



Canada's national laboratory  
for particle and nuclear physics  
and accelerator-based science

# Ab Initio Theory for All Medium-Mass Nuclei

Jason D. Holt

INPC September 12, 2016

## Collaborators

**S. R. Stroberg**

S. Bogner

H. Hergert

T. Morris

N. Parzuchowski

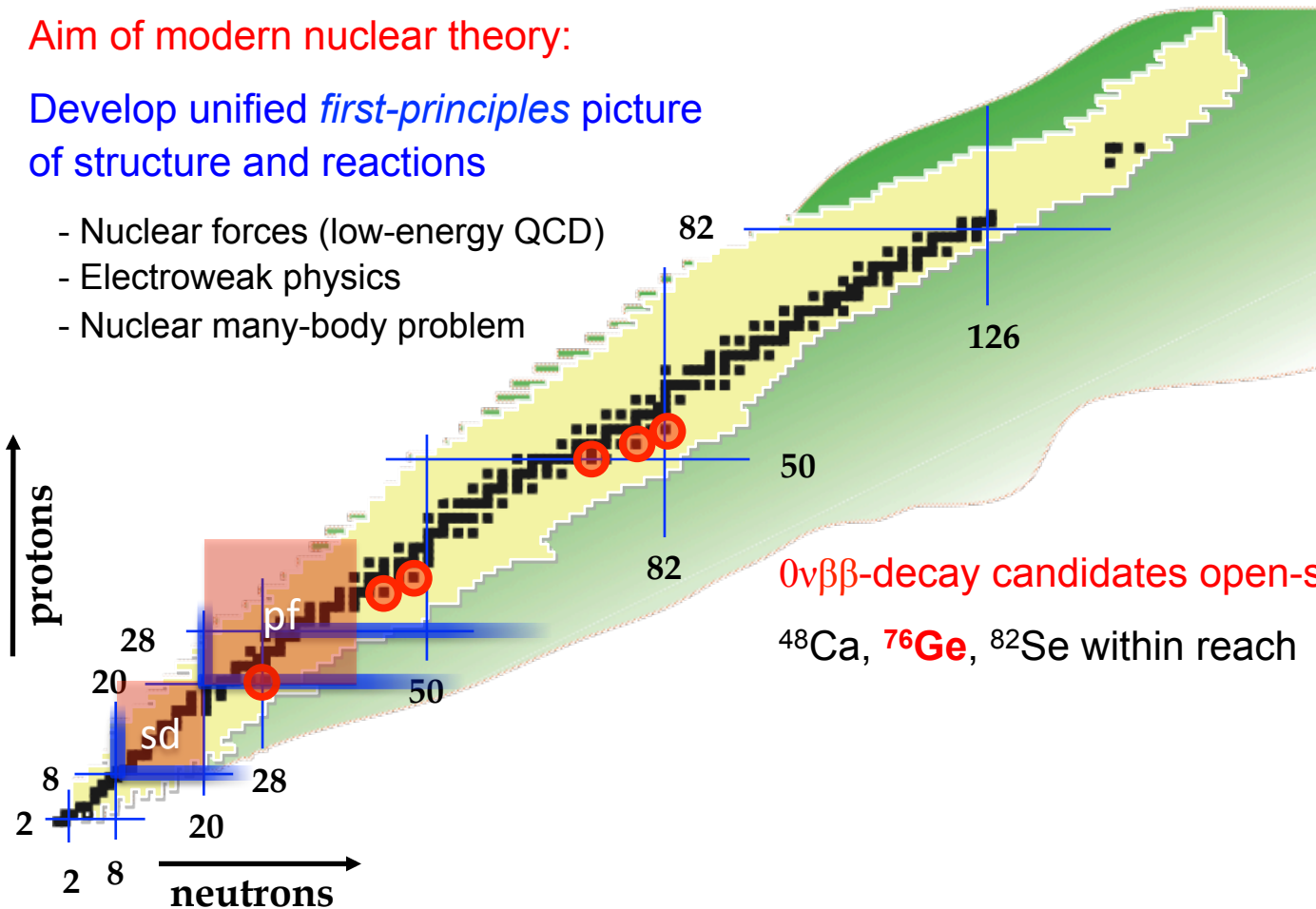
A. Schwenk



Aim of modern nuclear theory:

Develop unified *first-principles* picture of structure and reactions

- Nuclear forces (low-energy QCD)
- Electroweak physics
- Nuclear many-body problem



$0\nu\beta\beta$ -decay candidates open-shell medium/heavy-mass

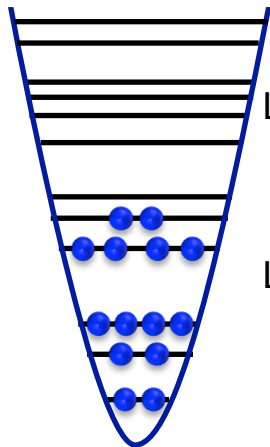
$^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$  within reach

Nucleus strongly interacting many-body system – full  $A$ -body problem impossible

$$H\psi_n = E_n\psi_n$$

**Large space:** controlled approximation to full Schrödinger Equation

Large-space approach



Limited range:  
Closed shell  $\pm 1$   
Even-even (spherical)

Limited properties:  
Ground states  
Some excited states

**Coupled Cluster**  
**In-Medium SRG**  
**Green's Function**  
**Perturbation Theory**

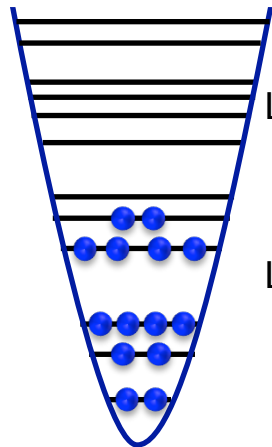
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**Valence space:** diagonalize *effective valence-space Hamiltonian*

**Large-space approach**

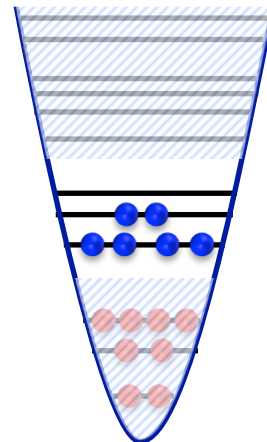


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**In-Medium SRG**  
**Coupled Cluster**  
Green's Function  
**Perturbation Theory**

**Valence-space approach**



All nuclei near  
closed shells

All properties  
Ground states  
Excited states  
Transitions



Nucleus strongly interacting many-body system – full  $A$ -body problem impossible

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## In-Medium SRG

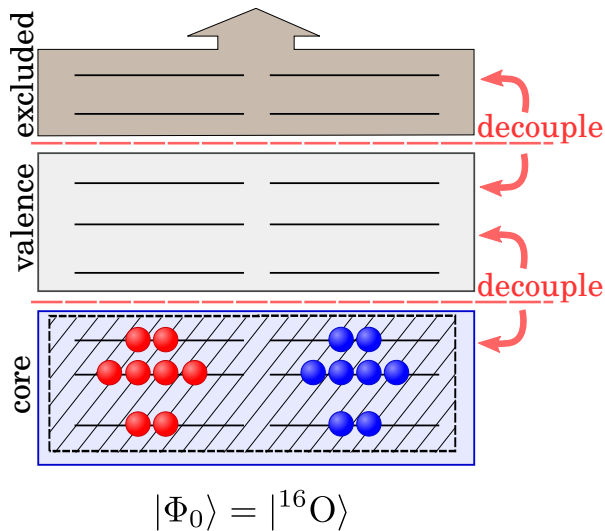
Can we achieve accuracy  
of large-space methods?

$$U = e^{\Omega}$$

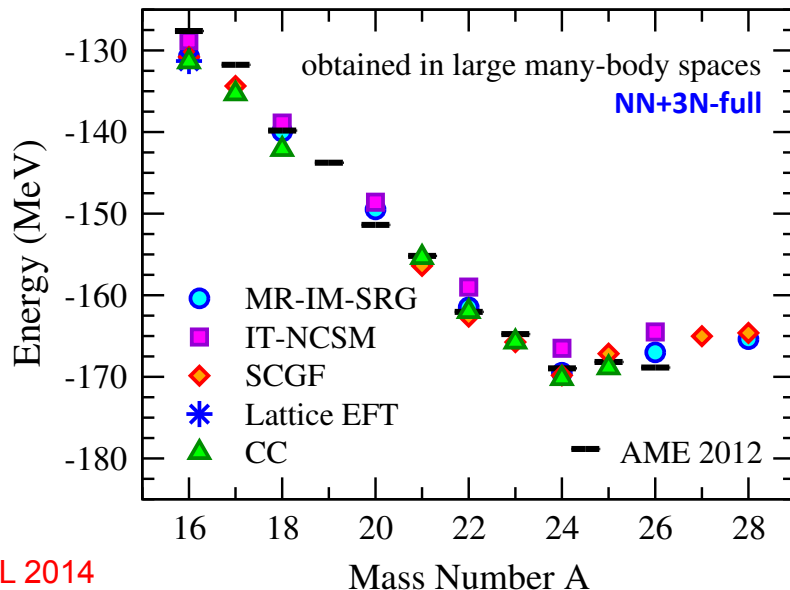
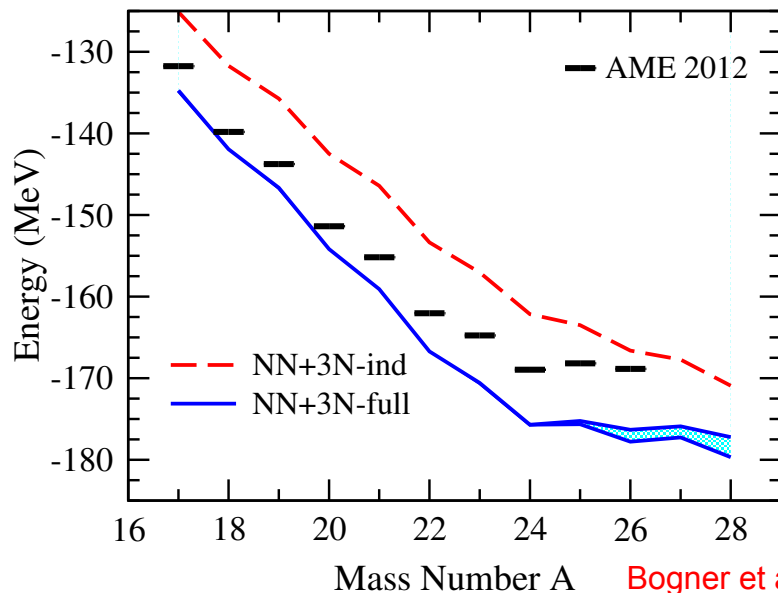
$$\tilde{H} = e^{\Omega} H e^{-\Omega}$$

$$\langle \tilde{\Psi}_n | P \tilde{H} P | \tilde{\Psi}_n \rangle = \langle \Psi_i | H | \Psi_i \rangle$$

$\langle P   H   P \rangle$	$\langle P   H   Q \rangle \rightarrow 0$
$\langle Q   H   P \rangle \rightarrow 0$	$\langle Q   H   Q \rangle$



