Effects Of Target Resonances In Low-energy

Nucleon Scattering from Weakly-bound Nuclei

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Why this study?

- Weakly bound radioactive nuclei now formed as RIBs
- Their low excitation spectra have particle emission resonant states.

Questions: with weakly bound nuclei in scattering (beam/target)

- What effect do such particle-emitting states have on cross sections?
- What physical considerations

  need be taken in modelling these resonances?

We seek answers using:

- The MCAS method is used to solve Lippmann-Schwinger equations

  Currently built for n+A and α+A coupled-channel systems.
- Compound system spectra found:

  all bound and (low excitation) resonant states of the cluster.
- The S-matrix is formed $\longrightarrow$ elastic and reaction cross sections.

The Lippmann-Schwinger equations:

The coupled-channel problem of a two-nucleus system (no resonant states)

- **In momentum space, with an interaction matrix, $\mathbf{V}^{\mathbf{J}^\pi}_{\mathbf{cc'}}(\mathbf{p},\mathbf{q})$, ($c$: unique quantum number sets), the multi-channel T-matrix is**

$$\mathbf{T}^{\mathbf{J}^\pi}_{\mathbf{cc'}}(\mathbf{p},\mathbf{q};\mathbf{E}) = \mathbf{V}^{\mathbf{J}^\pi}_{\mathbf{cc'}}(\mathbf{p},\mathbf{q}) + \mu\left[\sum_{\mathbf{c''}=\mathbf{1}}^{\mathtt{open}}\int_0^\infty \mathbf{V}^{\mathbf{J}^\pi}_{\mathbf{cc''}}(\mathbf{p},\mathbf{x})\frac{\mathbf{x}^2}{\mathbf{k}^2_{\mathbf{c''}}-\mathbf{x}^2+\mathbf{i}\varepsilon}\mathbf{T}^{\mathbf{J}^\pi}_{\mathbf{c''c'}}(\mathbf{x},\mathbf{q};\mathbf{E})\,\mathbf{dx} - \sum_{\mathbf{c''}=\mathbf{1}}^{\mathtt{closed}}\int_0^\infty \mathbf{V}^{\mathbf{J}^\pi}_{\mathbf{cc''}}(\mathbf{p},\mathbf{x})\frac{\mathbf{x}^2}{\mathbf{h}^2_{\mathbf{c''}}+\mathbf{x}^2}\mathbf{T}^{\mathbf{J}^\pi}_{\mathbf{c''c'}}(\mathbf{x},\mathbf{q};\mathbf{E})\,\mathbf{dx}\right]$$

- **For an n+A system, $\mathbf{c} = (\ell\tfrac{1}{2})\,\mathbf{jI};\mathbf{J}^\pi$; and with $\mathbf{E_c}$: target state energies**

$$\mathbf{k}^2_{\mathbf{c}} = \mu\,(\mathbf{E}-\mathbf{E_c}) \quad ; \quad \mathbf{h}^2_{\mathbf{c}} = \mu\,(\mathbf{E_c}-\mathbf{E})\,.$$

Some details of the MCAS approach:

References: 1. K. Amos *et al.*, Nucl. Phys. A728, 65 (2003).

2. L. Canton *et al.* Phys. Rev. C 83, 04763 (2011).

- **Pauli principle:** **coupled-channels and local interactions — can violate P.P.**
- **Overcome:** **use orthogonalizing pseudo-potentials (OPPs)**

$$\mathbf{V_{cc'}(p,q) \longrightarrow V_{cc'}(p,q) \; + \; \lambda A_c(p) A_{c'}(q)}$$

$\mathbf{A_c(p)}$ **: bound state functions for occupied orbits (in $\mathbf{V_{c,c}}$)**

- **Effect:** **New Hamiltonian has no solutions involving occupied states**
- **If a target state is a resonance:** **Replace excitation energy ($\mathbf{E_c}$) with**

$$\mathbf{E_c \quad \longrightarrow \quad E_c + \Delta_c(E) + i\frac{1}{2}U(E)\Gamma_c}$$

where the shift that restores causality is given by

$$\mathbf{\Delta_c(E) = \frac{\Gamma_c}{2}\frac{1}{\pi}\mathcal{P}\int_0^{\infty} \frac{U(E')}{(E'-E)} dE'}$$

Why the scale factor, $\mathbf{U(E)}$?

- **Lorentzian forms:** can lead to non-physical features in results:
  1. Reaction cross sections

     $E \to 0$ (scattering threshold) may diverge, and
  2. bound states in the compound system spectrum

     may have spurious widths.

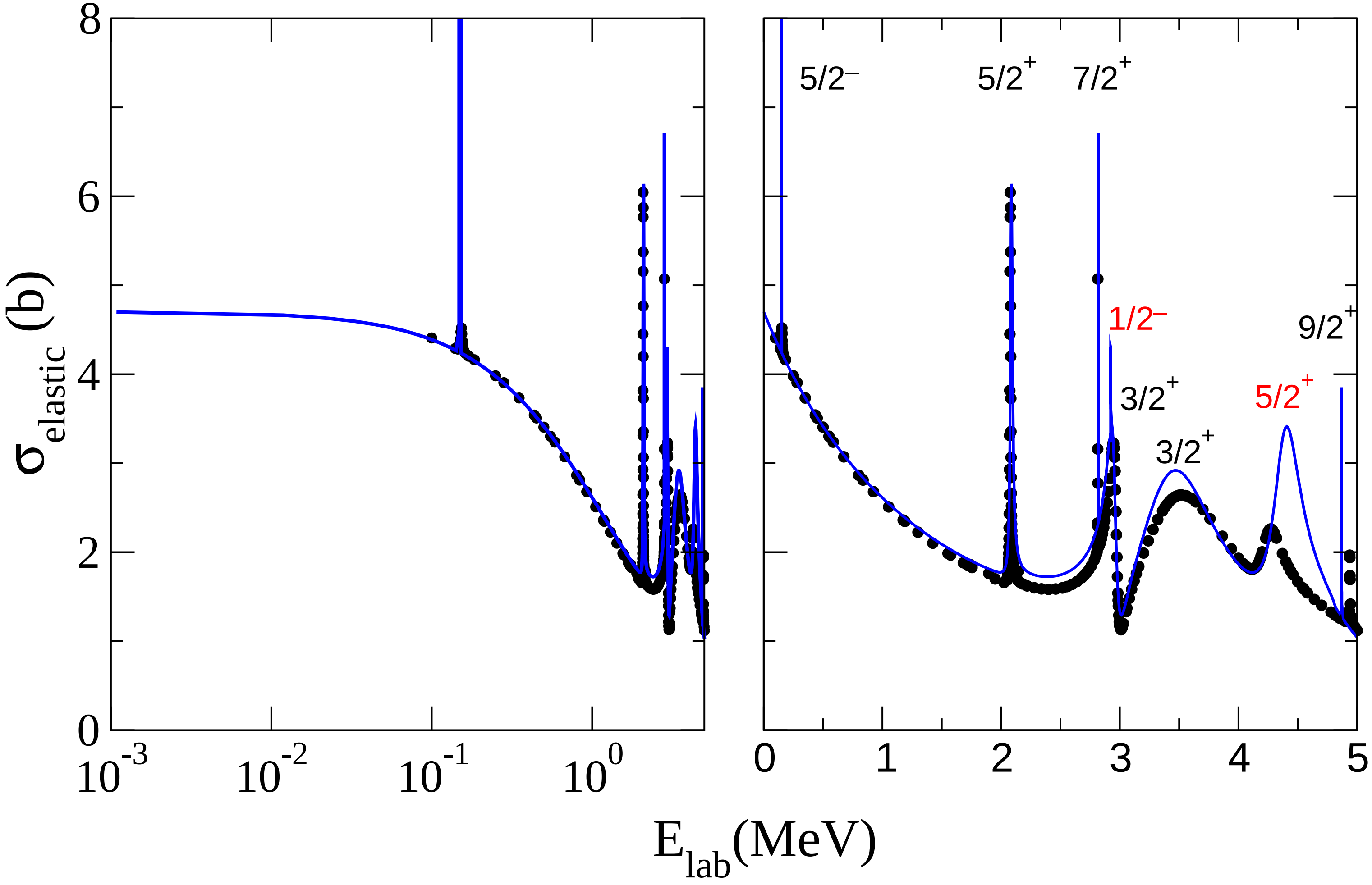
- **Cause of problems:** Lorentzians are non-zero below the threshold.

