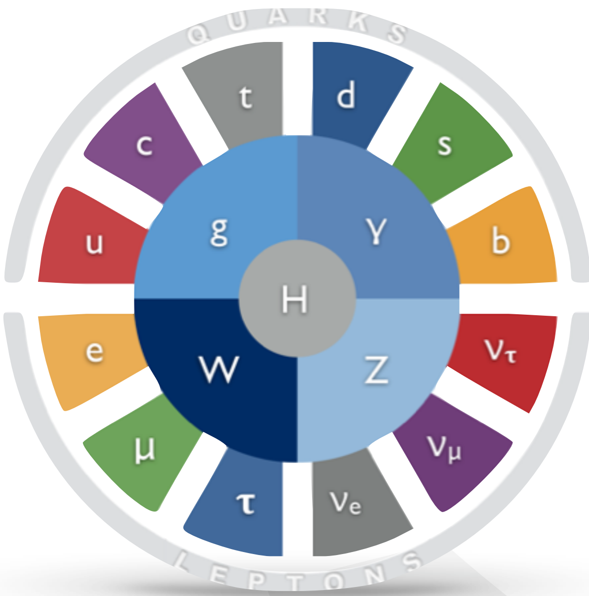
Nuclear Physics from the Ground Up

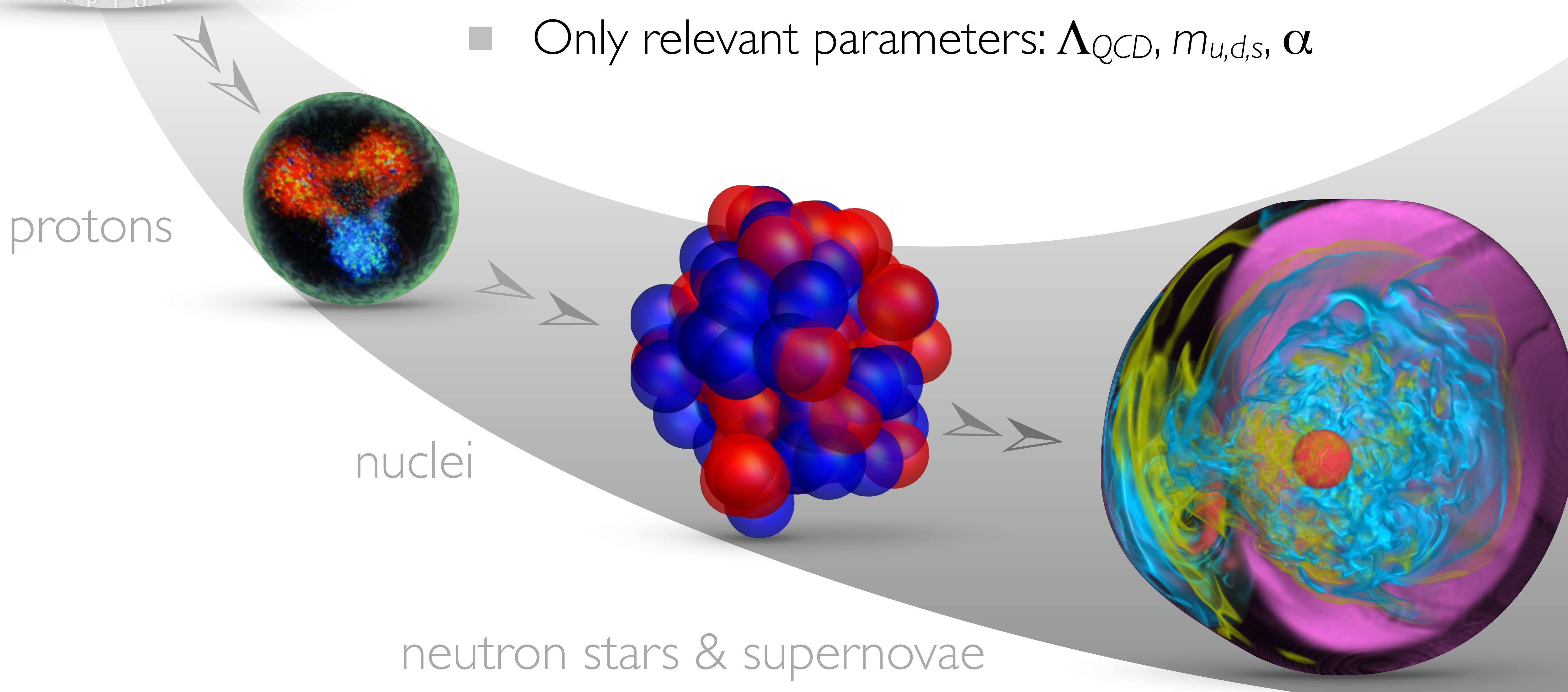
William Detmold, MIT

26th International Conference on Nuclear Physics, Adelaide, Sep 12th 2016

From Quarks to the Cosmos



- Complexity of nuclear physics emerges from the Standard Model
  - Same underlying physics at vastly different scales
  - EM, weak and strong (QCD) interactions
  - Only relevant parameters: Λ_{QCD}, $m_{u,d,s}$, α



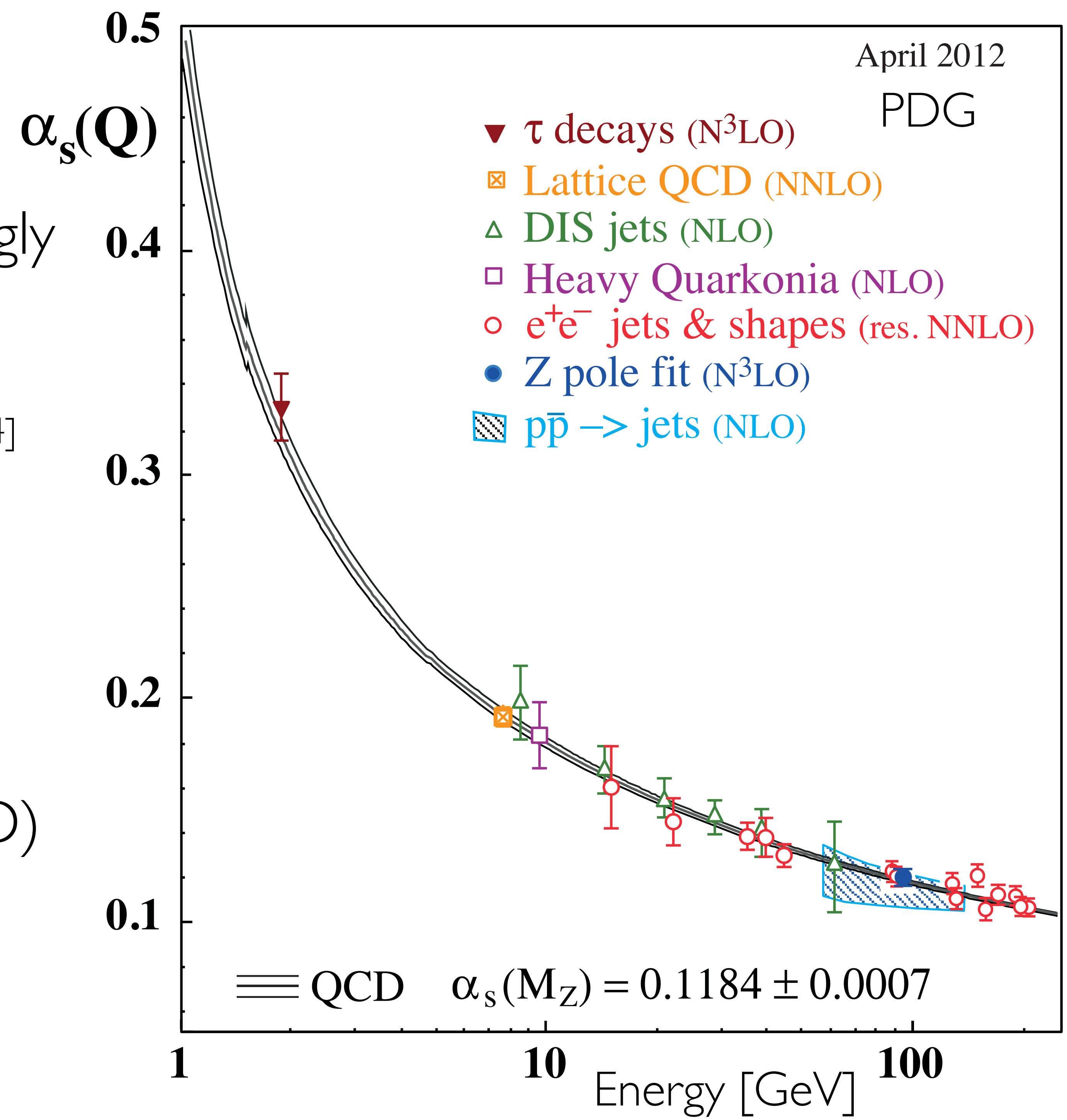
Quantitative QCD

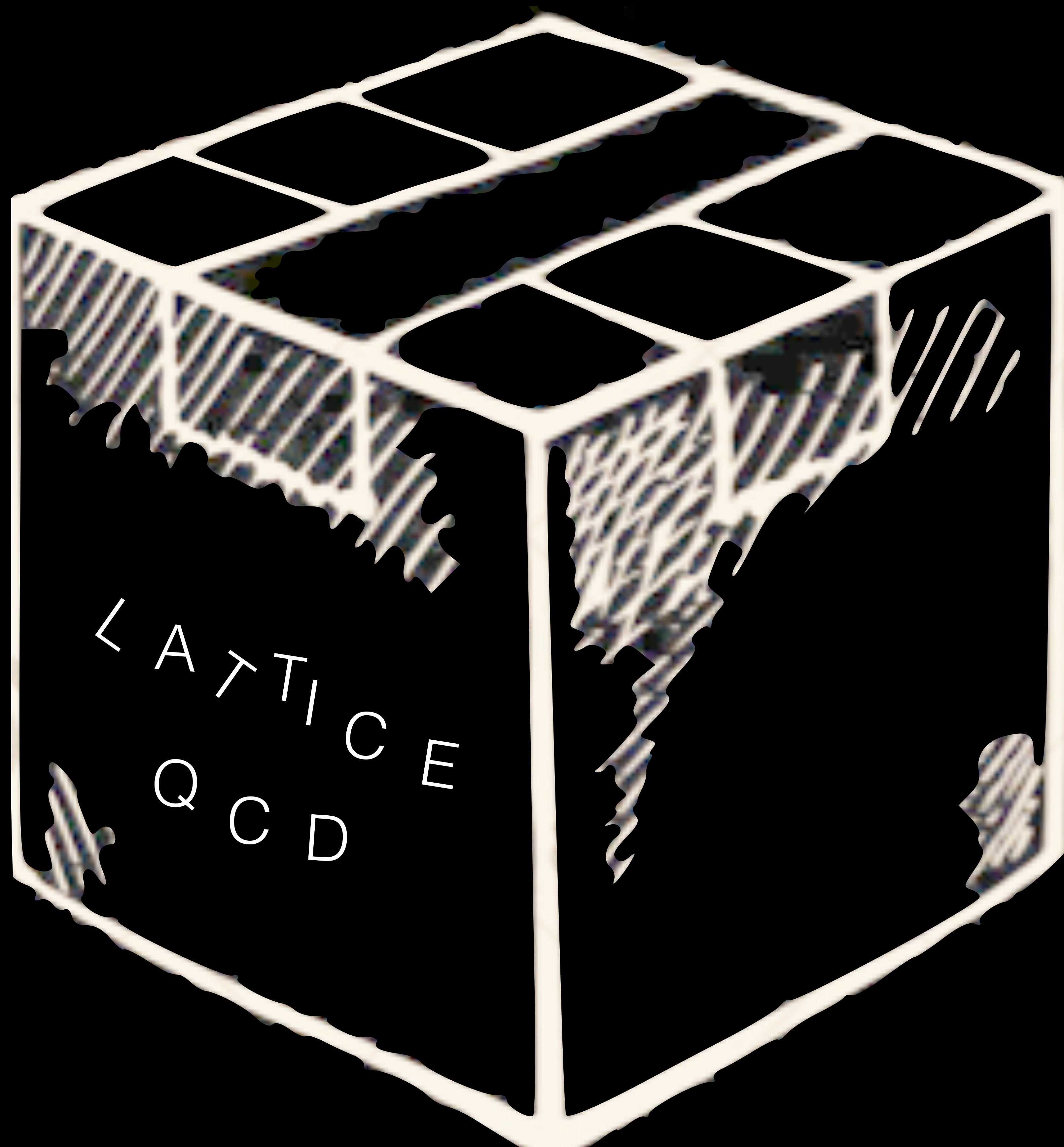
- QCD is the "strong force": quarks and gluons interact strongly
- Interaction strength depends on energy [Gross, Politzer, Wilczek, Nobel 2004]
- At high energy, can use perturbative theory

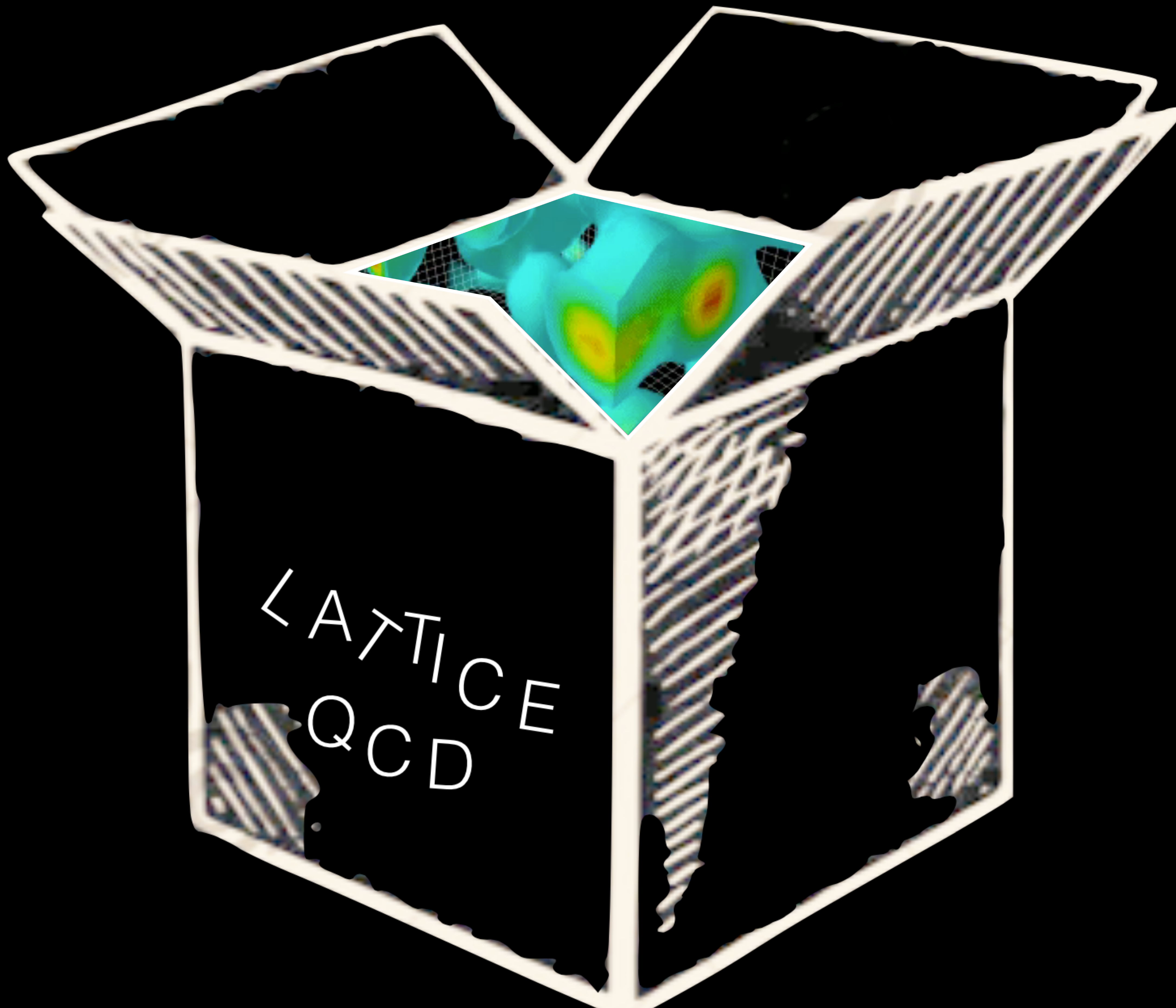
$$\mathcal{O}_{\text{exact}} = \mathcal{O}_0 + \mathcal{O}_1\alpha_s + \mathcal{O}_2\alpha_s^2 + \ldots$$

(also works spectacularly in QED)

- At low energies: need another approach




LATTICE
QCD

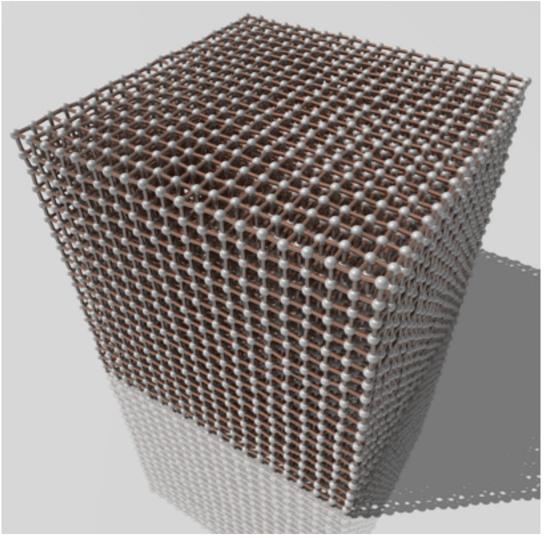

LATTICE
QCD

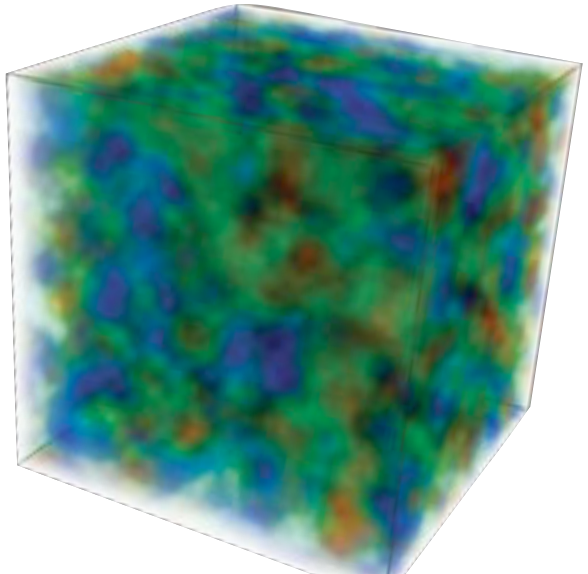
Quantum Chromodynamics

- Lattice QCD: tool to deal with quarks and gluons
  - Correlation functions as functional integral over quark and gluon d.o.f. on R_4

$$\langle \mathcal{O} \rangle \sim \int dA_\mu dq d\bar{q}\, \mathcal{O}[q, \bar{q}, A] e^{-S_{QCD}[q, \bar{q}, A]}$$

