Magnetic properties of the nucleon in a uniform background field

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Introduction

- The Magnetic Polarisability (β) is a fundamental property of a system of charged particles that describes the systems response to an external magnetic field.
- To calculate these on the Lattice we use,
  - the Background Field Method.

Outline

1. How is it done?
   - Background Field Method
2. Magnetic Polarisability
   - Correlator Ratios
   - Landau levels
   - Projections & Smearings
3. Results
   - Energy Shifts
   - Energy vs. Field Strength fits

Background Field Method

Background Field Method

- How is the uniform magnetic field put across the lattice?

$$\mathcal{D}'_\mu = \partial_\mu + g\,G_\mu + qe\,A_\mu qe\,A_\mu, \quad U'_\mu(x) = U_\mu(x)\,\mathrm{e}^{-i\,qe\,a\,A_\mu}$$

- Causes a shift in energy (small field limit) of the baryon.

$$E(B) = M - \vec{\mu}\cdot\vec{B} - \frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}(B^3)$$

- Magnetic moment μ and magnetic polarisability β.
- Use of periodic boundary conditions impose a quantisation condition:

$$qe\,B\,a^2 = \frac{2\,\pi\,k}{N_x\,N_y}$$

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Configurations

- Through the International Lattice Data grid and PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. D79 (2009) 034503.
  - Lattice Volume: $32^3 \times 64$
  - Non-perturbative $\mathcal{O}(a)$-improved Wilson quark action
  - Iwasaki gauge action
  - $2 + 1$ flavour dynamical-fermion QCD
  - Physical lattice spacing $a = 0.0907$ fm
  - $m_\pi = 413$ MeV
- Standard Interpolating Fields: $\chi_{p1} = \left(u^T\, C\, \gamma_5\, d\right) u$, $\chi_{n1} = \left(u^T\, C\, \gamma_5\, d\right) d$
- Dynamical QCD configurations only - 'sea' quarks experience no field.

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$$E(B) = M - \vec{\mu} \cdot \vec{B} - \frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}(B^3)$$

- Construct ratios of different spin and field direction different 2pt correlation functions.

$$R(B,t) = \left(\frac{G_\downarrow(B+,t) + G_\uparrow(B-,t)}{G_\downarrow(0,t) + G_\uparrow(0,t)}\right)\left(\frac{G_\downarrow(B-,t) + G_\uparrow(B+,t)}{G_\downarrow(0,t) + G_\uparrow(0,t)}\right)$$

- Then extract an effective energy in the standard manner.

$$\delta E(B) = \frac{1}{\delta t}\log\left(\frac{R(B,t)}{R(B,t+\delta t)}\right) = \frac{1}{2}\left(\frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2\right)$$

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Landau Levels

- A charged particle in an external magnetic field sits in a superposition of energy levels

$$E^2 = m^2 + |qe\,B|\,(2\,n + 1 - \alpha) + p_z^2$$

- Quarks are charged - quarks also have Landau levels!
- The Landau levels are closely grouped due to the small fields used.
- Takes longer in Euclidean time for levels above ground state to be exponentially suppressed.

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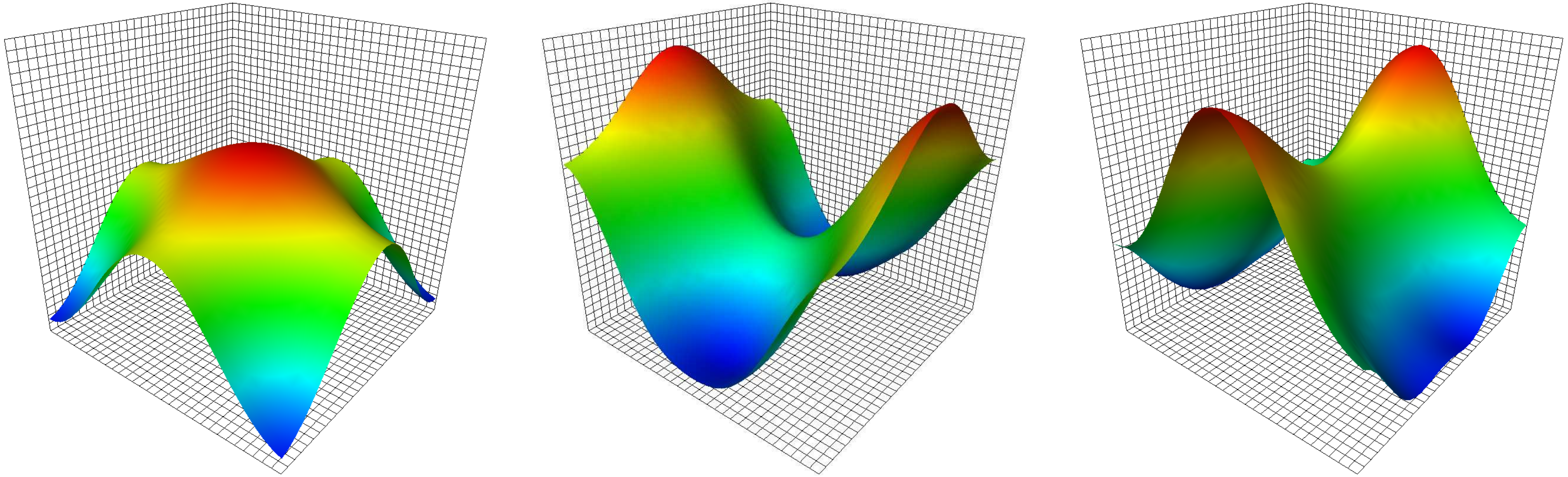
QED Eigenmodes



Lowest lying eigenmode probability densities of lattice Laplacian operator.

- Origin is centre of the x-y plane illustrated by bottom surface of the grid.
- Set source wavefunction to be exactly these modes, i.e. $\langle x|\psi\rangle = \sum_{i=1}^{n} \langle x|\nu_i\rangle$

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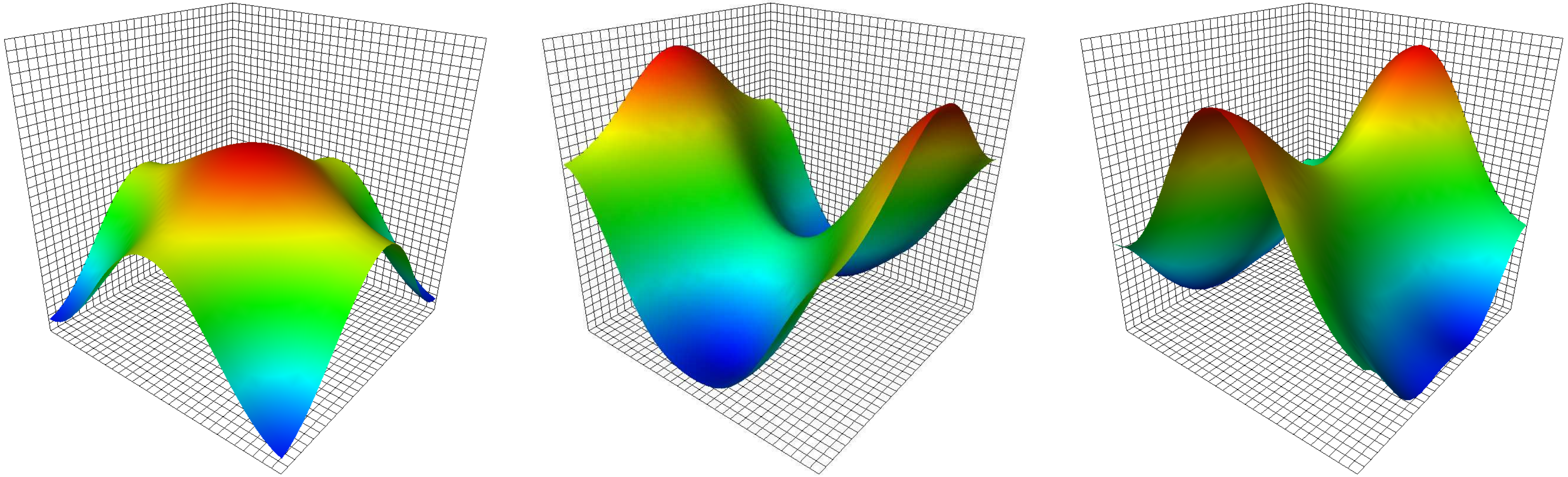

