The Strange and Light Quark Magnetic Moments of the $\Lambda(1405)$ Antikaon-nucleon Molecule

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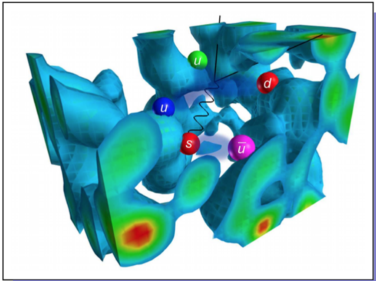

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Overview

- The strange-quark sector of the $\Lambda(1405)$
    - Lattice QCD results
    - Hamiltonian effective field theory (HEFT)
- The light-quark sector of the $\Lambda(1405)$
    - Loop contributions to magnetic moment
    - The graded symmetry approach
- Summary of the molecular nature of the $\Lambda(1405)$

The strange-quark sector of the $\Lambda(1405)$

The $\Lambda(1405)$

- The $\Lambda(1405)$ is an odd-parity hyperon, but is lighter than any other excited spin-1/2 baryon.
- Its unexpected position in the spectrum renders its structure mysterious. Descriptions range from
  - pure meson-baryon scattering, e.g. **arXiv:1301.5741**,
  - pure three-quark state (*uds*), e.g. **arXiv:1109.6716**,
  - or a combination of both, e.g. **arXiv:1411.3402**.
- Until recently, there has not been a convincing resolution to the structure of the $\Lambda(1405)$ that describes both scattering experiments and lattice QCD well.

The $\Lambda(1405)$ in lattice QCD

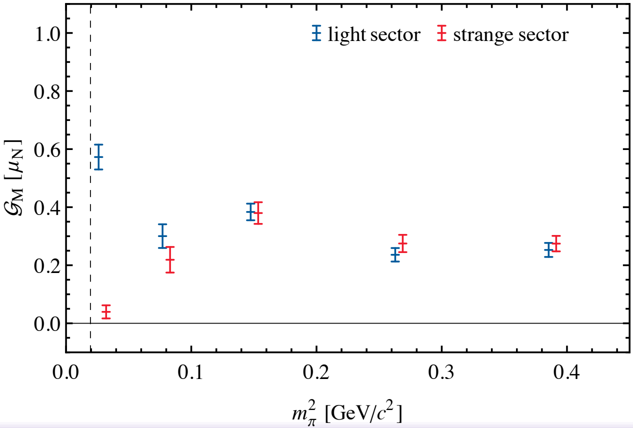
- It has recently emerged from lattice QCD that a component of the $\Lambda(1405)$ must be a bare three-quark state near the physical point, which becomes the dominant contribution at large pion mass **arXiv:1512.05831**, **arXiv:1607.05856**.

- Lattice QCD shows that the strange magnetic moment vanishes near the physical pion mass.
- This suggests there is a strong $\overline{K}N$ bound-state contribution to the $\Lambda(1405)$, since the kaon is spin 0 and in an S wave, and cannot contribute to the moment!

- Hamiltonian effective field theory (HEFT) analysis of the interacting states, as a function of pion mass, arrives at the same conclusion.

Lattice QCD results

- The strange magnetic form factor of the Λ(1405) vanishes near the physical pion mass.



Hamiltonian effective field theory

- The matrix Hamiltonian model was developed for analysing scattering states on the lattice as an alternative to Lüscher's method.
- The free parameters of the model (related to the low-energy coefficients of the chiral expansion) can be fit directly to lattice results $\longrightarrow$ more stable estimation of renormalised masses, decay widths, etc.
- The matrix Hamiltonian model is more straightforwardly generalisable to multi-channel interactions,
- It has the ability to provide information on interpreting the output spectrum of states at any quark-mass/volume.

