# Tests of Partial Dynamical Symmetries in Deformed and Transitional Nuclei

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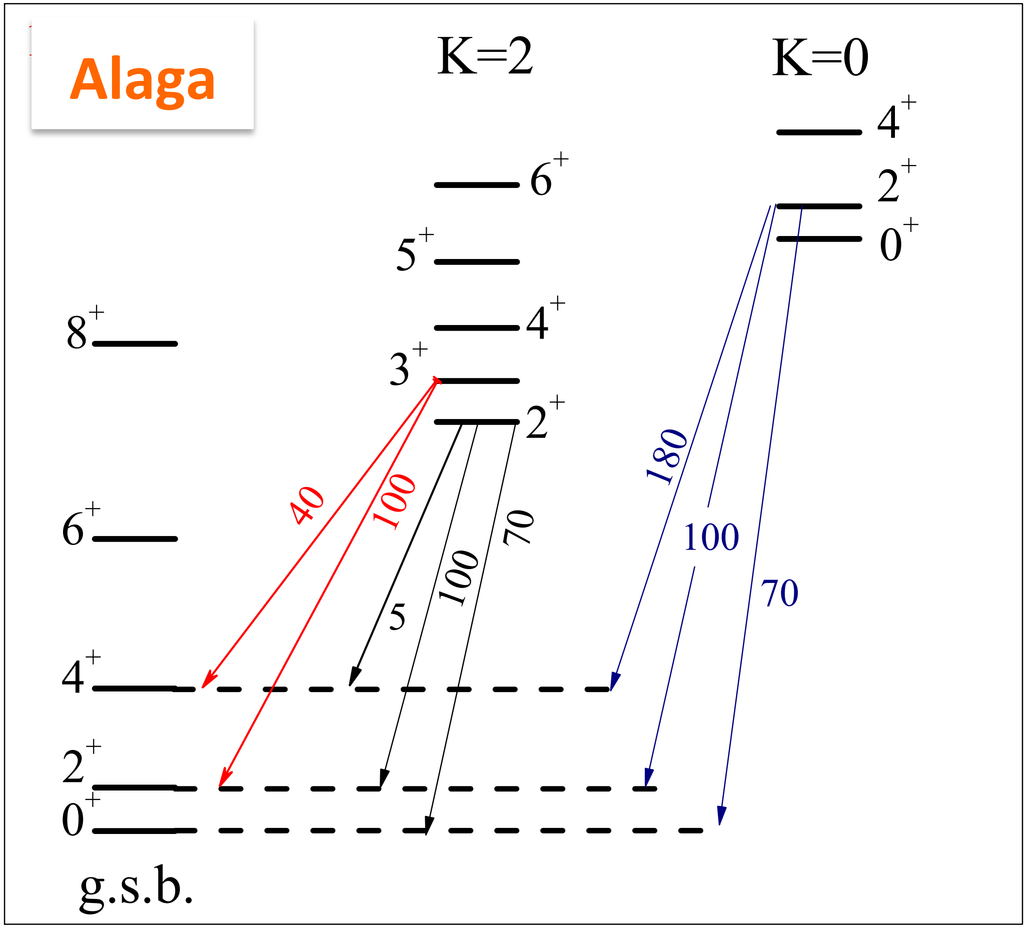
# E2 Branching ratios -- Alaga Rules
# Independent of structure except for K and separation of rotational and intrinsic degrees of freedom



# Alaga plus one-parameter bandmixing

| $J_i^\pi \rightarrow J_f^\pi$ | $^{168}$Er | ALAGA |
|---|---|---|
| $2_\gamma^+ \rightarrow 0^+$ | 56.2(11) | 70 |
| $2_\gamma^+ \rightarrow 2^+$ | 100 | 100 |
| $2_\gamma^+ \rightarrow 4^+$ | 7.3(4) | 5 |
| $3_\gamma^+ \rightarrow 2^+$ | 100 | 100 |
| $3_\gamma^+ \rightarrow 4^+$ | 62.6(14) | 40 |
| $4_\gamma^+ \rightarrow 2^+$ | 19.3(4) | 34 |
| $4_\gamma^+ \rightarrow 4^+$ | 100 | 100 |
| $4_\gamma^+ \rightarrow 6^+$ | 13.1(12) | 8.64 |
| $5_\gamma^+ \rightarrow 4^+$ | 100 | 100 |
| $5_\gamma^+ \rightarrow 6^+$ | 123(14) | 57.1 |
| $6_\gamma^+ \rightarrow 4^+$ | 11.2(10) | 26.9 |
| $6_\gamma^+ \rightarrow 6^+$ | 100 | 100 |
| $6_\gamma^+ \rightarrow 8^+$ | 37.6(72) | 10.6 |

Spin **De**creasing: **smaller** than Alaga

Spin **In**creasing: **larger** than Alaga

Deviations **increase** with J

These are signature characteristics of mixing of γ and ground band intrinsic excitations.

Introduce a mixing parameter, **Z** $_\gamma$.

