SU(3) flavour breaking effects

James Zanotti
The University of Adelaide

Asia-Pacific Few Body Conference, April 7 - 11, 2014, Hahndorf, Australia

CSSM/ QCDSF/UKQCD Collaborations

Adelaide

- Alex Chambers
- Jack Dragos
- Phiala Shanahan
- Tony Thomas
- Ross Young

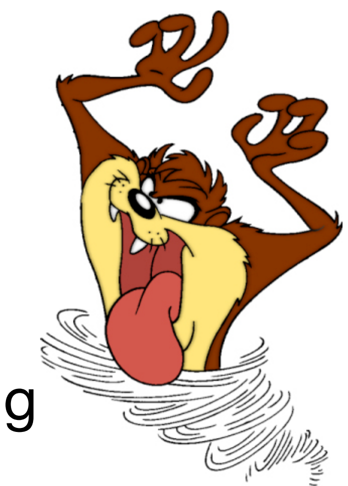
- A. Cooke (Edinburgh)
- R. Horsley (Edinburgh)
- Y. Nakamura (RIKEN, Kobe)
- H. Perlt (Leipzig)
- D. Pleiter (Jülich)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- A. Shiller (Leipzig)

Outline

- Introduction
- Tuning and simulation parameters
- Hyperon Results
    - Electromagnetic Form Factors [See talk by P. Shanahan Tuesday 4:10]
    - Axial Charges [See also talk by A. Chambers, Tuesday 5:30]
    - Momentum fractions
- Charge symmetry violation
    - Proton-neutron mass splitting
    - CSV in parton distribution functions
- Future: electromagnetic effects
- Summary

Motivation for Investigation of Hadron Structure

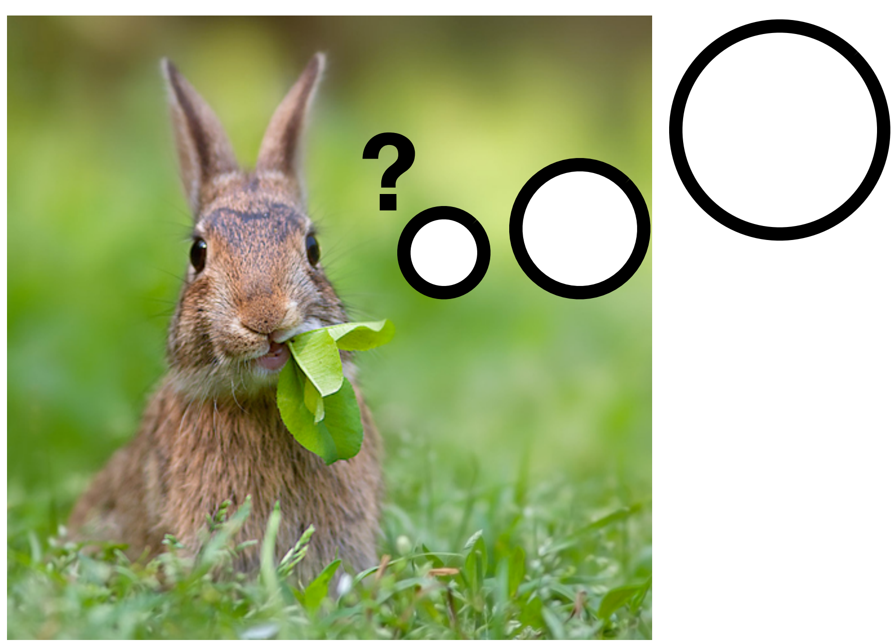
- We know the nucleon is not a point-like particle but in fact is composed of quarks and gluons
- But how are these constituents distributed inside the nucleon?
- How do they combine to produce its experimentally observed properties?
- For example
  - "Spin crisis": quarks carry on ~30% of the proton's spin
  - QCD vs QED effects in charge symmetry violation in nucleon properties, e.g



$M_n - M_p = 1.29333217(42)\,\mathrm{MeV}$ but $Q_p = +e,\ Q_n = 0$ vs $m_d > m_u$

- Understanding how the nucleon is built from its quark and gluon constituents remains one the most important and challenging questions in modern nuclear physics.
- Lattice has a big role to play in tackling these questions.

Lattice

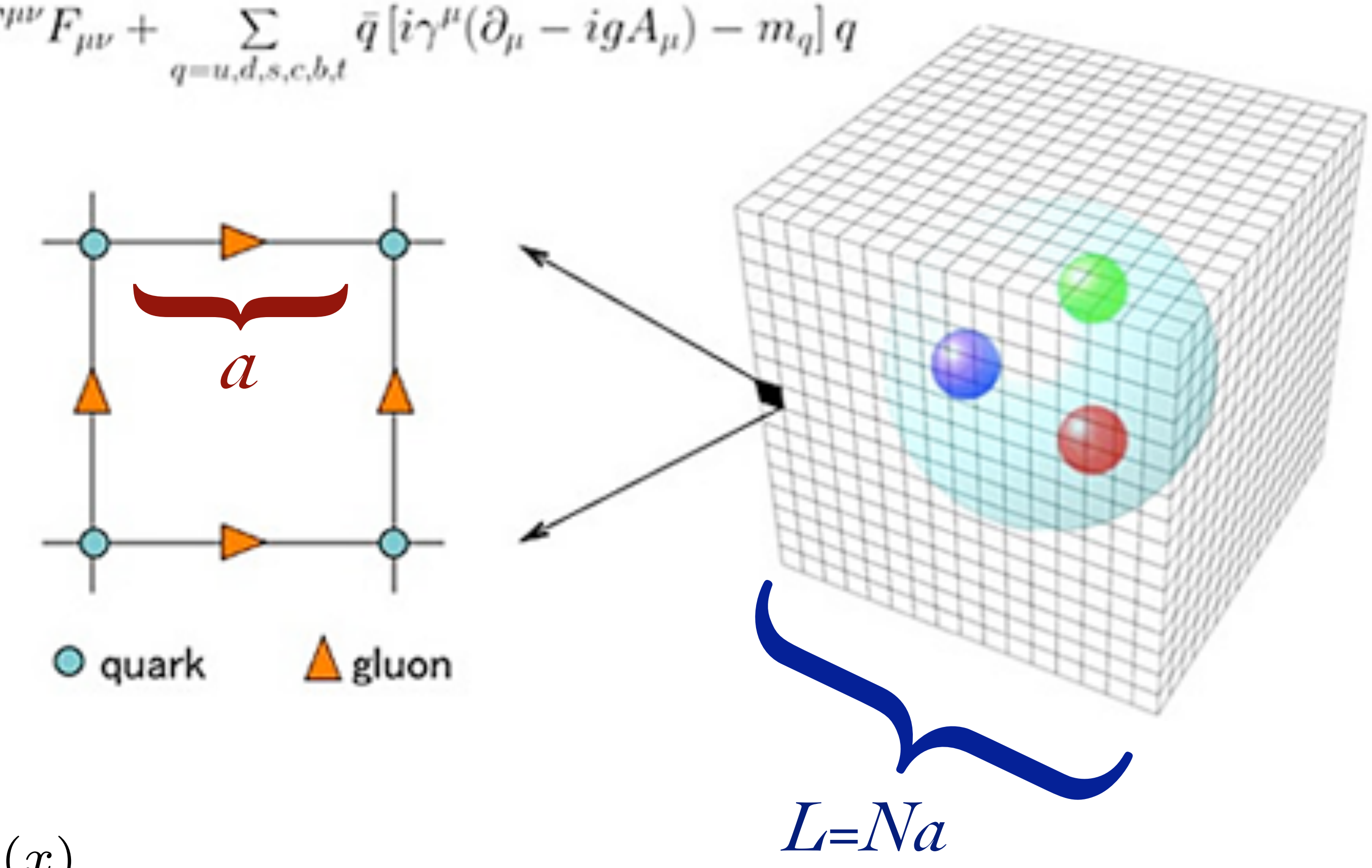


Lattice

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$

Take QCD



- Discretise space-time with lattice spacing *a* volume *L*^{3}*xT*

- Quark fields reside on sites $\psi(x)$

- Gauge fields on the links $U_\mu(x) = e^{-iagA_\mu(x)}$

- Approximate the full QCD path integral by Monte Carlo methods

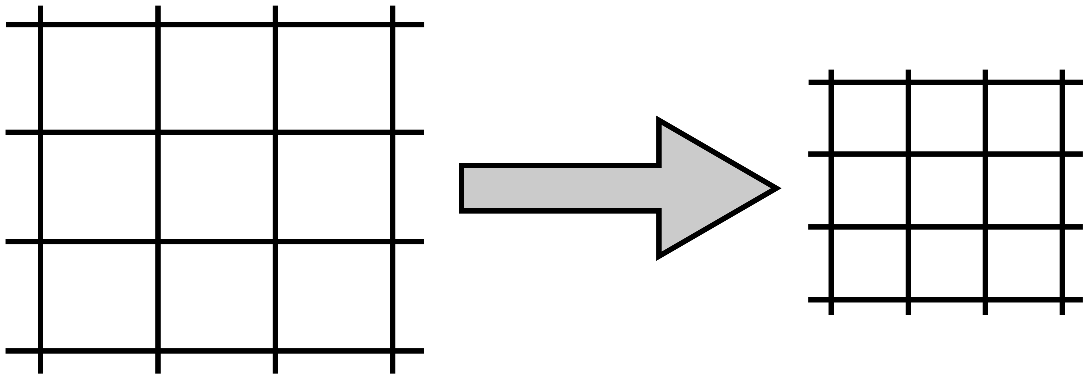
$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi\, \mathcal{O}[A,\bar{\psi},\psi]\, e^{-S[A,\bar{\psi},\psi]} \Rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\rm conf}}\sum_{i}^{N_{\rm conf}} \mathcal{O}([U^{[i]}])$$

With field configurations U_i distributed according to $e^{-S[U]}$

Put it on a supercomputer

Systematics of a Lattice Calculation

- Extrapolations:
  - Continuum
    - Unavoidable
    - Improved actions (errors O(a^2))
    - Finer lattice spacings



$$a \to 0$$

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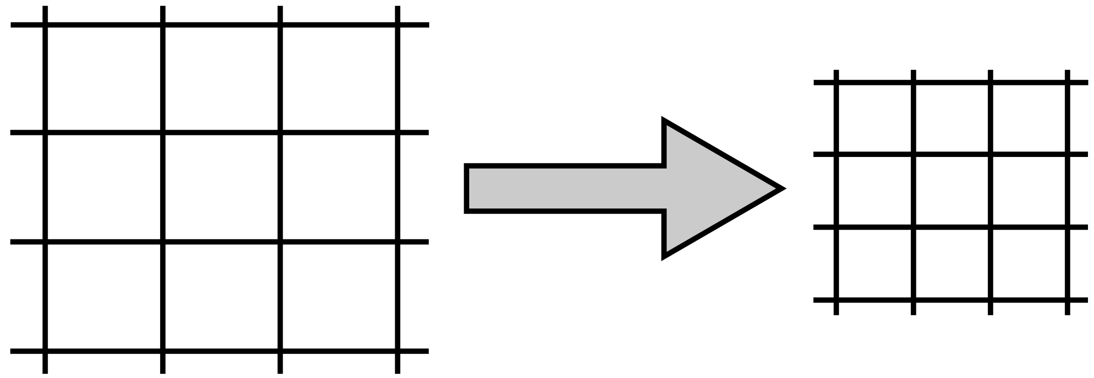

$$a \to 0$$

    - Unavoidable
    - Improved actions (errors O(a^2))
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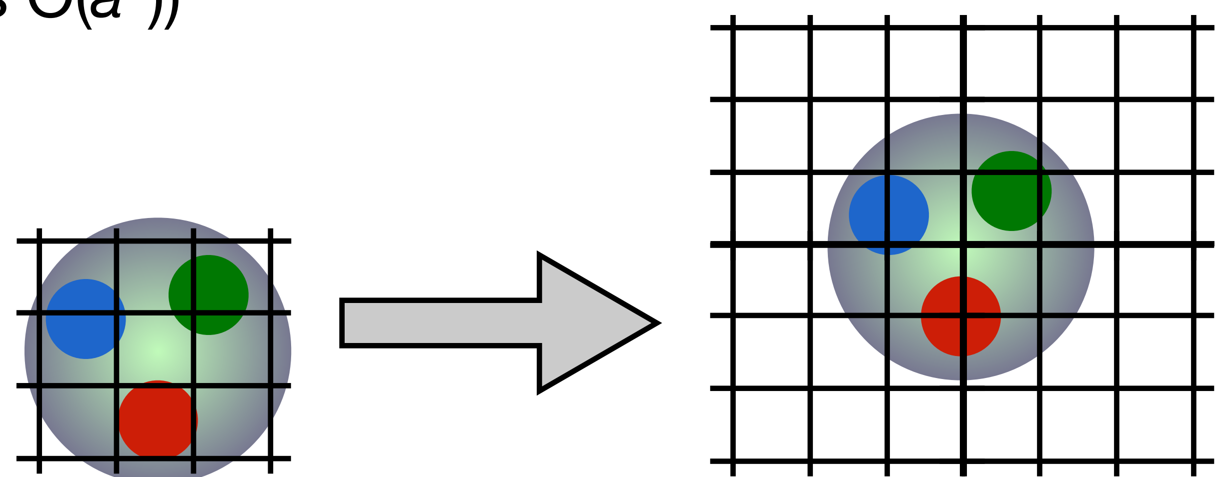
  - Finite volume



$$L \to \infty$$

    - Large volumes so effects are exponentially suppressed

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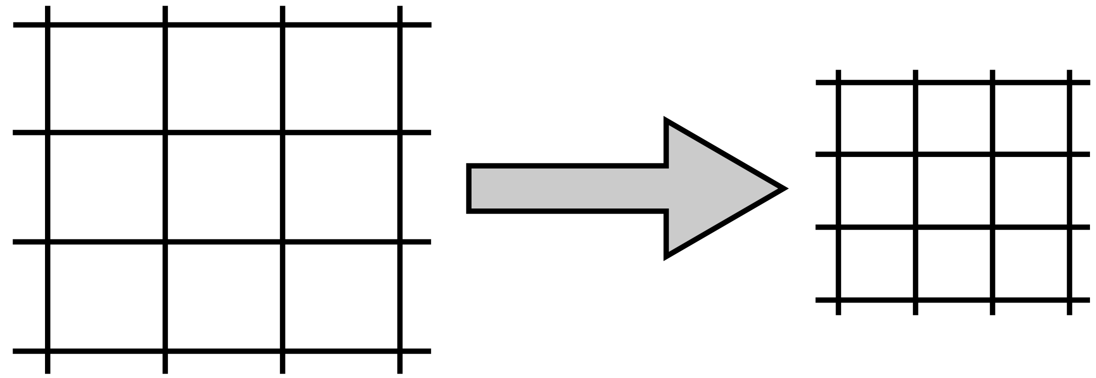

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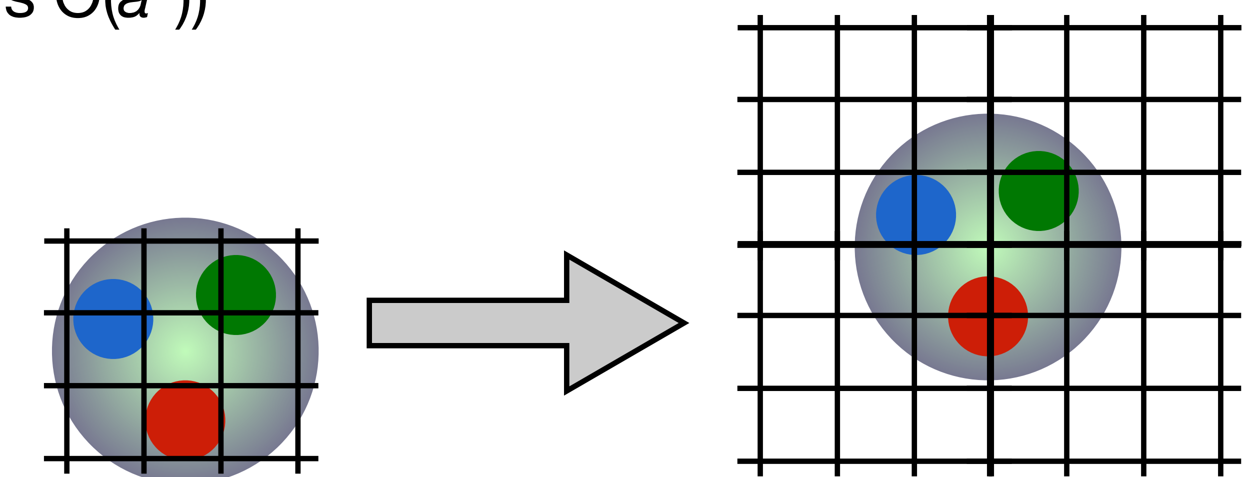
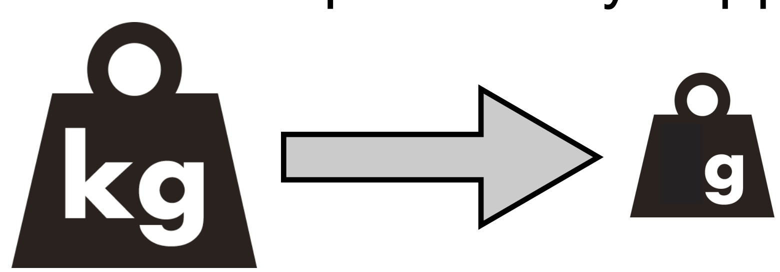


$$L \to \infty$$

  - Chiral
    - Chiral perturbation theory
    - Simulate at physical quark masses

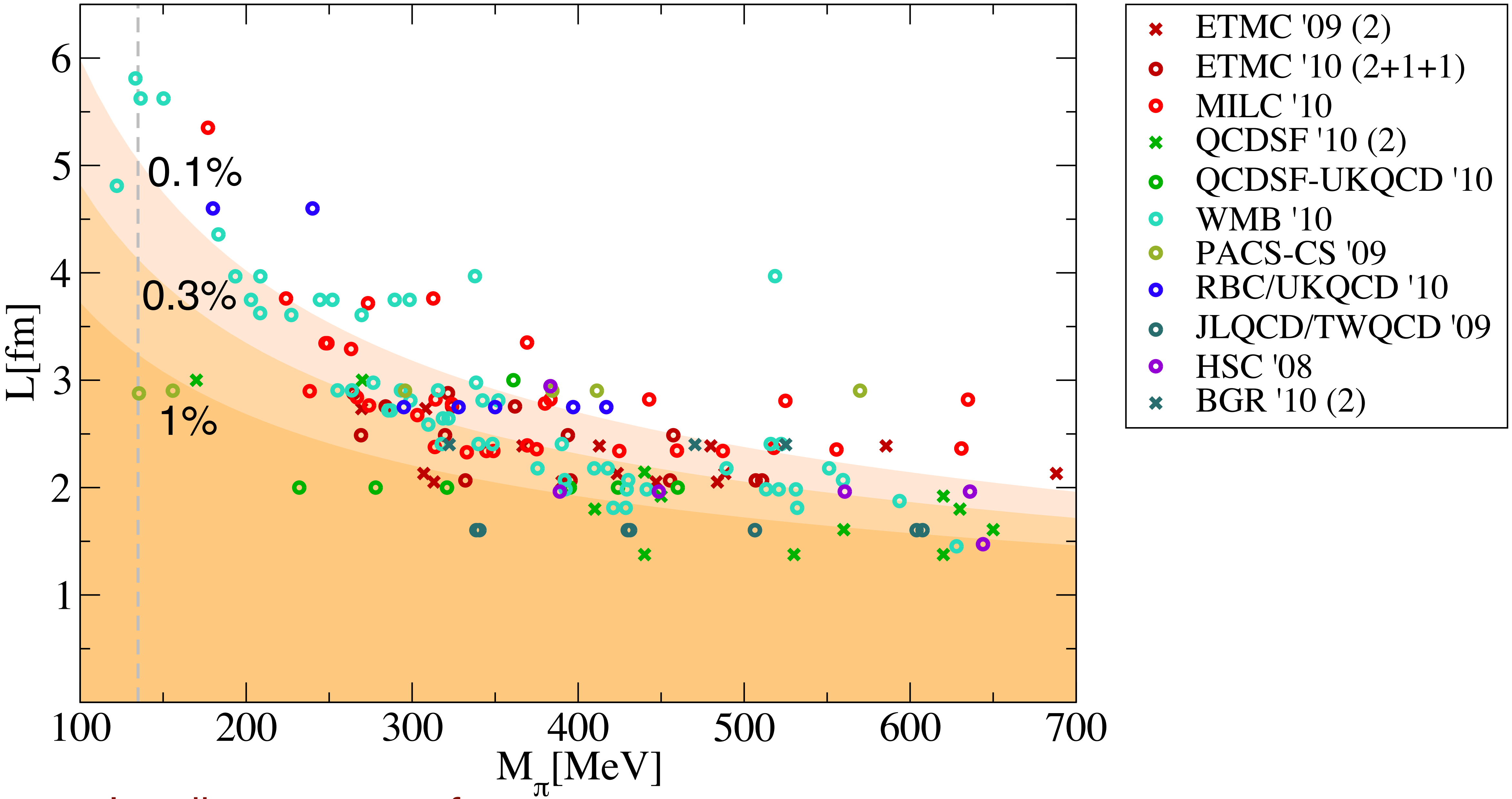


$$m_\pi \to 140 MeV$$

$$\mathrm{GOR} \implies m_\pi^2 \propto m_q$$

The Lattice Landscape

[Hoebling (Lattice 2010) 1102.0410]



- Leading sources of error:
  - Unphysically large quark masses
  - Finite Volume

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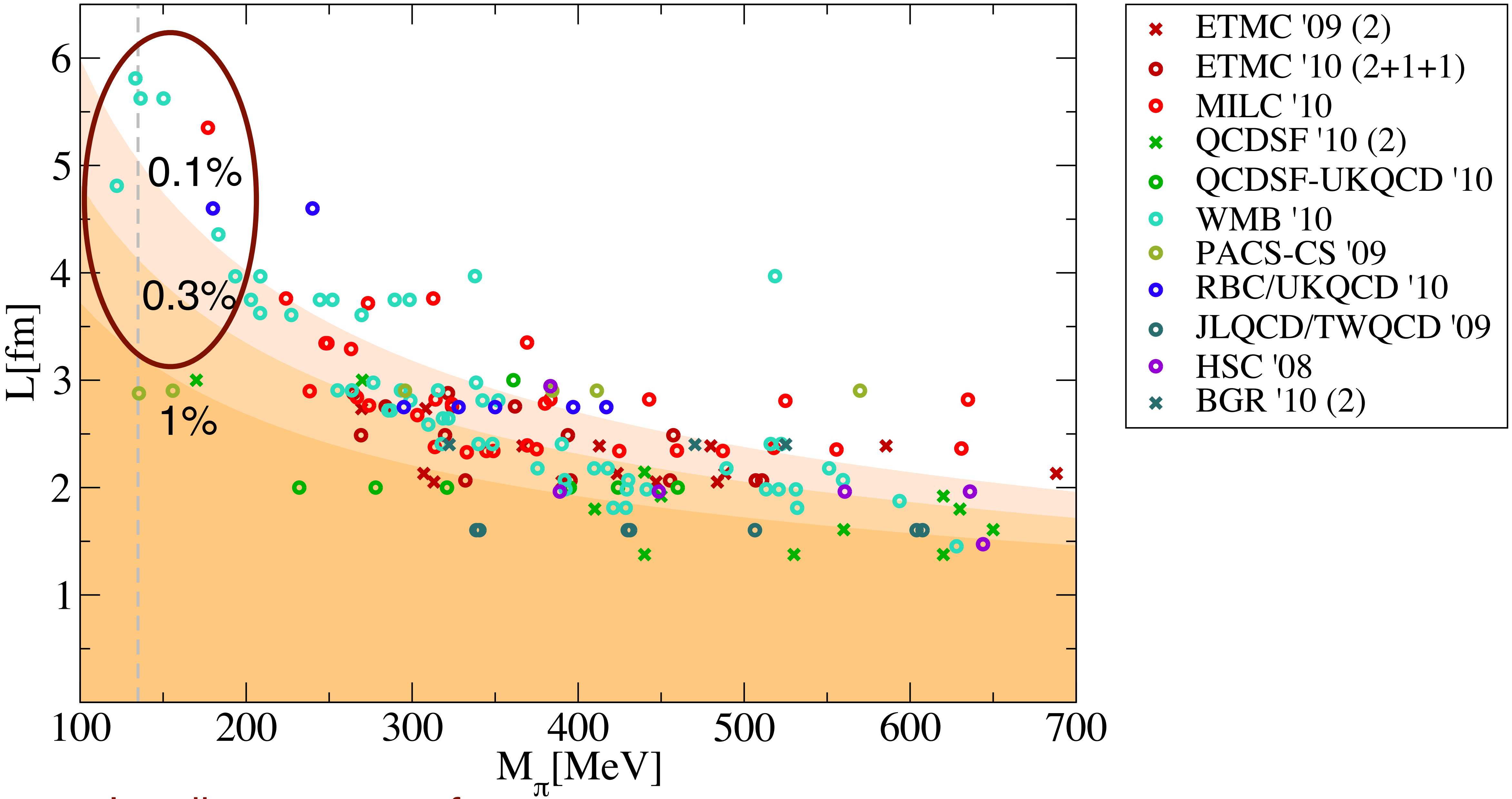