Hamiltonian effective field theory

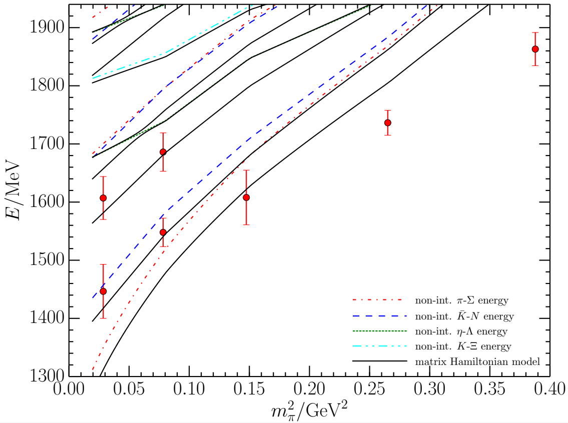
- The finite-volume Hamiltonian includes the interactions between a bare Λ(1405) mass, $m_{\rm bare} = m_0 + \alpha_0 m_\pi^2$, and the leading octet states: $\pi\Sigma$, $\overline{K}N$, $K\Xi$, $\eta\Lambda$.

$$H = \begin{pmatrix} m_{\rm bare} & g_{\pi\Sigma}(k_0) & g_{\overline{K}N}(k_0) & g_{K\Xi}(k_0) & g_{\eta\Lambda}(k_0) & \cdots \\ g_{\pi\Sigma}(k_0) & \omega_{\pi\Sigma}(k_0) & 0 & \cdots & & \\ g_{\overline{K}N}(k_0) & 0 & \omega_{\overline{K}N}(k_0) & 0 & \cdots & \\ g_{K\Xi}(k_0) & 0 & 0 & \omega_{K\Xi}(k_0) & & \\ g_{\eta\Lambda}(k_0) & 0 & 0 & 0 & \omega_{\eta\Lambda}(k_0) & \\ \vdots & \vdots & \vdots & \vdots & & \ddots \end{pmatrix}$$

- The non-interacting energies are: $\omega_{MB}(k_n) = \sum_{i=M}^{B} \sqrt{k_n^2 + m_i^2}$.
- The zero cross-terms can be replaced with the appropriate Weinberg-Tomozawa scattering amplitudes.

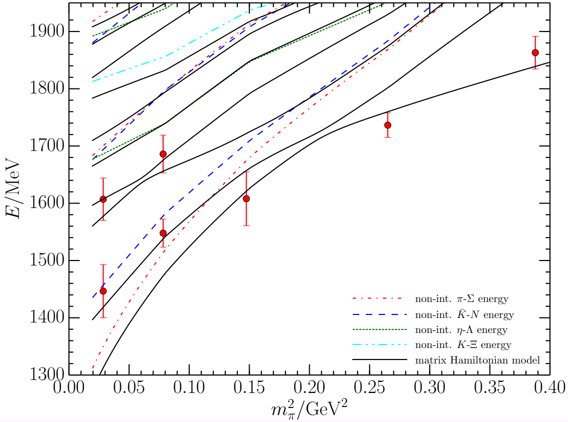
Hamiltonian effective field theory

- The absence of a bare Λ(1405) state means large pion mass results are hard to reproduce.



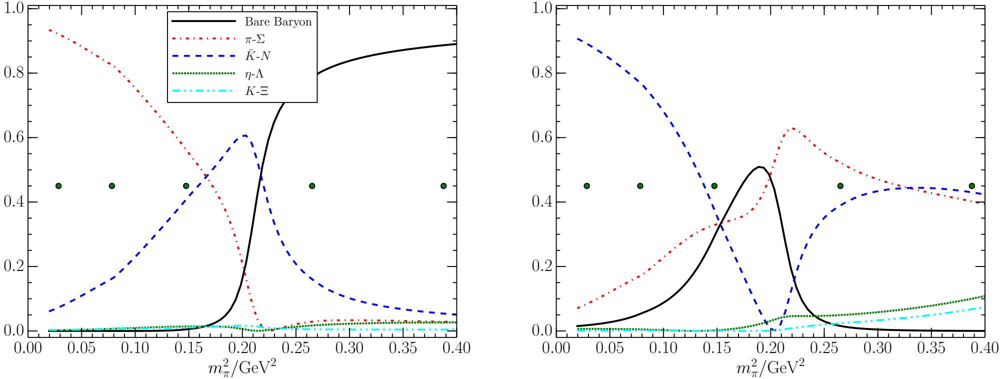
Hamiltonian effective field theory

- The addition of a bare Λ(1405) state means all pion mass results can be reproduced.



Hamiltonian effective field theory

- For the lightest pion mass, the Λ(1405) is dominated by a bound $\overline{K}N$ state.

