

Comparison between ${}^9\text{Li}$ and ${}^{10}\text{Be}$ nuclei from view point of nuclear structure and reaction

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Collaborators

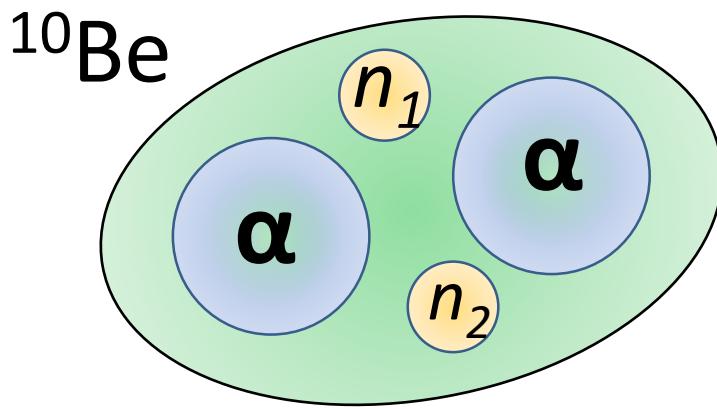
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N. Itagaki (YITP, Kyoto Univ.)

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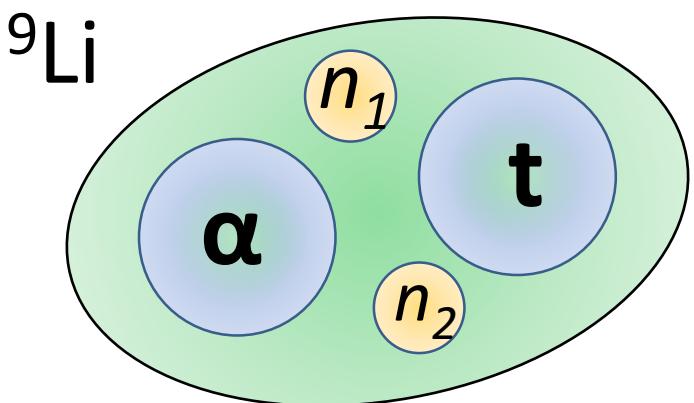
- *Introduction*
- *Stochastic Multi-configuration mixing Method*
 - Excitation energy & Radius & Transition strength
- *Microscopic Coupled Channel (MCC) Method
with complex G-matrix interaction*
 - Channel Coupling (CC) effects
- *Summary*

Comparison of ^{10}Be and ^9Li nuclei



The ^{10}Be and ^9Li nuclei are well described by assuming the $\alpha+\alpha(t)+n+n$ 4-body cluster model.

*N. Itagaki and S. Okabe, PRC61, 044306 (2000),
K. Arai et. al., PTPS142, 97 (2001),
Y. En'yo and T. Suhara, PRC85, 024303 (2012), and so on.*

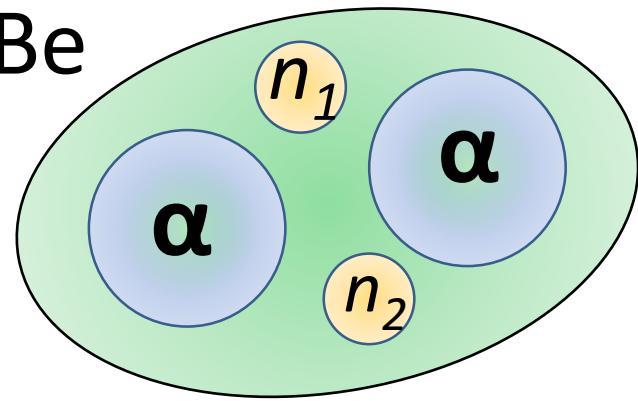


The similarity of them is often discussed from the view point of **the nuclear structure**.

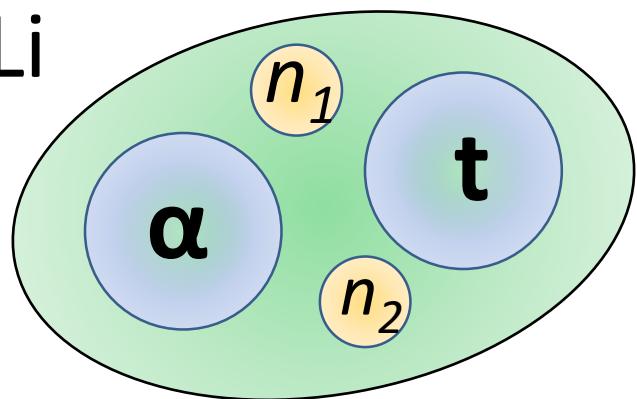
*T. Suhara and Y. En'yo, PTP123, 303 (2010),
T. Furumoto, T. Suhara and N. Itagaki, PRC87, 064320 (2013)
Y. En'yo, arXiv 1604.01453 (2016)*

Comparison of ^{10}Be and ^9Li nuclei

^{10}Be

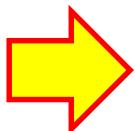


^9Li



However, the similarity and/or dissimilarity of them are **NOT** discussed from the view point of the nuclear reaction.

- Mass is different
- Coulomb strength is different



We try it!