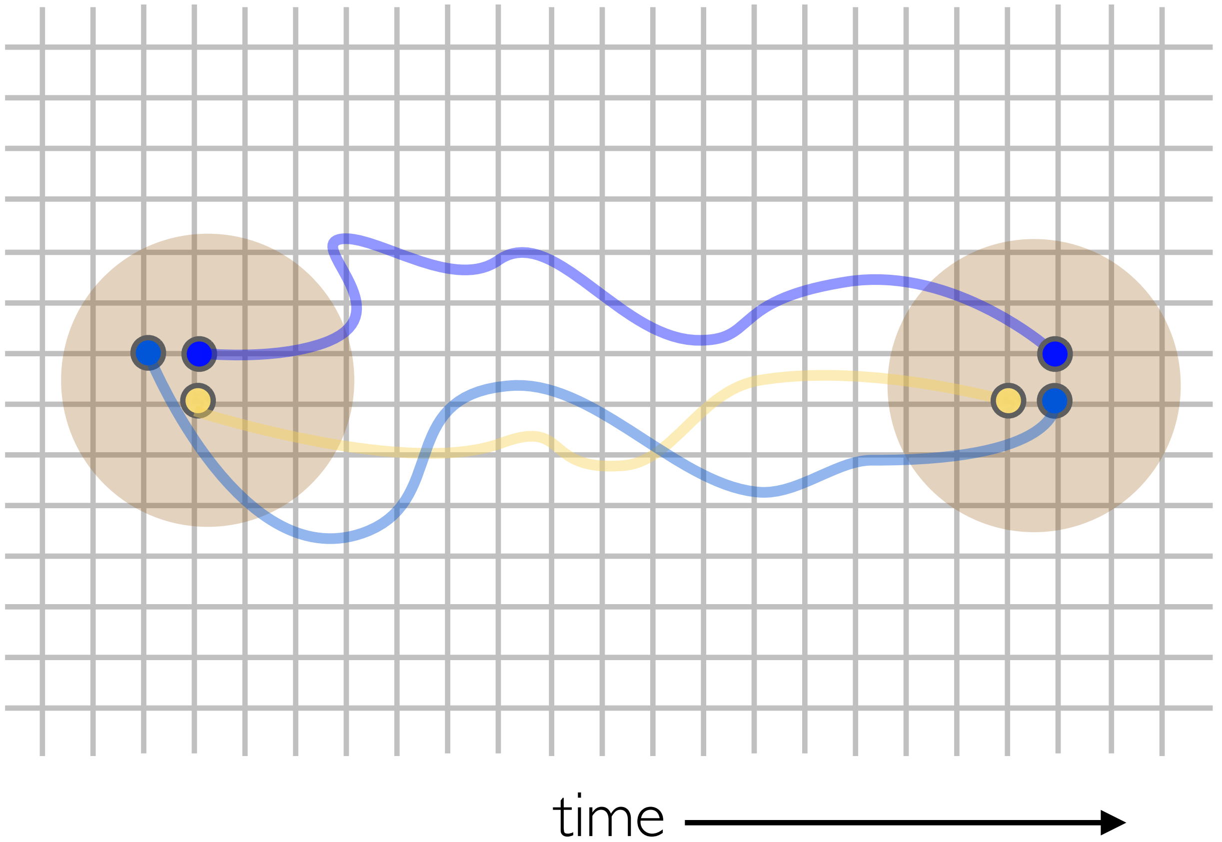
  - Discretise and compactify system
    - Finite but large number of d.o.f (10^{12})
  - Integrate via importance sampling (average over important configurations)
  - Undo the harm done in previous steps
    - Lattice QCD ⇒ QCD

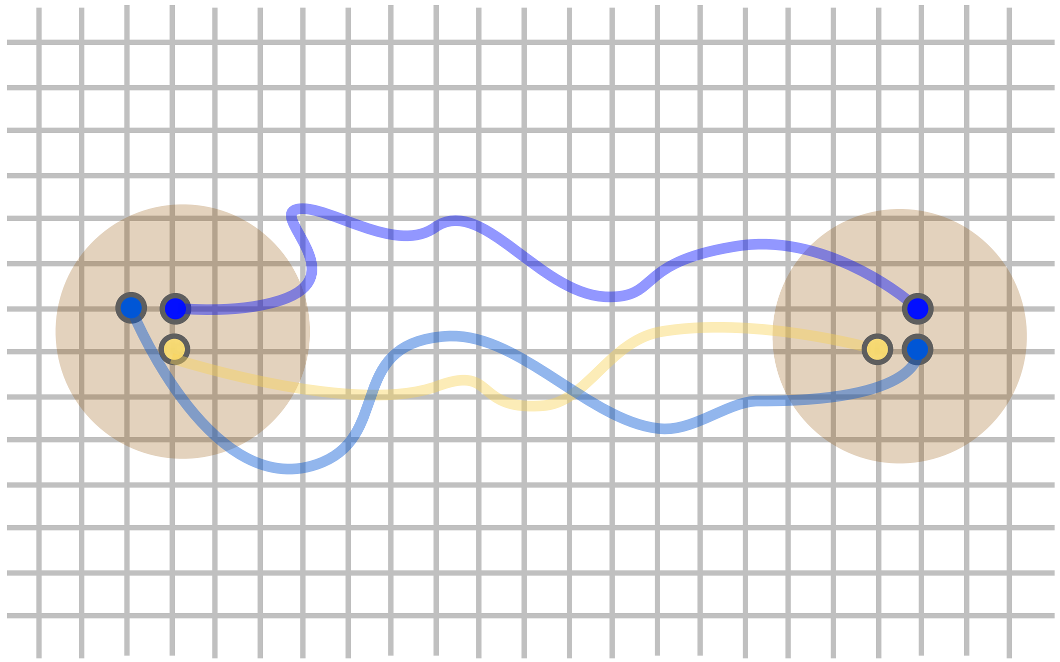




Spectroscopy

- How do we calculate the proton mass?
- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- QCD adds all the quark anti-quark pairs and gluons automatically: only eigenstates with correct q#'s propagate





- Correlation decays exponentially with distance

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

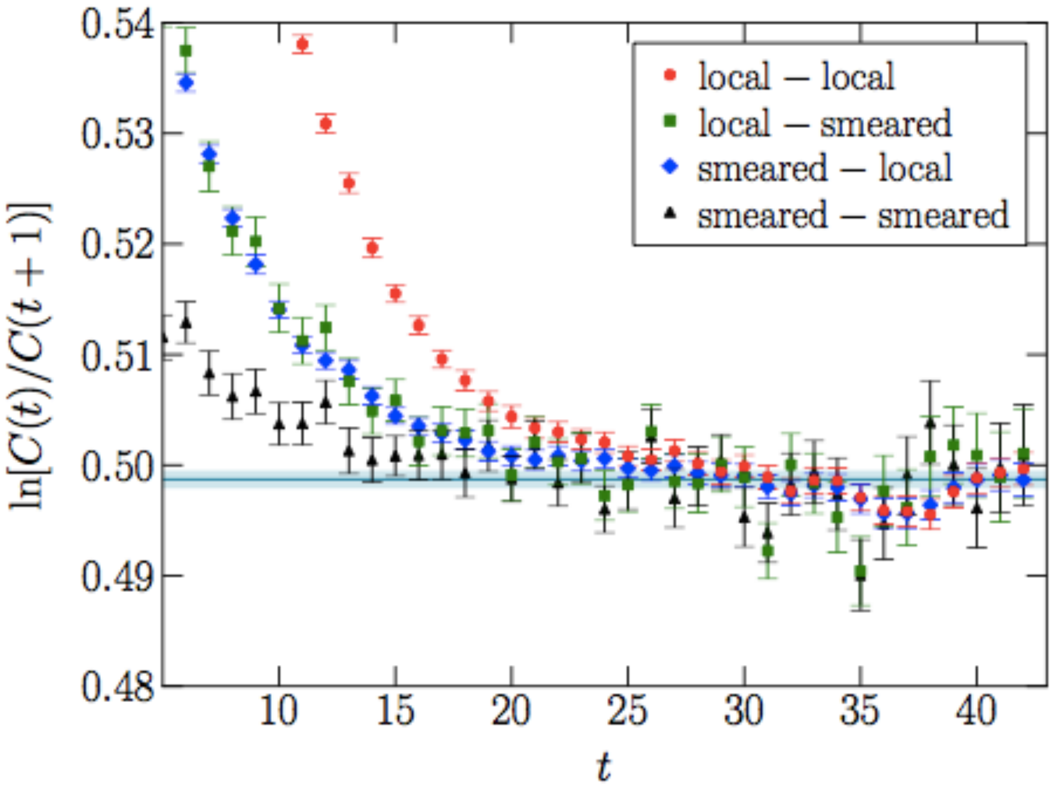
n ← all eigenstates with q#'s of proton

at late times

$$\rightarrow Z_0 \exp(-E_0 t)$$

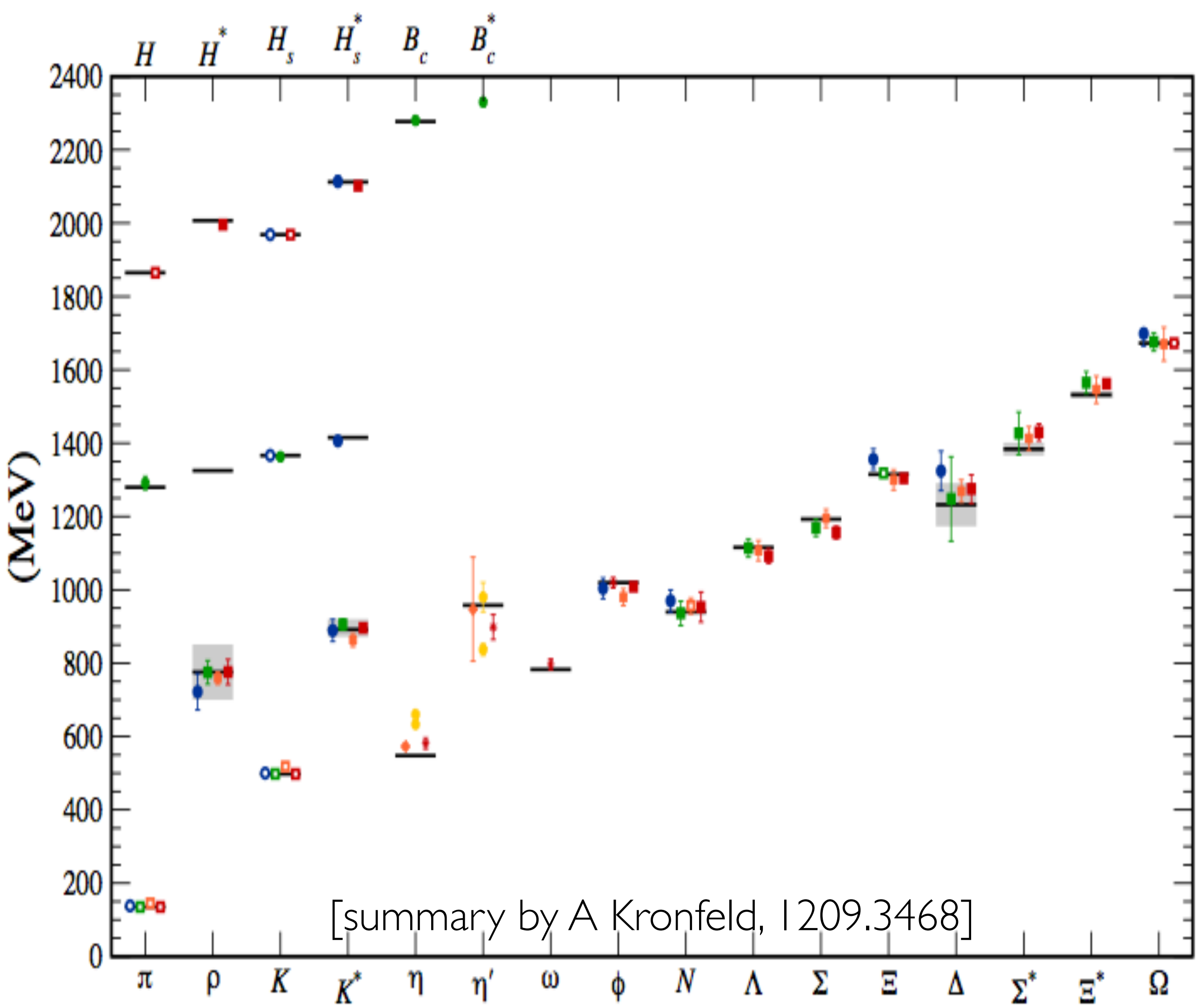
- Ground state mass revealed through "effective mass plot"