- **Remedy:** use a scaling function on the Lorentzian form.

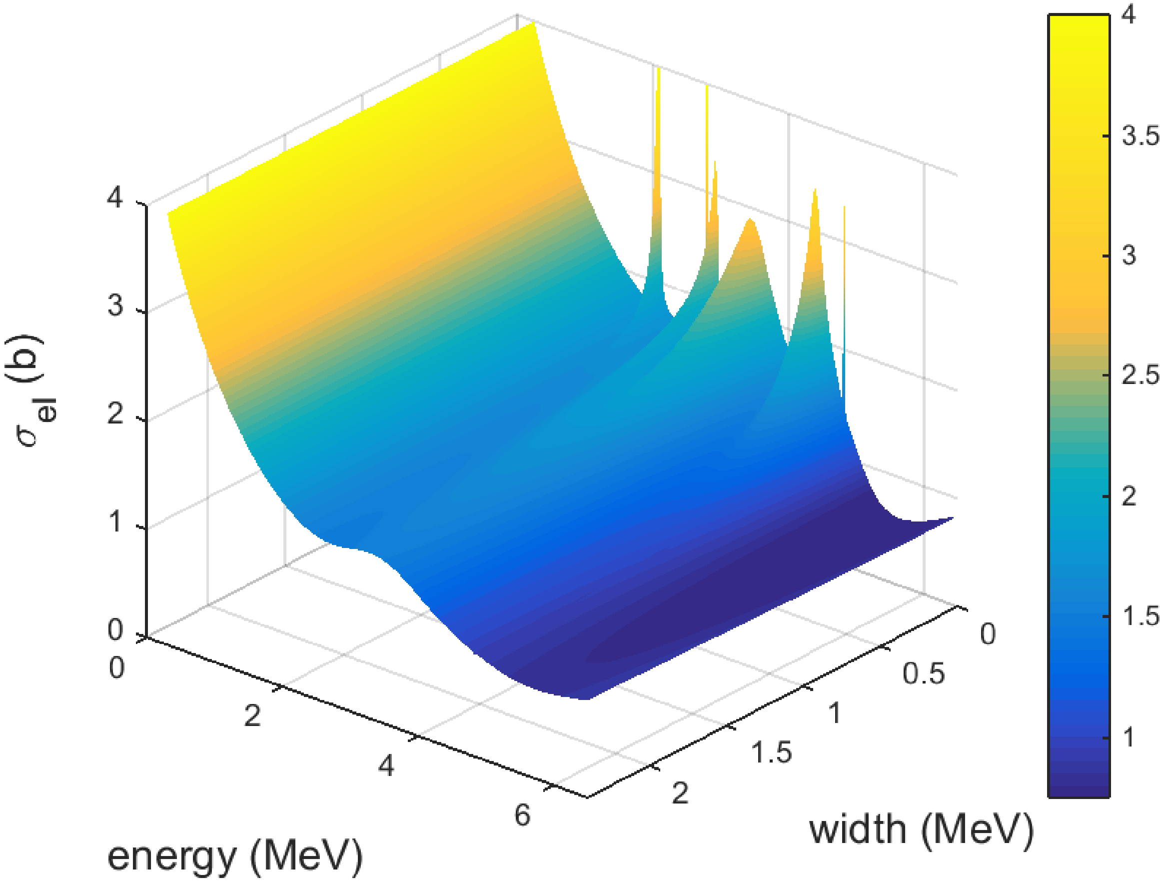
- **Restore causality:** Add the energy-dependent shift, $\mathbf{\Delta_c(E)}$

- **Scale function conditions:**
  1. $\mathbf{U(E) = 0}$ and $\mathbf{\frac{dU(E)}{dE} = 0}$ for $\mathbf{E \leq 0}$
  2. $\mathbf{U(E) \to 0}$ as $\mathbf{E \to \infty}$
  3. $\mathbf{U(E_c) = 1}$

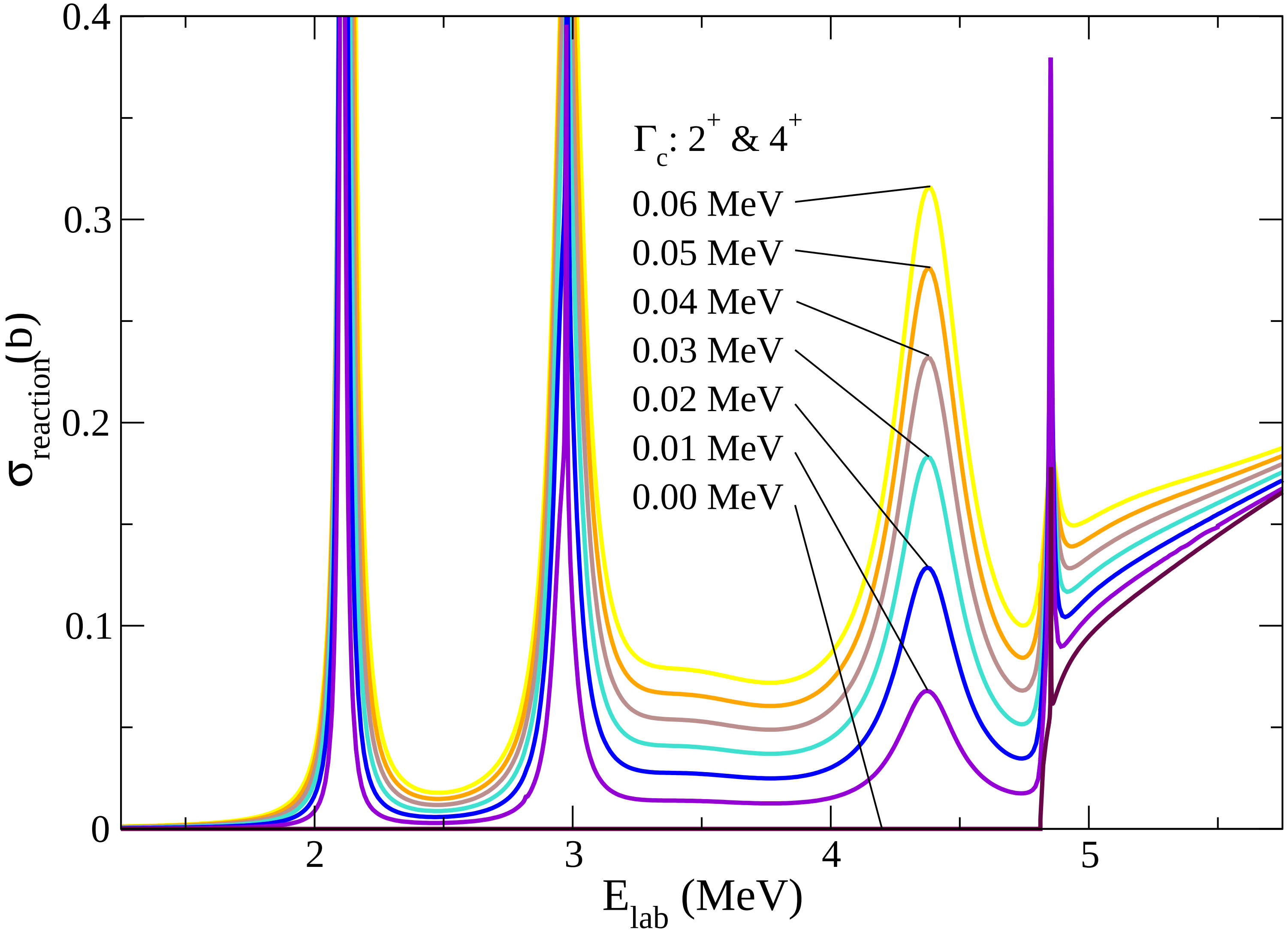
Case: n+^{12}C scattering – 3 states, $0^+_{g.s.}, 2^+_1, 0^+_2$, no widths