Formalism (Structure)

Brink model + Stochastic multi-configuration mixing

$$\Phi^{J^\pi M} = \sum_K \sum_i c_{i,K} \Psi_i^{J^\pi MK}$$

$$\Psi_i^{J^\pi MK} = P^\pi P^{JMK} [A\{\phi_\alpha(\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4, \mathbf{R}_1) \phi_\alpha(\mathbf{r}_5 \mathbf{r}_6 \mathbf{r}_7 \mathbf{r}_8, \mathbf{R}_2) \phi_{n_1}(\mathbf{r}_9, \mathbf{R}_3) \phi_{n_2}(\mathbf{r}_{10}, \mathbf{R}_4)\}]$$

$$\phi(\mathbf{r}_j, \mathbf{R}_k) = \exp\left[-\nu\left(\mathbf{r}_j - \frac{\mathbf{R}_k}{\sqrt{\nu}}\right)^2\right] \chi_j \quad \chi_j : \text{spin and isospin parts}$$

Hamiltonian

$$\hat{H} = \sum_{i=1}^A \hat{t}_i - \hat{T}_{c.m.} + \sum_{i>j} \hat{v}_{ij}$$

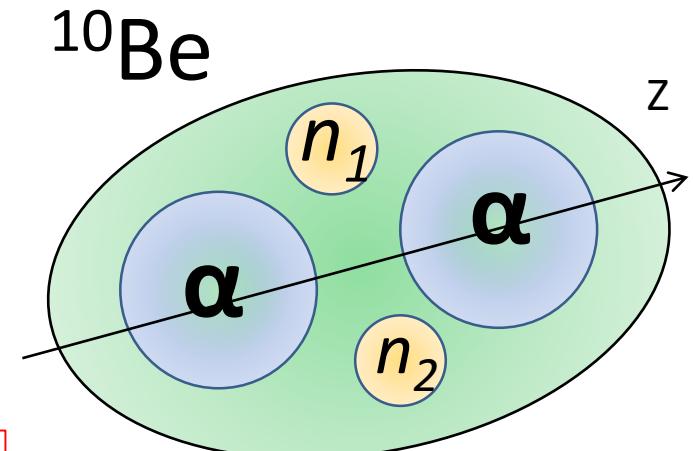
$$\nu = 0.235$$

$$M = 0.60$$

$$B = H = 0.08$$

Effective NN interaction

Central : Volkov No.2 (W=1-M)
 LS : G3RS ($V_{LS}=2000\text{MeV}$)



Formalism (Structure)

Brink model + Stochastic multi-configuration mixing

$$\Phi^{J^\pi M} = \sum_K \sum_i c_{i,K} \Psi_i^{J^\pi MK}$$

$$\Psi_i^{J^\pi MK} = P^\pi P^{JMK} [A\{\phi_\alpha(\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4, \mathbf{R}_1) \phi_t(\mathbf{r}_5 \mathbf{r}_6 \mathbf{r}_7, \mathbf{R}_2) \phi_{n_1}(\mathbf{r}_8, \mathbf{R}_3) \phi_{n_2}(\mathbf{r}_9, \mathbf{R}_4)\}]$$

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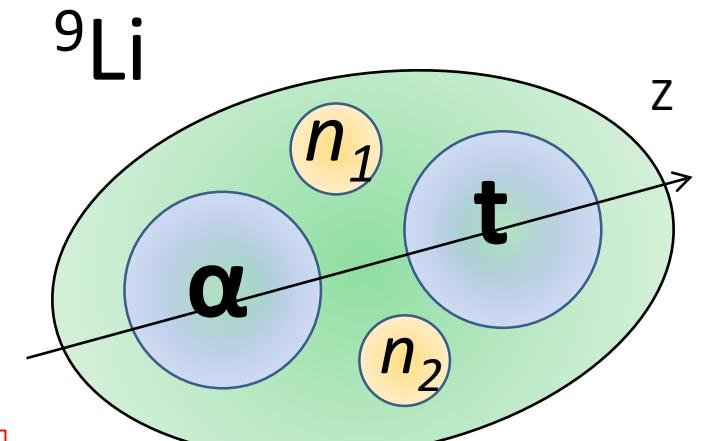
$$\nu = 0.235$$

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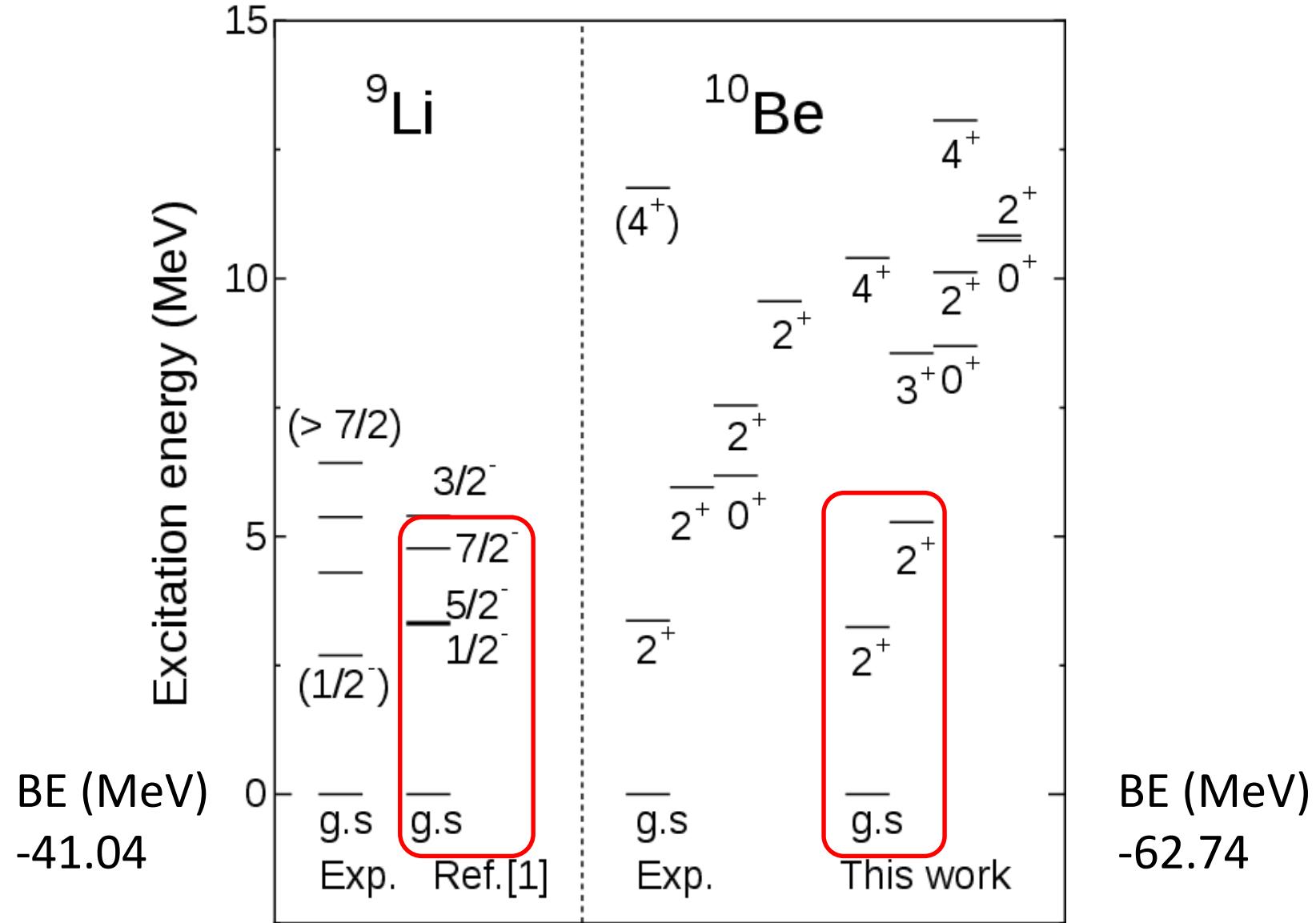
$$B = H = 0.08$$