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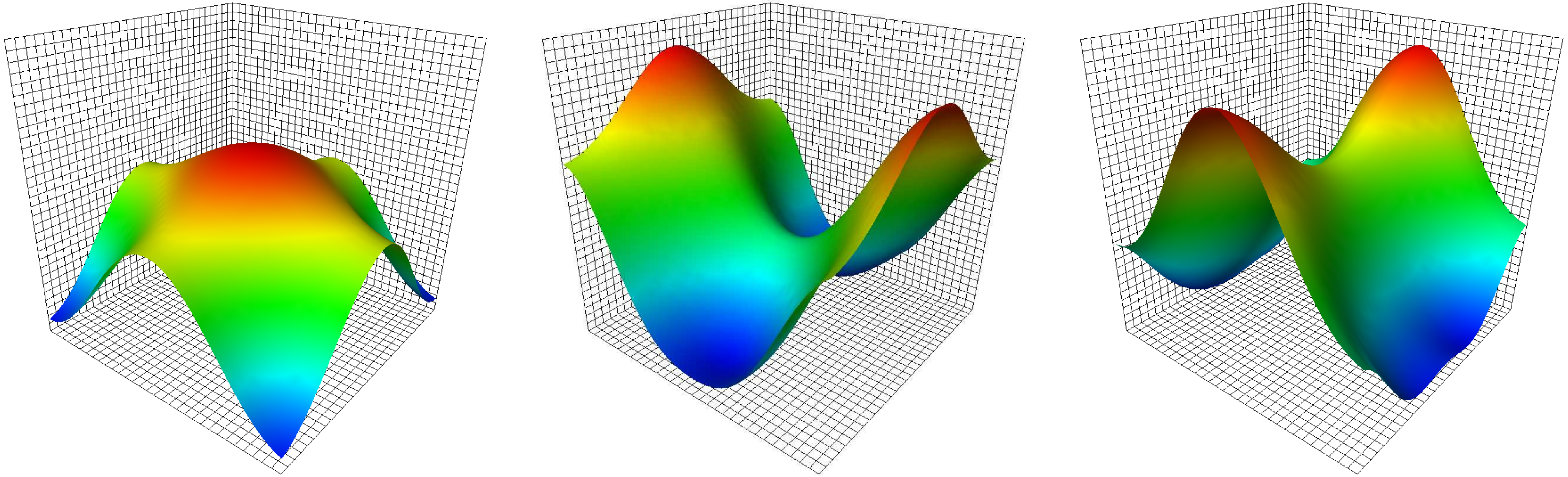

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- Define QED eigenmode projection operator

$$P^n_{QED}(x,y) = \sum_{i=1}^{n=\left|3\,q_f\,k_d\right|} \langle x|\nu_i\rangle\,\langle\nu_i|y\rangle$$

- Also define QCD+QED eigenmode projection operator

$$P^n_{QCD+QED}(x,y) = \sum_{i=1}^{n=n_{max}} \langle x|\lambda_i\rangle\,\langle\lambda_i|y\rangle$$

- and project the propagator

$$S(x,y,I,J) = P_I(x,z)\,S(z,z')\,P^{\dagger}_J(z',y)$$

I and J describe which projection operator is used, i.e. I=QED or I=QCD+QED.

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Smearing

- Can apply standard Gaussian smearing in any combination of spatial dimensions.
- Previous results found that a background field can change the wavefunction's spatial extent.
- Can be combined with eigenmode projections.
- Smearing along field axis found to be essential to remove excited state contamination.
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Fit window selection

$$\delta E(B) = \left(\frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 \right)$$