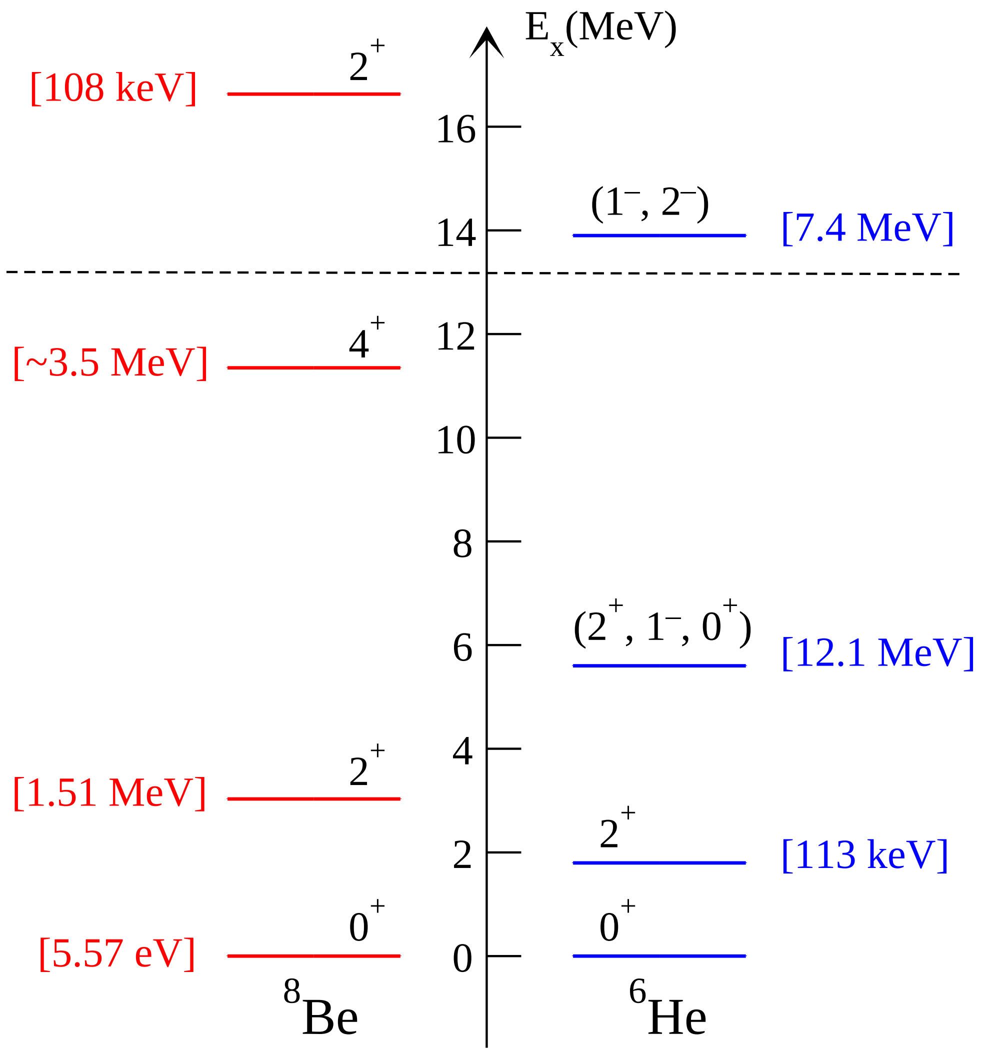
n+^{12}C elastic scattering — adding widths to $2_1^+, 0_2^+$ states



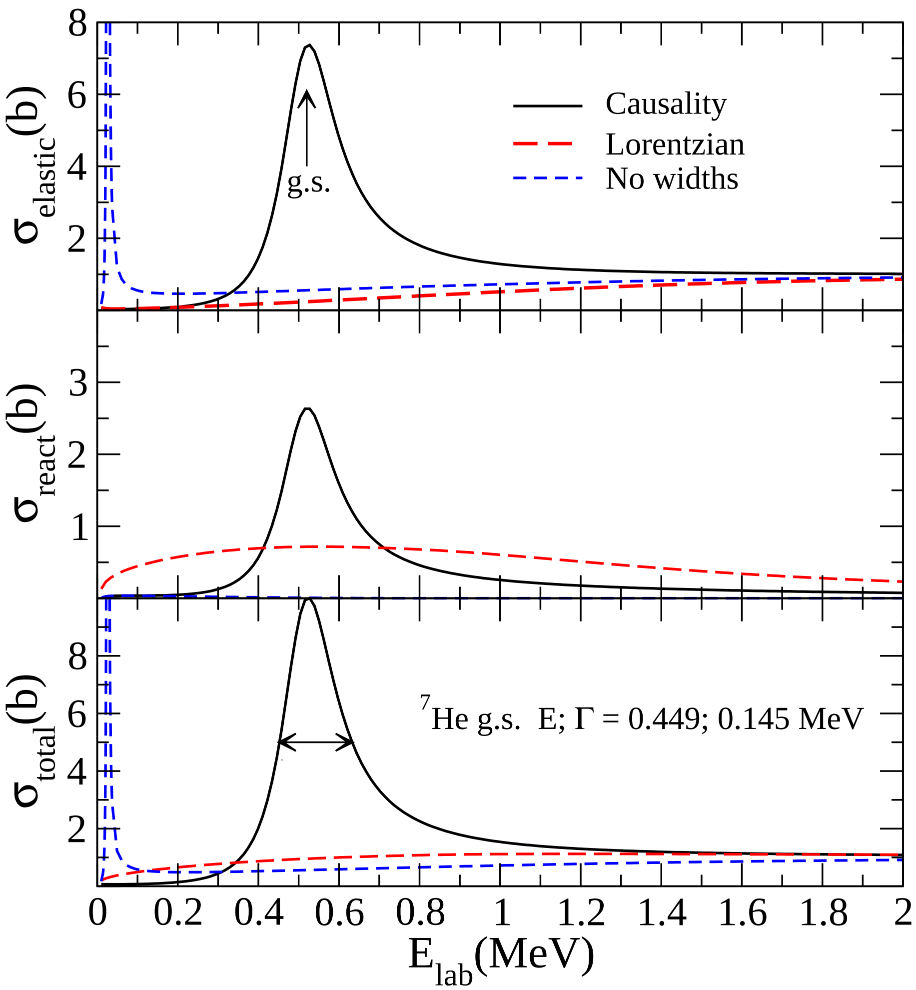
n+^{12}C reaction cross sections — small widths



