Role of DCSB in the Pion & Nucleon

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The Pion – Nature's strong messenger



- Hideki Yukawa in 1935 postulated a strongly interacting particle of mass ~ 100 MeV
 - Yukawa called this particle a "meson"
- Cecil Powell in 1947 discovered the π-meson from cosmic ray tracks in a photographic emulsion – a technique Cecil developed





- Cavendish Lab had said method is incapable of *"reliable and reproducible precision measurements"*
- The measured *pion* mass was: 130 150 MeV
- Both Yukawa & Powell received Nobel Prize in 1949 and 1950 respectively
- Discovery of pion was beginning of particle physics; before long there was the particle *zoo*

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- Today the pion is understood as both a bound state of a dressed-quark and a dressed-antiquark in QFT and the Goldstone mode associated with DCSB in QCD
- This dichotomous nature has numerous ramifications, e.g.:

 $m_{
ho}/2 \sim M_N/3 \sim 350 \,\mathrm{MeV}$ however $m_{\pi}/2 \simeq 0.2 \times 350 \,\mathrm{MeV}$

- The pion is unusually light, the key is dynamical chiral symmetry breaking
 - in coming to understand the pion's lepton-like mass, DCSB (and confinement) has been exposed as the origin of more than 98% of the mass in the visible Universe
- A unifying challenge of Hadron Physics is to:
 - discover the nature of *confinement* and its relation to *dynamical chiral symmetry breaking*
- Computation is critical as data alone can only reveal limited aspects about the theory underlying the strong interaction





QCD's Dyson-Schwinger Equations



- The equations of motion of QCD \iff QCD's Dyson–Schwinger equations
 - an infinite tower of coupled integral equations
 - tractability \implies must implement a symmetry preserving truncation
 - The most important DSE is QCD's gap equation \implies quark propagator



• ingredients - dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i \not p + M(p^2)}$$

• S(p) has correct perturbative limit

- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
 - no real mass shell \Longrightarrow confinement



Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
 - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
 - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$



- Light-front quantization \implies light-front WFs; many remarkable properties:
 - frame-independent; probability interpretation as close as QFT gets to QM
 - boosts are kinematical not dynamical
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

$$arphi(x) = \int d^2 \vec{k}_{\perp} \; \psi(x, \vec{k}_{\perp}) \; ,$$

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Pion's Parton Distribution Amplitude



- pion's PDA $\varphi_{\pi}(x)$: is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state
 - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2



PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude



• The pion's PDA is defined by

$$f_{\pi} \,\varphi_{\pi}(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \,\delta\left(k^+ - x \,p^+\right) \operatorname{Tr}\left[\gamma^+ \gamma_5 \, S(k) \,\Gamma_{\pi}(k,p) \, S(k-p)\right]$$

- $S(k) \Gamma_{\pi}(k, p) S(k p)$ is the pion's Bethe-Salpeter wave function
 - in the non-relativistic limit it corresponds to the Schrodinger wave function
- φ_π(x): is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front
- Pion PDA is a *scale dependent* non-perturbative quantity, which e.g., alters the Q² dependence of pion form factor in the asymptotic regime

$$Q^2 F_{\pi}(Q^2) \xrightarrow{Q^2 \to \infty} 16 \pi f_{\pi}^2 \alpha_s(Q^2)$$



QCD Evolution & Asymptotic PDA



ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \, \varphi(x,\mu) = \int_0^1 dy \, V(x,y) \, \varphi(y,\mu)$$

This evolution equation has a solution of the form

$$\varphi_{\pi}(x,Q^2) = \frac{6 x (1-x)}{1+\sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1)}$$

- α = 3/2 because in Q² → ∞ limit QCD is invariant under the collinear conformal group SL(2; ℝ)
- Gegenbauer- $\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$
- The coefficients of the Gegenbauer polynomials, a^{3/2}_n(Q²), evolve logarithmically to zero as Q² → ∞: φ_π(x) → φ^{asy}_π(x) = 6 x (1 − x)
- At what scales is this a good approximation to the pion PDA?