- To choose where to fit and obtain polarisability values, a number of factors are considered.
  - The constant fits to the energy shifts as function of time.
  - The fits to energy shifts as function of field strength, i.e. to
  - We only consider the same fit window across all field strengths.
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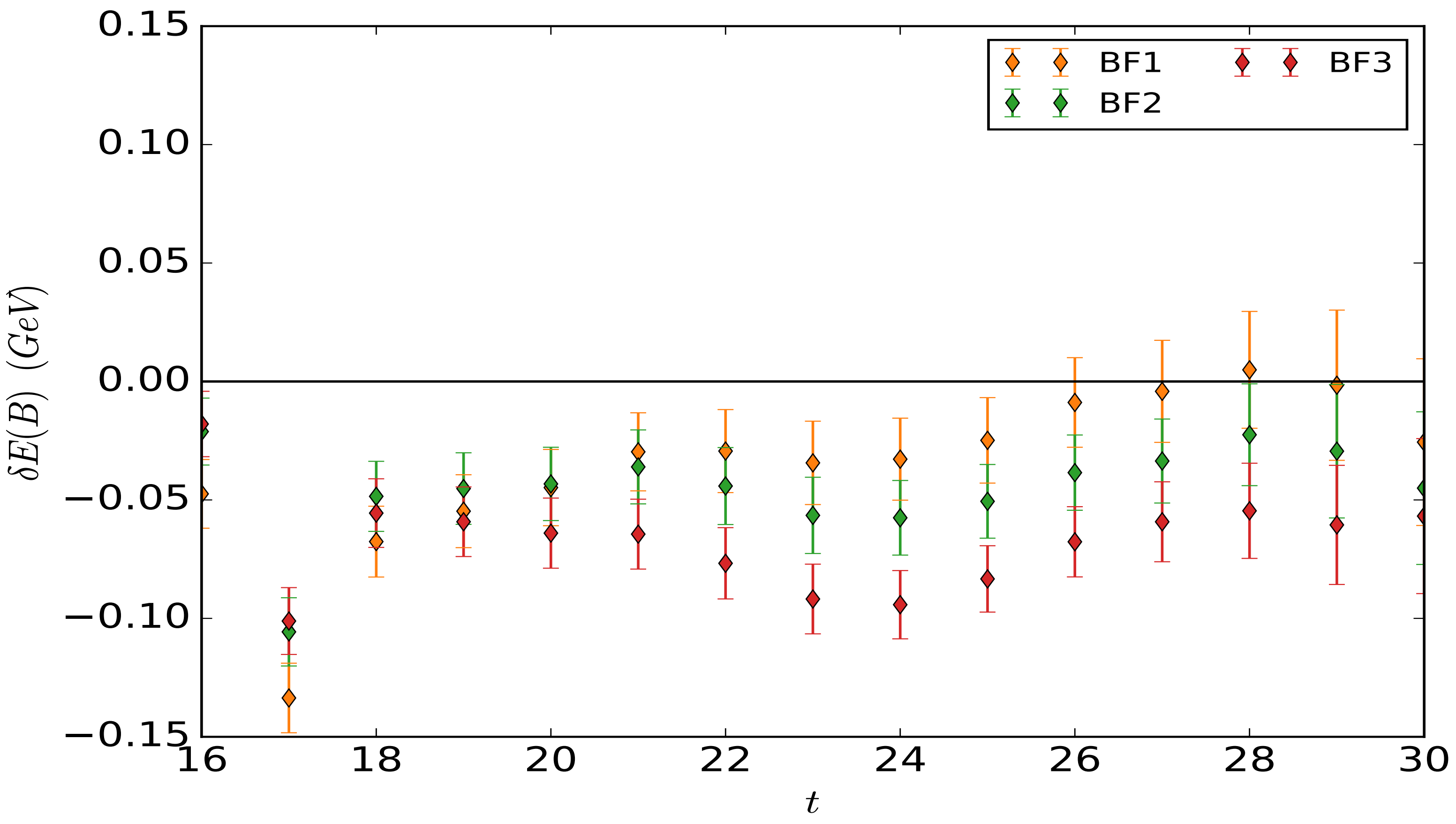
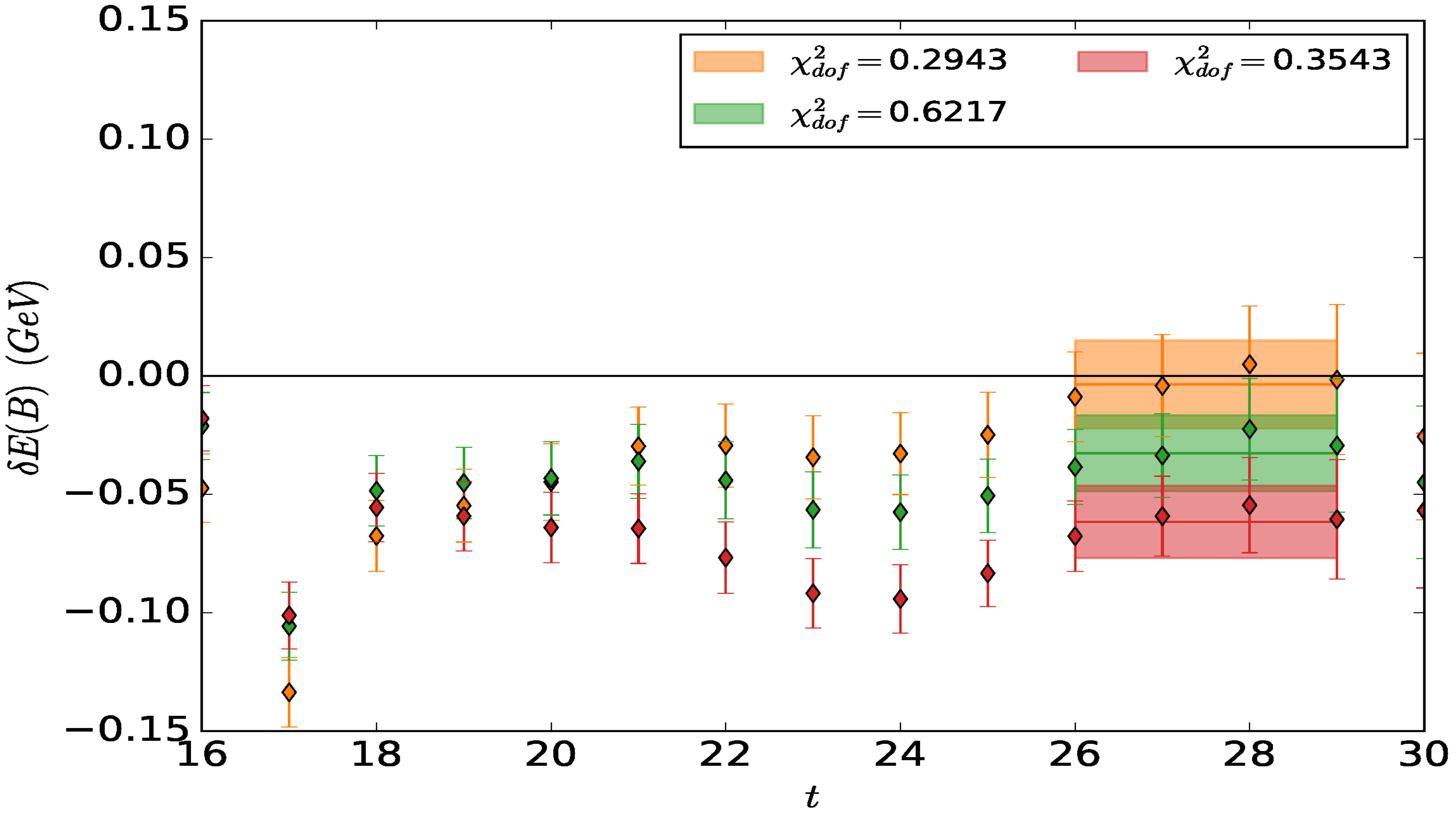


Smeared Source to QED eigenmode projected sink neutron energy shift

- For the first time it is possible to get plateaus in the energy shifts by using the projection methods.

