Hadron Structure using the Feynman-Hellmann Theorem

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Outline

Aim to demonstrate advantages of a Feynman-Hellmann approach in lattice QCD

Previously applied in forward case to hadron spin decomposition

Want to extend to non-forward matrix elements

Electromagnetic form factors of the pion and nucleon

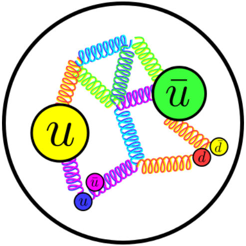
Large momentum transfers

Note: Exploratory calculations
$\rightarrow$ full systematic analysis not yet performed

Electromagnetic Form Factors

Hadrons are composite particles

How is charge/magnetisation distributed?

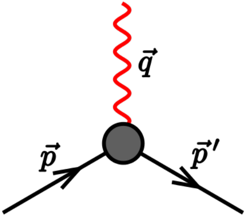


Need to determine vector matrix elements

$$\langle H(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, H(\mathbf{p}) \rangle = \ldots ?$$

Full analytic form unknown

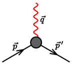
Parameterise amplitude by form factors



e.g. pion has single form factor

$$\langle \pi(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, \pi(\mathbf{p}) \rangle \propto F_\pi(Q^2), \quad Q^2 = -(p' - p)^2$$

- Fourier transform of transverse charge density
- Slope at $Q^2 = 0 \rightarrow$ charge radius



$$\langle\, H(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, H(\mathbf{p}) \,\rangle = \ldots F(Q^2) + \ldots \qquad Q^2 = (p' - p)^2$$

What can experiment tell us?

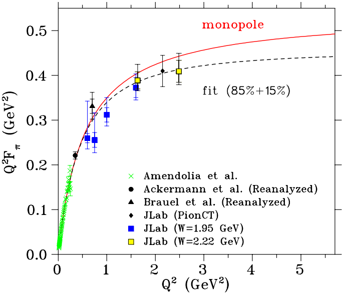
- Low Q^2: π^+ scattering by atomic e^-
- High Q^2: π electroproduction off nucleon

High-Q^2 measurement difficult

High-Q^2 data $\Longrightarrow$ "fine-detail" information

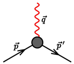
Ongoing experimental efforts

e.g. 12 GeV upgrade at JLab $\rightarrow F_\pi(Q^2)$ up to 6 GeV



[Jefferson Lab (PRC 2008)]

Electromagnetic Form Factors — Proton



$$\langle\, H(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, H(\mathbf{p}) \,\rangle = \ldots F(Q^2) + \ldots \qquad Q^2 = (p' - p)^2$$

Write nucleon matrix element in terms of Sachs EM form factors

$$\langle\, N(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, N(\mathbf{p}) \,\rangle = \ldots G_E(Q^2) + \ldots G_M(Q^2)$$

Non-relavistically, Fourier Transforms of charge/magnetisation

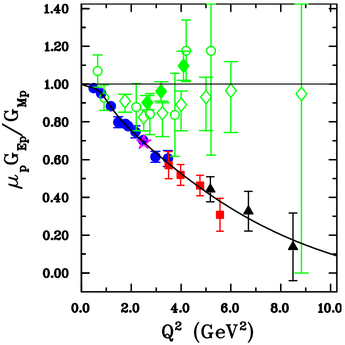
For the proton

- Early experiments → low G_E sensitivity
- Double polarisation → $\frac{G_E}{G_M}$ directly

Zero crossing in $(G_E/G_M)_p$?

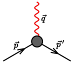
Central negatively charged region?

Require more high-Q^2 data



[Punjabi et al. (EPJA 2015)]

Electromagnetic Form Factors — Lattice

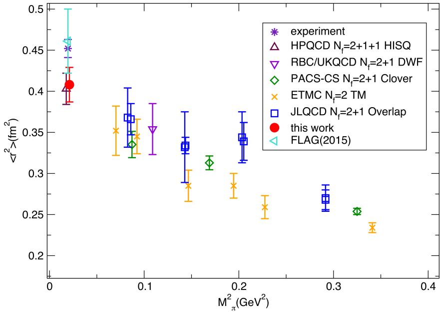


$$\langle H(\mathbf{p}') \,|\, \mathcal{J}(0) \,|\, H(\mathbf{p}) \rangle = \ldots F(Q^2) + \ldots \qquad Q^2 = (p' - p)^2$$

What can lattice tell us?