• E.g., AdS/QCD find $\varphi_{\pi}(x) \sim x^{1/2} (1-x)^{1/2}$ at $Q^2 = 1 \text{ GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2}/a_2^{3/2} \sim 10\%$

Pion PDA from the DSEs





Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA

- scale of calculation is given by renormalization point $\xi = 2 \text{ GeV}$
- A realization of DCSB on the light-front
- ERBL evolution demonstrates that the pion's PDA remains broad & concave for all accessible scales in current and conceivable experiments
- **)** Broading of PDA influences the Q^2 evolution of the pion's EM form factor

Pion PDA from Lattice QCD





- however this expansion is guaranteed to converge rapidly only when $Q^2 \rightarrow \infty$
- method results in a *double-humped* pion PDA not supported by BSE WFs
- Advocate using a generalized expansion

$$\varphi_{\pi}(x,Q^2) = N_{\alpha} x^{\alpha} (1-x)^{\alpha} \left[1 + \sum_{n=2,4,\dots} a_n^{\alpha+1/2}(Q^2) C_n^{\alpha+1/2}(2x-1) \right]$$

• Find $\varphi_{\pi} \simeq x^{\alpha}(1-x)^{\alpha}$, $\alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

Pion PDA from Lattice QCD

- Currently, lattice QCD can determine only one non-trivial moment, e.g.: $\int_{0}^{1} dx (2x-1)^{2} \varphi_{\pi}(x) = 0.27 \pm 0.04$
 - [V. M. Braun et al., Phys. Rev. D 74, 074501 (2006)]
 - scale is $Q^2 = 4 \,\mathrm{GeV}^2$
 - Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_{\pi}(x,Q^2) = \frac{6 x (1-x)}{1+\sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1)}$$

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Pion PDA from Lattice QCD – updated



Most recent lattice QCD moment:

$$\int_{0}^{1} dx \, (2 \, x - 1)^{2} \varphi_{\pi}(x) = 0.2361 \, (41) \, (39) \, (?)$$

[V. M. Braun, et al., Phys. Rev. D 92, no. 1, 014504 (2015)]

DSE prediction:

$$\int_{0}^{1} dx \, (2 \, x - 1)^2 \varphi_{\pi}(x) = 0.251$$

- Near complete agreement between DSE prediction and latest lattice QCD result
- Conclude that the pion PDA is a broad concave function
 - double humped distributions are no longer tenable



When is the Pion's PDA Asymptotic





• Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100 \text{ GeV}^2$, compared with $\varphi_{\pi}^{\text{asy}}(x) = 6 x (1 - x)$

• Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors

• Importantly, $\varphi_{\pi}^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_{\pi}(x)$ when pion valence quark PDF satisfies: $q_{v}^{\pi}(x) \sim \delta(x)$

• This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





LO QCD evolution of momentum fraction carried by valence quarks

$$\left\langle x \, q_v(x) \right\rangle(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \left\langle x \, q_v(x) \right\rangle(Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

• therefore, as $Q^2 \to \infty$ we have $\langle x \, q_v(x) \rangle \to 0$ implies $q_v(x) \propto \delta(x)$

At LHC energies valence quarks still carry 20% of pion momentum

• the gluon distribution saturates at $\langle x g(x) \rangle \sim 55\%$

Asymptotia is a long way away!

Pion Elastic Form Factor



- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_{\pi}(Q^2)$ at $Q^2 \approx 6 \,\mathrm{GeV^2}$
 - magnitude of this product is determined by strength of DCSB at all accessible scales

The QCD prediction can be expressed as



- Find consistency between the *direct pion form factor calculation* and the QCD hard-scattering formula – if DSE pion PDA is used
 - 15% disagreement may be explained by higher order/higher-twist corrections
- Predict that QCD power law behavior with QCD's scaling law violations sets in at $Q^2 \sim 8 \,\mathrm{GeV^2}$ [Featured in 2015 NP Long Range Plan]