Neutron Energy Shifts for polarisability



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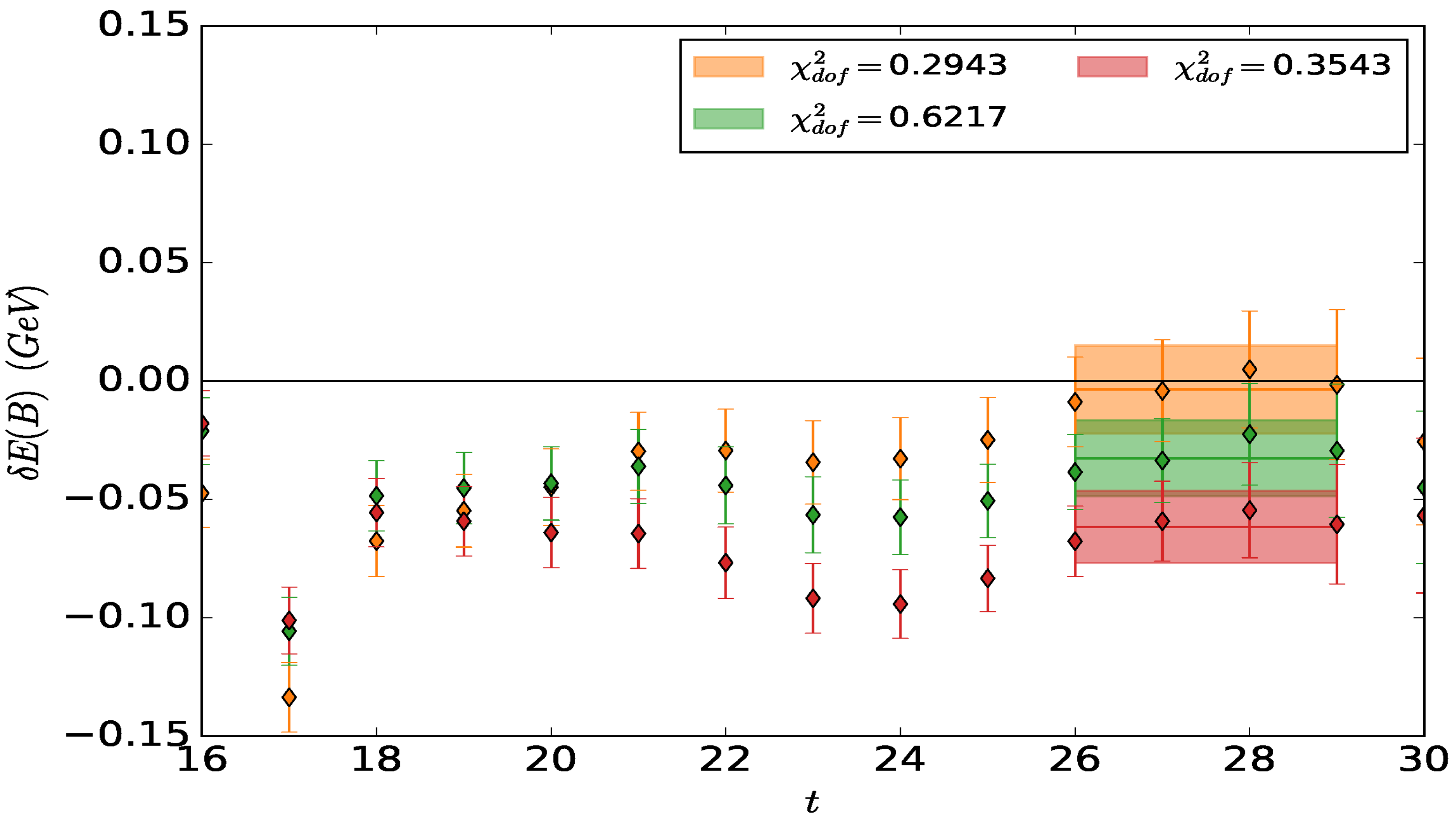

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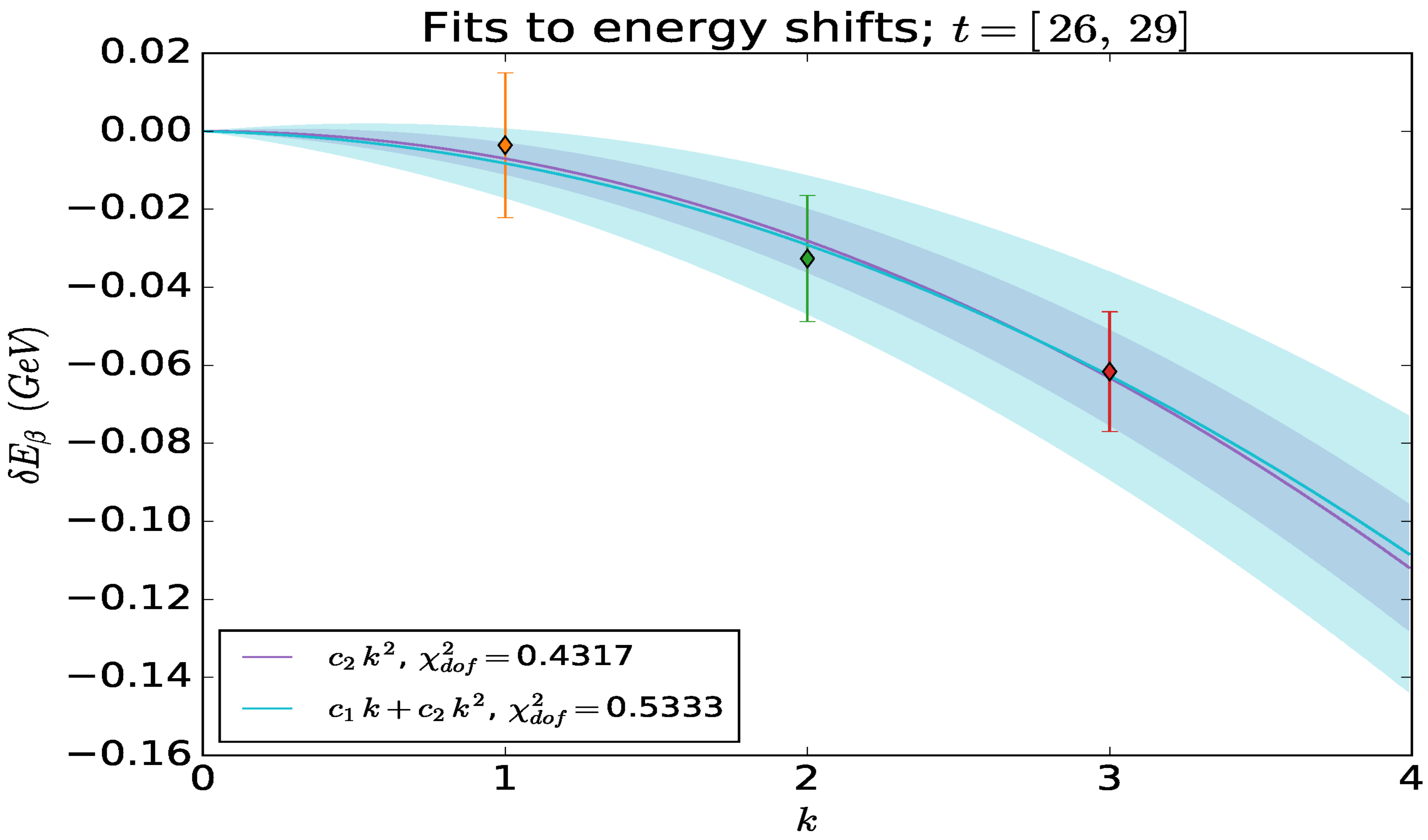
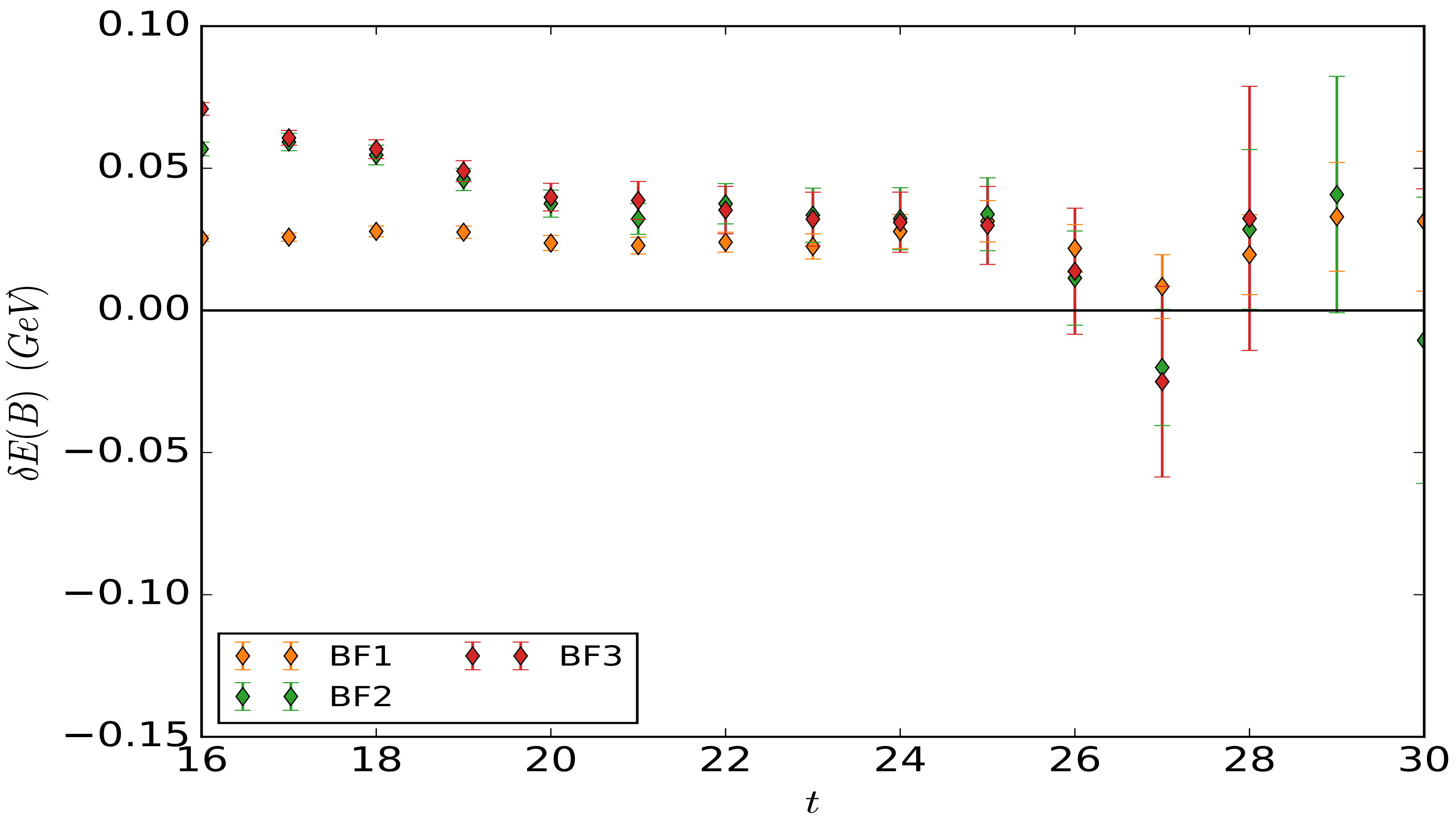
Neutron