Large/valence-space methods with **same SRG-evolved NN+3N-full forces**

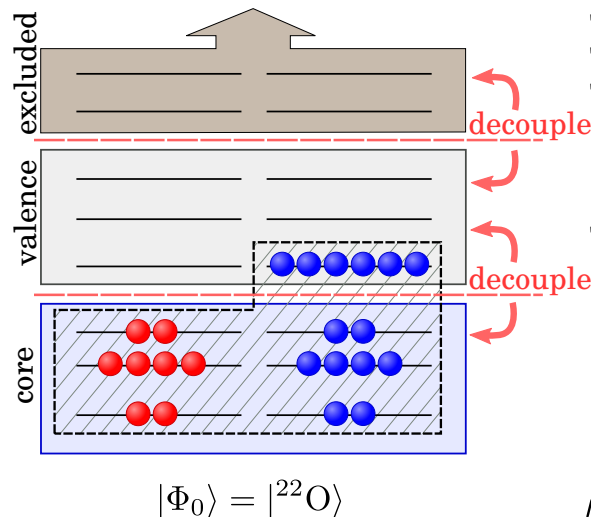
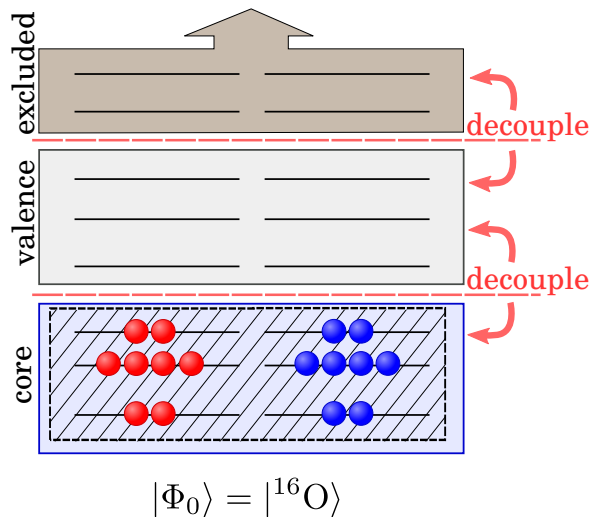


Agreement between all methods with same input forces

Hebeler, JDH, Menéndez, Schwenk, ARNPS 2015

Discrepancy between valence/large-space results

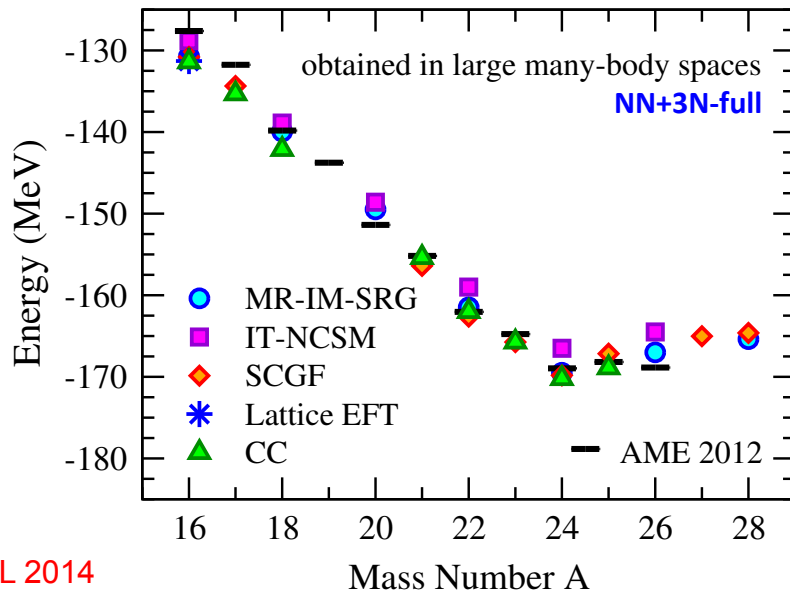
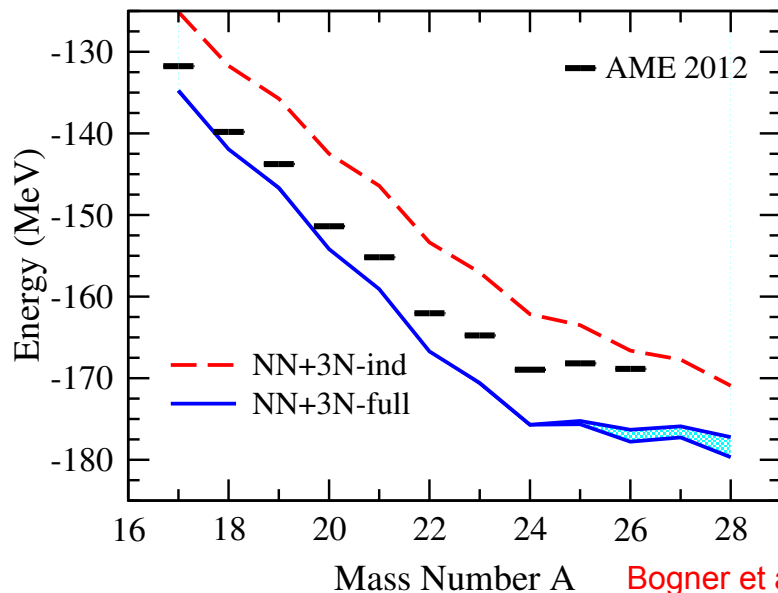
With more valence nucleons, new reference becomes more accurate



**Targeted Normal Ordering:** take nearest closed shell as new reference

Still decouple *sd* valence space in IMSRG

Large/valence-space methods with **same SRG-evolved NN+3N-full forces**

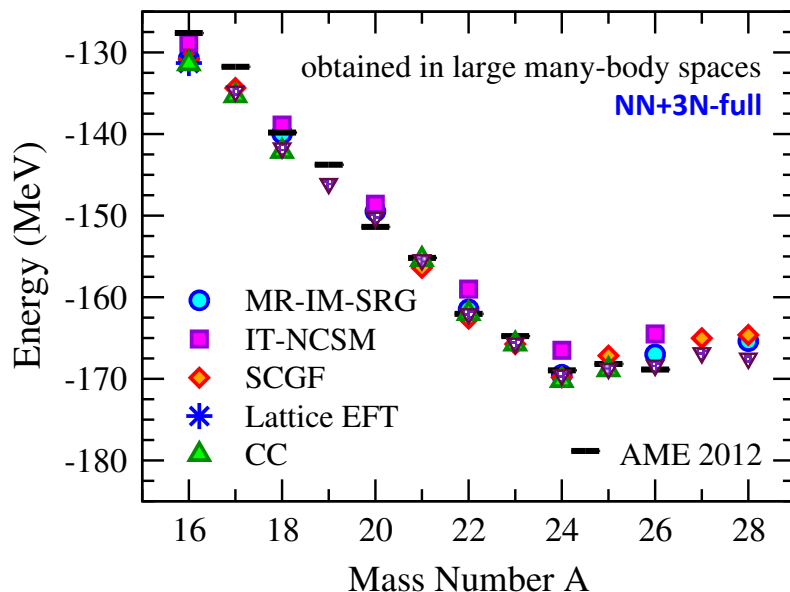
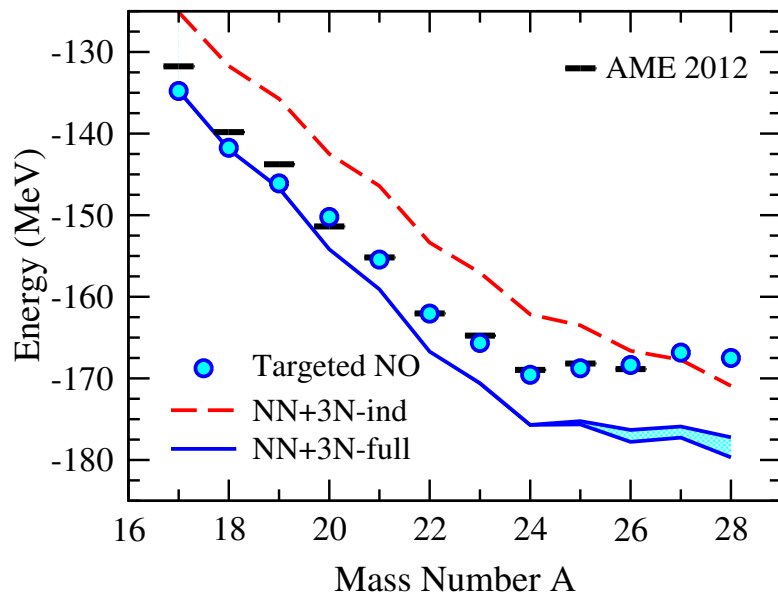


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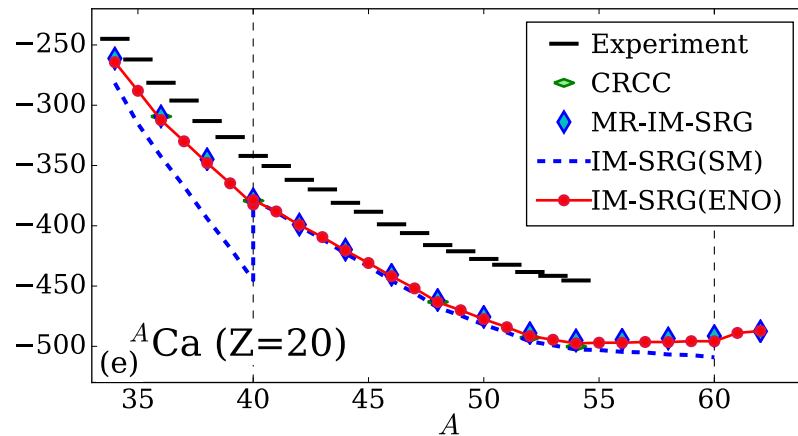
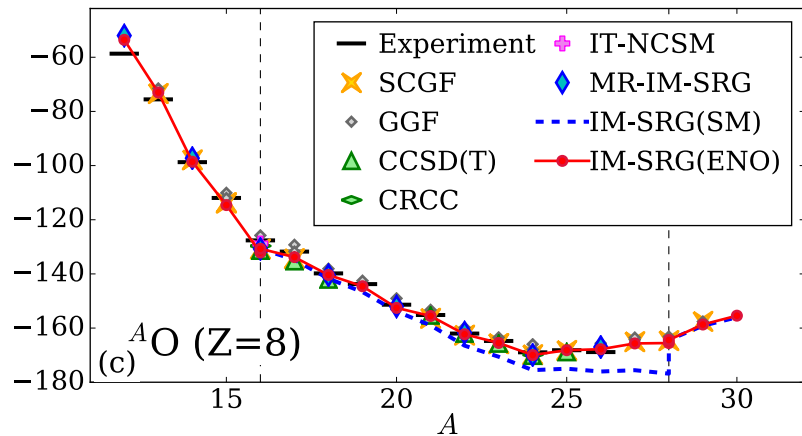
Agreement between all methods with same input forces