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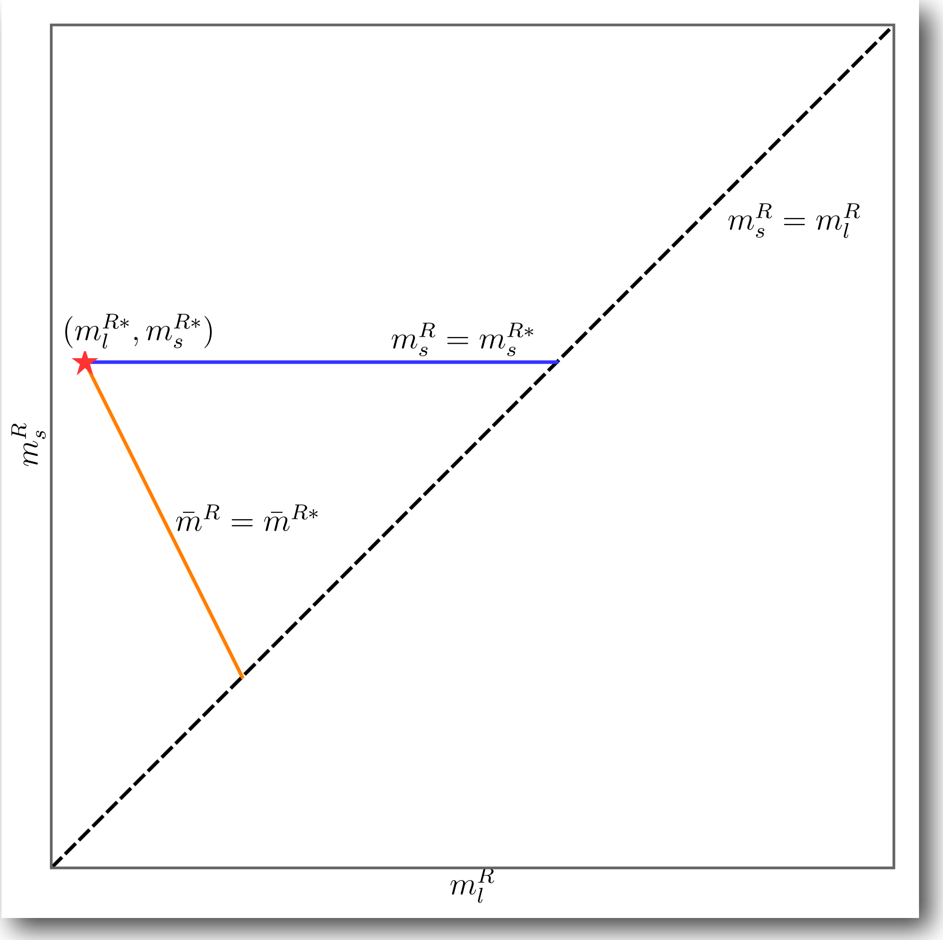
QCDSF Lattice Set-Up

- N_f =2+1 O(a)-improved Clover fermions (“SLiNC” action)
- Tree-level Symanzik gluon action (plaq + rect)
- Results from a single lattice spacing (a~0.074fm),
- Simulations and preliminary results becoming available at a~0.06fm
- Pion masses down to 220MeV
- Novel method for tuning the quark masses [arXiv:1003.1114 (PLB), 1102.5300 (PRD)]

Tuning 2+1

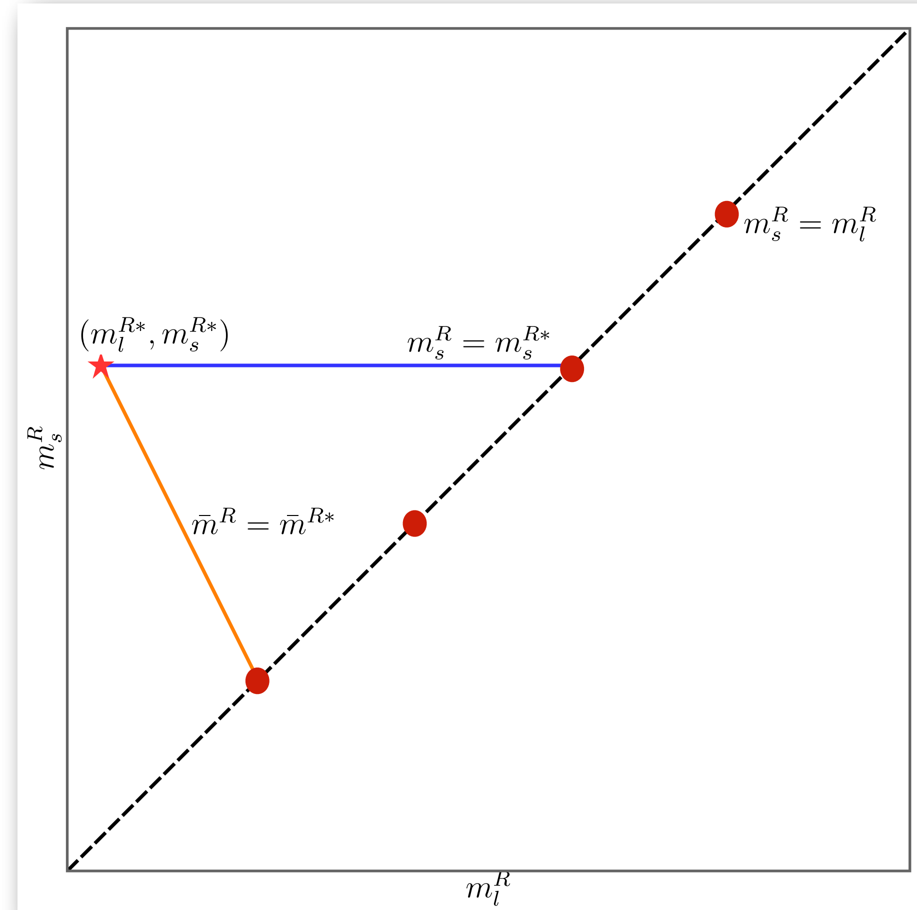
- Need to choose a path to physical point



Tuning 2+1

- Need to choose a path to physical point

- Start from a point on the SU(3)-symmetric line



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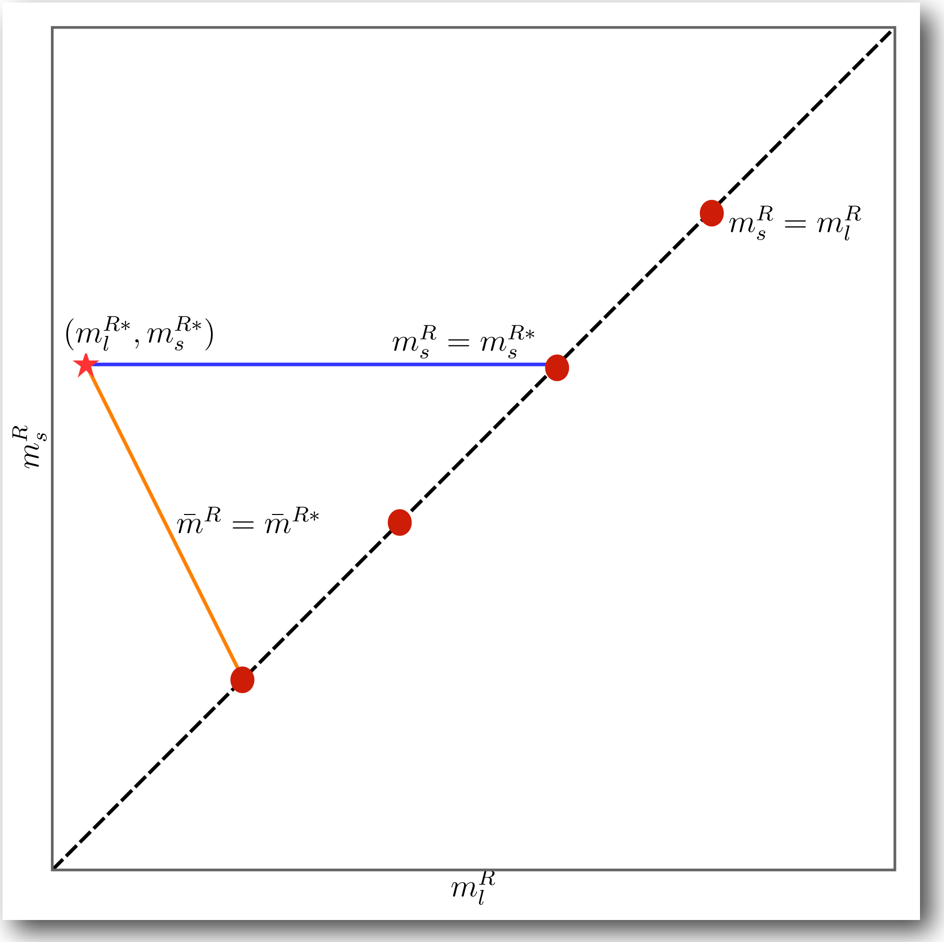
- Extrapolate to physical point by
    - e.g. keeping m_s fixed



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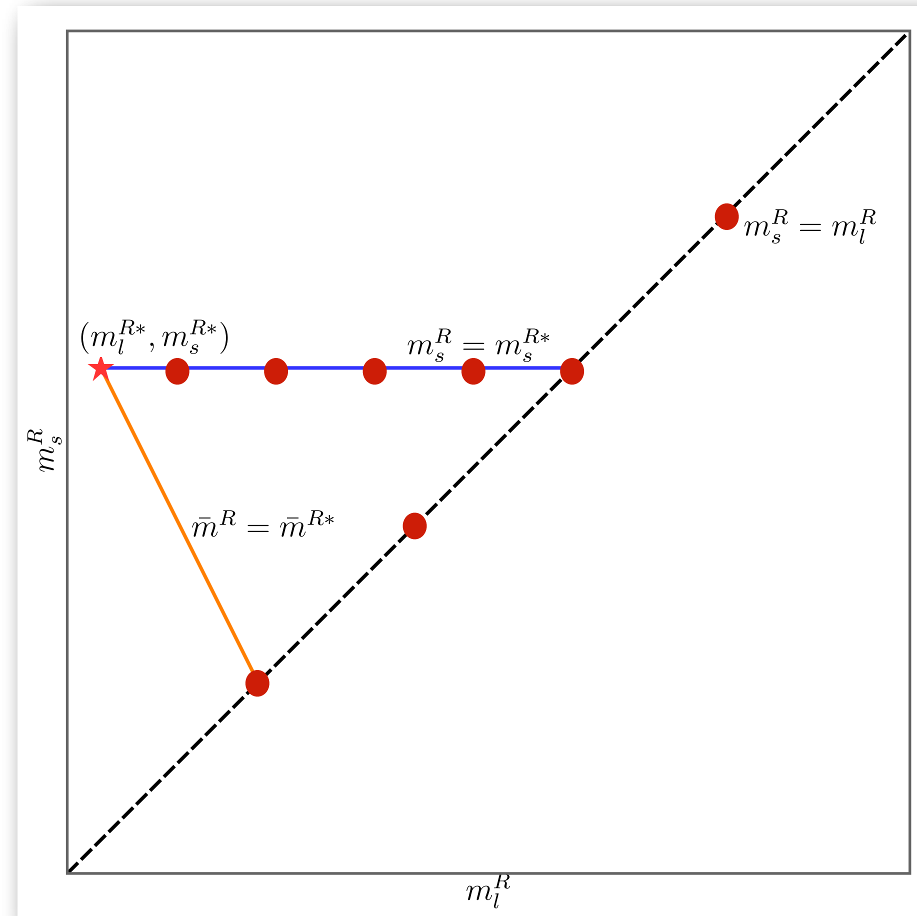
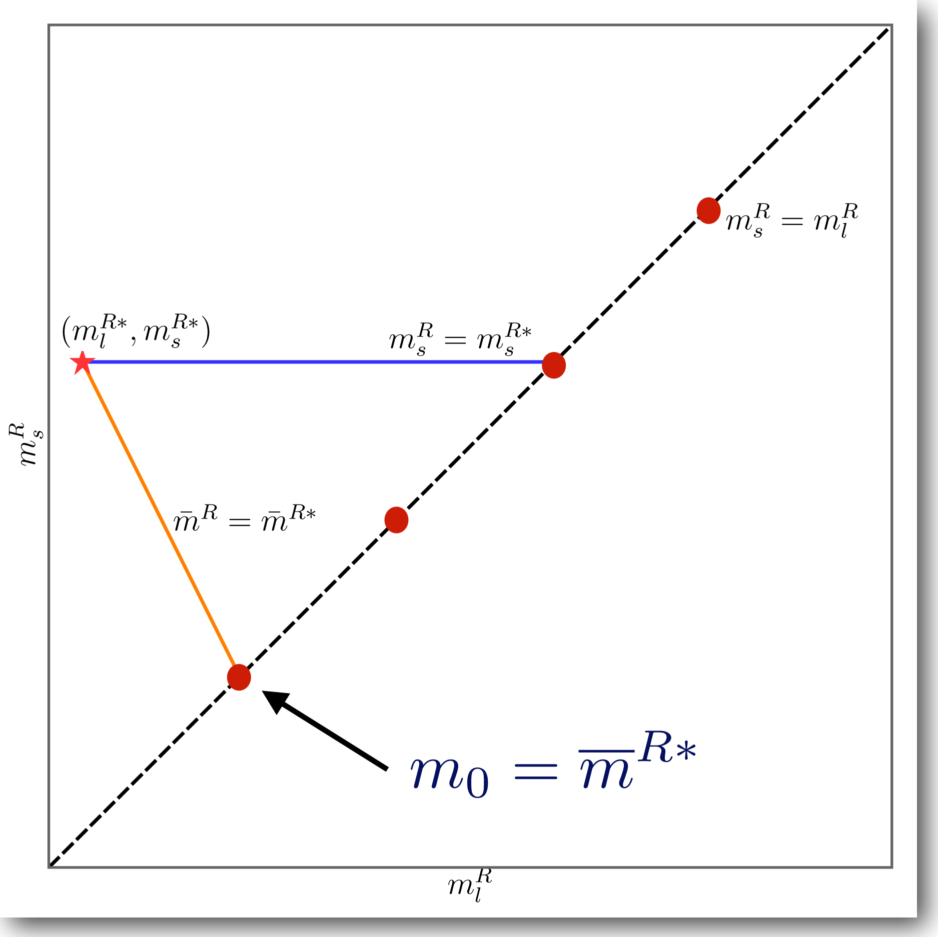


Tuning 2+1

- Need to choose a path to physical point
- Start from a point on the SU(3)-symmetric line
- Our choice is
  - to keep the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}(2m_l^R + m_s^R)$$

  - at its physical value $\overline{m}^{R*}$



- Several benefits:
  - Any flavour singlet quantity can be used to set the scale $(r_0, X_\pi, X_N, t_0, w_0, ...)$
  - Simplified SU(3)-flavour expansions [arXiv:1102.5300 (PRD)]
  - Simple tuning of quark mass (e.g. from ratio of singlets $\frac{X_\pi}{X_N}$)

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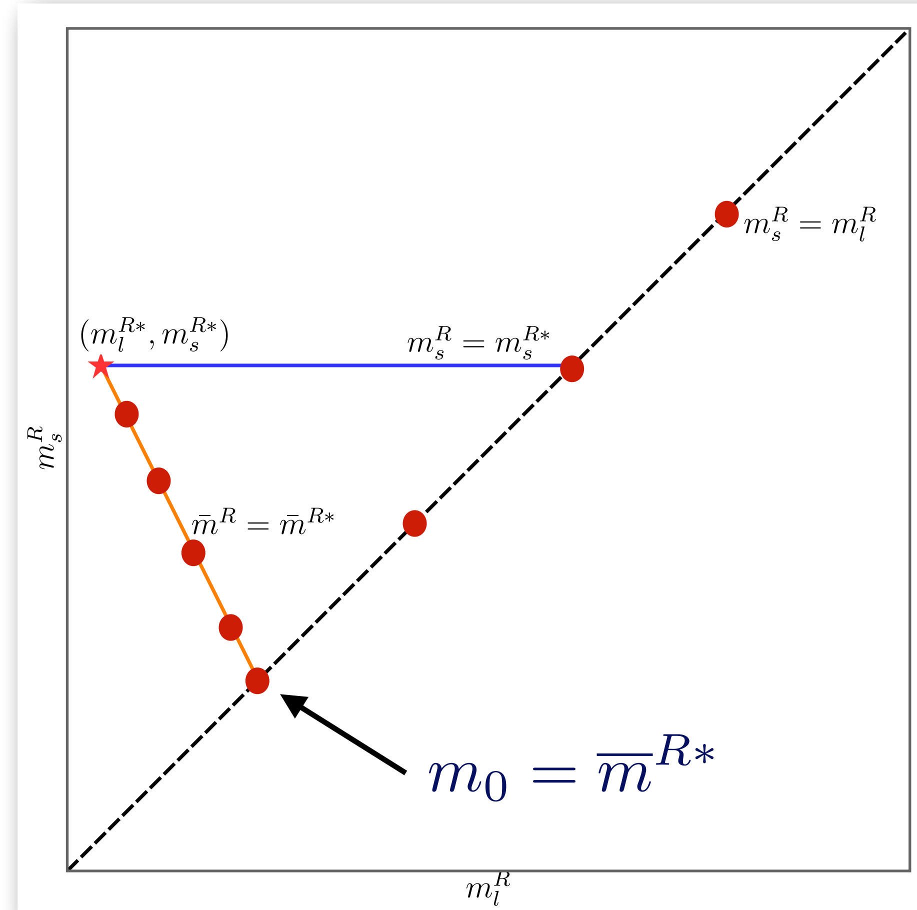

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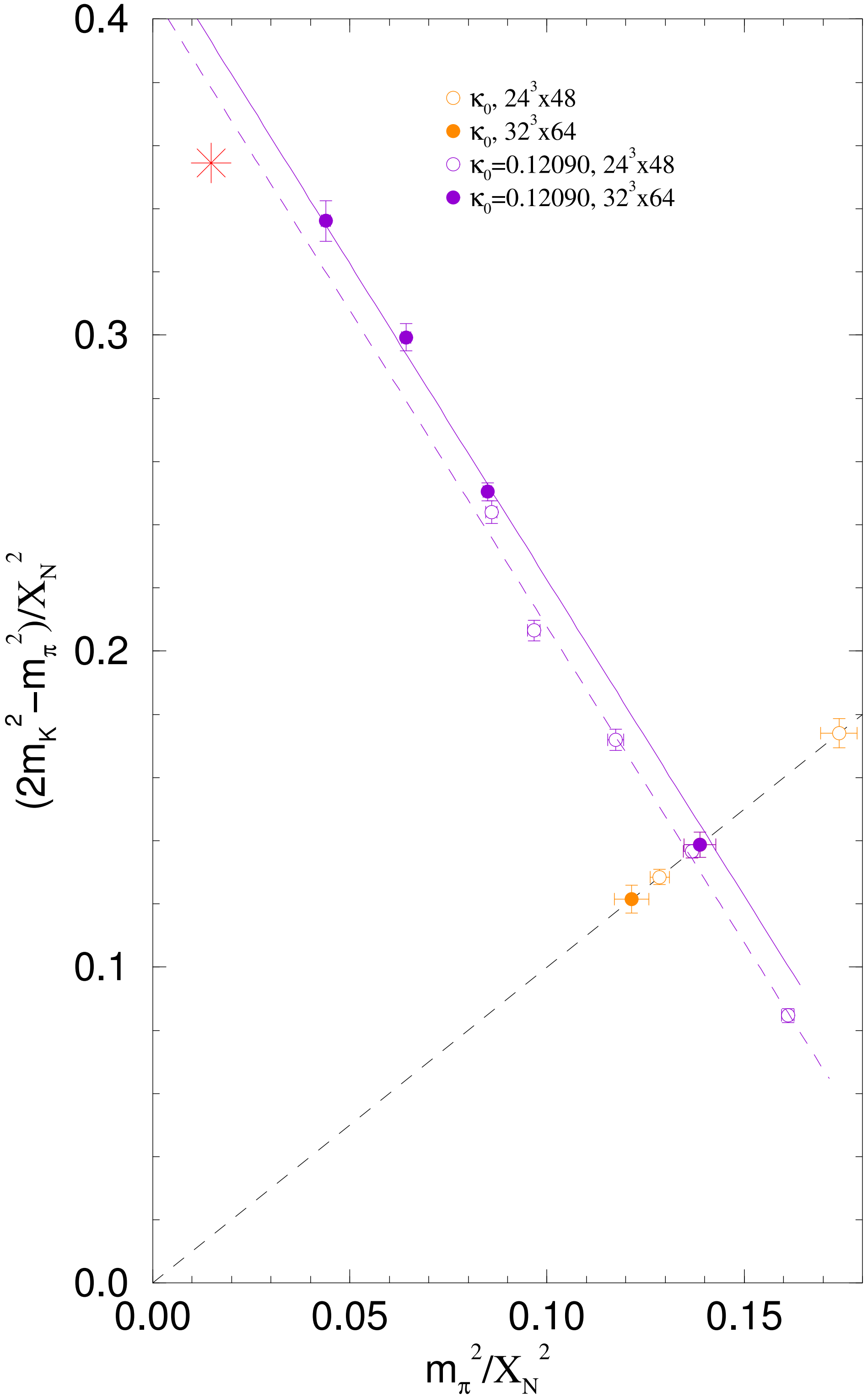
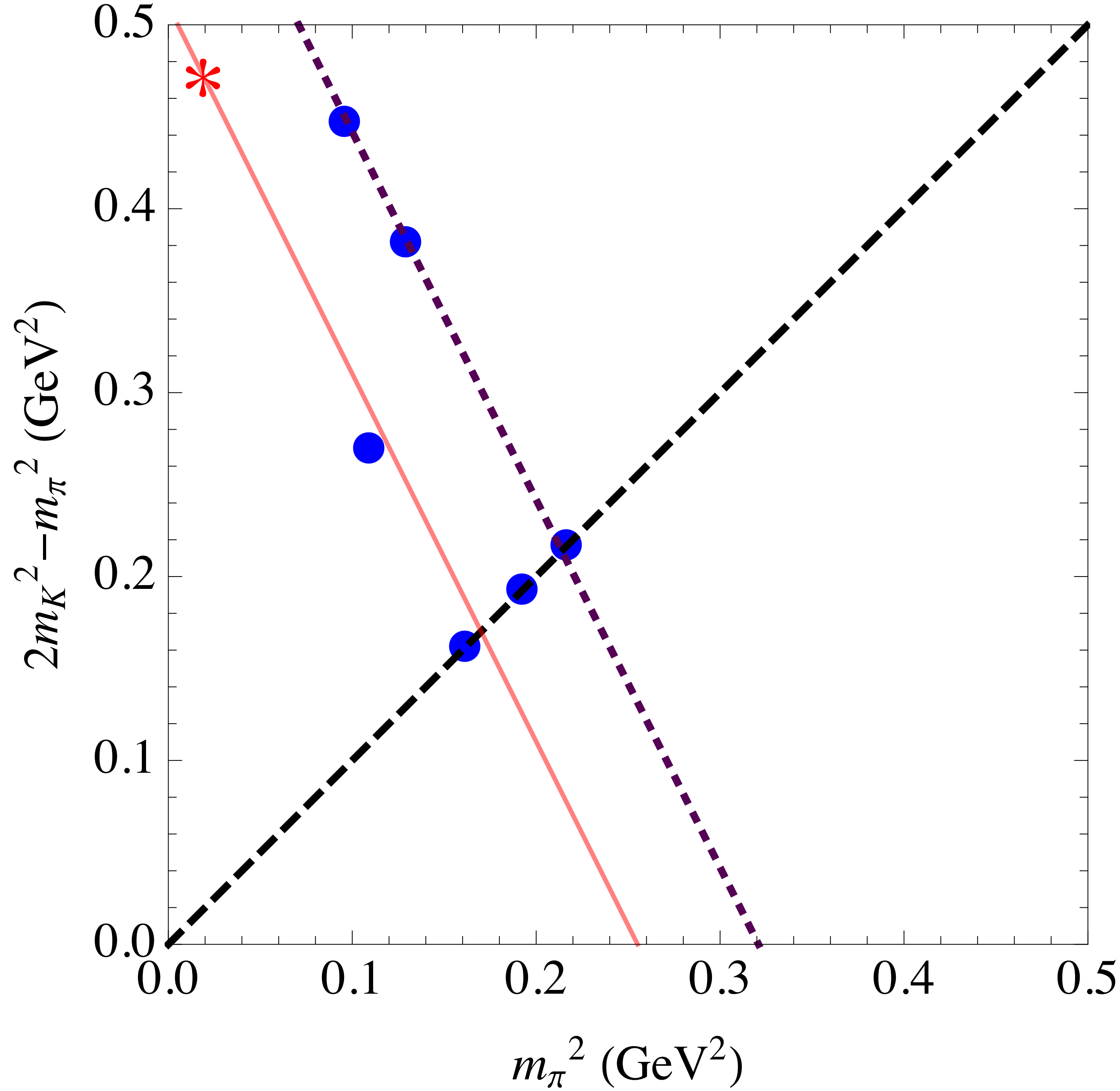
$m_l^R - m_s^R$ plane