Effective NN interaction

Central : Volkov No.2 (W=1-M)
 LS : G3RS ($V_{LS}=2000\text{MeV}$)



Excitation energy



Radius & transition strength

^9Li

Theory

Radius 2.459 fm
 $B(\text{IS}2)$ ($1/2_1 \rightarrow \text{g.s.}$) 38.51 fm^4
 ($5/2_1 \rightarrow \text{g.s.}$) 12.79 fm^4
 ($7/2_1 \rightarrow \text{g.s.}$) 15.78 fm^4

Experiment

Radius 2.3-2.5 fm [1,2]

[1] I. Tanihata et al., PLB206, 592 (1998)

[2] A. V. Dobrovolsky et al., NPA766, 1 (2006)

^{10}Be

Theory

Radius 2.559 fm
 $B(\text{IS}2)$ ($2_1 \rightarrow \text{g.s.}$) 46.84 fm^4
 ($2_2 \rightarrow \text{g.s.}$) 0.3551 fm^4
 $B(\text{E}2)$ ($2_1 \rightarrow \text{g.s.}$) $11.27 \text{ e}^2\text{fm}^4$
 ($2_1 \rightarrow 0_2$) $0.29 \text{ e}^2\text{fm}^4$
 ($2_2 \rightarrow \text{g.s.}$) $0.7429 \text{ e}^2\text{fm}^4$

Experiment

Radius 2.3-2.5 fm [1,3,4]

$B(\text{E}2)$ ($2_1 \rightarrow \text{g.s.}$) $10.24 \pm 0.97 \text{ e}^2\text{fm}^4$ [5]

 ($2_1 \rightarrow 0_2$) $0.64 \pm 0.23 \text{ e}^2\text{fm}^4$ [5]

[3] E. Liatard et al., Europhys. Lett. 13, 401 (1990)

[4] R.E. Warner, et al., PRC64, 044611 (2001)

[5] F. Ajzenberg-Selvo and J.H. Kelley, NPA506, 1 (1990)

Formalism (Reaction)

Microscopic Coupled Channel (MCC)

$$[T_R + U_{\alpha\alpha}(\mathbf{R}) - E_\alpha] \chi_\alpha(\mathbf{R}) = - \sum_{\beta \neq \alpha}^N U_{\alpha\beta}(\mathbf{R}) \chi_\beta(\mathbf{R})$$

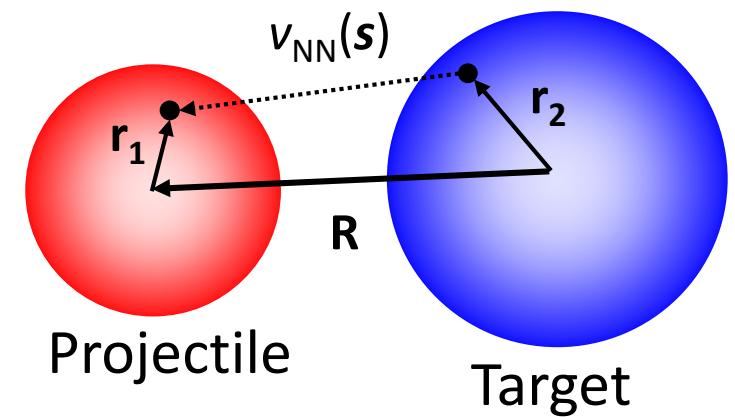
The diagonal and coupling potentials are derived from microscopic view point.

$$U_{\alpha\beta}(\mathbf{R}) = \int \underline{\rho}_{ik}^{(P)}(\mathbf{r}_1) \underline{\rho}_{jl}^{(T)}(\mathbf{r}_2) v_{NN}(\mathbf{s}; \rho, E) d\mathbf{r}_1 d\mathbf{r}_2$$