Hebeler, JDH, Menéndez, Schwenk, ARNPS 2015

Capture 3N forces between valence nucleons

**“Targeted normal ordering” results agree well with large-space methods**

**Targeted valence space agrees to 1% with all large-space methods** (where calculations exist)



Stroberg et al., arXiv:1607.03229

Extend beyond standard  $sd/pf$  shells

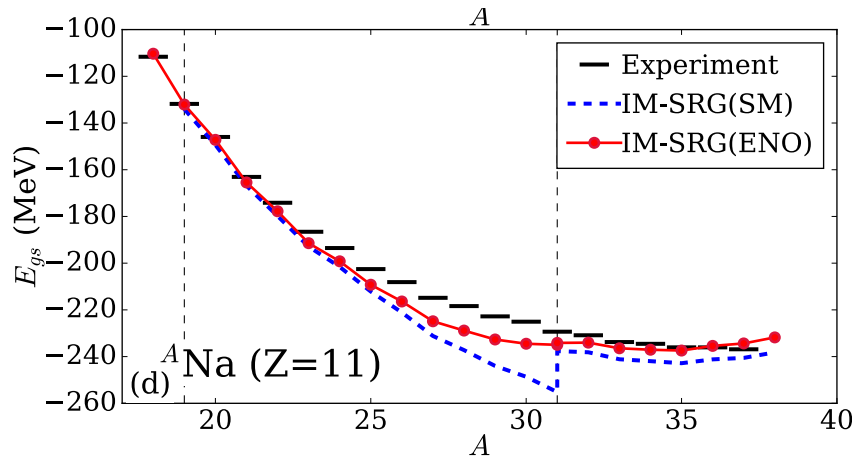
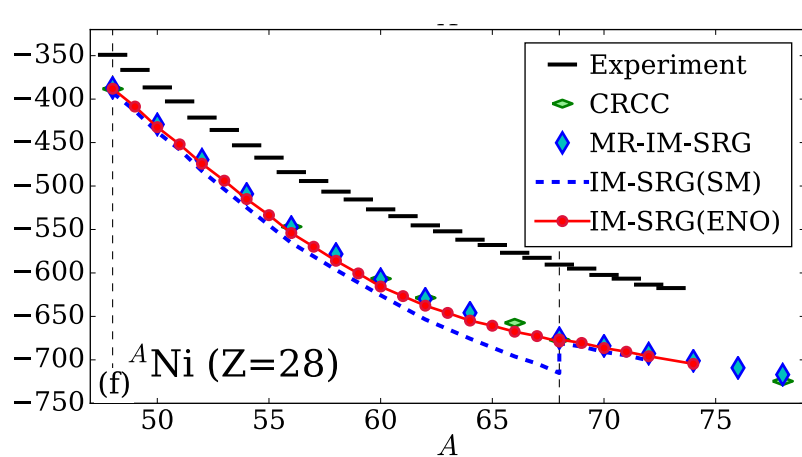
Agreement with experiment deteriorates for heavy chains (due to input Hamiltonian)

Significant gain in applicability with little/no sacrifice in accuracy

**Low computational cost:**  $\sim 1$  node-day/nucleus



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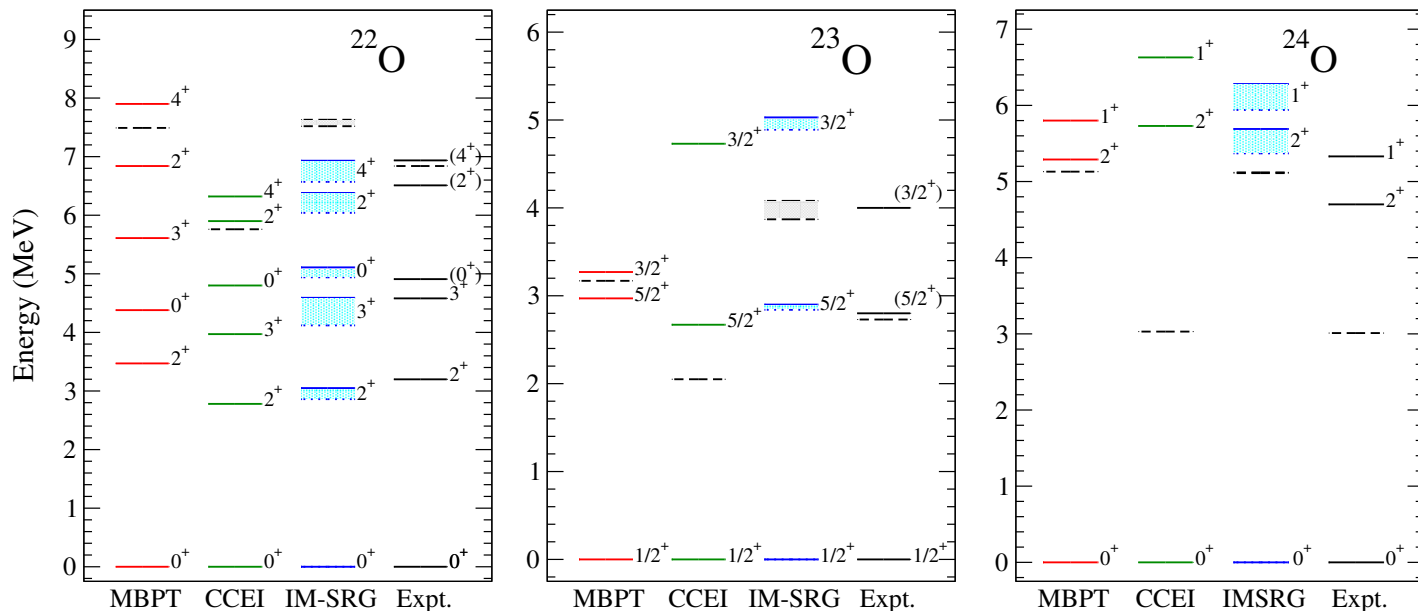
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## Neutron-rich oxygen spectra from existing shell-model approaches

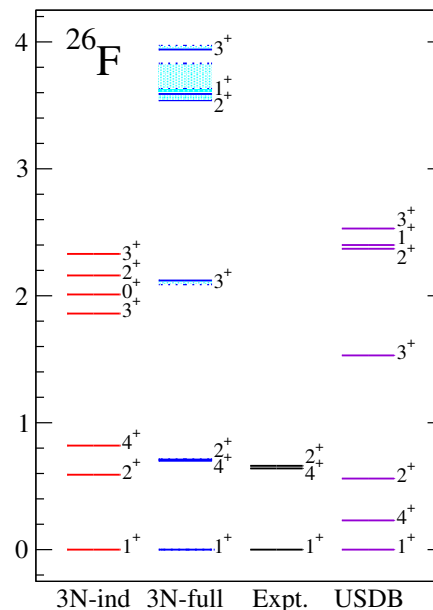
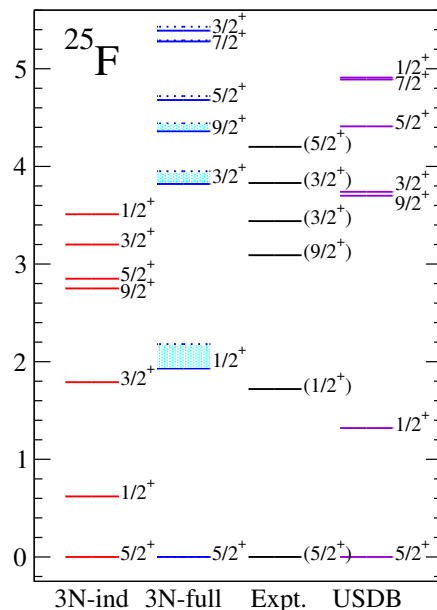
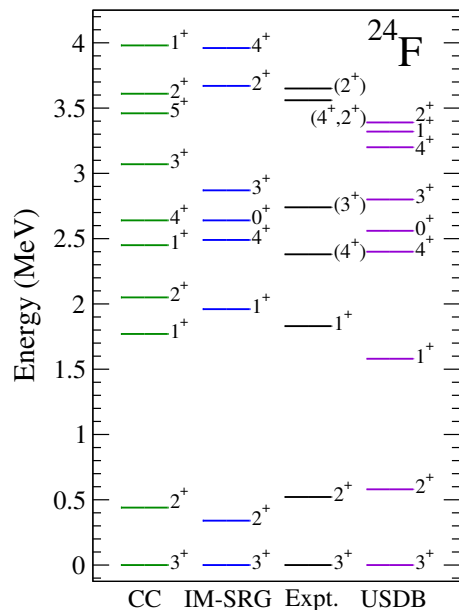


Hebeler, JDH, Menéndez, Schwenk, ARNPS 2015

**MBPT** in extended valence space

**IM-SRG/CCEI** spectra agree within  $\sim 300$  keV

Fluorine spectroscopy: **NN+3N-ind** and **NN+3N-full**, **Full CC**

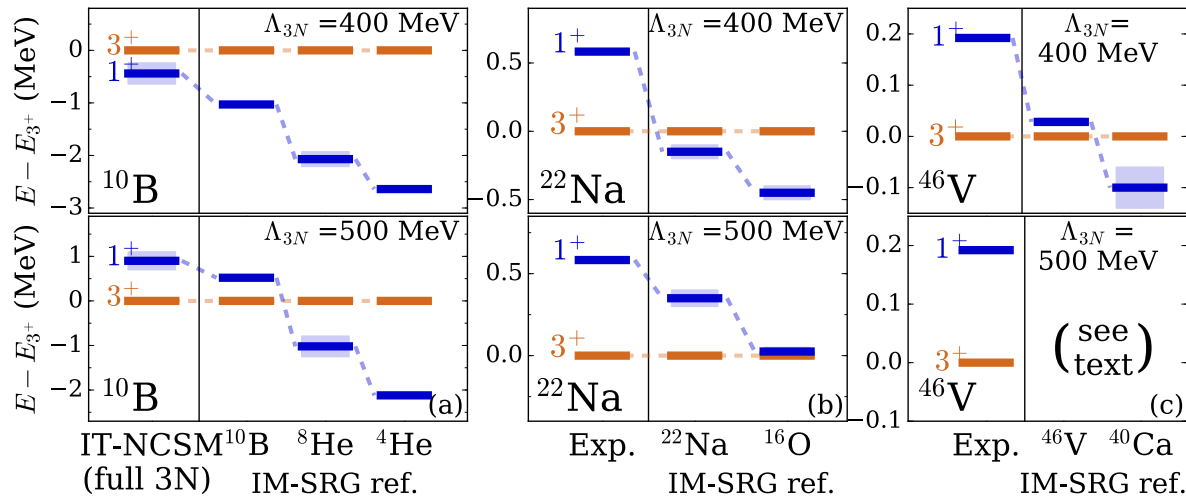
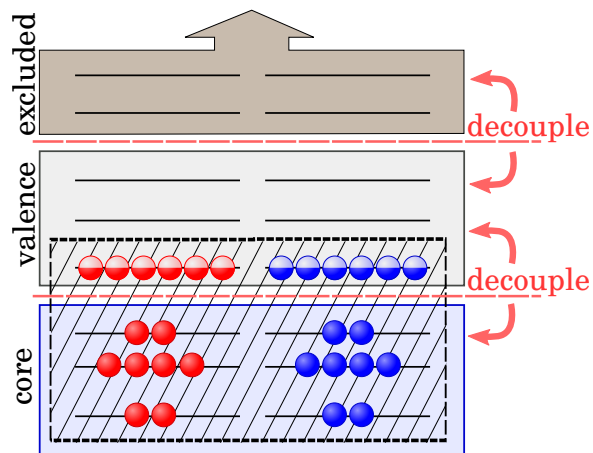