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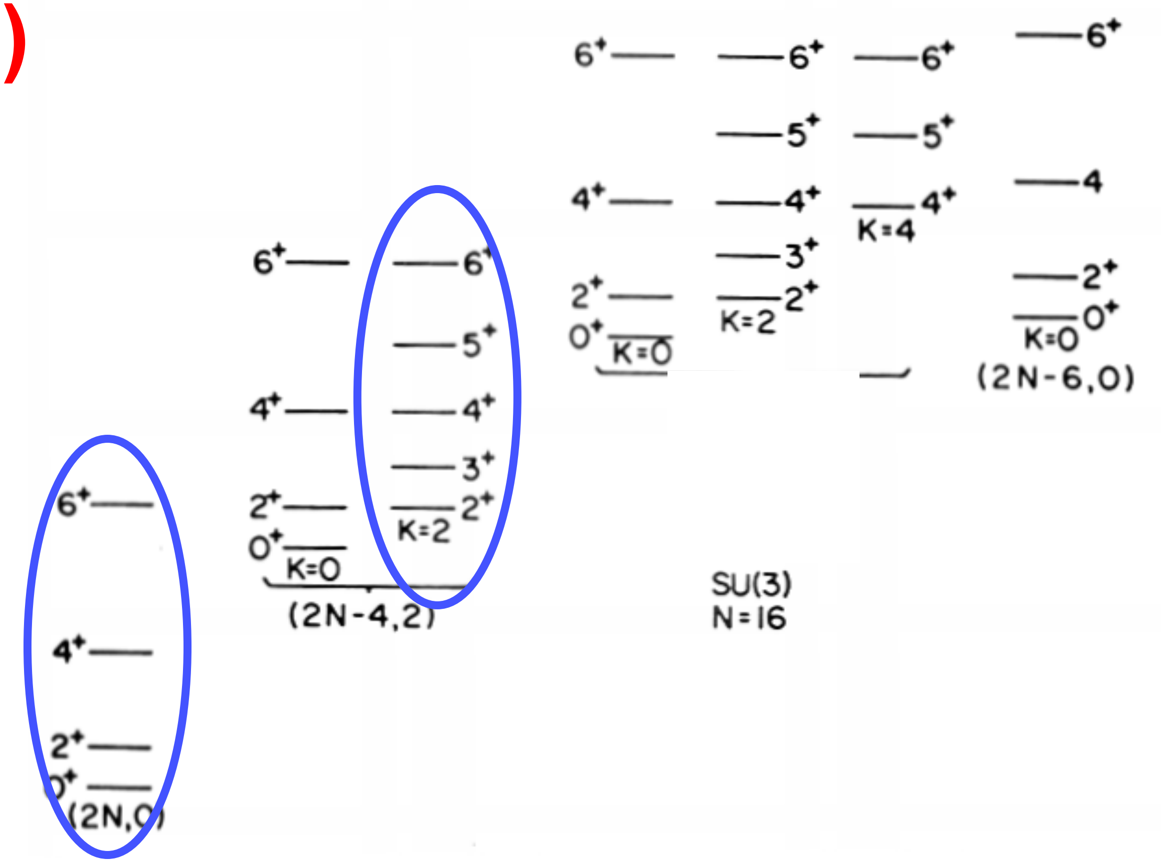
| $J_i$ | $J_f$ | Correction factor $\gamma \rightarrow g$ |
|---|---|---|
| $J_f$-2 | $J_f$ | $1 + (2J_f + 1)Z_\gamma$ |
| $J_f$-1 | $J_f$ | $1 + (J_f + 2)Z_\gamma$ |
| $J_f$ | $J_f$ | $1 + 2Z_\gamma$ |
| $J_f$+1 | $J_f$ | $1 - (J_f - 1)Z_\gamma$ |
| $J_f$+2 | $J_f$ | $1 - (2J_f + 1)Z_\gamma$ |

Usually the deviations from the Alaga rules can be accounted for by a single **$Z_\gamma$**.

