^{6}Li structure information from ^{2}H$(\alpha,\alpha)^{2}$H scattering

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Multi-channel algebraic scattering (MCAS)

Low-energy, two-body, light-mass, Lippmann-Schwinger scattering

$$\begin{aligned} T^{J^\pi}_{cc'}(p,q;E) &= V^{J^\pi}_{cc'}(p,q) \\ &+ \mu\left[\sum_{c''=1}^{\text{open}} \int_0^\infty V^{J^\pi}_{cc''}(p,x)\frac{x^2}{k^2_{c''}-x^2+i\epsilon}T^{J^\pi}_{c''c'}(x,q;E)\,dx \right. \\ &\left. - \sum_{c''=1}^{\text{closed}} \int_0^\infty V^{J^\pi}_{cc''}(p,x)\frac{x^2}{h^2_{c''}+x^2}T^{J^\pi}_{c''c'}(x,q;E)\,dx\right] \end{aligned}$$

If we assume the T-matrix can be split, viz.

$$T_{cc'} = -\sum_p |\chi_{cp}\rangle \frac{1}{[1-\eta_p]\,\eta_p} \langle\chi_{c'p}| \ ,$$

we are led to a separable potential

$$V_{cc'} = -\sum_p |\chi_{cp}\rangle \frac{1}{\eta_p} \langle\chi_{c'p}| \ ,$$

and we do this in reverse.

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MCAS includes of the Pauli exclusion principle in $V_{cc'}$, even for targets defined by collective models.

This is done with orthogonalising pseudo potentials:

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r') + \lambda A_c(r) A_{c'}(r')\delta_{c,c'}$$

Many low-energy reactions of light-mass nuclei with α-particles are of interest for nuclear structure and astrophysics.

So far, the MCAS formalism has only considered nucleon projectiles.

Changing the angular momentum algebra and interaction potential, we investigate $X(\alpha, \alpha)X$ reactions at energies below where excitation of the α-particle is relevant.

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The basic potential ingredients:

$$V_{cc}(r) = \left[\left(V_0 \delta_{cc'} + V_{\ell\ell}[\ell \cdot \ell] + V_{II}[I \cdot I] \right) f(r) + V_{\ell I} \frac{1}{r} \frac{df(r)}{dr} \right]_{cc'} + V_{\text{mono}}\, \delta_{cc'}\, \delta_{J^\pi=0}\, f(r)$$

where

$$f(r) = \frac{1}{1 + exp\left(\frac{r-R}{a}\right)}$$

^{2}H$(\alpha,\alpha)^{2}$H

Consider an effective two-body model of ^{2}H$(\alpha,\alpha)^{2}$H scattering.

Deuterium treated as a rigid drop of nuclear matter, with axial, permanent deformation as

$$R(\theta,\phi) = R_0\left[1+\sum_{L(\geq 2)}\sqrt{\frac{4\pi}{2L+1}}\,\beta_L\left[\mathbf{Y}_L(\hat{r})\cdot\mathbf{Y}_L(\hat{\Upsilon})\right]\right] = R_0\left[1+\epsilon\right] .$$

$f(r)$ is expanded to second order in ϵ.

The spectrum of deuterium is restricted to the $^{3}S_{1}$ 1^{+} ground state and the hypothetical $^{1}S_{0}$ 0^{+} state, bound by 67 keV*.

*S. B. Borzakov, N. A. Gundorin, Y. N. Pokotilovski, Phys. Part. Nuclei Lett. 12, 536 (2015).