Stroberg et al., PRC 2016

IMSRG: **competitive with phenomenology**, good agreement with data

Long-standing puzzle: 3p+3n above  $^{16}\text{O}/^{40}\text{Ca}$ : same  $1^+/3^+$  inversion as in  $^{10}\text{B}$

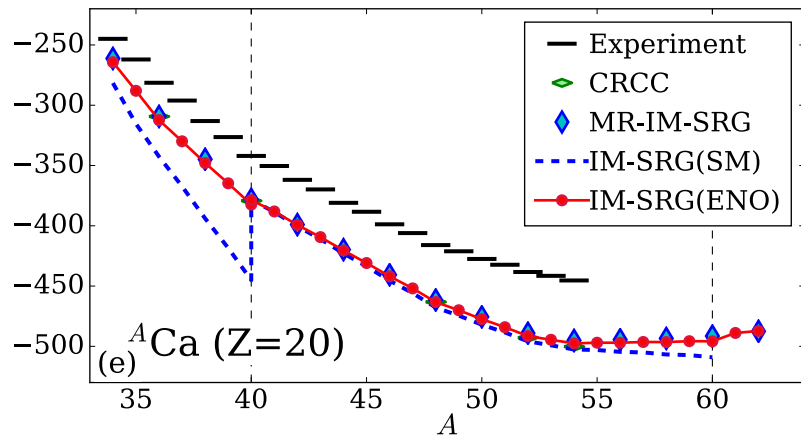
Clear improvement with targeted valence space approach – agreement with NCSM for  $^{10}\text{B}$



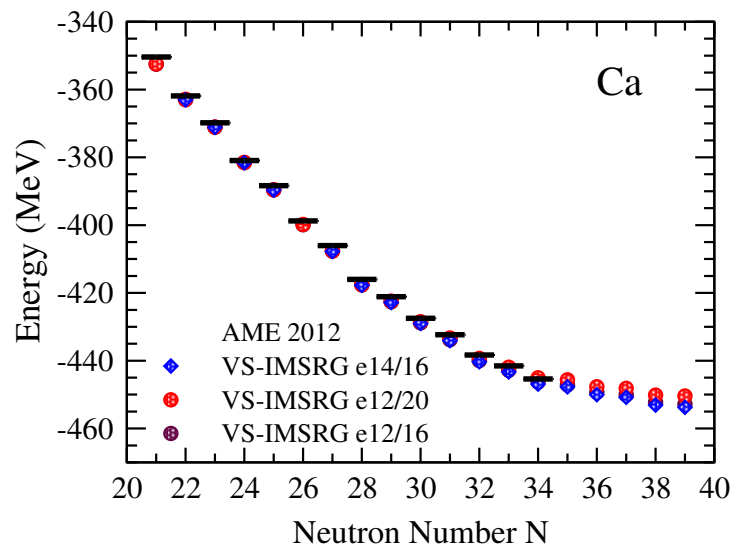
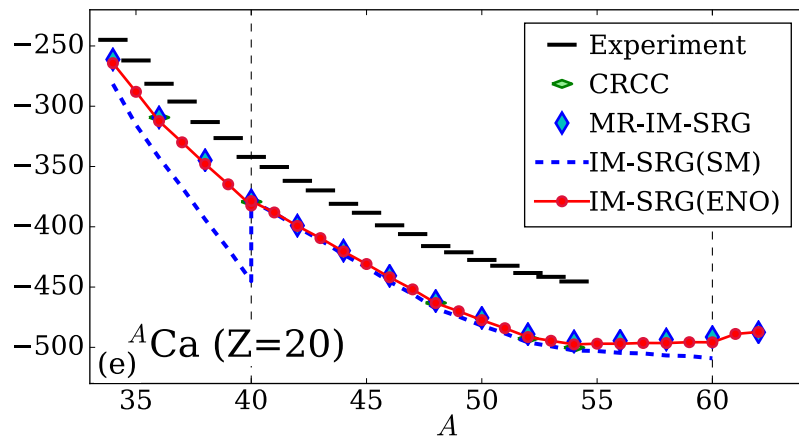
Stroberg et al., arXiv:1607.03229

Similar improvement in medium mass: first ab initio prediction of  $3^+/1^+$  ordering in  $^{22}\text{Na}$ ,  $^{46}\text{V}$

## New input NN+3N forces which reproduce saturation



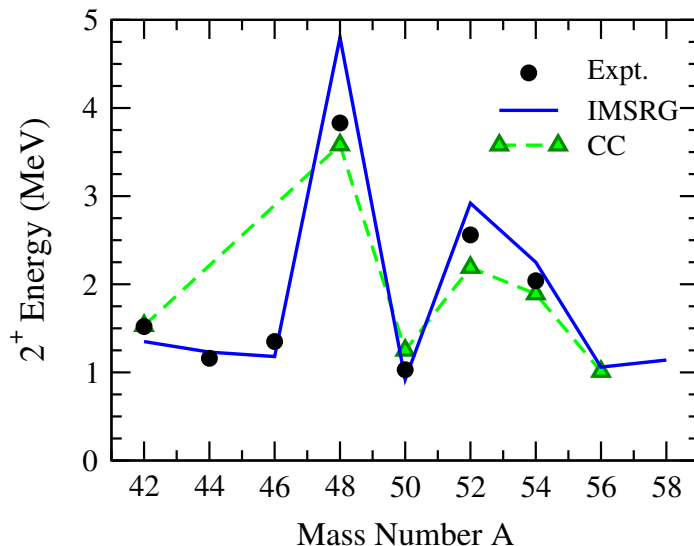
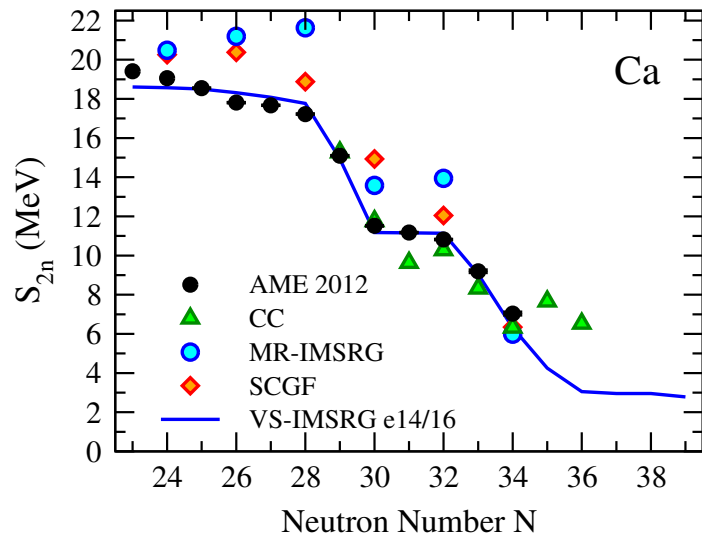
## New input NN+3N forces which reproduce saturation



Find remarkable improvement with respect to experimental data



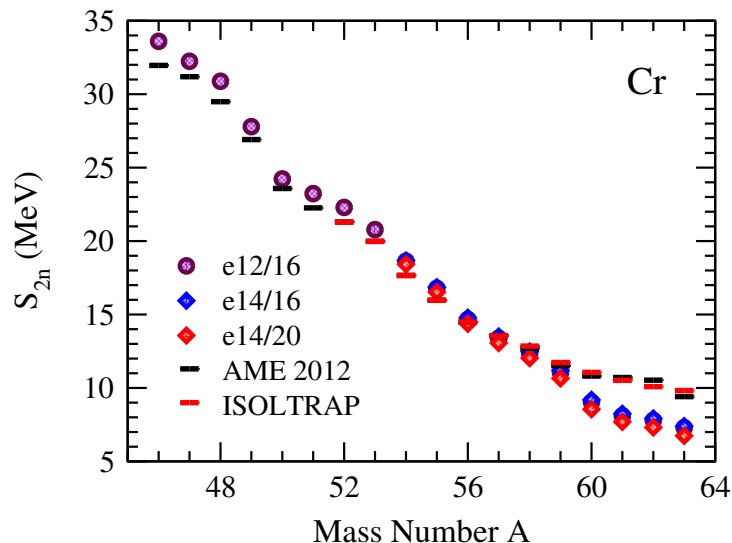
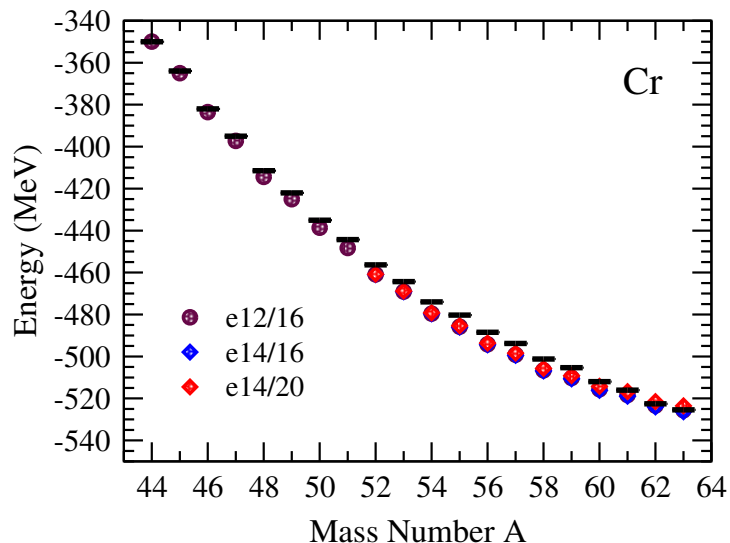
## New input NN+3N forces which reproduce saturation