# Another Approach: Dynamical Symmetry

**SU(3)**
$(\lambda,\mu)$

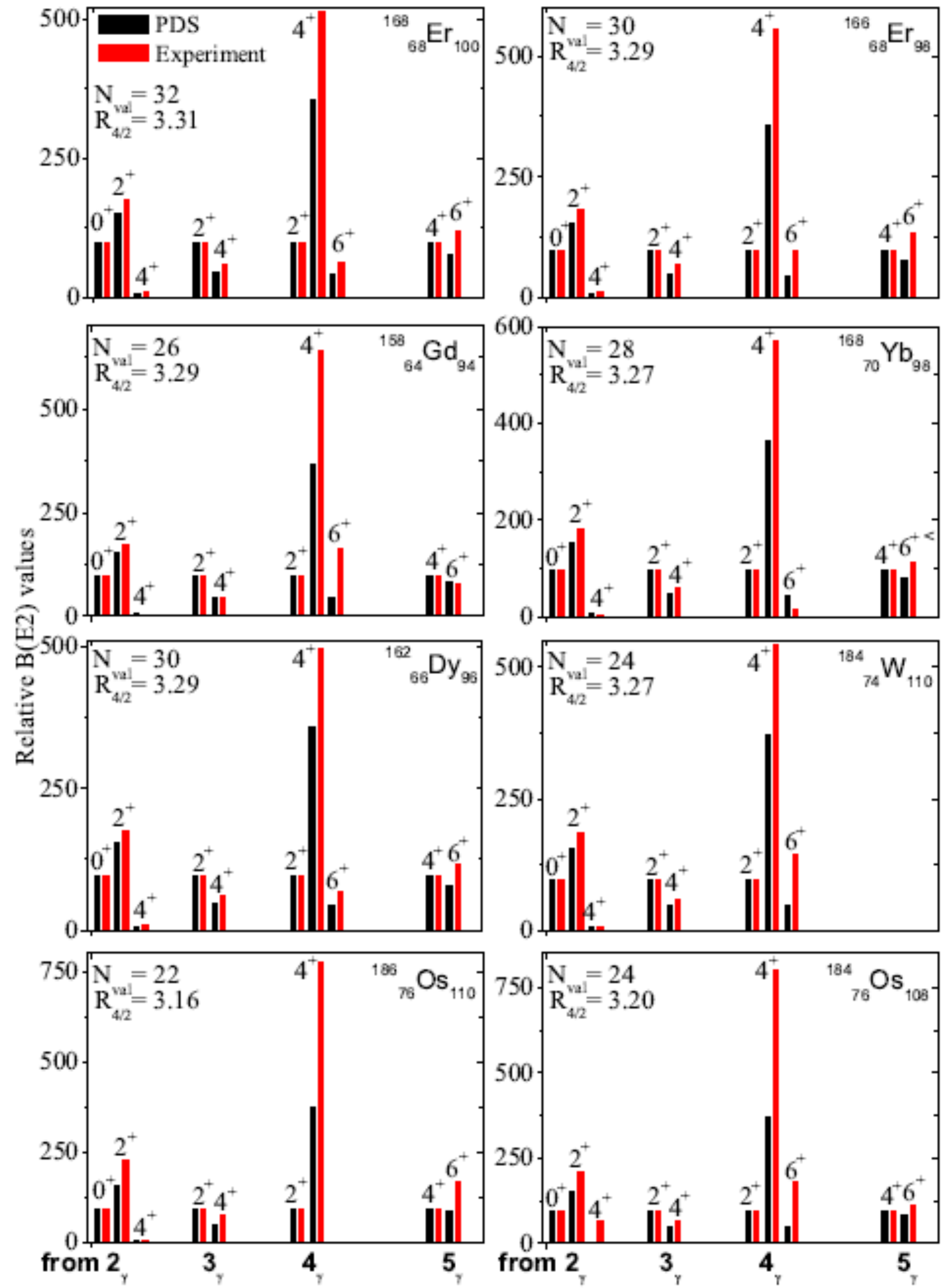



**Partial Dynamical Symmetry(PDS):**
**ONLY $\gamma$ and ground bands are pure SU(3) –**
**NO mixing whatsoever.**

# INTERBAND
# γ — ground

**Extensive test: 47(22) rare earth nuclei**

**Overall good agreement for well-deformed nuclei**



| $J_i^\pi \to J_f^\pi$ | $^{168}$Er | ALAGA | $Z_\gamma = 0.035$ | PDS |
|---|---|---|---|---|
| $2_\gamma^+ \to 0^+$ | 56.2(11) | 70 | 56.9 | 64.3 |
| $2_\gamma^+ \to 2^+$ | 100 | 100 | 100 | 100 |
| $2_\gamma^+ \to 4^+$ | 7.3(4) | 5 | 7.6 | 6.3 |
| $3_\gamma^+ \to 2^+$ | 100 | 100 | 100 | 100 |
| $3_\gamma^+ \to 4^+$ | 62.6(14) | 40 | 62.9 | 49.3 |
| $4_\gamma^+ \to 2^+$ | 19.3(4) | 34 | 20.2 | 28.1 |
| $4_\gamma^+ \to 4^+$ | 100 | 100 | 100 | 100 |
| $4_\gamma^+ \to 6^+$ | 13.1(12) | 8.64 | 16.0 | 12.5 |
| $5_\gamma^+ \to 4^+$ | 100 | 100 | 100 | 100 |
| $5_\gamma^+ \to 6^+$ | 123(14) | 57.1 | 117 | 79.6 |
| $6_\gamma^+ \to 4^+$ | 11.2(10) | 26.9 | 11.0 | 20.3 |
| $6_\gamma^+ \to 6^+$ | 100 | 100 | 100 | 100 |
| $6_\gamma^+ \to 8^+$ | 37.6(72) | 10.6 | 23.6 | 18.0 |

**PDS always closer to data than Alaga. But does not go far enough.**

**PDS simulates bandmixing without mixing.**

**Why differs from Alaga? PDS (from IBA) valence space model: predictions $N_{val}$– dep.**

**Why do two such different descriptions (mixing and PDS) give basically similar predictions?**

# Formalisms and formulas seem completely different, with different functional dependence

**Bandmixing B(E2)'s (Squares give corrections to Alaga)**

| $J_i$ | $J_f$ | Correction factor $\gamma \to g$ |
|---|---|---|
| $J_f$-2 | $J_f$ | $1 + (2J_f + 1)Z_\gamma$ |
| $J_f$-1 | $J_f$ | $1 + (J_f + 2)Z_\gamma$ |
| $J_f$ | $J_f$ | $1 + 2Z_\gamma$ |
| $J_f$+1 | $J_f$ | $1 - (J_f - 1)Z_\gamma$ |
| $J_f$+2 | $J_f$ | $1 - (2J_f + 1)Z_\gamma$ |