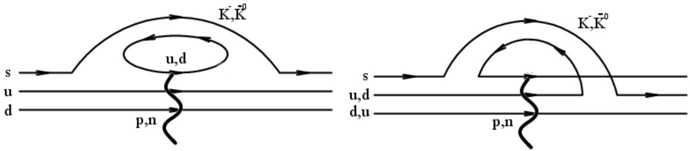


Eigenstate 1 Eigenstate 2

The light-quark sector of the $\Lambda(1405)$

The light-quark sector of the Λ(1405)

- **If** the Λ(1405) is dominated by $\overline{K}N$ loops near the physical pion mass, we expect its magnetic moment, μ, to be strongly related to that of the nucleon.
- We perform a lattice QCD calculation. But we need to account for missing disconnected loops.



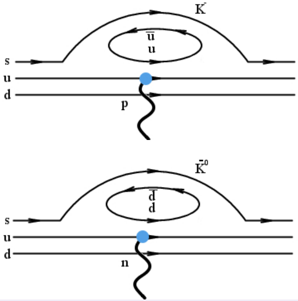
Λ^*

- We consider first a flavour-singlet Λ^*, but we will consider the octet component of the Λ(1405) as well (see **arXiv:1109.6716**).

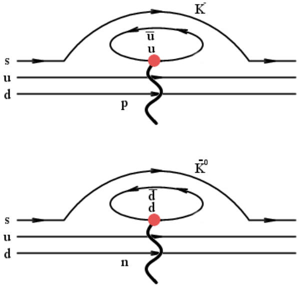
Loop contributions to magnetic moment

- For the disconnected loop contributions in both $\Lambda^* \to K^- p$ and $\Lambda^* \to \overline{K}^0 n$, valence couplings are present, but sea couplings are not.

present in lattice calc.

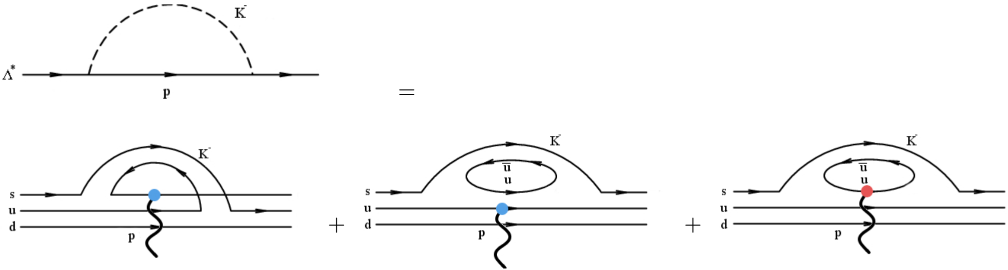


not present in lattice calc.



Loop contributions to magnetic moment

- Therefore, we want the partially quenched coupling, for example, for the process $\Lambda^* \to K^- p$, where



- We need to cancel off the last graph, which represents half the disconnected *u*-sector. But how much of the coupling is in this graph?

The graded symmetry approach

- One can generalise the baryon field from effective field theory, to a three-index tensor, $\mathcal{B}_{ijk}$, which can intuitively isolate the required quark flavour lines.
- The disconnected loops are handled by ghost quark counterparts $(\tilde{u},\ \tilde{d},\ \tilde{s})$ which are added to extract the quenched couplings.
- We extend this formalism to consider the singlet-octet and singlet-singlet components as well **arXiv:1509.08226**. The original octet-octet formalism can be found in **arXiv:9605034**.
- We create an anti-symmetrized field that contains the singlet

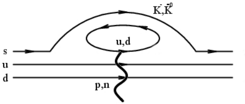
$$\mathcal{B}^S_{ijk} = \frac{1}{3\sqrt{2}}\Big(+ \epsilon_{ijk'} B_{kk'} + \epsilon_{jkk'} B_{ik'} + \epsilon_{kik'} B_{jk'} - \epsilon_{jik'} B_{kk'} - \epsilon_{kjk'} B_{ik'} - \epsilon_{ikk'} B_{jk'} \Big)$$

The graded symmetry approach

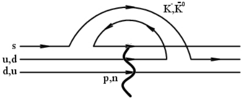
- The interaction Lagrangian takes the following form

$$\mathcal{L}_S = 2\sqrt{2}\, g_s\, (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})}\, \overline{\mathcal{B}}^{S*}_{kji}\gamma_\mu\, A^{\mu}_{kk'}\, \mathcal{B}_{ijk'}$$

- $\Lambda^* \to \tilde{K}^- \tilde{\Lambda}^+_{p,\tilde{u}}$ and $\Lambda^* \to \overline{\tilde{K}}^0 \tilde{\Lambda}^0_{n,\tilde{d}}$ correspond to the disconnected loop contributions, each with a strength of $\frac{2}{3} g_s^2$.