Find remarkable improvement with respect to experimental data

New ab initio predictions for shell closures in neutron-rich Ca

## New input NN+3N forces which reproduce saturation



Find remarkable improvement for experimental data

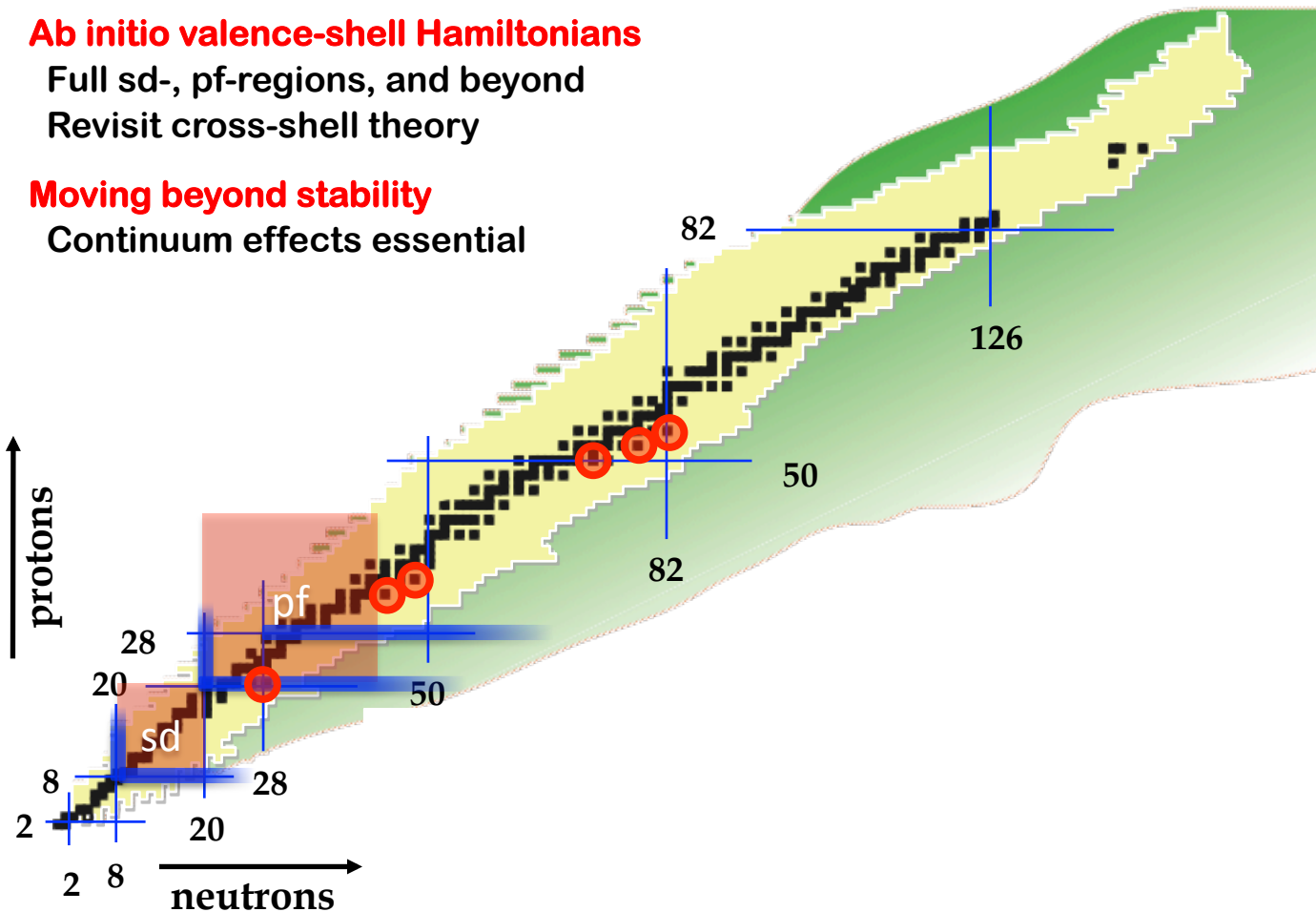
New ab initio predictions in Cr isotopes – compares well with new ISOLTRAP data

## Ab initio valence-shell Hamiltonians

Full sd-, pf-regions, and beyond  
Revisit cross-shell theory

## Moving beyond stability

Continuum effects essential

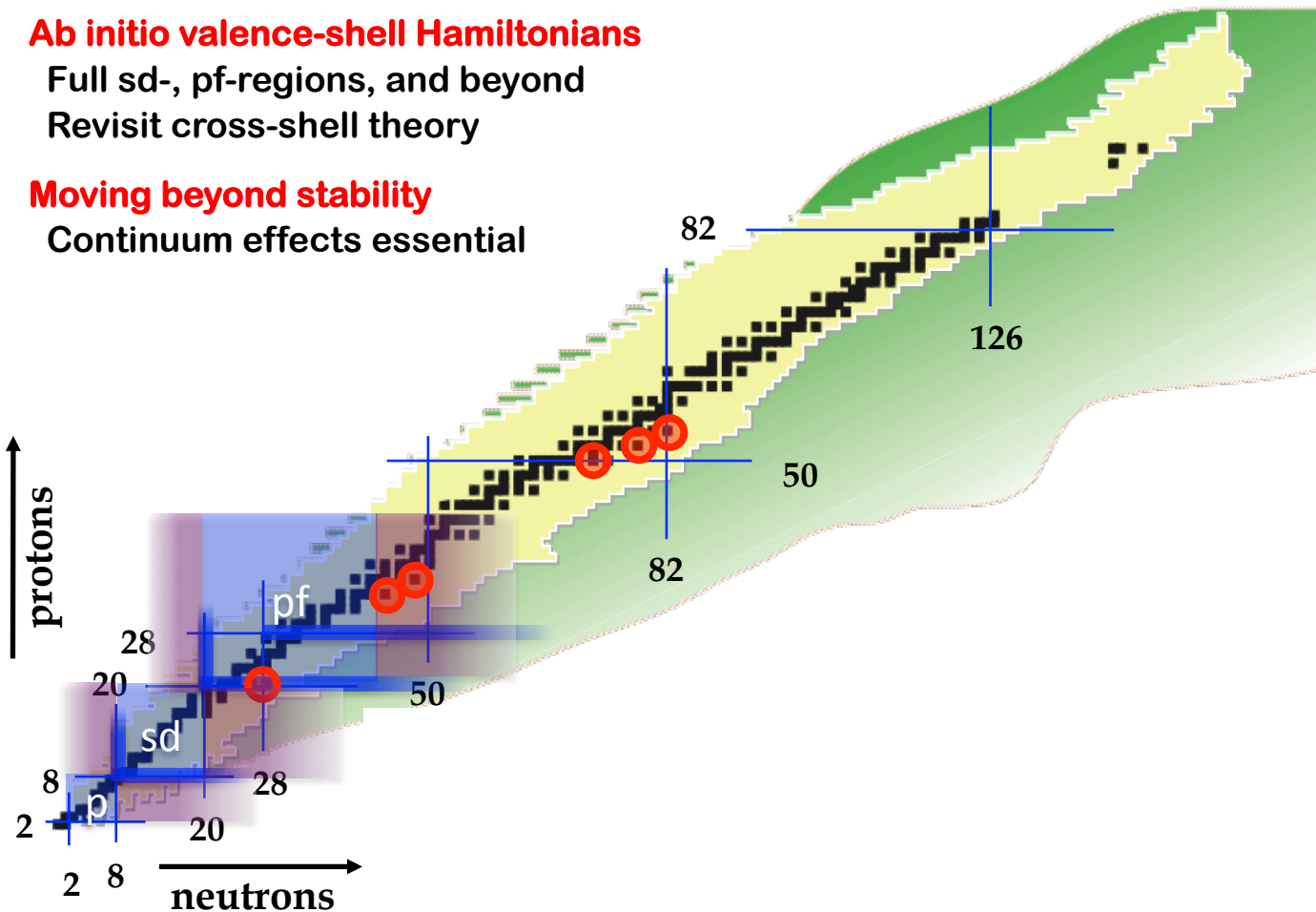


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## Fundamental physics

Effective electroweak operators underway

Effective  $0\nu\beta\beta$  decay operator

Superaligned  $\beta$  decay

Dark-matter scattering

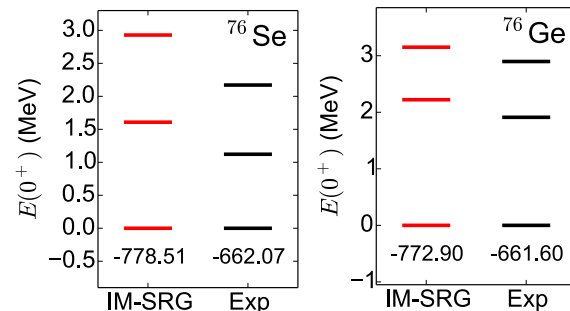
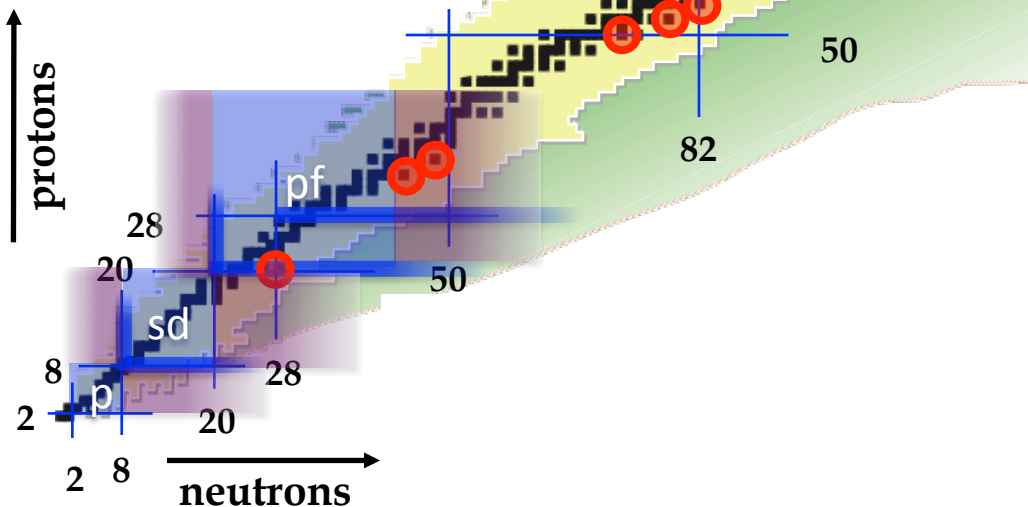
## Path to ab initio $^{76}\text{Ge}$ NME

Benchmark with large-space for  $^{48}\text{Ca}$  ( $2\nu\beta\beta$ )

Multiple predictions for  $0\nu\beta\beta$  in  $^{48}\text{Ca}$

Valence-space IMSRG calculation of  $^{76}\text{Ge}$

Quantify uncertainties



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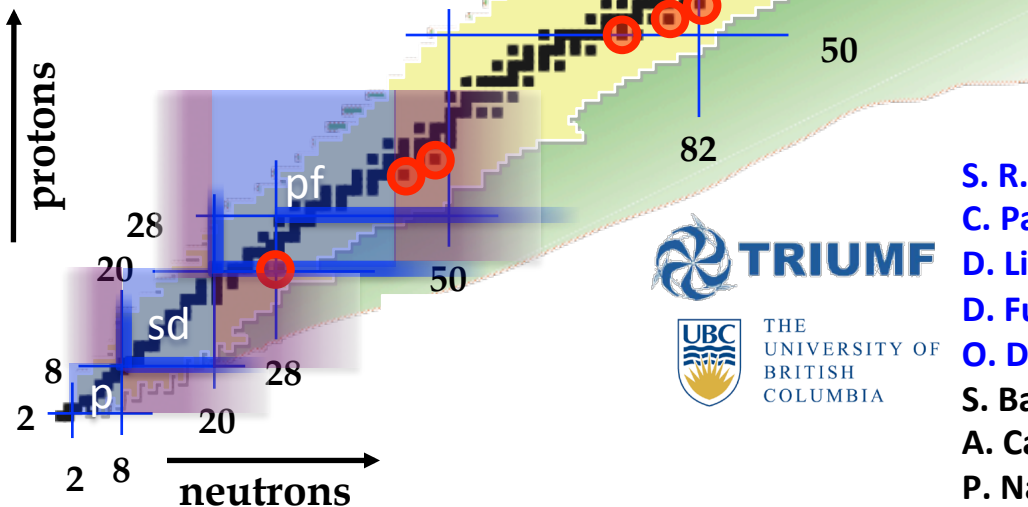
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S. R. Stroberg  
C. Payne  
D. Livermore  
D. Fullerton  
O. Drozdowski  
S. Bacca  
A. Calci  
P. Navrátil