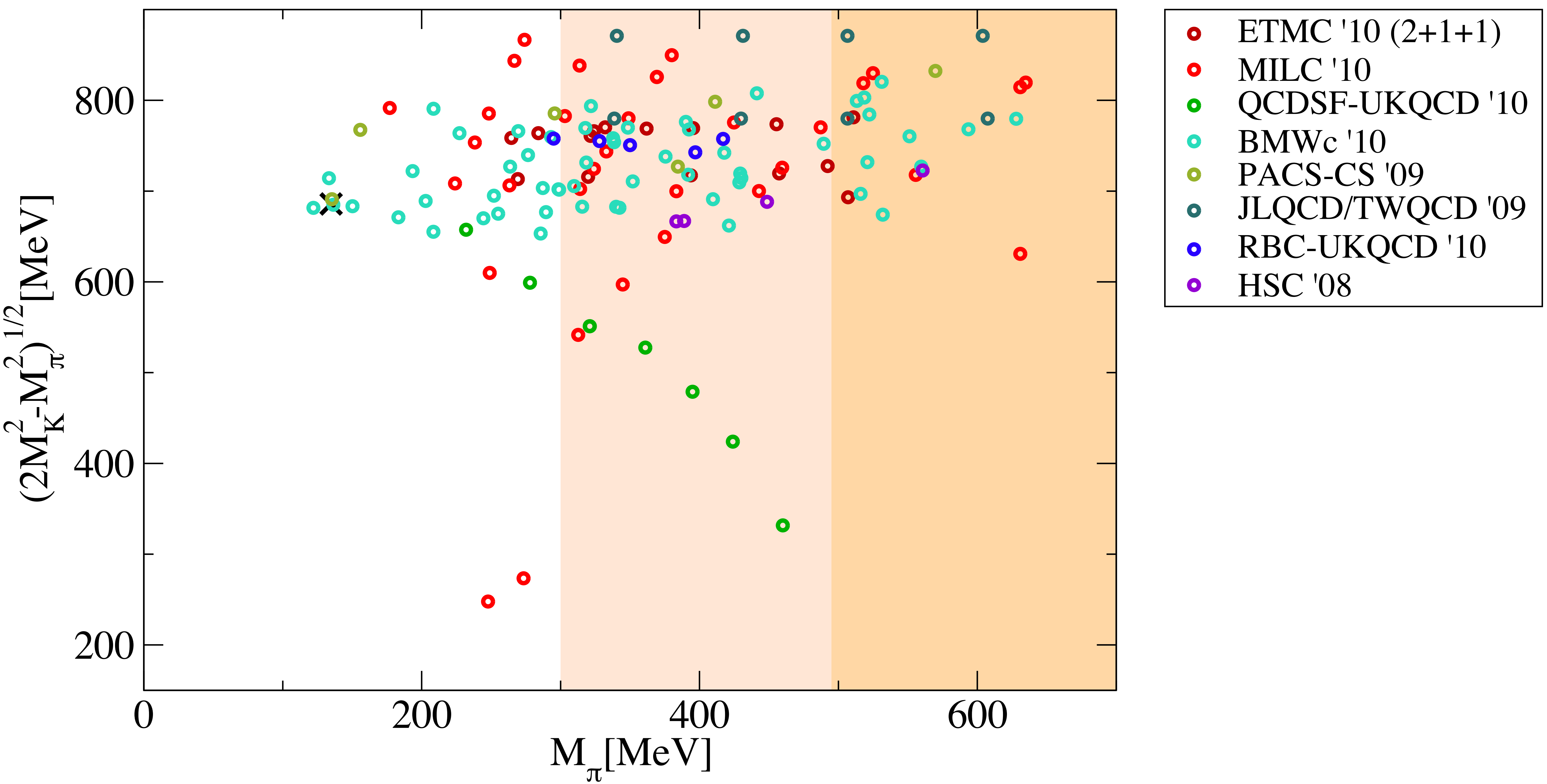
[R.Horsley]



[P.Shanahan]

Landscape

C. Hoebling, plenary talk Lattice 2010, arXiv:1102.0410



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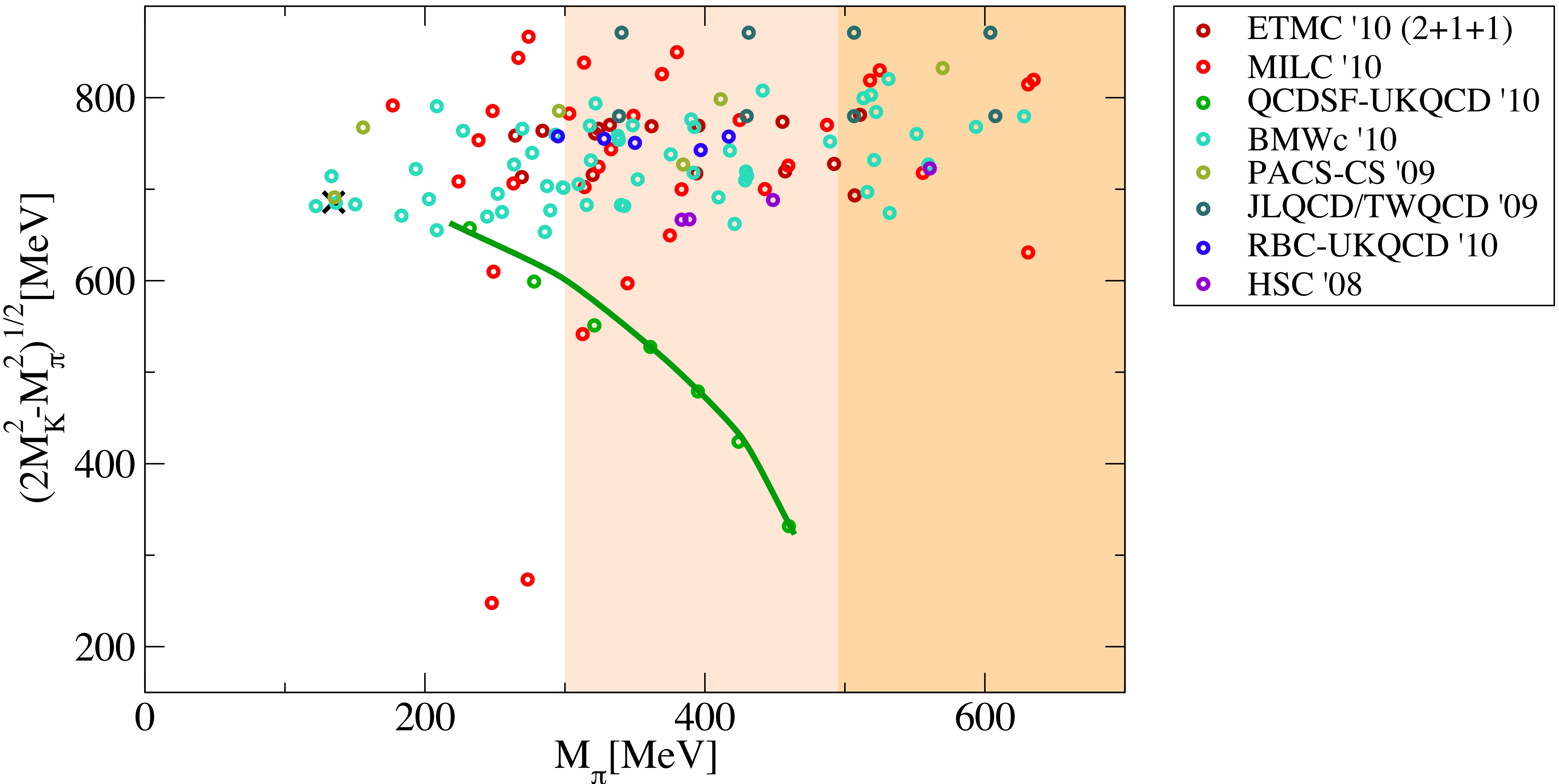
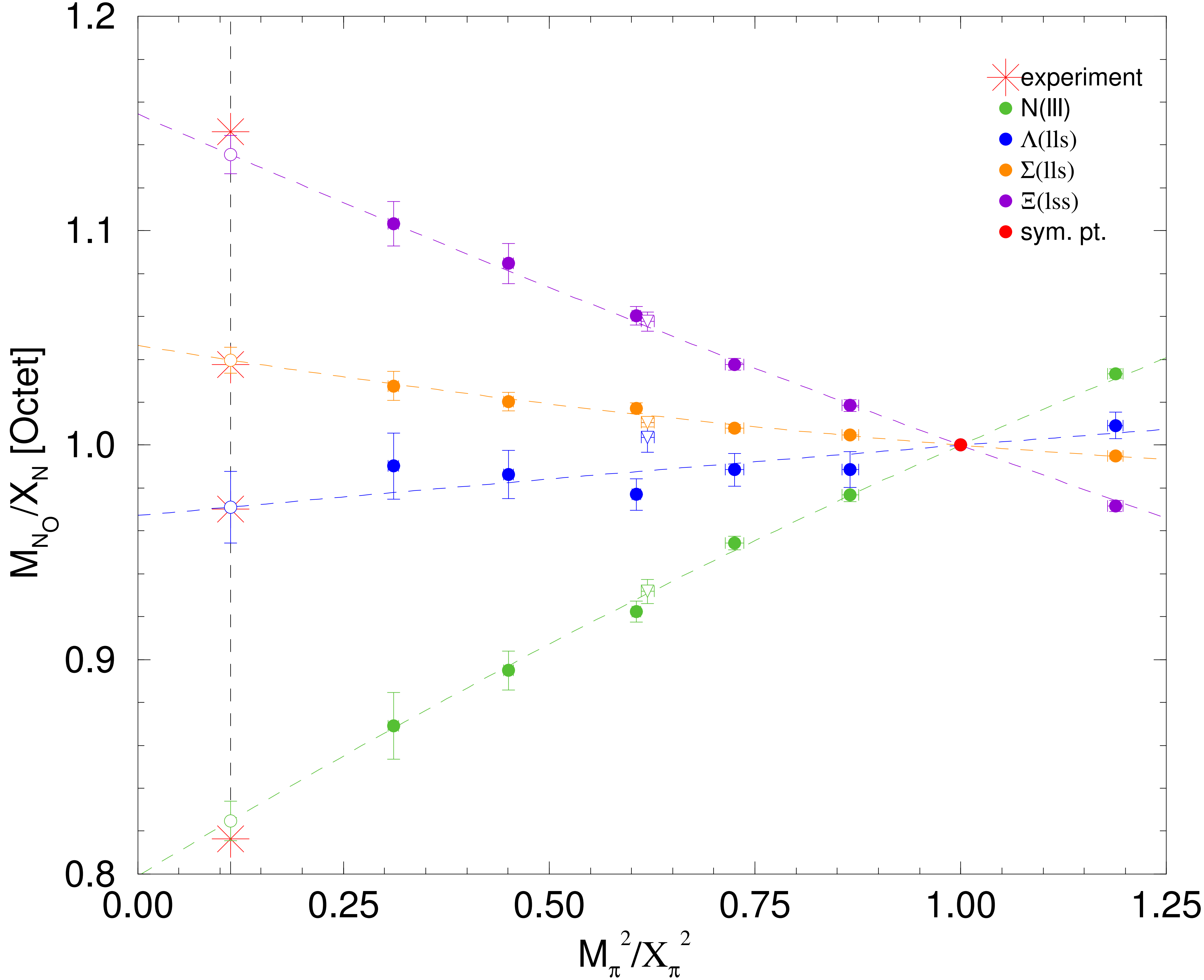


ie $m_\pi \searrow m_\pi^*$ and $m_K \nearrow m_K^*$

Baryon Octet ‘fan plot’



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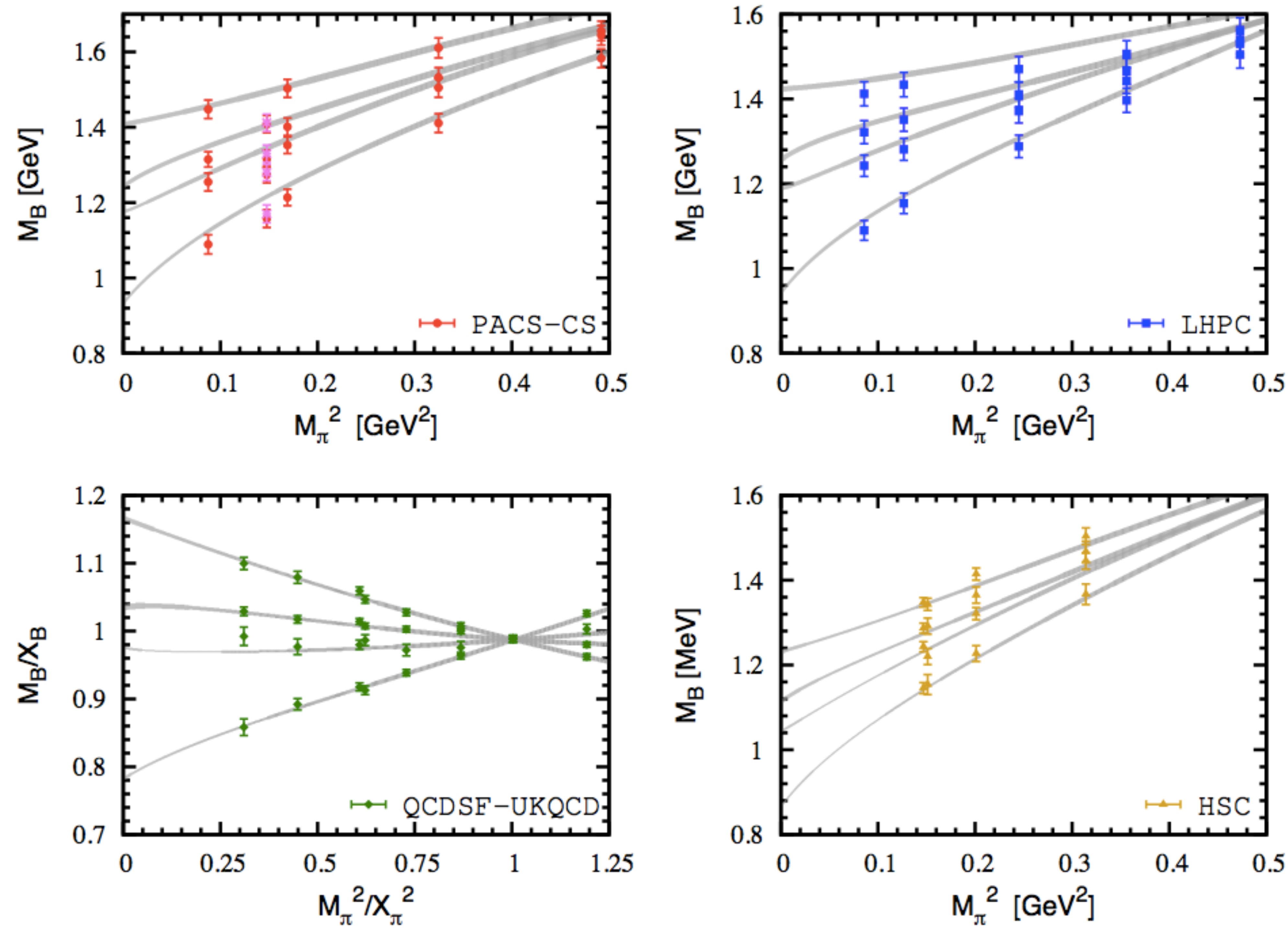


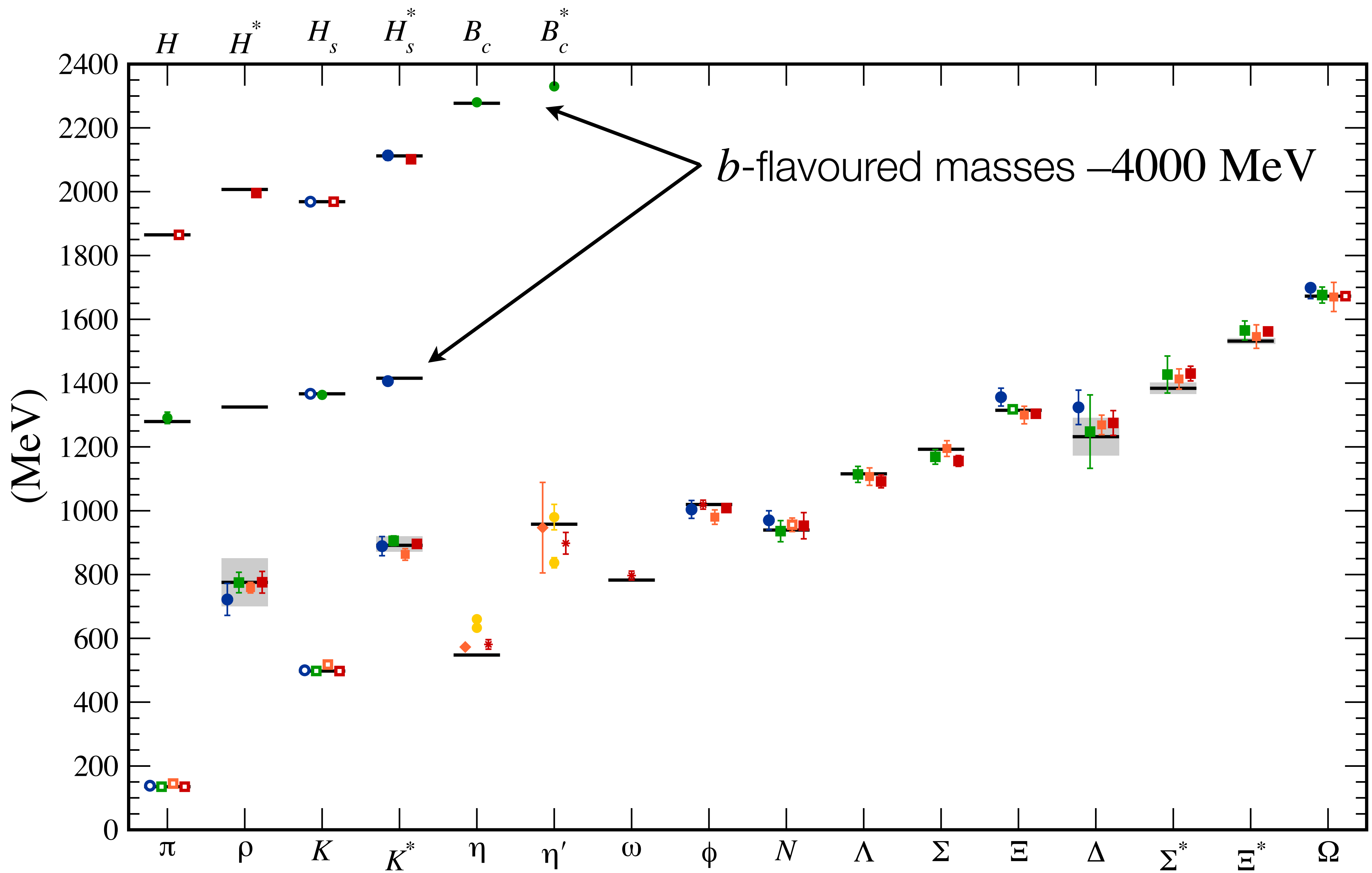
Octet baryon masses: NNNLO ChPT

X.-L. Ren et al., JHEP 12 (2012) 073

QCD Hadron Spectrum

Plot from A. Kronfeld [1203.1204]

$\pi \ldots \Omega$: BMW, MILC, PACS-CS, QCDSF;
η-η': RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler-Woloshyn



Excellent agreement between different collaborations/lattice formulations

Hyperon Axial Charges

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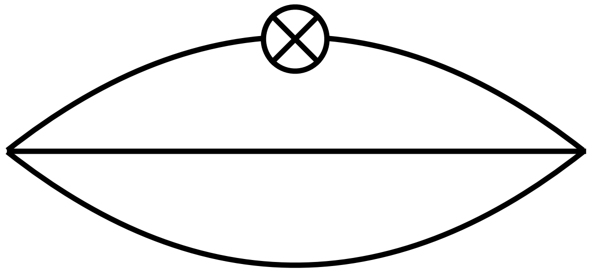
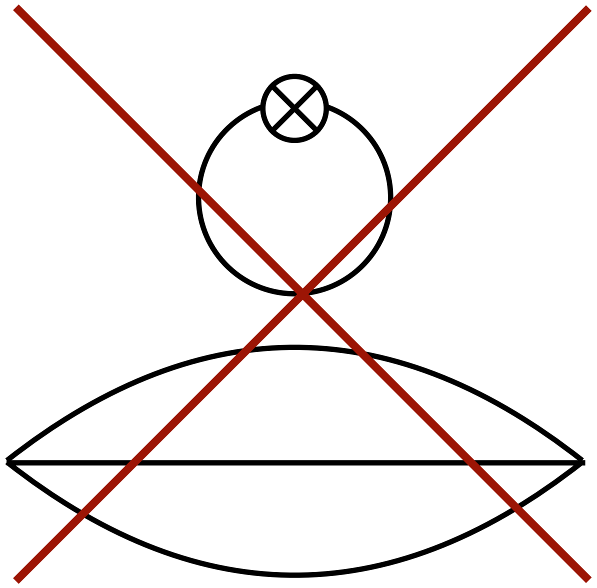




Only quark line connected contributions

Hyperon Axial Charges

- Important for low-energy effective field theory description of octet baryons
- SU(3)$_f$:

$$g_{A\,NN} = F + D, \qquad g_{A\,\Xi\Xi} = F - D, \qquad g_{A\,\Sigma\Sigma} = F,$$
$$g_{A\,\Lambda\Xi} = F - \frac{1}{3}D, \qquad g_{A\,\Sigma\Xi} = F + D,$$
$$g_{A\,\Lambda N} = F + \frac{1}{3}D, \qquad g_{A\,\Sigma N} = F - D, \qquad g_{A\,\Lambda\Sigma} = D.$$

- D and F enter chiral expansion of every baryonic quantity (e.g. masses, hyperon semi-leptonic decays, B-B' scattering phase shifts, ...)
- Poorly (or not at all) determined experimentally
- Quark Model F=0.46 , D=0.68 [K.-S.Choi, 1005.0337]
- Fits to Hyperon beta decay F=0.46 , D=0.8 [Close & Roberts, PLB316, 165 (1993)]
- ChPT, Large N$_c$ predicts

$$0.3 \leq g_{\Sigma\Sigma} \leq 0.55 \qquad 0.18 \leq -g_{\Xi\Xi} \leq 0.36$$

Hyperon Spin Content

- Proton “Spin Crisis”: only 33(3)(5)% of the proton spin carried by quarks
- Is this suppression a property of the nucleon, or a universal feature?