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The Nucleon



Nucleon Electromagnetic Form Factors





- Provide vital information on the distribution of charge and magnetization within hadrons and nuclei
 - form factors also directly probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
 - proton radius puzzle
 - $\mu_p G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
 - flavor decomposition and evidence for diquark correlations
 - meson-cloud effects
 - seeking verification of perturbative QCD scaling predictions & scaling violations

Form Factors in Conformal Limit



- At asymptotic energies hadron form factors factorize into parton distribution amplitudes & a hard scattering kernel [Farrar, Jackson; Lepage, Brodsky; Radyushkin, Efremov]
 - only the valence Fock state ($\bar{q}q$ or qqq) can contribute as $Q^2 \rightarrow \infty$
 - both confinement and asymptotic freedom are important in this limit
 - Most is known about $\bar{q}q$ bound states, e.g., for the pion:



Diquarks & quark-sector form factors



Nucleon wave function obtained from a Poincaré covariant Faddeev equation



- a *prediction* of these approaches is that owing to DCSB in QCD strong *non-pointlike* diquark correlations exist within baryons
- scalar diquarks dominant for nucleon has numerous empirical consequences
- For example, consider the flavor separated proton form factors: *u* and *d* quark-sectors have very *different scaling behavior* <sup>I.C. Cloët, W. Bentz, A. W. Thomas, Phys. F ^{I.O}
 ^{I.O}</sup>
- Prima facie a remarkable result, but naturally explained by dominance of scalar diquarks
 - in the proton (*uud*) the *d* quark is "bound" inside a scalar diquark [*ud*] 70% of the time – therefore *d*-quark receives additional $1/Q^2$



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Proton G_E/G_M **Ratio**



Quark anomalous magnetic moment required for good agreement with data

- important for low to moderate Q^2
- power law suppressed at large Q^2

Feedback with EM form factor measurements helps constrain the quarkphoton vertex and – via DSEs – the quark-gluon vertex ... current focus of much continuum and lattice analysis

• knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs \Leftrightarrow confinement

Neutron G_E/G_M **Ratio**



- Dressed quark anomalous chromo- (⇒ electro-) magnetic moment has only a minor impact on neutron Sachs form factor ratio *cancellations*
- The DSE *prediction* was confirmed on domain $1.5 \leq Q^2 \leq 3.5 \,\text{GeV}^2$
 - shortcomings in other approaches have been exposed
- Predict a zero-crossing in G_{En}/G_{Mn} at $Q^2 \sim 11 \,\mathrm{GeV^2}$
 - turn over in G_{En}/G_{Mn} will be tested at Jefferson Lab

Prediction has provided the impetus for neutron experiments at large Q²
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Proton G_E form factor & DCSB



[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. 111, 101803 (2013)]



Find that slight changes in $M(p^2)$ on the domain $1 \leq p \leq 3 \text{ GeV}$ have a striking effect on the G_E/G_M proton form factor ratio

• strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD

• Zero in
$$G_E = F_1 - \frac{Q^2}{4M_N^2}F_2$$
 largely determined by evolution of $Q^2 F_2$

- F₂ is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment *vanishes in conformal limit*
- the quicker the perturbative/conformal regime is reached the quicker $F_2 \rightarrow 0$

Conclusion



- Find that DCSB drives numerous effects in QCD, e.g.:
 - hadron masses & confinement
 - $Q^2 F_{\pi}(Q^2)$ peaks at $6 \,\mathrm{GeV}^2$
 - predict that QCD power law behaviour sets in at $Q^2 \sim 8 \,{\rm GeV^2}$
 - dressed quarks have large anomalous chromo- & electro-magnetic moments
 - diquark correlations important for nucleon structure
 - e.g. location of zero's in form factors - G_{Ep}, F^d_{1p}, etc – provide tight constraints on QCD dynamics
- Research has stimulated numerous experiments, e.g.:
 - pion and kaon form factors
 - neutron form factors at large Q^2



 $(GeV^2$