$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\to\infty} E_0$$

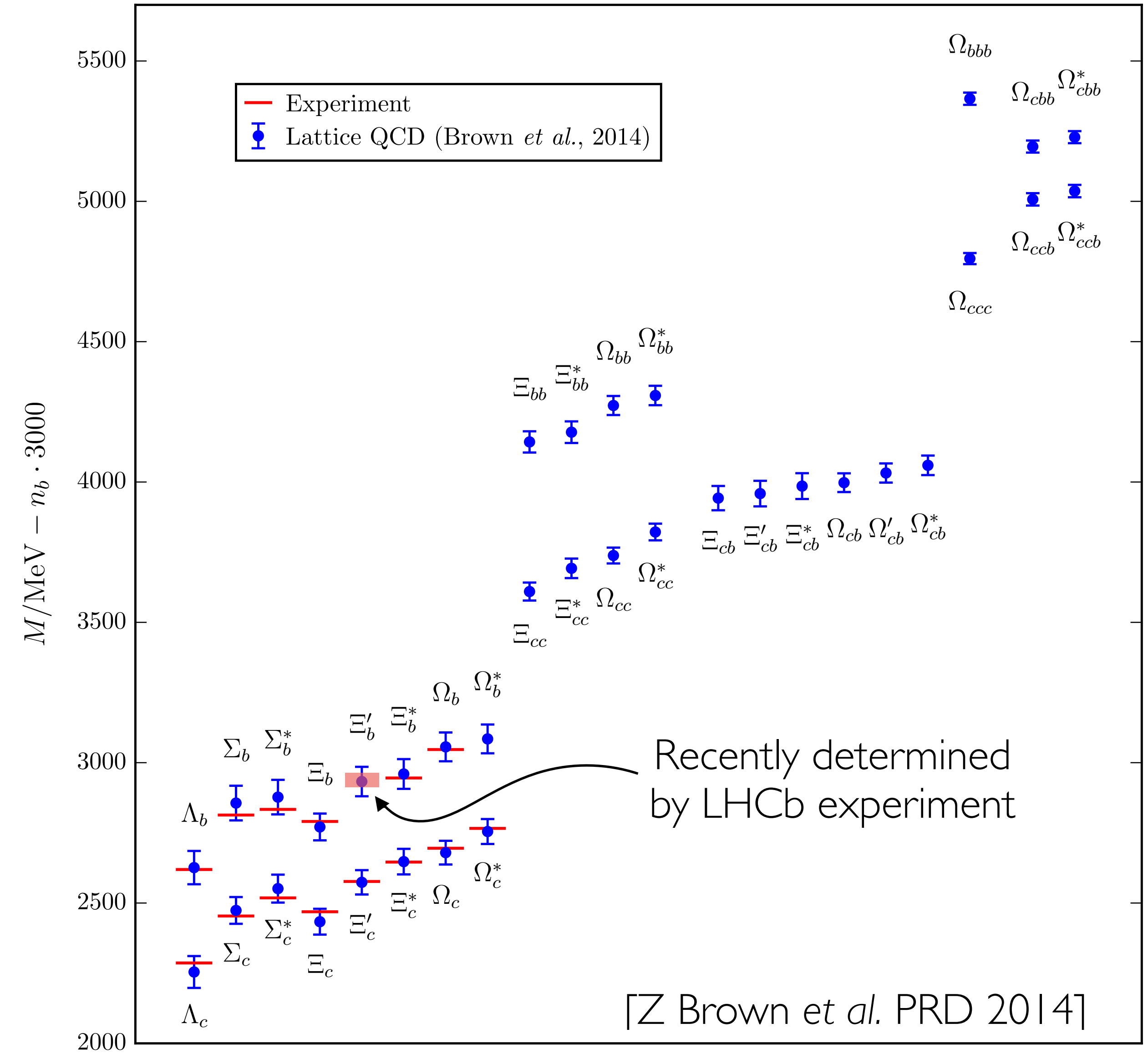


QCD Spectrum

- 30 years of developments
- Ground state hadron spectrum reproduced

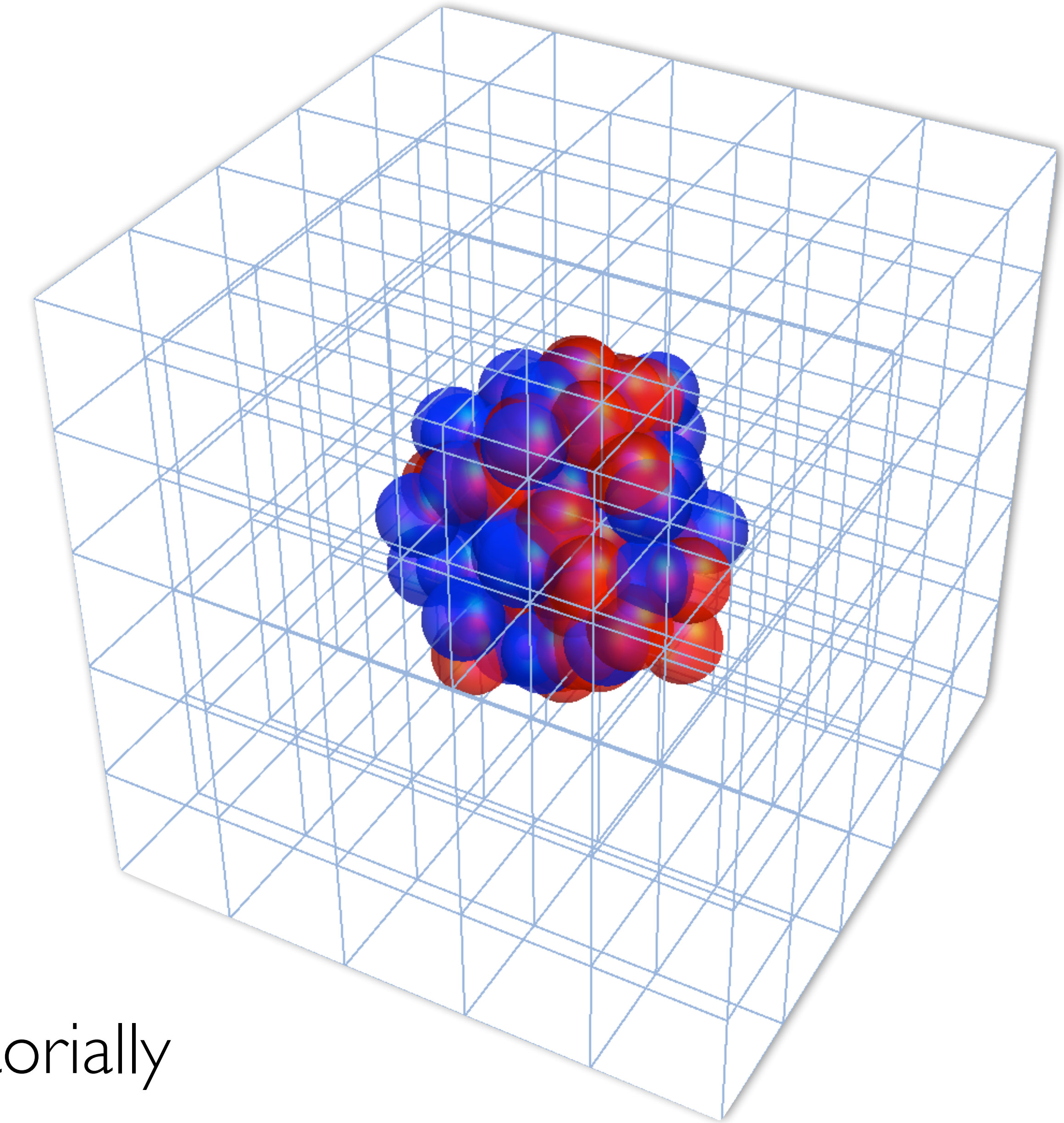


- Predictions for new states with controlled uncertainties

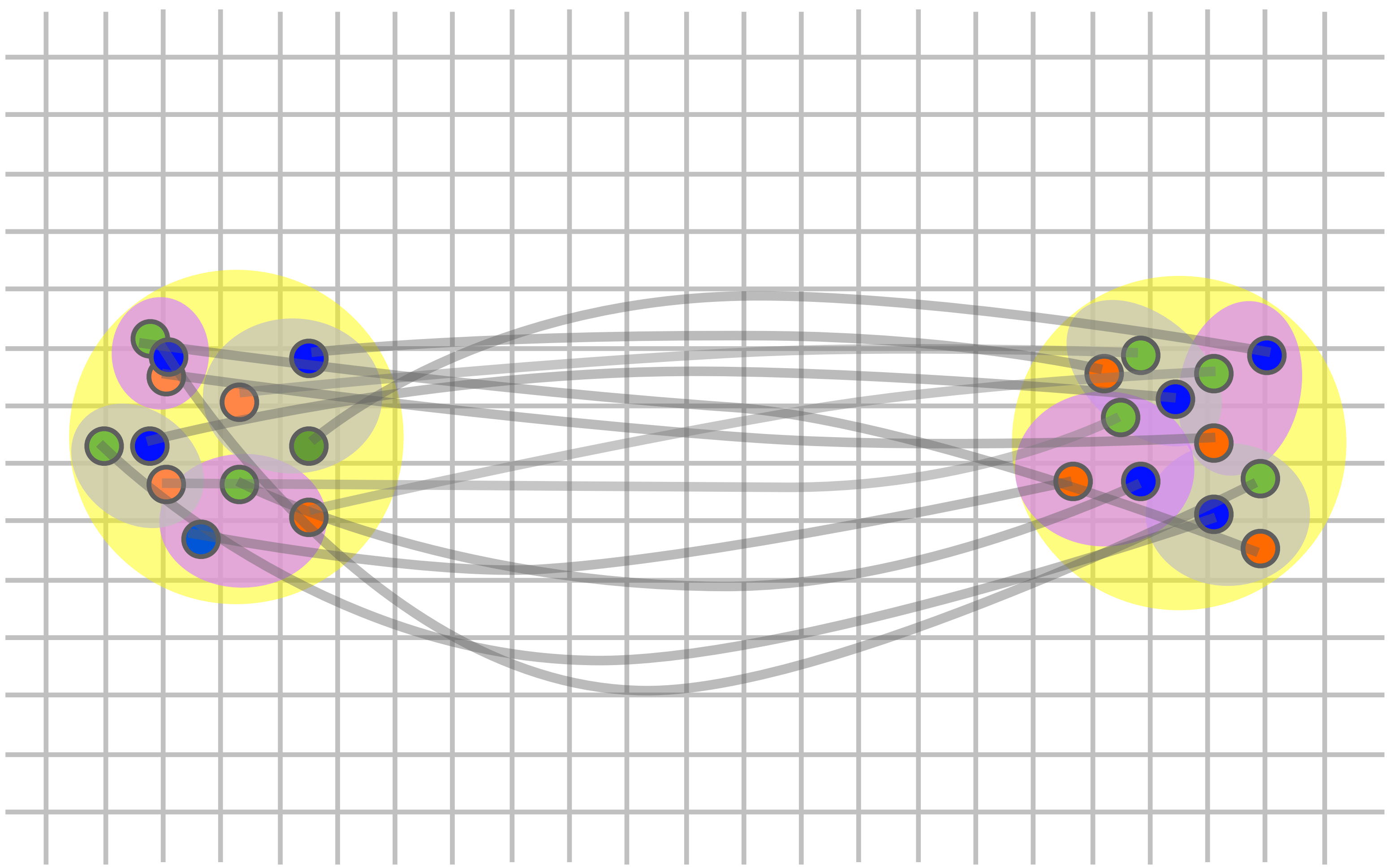


QCD for Nuclear Physics

- Move on to nuclei!
- In practice: a hard problem
  - Physics gets complicated!
- At least two exponentially difficult challenges
  - Noise: probabilistic method so statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially



- Quarks need to be tied together in all possible ways
  - $N_{\mathrm{contractions}} = N_u! N_d! N_s!$

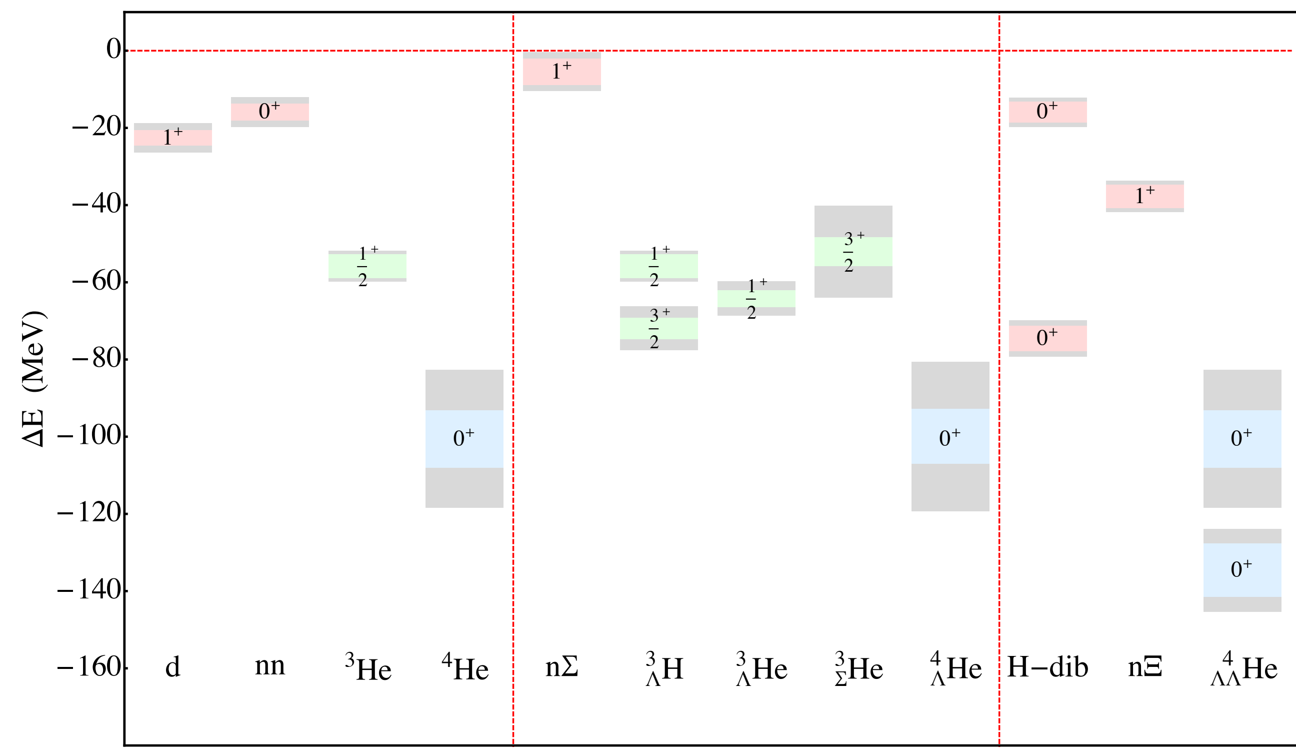


- Managed using algorithmic trickery [WD & Savage, WD & Orginos; Doi & Endres]
  - Study up to N=72 pion systems, A=5 nuclei

Unphysical nuclei

- NPLQCD collaboration
- Case study QCD with unphysical quark masses
  - $m_\pi \sim 800$ MeV, $m_N \sim 1{,}600$ MeV
  - $m_\pi \sim 450$ MeV, $m_N \sim 1{,}200$ MeV

- NPLQCD collaboration
- Case study QCD with unphysical quark masses
  - m_{π}~800 MeV, m_N~1,600 MeV
  - m_{π}~450 MeV, m_N~1,200 MeV



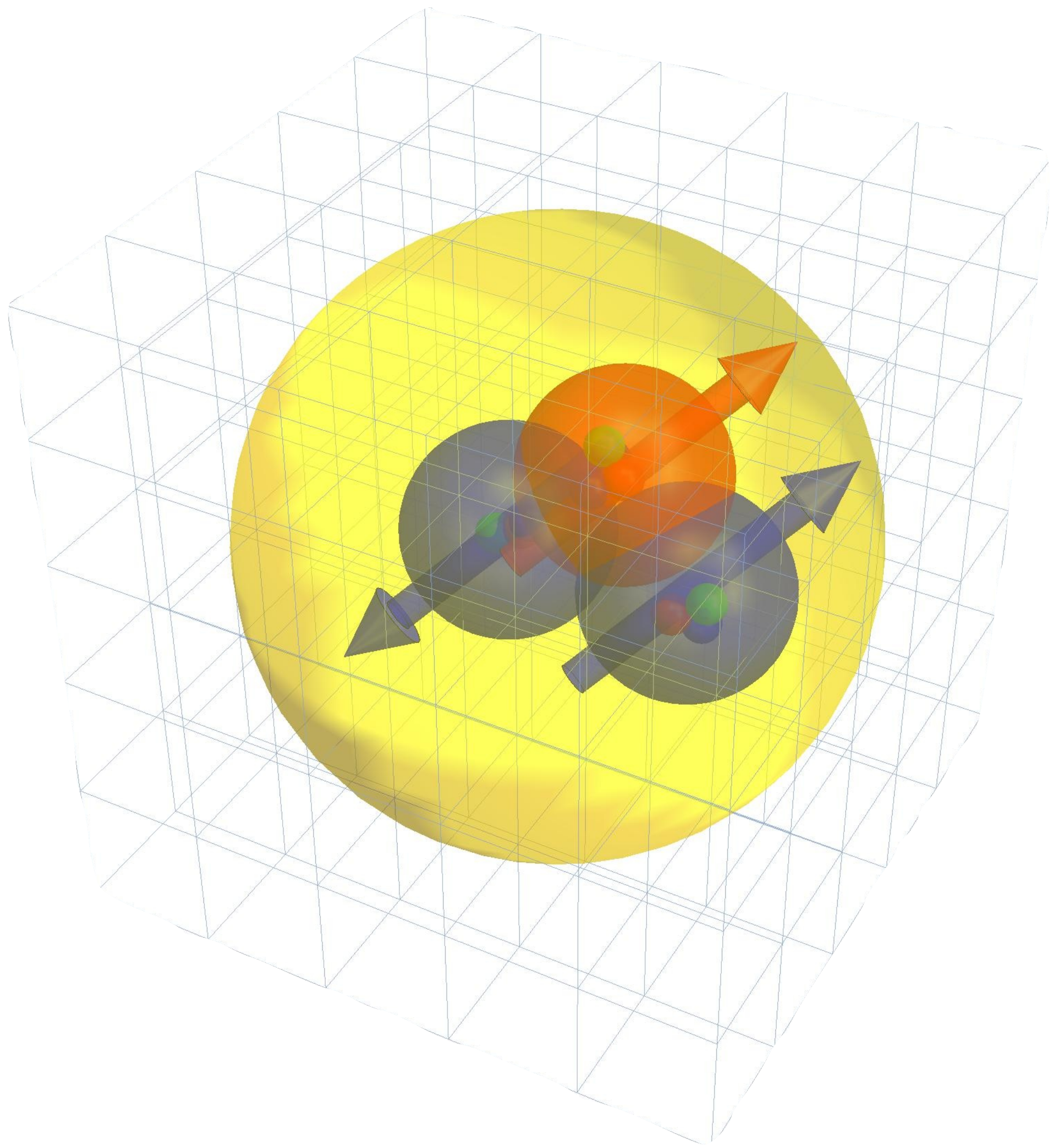
1. Spectrum of light nuclei (A<5)
[PRD **87** (2013), 034506]

Unphysical nuclei

- NPLQCD collaboration
- Case study QCD with unphysical quark masses
  - m_π~800 MeV, m_N~1,600 MeV
  - m_π~450 MeV, m_N~1,200 MeV

1. Spectrum of light nuclei (A<5)
   [PRD **87** (2013), 034506]

2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014),PRD 92, 114502 (2015)]



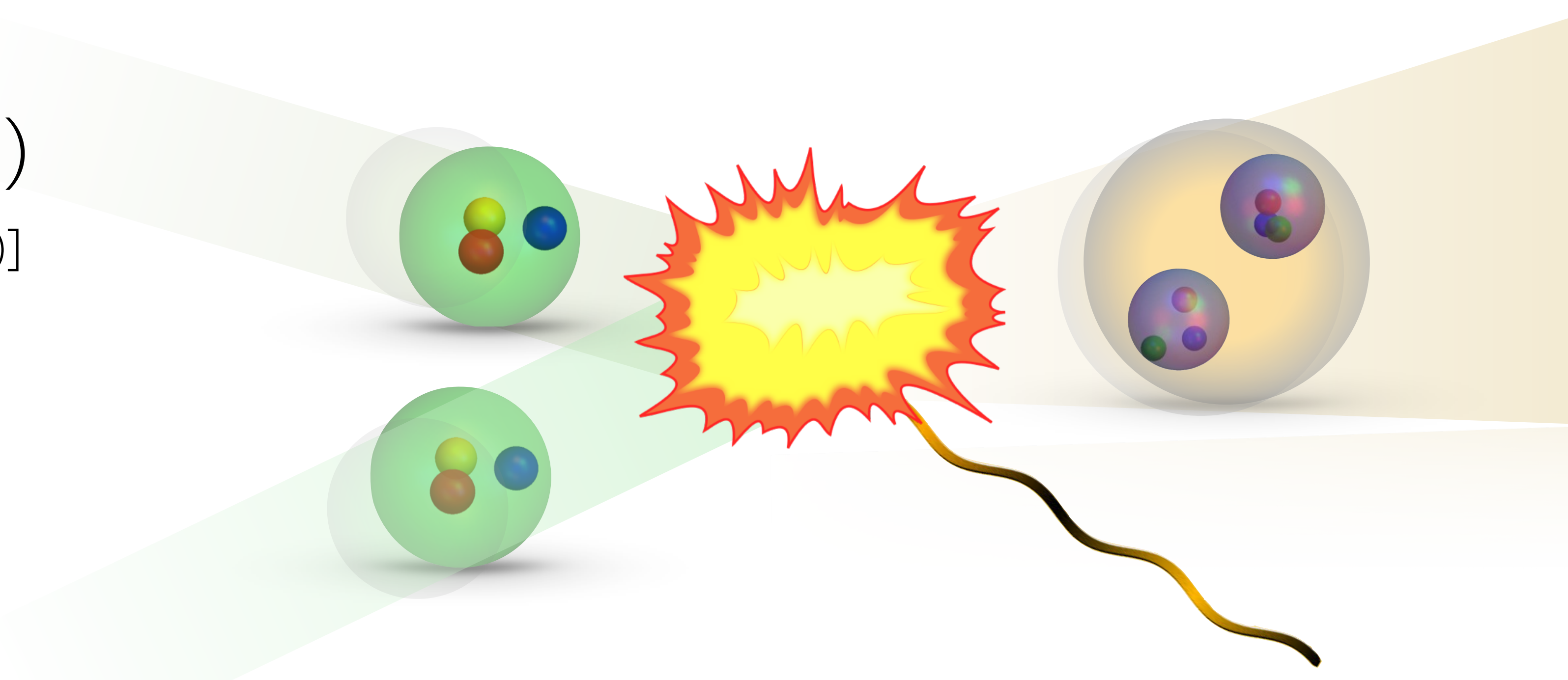
Unphysical nuclei

- NPLQCD collaboration
- Case study QCD with unphysical quark masses
  - m_π~800 MeV, m_N~1,600 MeV
  - m_π~450 MeV, m_N~1,200 MeV

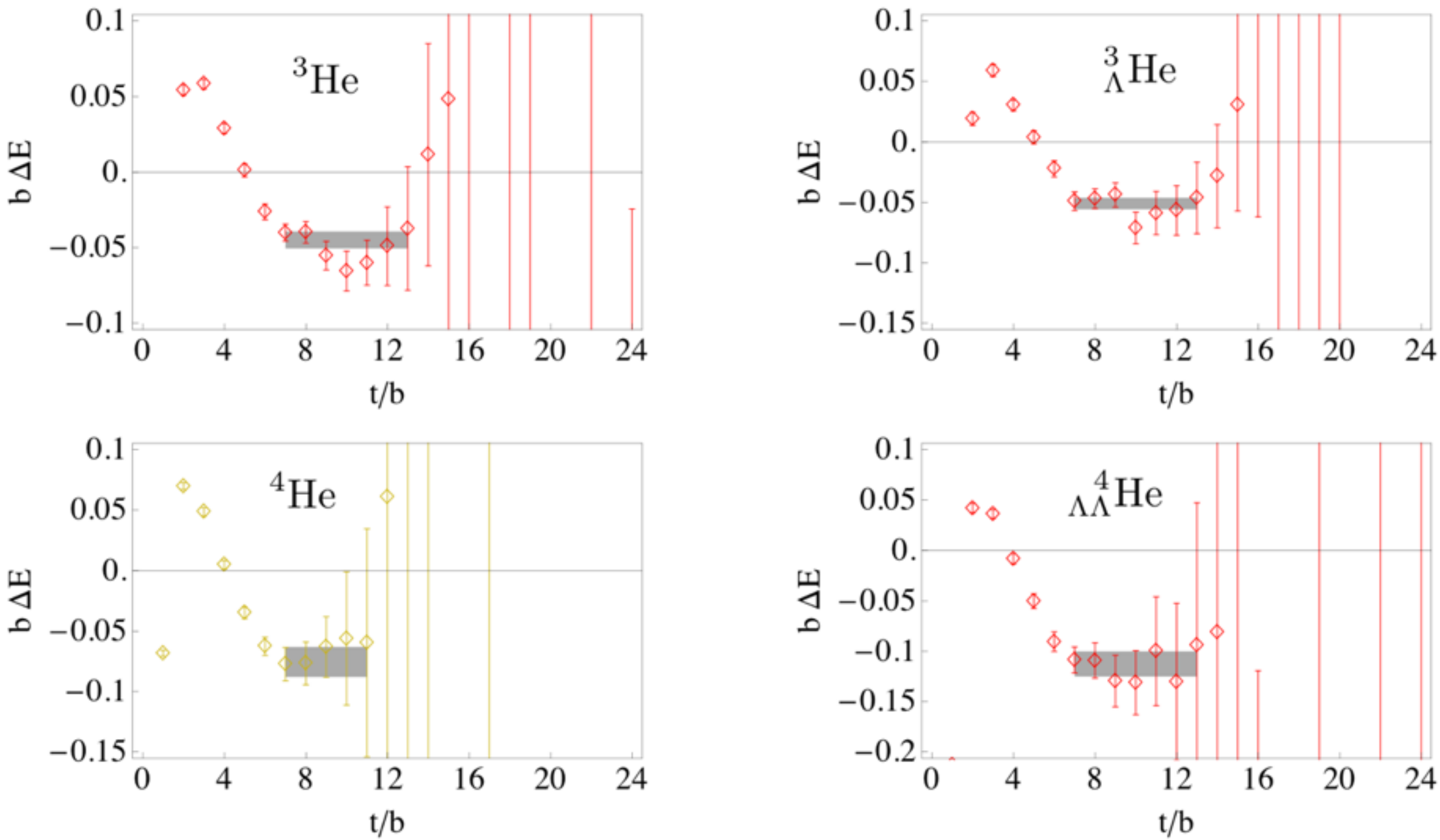
1. Spectrum of light nuclei (A<5)
   [PRD **87** (2013), 034506]