See also talk by A. Chambers, Tuesday 5:30 for other hadrons

- Do we observe $SU(3)_f$ breaking effects?

Nucleon Axial Charge, gA

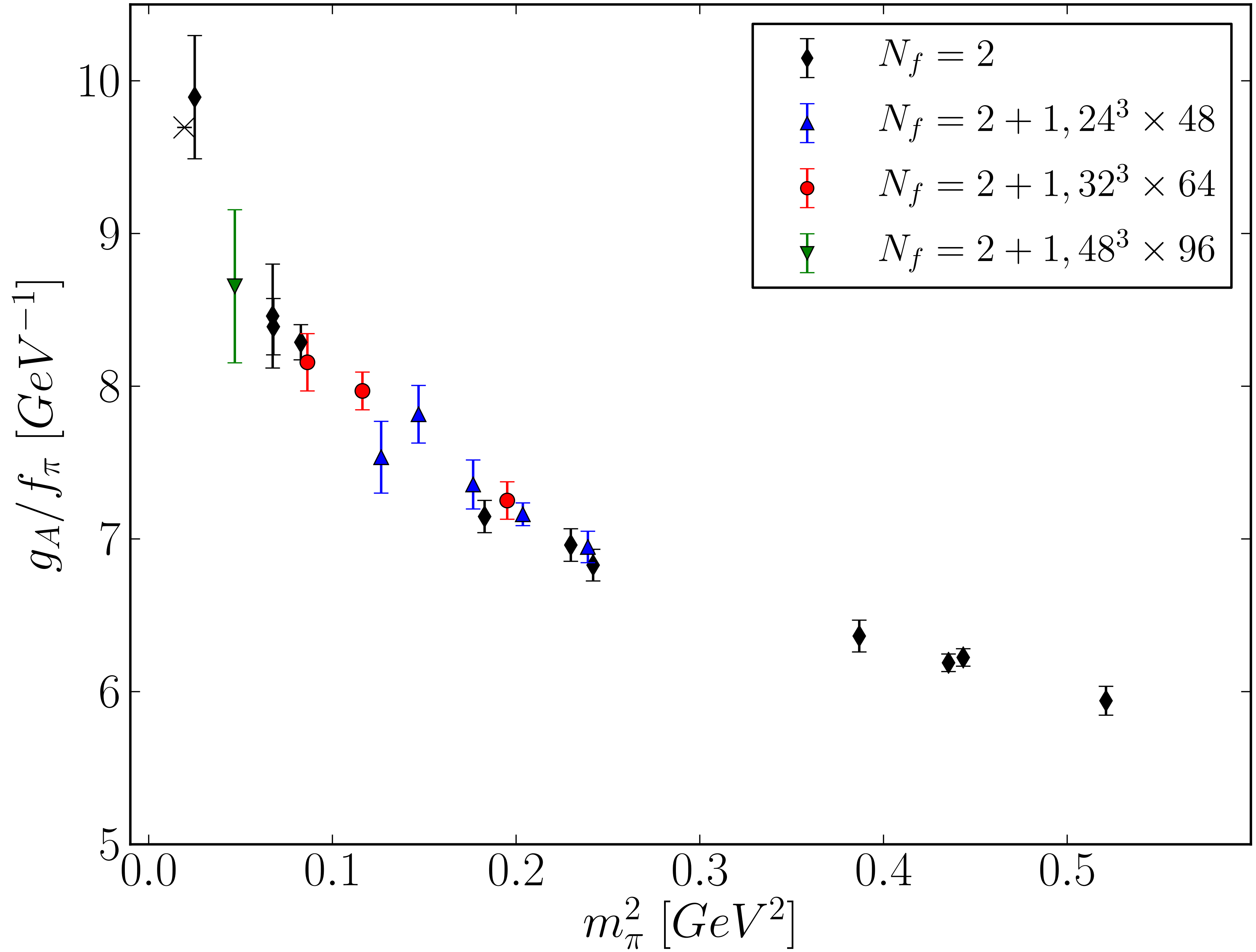
- Z_A almost complete (~0.85)

a~0.074fm

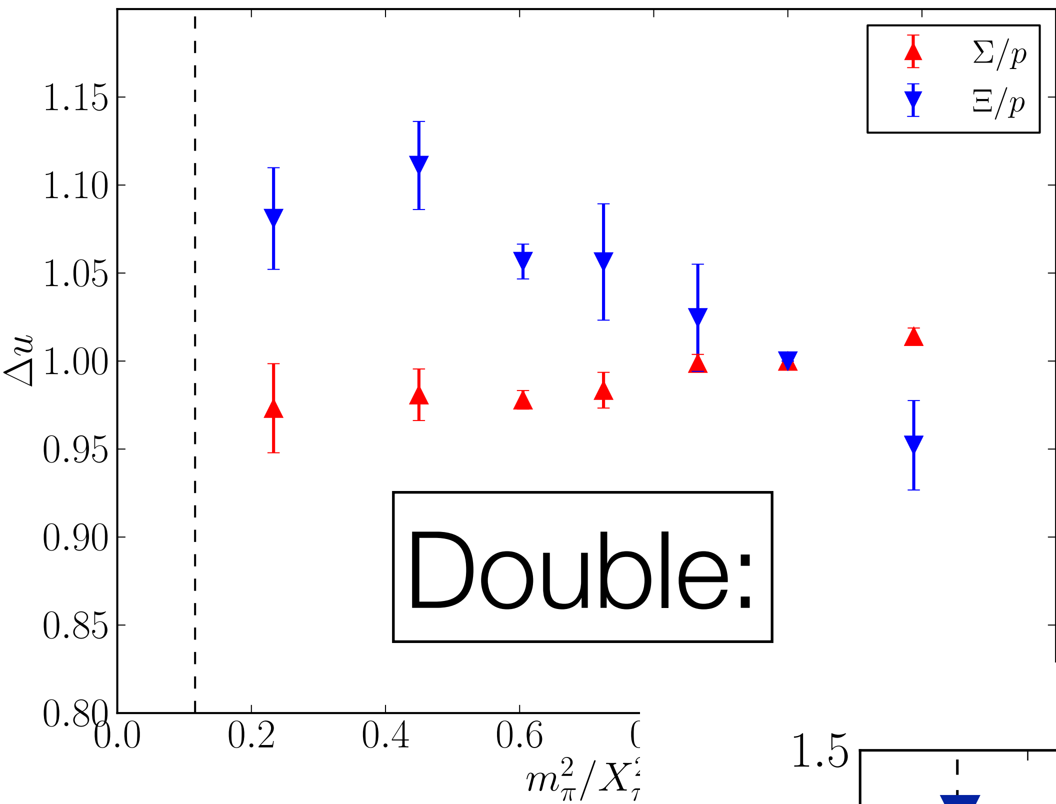
- Cancels in ratio g_A/f_π

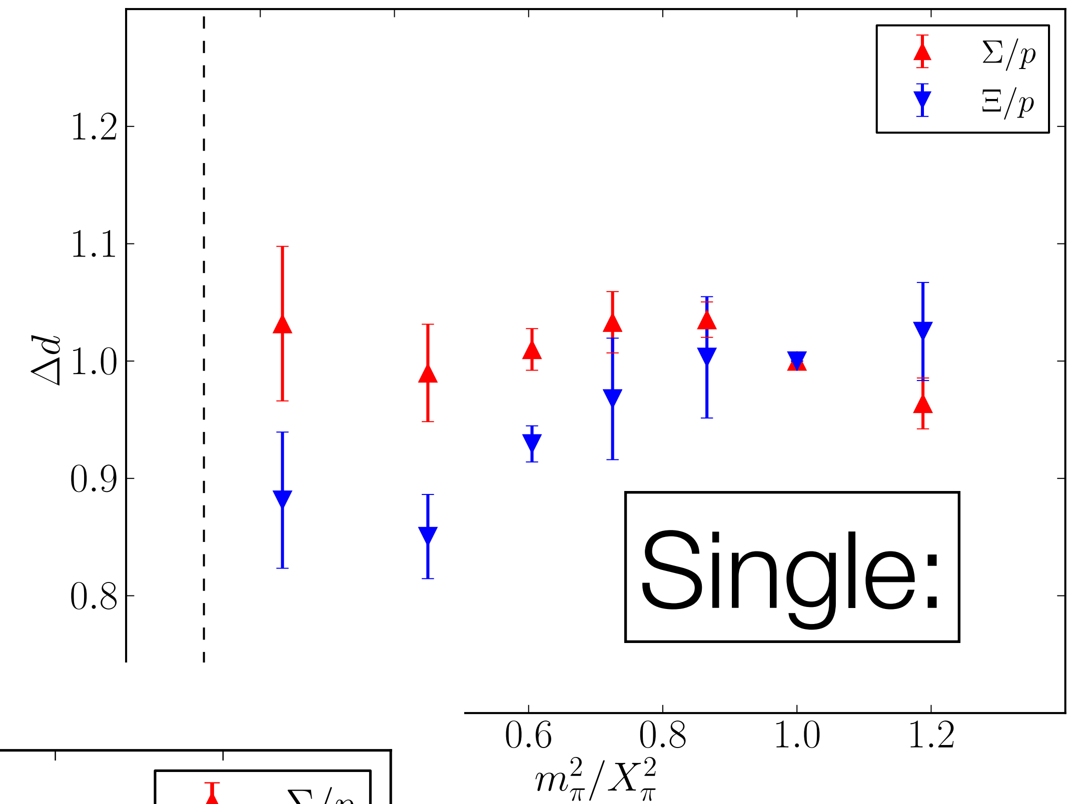
- Compare with N_f=2

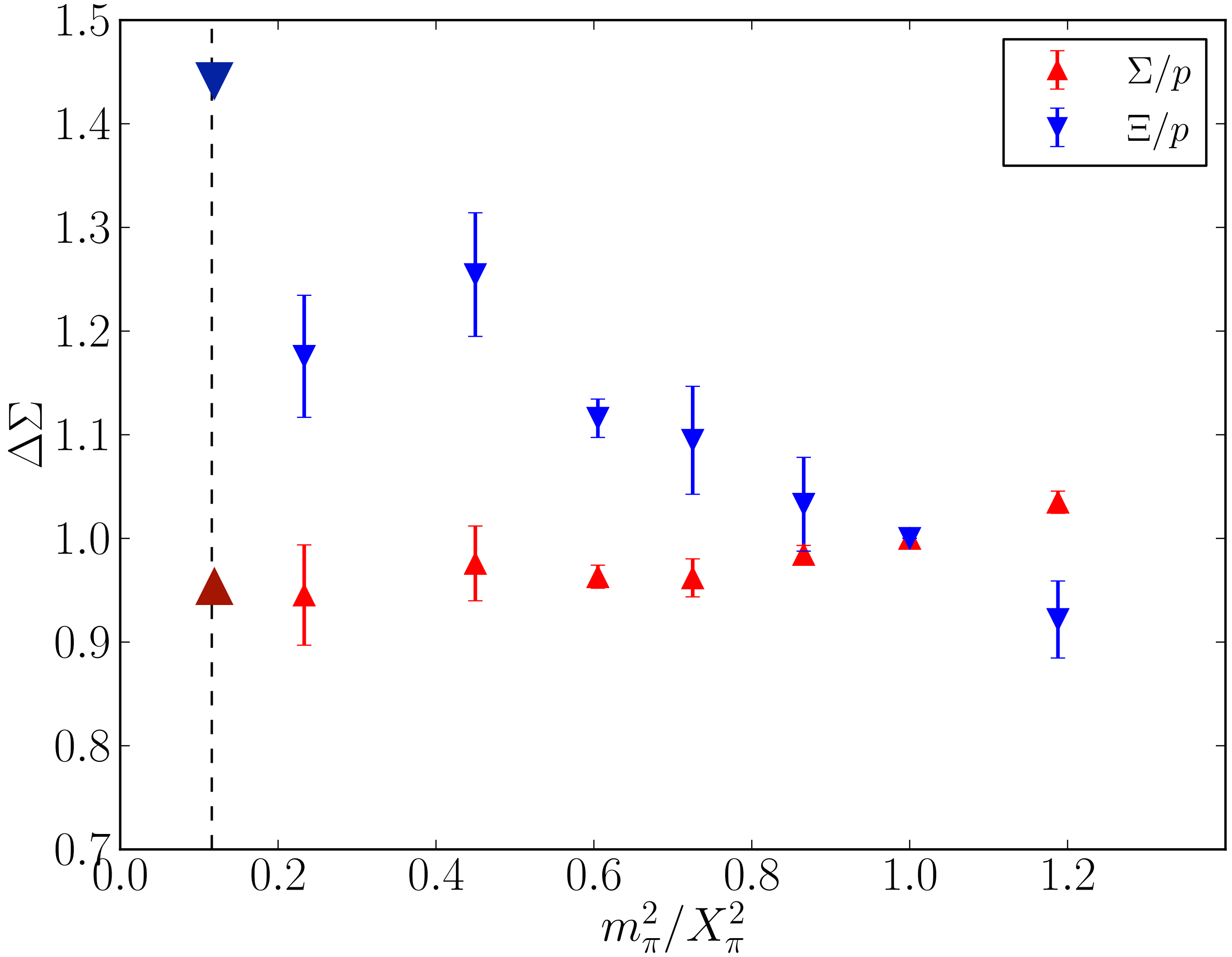
[QCDSF: 1302.2233 (PLB)]



Quark Spin Contributions

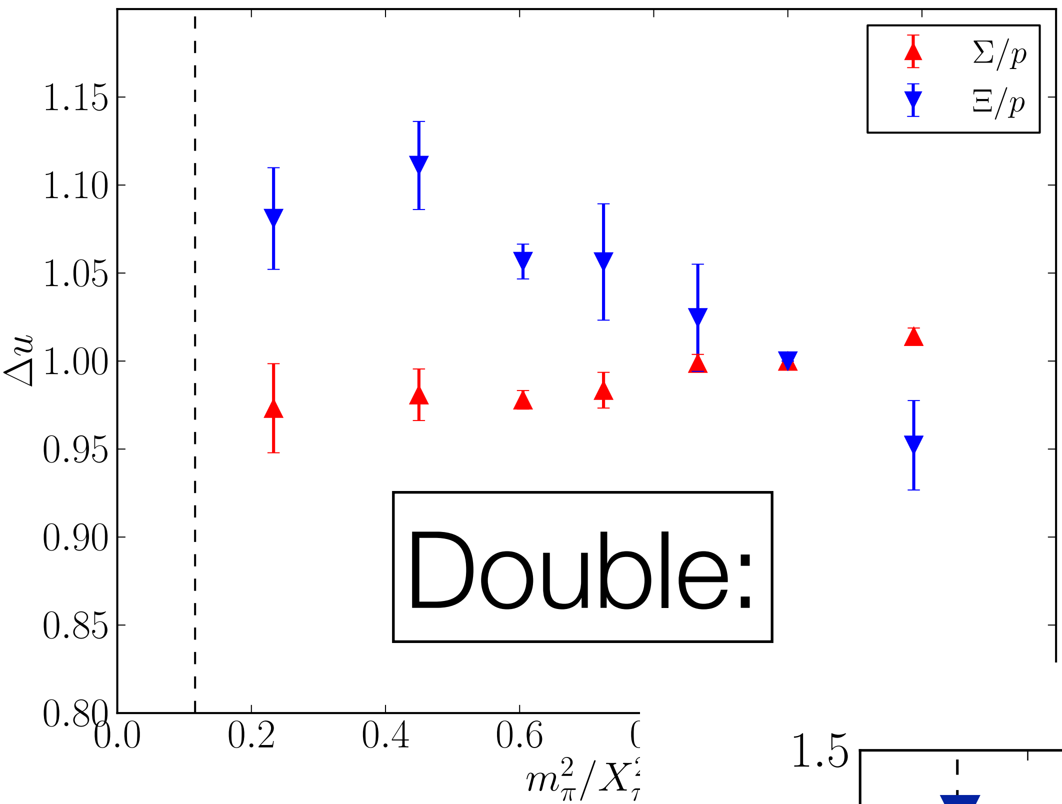


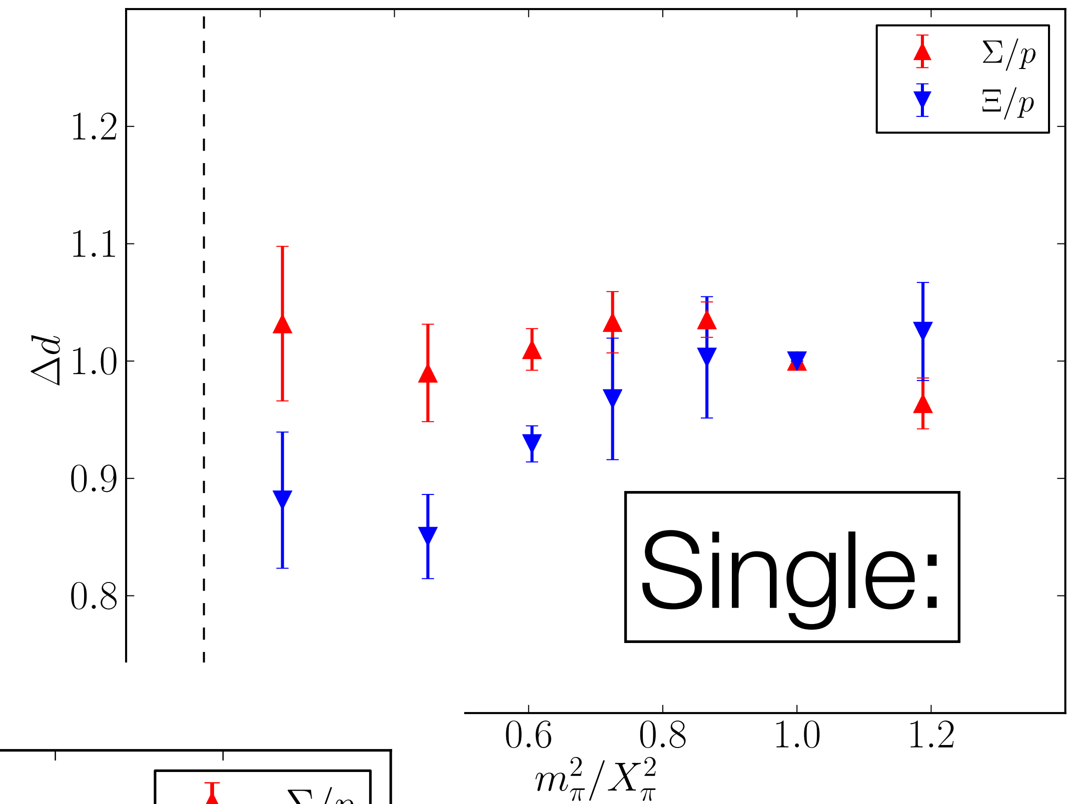


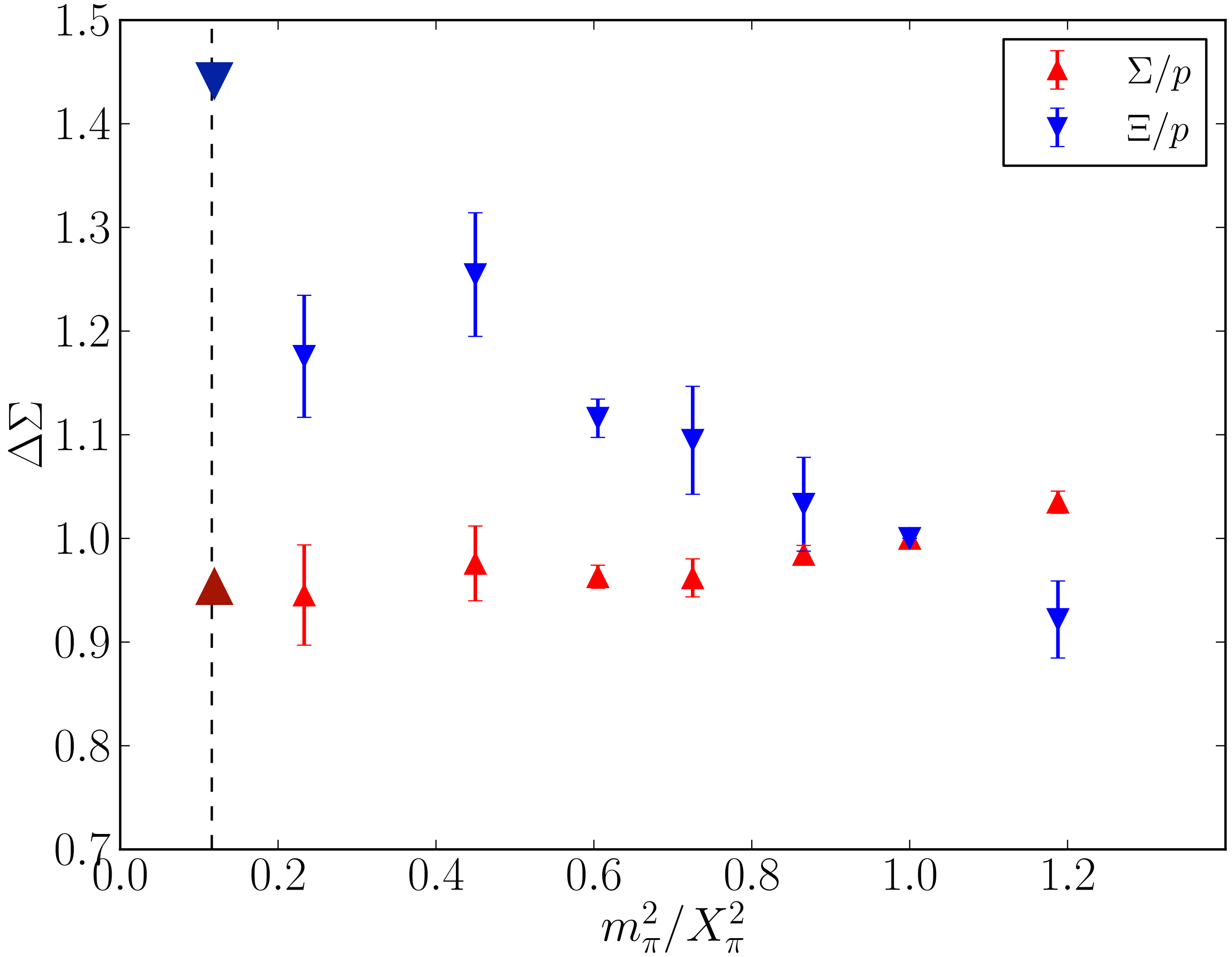


[c.f. Shanahan, Thomas, Young (PRL, 2013)]

Quark Spin Contributions







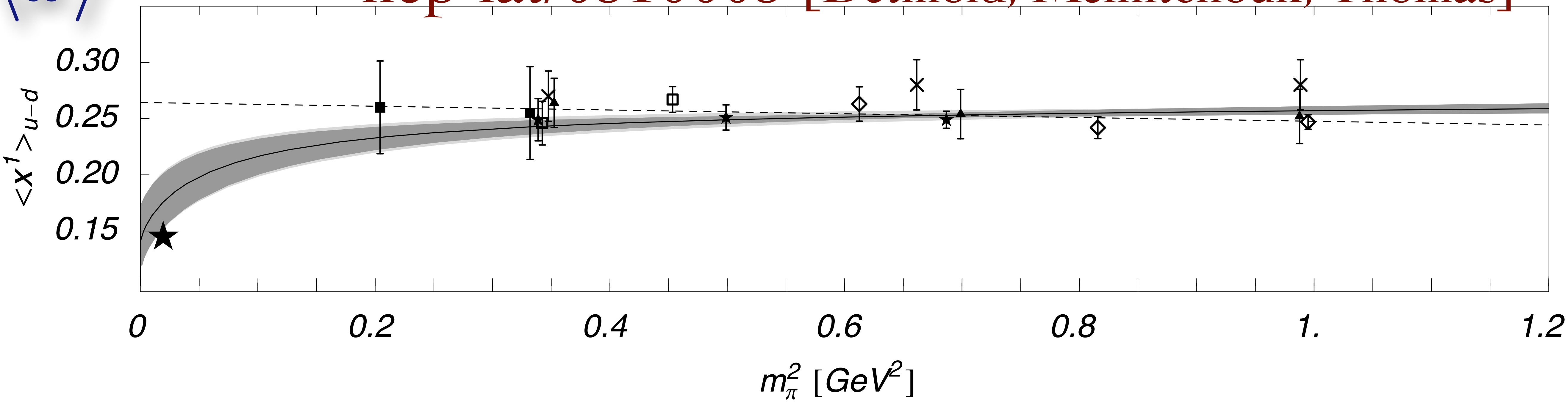
[c.f. Shanahan, Thomas, Young (PRL, 2013)]