**PDS Interband B(E2)s**

$$B(E2:\gamma, J \to g, J+2) = \theta^2 \frac{2}{3} N \frac{(J-1)J}{4(2J+1)(2J+3)}$$

$$\times \; \frac{2(N-1)(2N-J-2)(2N+J-1)(2N+J+1)(2N+J+3)}{N(2N-3)(2N-1)[8(N-1)^2 - J(J+1)]}$$

# Bandmixing correction factors to Alaga rules

From this table, for J = 4 and 2

| $J_i$ | $J_f$ | Correction factor $\gamma \to g$ |
|---|---|---|
| $J_f$-2 | $J_f$ | $1 + (2J_f + 1)Z_\gamma$ |
| $J_f$-1 | $J_f$ | $1 + (J_f + 2)Z_\gamma$ |
| $J_f$ | $J_f$ | $1 + 2Z_\gamma$ |
| $J_f$+1 | $J_f$ | $1 - (J_f - 1)Z_\gamma$ |
| $J_f$+2 | $J_f$ | $1 - (2J_f + 1)Z_\gamma$ |

$$\mathrm{CF}_{BM}(2^+_\gamma \to 4^+_{gr}/2^+_\gamma \to 2^+_{gr}) = \left(\frac{1+9Z_\gamma}{1+2Z_\gamma}\right)^2$$

Since $Z_\gamma$ is small, ~ 0.04, drop quadratic terms, expand denominator

$$\mathrm{CF}_{BM} = (1 + 18\,Z_\gamma)/(1 + 4Z_\gamma) \quad = \quad (1 + 18\,Z_\gamma) \text{ x } (1 - 4Z_\gamma)$$

Again, drop quadratic terms

$$\mathrm{CF}_{BM} = 1 + 14Z_\gamma$$

# B(E2) values in the PDS

$$B(E2:\gamma,J\to g,J+2)=\theta^2\frac{2}{3}N\frac{(J-1)J}{4(2J+1)(2J+3)}$$

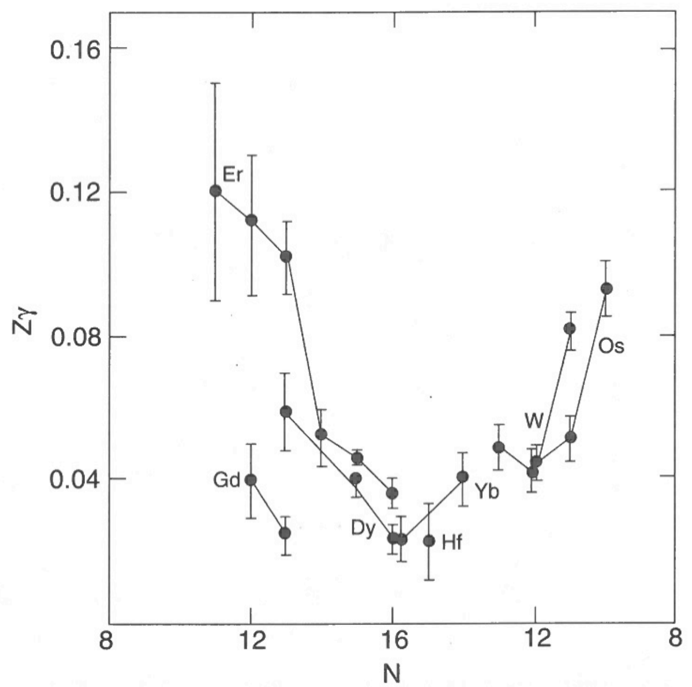
$$\mathrm{x}\;\frac{2(N-1)(2N-J-2)(2N+J-1)(2N+J+1)(2N+J+3)}{N(2N-3)(2N-1)[8(N-1)^2-J(J+1)]}$$

$$B(E2:\gamma,J\to g,J)=\theta^2\frac{2}{3}N\frac{3(J-1)(J+2)}{2(2J-1)(2J+3)}$$

$$\mathrm{x}\;\frac{2(N-1)(2N-J-2)(2N-J)(2N+J-1(2N+J+1)}{N(2N-3)(2N-1)[8(N-1)^2-J(J+1)]}$$

$$CF_{PDS}=\frac{2N+5}{2N-2}$$

$$CF_{PDS}=1+7/2N$$




$Z_\gamma$ minimizes near mid-shell where separation of rotational and vibrational degrees of freedom is best.