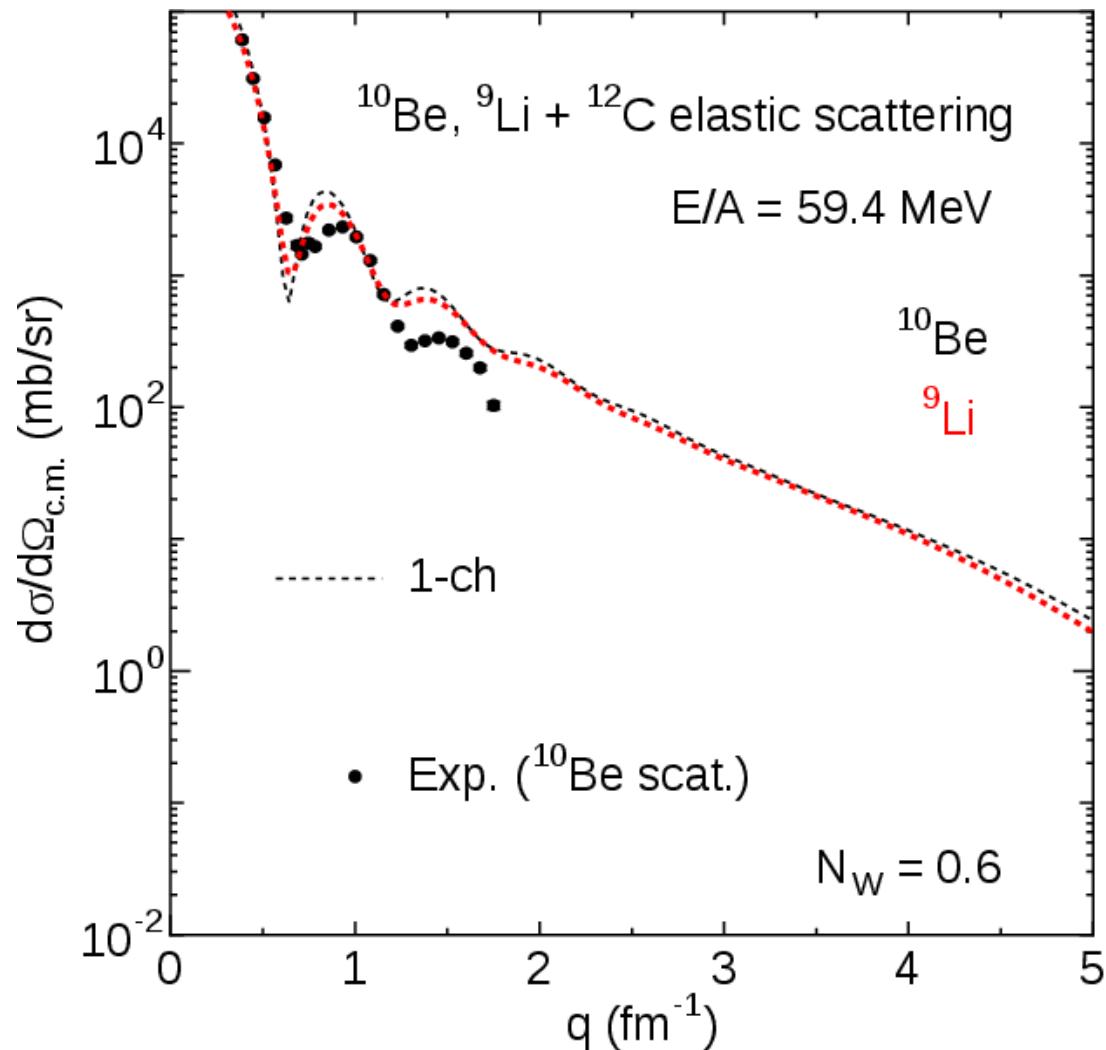
transition density MPa

Transition density

$$\underline{\rho}_{ik}(\mathbf{r}) = \left\langle \varphi_i(\xi) \left| \sum_i \delta(\mathbf{r}_i - \mathbf{r}) \right| \varphi_k(\xi) \right\rangle$$

