Compositeness of hadrons from effective field theory

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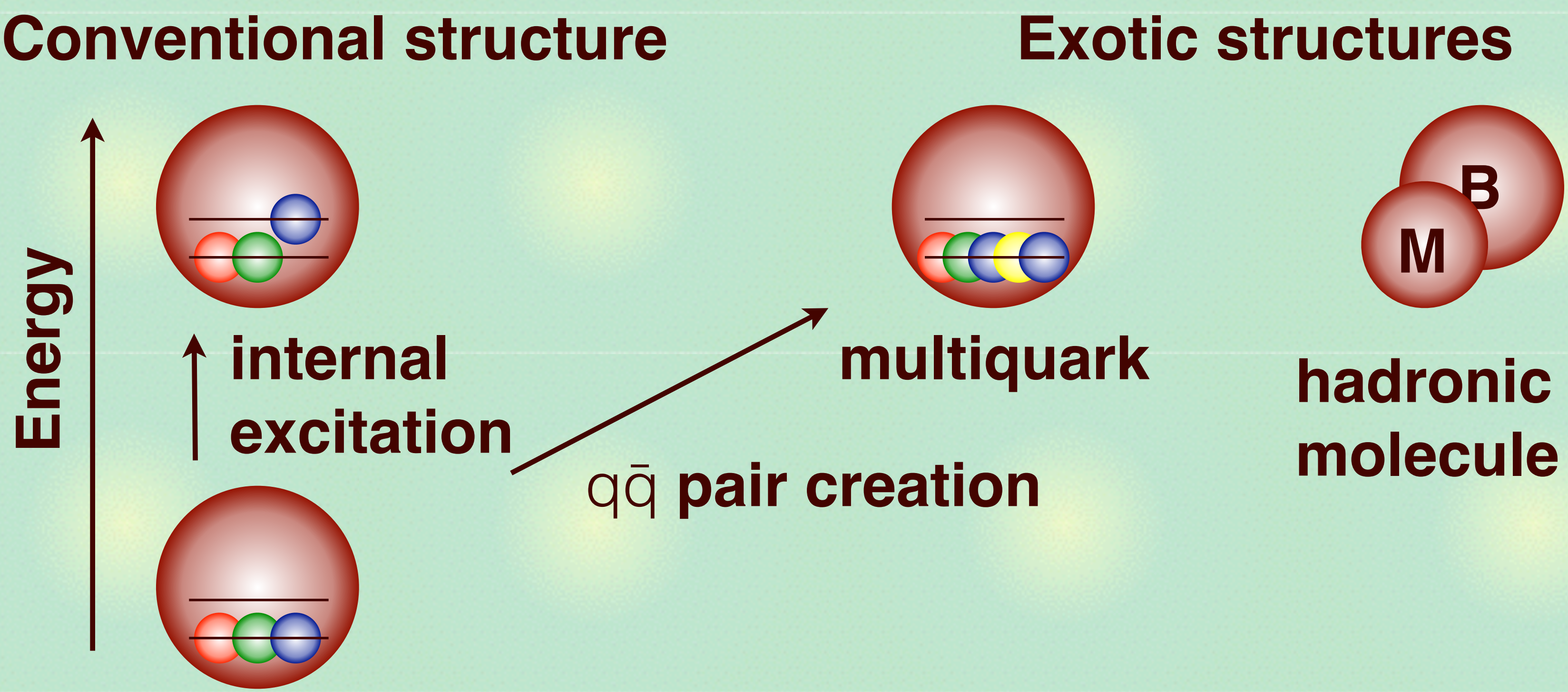
Method to study the internal structure

Internal structure of excited hadrons?