$Z_\gamma \sim 1/2N$

Hence:

$CF_{PDS} \sim 1 + 7/2N \quad \sim \quad 1 + 7\,Z_\gamma$

Compare $\quad CF_{BM} \sim \quad 1 + 14\,Z_\gamma$

**PDS correction factors similar in form to BM but about half as strong. $Z_\gamma \sim 1/2N$ → parameter-free bandmixing based on connection to valence nucleon number**

| $B(E2)$ ratio $(J_i$ - $J_f)/(J_i$ - $J'_f)$ | Bandmixing | PDS |
|---|---|---|
| $\frac{2-0}{2-2}$ | 1 - $6Z_\gamma$ | 1 - $3Z_\gamma$ |
| $\frac{2-4}{2-2}$ | $1 + 14Z_\gamma$ | $1 + 7Z_\gamma$ |
| $\frac{3-4}{3-2}$ | $1 + 14Z_\gamma$ | $1 + 7Z_\gamma$ |
| $\frac{4-2}{4-4}$ | 1 - $14Z_\gamma$ | 1 - $7Z_\gamma$ |
| $\frac{4-6}{4-4}$ | $1 + 22Z_\gamma$ | $1 + 11Z_\gamma$ |
| $\frac{5-6}{5-4}$ | $1 + 22Z_\gamma$ | $1 + 11Z_\gamma$ |
| $\frac{6-4}{6-6}$ | 1 - $22Z_\gamma$ | 1 - $11Z_\gamma$ |
| $\frac{6-8}{6-6}$ | $1 + 30Z_\gamma$ | $1 + 15Z_\gamma$ |

# Conclusions

- Both BM and PDS successfully account for iconic $\gamma$ −> G band B(E2) values
- Physics seemingly different: Rot-Vib Interaction (BM) and finite nucleon number effects
- Derived connection between the two concepts. Can predict BM simply from shell structure

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**Collaborators**

**R. Burcu Cakirli (Istanbul)**

**A. Couture (LANL)**

**Jan Jolie (Koeln)**

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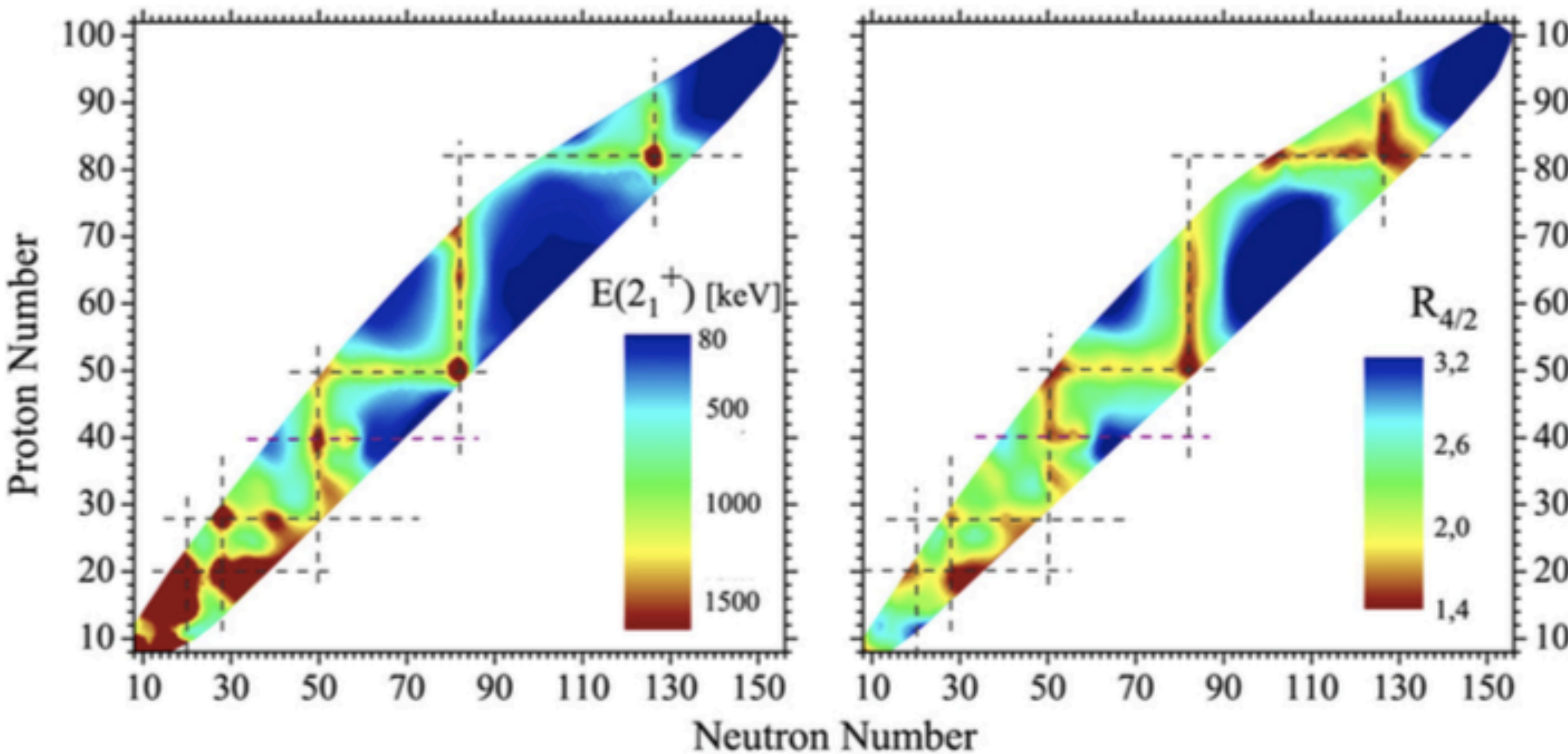
# BACKUPS

Dynamical Symmetries (DS) are spectrum-generating algebras that provide simple predictions for collective behavior in nuclei. They usually describe idealized solutions that can serve as benchmarks but which are seldom realized in actual nuclei. The upshot, historically, has been to solve specific parameterized collective Hamiltonians that break these symmetries. Such approaches have often been very successful and parameter efficient.