Spin suppression perhaps not universal

$$\langle x \rangle$$

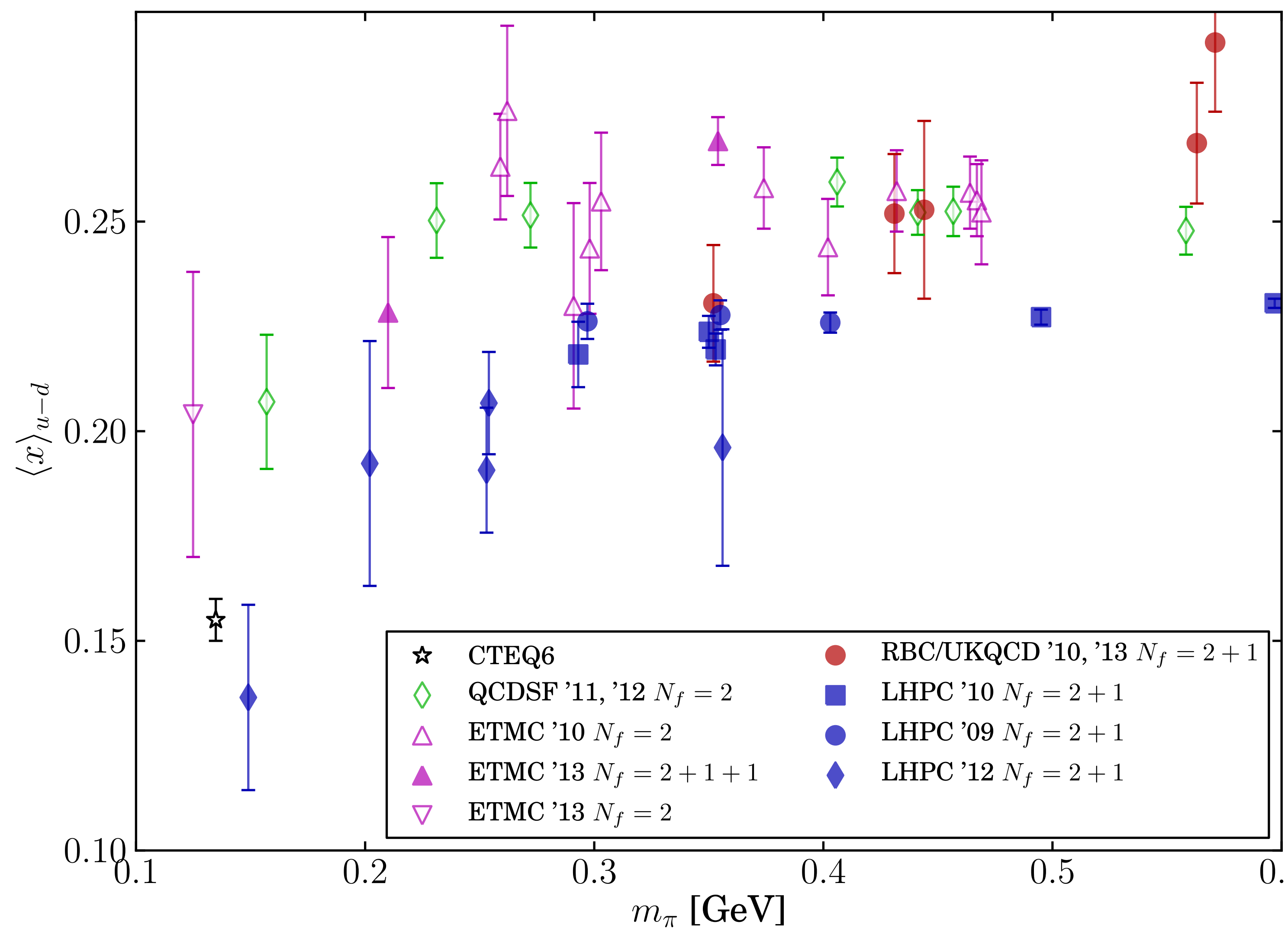
$\langle x \rangle$

hep-lat/0310003 [Detmold, Melnitchouk, Thomas]



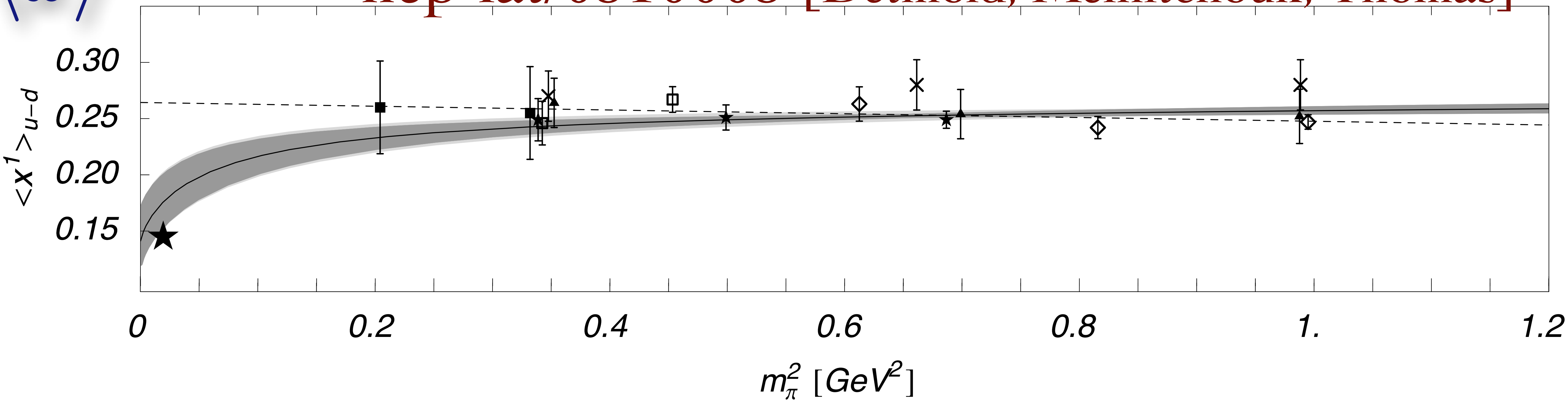
- Notorious for producing lattice results ≈ 2x too large for isovector nucleon
  - Will it ever bend down?

2013 update: S. Syritsyn (Lattice review)



$\langle x \rangle$

hep-lat/0310003 [Detmold, Melnitchouk, Thomas]



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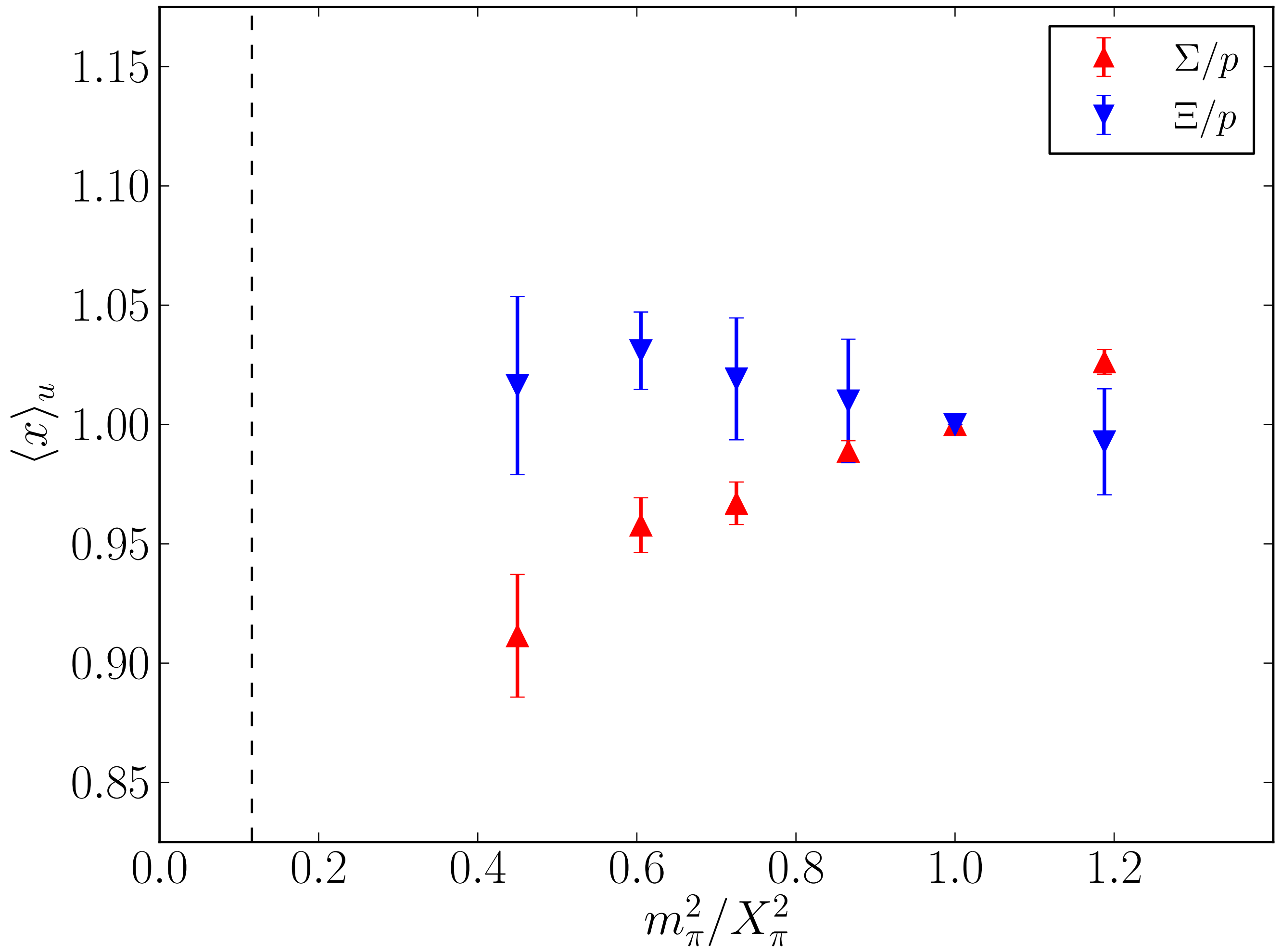
Hyperon momentum fractions

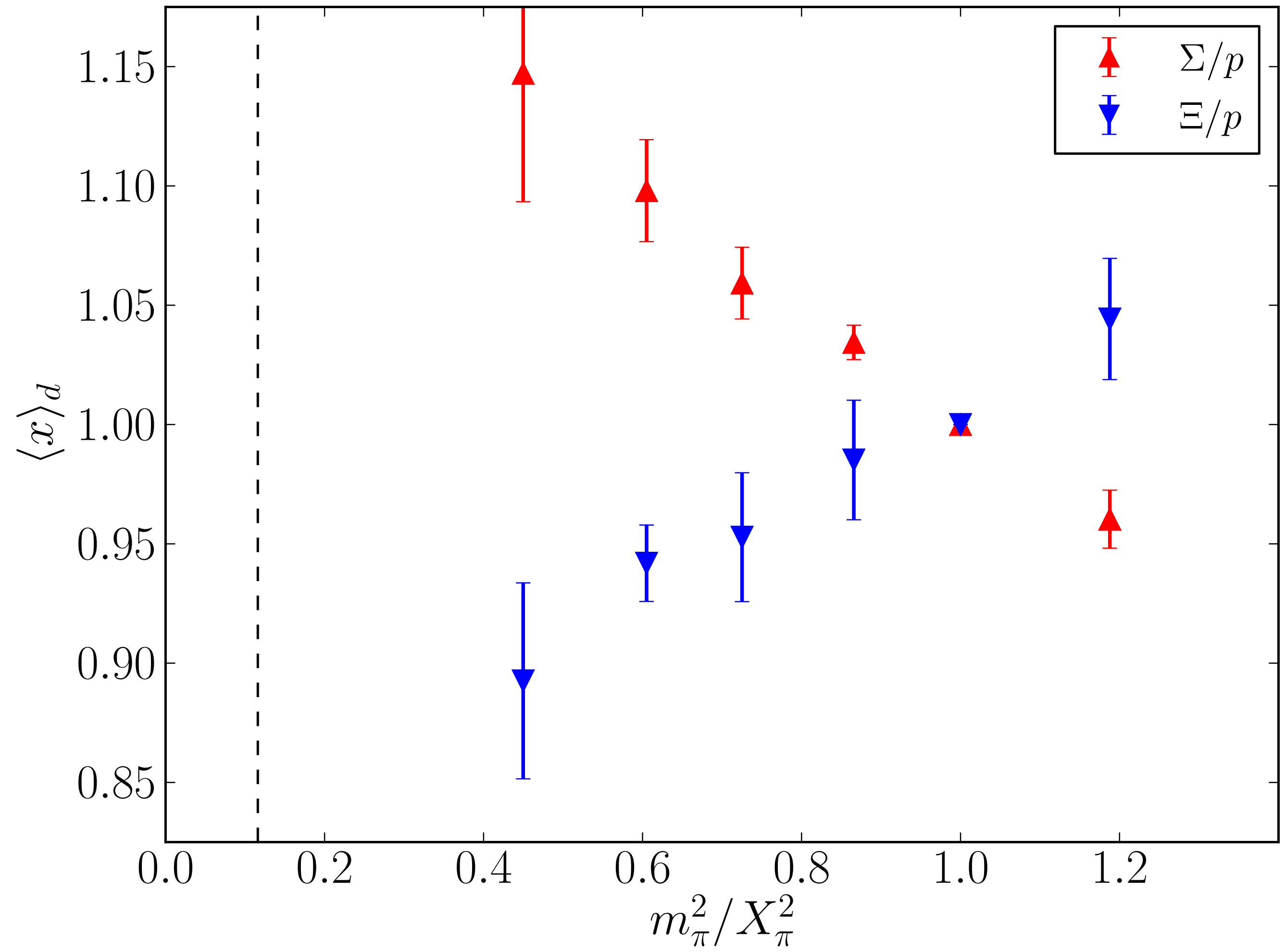
- Nucleon (& pion) momentum fractions have received much attention for many years
- What about SU(3) breaking effects?
- How is the momentum of the Hyperon distributed amongst light and strange quarks?

Hyperon Momentum Fractions

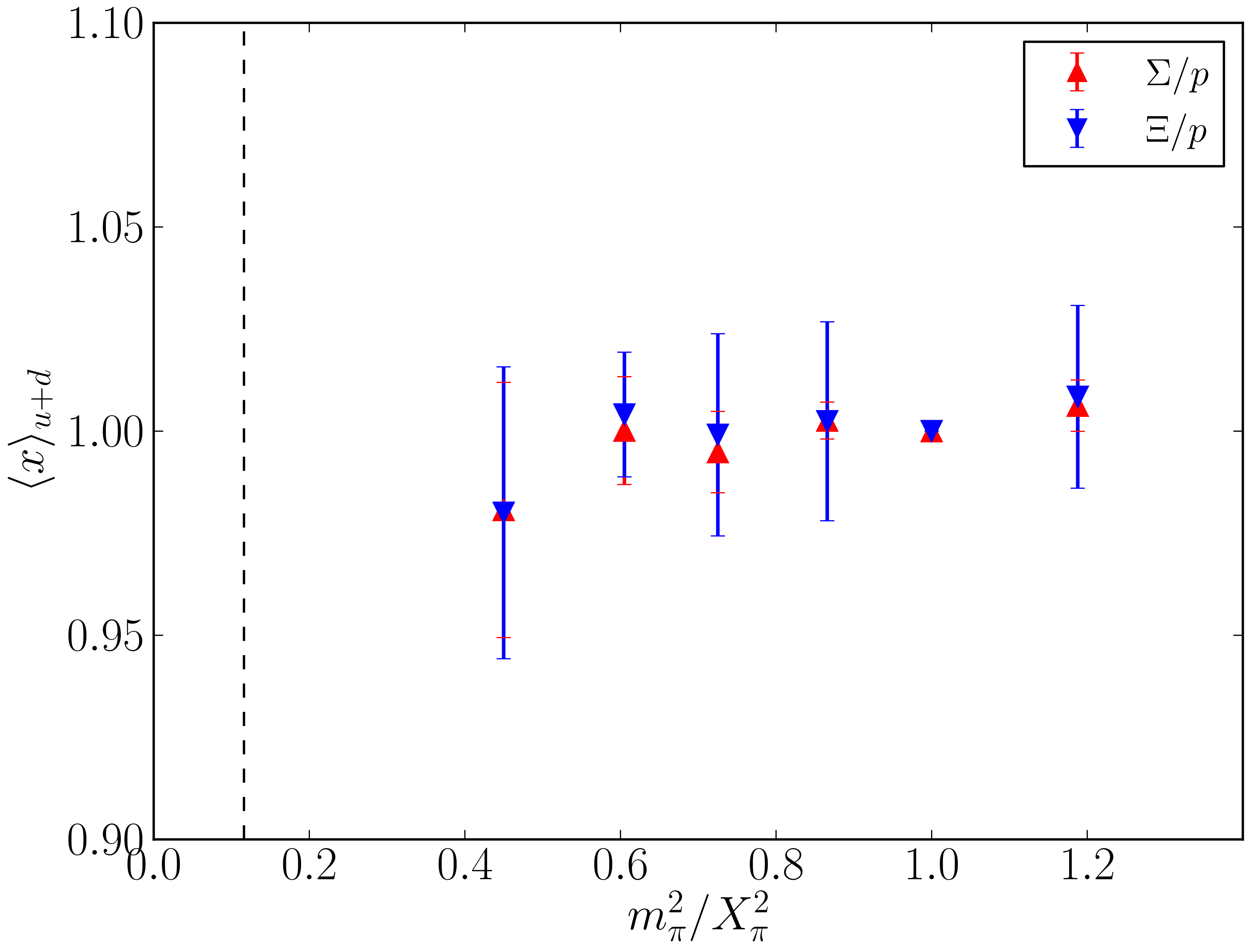
$$\frac{\langle x \rangle_{u_\Sigma}}{\langle x \rangle_{u_p}} \quad \frac{\langle x \rangle_{s_\Xi}}{\langle x \rangle_{u_p}}$$

$$\frac{\langle x \rangle_{s_\Sigma}}{\langle x \rangle_{d_p}} \quad \frac{\langle x \rangle_{u_\Xi}}{\langle x \rangle_{d_p}}$$





Hyperon Momentum Fractions



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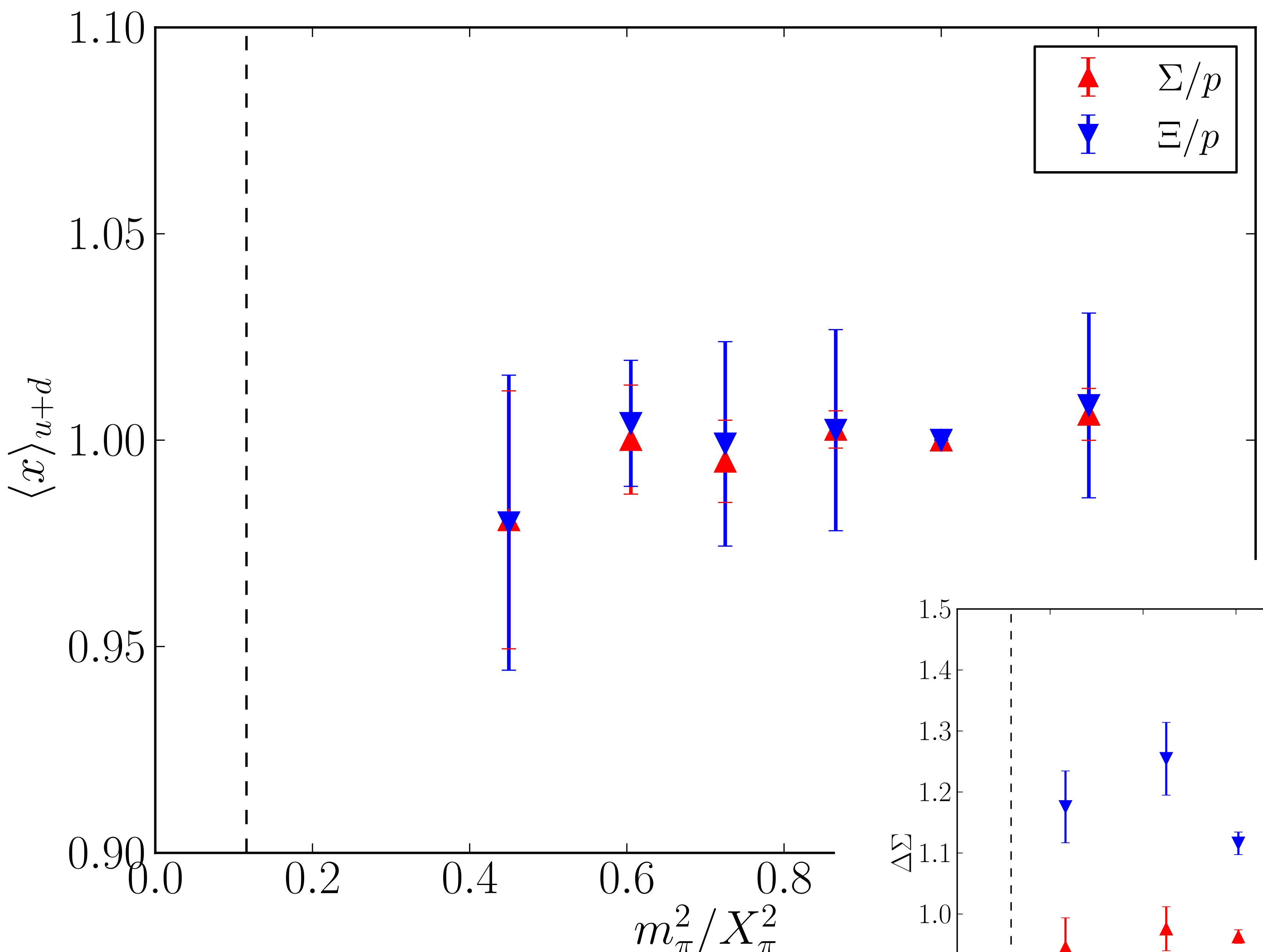
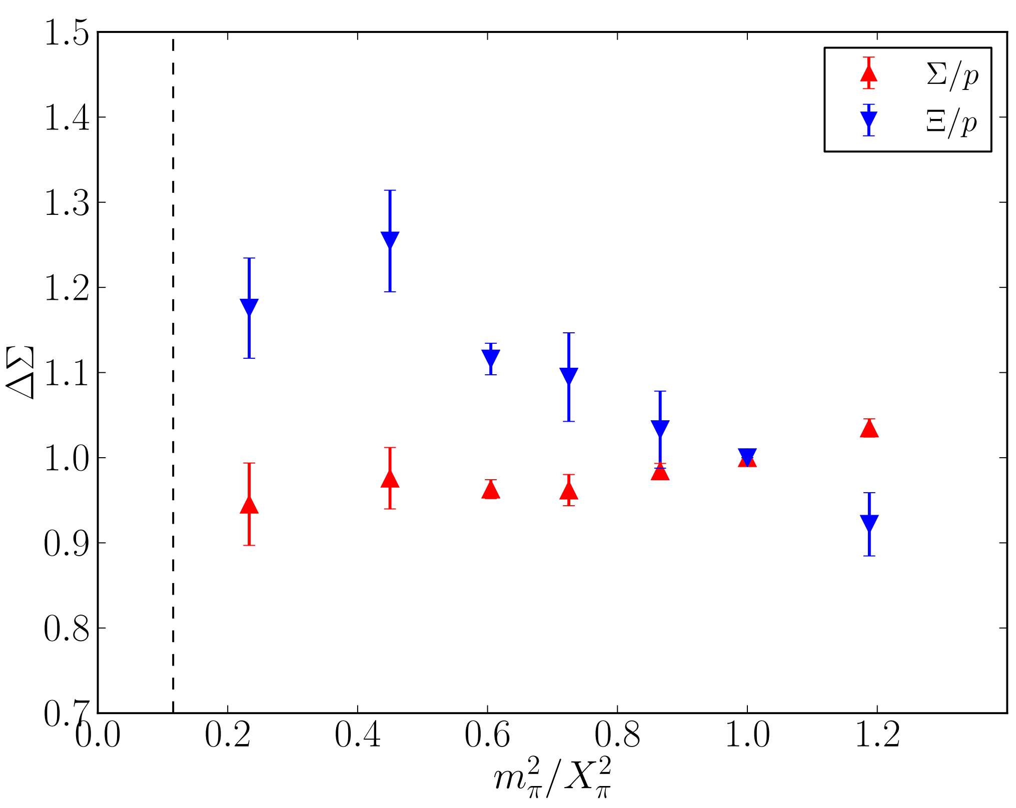