Fits to energy shifts; $t = [26, 29]$
δE_β (GeV)
k
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
0
1
2
3
4
$c_2\, k^2,\ \chi^2_{dof} = 0.4317$
$c_1\, k + c_2\, k^2,\ \chi^2_{dof} = 0.5333$



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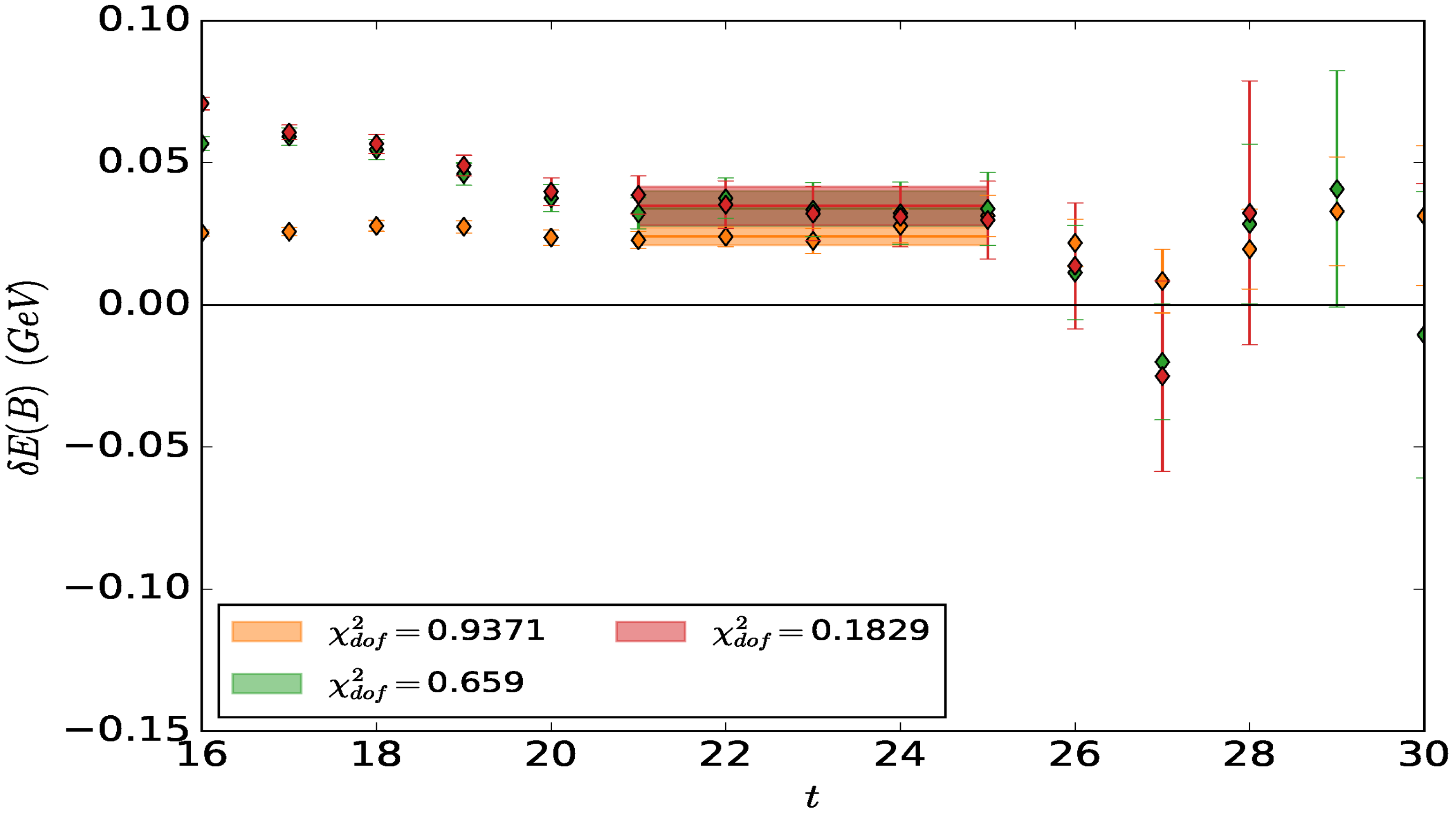
Proton Energy Shifts for polarisability