2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014),PRD 92, 114502 (2015)]

3. Nuclear reactions: np→dγ
   [PRL **115**, 132001 (2015)]



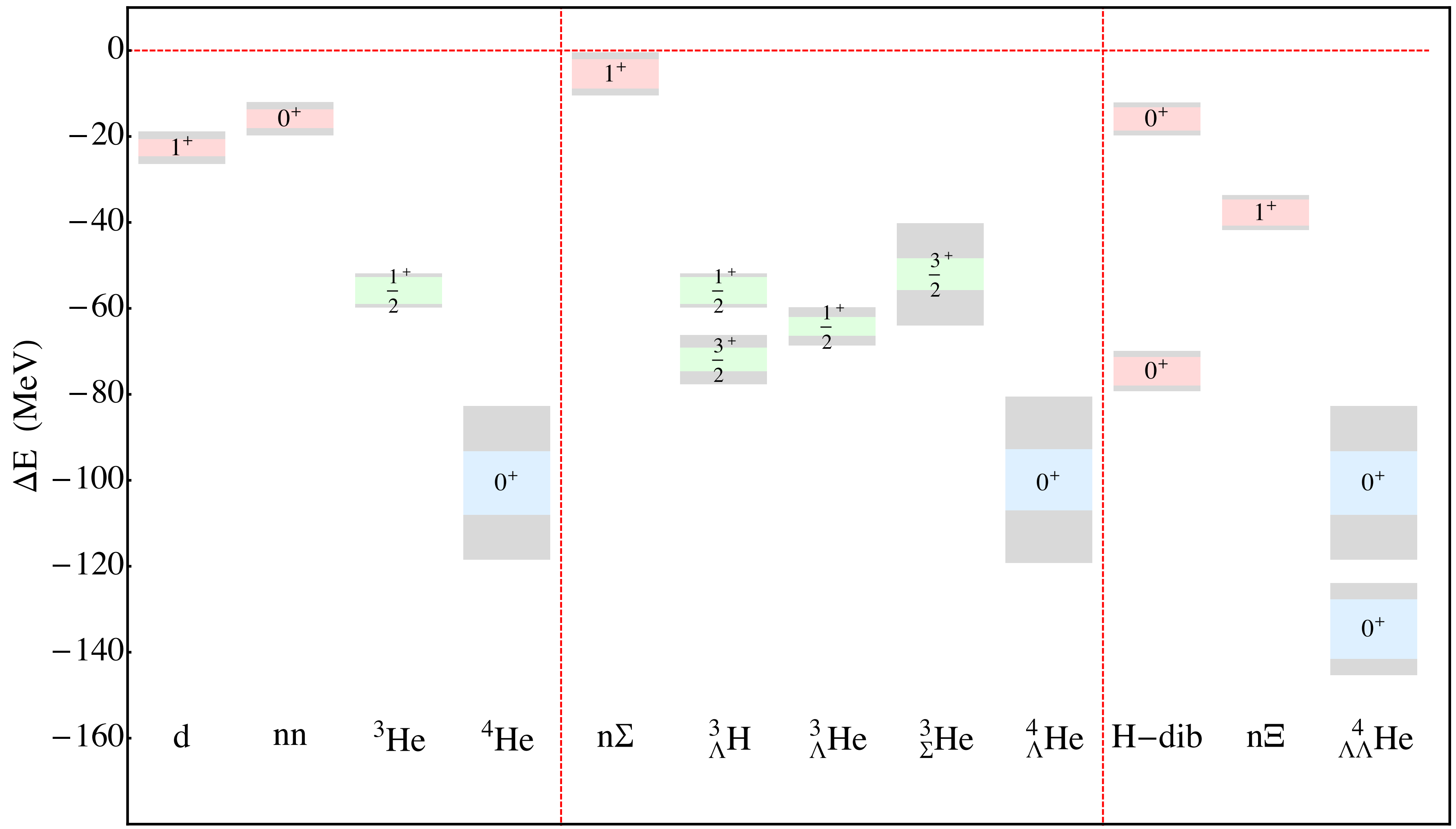
Binding energies



[NPLQCD PRD **87** (2013), 034506]

Light nuclei

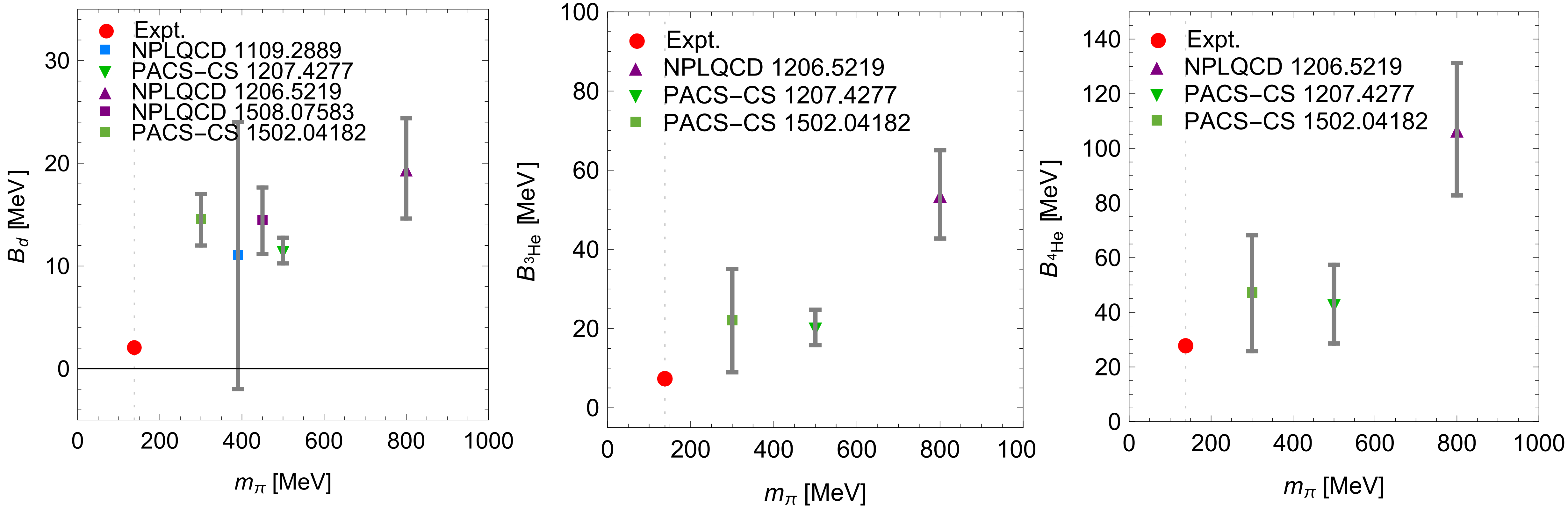
- Light hypernuclear binding energies @ m_π=800 MeV



- More states bound; deeper bindings; more like quark nuggets?

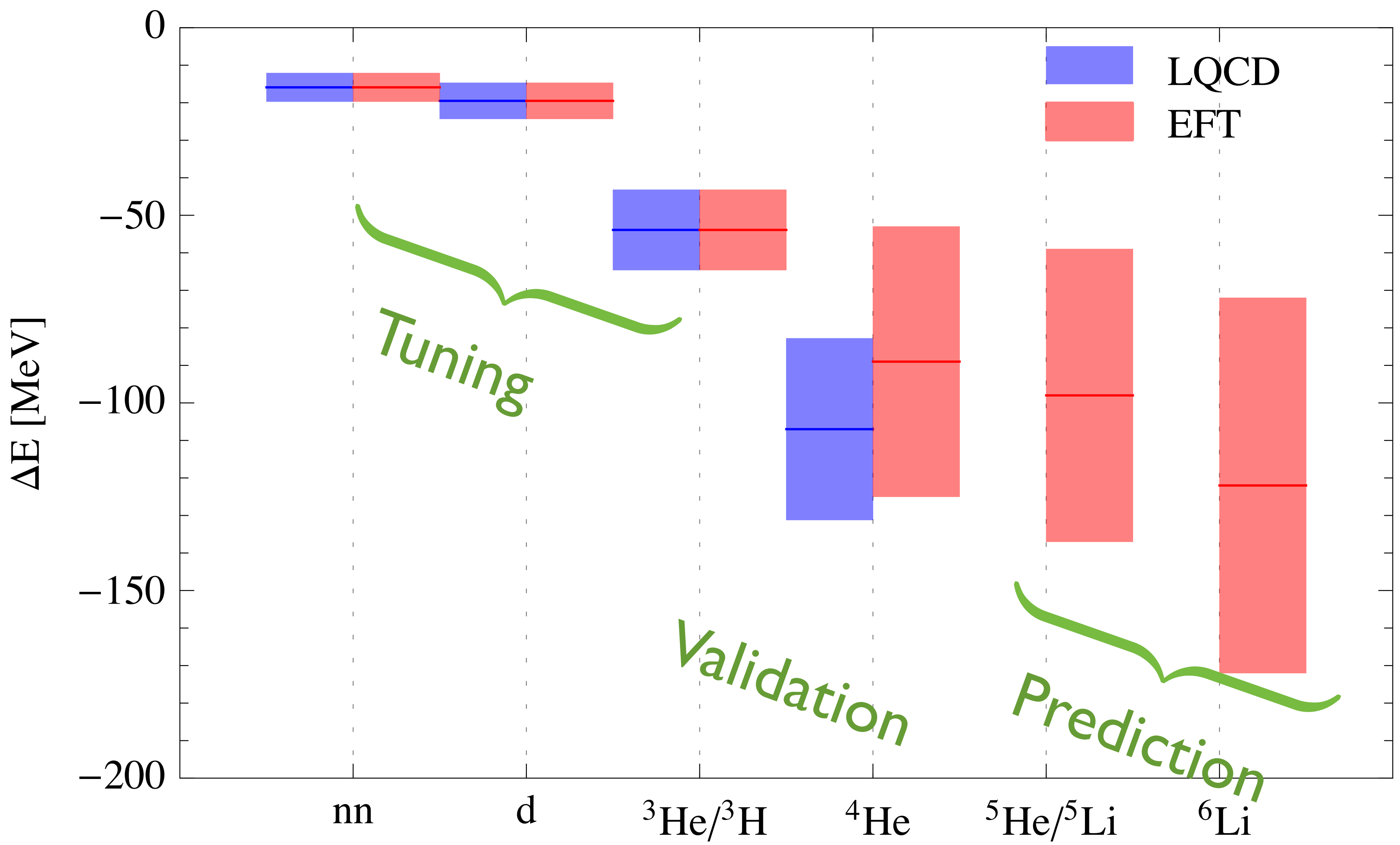
[NPLQCD PRD **87** (2013), 034506]

- Binding energies of few-nucleon systems

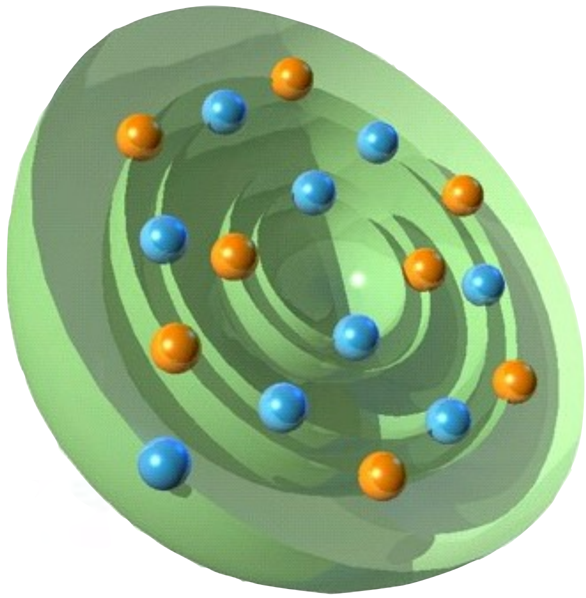


- Obviously more calculations needed at light masses

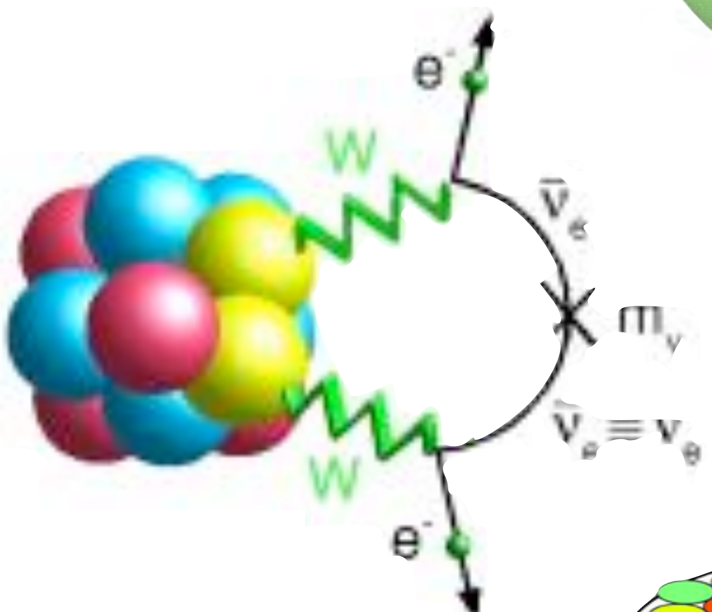
- Combining LQCD and pionless EFT [Barnea et al, PRL 2015]

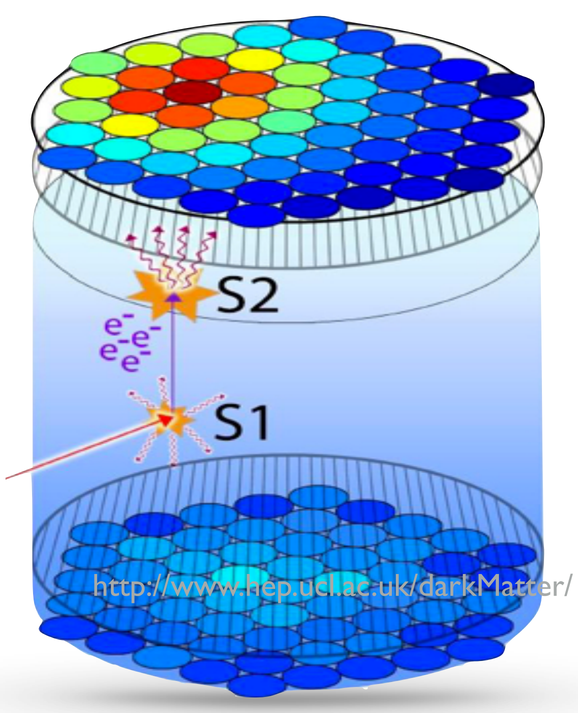


- More detailed matchings possible (FV spectrum,…)



- Nuclear matrix elements important in many contexts
  - Probes of nuclear structure
  - Neutrino-nucleus scattering
  - Tests of fundamental symmetries
  - Dark matter direct detection
  - ….



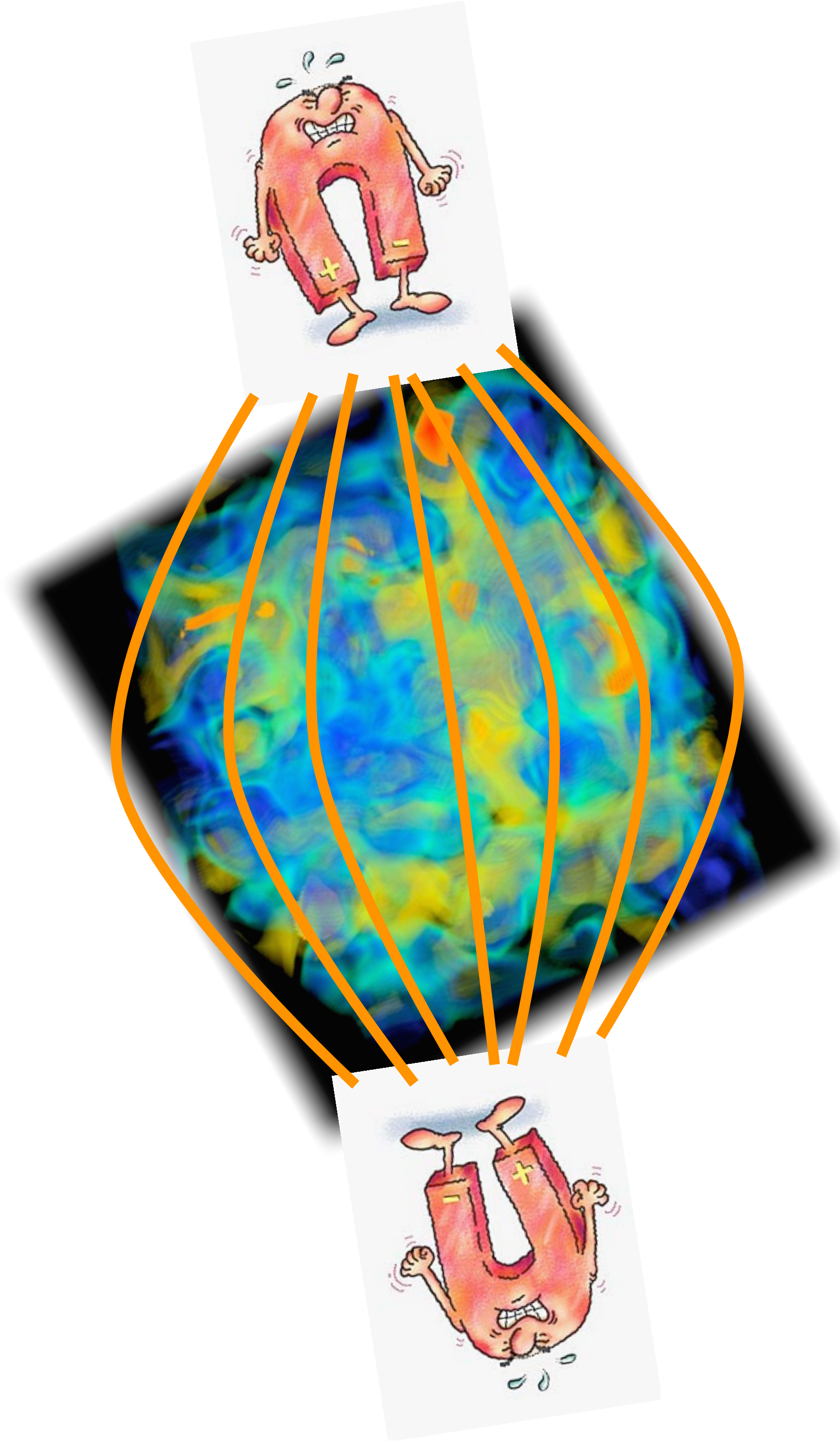


External field method

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moments, polarisabilities, …
- Not restricted to simple EM fields