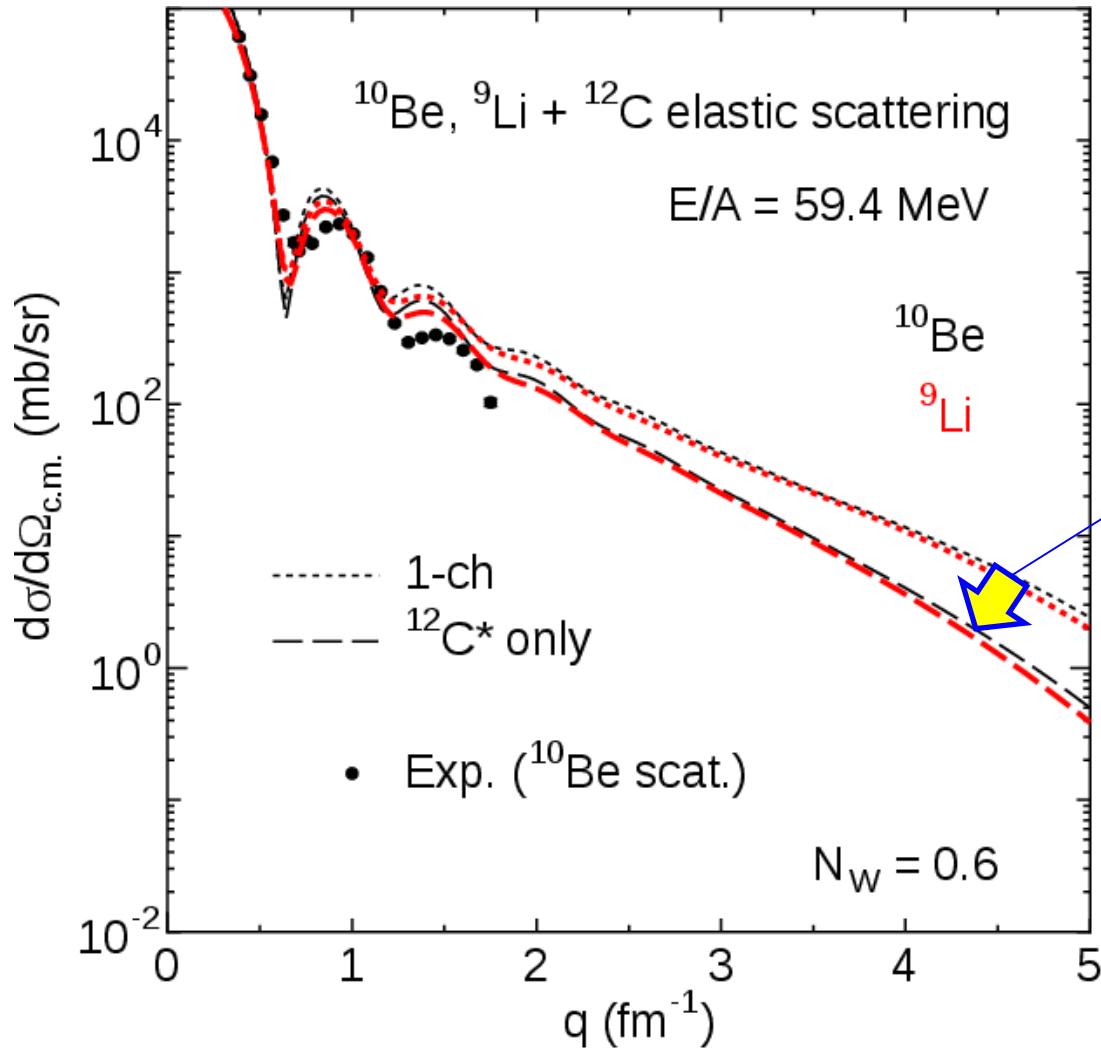


Elastic cross section of ${}^9\text{Li}$, ${}^{10}\text{Be}$ + ${}^{12}\text{C}$



$$U = V + i\underline{N}_W W$$

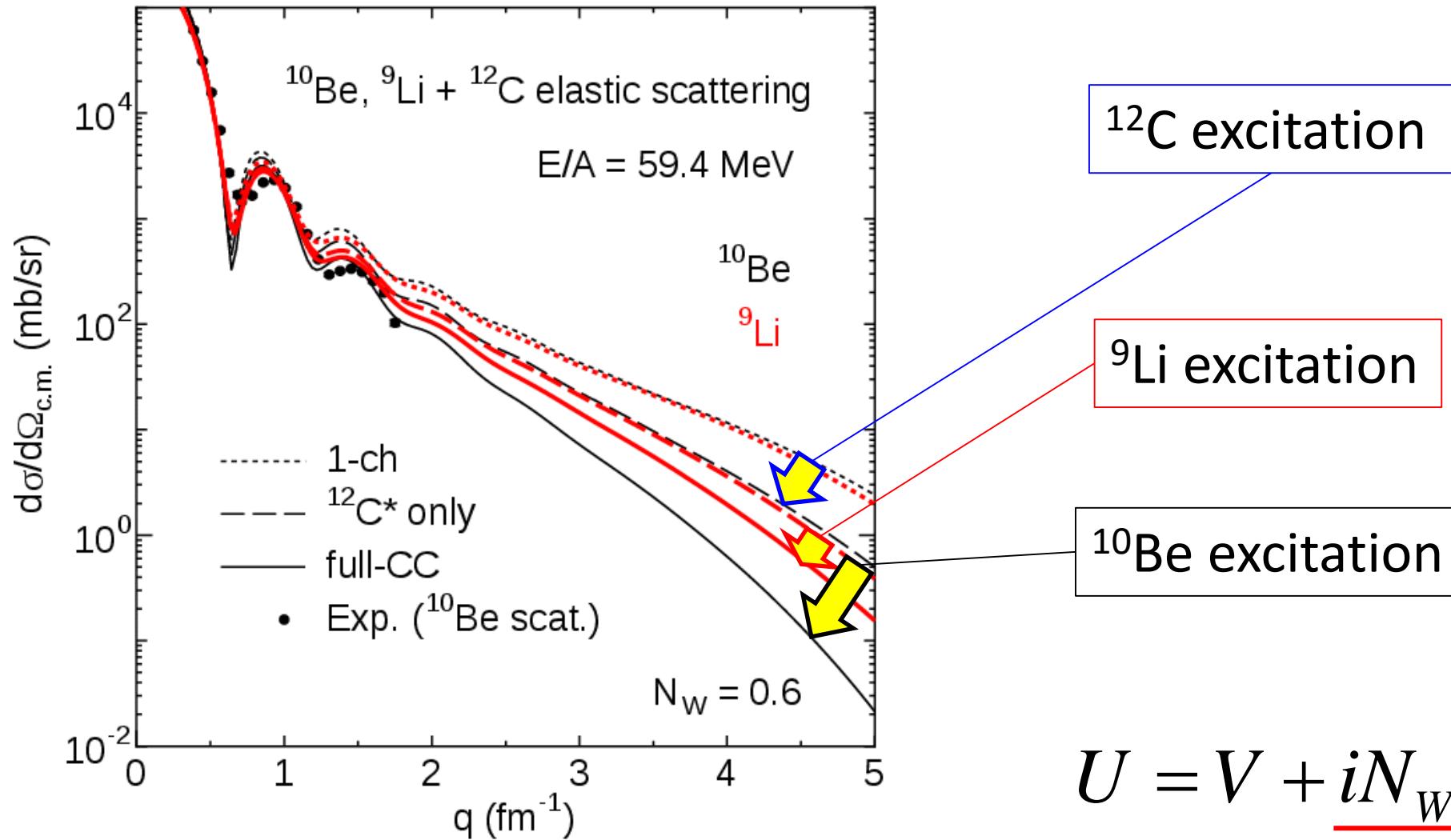
Elastic cross section of ${}^9\text{Li}$, ${}^{10}\text{Be} + {}^{12}\text{C}$