H. Hergert  
S. Bogner  
T. Morris  
N. Parzuchowski



J. Simonis  
A. Schwenk



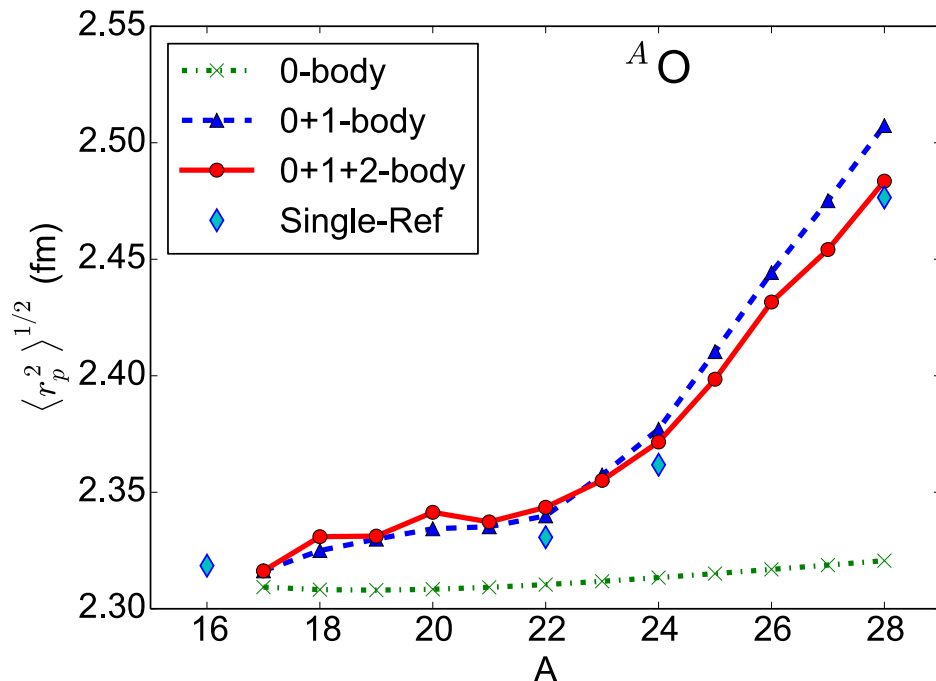
東京大学  
THE UNIVERSITY OF TOKYO

J. Menéndez



General **scalar operators** developed for valence-space IMSRG

$$\tilde{R}^2 = UR^2U^\dagger \quad \langle R^2 \rangle = \langle \Phi_0 | \tilde{R}^2 | \Phi_0 \rangle + \langle \Phi_{\text{SM}} | \tilde{R}^2 | \Phi_{\text{SM}} \rangle$$

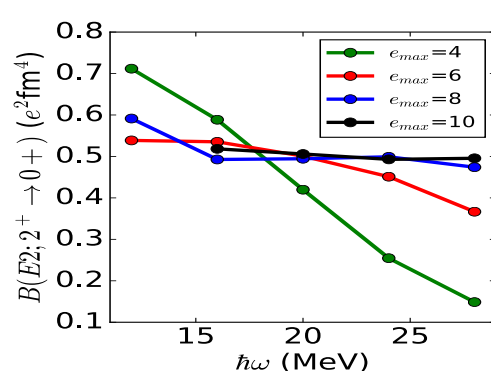
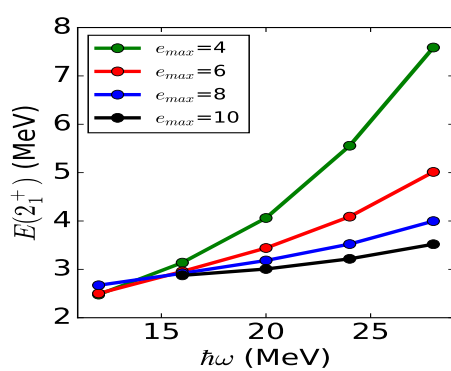
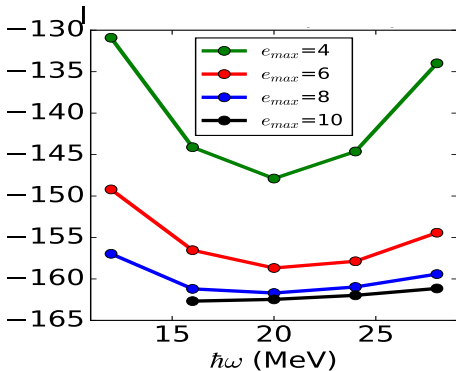
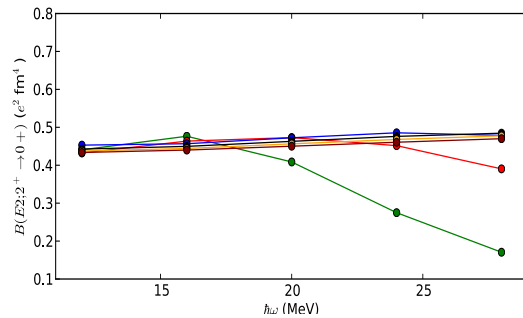
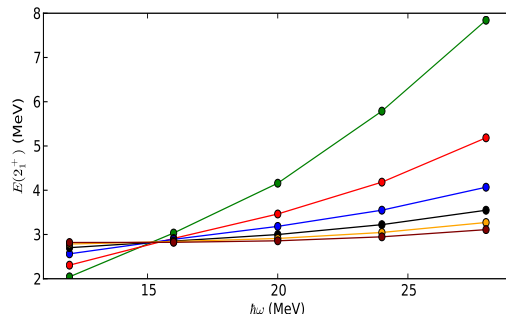
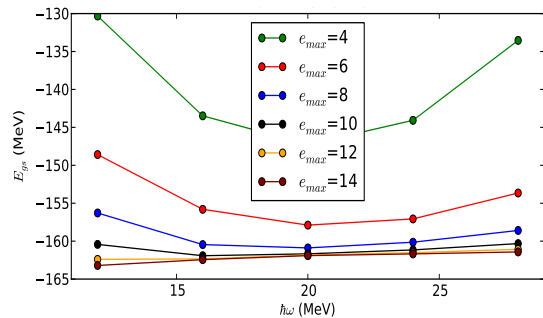


Agreement with SR-IMSRG; two-body contribution minor

General **one-body tensor operators** developed for valence-space IMSRG

$$\tilde{\mathcal{O}} = e^{\Omega} \mathcal{O} e^{-\Omega} = \mathcal{O} + [\Omega, \mathcal{O}] + [\Omega, [\Omega, \mathcal{O}]] + \dots$$

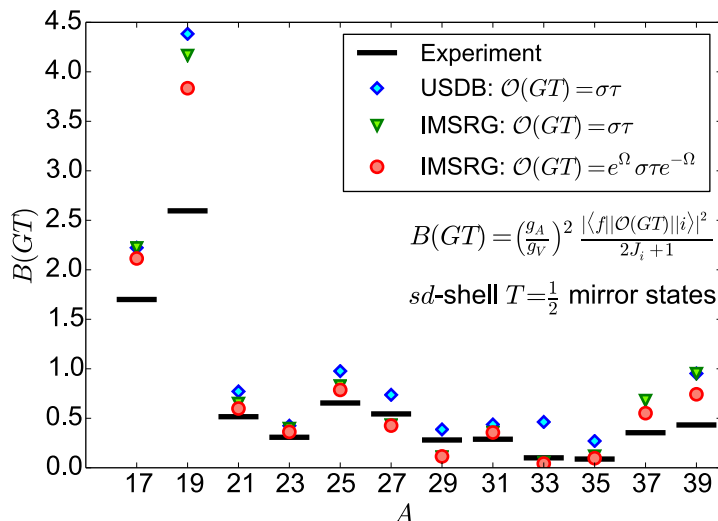
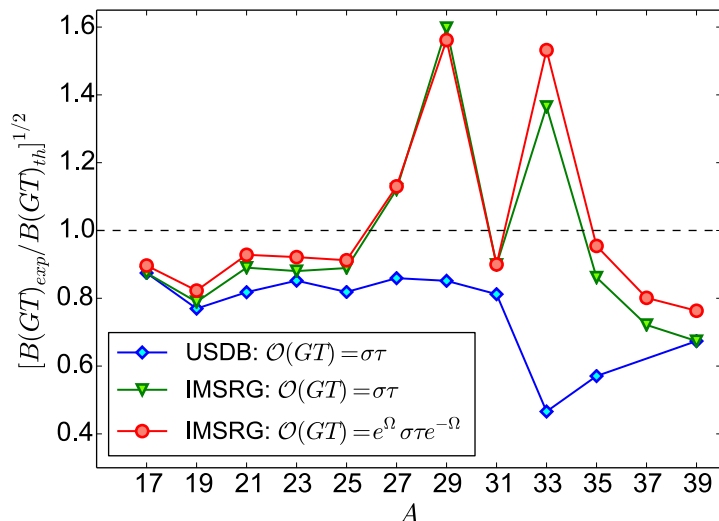
Parzuchowski, Stroberg et al., in prep



Agreement with EOM-IMSRG; benchmarks also underway with EOM Coupled-Cluster

General **one-body tensor operators** developed for valence-space IMSRG: Gamow-Teller

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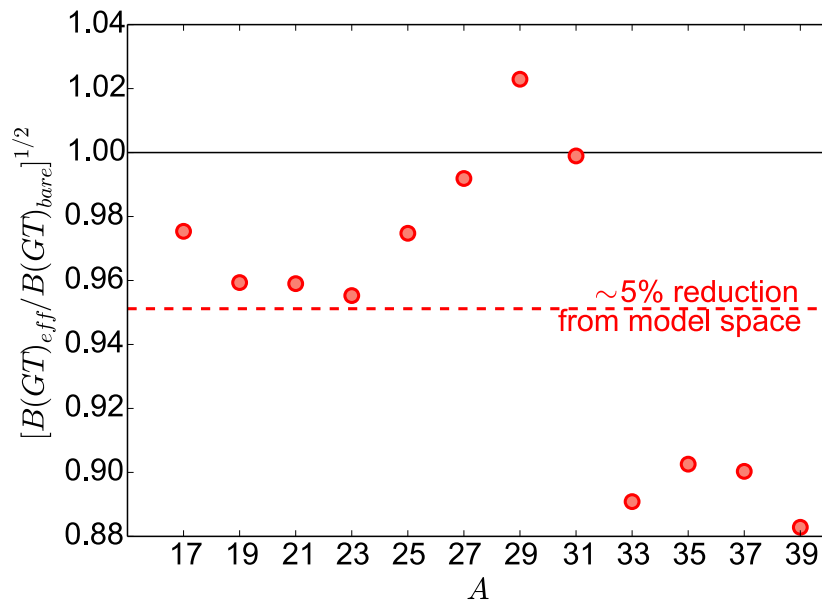
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**First ab initio valence-space calculations of GT transition rates**