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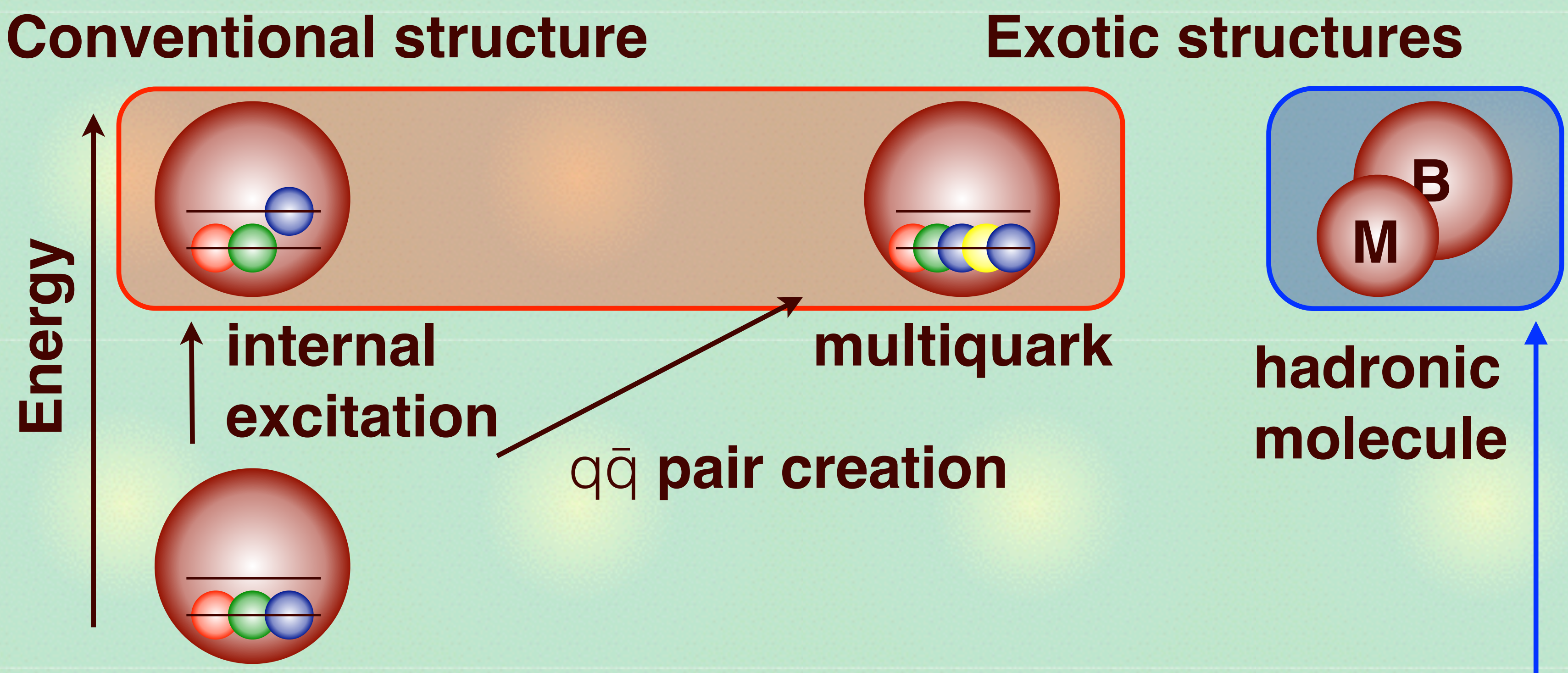

- Weak binding relation: observables -> compositeness

S. Weinberg, Phys. Rev. 137, B672 (1965)

c.f. Talk by
T. Sekihara (Fri)

Weak binding relation for stable states

Compositeness of s-wave **weakly bound** state ($R \gg R_{typ}$)

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$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

a_0: **scattering length,** r_e: **effective range**
$R = (2\mu B)^{-1/2}$: **radius of wave function**
R_{typ}: **length scale of interaction**
X: **probability of finding composite component**

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- **deuteron is** NN **composite (**$a_0 \sim R \gg r_e$**) —>** $X \sim 1$
- **internal structure from observable**
- **no nuclear force potential / wavefunction of deuteron**

Note: applicable only for stable states

Effective field theory

Low-energy scattering with near-threshold bound state

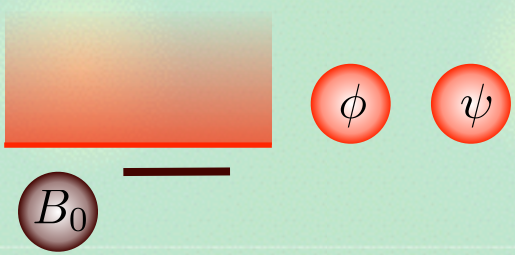
- Nonrelativistic EFT with contact interaction