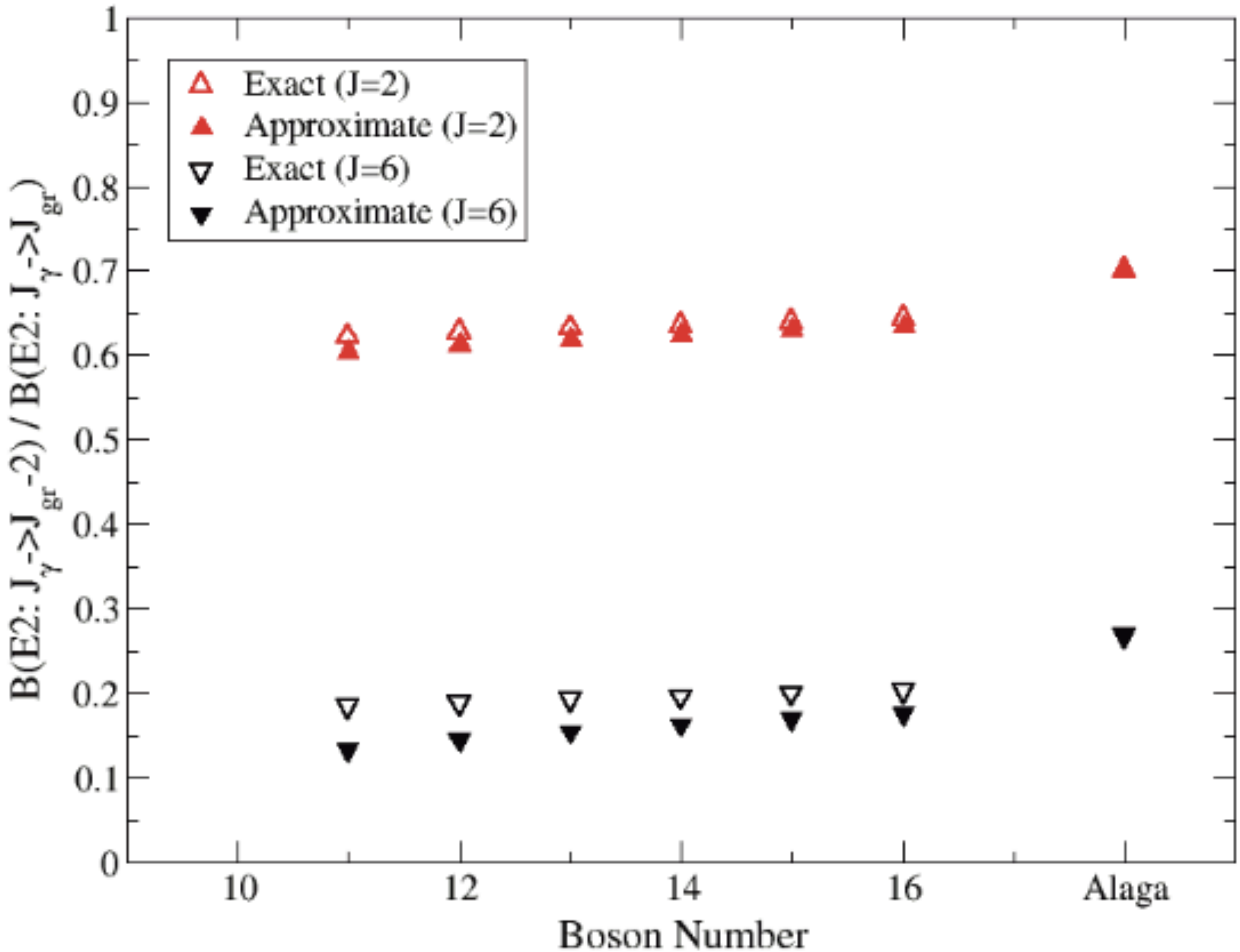
However, an alternate approach, barely tested until recently, has been that of Partial Dynamical Symmetries (PDS) in which some of the properties of the parent symmetry are exactly preserved while others are arbitrarily broken. Many key predictions of PDSs are parameter-free. We will discuss the first extensive tests of this concept, focusing on deformed and transitional nuclei from A~ 100 to A~ 240, and discuss their implications for the sensitivity of collective structure to valence nucleon number and for the mixing of intrinsic configurations.