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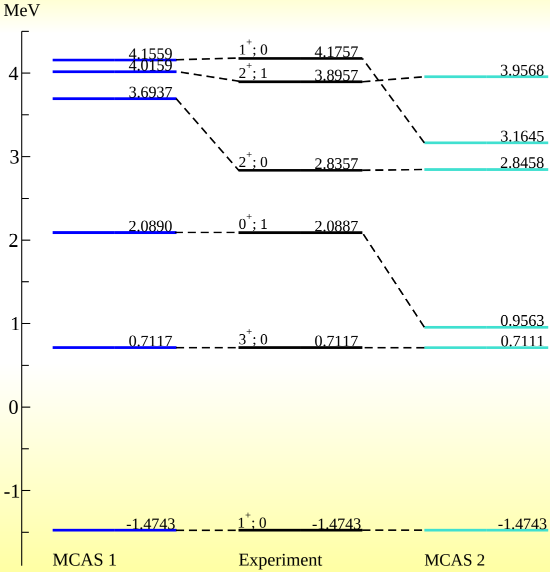
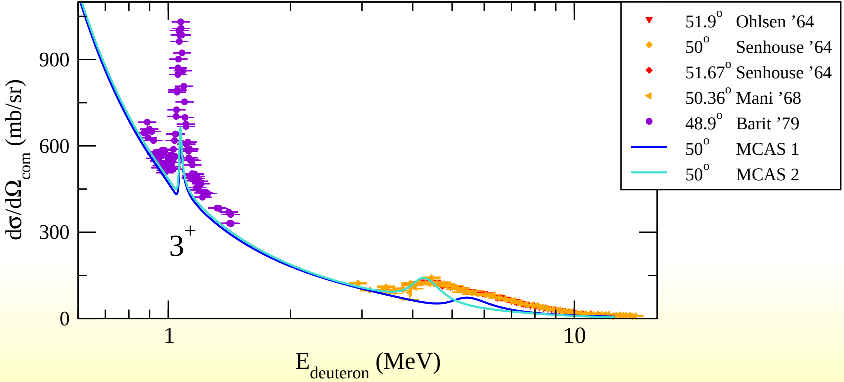
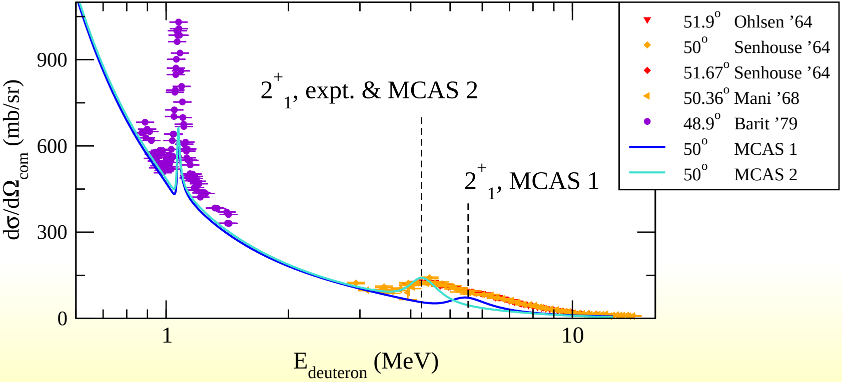
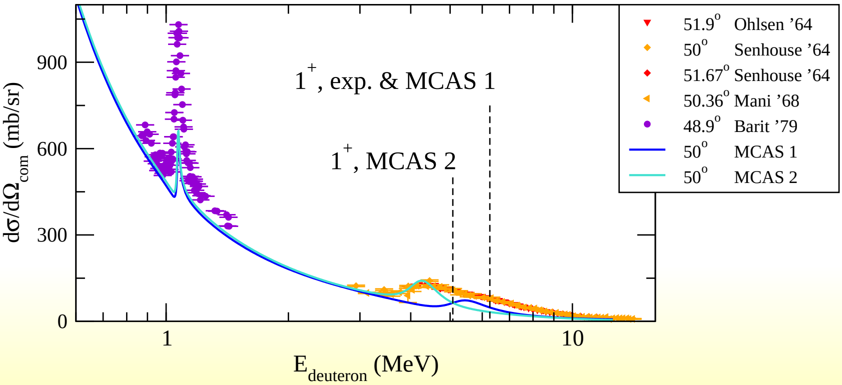
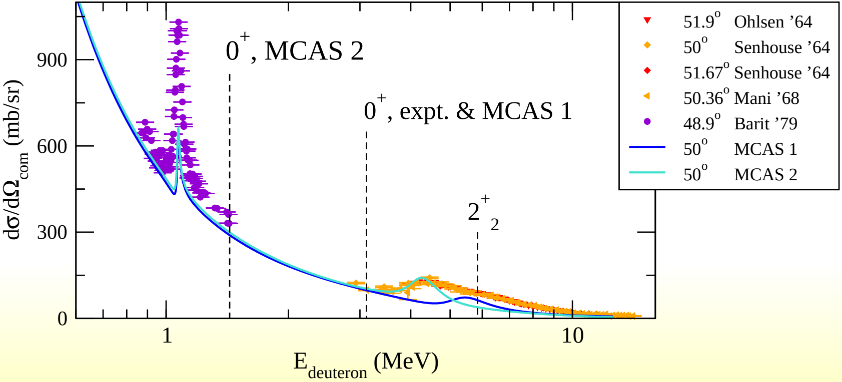