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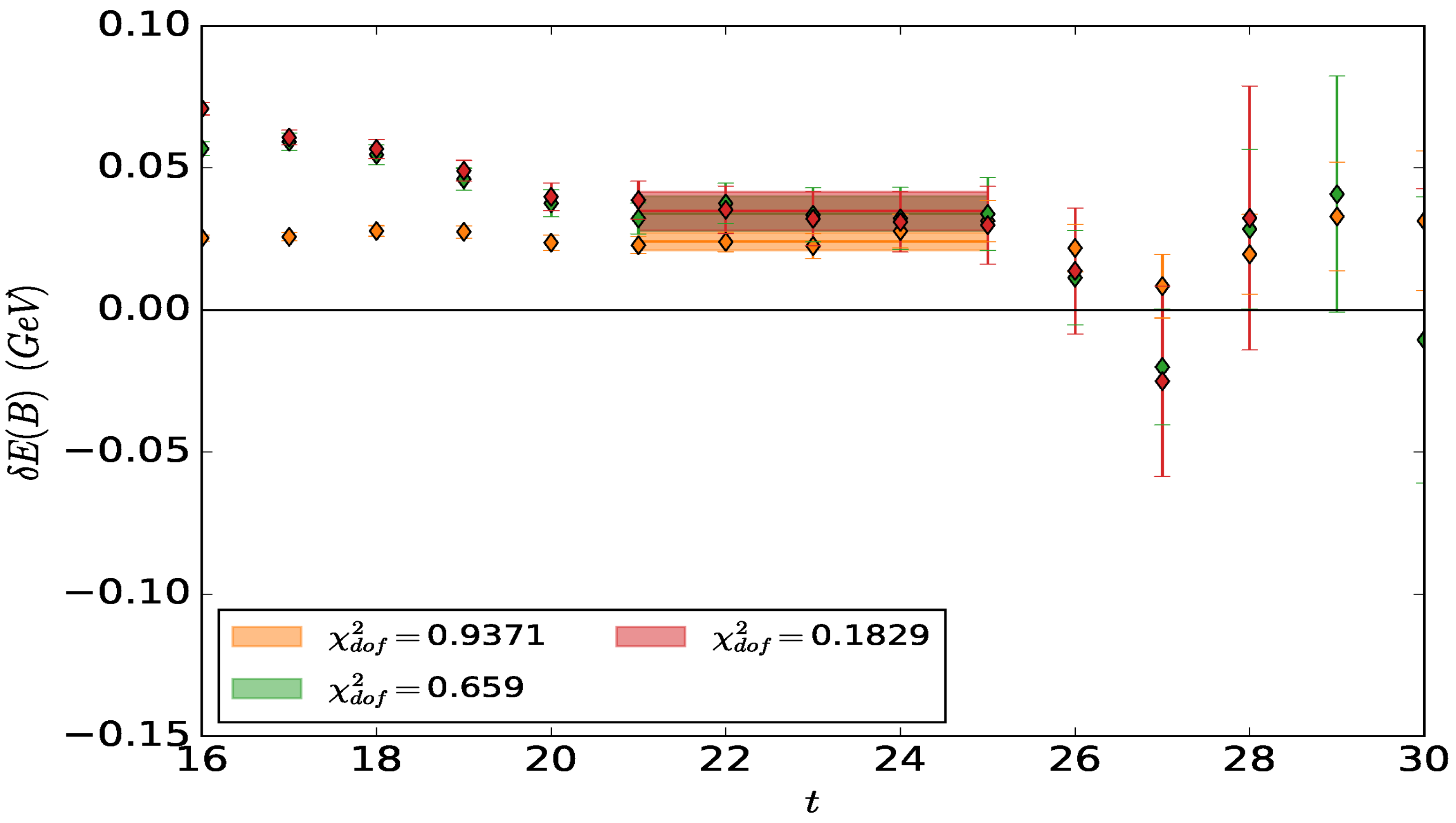

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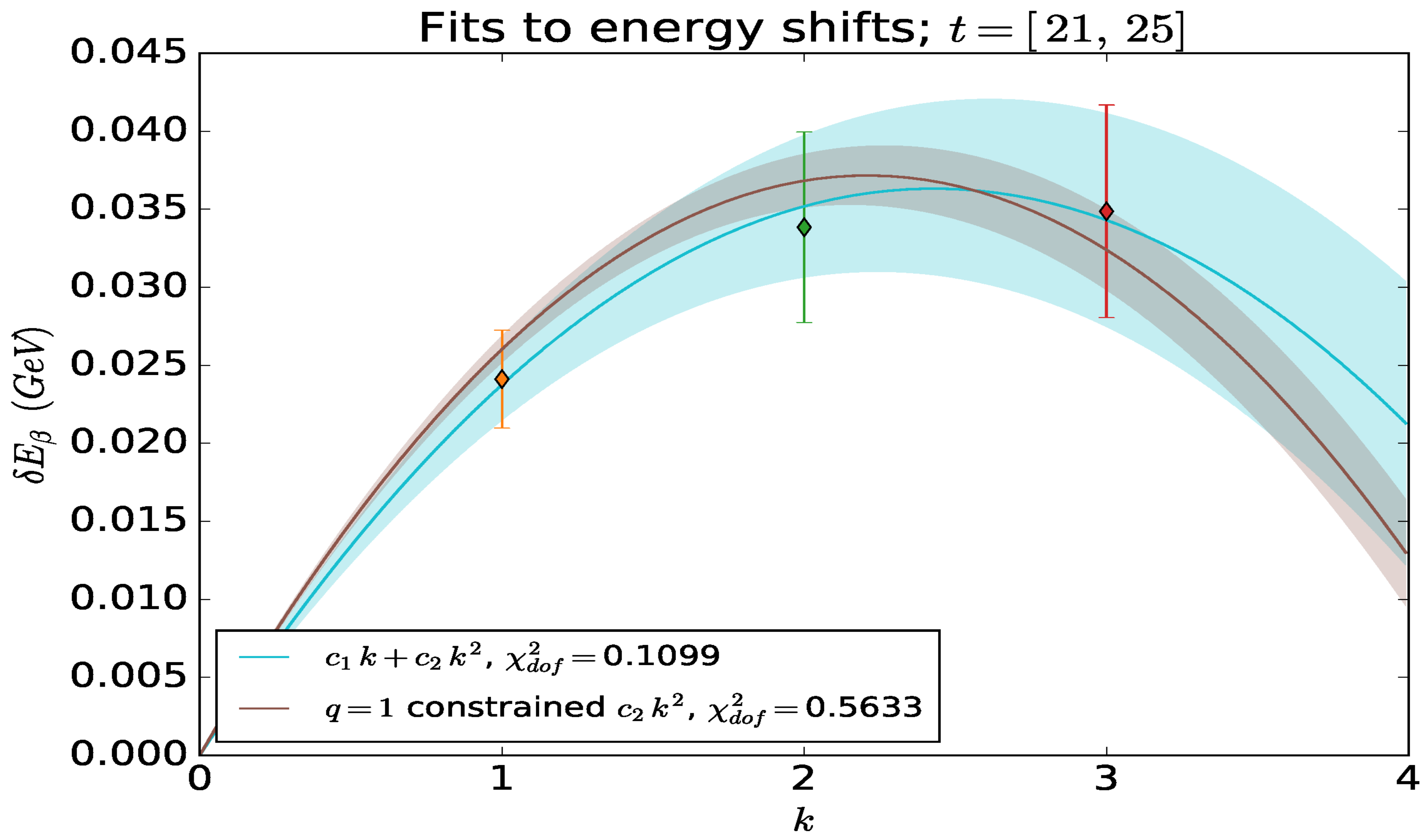
Proton