Small renormalization effect, but (mostly) reasonable agreement with experiment

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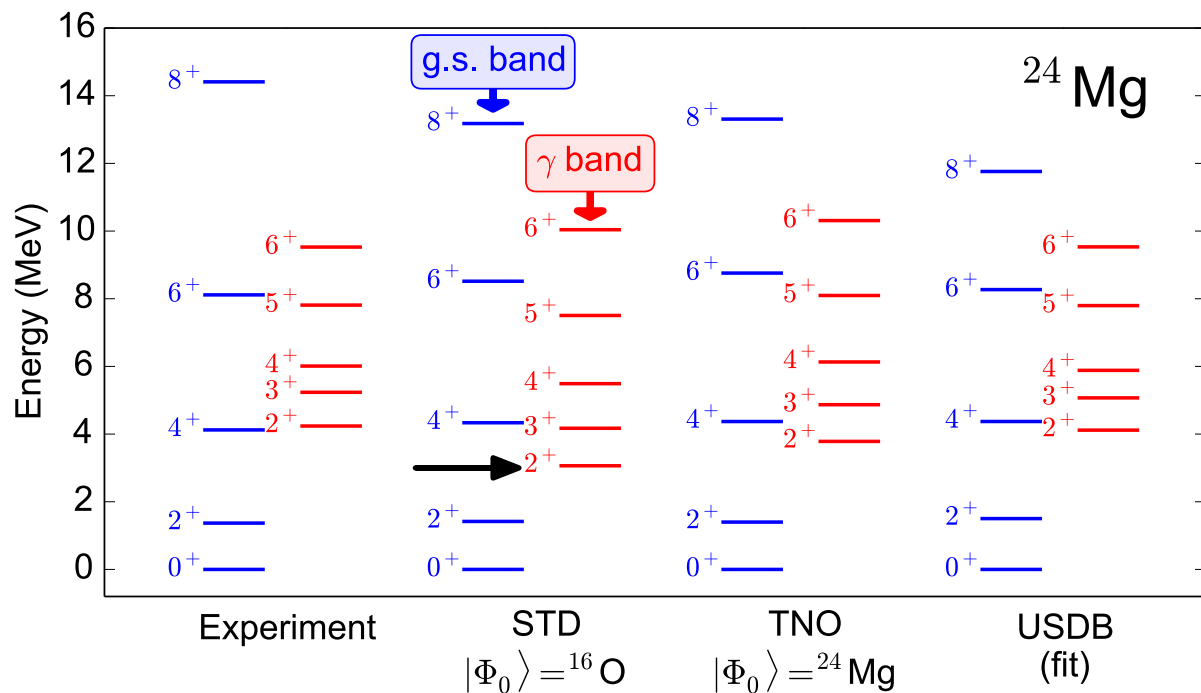
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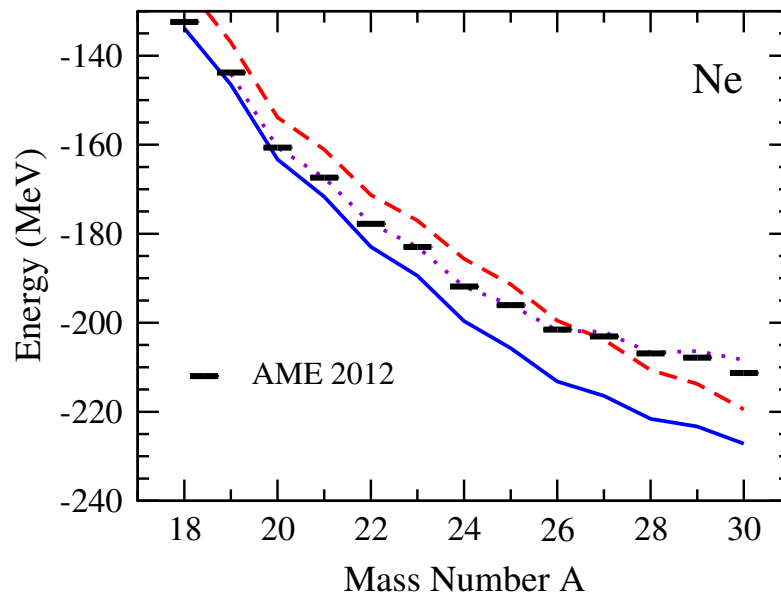
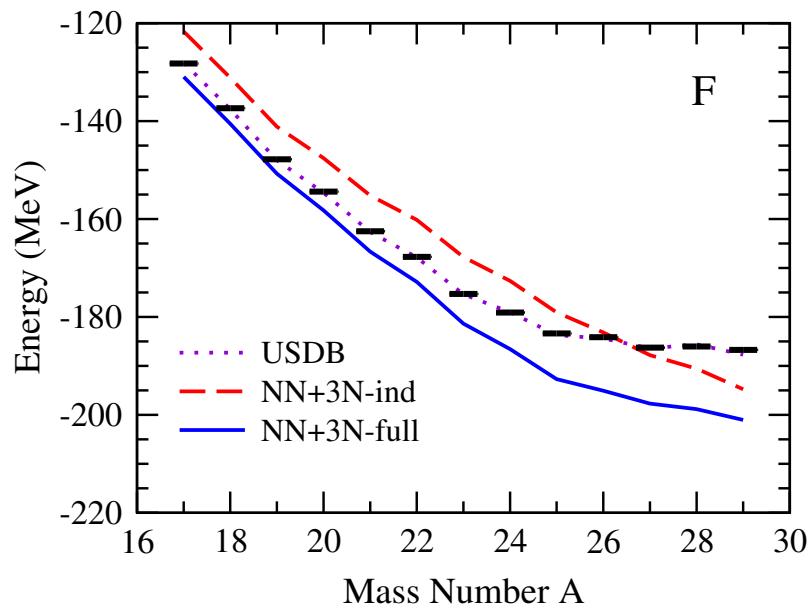
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Prediction of ground-state and gamma bands

Compare with phenomenology in *sd*-shell nuclei



## Valence-space IMSRG results for open-shell fluorine and neon isotopes

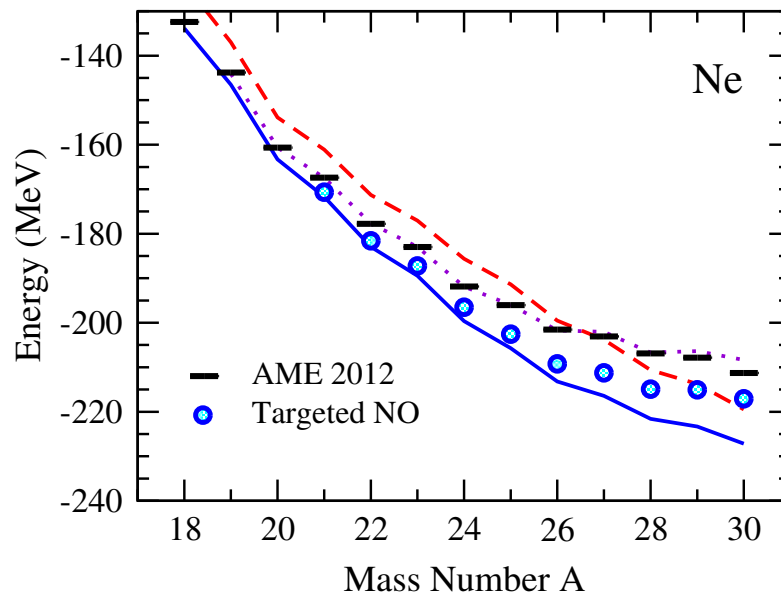
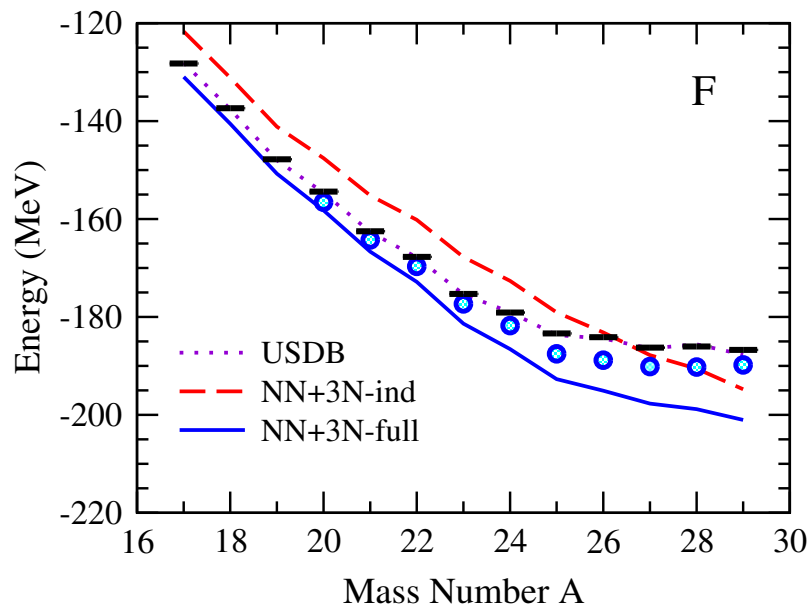


3N forces improve experimental agreement; significant overbinding

Stroberg et al., PRC 2016



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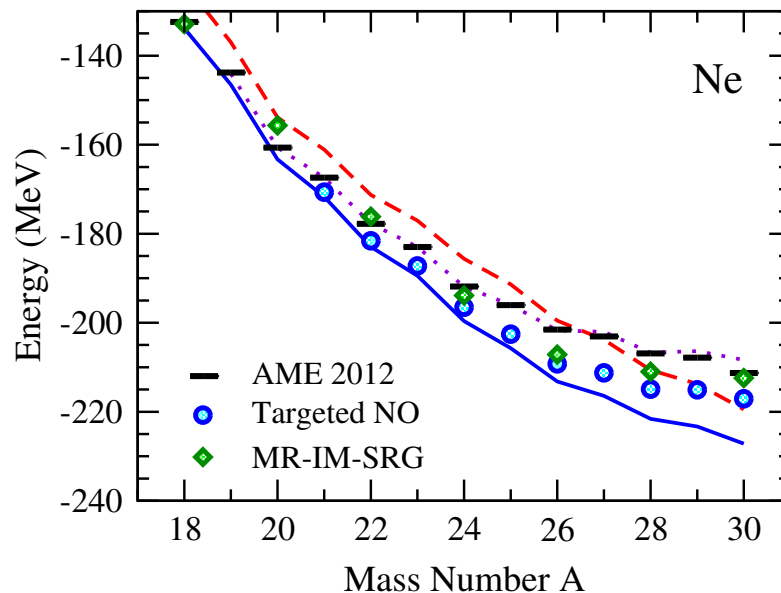
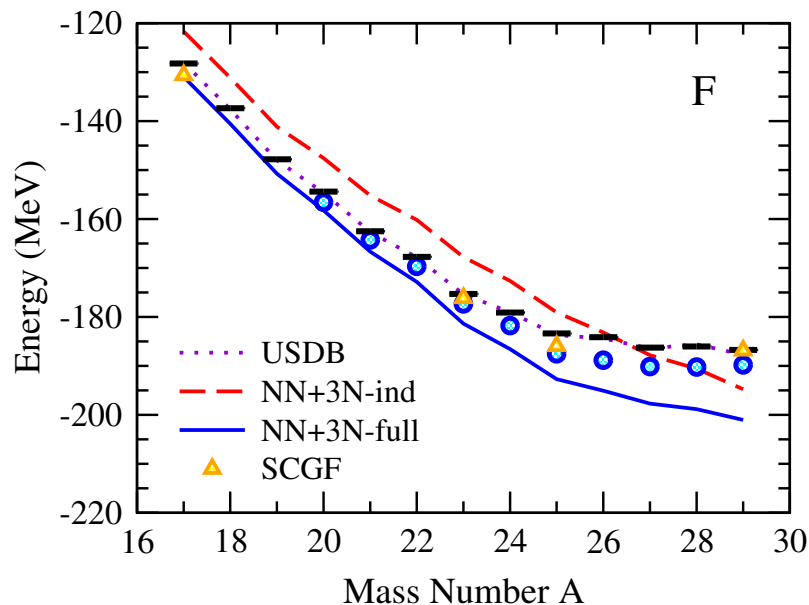