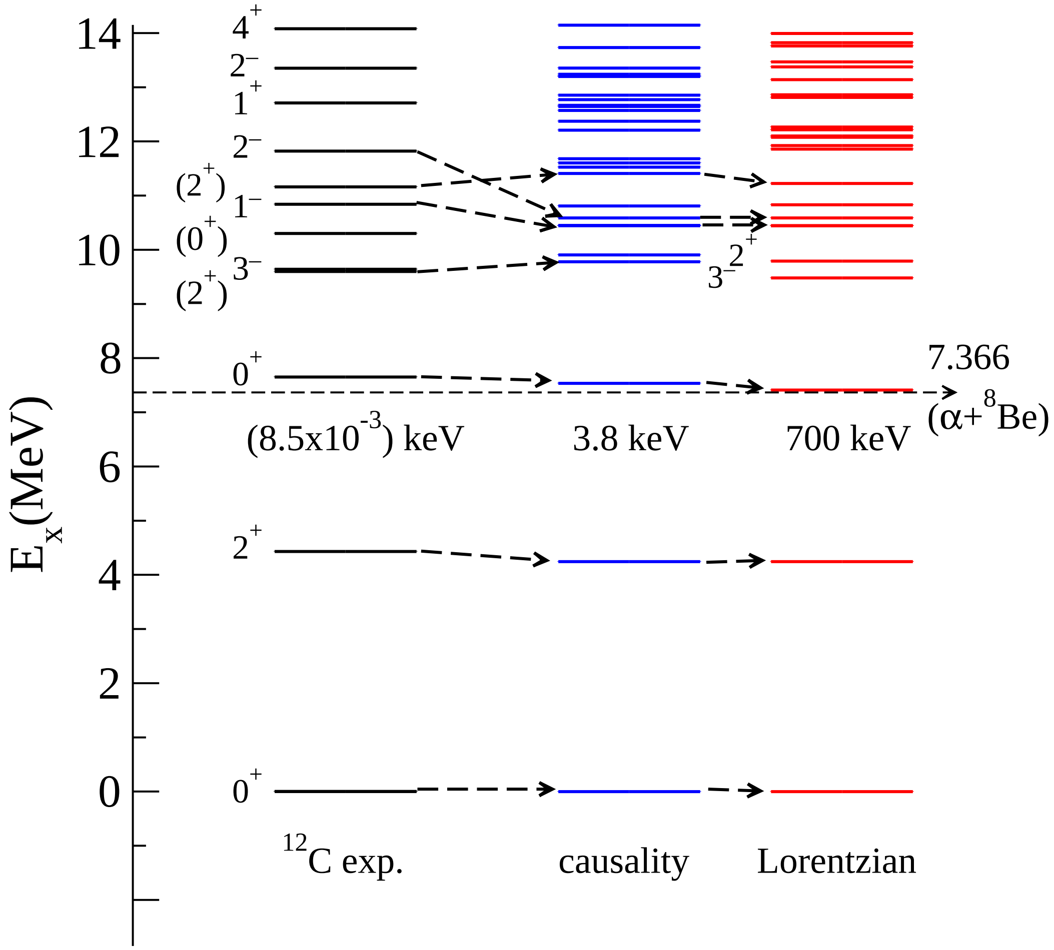
Target nuclei:



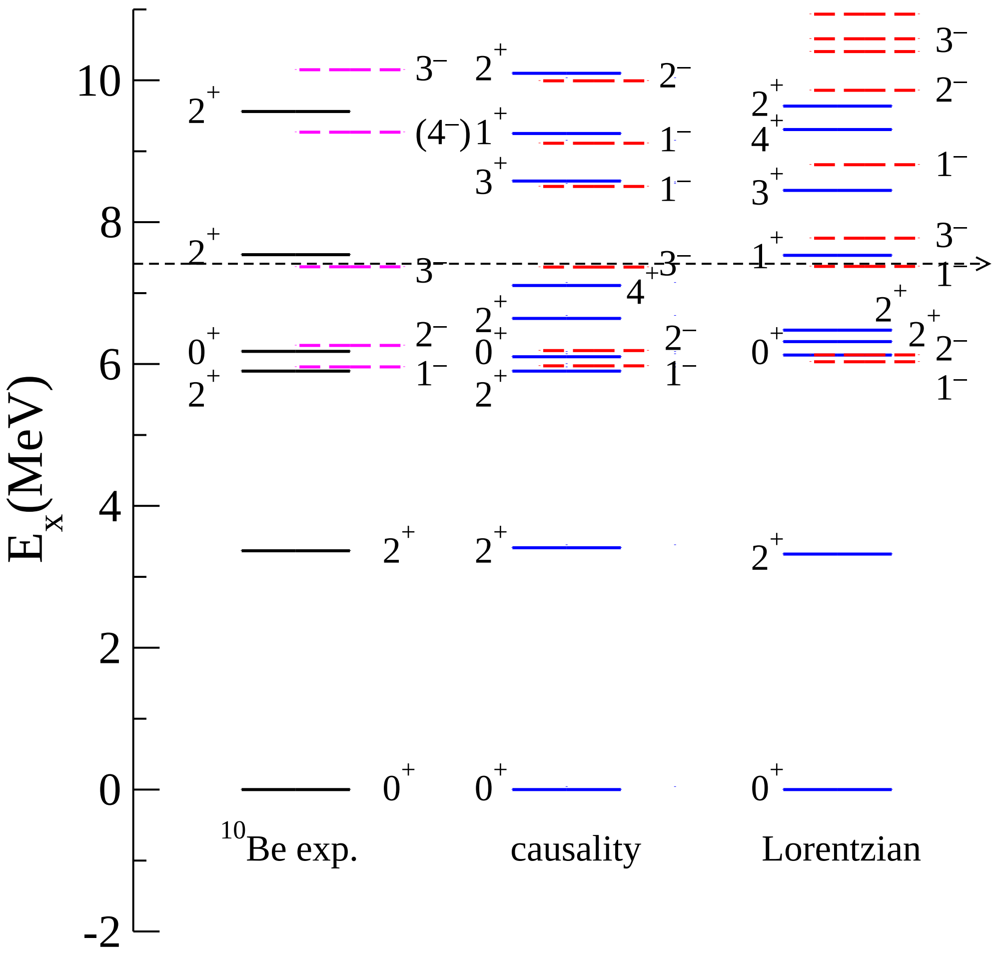
Case: ^{7}He (n+^{6}He) — g.s. $\frac{3}{2}^-$: E = 0.445 MeV (0.52 lab.); $\Gamma = 0.15$ MeV