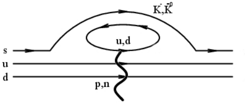
$\sim \frac{2}{3} \times g_s^2$



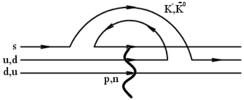
$\sim \frac{1}{3} \times g_s^2$

The graded symmetry approach

- In the case of the octet component of the $\Lambda(1405)$, we have the ghost processes: $\Lambda \to \tilde{K}^- \tilde{\Lambda}^+_{p,\tilde{u}}$ and $\Lambda \to \overline{\tilde{K}}^0 \tilde{\Lambda}^0_{n,\tilde{d}}$. We find that



$$\sim \frac{2}{3} \times \frac{1}{3}(D+3F)^2$$



$$\sim \frac{1}{3} \times \frac{1}{3}(D+3F)^2$$

- The ratio between the disconnected and total octet component is the same factor of $\frac{2}{3}$!

The graded symmetry approach

- The weight of the disconnected part, in this case, is independent of representation.
- Therefore, the light-quark sector magnetic moments of the $\Lambda(1405)$ on the lattice are expected to be:

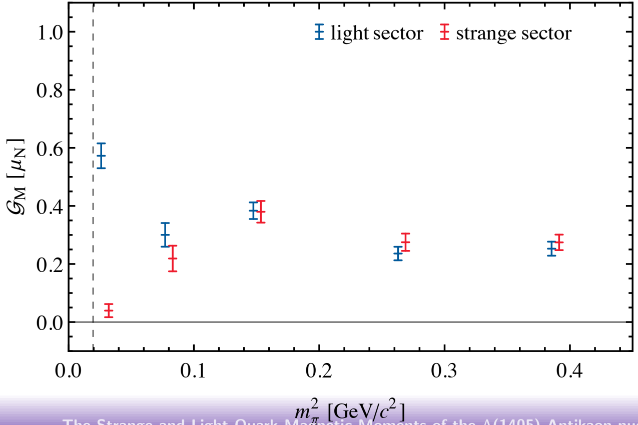
$$\langle \Lambda^* | \mu_u | \Lambda^* \rangle^{\mathrm{conn}} = \frac{1}{2}(2u_p + u_n - \frac{2}{3}u_p)$$

$$\langle \Lambda^* | \mu_d | \Lambda^* \rangle^{\mathrm{conn}} = \frac{1}{2}(2d_n + d_p - \frac{2}{3}d_n)$$

- Checking, we find that: $\frac{1}{2}(2u_p + u_n) = 1.03(2)\,\mu_N$,
- $\frac{1}{2}(2u_p + u_n - \frac{2}{3}u_p) = 0.63(2)\,\mu_N$,
- Whereas the $\Lambda(1405)$ magnetic moment on the lattice is: $\mu_{\Lambda*}^{conn} = 0.58(5)\,\mu_N$. Success!

The graded symmetry approach

- The light quark contribution to the Λ(1405) observed in lattice QCD has its origin in the nucleon of the $\bar{K}N$ molecule.



- We examined magnetic moment in the strange and light quark sectors of the $\Lambda(1405)$.
- We found that both lattice QCD and Hamiltonian effective field theory agree that the $\Lambda(1405)$ is dominated by a bound $\overline{K}N$ state near the physical point.
- It is clear that a bare three-quark state is vital for describing the behaviour of lattice QCD at large pion masses.
- We extended the graded symmetry approach to include flavour-singlet interactions, which describe a significant portion of the $\Lambda(1405)$.
- Both octet and singlet components of the $\Lambda(1405)$ receive a disconnected contribution $\frac{2}{3}$ the size of the total contribution. This makes the light sector lattice result consistent with the $\overline{K}N$ picture.

Acknowledgments

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