[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

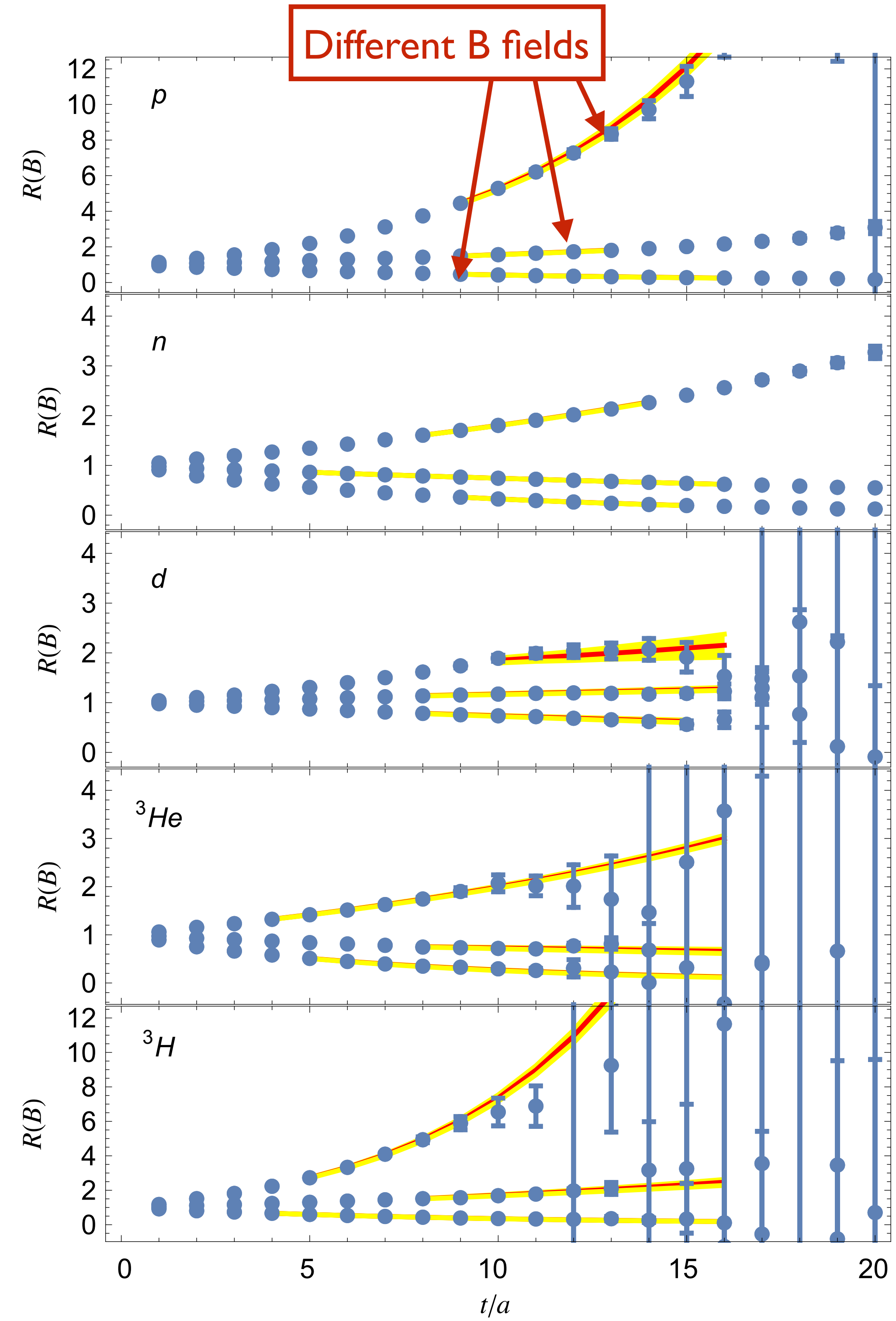
- Magnetic field in *z*-direction (strength quantised by lattice periodicity)
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

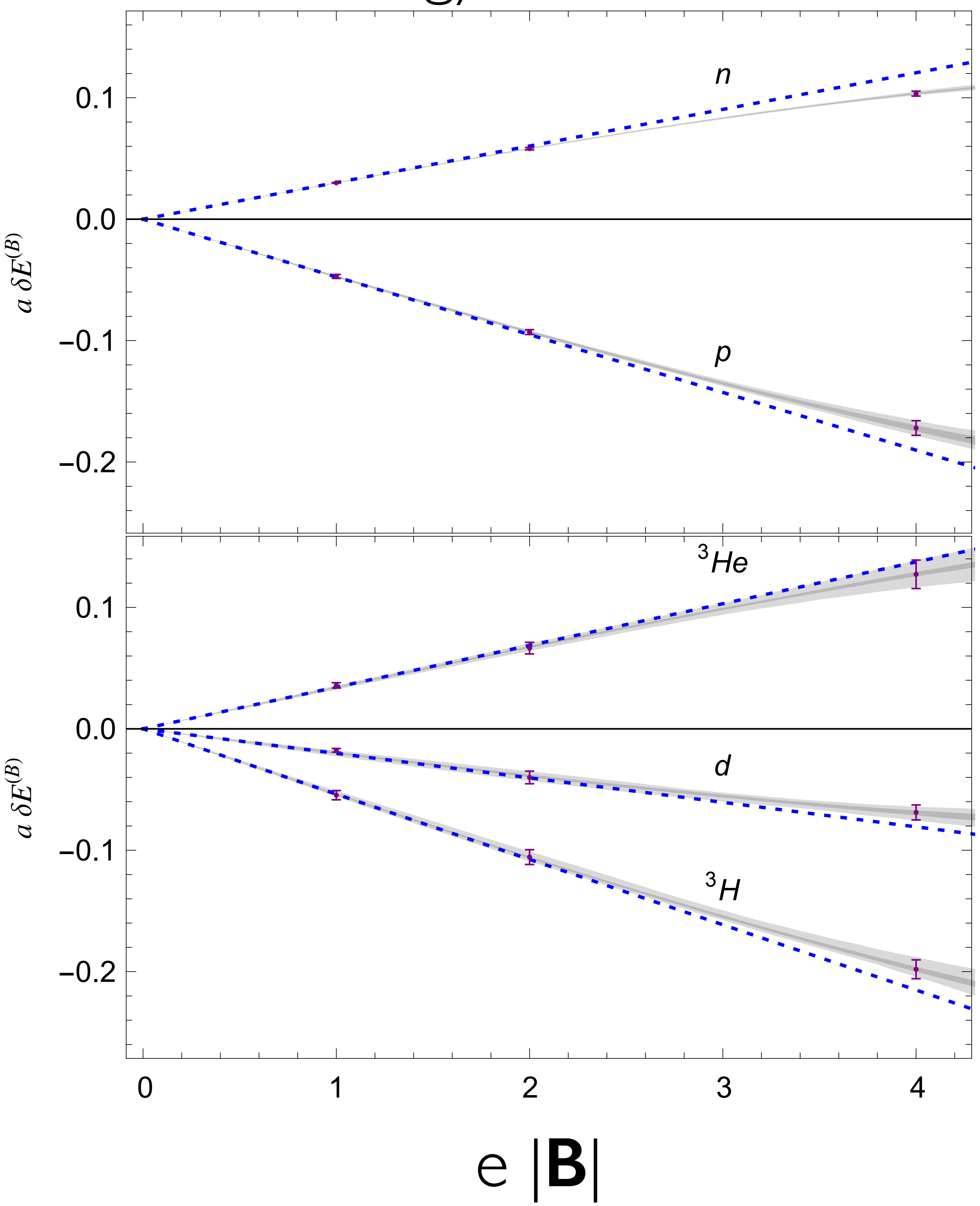
$$R(B) = \frac{C_j^{(B)}(t)\ C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\ C_j^{(0)}(t)} \overset{t\to\infty}{\longrightarrow} Z e^{-\delta E^{(B)} t}$$

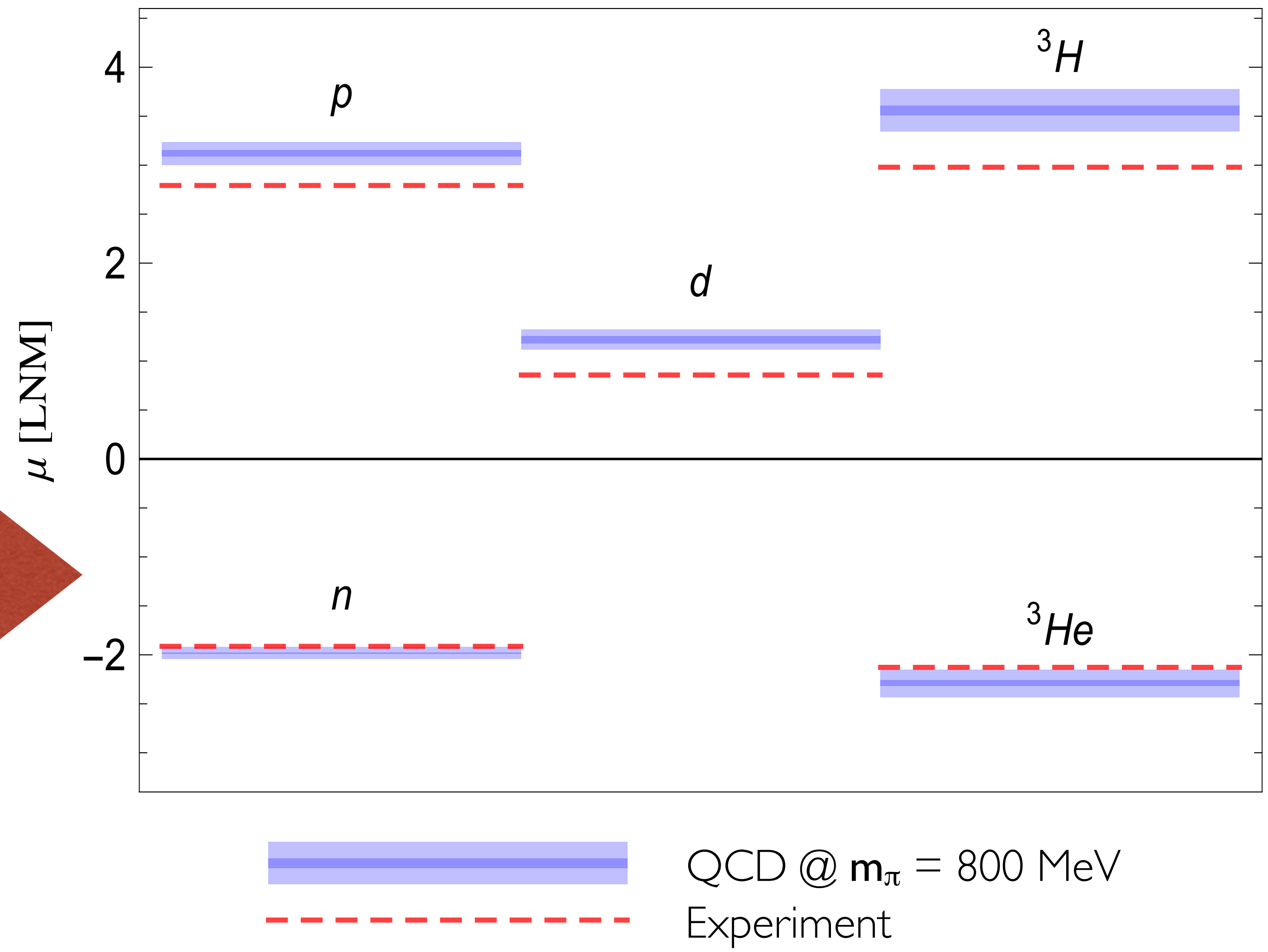
- Careful to be in single exponential region of each correlator



[NPLQCD PRL **113**, 252001 (2014)]

Energy shift vs B





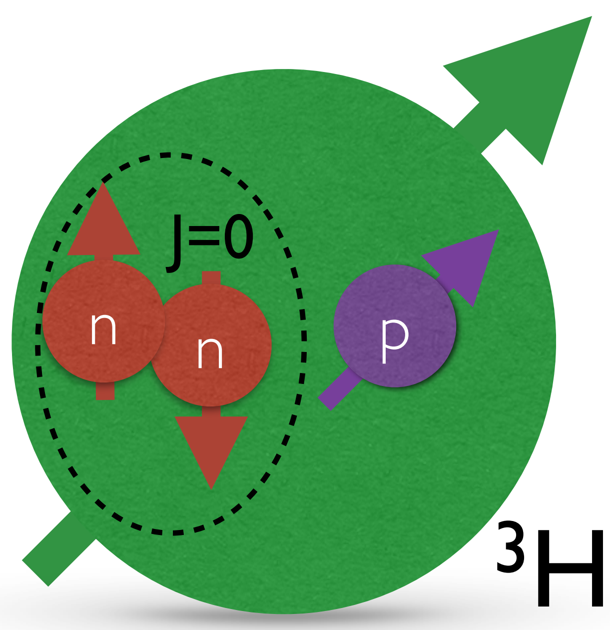
	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

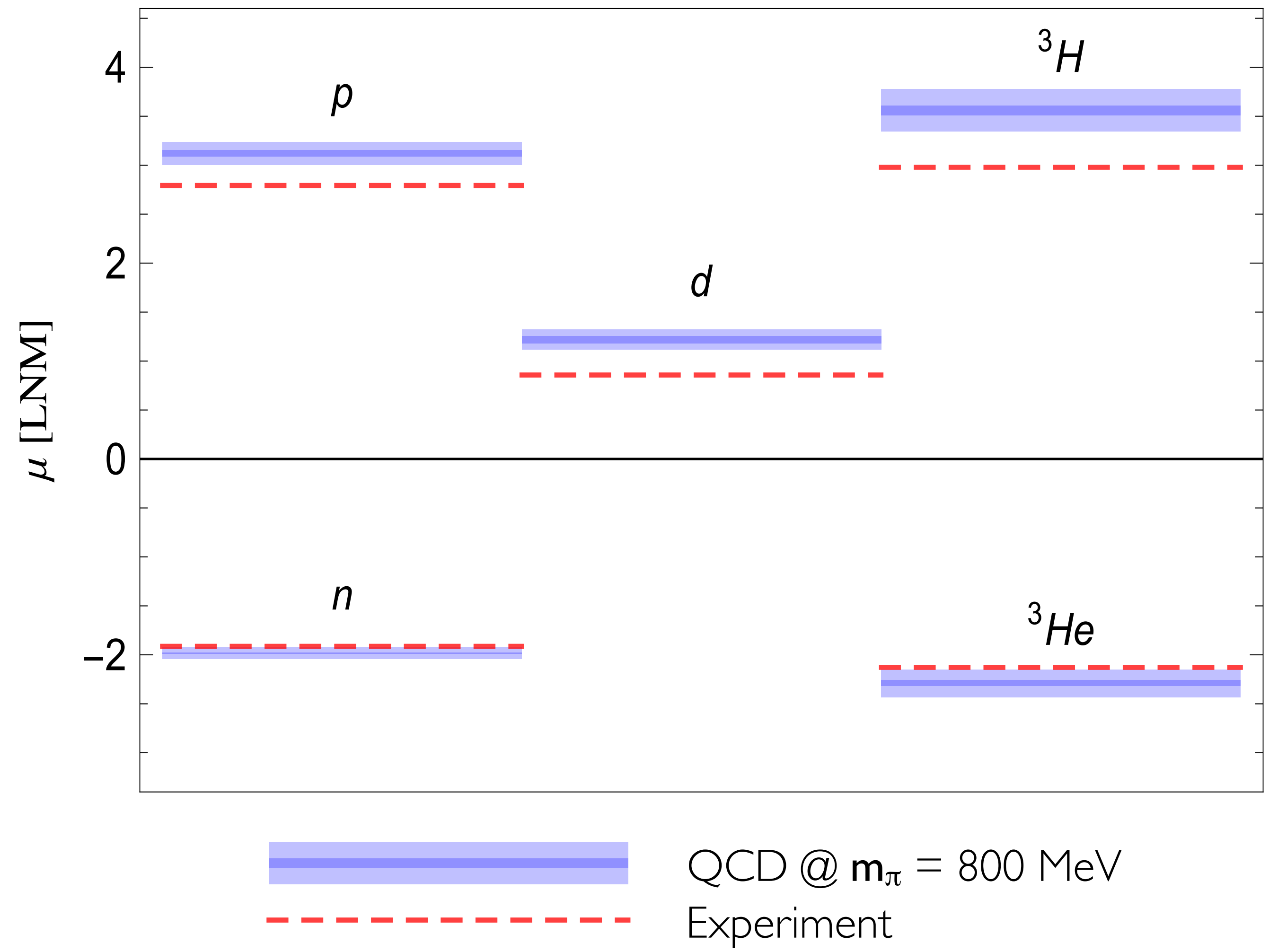
[NPLQCD PRL **113**, 252001 (2014)]