${}^{12}\text{C}$ excitation

$$U = V + i\underline{N}_W W$$

Elastic cross section of ${}^9\text{Li}$, ${}^{10}\text{Be} + {}^{12}\text{C}$



Elastic channel, Coupling & Dynamical Polarization Potentials for ${}^9\text{Li}$, ${}^{10}\text{Be}$ + ${}^{12}\text{C}$ systems

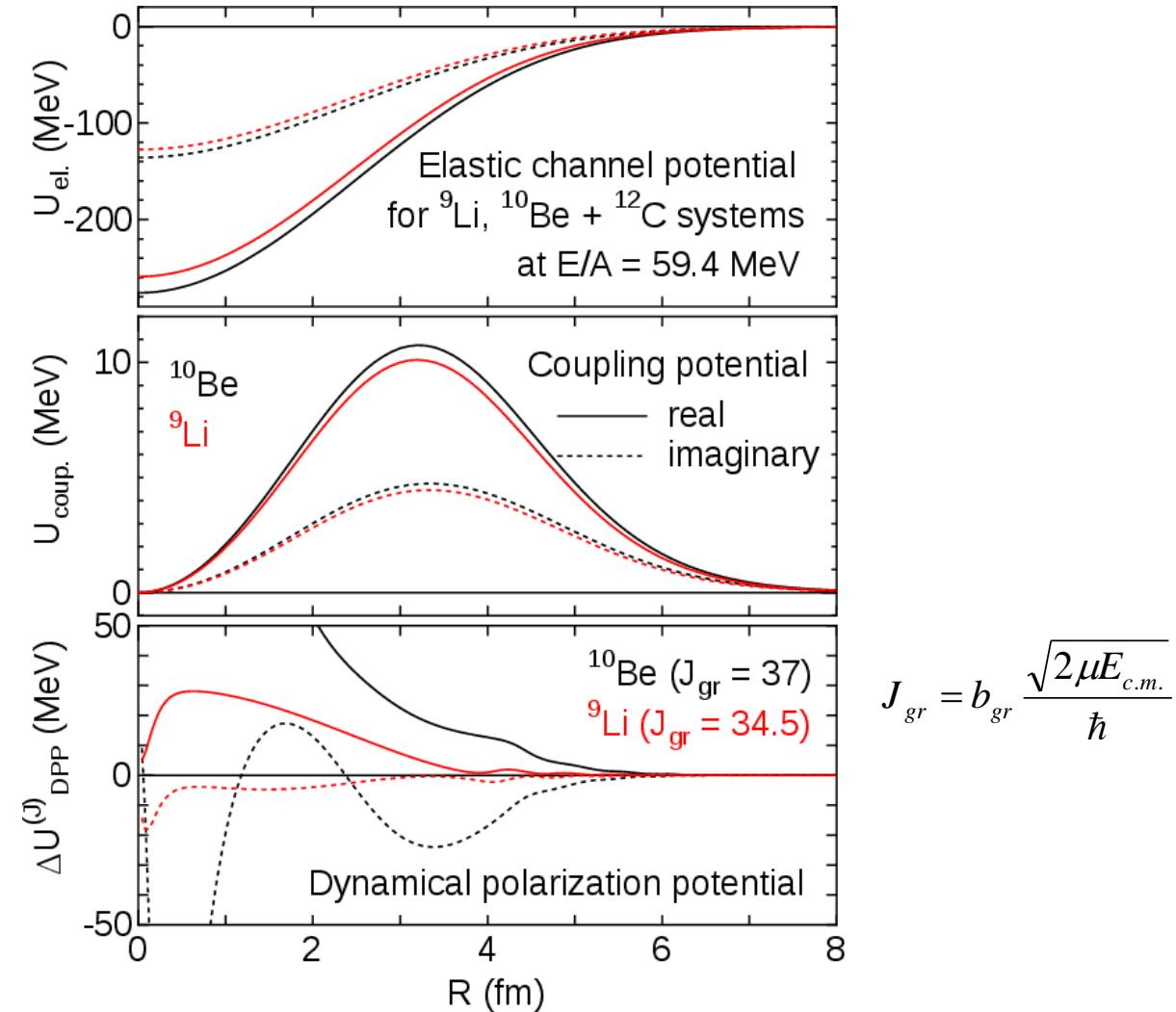
CC equation & Form Factor

$$[T_R - E_\alpha] \chi_{\alpha L}^{(J')}(R) = - \sum_{\alpha', L'} F_{\alpha L, \alpha' L'}^{(J')}(R) \chi_{\alpha' L'}^{(J')}(R), \quad (8)$$

$$\begin{aligned} F_{\alpha L, \alpha' L'}^{(J')}(R) \\ \equiv F_{\alpha S(I_1 I_2) L, \alpha' S'(I'_1 I'_2) L'}^{(J')}(R) \\ = \sum_{\lambda} i^{L+L'-\lambda} (-1)^{S+L'-J'-\lambda} \hat{L} \hat{L}' W(SLS'L' : J'\lambda) \\ \times (L0L'0|\lambda0) U_{\alpha S(I_1 I_2), \alpha' S'(I'_1 I'_2)}^{(\lambda)}(R), \end{aligned} \quad (9)$$