MeV
4
3
2
1
0
-1
4.1559
4.0159
3.6937
2.0890
0.7117
-1.4743
1+; 0
4.1757
2+; 1
3.8957
2+; 0
2.8357
0+; 1
2.0887
3+; 0
0.7117
1+; 0
-1.4743
3.9568
3.1645
2.8458
0.9563
0.7111
-1.4743
MCAS 1
Experiment
MCAS 2


900
600
300
0
dσ/dΩ$_{com}$ (mb/sr)
1
10
E$_{deuteron}$ (MeV)
3$^+$
51.9° Ohlsen '64
50° Senhouse '64
51.67° Senhouse '64
50.36° Mani '68
48.9° Barit '79
50° MCAS 1
50° MCAS 2



2+1, expt. & MCAS 2
2+1, MCAS 1
dσ/dΩcom (mb/sr)
Edeuteron (MeV)
0
300
600
900
1
10
51.9° Ohlsen '64
50° Senhouse '64
51.67° Senhouse '64
50.36° Mani '68
48.9° Barit '79
50° MCAS 1
50° MCAS 2


51.9^o Ohlsen '64
50^o Senhouse '64
51.67^o Senhouse '64
50.36^o Mani '68
48.9^o Barit '79
50^o MCAS 1
50^o MCAS 2
1$^+$, exp. & MCAS 1
1$^+$, MCAS 2
dσ/dΩ$_{com}$ (mb/sr)
900
600
300
0
1
10
E$_{deuteron}$ (MeV)


0+, MCAS 2
0+, expt. & MCAS 1
2+2
dσ/dΩcom (mb/sr)
E deuteron (MeV)
0
300
600
900
1
10
51.9° Ohlsen '64
50° Senhouse '64
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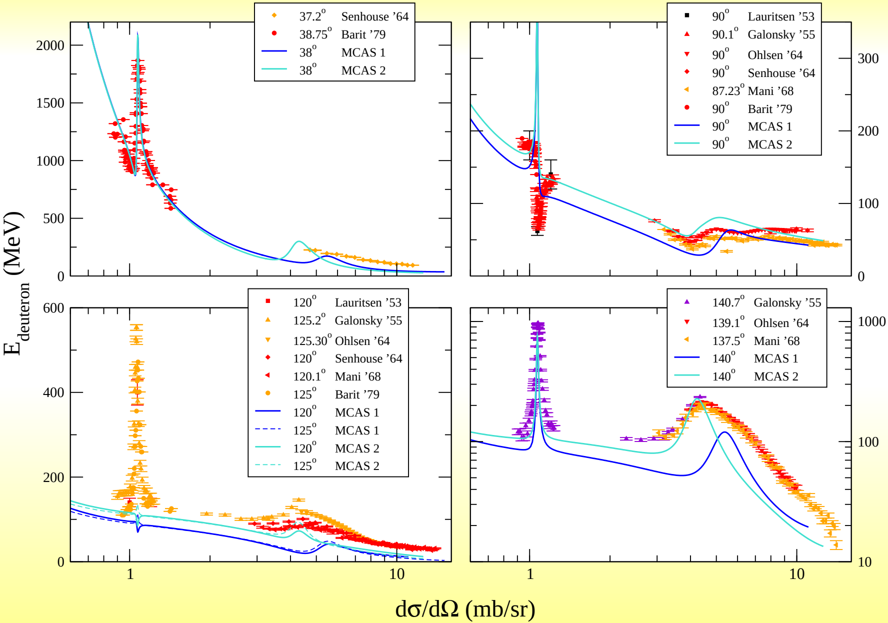
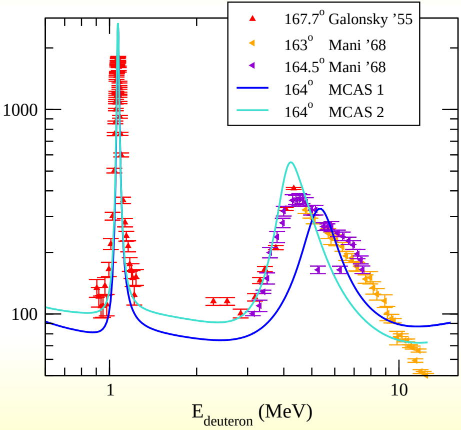
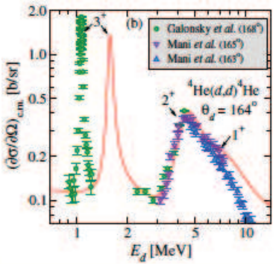
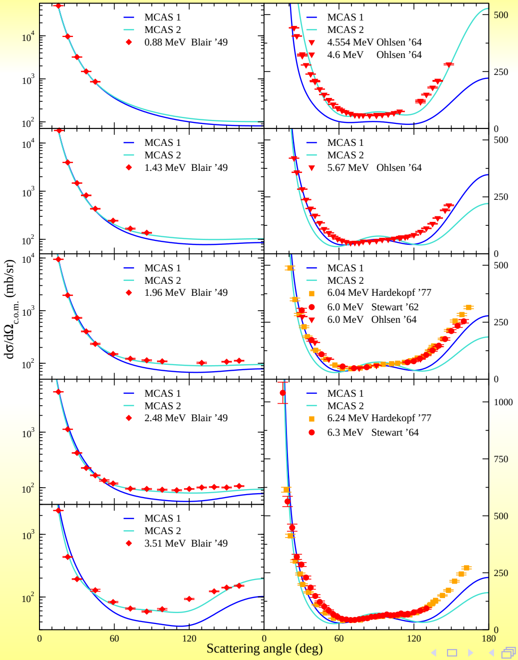
50.36° Mani '68
48.9° Barit '79
50° MCAS 1
50° MCAS 2