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

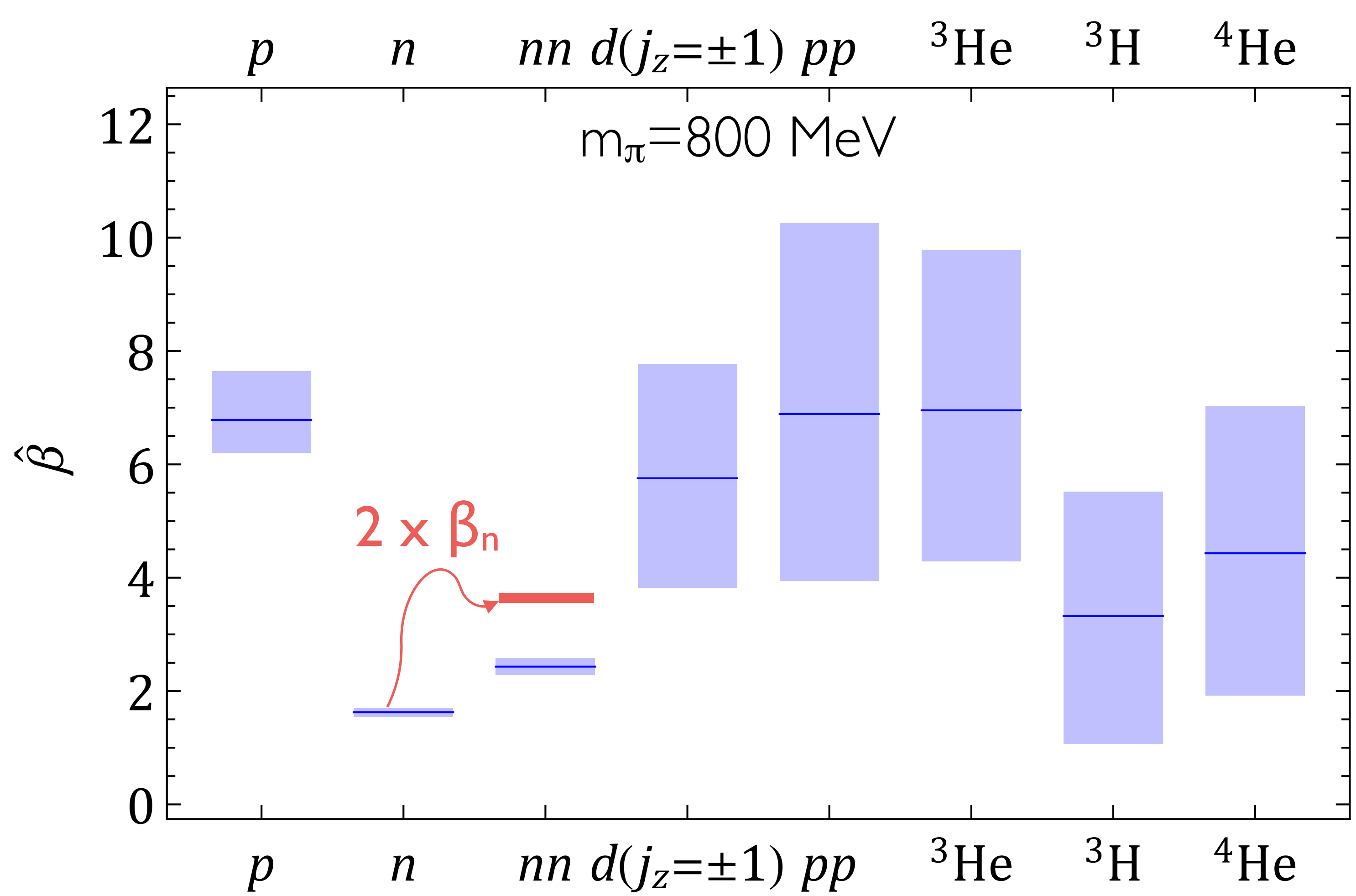
Magnetic Polarisabilities

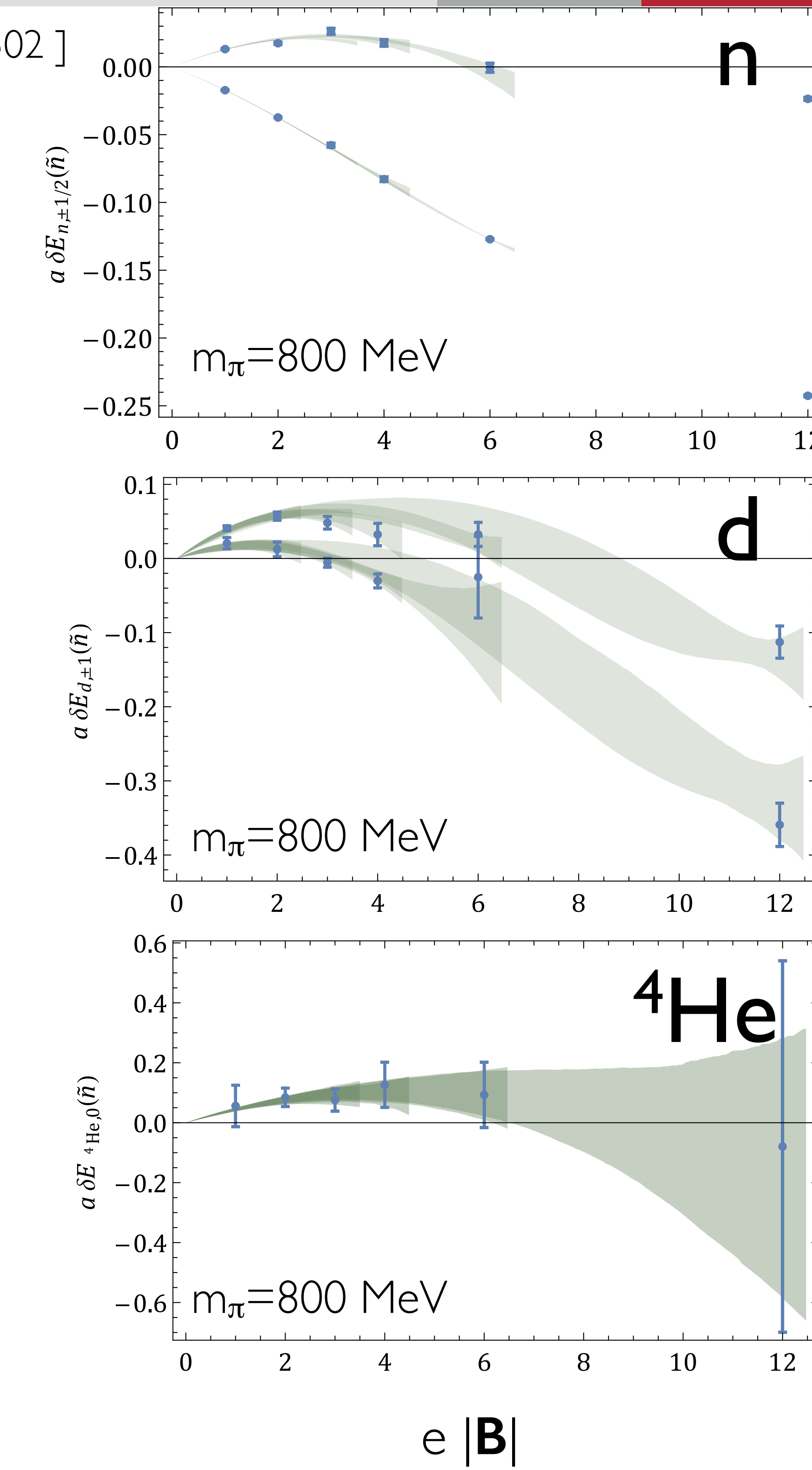
[NPLQCD Phys.Rev. D92 (2015), 114502]

- Second order shifts

$$E_{h;j_z}(\mathbf{B}) \ = \ \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij}B_iB_j\rangle + \ldots$$

- Care required with Landau levels
- Polarisabilities (dimensionless units)

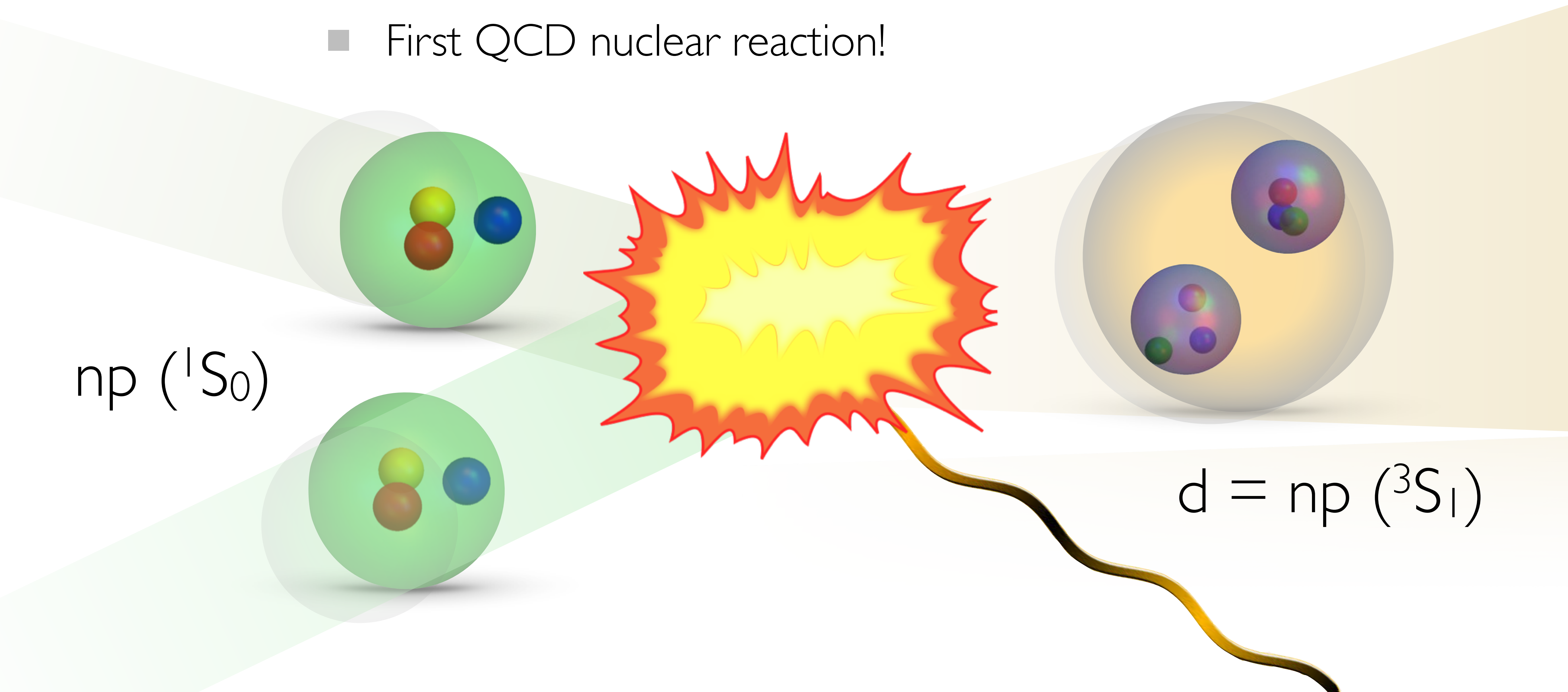




Thermal Neutron Capture Cross-Section

[NPLQCD PRL 115, 132001 (2015)]

- Thermal neutron capture cross-section: np→dγ
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: nucleus is not just nucleons
  - First QCD nuclear reaction!



np→dγ in pionless EFT

$$Z_d = 1/\sqrt{1-\gamma_0 r_3}$$

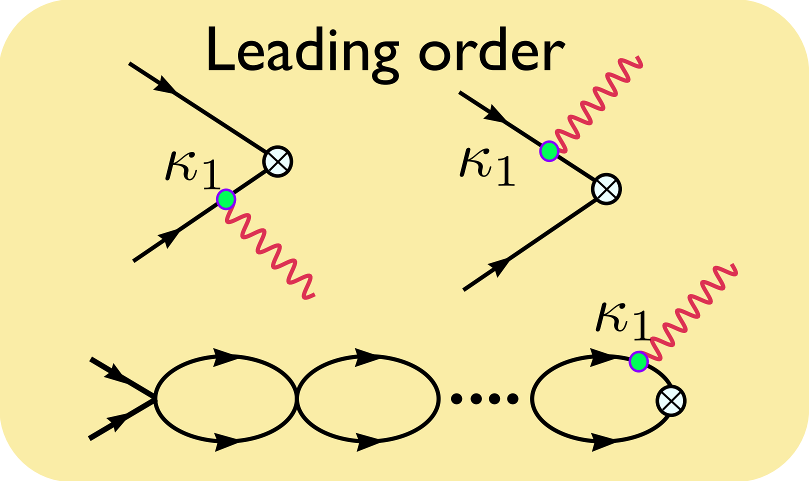
- Cross-section at threshold calculated in pionless EFT

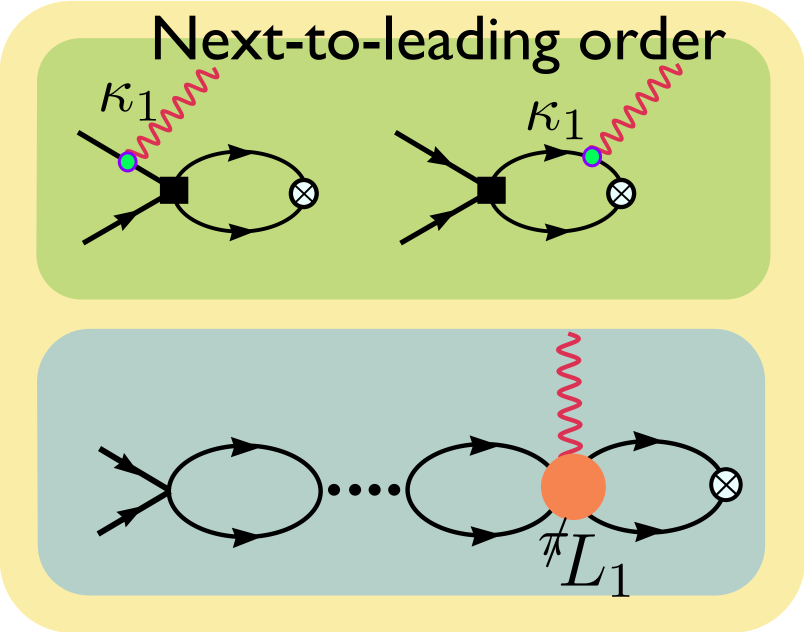
$$\sigma(np\to d\gamma) = \frac{e^2(\gamma_0^2+|\mathbf{p}|^2)^3}{M^4\gamma_0^3|\mathbf{p}|}|\tilde{X}_{M1}|^2+\ldots$$

- EFT expansion at LO given by mag. moments NLO contributions from short-distance two nucleon operators

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2-i|\mathbf{p}|} \times\left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2+|\mathbf{p}|^2}\left(\gamma_0-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2\right)+\frac{\gamma_0^2}{2}l_1\right]$$

- Phenomenological description with 1% accuracy for E< 1MeV
  - Short distance (MEC) contributes ~10%

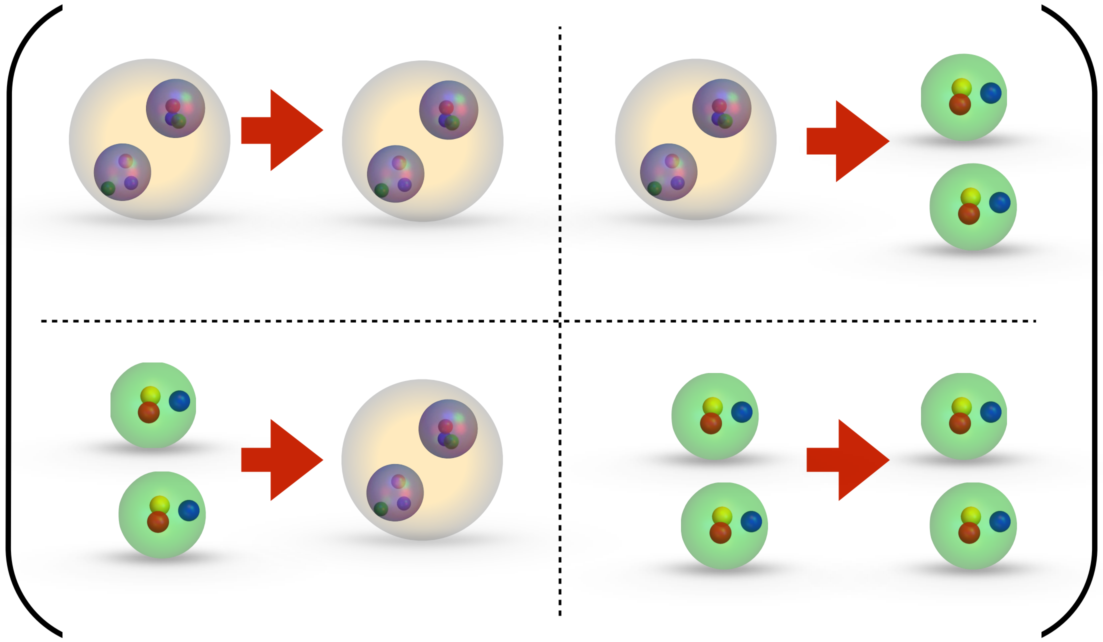




MECs: Riska, Phys.Lett. B38 (1972) 193
Hokert et al, Nucl.Phys. A217 (1973) 14

EFT: Chen et al.,Nucl.Phys. A653 (1999) 386
Chen et al, Phys.Lett. B464 (1999) 1
Rupak Nucl.Phys. A678 (2000) 405

- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ *np* systems

H = 

- Wigner SU(4) super-multiplet symmetry relates $^3S_1$ and $^1S_0$ states
  - Shift of eigenvalues determined by transition amplitude

$$\Delta E_{^3S_1,^1S_0} = \mp \left(\kappa_1 + \overline{L}_1\right) \frac{eB}{M} + \ ...$$

- More generally eigenvalues depend on transition amplitude [WD, Savage 2004]

[NPLQCD PRL **115**, 132001 (2015)]

np→dγ

- Iz=Jz=0 correlation matrix

Lattice correlator with $^3S_1$ source and $^1S_0$ sink

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

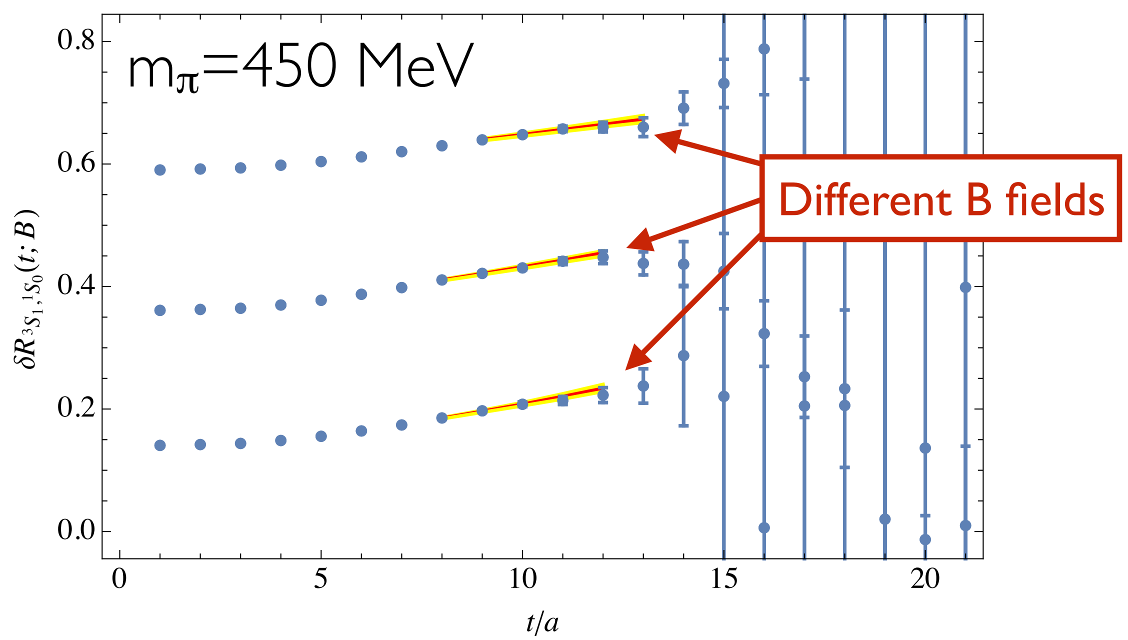
$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \xrightarrow{t\to\infty} \hat{Z}\exp\left[2\ \Delta E_{^3S_1,^1S_0}t\right]$$

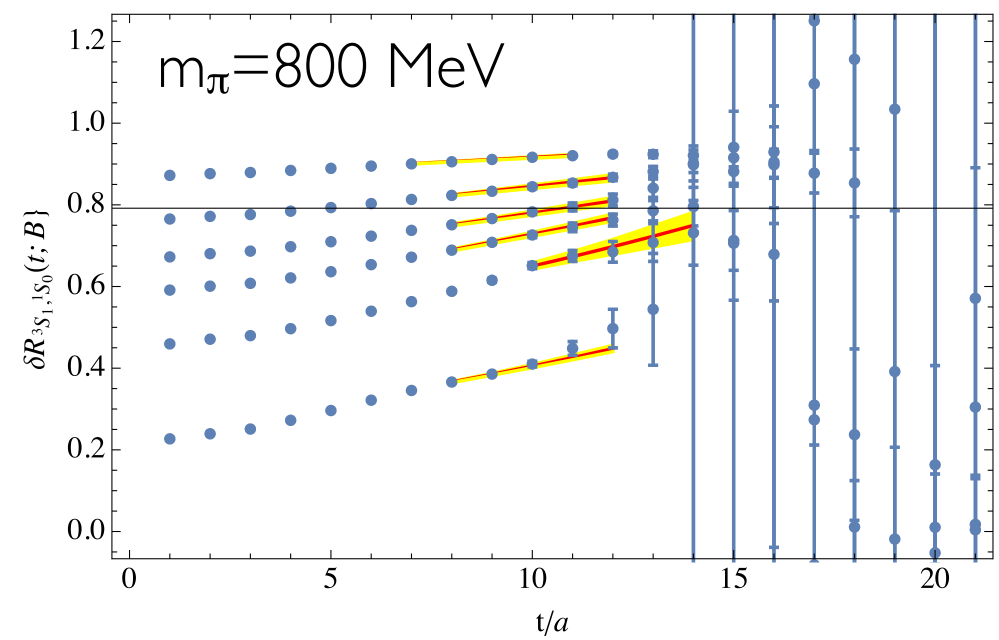
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \to A\ e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

$$\begin{aligned}\delta E_{^3S_1,^1S_0} &\equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\to 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)\end{aligned}$$