Charge Symmetry Violation

M_n - M_p

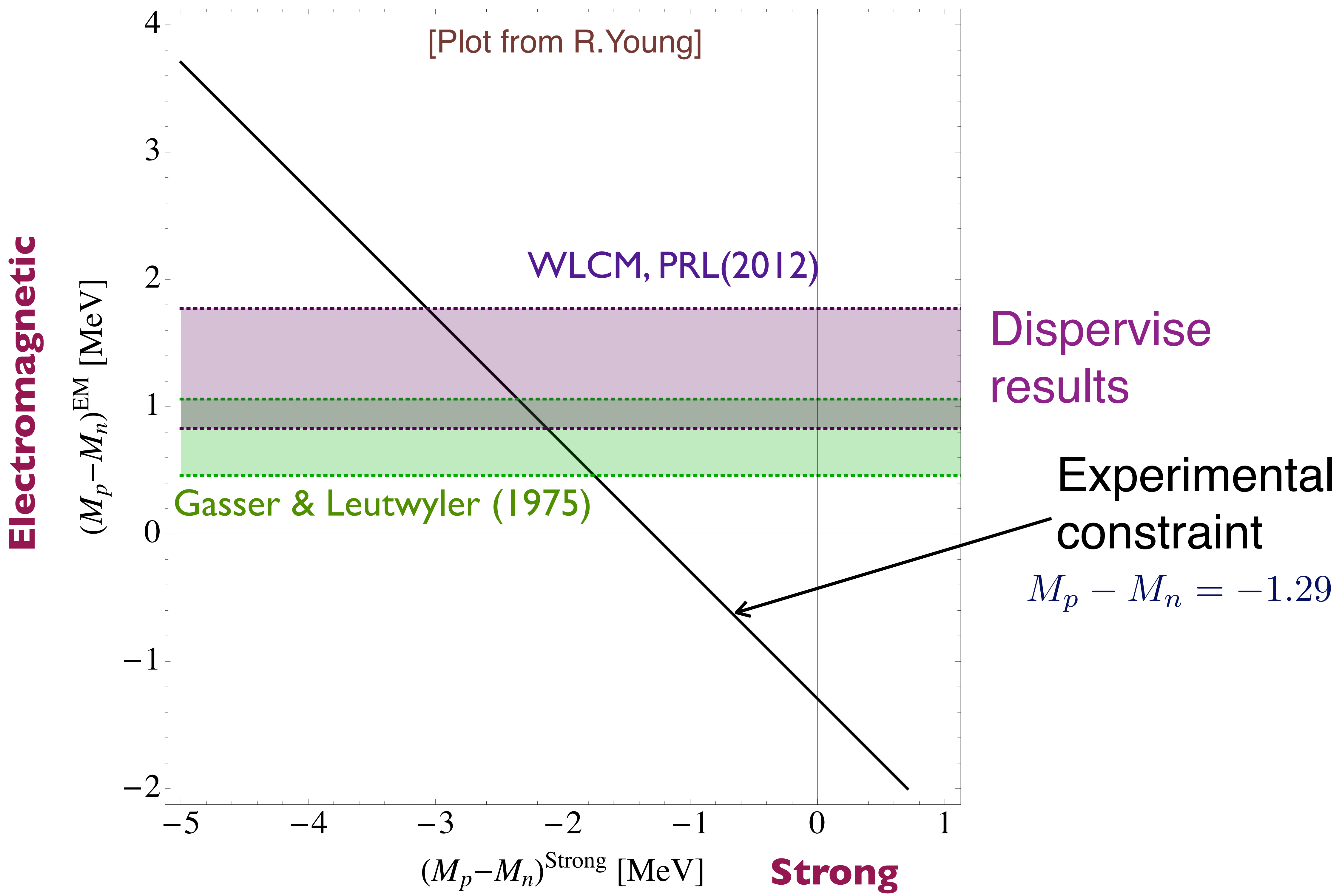
- Proton-neutron symmetry is exact if
  - up-down quark masses degenerate $m_u = m_d$
  - quark EM charges equal $Q_u = Q_d$
- Nature: $M_n - M_p = 1.29333217(42)\,\mathrm{MeV}$ [CODATA PDG (2012)]
- Given only EM effects, would expect

$$M_p > M_n$$

- The contribution from $m_d - m_u$ is comparable in size, but opposite in sign
- Neutron lifetime sensitive to M_n-M_p → implications for Big Bang Nucleosynthesis

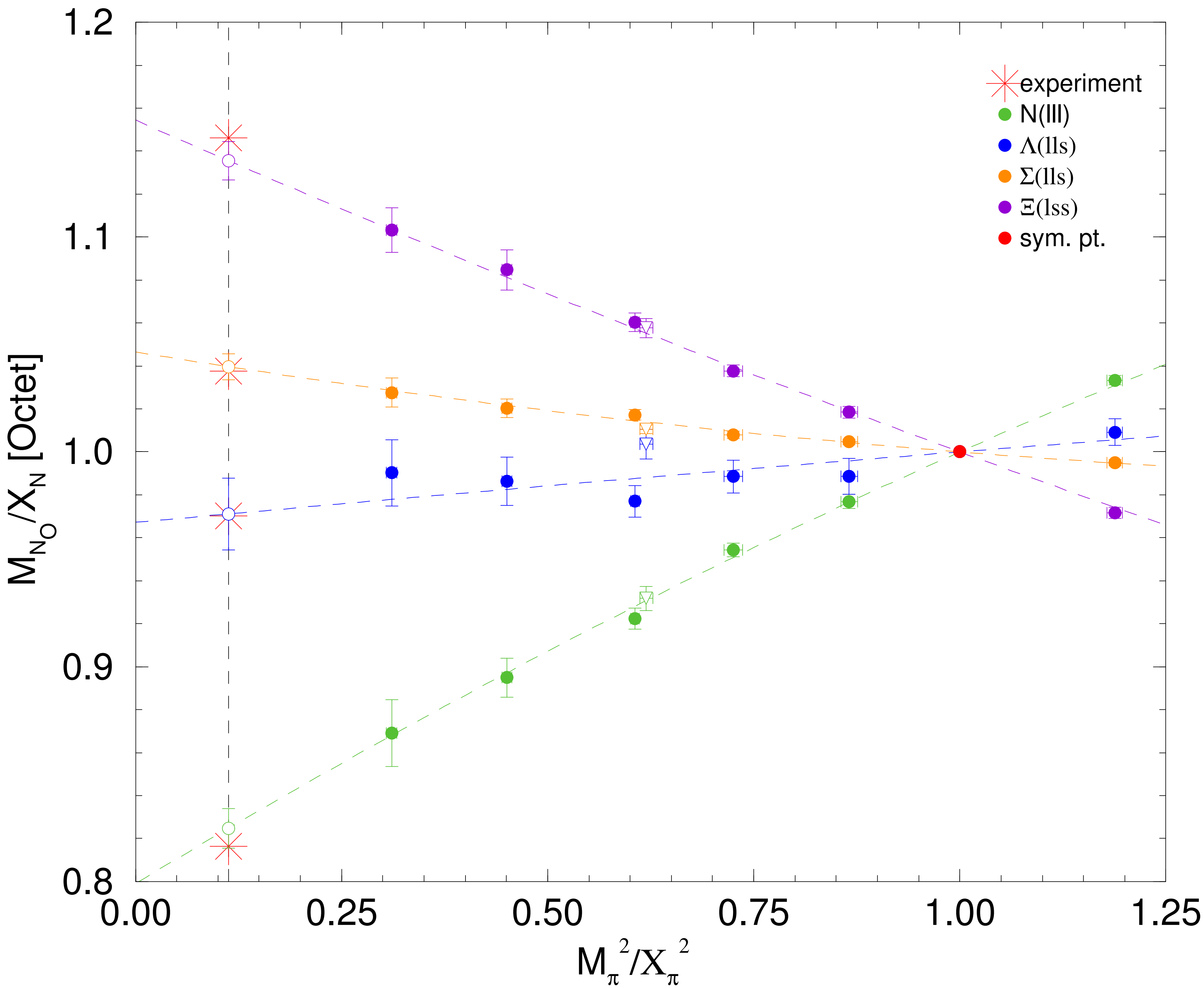
M_n - M_p

- Precise separation of QCD and QED contributions still under investigation



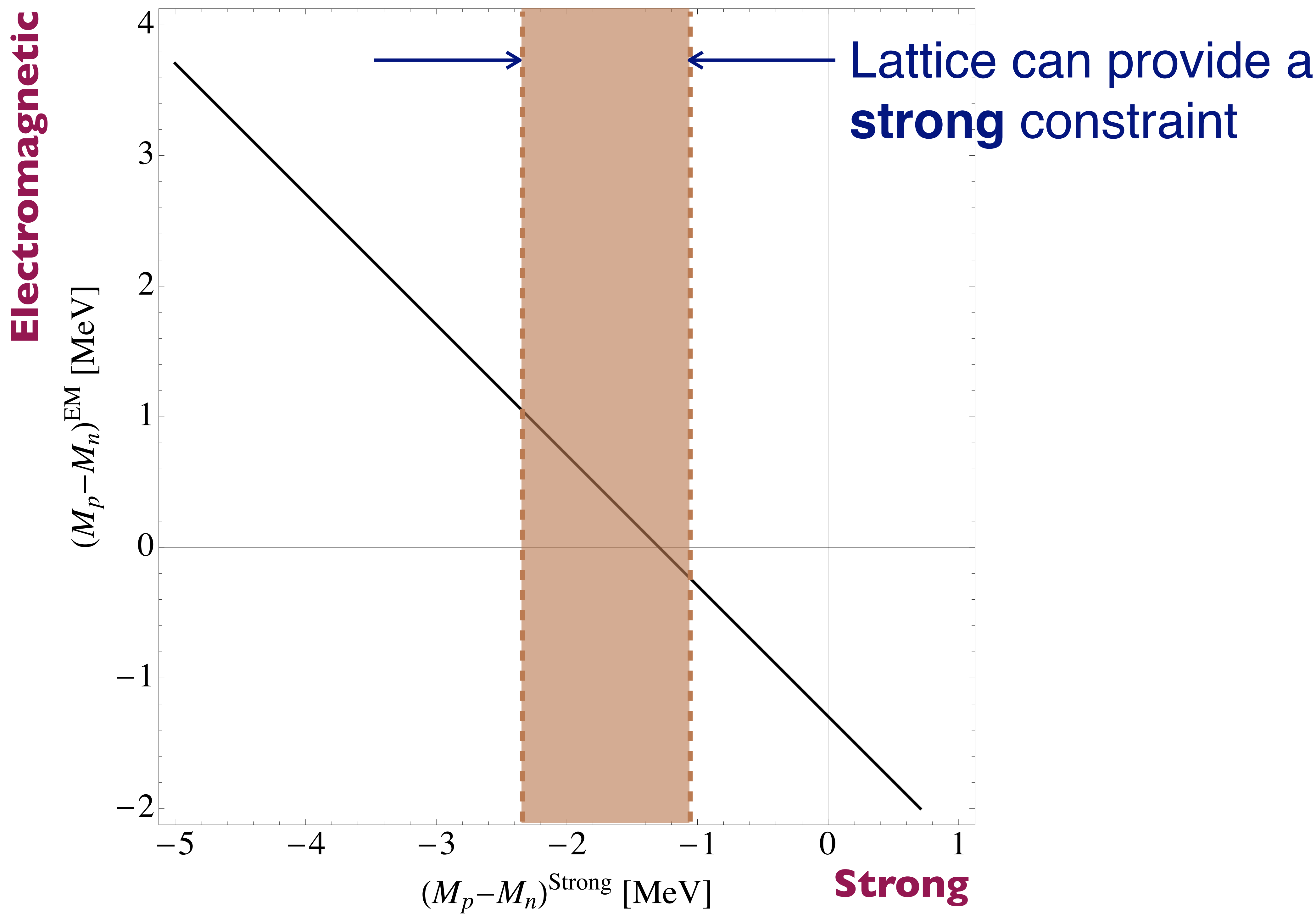
Mn - Mp

- But lattice is able to map out the quark mass dependence of hadronic observables
- Recall:



No new parameters required to break SU(2) (also in EFT)

M_n - M_p



Electromagnetic
$(M_p - M_n)^{EM}$ [MeV]
4
3
2
1
0
−1
−2
−5
−4
−3
−2
−1
0
1
$(M_p - M_n)^{Strong}$ [MeV]
Strong
Lattice can provide a **strong** constraint

M_n - M_p

- But all dynamical lattice simulations have $m_u = m_d$
- Study mass variation with partially-quenched valence quarks
  - sea quark mass m_q
  - valence quark mass μ_q
  - mass expansions in terms of $\delta\mu_q = \mu_q - m_0$ have the same coefficients as the full theory, e.g.

[1102.5300 (PRD)]

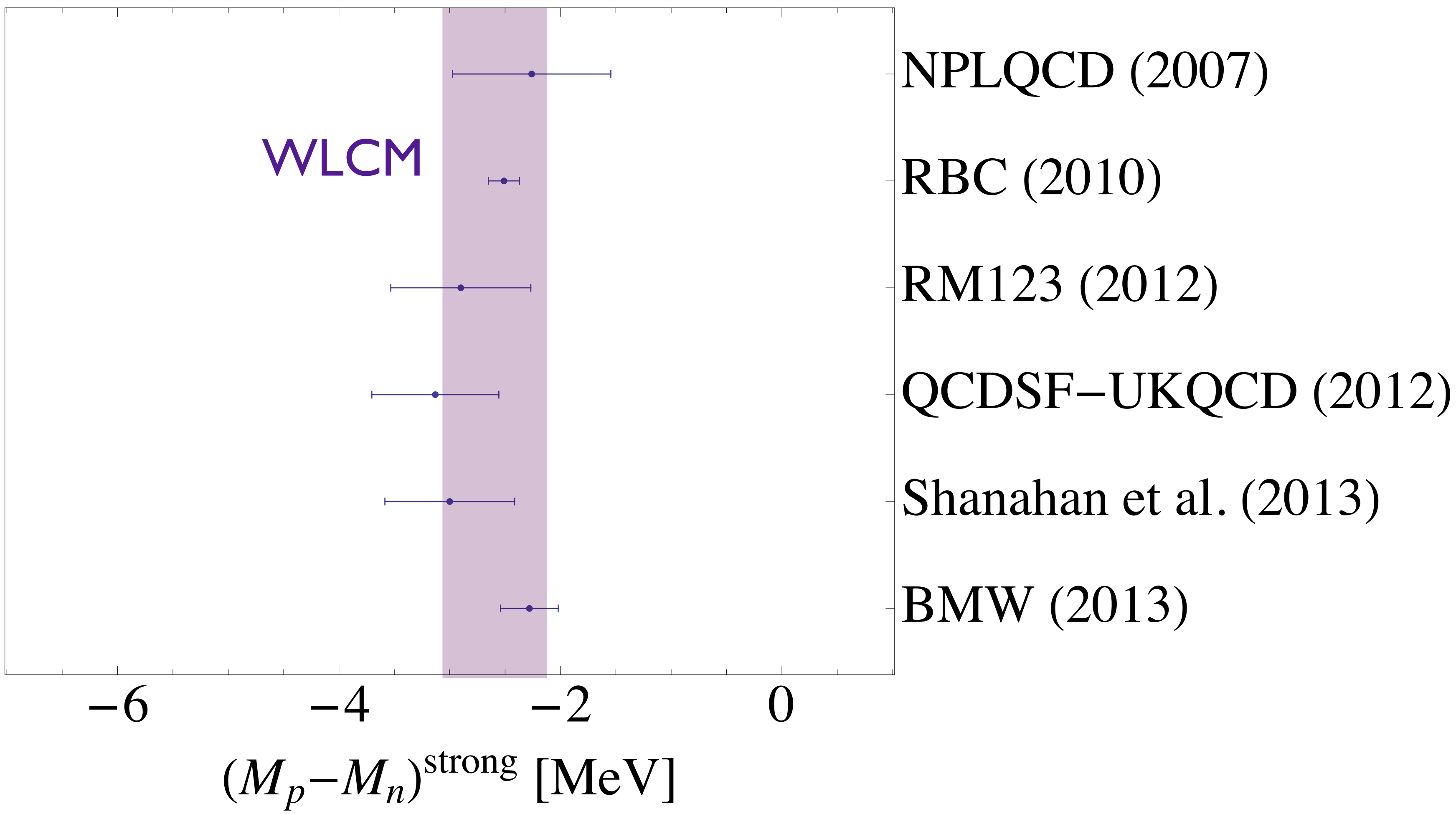
$$
\begin{aligned}
M_N &= M_0 + 3A_1\delta\mu_l + B_0\delta m_l^2 + 3B_1\delta\mu_l^2 \\
M_\Lambda &= M_0 + A_1(2\delta\mu_l + \delta\mu_s) - A_2(\delta\mu_s - \delta\mu_l) + B_0\delta m_l^2 \\
&\quad + B_1(2\delta\mu_l^2 + \delta\mu_s^2) - B_2(\delta\mu_s^2 - \delta\mu_l^2) + B_4(\delta\mu_s - \delta\mu_l)^2 \\
M_\Sigma &= M_0 + A_1(2\delta\mu_l + \delta\mu_s) + A_2(\delta\mu_s - \delta\mu_l) + B_0\delta m_l^2 \\
&\quad + B_1(2\delta\mu_l^2 + \delta\mu_s^2) + B_2(\delta\mu_s^2 - \delta\mu_l^2) + B_3(\delta\mu_s - \delta\mu_l)^2 \\
M_\Xi &= M_0 + A_1(2\delta\mu_l + \delta\mu_s) - A_2(\delta\mu_s - \delta\mu_l) + B_0\delta m_l^2 \\
&\quad + B_1(\delta\mu_l^2 + 2\delta\mu_s^2) - B_2(\delta\mu_s^2 - \delta\mu_l^2) + B_3(\delta\mu_s - \delta\mu_l)^2 .
\end{aligned}
$$

- Use SU(3) symmetry in relation to hyperon masses

M_n - M_p

- Progress by several collaborations using such techniques in determining the $(m_d - m_u)$ contribution

[Plot from R.Young]



$$M_n - M_p = 3.13(55) \text{ MeV}$$

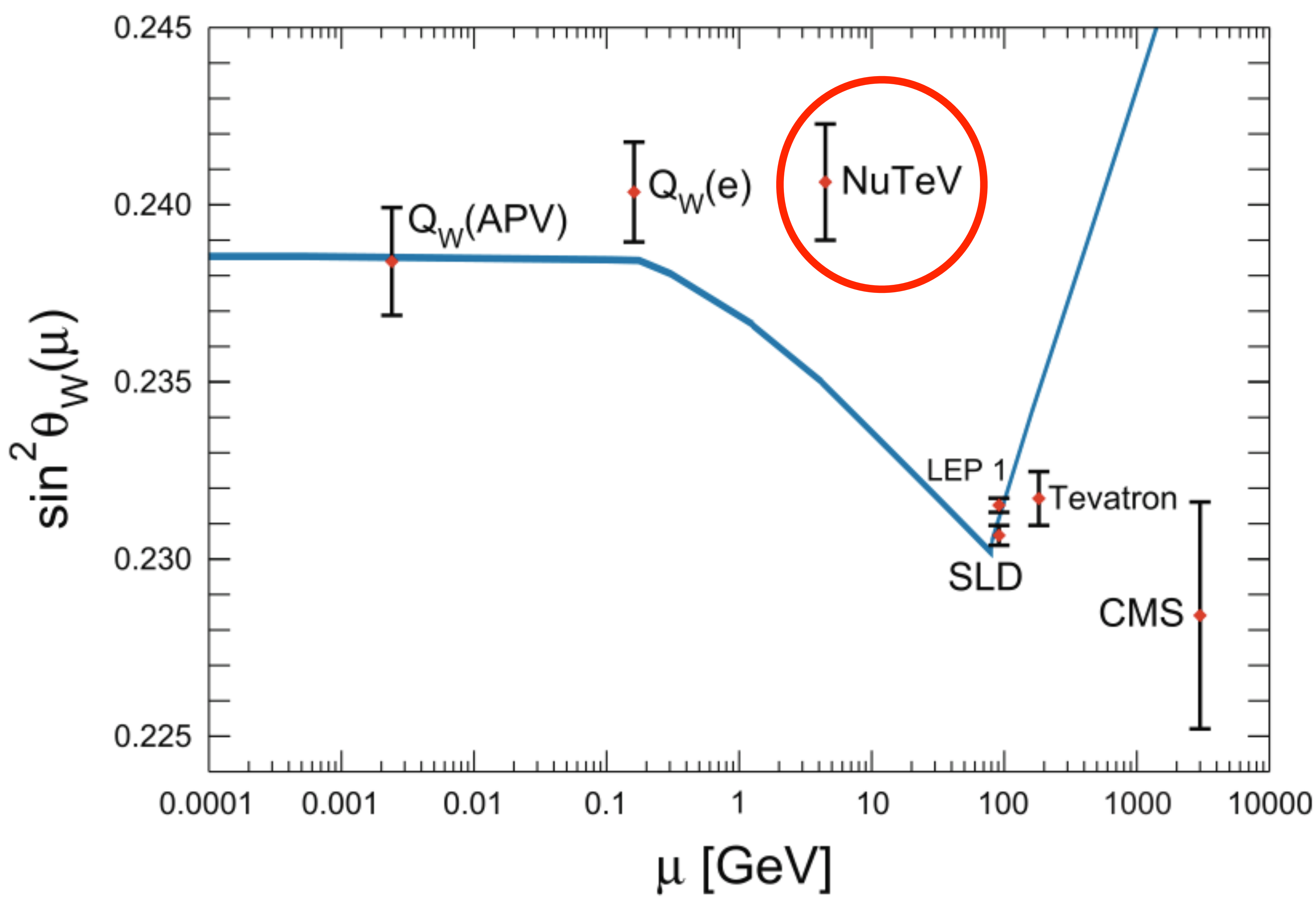
- QCDSF (1206.3156):

$$M_{\Sigma^-} - M_{\Sigma^+} = 8.10(136) \text{ MeV}$$

$$M_{\Xi^-} - M_{\Xi^0} = 4.98(85) \text{ MeV}$$

NuTeV & $\sin^2 \theta_W$

- NuTeV report a 3-sigma discrepancy from the Standard Model



Relies on assumption that CSV is negligible

CSV in Parton Distribution Functions

- Under charge symmetry

$$u^p(x) = d^n(x)$$
$$d^p(x) = u^n(x)$$

- Many experiments make this assumption (e.g. NuTeV)

- Use Lattice simulations to constrain the violation of charge symmetry

$$\delta u(x) = u^p(x) - d^n(x)$$
$$\delta d(x) = d^p(x) - u^n(x)$$

- Lattice, however, can only access (the lowest couple of) moments

$$\langle x^{m-1} \rangle = \int_0^1 dx\, x^{m-1} \left[q(x) + (-1)^m \bar{q}(x)\right]$$

- Our aim is then to determine (for second moment)

$$\delta u = \langle x \rangle_u^p - \langle x \rangle_d^n$$
$$\delta d = \langle x \rangle_d^p - \langle x \rangle_u^n$$

(Similar for moments of spin-dependent PDFs)

CSV in Parton Distribution Functions

- For small isospin breaking $m_\delta = (m_d - m_u)$

$$\delta u \simeq \frac{m_\delta}{2}\left[\left(-\frac{\partial\langle x\rangle_u^p}{\partial m_u}+\frac{\partial\langle x\rangle_u^p}{\partial m_d}\right)-\left(-\frac{\partial\langle x\rangle_d^n}{\partial m_u}+\frac{\partial\langle x\rangle_d^n}{\partial m_d}\right)\right]$$

CSV in Parton Distribution Functions

- For small isospin breaking $m_\delta = (m_d - m_u)$

$$\delta u \simeq \frac{m_\delta}{2}\left[\left(-\frac{\partial\langle x\rangle_u^p}{\partial m_u} + \frac{\partial\langle x\rangle_u^p}{\partial m_d}\right) - \left(-\frac{\partial\langle x\rangle_d^n}{\partial m_u} + \frac{\partial\langle x\rangle_d^n}{\partial m_d}\right)\right]$$