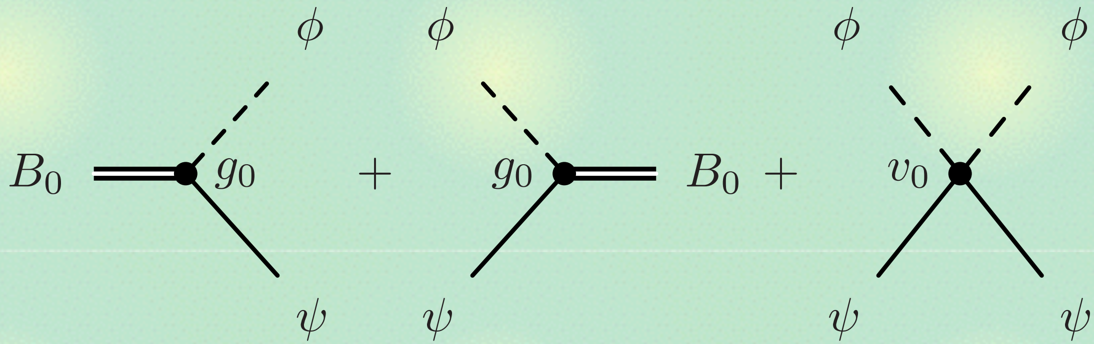
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

$$H_{\text{free}} = \int d\boldsymbol{r}\left[\frac{1}{2M}\nabla\psi^\dagger\cdot\nabla\psi + \frac{1}{2m}\nabla\phi^\dagger\cdot\nabla\phi + \frac{1}{2M_0}\nabla B_0^\dagger\cdot\nabla B_0 + \nu_0 B_0^\dagger B_0\right],$$

$$H_{\text{int}} = \int d\boldsymbol{r}\left[g_0\left(B_0^\dagger\phi\psi + \psi^\dagger\phi^\dagger B_0\right) + v_0\psi^\dagger\phi^\dagger\phi\psi\right]$$





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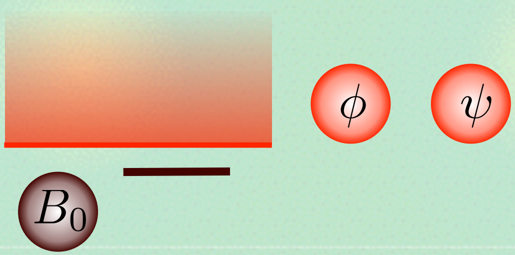

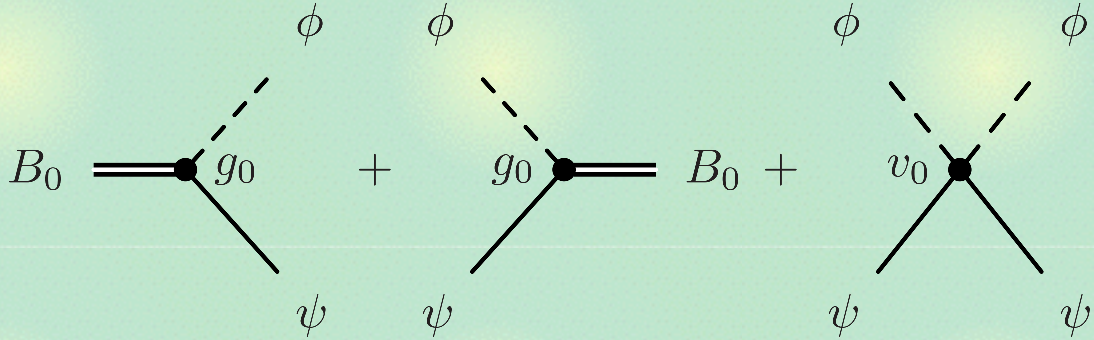

- **cutoff** : $\Lambda \sim 1/R_{\mathrm{typ}}$ **(interaction range of microscopic theory)**
- **At low momentum** $p \ll \Lambda$**, interaction ~ contact**

Compositeness and “elementariness”