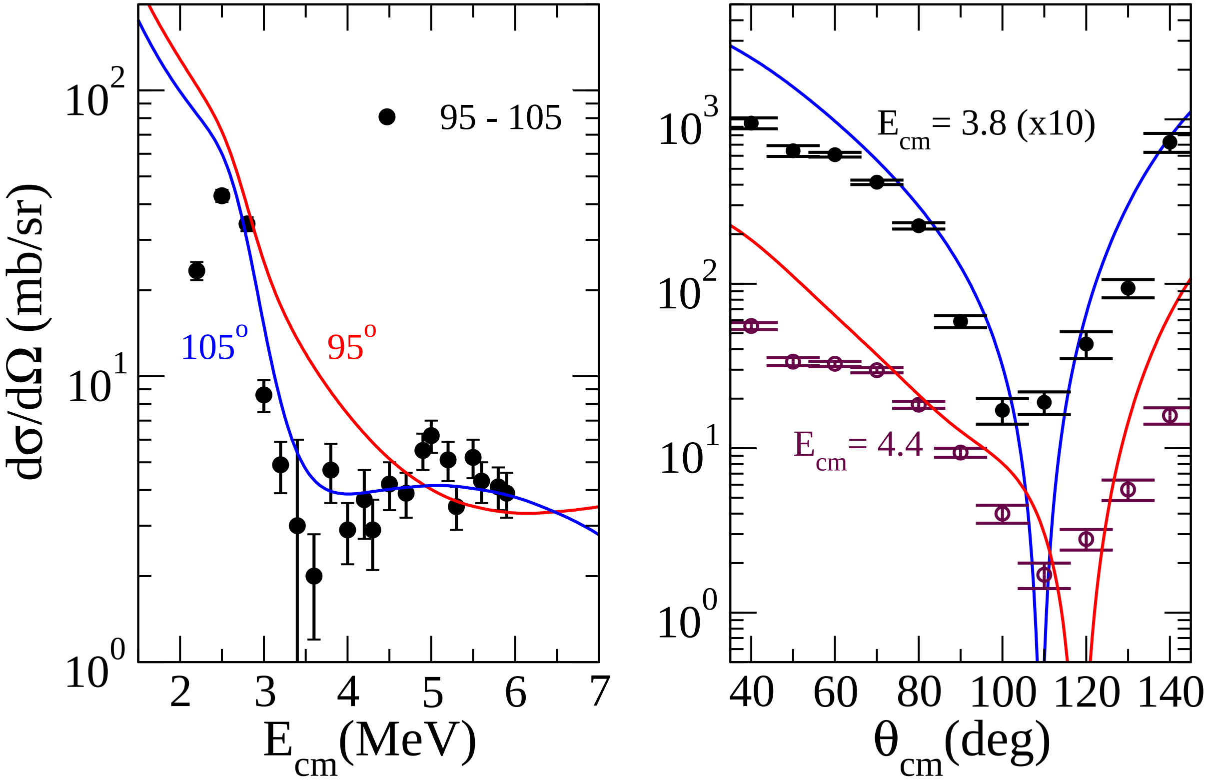
Case: Spectra of ^{12}C (α+^{8}Be)



Case: Spectra of ^{10}Be (α+^{6}He)



Case: α+^{6}He scattering



Data: D. Suzuki ***et al.*** **Phys. Rev. C 87, 054301 (2013)**

Conclusions:

MCAS: developed to solve coupled-channel two-nucleus cluster problems

This method:

- accounts for target state particle instability
- is free of any unphysical behaviour at the scattering threshold
- satifies the Pauli principle (via OPP)
- conserves causality.
- allows scaling the usual Lorentz form of a resonant target state

  (to meet physical requirements while preserving causality)
- yields compund spectra and scattering cross sections

  (Results are sensitive to target state particle instability)

Structure and scattering involving weakly-bound RIBs will be influenced.

References:

- **Basic MCAS: K. Amos** *et al.* **Nucl. Phys. A728, 65 (2003).**
- **Pauli principle violation and coupled-channel calculations:**

  L. Canton *et al.* **Phys. Rev. Lett. 94, 122503 (2005).**

  K. Amos *et al.* **Phys. Rev. C. 72, 064604 (2005).**
- **Predicting resonances in drip-line spectra:**

  L. Canton *et al.* **Phys. Rev. Lett. 96, 072502 (2006).**
- **Non-local effective potentials from MCAS:**

  P. Fraser *et al.* **Euro. Phys. J. A 35, 69 (2008).**
- **MCAS with particle unstable target states:**

  P. Fraser *et al.* **Phys. Rev. Letts. 101, 242501 (2008).**

  L. Canton *et al.* **Phys. Rev. C 83, 047603 (2011)**

MCAS details: **Optimal functions, $\hat{\chi}_{cn}(q)$, from Sturmians,**

$$|\hat{\chi}_{\mathbf{cn}}\rangle = \sum_{\mathbf{c'}} \mathbf{V_{cc'}}\, |\mathbf{\Phi_{c'n}}\rangle \;\; ; \;\; \sum_{\mathbf{c'}} \mathbf{G_c^{(0)}} \mathbf{V_{cc'}}\, |\mathbf{\Phi_{c'n}}\rangle = -\eta_{\mathbf{n}}\, |\mathbf{\Phi_{cn}}\rangle$$

- **Expand:** **the potential matrix:** $\mathbf{V_{cc'}}(\mathbf{p},\mathbf{q}) \sim \sum_{\mathbf{n}} \chi_{\mathbf{cn}}(\mathbf{p})\, \eta_{\mathbf{n}}^{-1}\, \chi_{\mathbf{c'n}}(\mathbf{q})$
- **Leads to:** **a multi-channel S-matrix:** $\mathbf{S_{cc'}} = \delta_{\mathbf{cc'}} - \mathbf{i}\pi\mu\sqrt{\mathbf{k_c k_{c'}}}\, \mathbf{T_{cc'}}$

  where

$$\mathbf{T_{cc'}} = \sum_{\mathbf{n,n'}} \chi_{\mathbf{cn}}(\mathbf{p}) \left([\boldsymbol{\eta} - \mathbf{G_0}]^{-1} \right)_{\mathbf{nn'}}\, \chi_{\mathbf{c'n'}}(\mathbf{q})$$

- **Involves:** **a resolvent matrix with**

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c''=1}^{\text{open}} \int_0^\infty \chi_{c''n}(x) \frac{x^2}{k_{c''}^2 - x^2 + i\epsilon} \chi_{c''n'}(x)\, dx \right.$$
$$\left. - \sum_{c''=1}^{\text{closed}} \int_0^\infty \chi_{c''n}(x) \frac{x^2}{h_{c''}^2 + x^2} \chi_{c''n'}(x)\, dx \right] \; ; \; [\boldsymbol{\eta}\,]_{nn'} = \eta_n\, \delta_{nn'}$$

What are Sturmians?