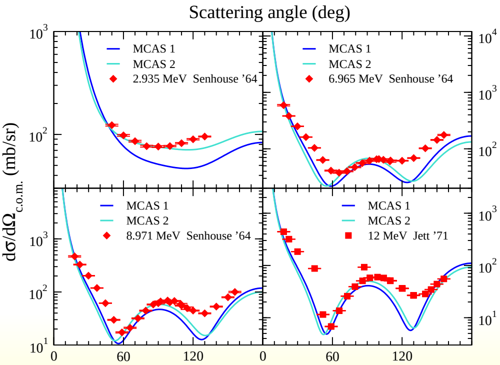

37.2° Senhouse '64
38.75° Barit '79
38° MCAS 1
38° MCAS 2
90° Lauritsen '53
90.1° Galonsky '55
90° Ohlsen '64
90° Senhouse '64
87.23° Mani '68
90° Barit '79
90° MCAS 1
90° MCAS 2
120° Lauritsen '53
125.2° Galonsky '55
125.30° Ohlsen '64
120° Senhouse '64
120.1° Mani '68
125° Barit '79
120° MCAS 1
125° MCAS 1
120° MCAS 2
125° MCAS 2
140.7° Galonsky '55
139.1° Ohlsen '64
137.5° Mani '68
140° MCAS 1
140° MCAS 2
E_deuteron (MeV)
dσ/dΩ (mb/sr)

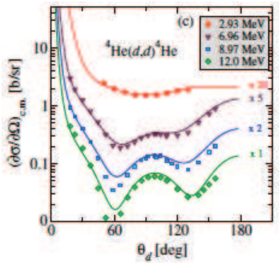




Right panel: G. Hupin, S. Quaglioni, P. Navrátil, PRL 114, 212502 (2015)


MCAS 1
MCAS 2
0.88 MeV Blair '49
1.43 MeV Blair '49
1.96 MeV Blair '49
2.48 MeV Blair '49
3.51 MeV Blair '49
4.554 MeV Ohlsen '64
4.6 MeV Ohlsen '64
5.67 MeV Ohlsen '64
6.04 MeV Hardekopf '77
6.0 MeV Stewart '62
6.0 MeV Ohlsen '64
6.24 MeV Hardekopf '77
6.3 MeV Stewart '64
dσ/dΩ_c.o.m. (mb/sr)
Scattering angle (deg)





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Conclusions

Much data exists for ^{2}H(α, α)^{2}H elastic scattering.

Most theoretical analyses have been fits with no structure input.

In light of a recent 6-nucleon study of the data, we have used an effective 2-body potential to analyse the data.

The correct list of states is recreated, with energies in reasonable agreement with the ^{6}Li data.

The non-resonant part of the cross section, the 3^+ resonance and the 2^+ resonance are well recreated at most angles (and energies). The 1^+ resonance is not.

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Physicist for hire: have laptop, will travel.

Outline