Eigenstates

$$H_{\mathrm{free}}|\,B_0\,\rangle = \nu_0|\,B_0\,\rangle, \quad H_{\mathrm{free}}|\,\boldsymbol{p}\,\rangle = \frac{p^2}{2\mu}|\,\boldsymbol{p}\,\rangle$$

free (discrete + continuum)

$$(H_{\mathrm{free}} + H_{\mathrm{int}})|\,B\,\rangle = -B|\,B\,\rangle$$

full (bound state)

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- normalization of |B> + completeness relation

$$\langle\,B\,|\,B\,\rangle = 1, \quad 1 = |\,B_0\,\rangle\langle\,B_0\,| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\,\boldsymbol{p}\rangle\langle\boldsymbol{p}\,|$$

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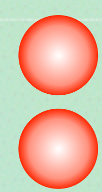
- projections onto bare states

$$1 = Z + X, \quad \underline{Z \equiv |\langle\,B_0\,|\,B\,\rangle|^2}, \quad \underline{X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\,\boldsymbol{p}\,|\,B\,\rangle|^2}$$

"elementariness" **compositeness**

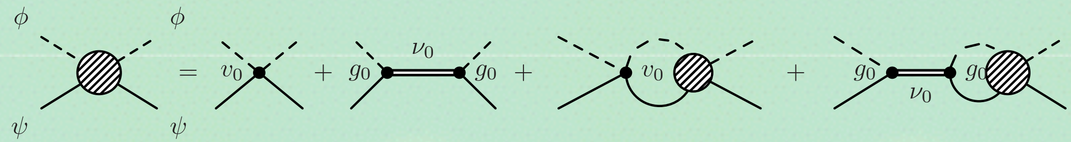


Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

ΨΦ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E-\nu_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

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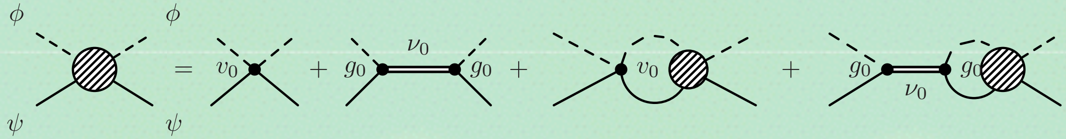

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Compositeness X <— v(E), G(E)

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

$$X = \left\{1 + G^2(-B) v'(-B) \left[G'(-B)\right]^{-1}\right\}^{-1}$$

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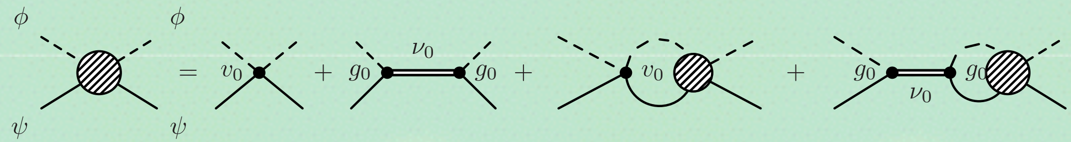

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$1/R=(2\mu B)^{1/2}$ expansion: leading term <— X

$$a_0 = -f(E=0) = R\left\{\underbrace{\frac{2X}{1+X}}_{\text{renormalization independent}} + \overline{\mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)}\right\}$$ renormalization dependent

If $R \gg R_{\mathrm{typ}}$, correction terms neglected: X <— (B, a_0)

Introduction of decay channel

Introduce decay channel

$$H'_{\mathrm{free}} = \int d\boldsymbol{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right],$$

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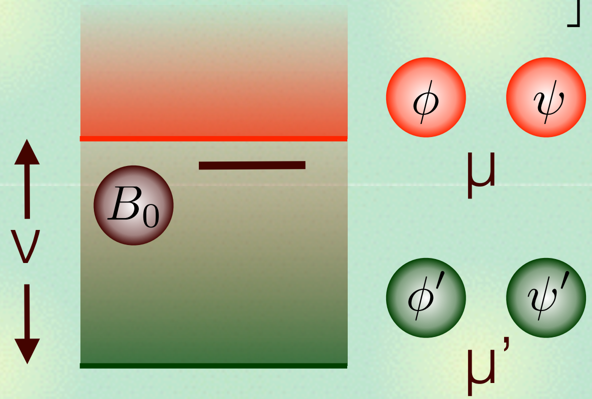