Low-Mid Q^2: Good progress

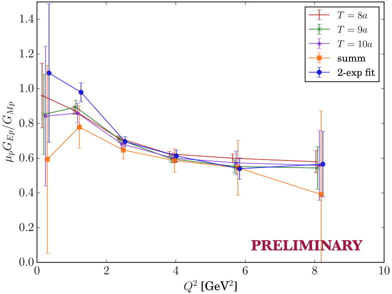
- Approaching physical charge radii



[Kakazu et al. (Lattice 2016)]

High Q^2: More Difficult

- Improvements with momentum-smeared operators

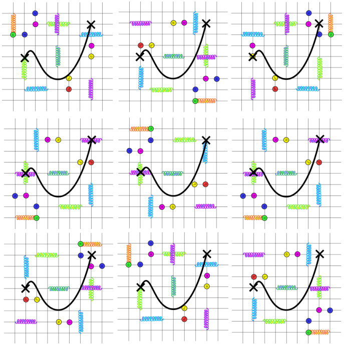


[Syritsyn et al. (Lattice 2016)]

Lattice QCD

Lattice QCD — Numerically estimate path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathrm{D}\psi\, \mathrm{D}\bar{\psi}\, \mathrm{D}A\, \mathcal{O}\, e^{-S} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \overline{\mathcal{O}}[A_i]$$



- Generate gauge fields → Monte Carlo
- Quark props in $\overline{\mathcal{O}}$ → invert Dirac matrix

e.g. Hadron energies from 2-point functions

$$\int \mathrm{d}\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle\, \chi'(\mathbf{x}, t)\chi(0)\, \rangle \overset{\text{large } t}{\propto} e^{-E(\mathbf{p})t}$$

Normally extract matrix elements from 3-point functions

Alternative approach → Feynman-Hellmann

Feynman-Hellmann Recipe (Forward Case)

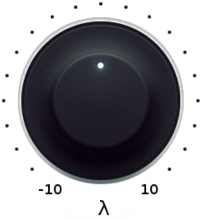
How to calculate $\langle H(\mathbf{p}) | \mathcal{O}(0) | H(\mathbf{p}) \rangle$?

1. **Add term to Lagrangian**

$$\mathcal{L} \to \mathcal{L} + \lambda \mathcal{O}$$

2. **Measure hadron energy while changing λ**

$$\int \mathrm{d}x\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \chi'(x)\chi(0) \rangle \overset{\text{large } t}{\propto} e^{-E_H(\lambda,\mathbf{p})t}$$



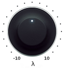
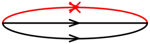
3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda,\mathbf{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\mathbf{p})} \langle H(\mathbf{p}) | \mathcal{O}(0) | H(\mathbf{p}) \rangle$$

Calculation of matrix element $\to$ hadron spectroscopy

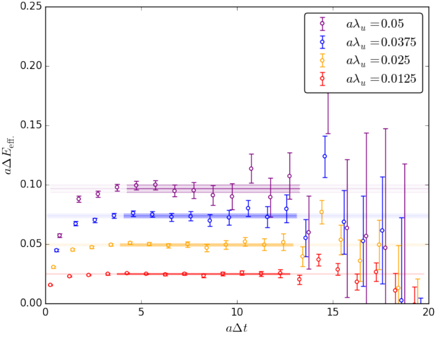
Only need to calculate two-point functions!

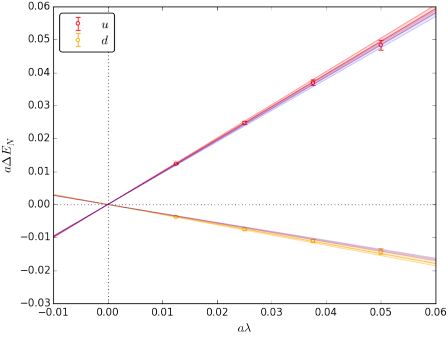
Quark Axial Charges in the Nucleon

$\mathcal{L} \to \mathcal{L} + \lambda\mathcal{O}$   $\partial E_H/\partial\lambda|_{\lambda=0} \propto \langle\, H(\mathbf{p}) \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \,\rangle$