Dynamical Polarization Potential (DPP)

$$\Delta U_{\text{DPP}}^{(J)}(R) = \sum_{\beta \neq 0, L'} F_{0J, \beta L'}^{(J)}(R) \chi_{\beta L'}^{(J)}(R) / \chi_{0J}^{(J)}(R), \quad (14)$$



Summary

- Application of microscopic cluster and folding models to the ^{10}Be & ^9Li nuclei
- Comparison with ^9Li & ^{10}Be nuclei
 - **Similarity** : Radius, Binding energy, Transition strength
 - **Dissimilarity**: Channel coupling (CC) effect on the elastic cross section
- The CC effect
 - The dissimilarity is uncovered by **Dynamical Polarization Potential (DPP)**
 - **Angular momentum algebra** are important
 - in the comparison with ^9Li & ^{10}Be scatterings

Back up

Formalism (Outline)

Our model consists of the **structure** and **reaction** parts

“Structure”

Brink model + Stochastic multi-configuration mixing



- Binding energy
- Radius
- Transition strength

Transition density



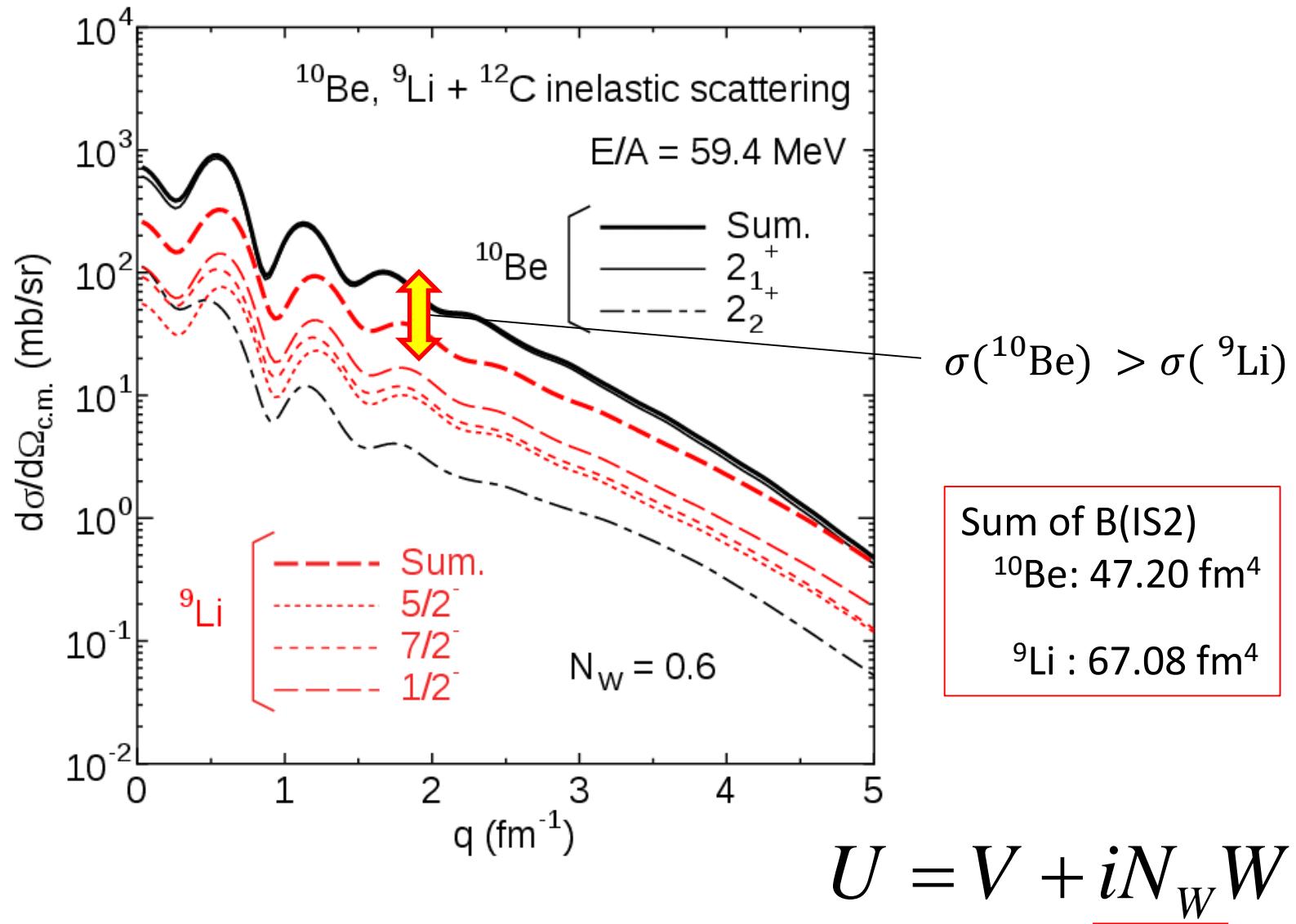
“Reaction”

Microscopic Coupled Channel (MCC)