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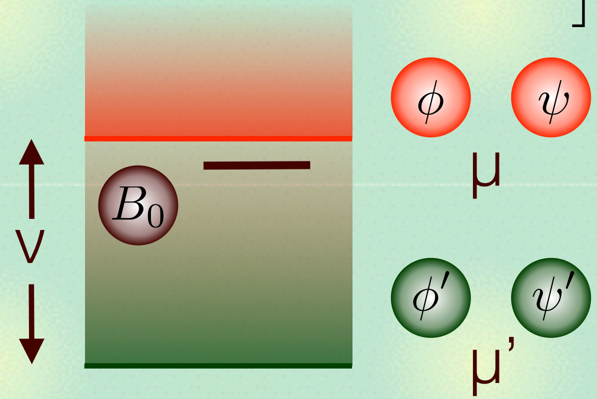
Quasi-bound state: complex eigenvalue

$$H = H_{\rm free} + H'_{\rm free} + H_{\rm int} + H'_{\rm int}$$

$$H|QB\rangle = E_{QB}|QB\rangle, \quad E_{QB}\in\mathbb{C}$$



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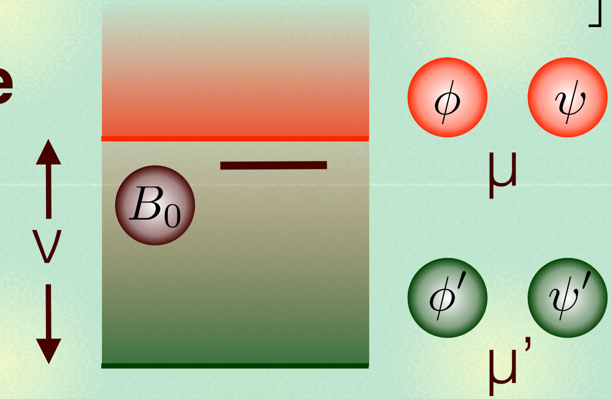

Generalized relation: correction term <— threshold difference

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \underline{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}\in\mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If $|R| \gg (R_{\rm typ}, l)$ correction terms neglected: X <— (E_{QB}, a_0)

Application

Generalized weak binding relation X <— (E_{QB}, a_0)

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- Λ(1405) **(higher) pole position and** $\bar{\mathrm{K}}$N **scattering length**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012), …

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- E_{QB} = -10 -26i **MeV —>** |R| ~ 2 fm **—> small correction term**

  $$\left|\frac{R_{\text{typ}}}{R}\right| \lesssim 0.12, \quad \left|\frac{l}{R}\right|^3 \lesssim 0.16$$ **(rho exchange,** πΣ **threshold)**

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systematic error

Ref.	E_{QB} (MeV)	a_0 (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	U
[45]	$-10-i26$	$1.39-i0.85$	$1.2+i0.1$	1.0	0.5
[46]	$-4-i8$	$1.81-i0.92$	$0.6+i0.1$	0.6	0.0
[47]	$-13-i20$	$1.30-i0.85$	$0.9-i0.2$	0.9	0.1
[48]	$2-i10$	$1.21-i1.47$	$0.6+i0.0$	0.6	0.0
[48]	$-3-i12$	$1.52-i1.85$	$1.0+i0.5$	0.8	0.6

$$\tilde{X} = \frac{1-|Z|+|X|}{2}$$

$$U = |Z| + |X| - 1$$

Λ(1405) **is** $\bar{\text{K}}$N **composite <— observables**

Summary

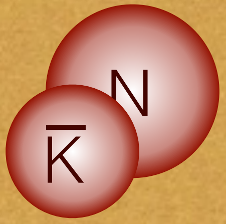
- **Compositeness of near-threshold bound state can be determined only by observables.**

  S. Weinberg, Phys. Rev. 137, B672 (1965)

- **Weak binding relation can be generalized to unstable states with effective field theory.**

$$a_0 = R\left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **Precise determination of the pole position and scattering length shows that Λ(1405) is dominated by $\bar{\mathrm{K}}$N composite component.**



Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016), arXiv:1607.01899[hep-ph]