# The Magnificent Evolution of Structure



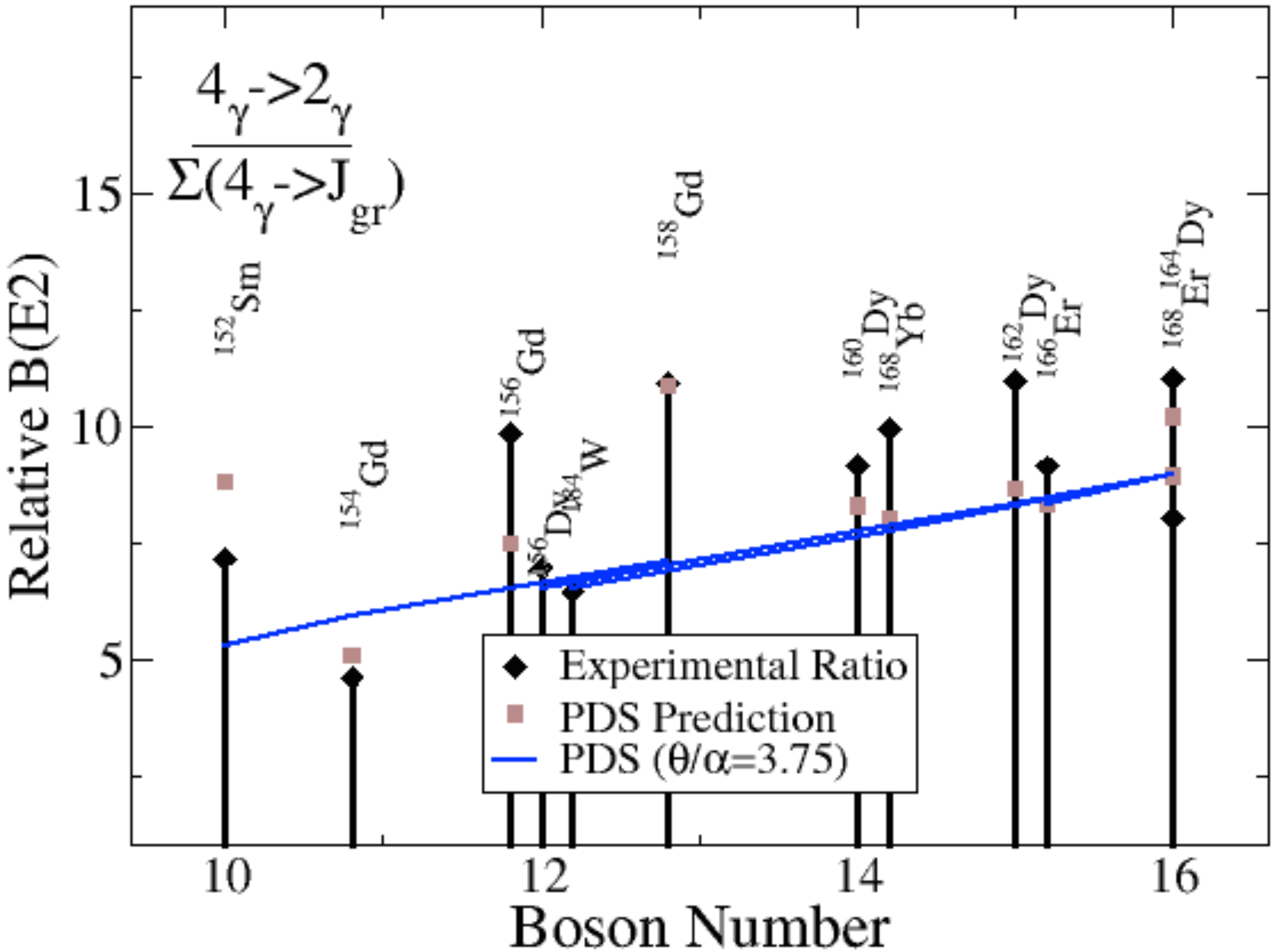

**Challenge: how can these complex many-body systems exhibit such regular patterns?**

**Nucleonic and "system" perspectives: Shell structure and collectivity.**

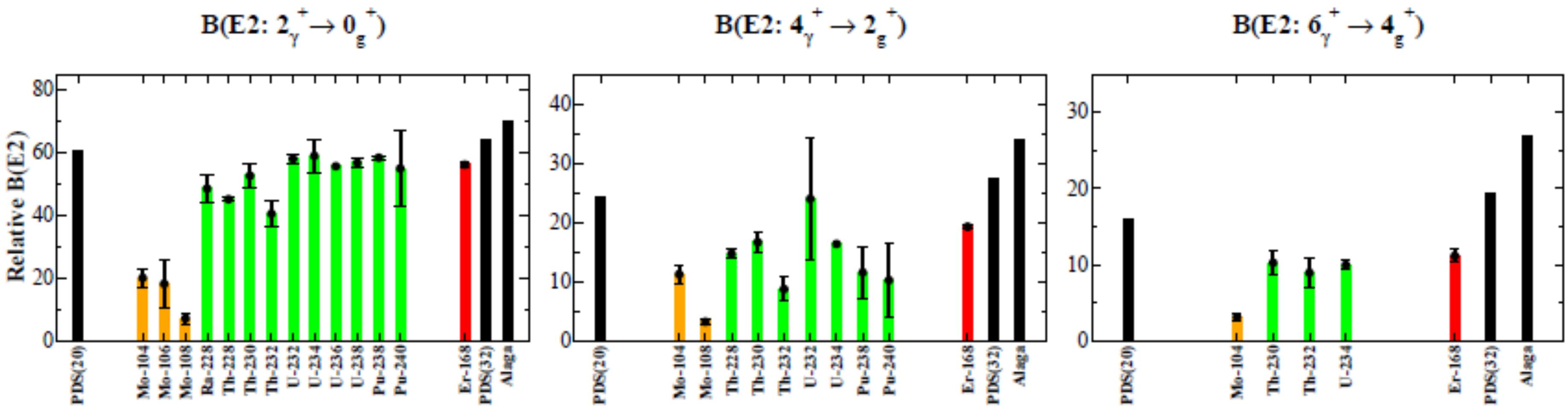


Exact (J=2)
Approximate (J=2)
Exact (J=6)
Approximate (J=6)
B(E2: J_γ->J_gr-2) / B(E2: J_γ->J_gr)
Boson Number
10
12
14
16
Alaga
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