- Two body Hamiltonian $\mathbf{H} = \mathbf{H_0} + \mathbf{V}$

- Standard Schrödinger equation $(\mathbf{E} - \mathbf{H_0})\,\mathbf{\Psi_E} = \mathbf{V}\mathbf{\Psi_E}$

  (Here E is the spectral variable, and Ψ_E is the eigenstate)

- Sturmians are the eigensolutions of $(\mathbf{E} - \mathbf{H_0})\,\mathbf{\Phi_i}(\mathbf{E}) = \frac{\mathbf{V}}{\eta_\mathbf{i}(\mathbf{E})}\mathbf{\Phi_i}(\mathbf{E})$

  (Here E is a parameter, and $\eta_i(E)$ is a potential scale)

- Spectrum consists of all rescalings that give solution to the equation

  (for a given energy and well-defined boundary conditions).

  - The energy E here can be chosen as any useable value
  - But of practical importance: use a negative value.
  - A single value suffices: the objective is to define a basis expansion set

    (for use with an energy independent set of interactions)

Finding resonances

- **Rapid determination of all resonances (no matter how narrow)**

$$1 - S_{\rm el} = i\pi\mu \sum_{nn'=1}^{M} k\, \hat{\chi}_{1n}(k) \left[(\boldsymbol{\eta} - \mathbf{G}_0)^{-1} \right]_{nn'} \hat{\chi}_{1n'}(k)$$

$$= i\pi\mu \sum_{nn'=1}^{M} k\, \frac{\hat{\chi}_{1n}(k)}{\sqrt{\eta_n}} \left[\left(\mathbf{1} - \boldsymbol{\eta}^{-\frac{1}{2}} \mathbf{G}_0 \boldsymbol{\eta}^{-\frac{1}{2}} \right)^{-1} \right]_{nn'} \frac{\hat{\chi}_{1n'}(k)}{\sqrt{\eta_{n'}}}$$

- **To find resonances: diagonalize the complex, symmetric matrix**

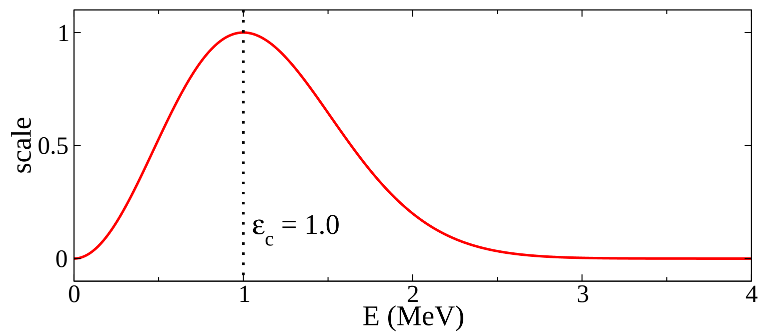
$$\sum_{n'=1}^{N} \left[\boldsymbol{\eta}^{-\frac{1}{2}} \right]_{nn} [\mathbf{G}_0]_{nn'} \left[\boldsymbol{\eta}^{-\frac{1}{2}} \right]_{n'n'} \tilde{Q}_{n'r} = \zeta_r \tilde{Q}_{nr} \,,$$

- **Resonances: occur at energies for which $\Re(\zeta_r) = 1$**
- **Resonance widths relate to the imaginary part of ζ_r**

Choice of scaling function:

- **Conditions: on a scaling of resonance widths**

1. $U(E)$ and $\frac{dU(E)}{dE} = 0$ for $E \leq 0$
2. $U(E) \to 0$ as $E \to \infty$
3. $U(E_c) = 1$
4. Example of Wigner function form $\longrightarrow$



- **A model choice for $U(E)$: a modified Wigner distribution**

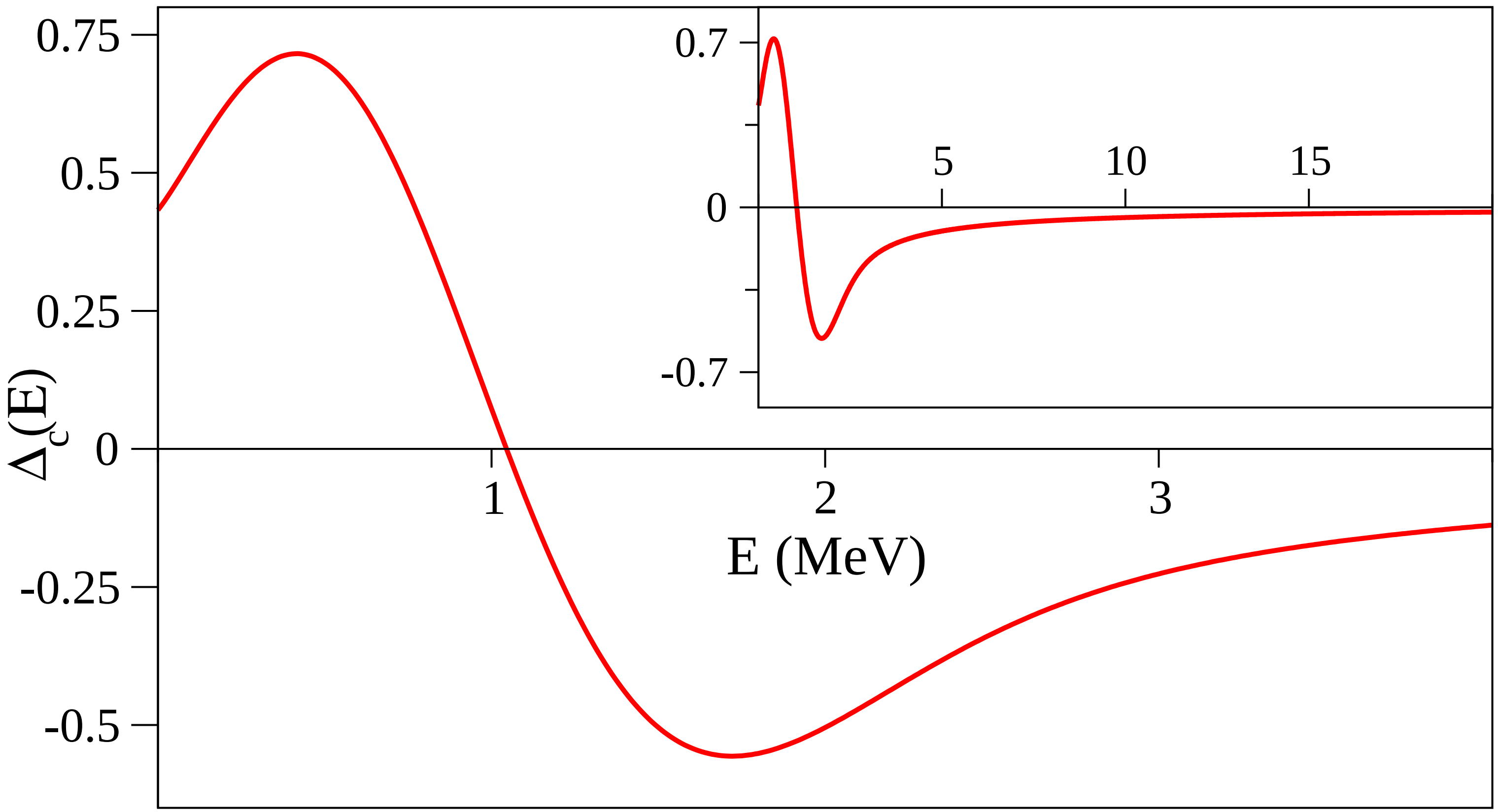
$$U(E) = e^{m} \left(\frac{E}{E_c} \right)^{n} e^{-m\left(\frac{E}{E_c}\right)^{n}} \mathcal{H}(E)$$