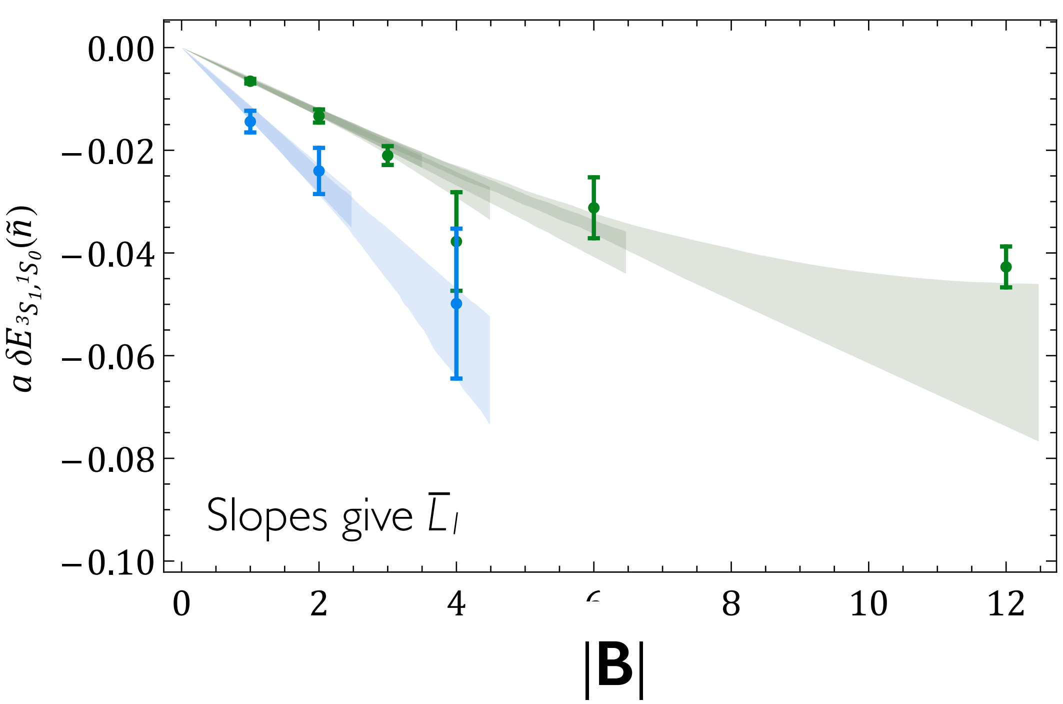
[NPLQCD PRL **115**, 132001 (2015)]

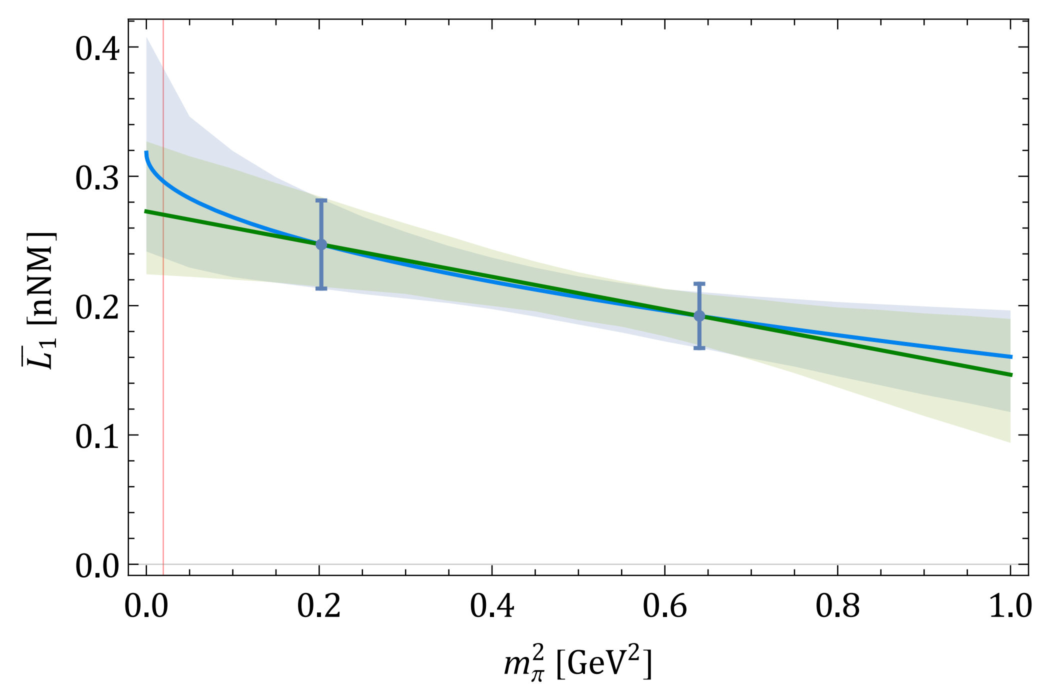
Correlator ratios





Field strength & mass dependence





[NPLQCD PRL **115**, 132001 (2015)]

np→dγ

- Extracted short-distance contribution at physical mass

$$\overline{L}_1^{\rm lqcd} = 0.285(\,^{+63}_{-60}\,)\ {\rm nNM} \qquad l_1^{\rm lqcd} = -4.48(\,^{+16}_{-15}\,)\ {\rm fm}$$

- Combine with phenomenological nucleon magnetic moment, scattering parameters at incident neutron velocity v=2,200 m/s

$$\sigma^{\rm lqcd}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\rm lqcd})\ {\rm mb}$$

$$\sigma^{\rm lqcd}(np \to d\gamma) = 332.4(\,^{+5.4}_{-4.7}\,)\ {\rm mb}$$

c.f. phenomenological value

$$\sigma^{\rm expt}(np \to d\gamma) = 334.2(0.5)\ {\rm mb}$$

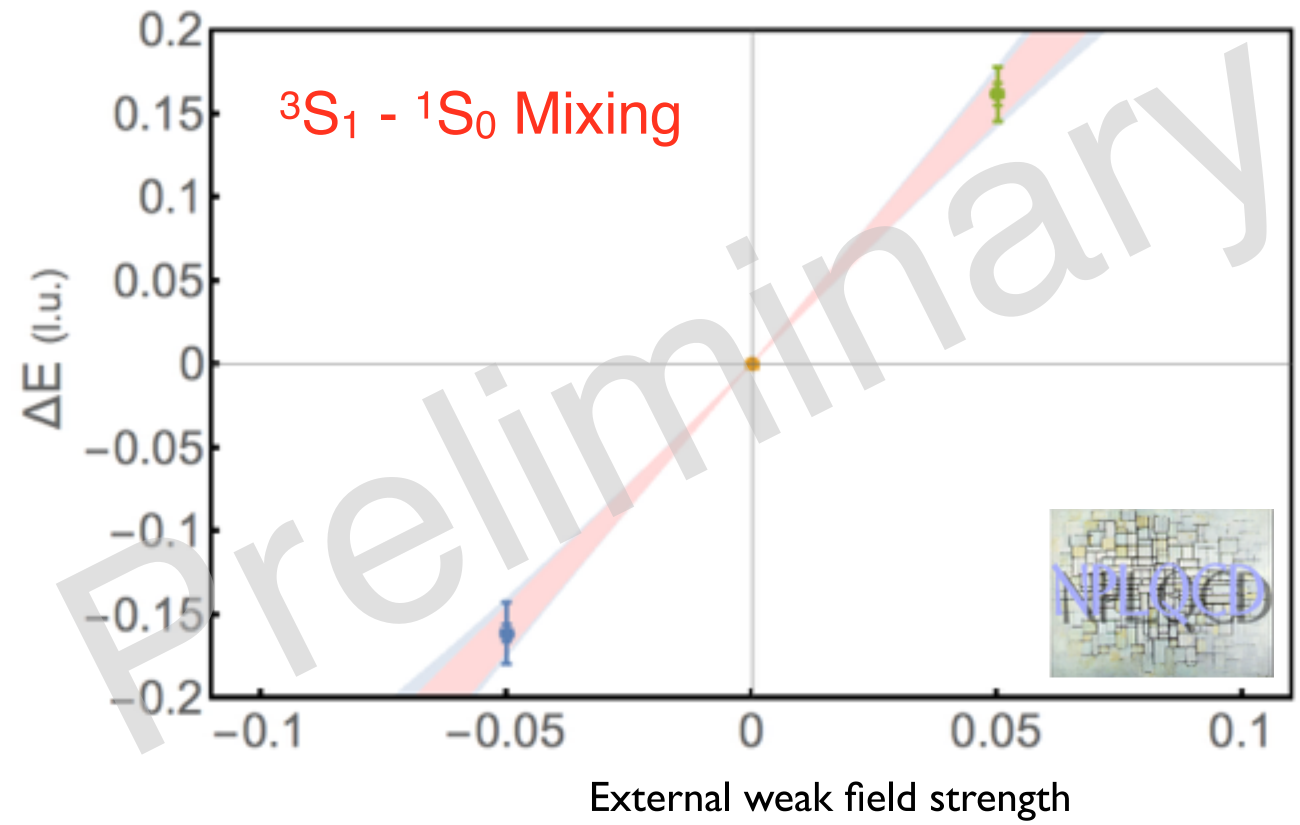
- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ {\rm MeV}}(np \to d\gamma) \sim 10\ {\rm mb}$$

[NPLQCD PRL **115**, 132001 (2015)]

Further matrix elements

- Background field approach to other cases
- Axial coupling to NN system
  - pp fusion: "Calibrate the sun"
  - Muon capture: MuSun @ PSI
  - $d\,\nu \rightarrow n\,n\,e^{+}$: SNO
- Quadrupole moments
- Axial form factors
- Scala matrix elements

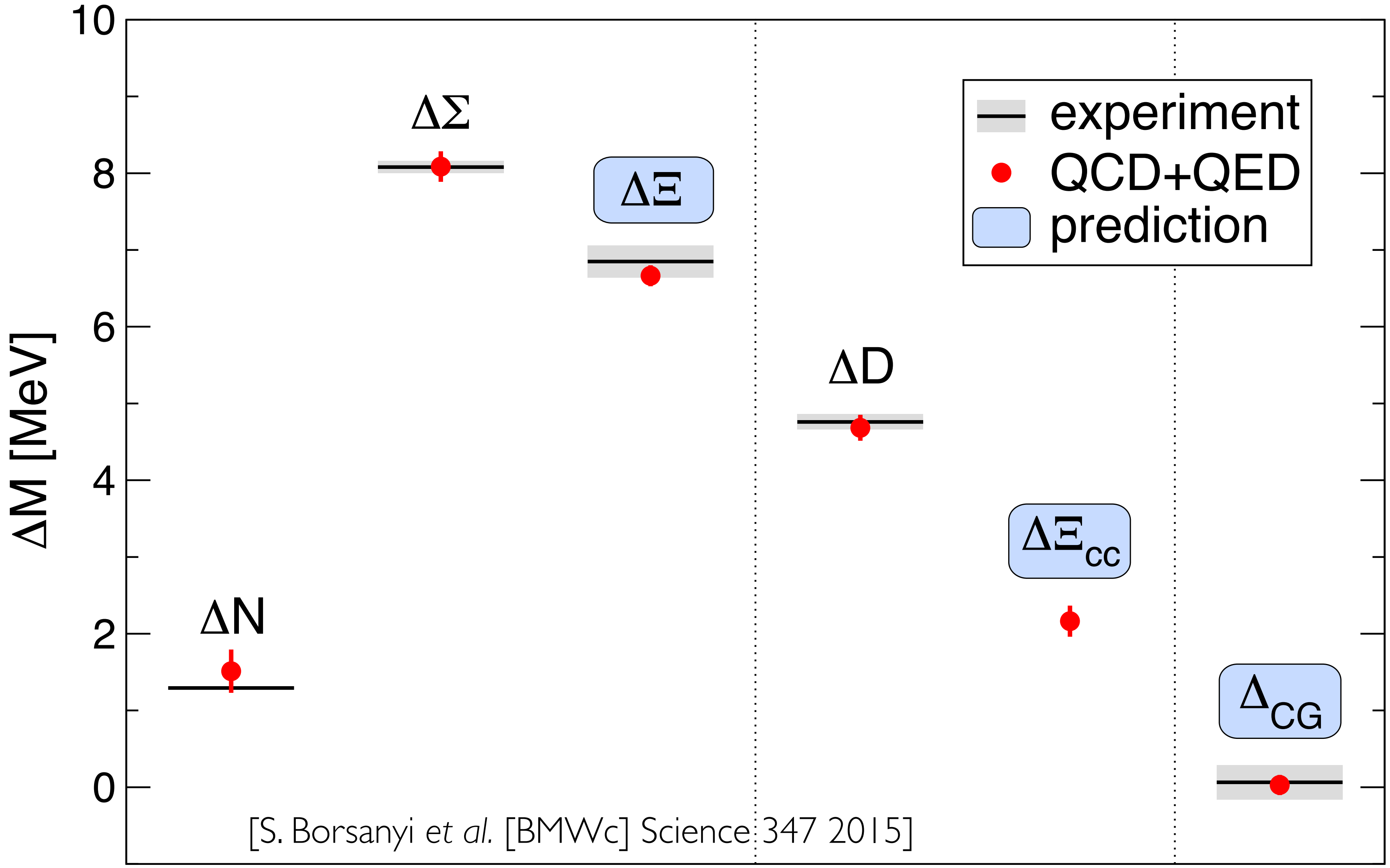


Nuclear physics from the ground up

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, ...)
  - Structure: magnetic moments and polarisabilities
  - Electroweak interactions: thermal capture cross-section
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Nuclear matrix elements important to experimental program
  - Learn many interesting things about nuclear physics along the way

Proton–neutron mass splitting

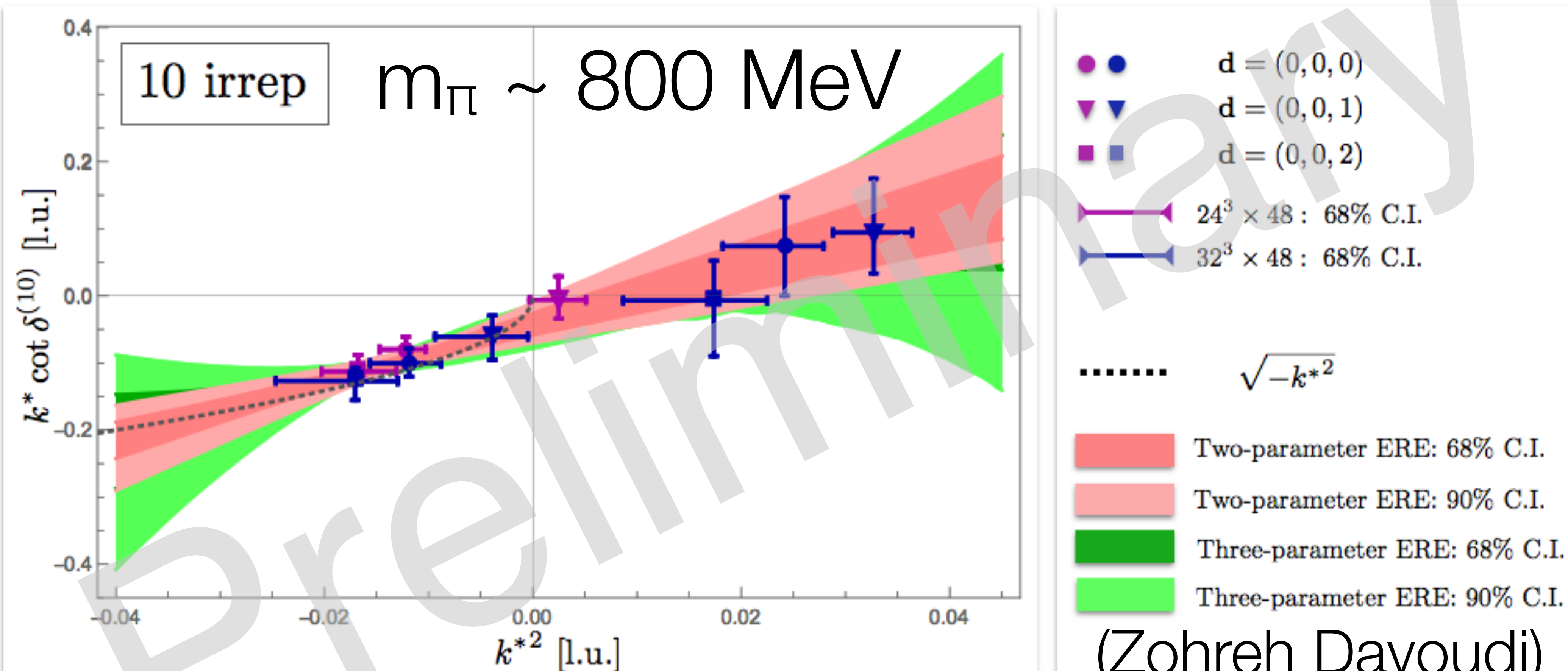
- Isospin mass splittings in QCD+QED



- Clarifies role of E&M effects and quark masses in M_n - M_p

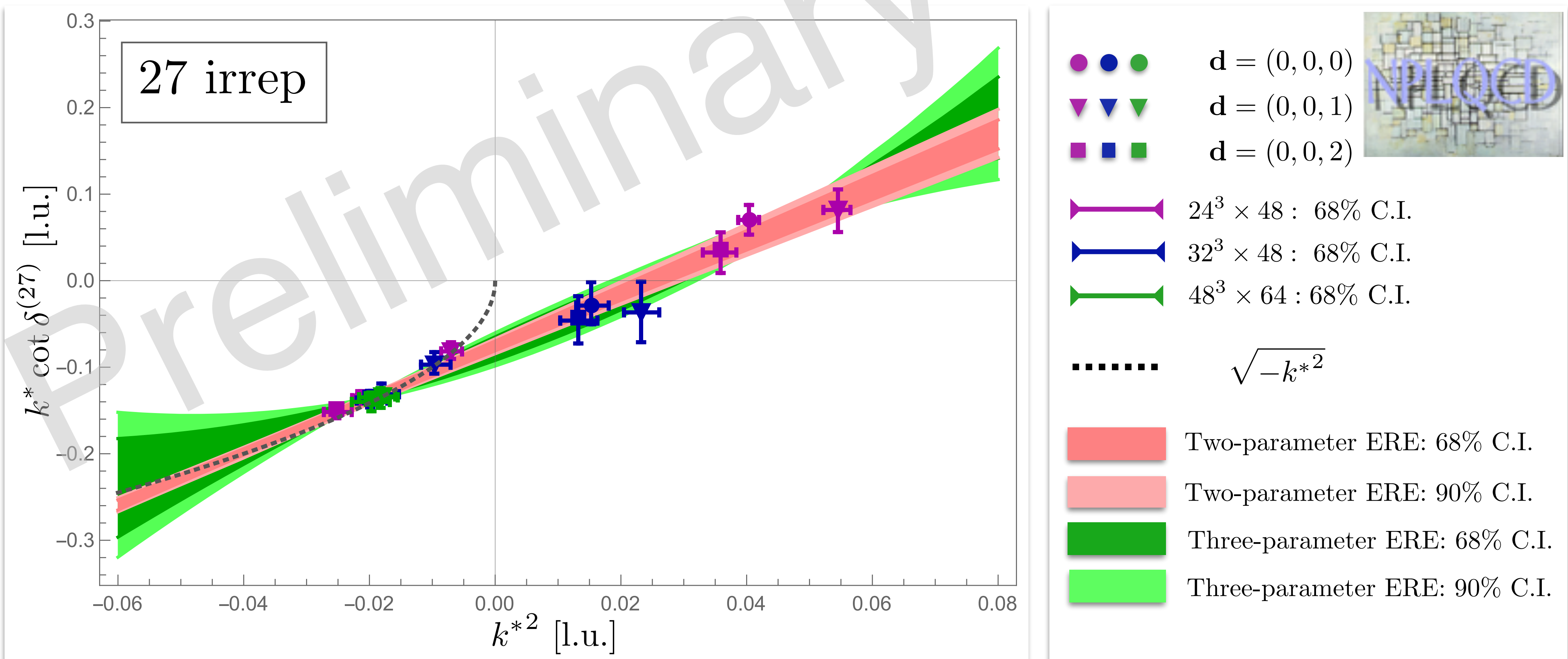
NN (YN,YY) scatteriing

- Scattering studied via finite-volume energy levels



- Clarifies role of E&M effects and quark masses in M_n - M_p

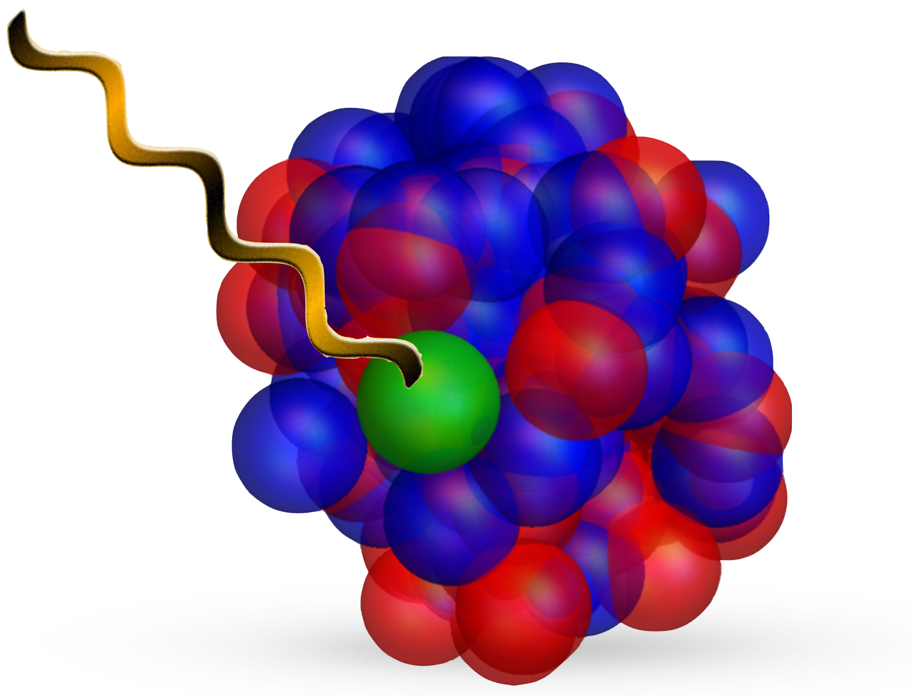
- Scattering studied via finite-volume energy levels