Previous application to quark axial charges

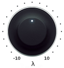
Do $\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q \implies \left.\frac{\partial E_N(\lambda)}{\partial\lambda}\right|_{\lambda=0} = \Delta q_{\text{conn.}}$

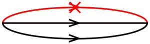


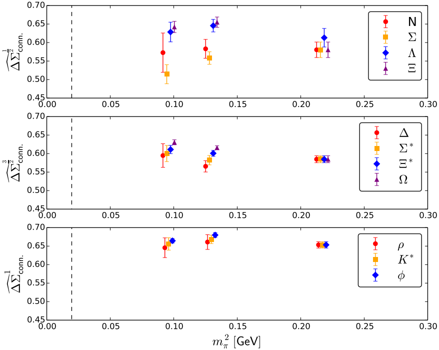


$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$

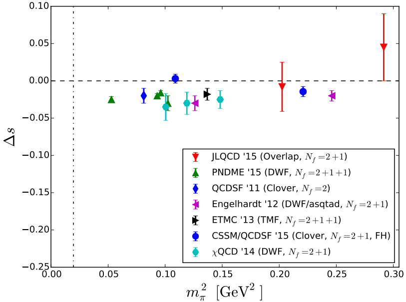
Quark Axial Charges in the Nucleon

$\mathcal{L} \to \mathcal{L} + \lambda \mathcal{O}$ 

 $\partial E_H / \partial \lambda |_{\lambda=0} \propto \langle\, H(\mathbf{p}) \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \,\rangle$



[Chambers et al. (PRD 2014)]



[Chambers et al. (PRD 2015)]

How do we extend method to non-forward matrix elements?

Feynman-Hellmann Recipe (Non-Forward Case)

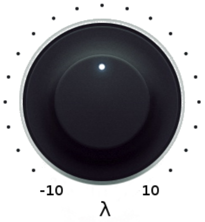
How to calculate $\langle\, H(\mathbf{p}') \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \,\rangle$?

1. **Add term to Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda\left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}}\right)\mathcal{O}(x)$$

2. **Measure hadron energy while changing λ**

$$G(\lambda; \mathbf{p}'; t) \overset{\text{large } t}{\propto} e^{-E_H(\lambda,\mathbf{p})t}$$



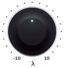
3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda, \mathbf{p}')}{\partial \lambda}\right|_{\lambda=0} \propto \langle\, H(\mathbf{p}') \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \,\rangle$$

Additional requirement $\to$ Restricted to Breit frame

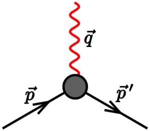
Will only perform connected calculations here

Electromagnetic Form Factors — Pion

$\mathcal{L} \to \mathcal{L} + \lambda \left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}} \right) \mathcal{O}$ 

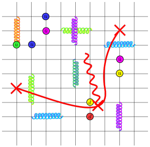
$\partial E_H / \partial\lambda|_{\lambda=0} \propto \langle H(\mathbf{p}') \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \rangle$

Want to calculate pion form factor



Flavour contributions to vector matrix element

$$\langle \pi(\mathbf{p}') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\mathbf{p}) \rangle = (p'_\mu + p_\mu) F^q_\pi(Q^2)$$



Add vector operator to Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}} \right) \bar{q}(x)\gamma_\mu q(x)$$

For temporal current insertion

$$\left.\frac{\partial E_\pi(\lambda, \mathbf{p})}{\partial\lambda}\right|_{\lambda=0} \stackrel{\mathbf{p}'=-\mathbf{p}}{=} F^q_\pi(Q^2)$$

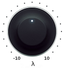
For spatial current insertion

$$\left.\frac{\partial E_\pi(\lambda, \mathbf{p})}{\partial\lambda}\right|_{\lambda=0} \stackrel{\mathbf{p}'=-\mathbf{p}}{=} 0$$

Choose this

Electromagnetic Form Factors — Pion

$$\mathcal{L} \to \mathcal{L} + \lambda \left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}} \right) \mathcal{O}$$



$$\partial E_H / \partial \lambda |_{\lambda=0} \propto \langle H(\mathbf{p}') \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \rangle$$