Fits to energy shifts; $t=[21, 25]$
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
δE_β (GeV)
0
1
2
3
4
k
$c_1\,k + c_2\,k^2$, $\chi^2_{dof} = 0.1099$
$q=1$ constrained $c_2\,k^2$, $\chi^2_{dof} = 0.5633$

- From the quadratic term extract the polarisability

$$\delta E(B) \propto -\frac{4\,\pi}{2}\,\beta\,B^2$$

- Nucleon polarisabilities are found to be

	Experiment ($m_\pi = 138$ MeV)	This Work ($m_\pi = 413$ MeV)
proton	$2.5(4) \times 10^{-4}$ fm^3	$1.15(24) \times 10^{-4}$ fm^3
neutron	$3.7(12) \times 10^{-4}$ fm^3	$1.31(38) \times 10^{-4}$ fm^3

- Potential exists to make interesting predictions from lattice QCD.

Future Work

- Expand to correlation matrix techniques
- Chiral extrapolations to confront experiment
- Fine tune source & sink combinations

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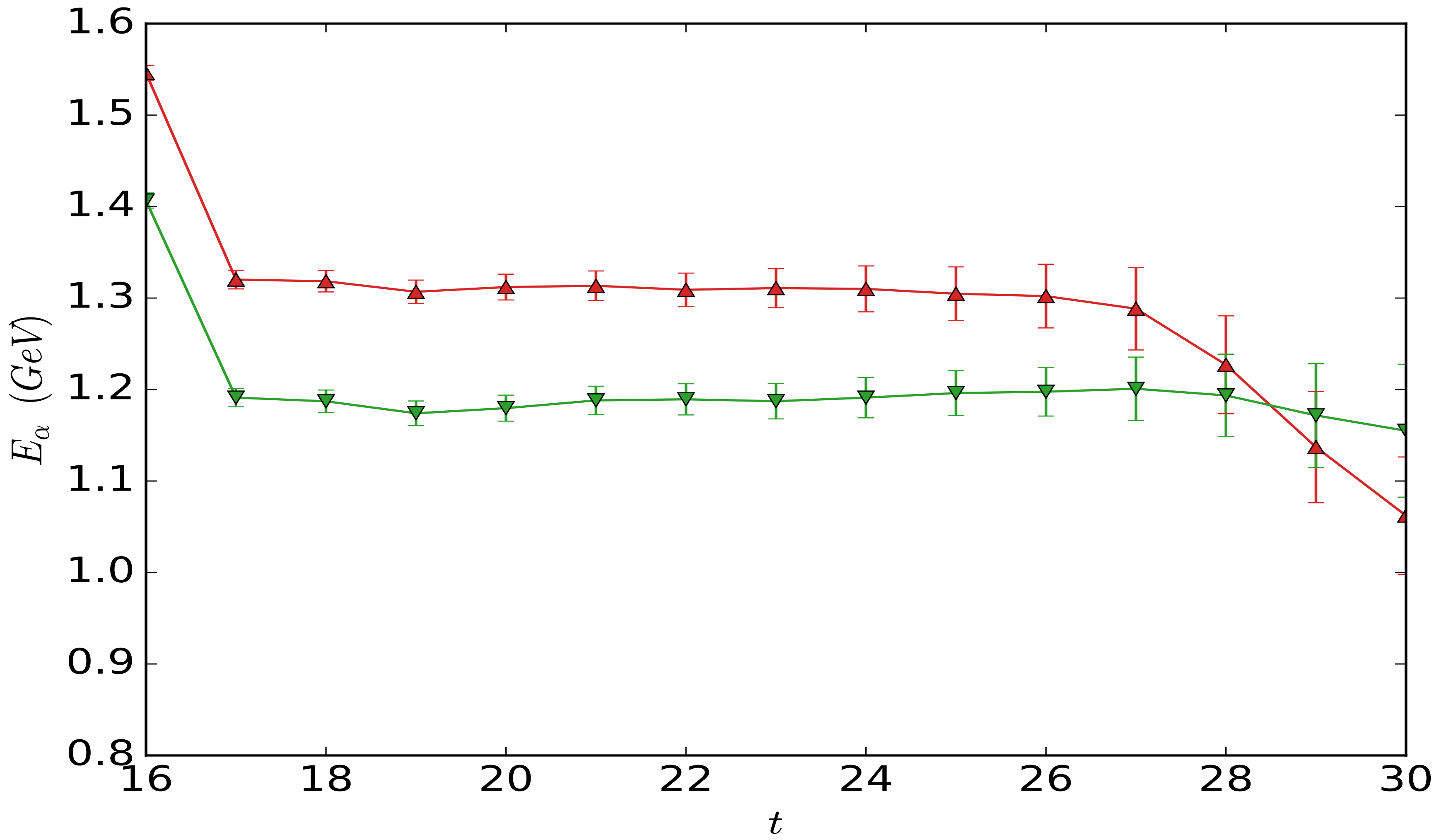
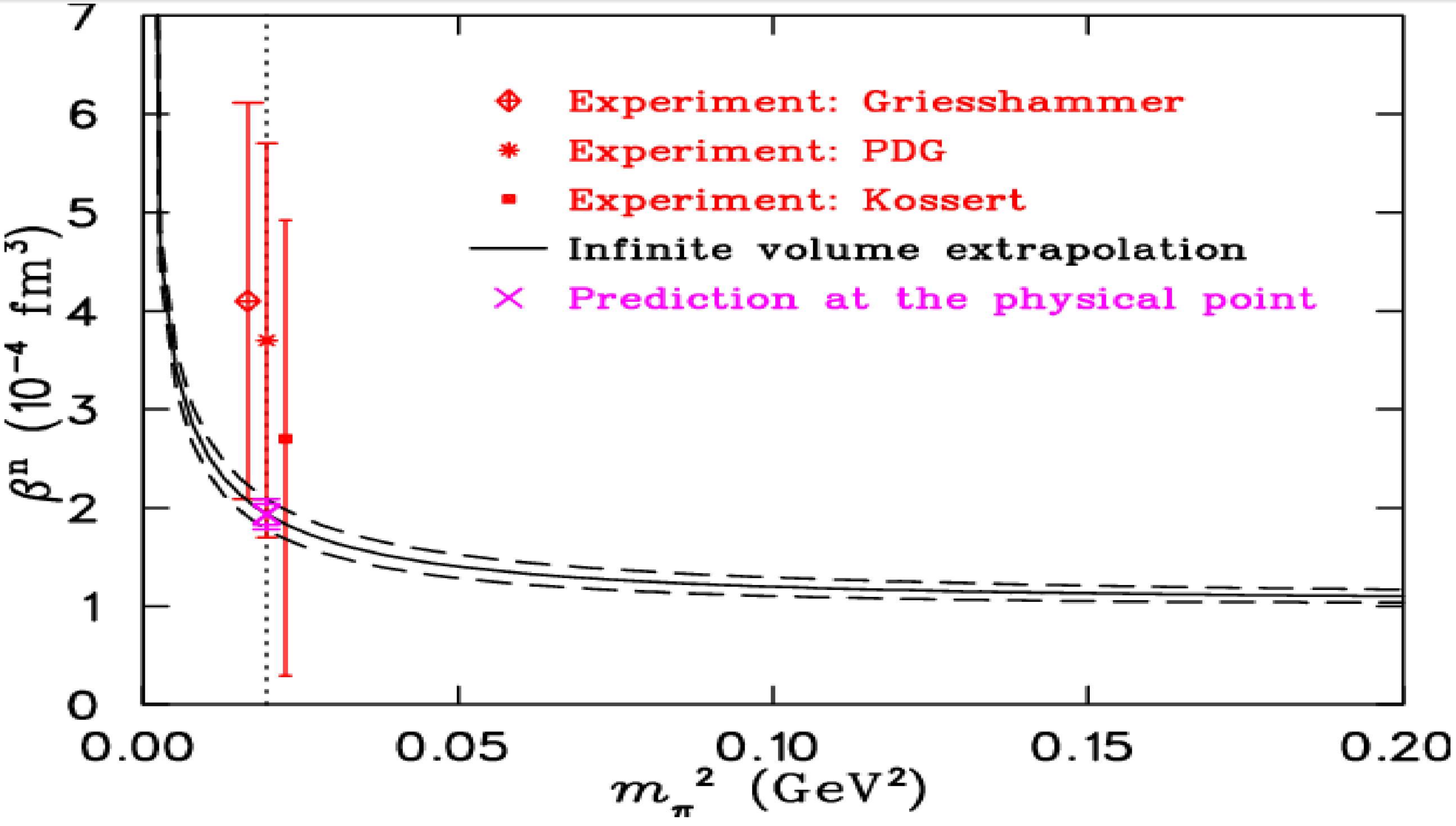
Effective energy of spin up & down components for neutron in smallest quantised field