m, n are positive integers

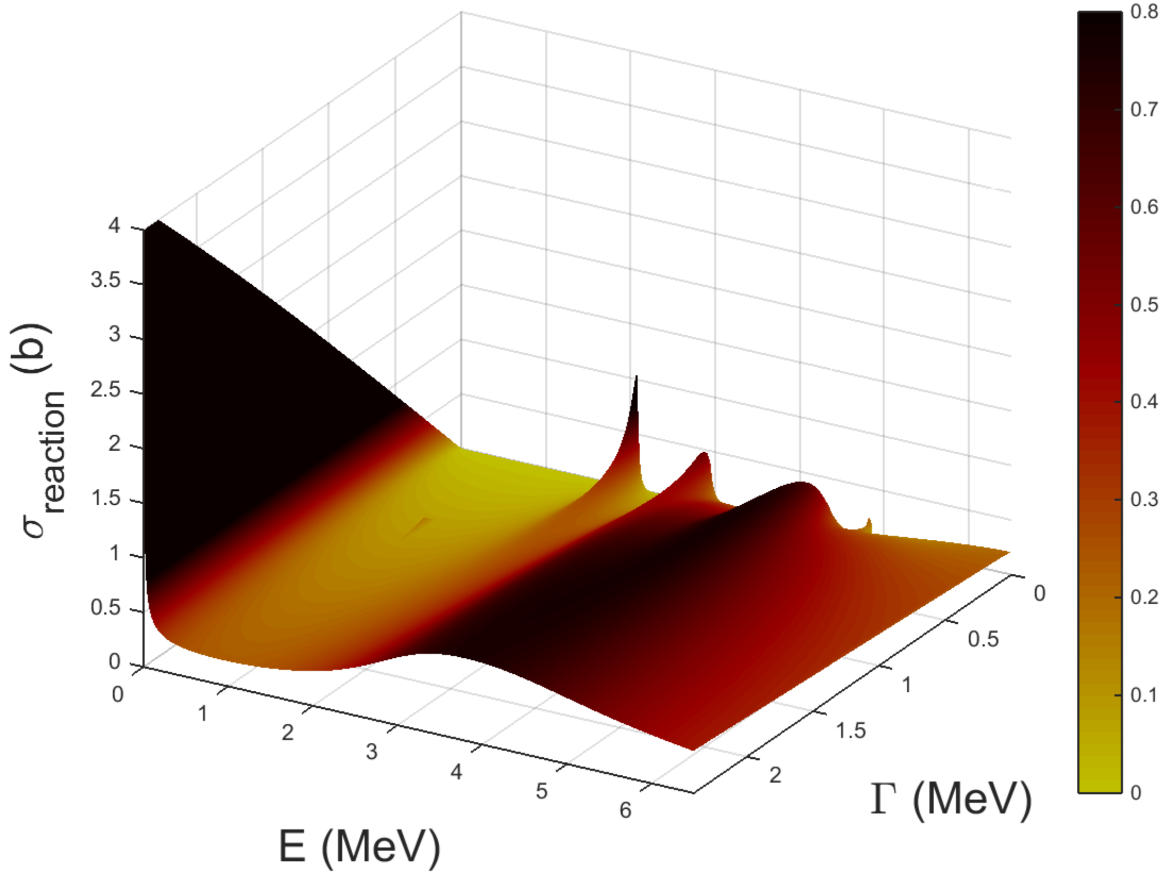
$\mathcal{H}(E)$ is the Heaviside function (No resonance effect for $E < 0$)

The energy shift $\Delta_c(E)$:

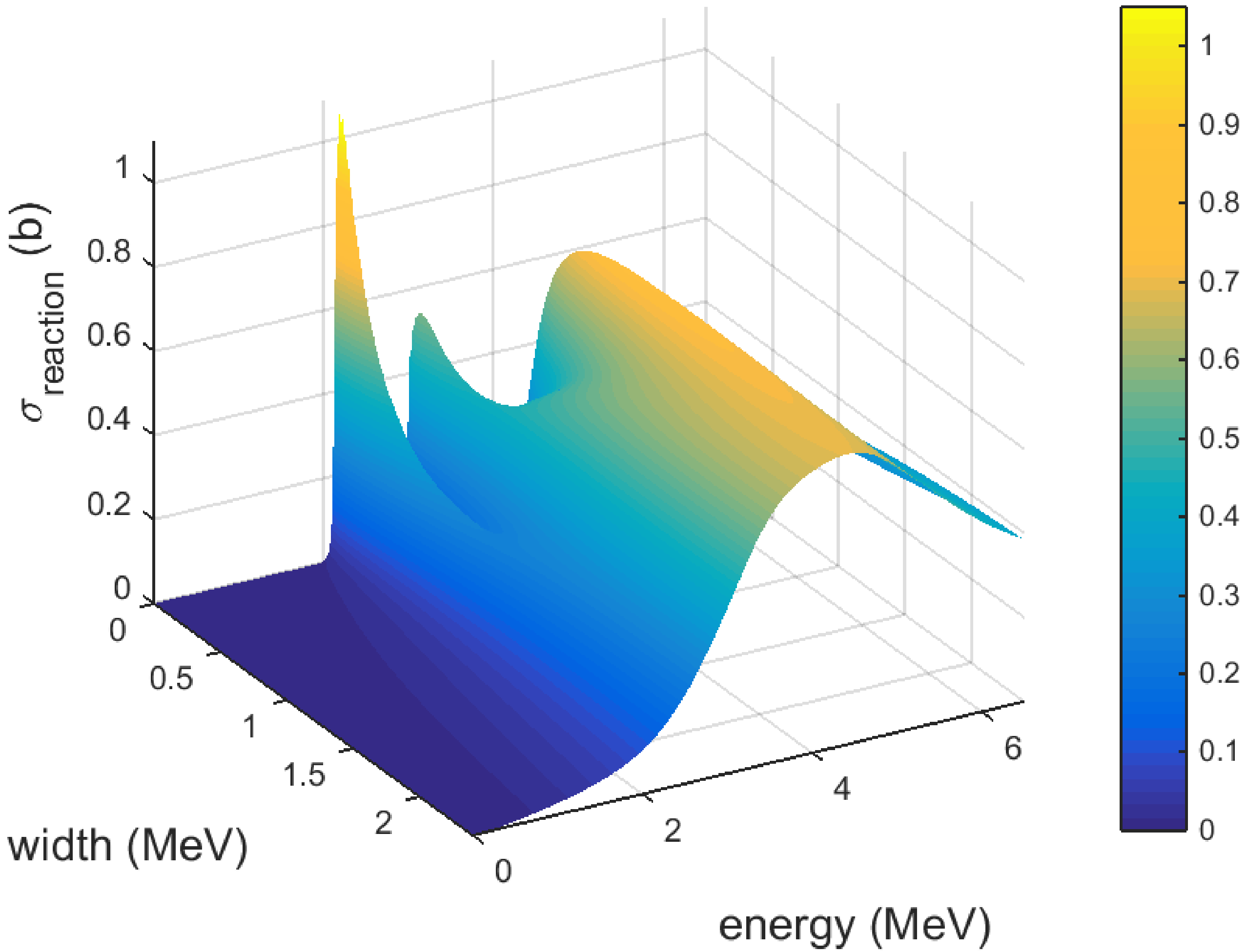
Case of Wigner scale function with $m = 1, n = 2, E_c = 1.0, \Gamma_c = 2.0$