$m_\pi \approx 470$ MeV $\quad N_{\text{conf}} = 750 \quad 32^3 \times 64 \quad \mathbf{q} = (2,0,0)$

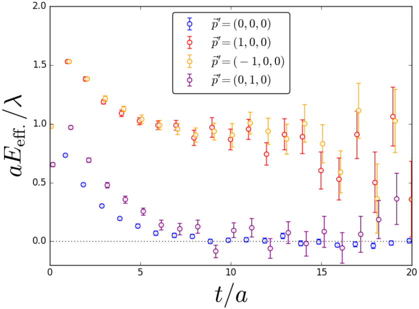
Require Breit frame kinematics

$$\mathbf{q} = (2,0,0) \implies \mathbf{p}' = (\pm 1, 0, 0)$$

Otherwise no signal at $\mathcal{O}(\lambda)$

Choose $\mathbf{q}^2$ points allowing $\mathbf{p}' = -\mathbf{p}$

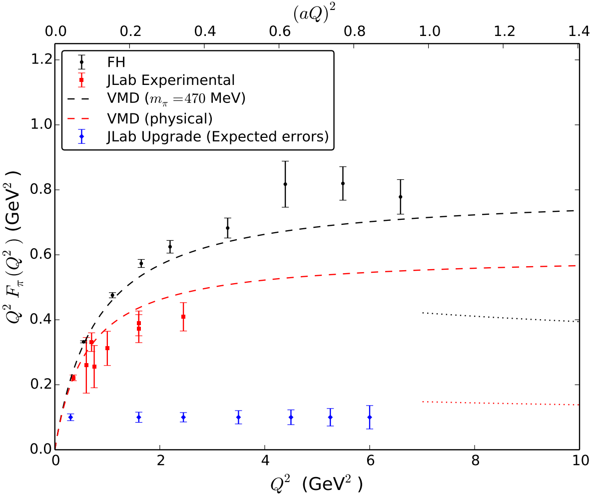
$$\mathbf{q}^2 = (4n)\frac{2\pi}{L} \quad n \in \mathbb{Z}^+$$



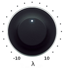
Minimises source/sink momentum for particular $\mathbf{q}^2 \to$ minimises noise

Electromagnetic Form Factors — Pion

$m_\pi \approx 470$ MeV $\approx 1000 - 1500$ configurations $32^3 \times 64$



Electromagnetic Form Factors — Nucleon

$\mathcal{L} \to \mathcal{L} + \lambda \left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}} \right) \mathcal{O}$  $\partial E_H / \partial \lambda |_{\lambda=0} \propto \langle\, H(\mathbf{p}') \,|\, \mathcal{O}(0) \,|\, H(\mathbf{p}) \,\rangle$

In the Breit frame, flavour contributions to form factors

$$\left\langle\, N_s(\mathbf{p}) \,\middle|\, \bar{q}(0)\gamma_{0/i}q(0) \,\middle|\, N_s(-\mathbf{p}) \,\right\rangle \propto G^q_{E/M}(Q^2)$$

Add vector operator to Lagrangian (identical to pion calculation)

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\mathbf{q}\cdot\mathbf{x}} + e^{-i\mathbf{q}\cdot\mathbf{x}} \right) \bar{q}(x)\gamma_\mu q(x)$$

For Temporal Current project unpolarised states

$$\left.\frac{\partial E_N(\lambda, \mathbf{p})}{\partial \lambda}\right|_{\lambda=0} \overset{\mathbf{p}'=-\mathbf{p}}{\propto} G_E(Q^2)$$