Chiral Extrapolations based on previous work



Magnetic polarisability of Neutron, compared with experimental results.

Magnetic Moment

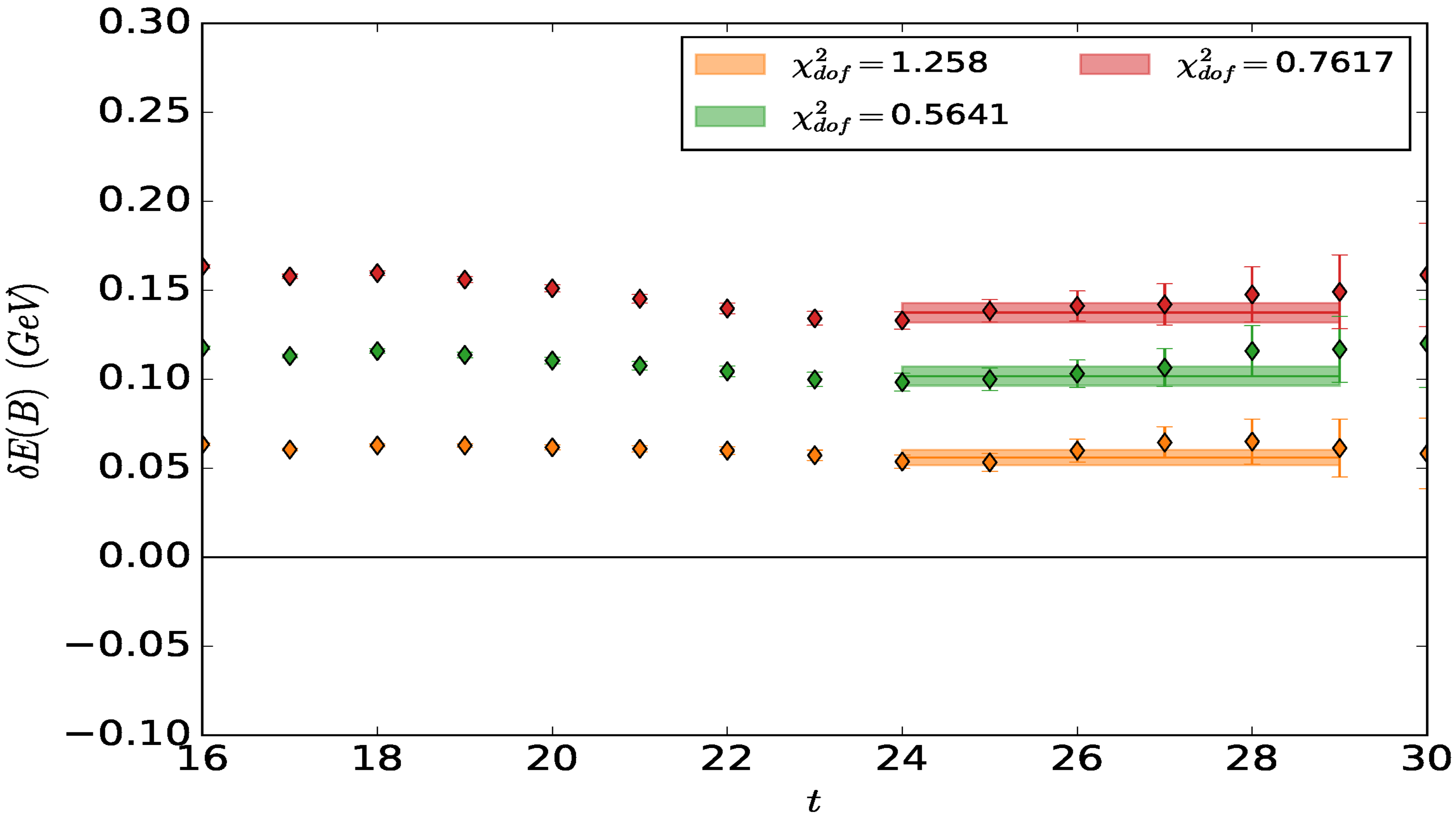
- Considerably easier than magnetic polarisability
- Take a different ratio

$$R(B,t) = \left(\frac{G_{\downarrow}(B-,t) + G_{\uparrow}(B+,t)}{G_{\downarrow}(B+,t) + G_{\uparrow}(B-,t)}\right)$$

- to get an energy shift of

$$\delta E_{\mu}(B) = -\mu\, B + \mathcal{O}\left(B^3\right)$$

Magnetic Moment



Energy shift for magnetic moment of the neutron.

Magnetic Moment

- Extract magnetic moment from linear term
- Background field results are prelimnary only

	BFM ($m_\pi = 413$ MeV)	3PT ($m_\pi = 413$ MeV)
proton (β_p)	$2.184(22)\,\mu_N$	$2.244(61)\,\mu_N$
neutron (β_n)	$-1.371(14)\,\mu_N$	$-1.36(10)\,\mu_N$