n+^{12}C reaction cross section with no $U_c(E)$



n+^{12}C reaction cross sections — using $U_c(E)$



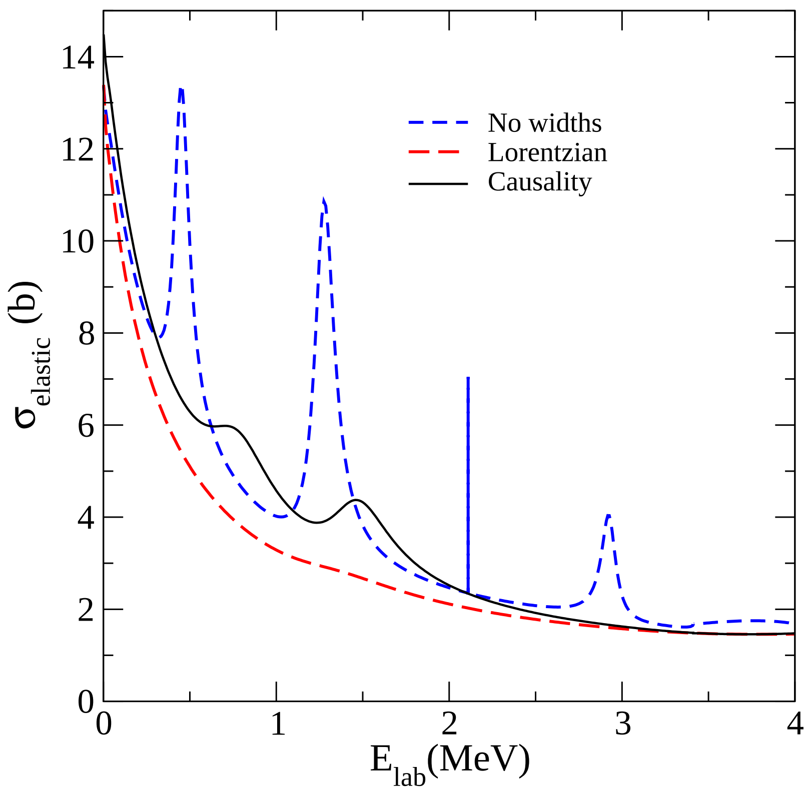
The low excitation (resonant) states of ^{9}Be (relative to the n+^{8}Be threshold)

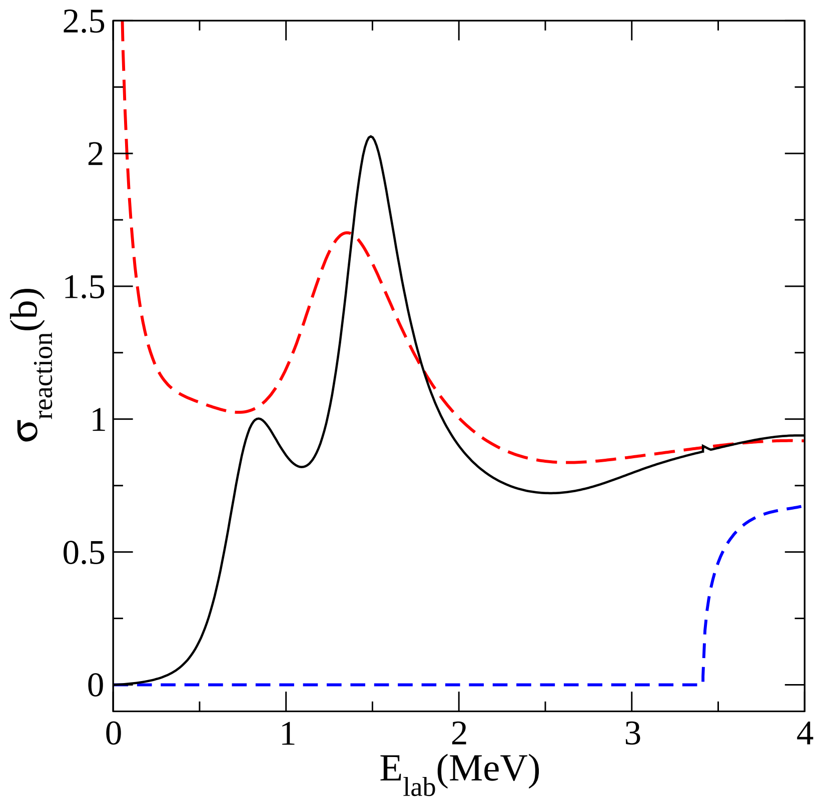
Energy centroids [widths] are in MeV

J^π	expt.	Causality	Lorentz
$\frac{3}{2}^-$	$-$1.665	$-$1.669	$-$1.668
$\frac{1}{2}^+$	0.018 [0.214]	$-$0.399	$-$0.399
$\frac{5}{2}^-$	0.764 [0.078]	2.499 [0.983]	1.991 [2.302]
$\frac{1}{2}^-$	1.11 [1.08]	0.773 [0.54]	0.445 [1.216]
$\frac{5}{2}^+$	1.384 [0.282]	1.337 [0.406]	1.184 [0.73]

Results shown in black lie within 0.5 MeV of measured ones.

Case: ^{9}Be ($n + {}^{8}$Be)





The low excitation (resonant) states of ^{7}He (relative to the n+^{6}He threshold)

Energy centroids [widths] are in MeV

$J^\pi =$	$\frac{3}{2}^-$	$\frac{5}{2}^-$	($\frac{1}{2}^-$)
Experiment	0.445	3.365 $\pm$ 0.09	6.425 $\pm$ 0.3
	[0.150$\pm$ 0.020]	[1.990$\pm$ 0.170]	[4 $\pm$1]
Causality	0.449 [0.145]	3.229 [0.073]	6.294 [10.17]
Lorentz	0.489 [0.177]	3.452 [0.219]	6.764 [0.096]

Results shown in black lie within 0.5 MeV of measured ones.

Case: α+^{8}Be scattering