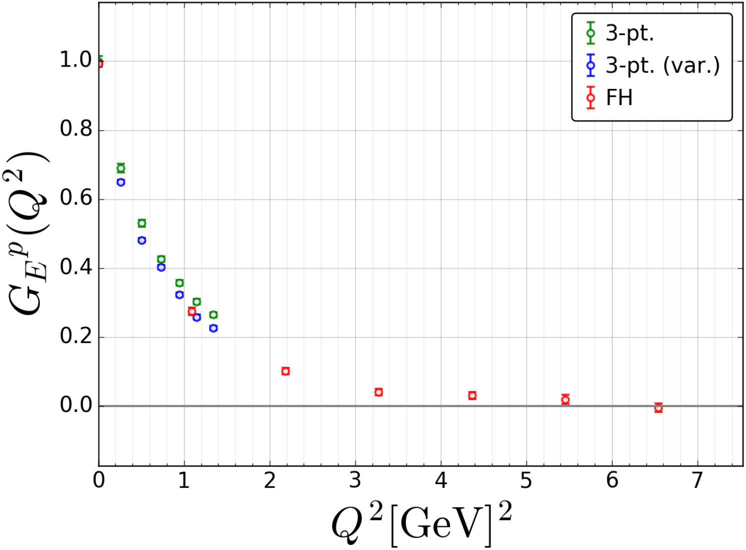
For Spatial Current project spin-up/down states

$$\left.\frac{\partial E_N(\lambda, \mathbf{p})}{\partial \lambda}\right|_{\lambda=0} \overset{\mathbf{p}'=-\mathbf{p}}{\propto} \pm G_M(Q^2)$$

Other Breit frame kinematics → combinations of G_E and G_M.

Electromagnetic Form Factors — Proton

$m_\pi \approx 470$ MeV $\approx 1000 - 1500$ configurations $32^3 \times 64$



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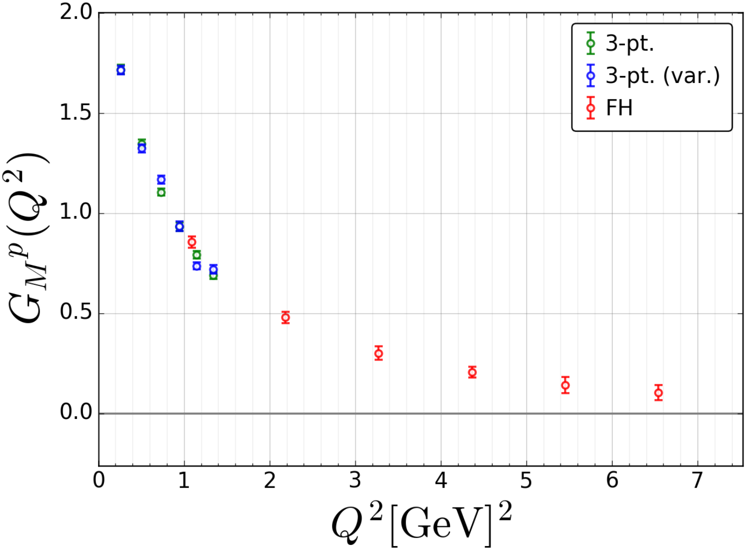

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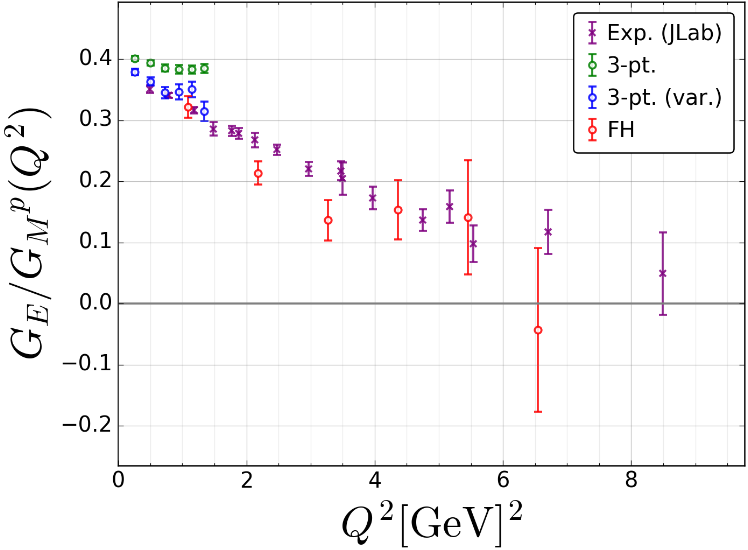

Summary

Exciting results and many successes from application of Feynman-Hellmann technique to non-forward matrix elements

Able to access much higher momentum transfers

Electromagnetic Form Factors — Proton

$m_\pi \approx 470$ MeV $\approx 1000 - 1500$ configurations $32^3 \times 64$