# PDS predictions: INTRA/INTER - VNN



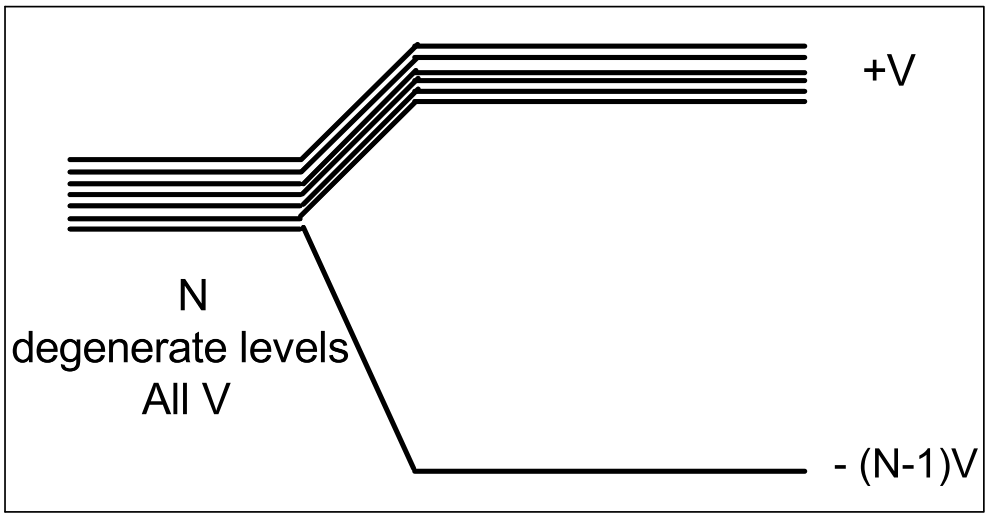
# Actinides and A ~ 100 as function of spin

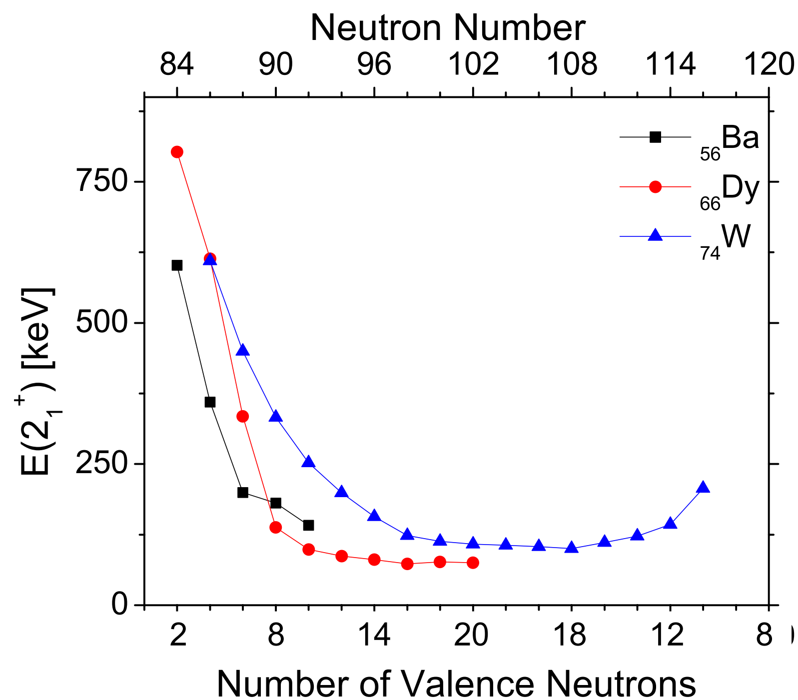



**Spin decreasing B(E2) values get smaller with increasing spin.**

**Clear signal of a mixing effect since the K mixing matrix elements increase with spin: $V_{mix}$ ~ $J_{init}$**

# Valence nucleon number (VNN) and Collectivity




Coherence: Increase of B(E2) values to ground state with increasing VNN