Charge symmetry

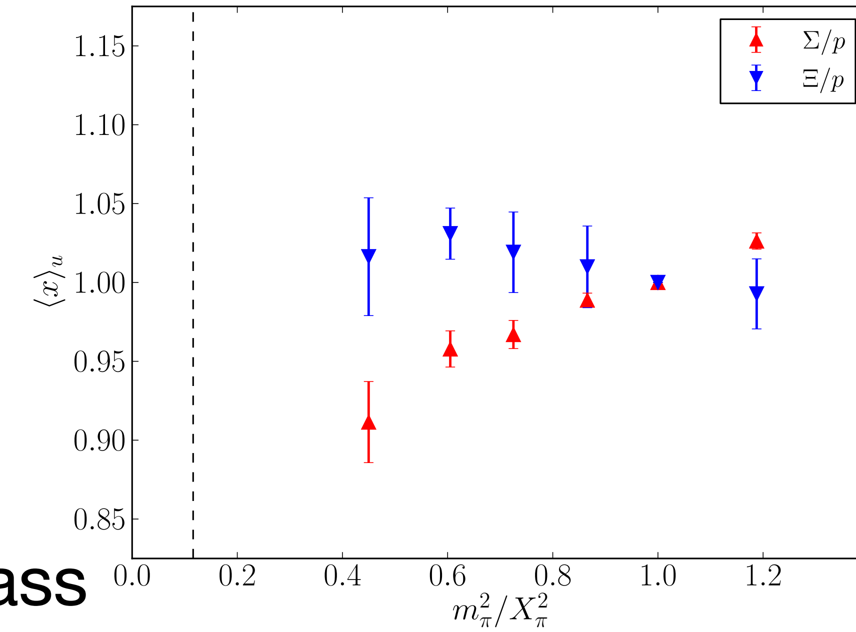
CSV in Parton Distribution Functions

- For small isospin breaking $m_\delta = (m_d - m_u)$

$$\delta u \simeq \frac{m_\delta}{2}\left[\left(-\frac{\partial\langle x\rangle_u^p}{\partial m_u} + \frac{\partial\langle x\rangle_u^p}{\partial m_d}\right) - \left(-\frac{\partial\langle x\rangle_d^n}{\partial m_u} + \frac{\partial\langle x\rangle_d^n}{\partial m_d}\right)\right]$$

$$\delta u \simeq m_\delta\left[-\frac{\partial\langle x\rangle_u^p}{\partial m_u} + \frac{\partial\langle x\rangle_u^p}{\partial m_d}\right]$$

CSV in Parton Distribution Functions



- Near the SU(3)-symmetric point

$$\langle x \rangle_u^p \simeq \langle x \rangle_u^{\Sigma^+} \simeq \langle x \rangle_s^{\Xi^0}$$

(uud) $d \leftrightarrow s$ (uus) $s \leftrightarrow u$ (ssu)

- Hence we can approximate the variation with quark mass

$$\frac{\partial \langle x \rangle_u^p}{\partial m_u} \simeq \frac{\langle x \rangle_s^{\Xi^0} - \langle x \rangle_u^p}{m_s - m_l} \qquad \frac{\partial \langle x \rangle_u^p}{\partial m_d} \simeq \frac{\langle x \rangle_u^{\Sigma^+} - \langle x \rangle_u^p}{m_s - m_l}$$

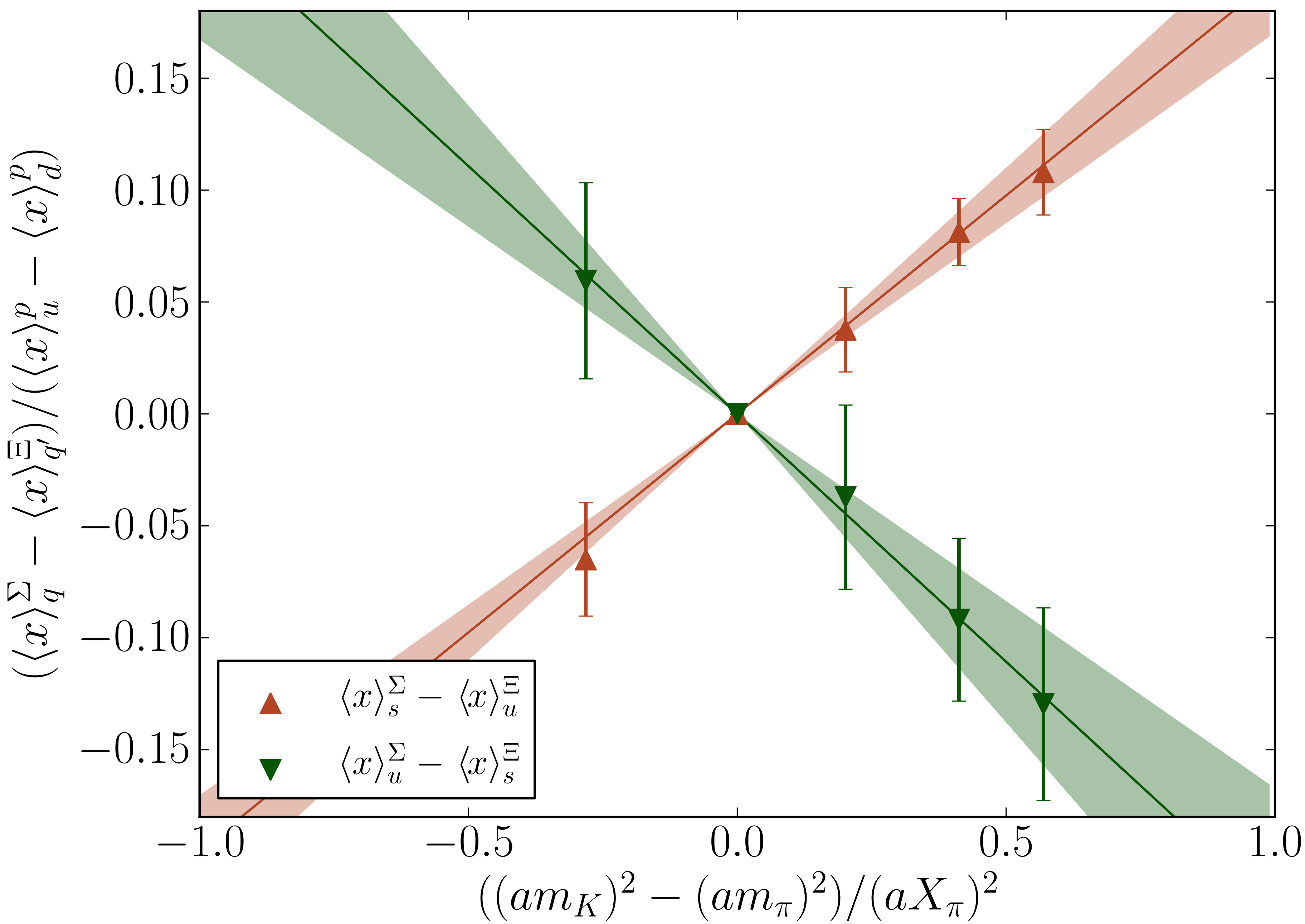
$$\Rightarrow \quad \delta u \simeq m_\delta \left[-\frac{\partial \langle x \rangle_u^p}{\partial m_u} + \frac{\partial \langle x \rangle_u^p}{\partial m_d} \right] \simeq m_\delta \frac{\langle x \rangle_u^{\Sigma^+} - \langle x \rangle_s^{\Xi^0}}{m_s - m_l}$$

- So we can use our earlier results for $\langle x \rangle_q^B$ around the SU(3)-symmetric point

CSV in Parton Distribution Functions

- Using our earlier results

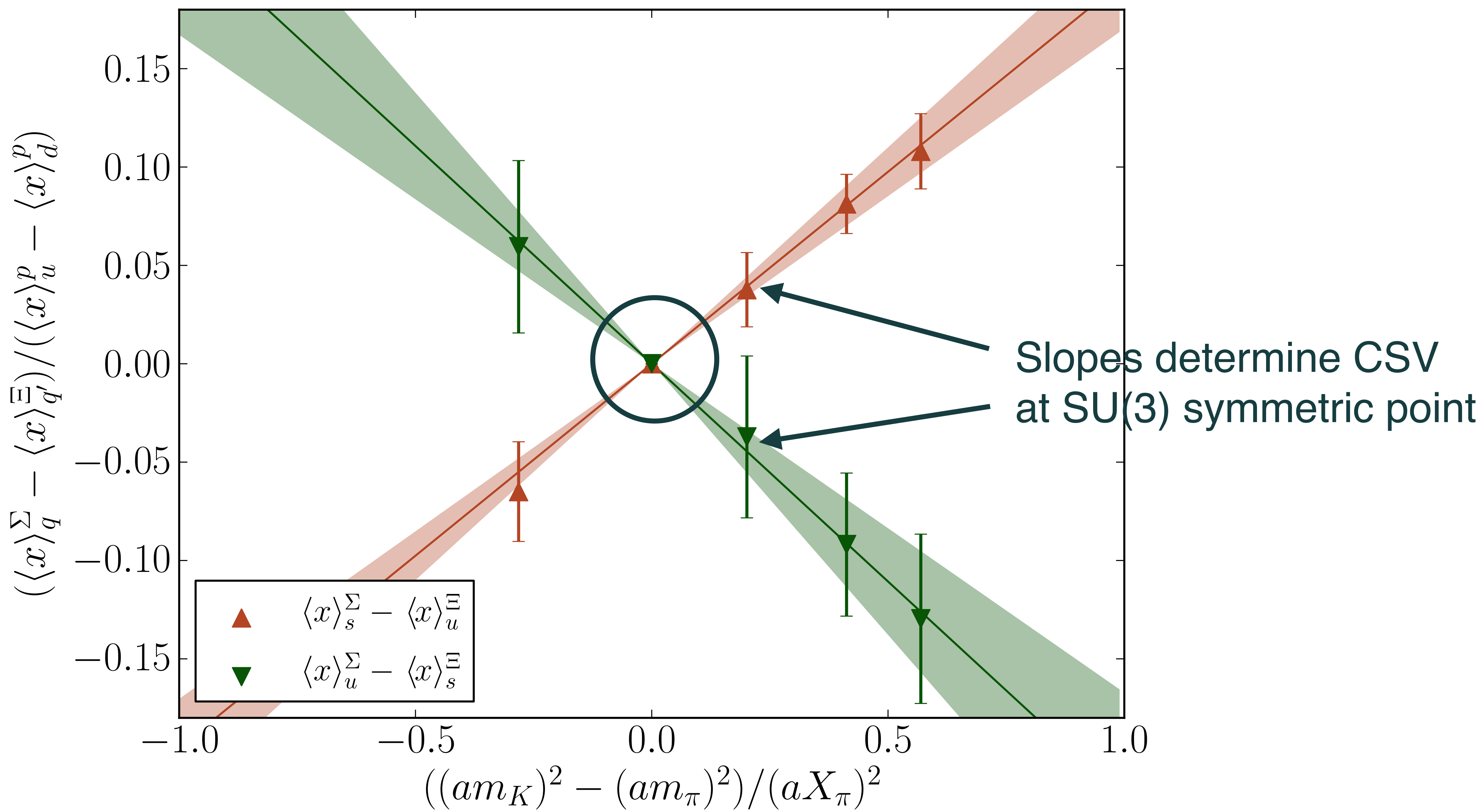
[CSSM/QCDSF, 1012.0215 (PRD)]



CSV in Parton Distribution Functions

- Using our earlier results

[CSSM/QCDSF, 1012.0215 (PRD)]

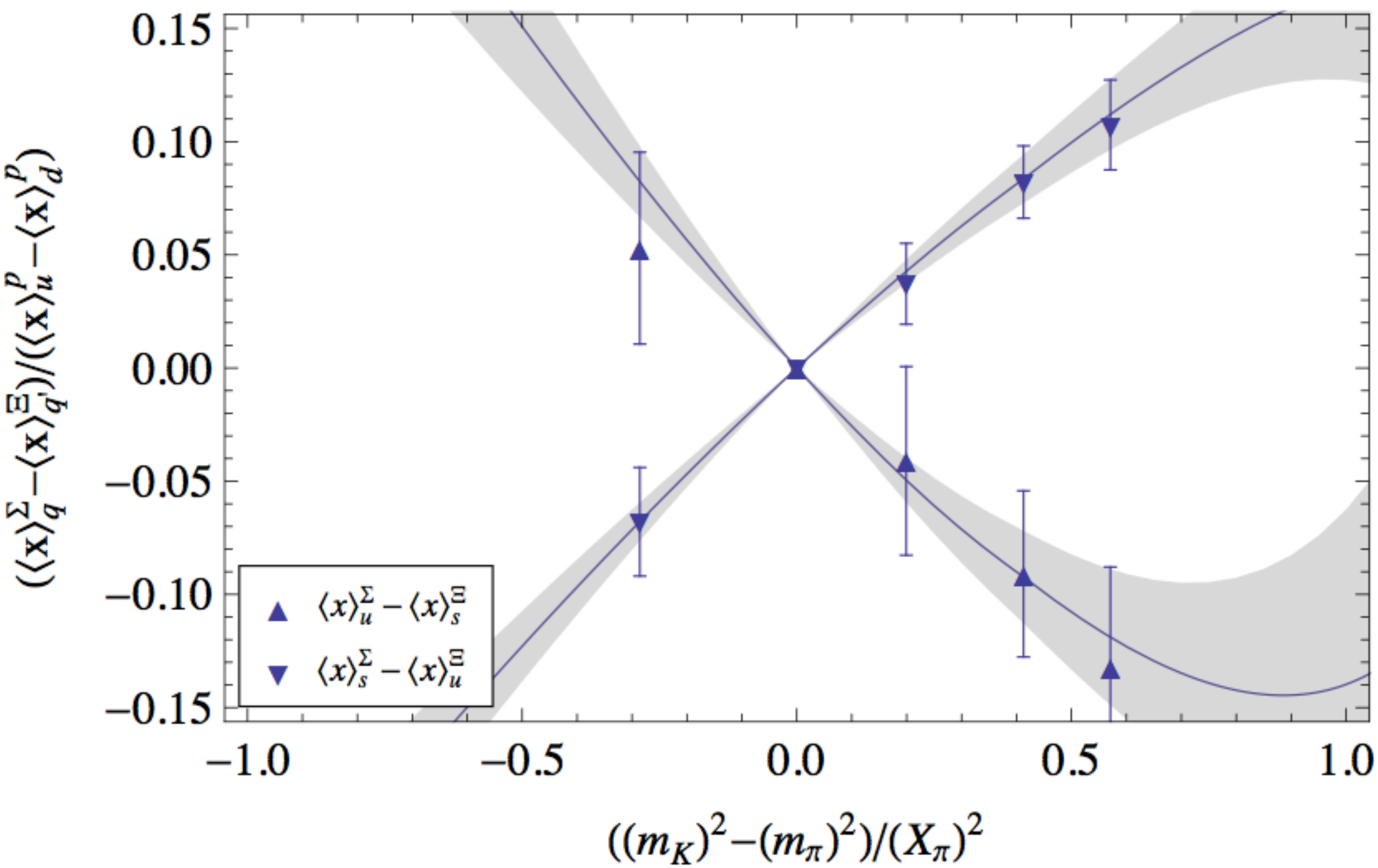


CSV in Parton Distribution Functions

- Chiral correction to obtain CSV at the physical point

$$\delta u = -0.0023(7)$$
$$\delta d = \ \ 0.0017(4)$$

Shanahan, Thomas & Young, PRD(2013)094515



- Reduce NuTeV Standard Model discrepancy by ~1 sigma

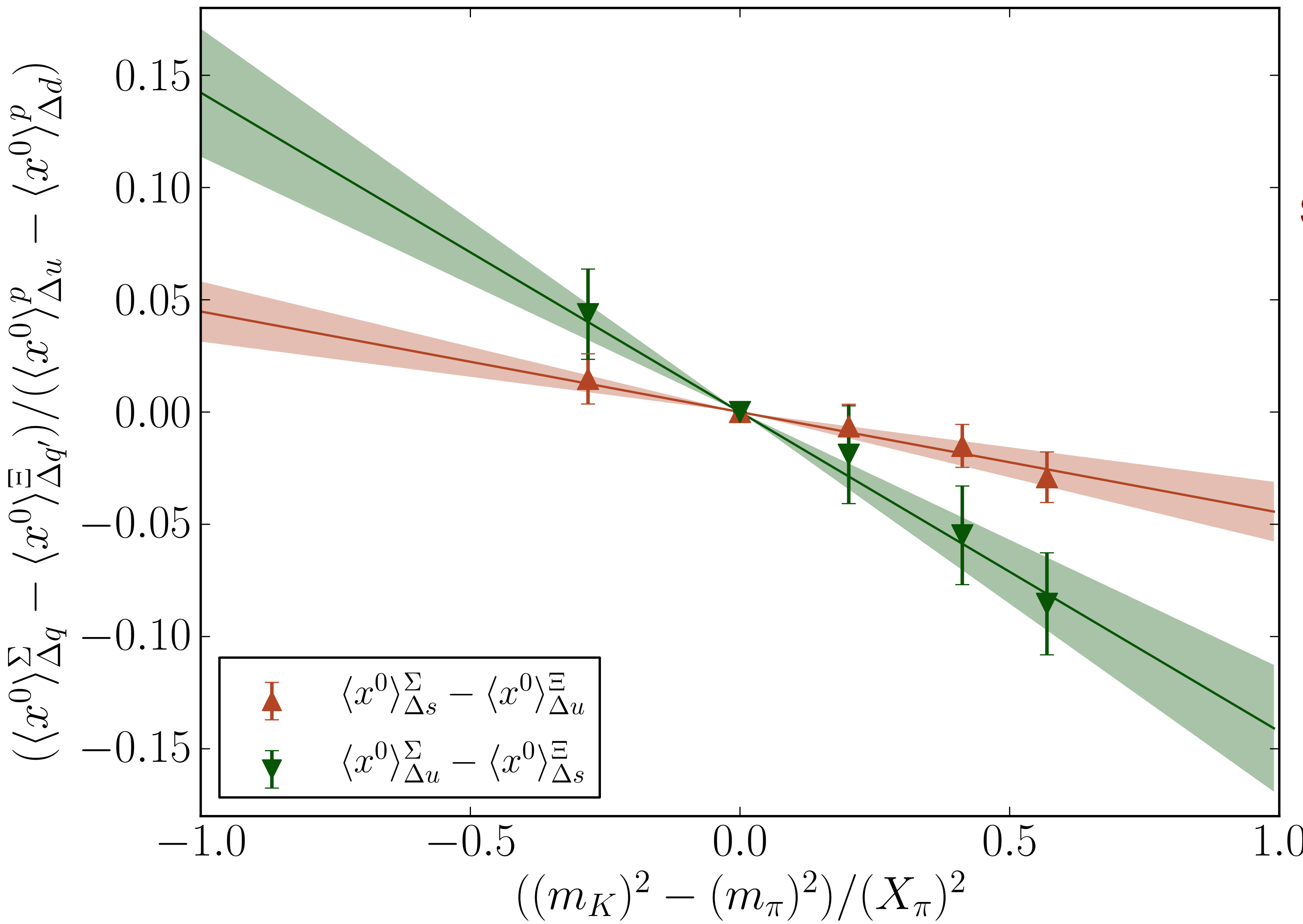
Spin-Dependent CSV

[1204.3492 (PLB)]

- Repeat procedure for

$$\delta\Delta u^m = \int_0^1 dx\ x^m[\Delta u^p(x) - \Delta d^n(x)]$$

$$\delta\Delta d^m = \int_0^1 dx\ x^m[\Delta d^p(x) - \Delta u^n(x)]$$



same sign for $\delta\Delta u$ and $\delta\Delta d$

~1% correction to Bjorken sum rule

QED Effects

- Good progress in understanding strong isospin-breaking effects
- QED effects may not be negligible and should be included
- Although in some cases, QED can be treated perturbatively, this is not always the case

  QCD+QED Lattice simulation

- Currently two main methods employed:
  - Quenched QED
  - Dynamical QED via reweighting
- Recent developments in pursuing
  - Full dynamical QED+QCD

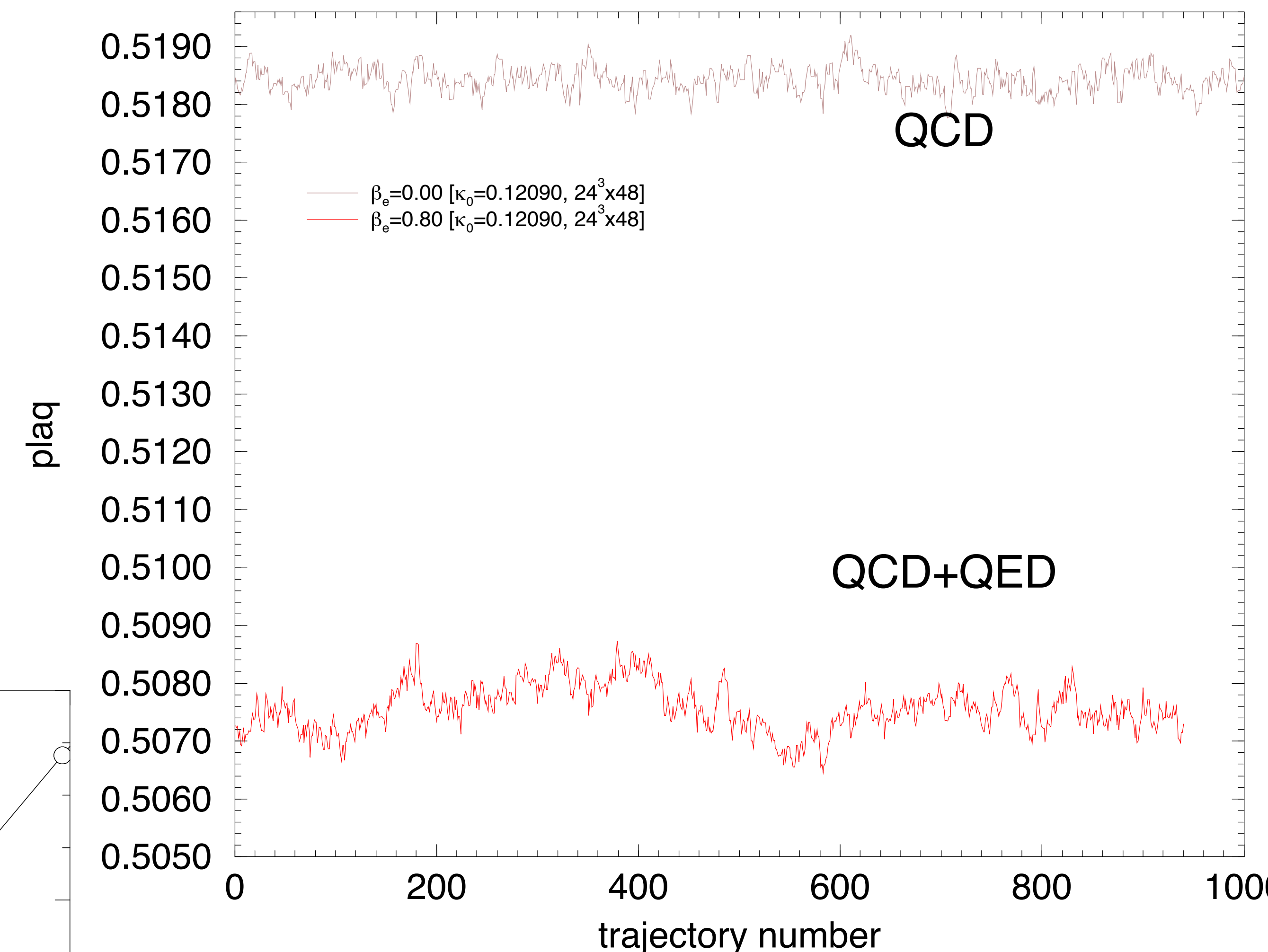
    [QCDSF, arxiv:1311.4554]