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**Further improvement from Targeted Normal Ordering**

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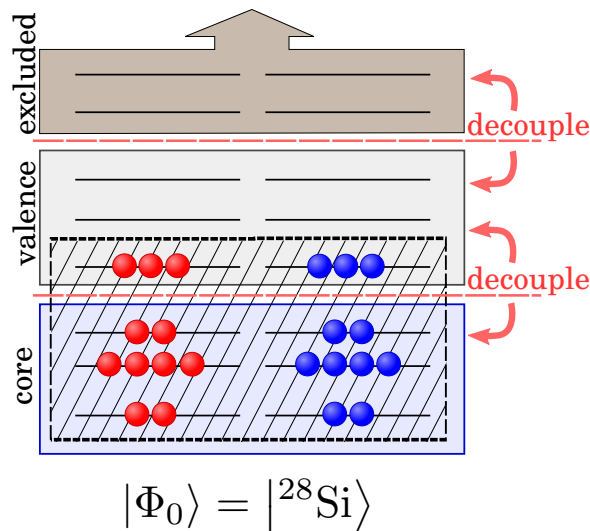
Stroberg et al., PRC 2016

Further improvement from Targeted Normal Ordering

**Minor loss in accuracy compared to SCGF and MR-IMSRG**

Long-standing puzzle:  $3p+3n$  from  $^{16}\text{O}/^{40}\text{Ca}$ , same  $1^+/3^+$  ground-state inversion as in  $^{10}\text{B}$

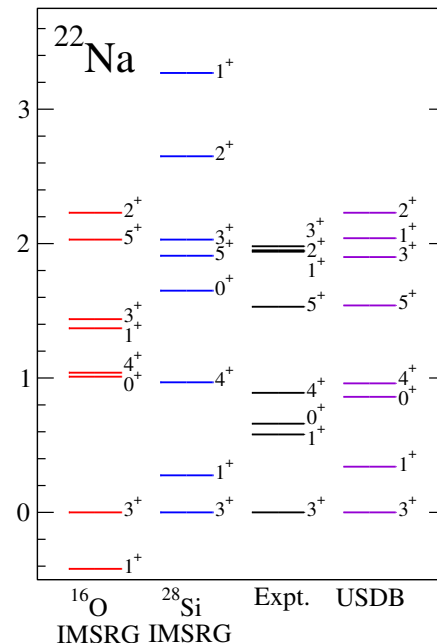
With 3N forces ab initio valence space (IMSRG, CCEI) still incorrect ground state



A fundamental energy observable in principle does not in fact, it implies that a large cutoff, with no offers the possibility of freedom. This decoupling to handle similar problems. The general purpose by David Gross [63]:

<sup>6</sup> For an early discussion

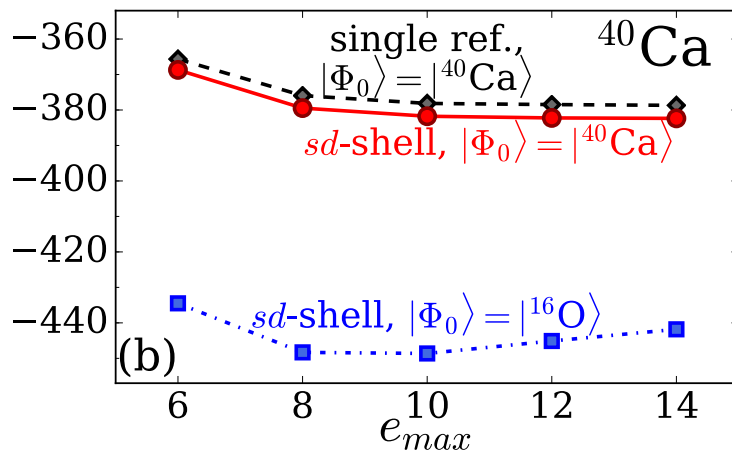
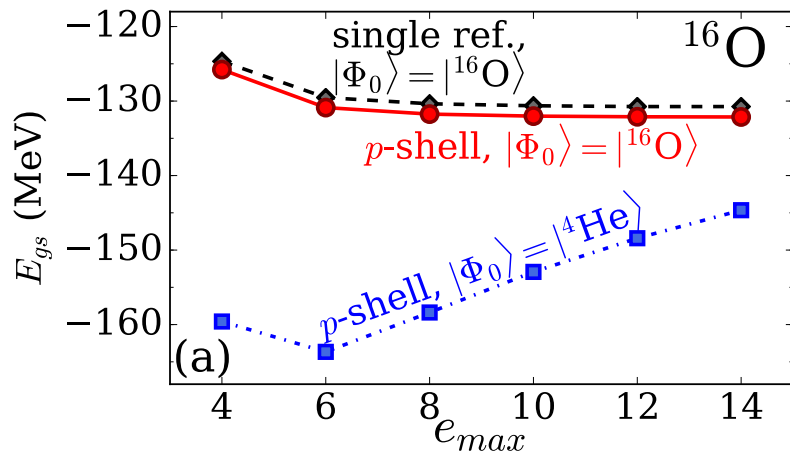
Navrátil, PRL (2007)



Far from closed shell –  $^{28}\text{Si}$  reference overestimates 3N

Results not converged with standard core reference

ENO converges as expected – small difference from single-reference



Use ensemble state as new reference, defined by the density matrix

$$\rho = \sum_i \alpha_i |\Phi_i\rangle \langle \Phi_i| \quad \langle \mathcal{O} \rangle = \text{Tr}(\rho \mathcal{O}) \implies \sum_p \text{Tr}(\rho a_p^\dagger a_p) = A$$

New definition of normal ordering:

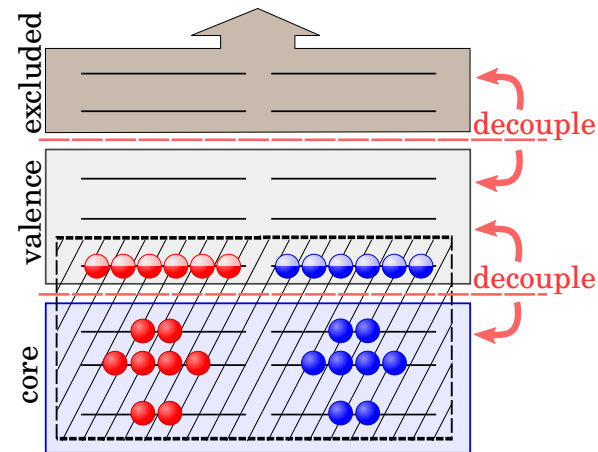
$$\text{Tr}(\rho \{a_1^\dagger \dots a_N\}) = \sum_i \alpha_i \langle \Phi_i | \{a_1^\dagger \dots a_N\} | \Phi_i \rangle = 0$$

And Wick contraction

$$\{\overline{a_p^\dagger a_q}\} = \sum_\alpha c_\alpha \langle \Phi_\alpha | a_p^\dagger a_q | \Phi_\alpha \rangle \equiv n_p \delta_{pq}$$

Can have fractional occupations

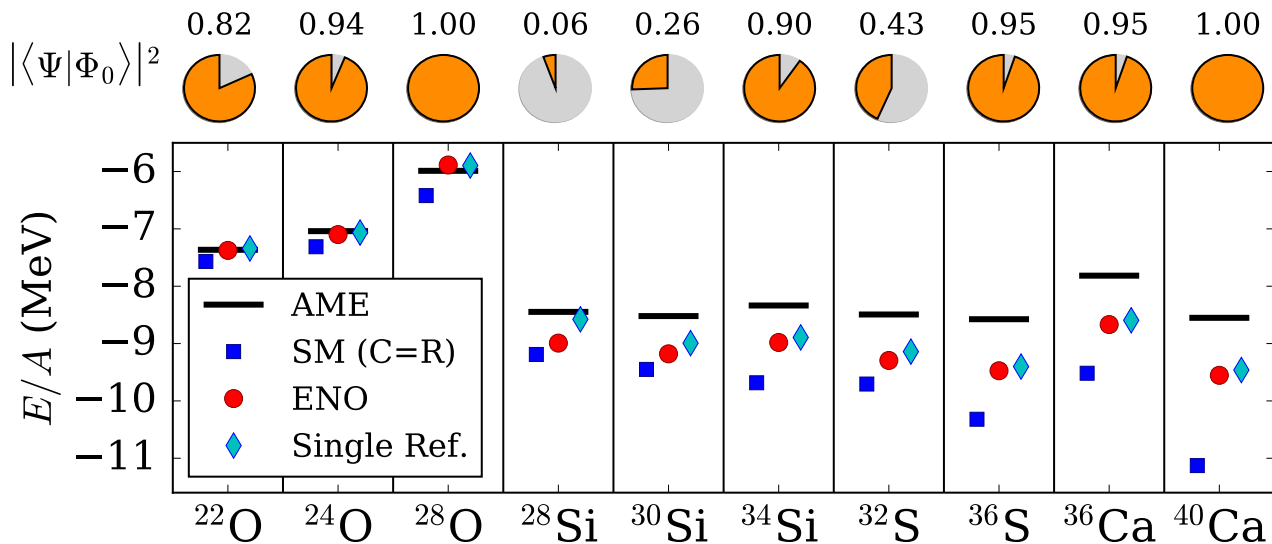
**No N-representability problem!**



$$\rho = \frac{1}{2} |^{16}\text{O}\rangle \langle ^{16}\text{O}| + \frac{1}{2} |^{28}\text{Si}\rangle \langle ^{28}\text{Si}|$$

Benchmark against SR-IMSRG results for closed sd-shell nuclei

Error from using core as reference grows far from core



Stroberg et al., arXiv:1607.03229

Targeted NO finds good agreement with SR-IMSRG

Experimental discrepancies due to deficiencies in initial Hamiltonians