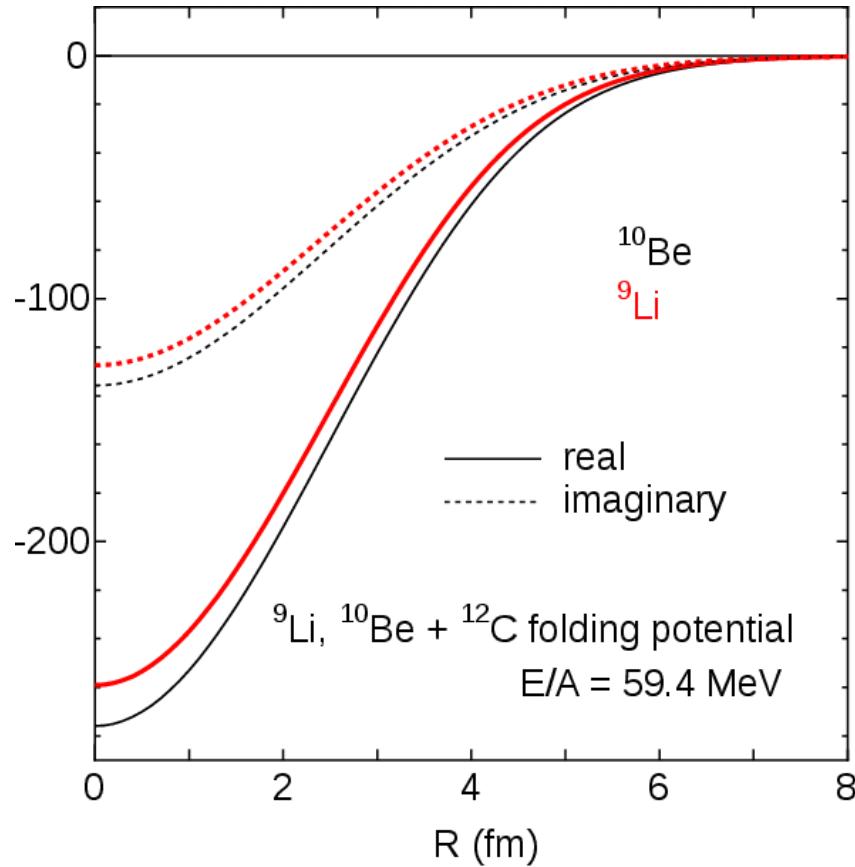


- Cross section

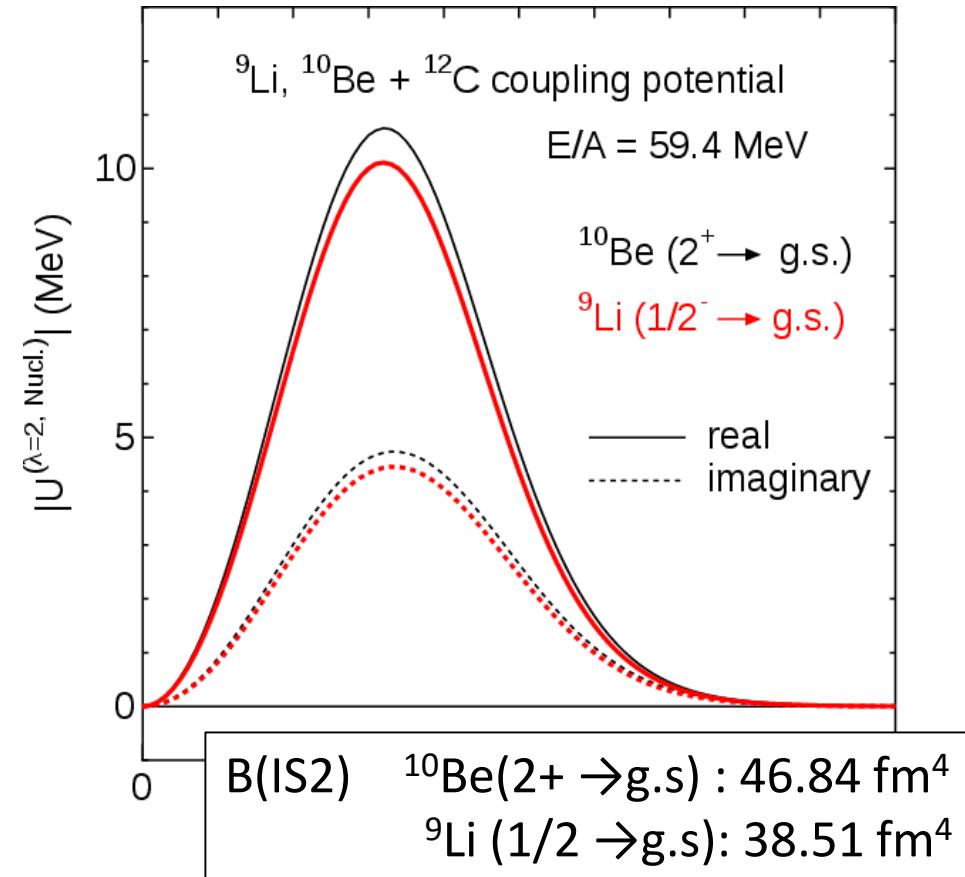
Inelastic cross section of ${}^9\text{Li}$, ${}^{10}\text{Be}$ + ${}^{12}\text{C}$



Potentials for ${}^9\text{Li}$, ${}^{10}\text{Be} + {}^{12}\text{C}$ systems



Elastic channel potential



Coupling potential

Dynamical Polarization Potential (DPP) for ${}^9\text{Li}$, ${}^{10}\text{Be} + {}^{12}\text{C}$ systems

CC equation & Form Factor

$$[T_R - E_\alpha] \chi_{\alpha L}^{(J')}(R) = - \sum_{\alpha', L'} F_{\alpha L, \alpha' L'}^{(J')}(R) \chi_{\alpha' L'}^{(J')}(R), \quad (8)$$

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$$J_{gr} = b_{gr} \frac{\sqrt{2\mu E_{c.m.}}}{\hbar}$$