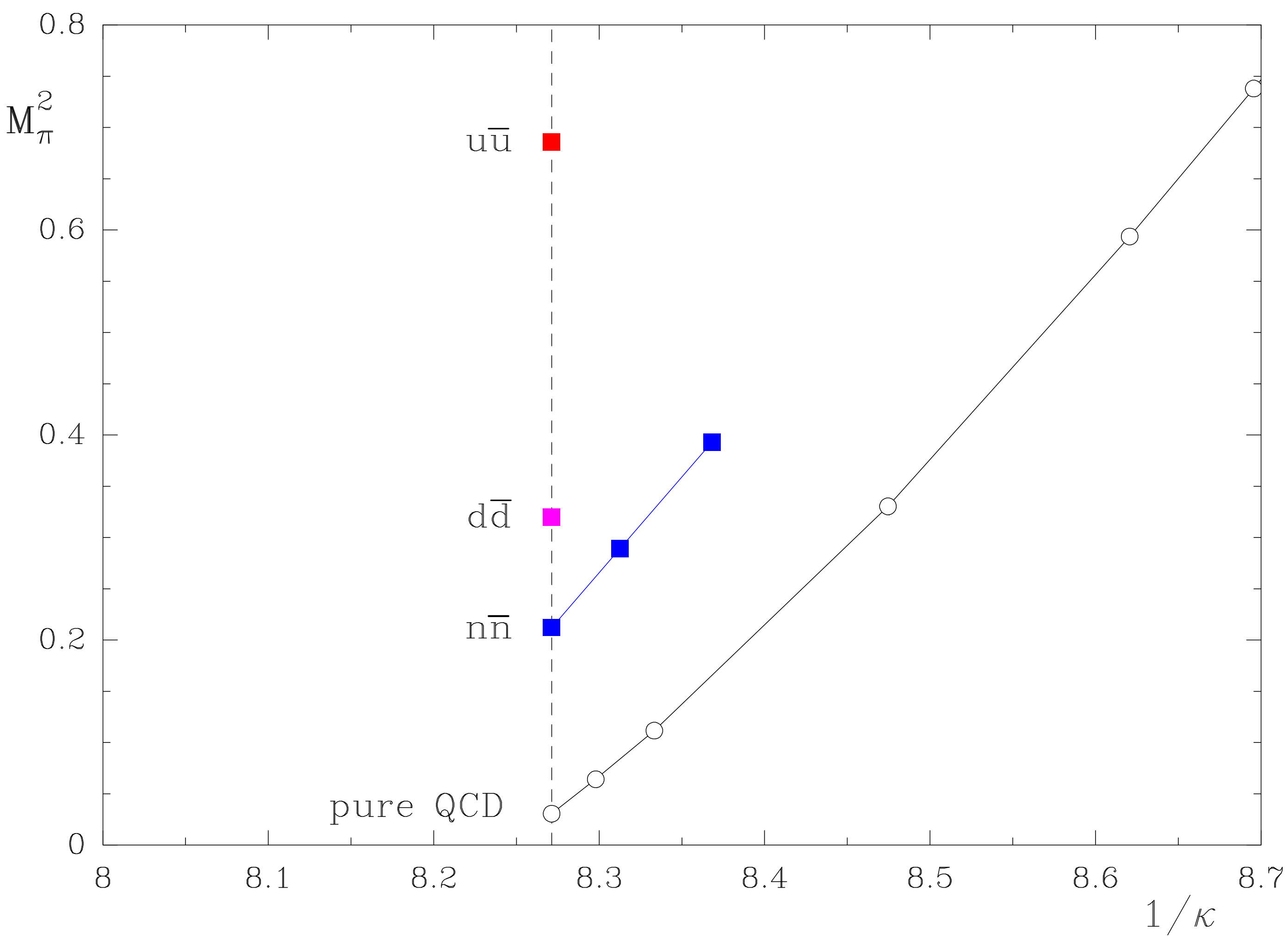
QED Effects

[QCDSF, arxiv:1311.4554]

- Simulations with dynamical QED +QCD fields now underway

- Tuning to find SU(3)-symmetric point





Summary

- Nf=2+1 simulations along $\overline{m} = \text{constant}$
  - Provide an excellent platform for investigating $SU(3)_f$ breaking effects
- Hyperon axial charges and spin content
  - $SU(3)_f$ breaking effects in quark spin contributions
  - “Spin crisis” not so severe for, e.g. Ξ
- Momentum fractions
  - Visible SU(3) breaking effects
  - Sum of connected quark contribution equal for hyperons (disconnected same)
- Charge Symmetry Violation
  - Effects becoming increasingly important for precision studies
  - Non-zero lattice result will have an impact on NuTeV, PVDIS, …

Backup

Flavour Expansions

- Step 1: S_3, SU(3) classification

$$\delta m_q = \bar{m} - m_l$$

Polynomial		S_3	$SU(3)$		
1	✓	A_1	1		
$(\overline{m} - m_0)$		A_1	1		
δm_s	✓	E^+		8	
$(\delta m_u - \delta m_d)$	✓	E^-		8	
$(\overline{m} - m_0)^2$		A_1	1		
$(\overline{m} - m_0)\delta m_s$		E^+		8	
$(\overline{m} - m_0)(\delta m_u - \delta m_d)$		E^-		8	
$\delta m_u^2 + \delta m_d^2 + \delta m_s^2$	✓	A_1	1		27
$3\delta m_s^2 - (\delta m_u - \delta m_d)^2$	✓	E^+		8	27
$\delta m_s(\delta m_d - \delta m_u)$	✓	E^-		8	27

- All the quark-mass polynomials up to $O(\delta m^3)$, classified by symmetry properties [shown here to $O(\delta m^2)$]

- A tick indicates relevant polynomials on constant mbar surface

Flavour Expansion

Gell-Man–Okubo

- Step 2: Mass hierarchy

  - Classify mass combinations according to their SU(3) representation, e.g.

$$1+1+1 \to 2+1$$

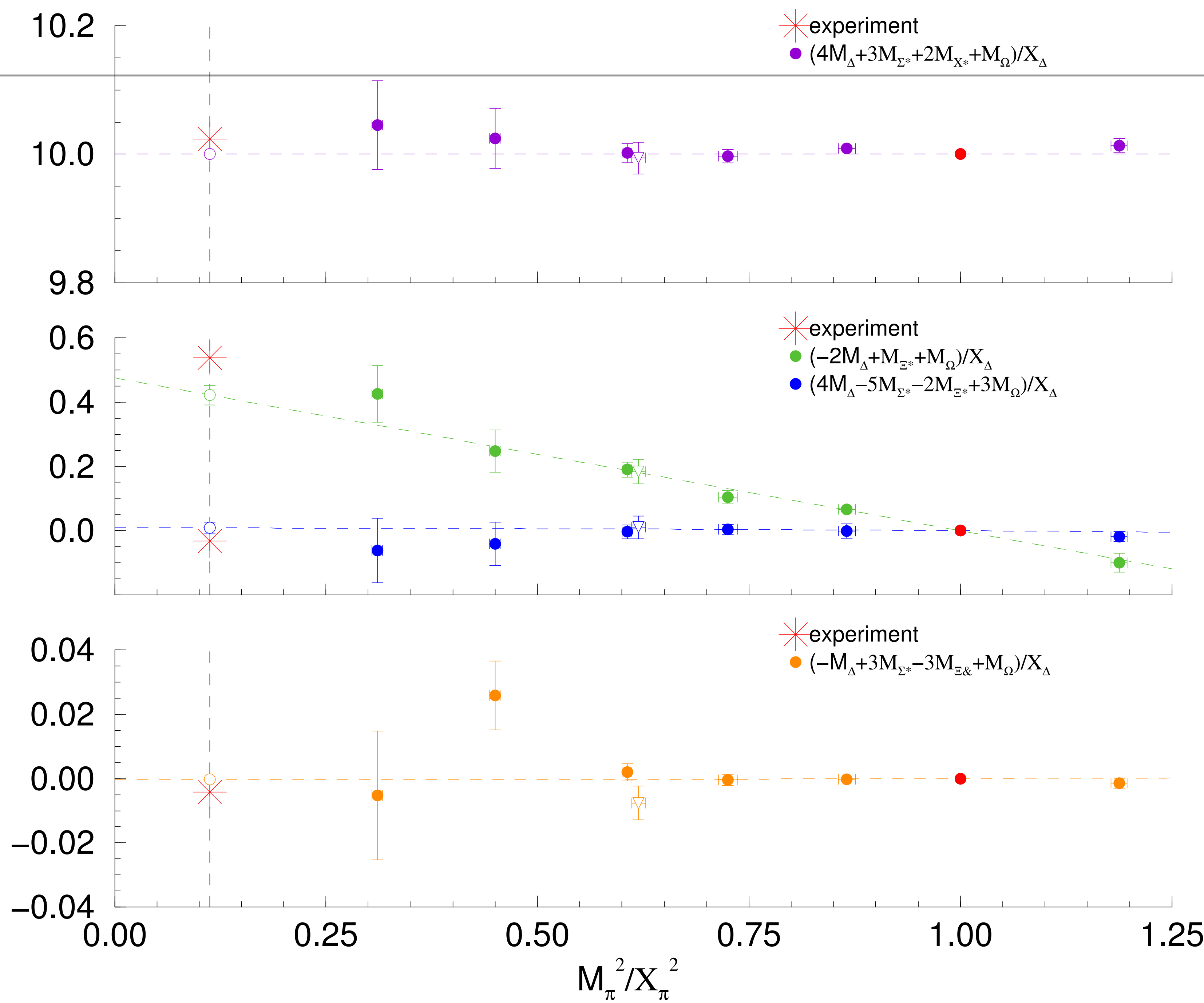
$SU(3)$	Mass Combination	Expansion					
1	$4M_\Delta + 3M_{\Sigma^*} + 2M_{\Xi^*} + M_\Omega$	1,		δm_l^2,	δm_l^3,	...	13.8 GeV
8	$-2M_\Delta + M_{\Xi^*} + M_\Omega$		δm_l,	δm_l^2,	δm_l^3,	...	0.742 GeV
27	$4M_\Delta - 5M_{\Sigma^*} - 2M_{\Xi^*} + 3M_\Omega$			δm_l^2,	δm_l^3,	...	−0.044 GeV
64	$-M_\Delta + 3M_{\Sigma^*} - 3M_{\Xi^*} + M_\Omega$				δm_l^3,	⋯	−0.006 GeV

  - Each additional factor δm gives order of magnitude reduction
    - rapidly converging Taylor expansion down to physical point

  - Invert to give flavour expansions for masses

Flavour Expansion

Gell-Man–Okubo



- Order of magnitude drop with each power of δm
- $(-2M_\Delta + M_{\Xi^*} + M_\Omega)/X_N$ dominated by linear term

Flavour Expansion

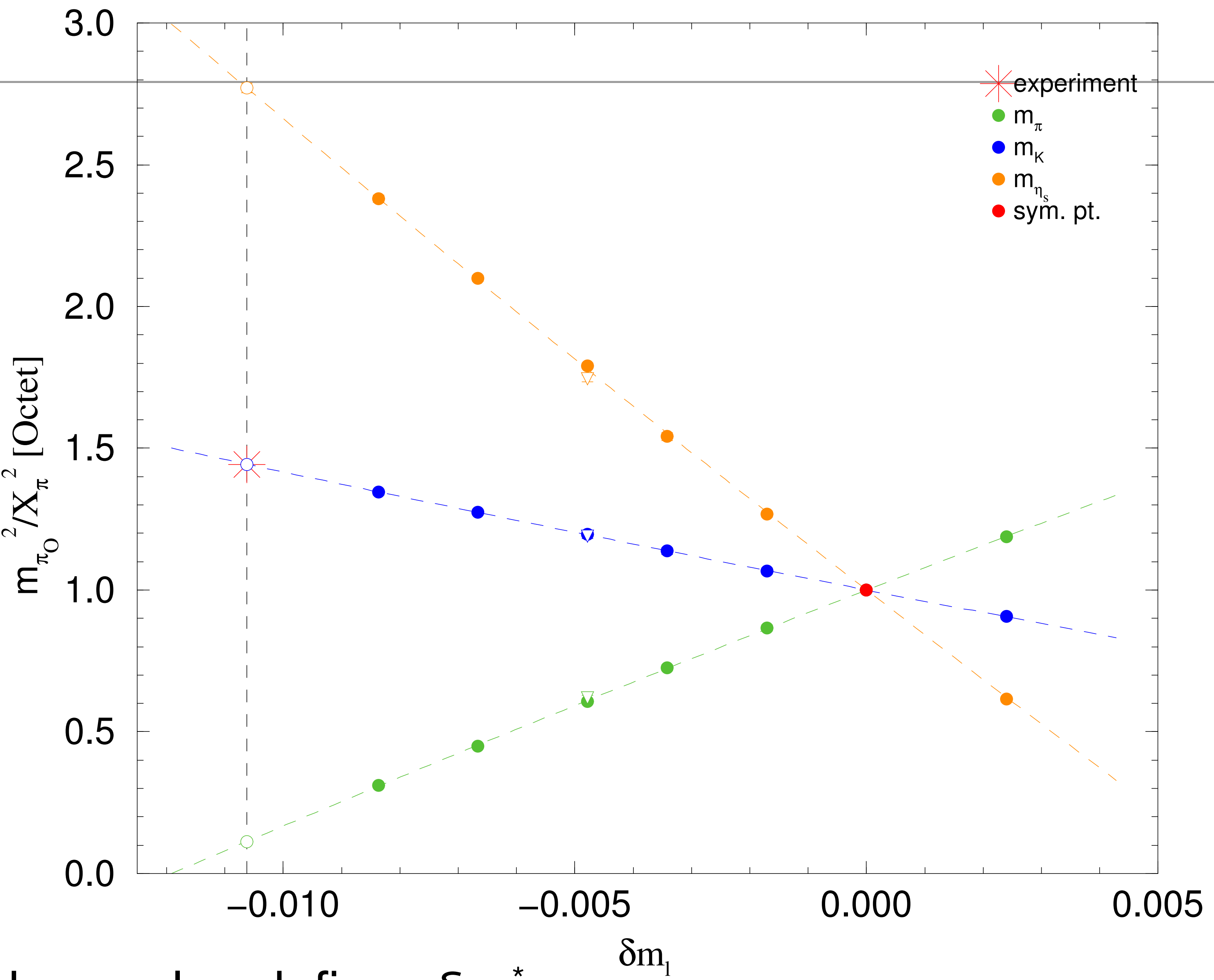
- Flavour expansion about the symmetric point (Gell-Mann–Okubo)

⇨ constrained fits for (pseudoscalar) meson octet:

$$
\begin{aligned}
M_\pi^2 &= M_0^2 + 2\alpha\,\delta m_l + (\beta_0 + 2\beta_1)\delta m_l^2 \\
M_K^2 &= M_0^2 - \alpha\,\delta m_l + (\beta_0 + 5\beta_1 + 9\beta_2)\delta m_l^2 \\
M_{\eta_s}^2 &= M_0^2 - 4\alpha\,\delta m_l + (\beta_0 + 8\beta_1)\delta m_l^2
\end{aligned}
$$

- Linear terms: 1 coefficient, Quadratic terms: 3 coefficients
- M_{η_s} : fictitious $s\bar{s}$ particle - but useful in a constrained fit
- Similar expansions for decay constants, f_π, f_K, ...

Pseudoscalar Meson Octet ‘fan plot’



- M_π^2/X_π^2 = phys. value defines δm^*
- Finite size effects cancel in ratio
- Quadratic terms appear to be small, $\beta_i \sim 0$

Flavour Expansion

- Flavour expansion about the symmetric point (Gell-Mann–Okubo)

constrained fits for baryon Octet:

$$
\begin{aligned}
M_N &= M_0 + 3A_1\,\delta m_l + (B_0 + 3B_1)\delta m_l^2 \\
M_\Lambda &= M_0 + 3A_2\,\delta m_l + (B_0 + 6B_1 - 3B_2 + 9B_4)\delta m_l^2 \\
M_\Sigma &= M_0 - 3A_2\,\delta m_l + (B_0 + 6B_1 + 3B_2 + 9B_3)\delta m_l^2 \\
M_\Xi &= M_0 - 3(A_1 - A_2)\,\delta m_l + (B_0 + 9B_1 - 3B_2 + 9B_3)\delta m_l^2
\end{aligned}
$$

- Decuplet:

$$
\begin{aligned}
M_\Delta &= M_0 + 3A\,\delta m_l + (B_0 + 3B_1)\delta m_l^2 \\
M_{\Sigma^*} &= M_0 + (B_0 + 6B_1 + 9B_2)\delta m_l^2 \\
M_{\Xi^*} &= M_0 - 3A\,\delta m_l + (B_0 + 9B_1 + 9B_2)\delta m_l^2 \\
M_\Omega &= M_0 - 6A\,\delta m_l + (B_0 + 12B_1)\delta m_l^2
\end{aligned}
$$

- Linear terms: Octet 2 coefficients, Decuplet 1 coefficient

Advantages (or maybe just interesting observations)

- $\overline{m}^R$ = const. means that as we extrapolate

$$m_l^R \searrow m_l^{R*} \text{ and } m_s^R \nearrow m_s^{R*}$$

$$\text{ie } m_\pi \searrow m_\pi^* \text{ and } m_K \nearrow m_K^*$$

  - so m_K is never heavier than its physical value

- Flavour singlet quantities (eg **r_0**) flat at symmetric point:
  - If X_S is a flavour singlet at the SU(3) symmetric point, then

$$\frac{\partial X_S}{\partial m_u} = \frac{\partial X_S}{\partial m_d} = \frac{\partial X_S}{\partial m_s}$$

  - and along our trajectory $dm_s = -dm_u - dm_d = -2dm_l$

$$dX_S = dm_u \frac{\partial X_S}{\partial m_u} + dm_d \frac{\partial X_S}{\partial m_d} + dm_s \frac{\partial X_S}{\partial m_s} = 0$$

Examples of Flavour Singlets

- Flavour singlet quantities flat at symmetric point and so will be closer to their extrapolated values at the physical point → useful as a scale

- Singlet quantities:
  - Octet baryons: (centre of mass)

$$X_N = \frac{1}{3}(m_N + m_\Sigma + m_\Xi) = 1.150\,\mathrm{GeV}$$

  - Decuplet baryons (centre of mass)

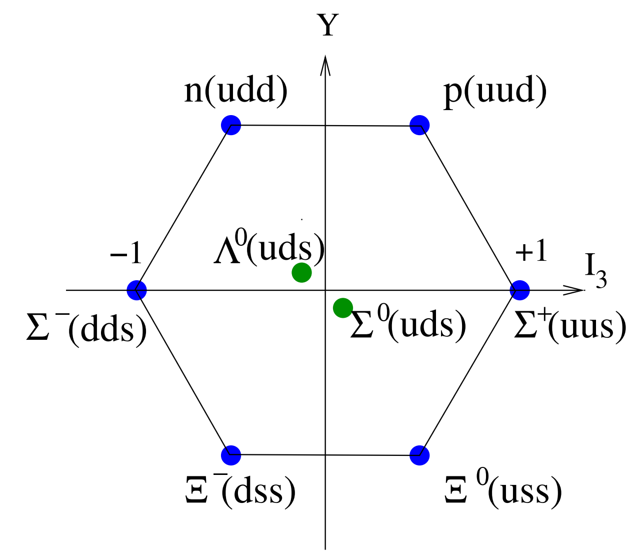
$$X_\Delta = \frac{1}{3}(2m_\Delta + m_\Omega) = 1.379\,\mathrm{GeV}$$

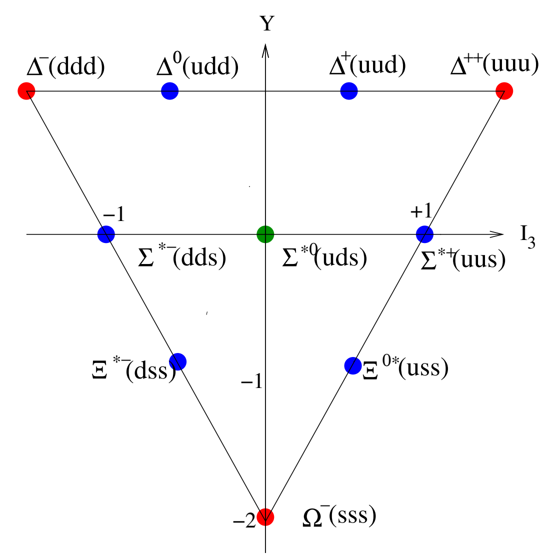
  - Gluonic:

$$X_r = \frac{1}{r_0} \qquad [r_0 = 0.5\,\mathrm{fm}?]$$

  - Others:

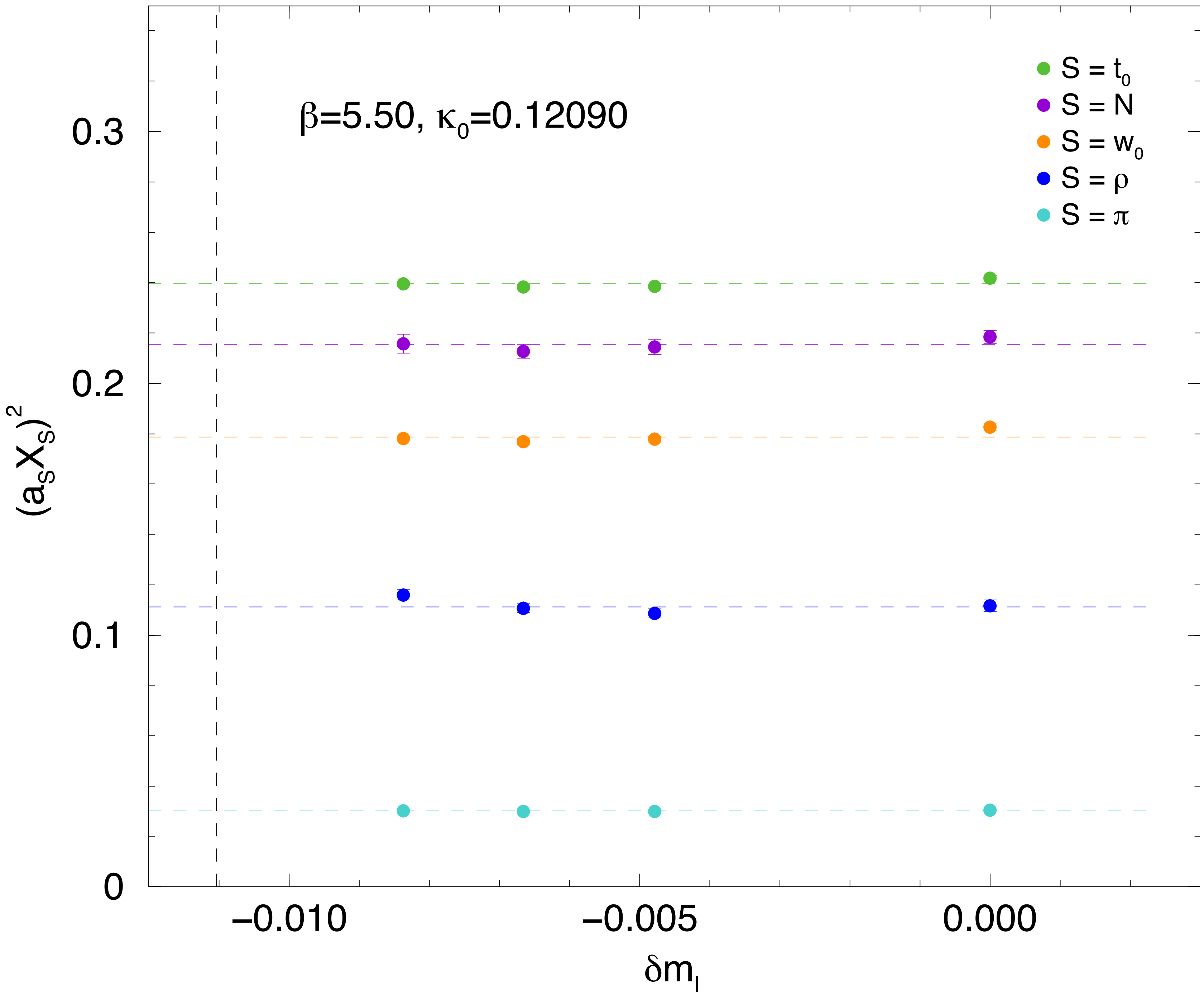
$$X_S = \begin{cases} \frac{1}{2}(m_\Sigma + m_\Lambda) \\ m_{\Sigma^*},\ \frac{1}{2}(m_\Delta + m_{\Xi^*}) \\ \sqrt{\frac{1}{3}(2m_K^2 + m_\pi^2)} & X_\pi \\ \frac{1}{3}(2m_{K^*} + m_\rho) & X_\rho \end{cases}$$





Singlets & Scale

[arXiv:1102.5300, 1311.5010]



Flat

a=0.074(3) fm