

# Source and sink operator

- NBS wave function and 4-point function

$$\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x} + \vec{r}, t) \underbrace{B_j(\vec{x}, t)}_{\text{equal}} | B=2, \vec{k} \rangle \text{ QCD eigenstate}$$

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | \underbrace{B_i(\vec{x} + \vec{r}, t)}_{\text{sink}} \underbrace{B_j(\vec{x}, t)}_{\text{source}} J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \dots$$

- Point type octet baryon field operator at **sink**

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with } \xi_i = \{c_i, \beta_i, \underline{x}\}$$

$$\Lambda_\alpha(x) = - \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha} \sqrt{\frac{1}{6}} [d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3)]$$

- Wall type **source** of two-baryon state

$$\text{e.g. } \overline{B}B^{(1)} = -\sqrt{\frac{1}{8}} \overline{\Lambda} \Lambda + \sqrt{\frac{3}{8}} \overline{\Sigma} \Sigma + \sqrt{\frac{4}{8}} \overline{N} \Xi \quad \text{for flavor-singlet}$$

# FAQ

1. Does your potential depend on the choice of **source**?
2. Does your potential depend on choice of **operator at sink**?
3. Does your potential  $U(r,r')$  or  $V(r)$  depends on **energy**?

# FAQ

1. Does your potential depend on the choice of **source**?
  - **No.** Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.
  
2. Does your potential depend on choice of **operator at sink**?
  - **Yes.** It can be regarded as the “**scheme**” to define a potential. Note that a potential itself is not physical observable. We will obtain **unique** result for physical observables irrespective to the choice, as long as the potential  $U(r,r')$  is deduced exactly.

# FAQ

3. Does your potential  $U(r,r')$  or  $V(r)$  depends on **energy**?

→ By definition,  $U(r,r')$  is non-local but energy **independent**. While, determination and validity of its leading term  $V(r)$  **depend** on energy because of the **truncation**.

However, we know that the dependence in  $NN$  case is **very small** (thanks to our choice of sink operator = point) and **negligible** at least at  $E_{lab.} = 0 - 90$  MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.