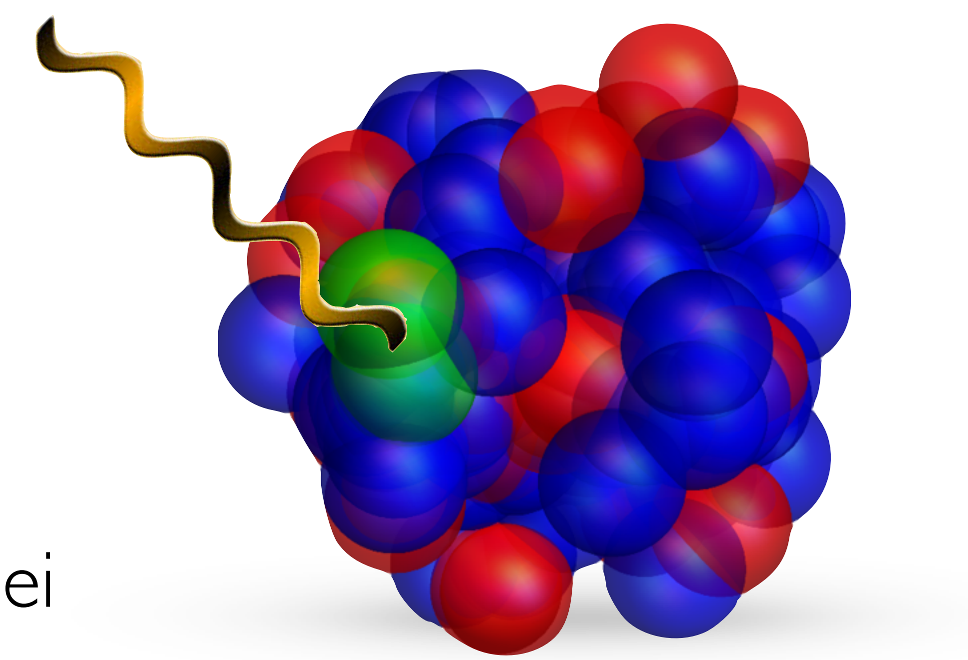


- Clarifies role of E&M effects and quark masses in M_n - M_p

External currents and nuclei

- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei





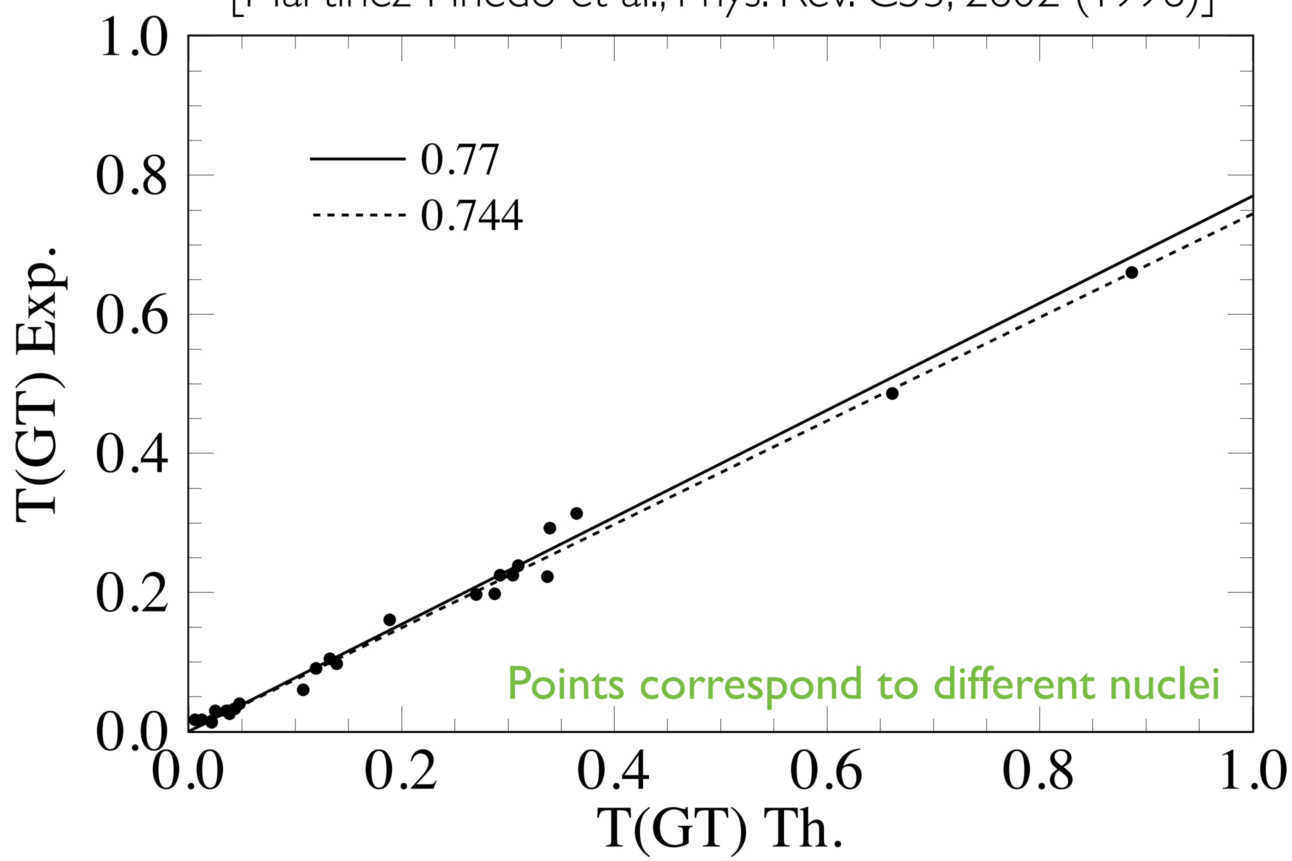
Nuclear uncertainties

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

  - Well measured for large range of nuclei (30<A<60)
  - Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
  - Matrix elements systematically off by 20–30%
  - "Correct" by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]



$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_\pm^k || i \rangle}{\sqrt{2J_i + 1}}$$

Nuclear sigma terms

- One possible DM interaction is through scalar exchange

$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

- Direct detection depends on nuclear matrix element

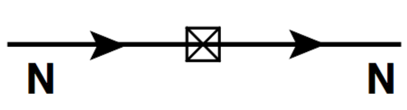
$$\sigma_{Z,N} \;=\; \overline{m}\langle Z, N(\mathrm{gs})|\; \overline{u}u + \overline{d}d \;|Z, N(\mathrm{gs})\rangle \;=\; \overline{m}\frac{d}{d\overline{m}} E_{Z,N}^{(\mathrm{gs})}$$

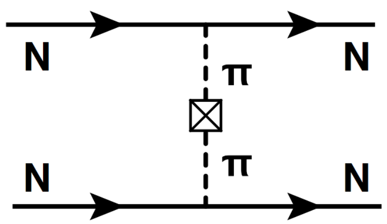
  - Accessible via Feynman-Hellman theorem

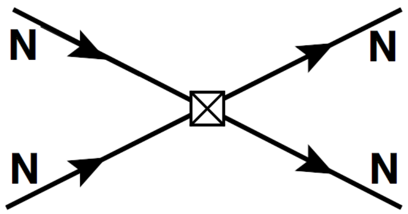
- At hadronic/nuclear level

$$\mathcal{L} \;\rightarrow\; G_F\;\overline{\chi}\chi\;\Big(\;\frac{1}{4}\langle 0|\overline{q}q|0\rangle\;\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\;+\;\frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \\ -\;\frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\;-\;4N^\dagger a_{S,\xi}N\right)\;+\;...\Big)$$

  - Contributions:

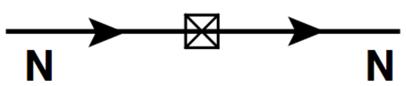






Nucleon sigma term

- Single nucleon contribution



calculated by many lattice groups

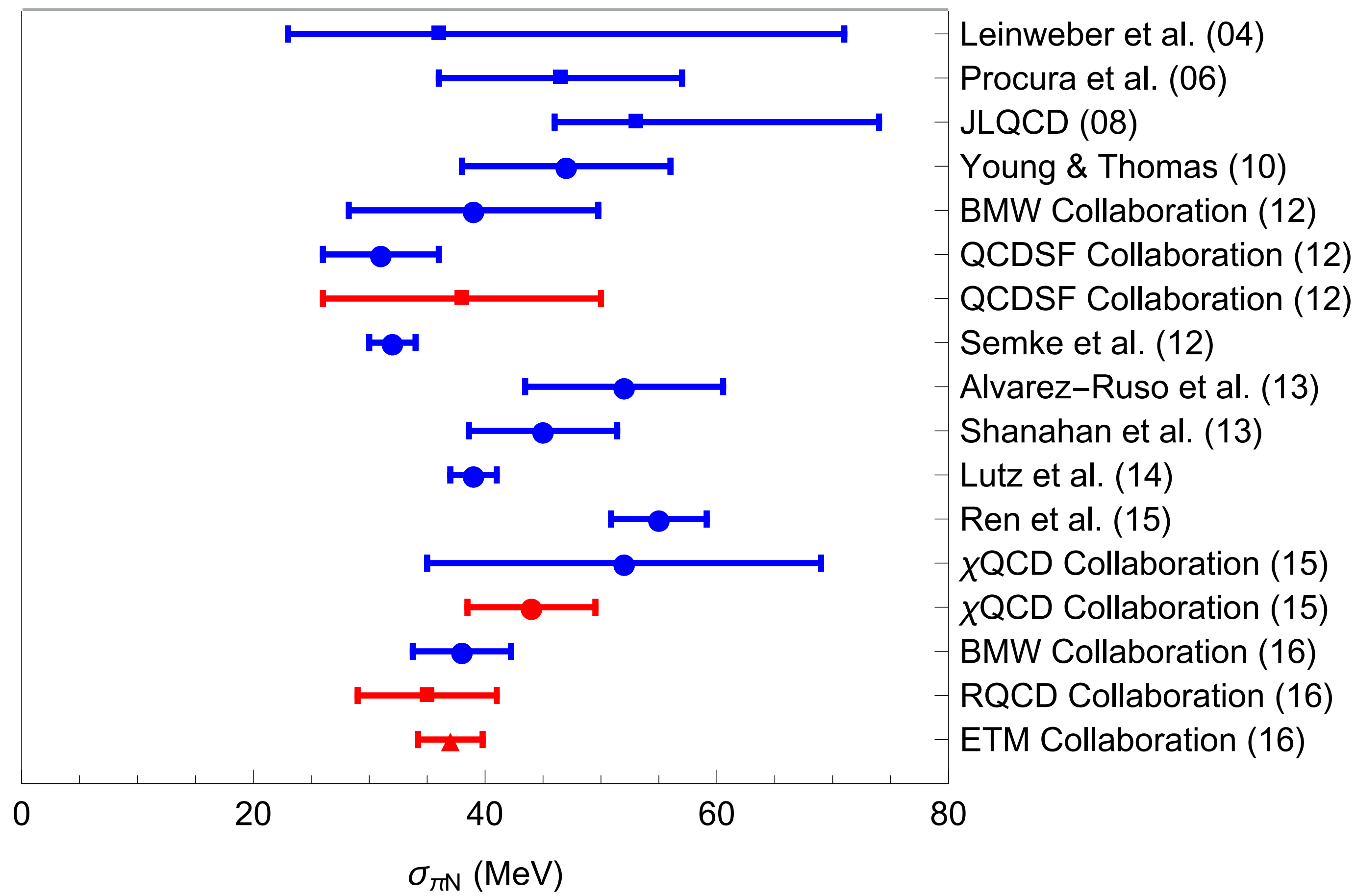
- Results stabilised

- Interesting ~3σ tension with recent πN dispersive analysis

[Hoferichter et al, PRL. **115** (2015) 092301]

$$\sigma_{\pi N} = (59.1 \pm 3.5)\,\mathrm{MeV}$$

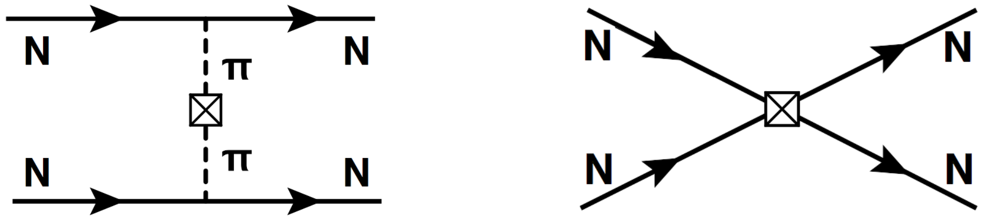
[P Shanahan 2016]



(a) $\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | \overline{u}u + \overline{d}d | N \rangle$

Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]



- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z, N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

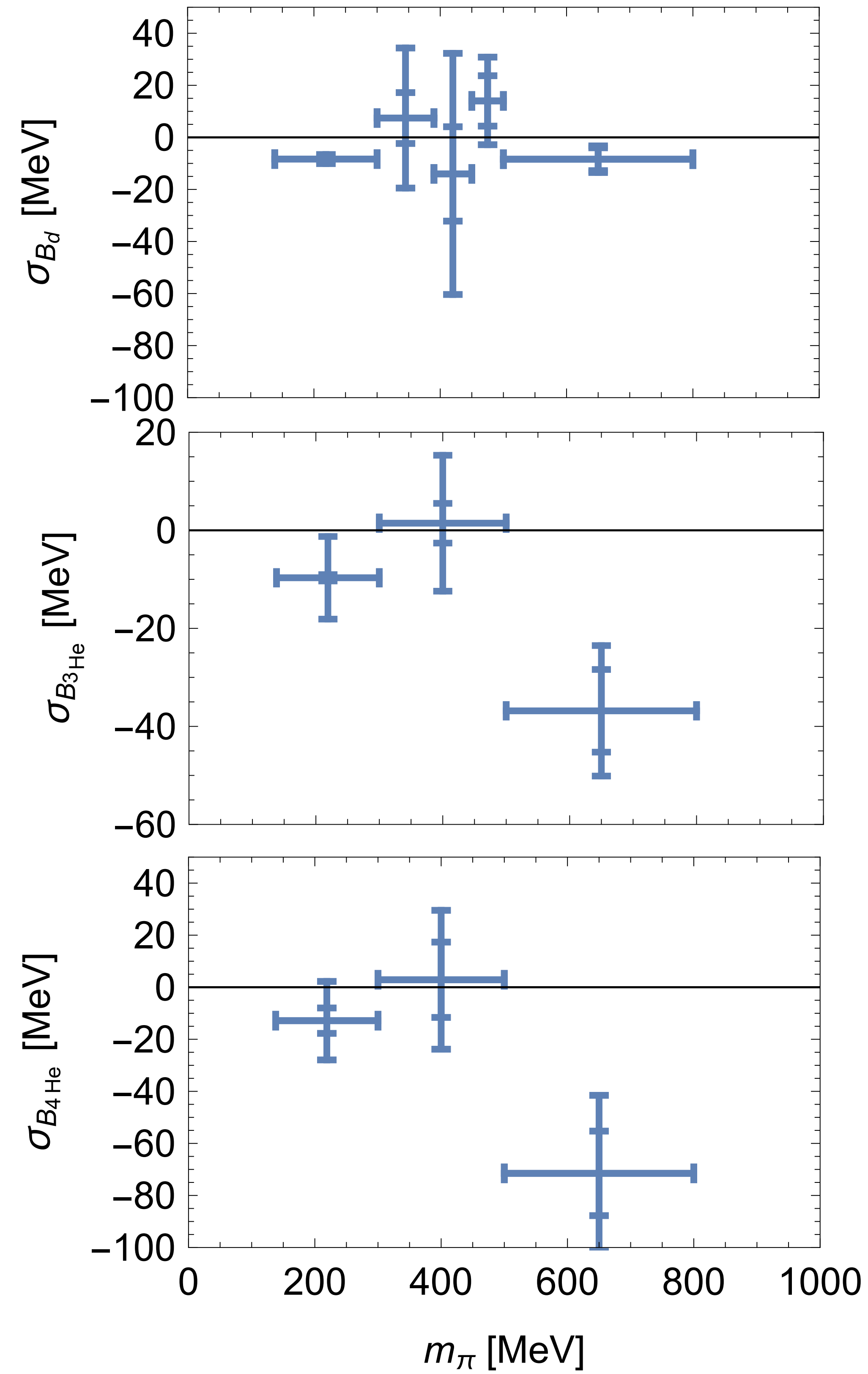
- Lattice calculations + physical point access this
  [NPLQCD, PRD **89** (2014) 074505]

[NPLQCD PRD 2014]

- Nuclear sigma terms

$$\sigma_{Z,N} = A\sigma_N + \sigma_{B_{Z,N}}$$
$$= A\sigma_N - \frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

crudely evaluate as finite difference



- Nuclear sigma terms

$$\sigma_{Z,N} = A\sigma_N + \sigma_{B_{Z,N}}$$
$$= A\sigma_N - \frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

crudely evaluate as finite difference

- Shift from coherent nucleon

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1$$
$$= -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

- O(10%) at most