$N(1400)$, $N(1535)$ and Pentaquark States

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Outline

- Constituent Quark Models
- Estimation of Ground State $q^4\bar{q}$ Messes
- Construction of $q^4\bar{q}$ States

Two Main Constituent Quark Models

- Hamiltonian for a N-quark system:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \frac{1}{2} C \sum_{i<j}^{N} (\vec{r}_i - \vec{r}_j)^2 + \sum_{i=1}^{N} m_i + H_{hyp} \tag{1}$$

- One-gluon-exchange H_{hyp}:

$$H_{hyp}^{OGE} = -C_G \sum_{i<j} \lambda_i^C \cdot \lambda_j^C \, \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{2}$$

- Goldstone-boson-exchange H_{hyp}:

$$H_{hyp}^{GBE} = -C_M \sum_{i<j} \lambda_i^F \cdot \lambda_j^F \, \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{3}$$

OGE Model to q^3 Baryons

- The $N(938)$ and $\Delta(1232)$ masses are estimated,

$$M_\Delta = 3m_q + \Delta m(\Delta) = 3m_q + 8C_G,$$
$$M_N = 3m_q + \Delta m(N) = 3m_q - 8C_G$$

- Let $M_N = 939$ MeV and $M_\Delta = 1232$ MeV, one gets,

$$m_q \approx 360 \text{ MeV}, \quad C_G \approx 18 \text{ MeV}$$

- The model reproduces well the mass of the ground state octet and decuplet baryons but fails to give a right order of $N(1440)$, $N(1520)$ and $N(1535)$.

Here we have used

$$\Delta m(q^3) = \langle \psi(q^3) | H_{hyp}^{OGE}(q^3) | \psi(q^3) \rangle,$$
$$\langle \psi(q^3)_{color} | \lambda_i^C \cdot \lambda_j^C | \psi(q^3)_{color} \rangle = -8/3,$$
$$\sum_{i<j} \langle \psi(q^3)_{spin} | \vec{\sigma}_i \cdot \vec{\sigma}_j | \psi(q^3)_{spin} \rangle = 3\ (-3),\ \text{for } S = 3/2\ (1/2)$$

GBE Model

- The $N(938)$ and $\Delta(1232)$ masses are estimated,

$$M_\Delta = 3m_q + \Delta m(\Delta) = 3m_q - 4C_M,$$
$$M_N = 3m_q + \Delta m(N) = 3m_q - 14C_M$$

- Fitting to $M_N = 939$ MeV and $M_\Delta = 1232$ MeV leads to,

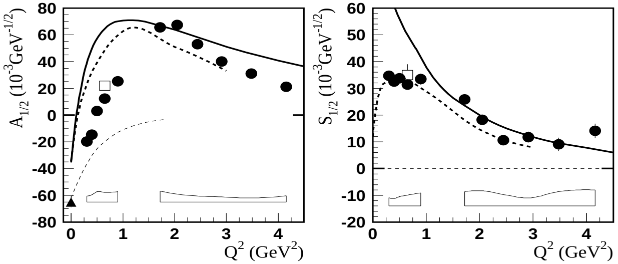
$$m_q \approx 450 \text{ MeV}, \quad C_M \approx 30 \text{ MeV}$$

- The model can bring the first excited $(L = 1)$ band of pentaquarks below the ground state $(L = 0)$ band;
- The first excited state $(L = 1)$ of the symmetry $[4]_{FS}[22]_F[22]_S$ is the lowest pentaquark, $M \sim 1400$ MeV;
- It is claimed that the $[4]_{FS}[22]_F[22]_S$ pentaquark state may be the Roper resonance.

$N_{1/2+}(1440)$

- In the traditional q^3 picture, the Roper $N_{1/2+}(1440)$ usually gets a mass ~ 100 MeV above the $N_{1/2-}(1535)$, but not 100 MeV below it.
- Roper resonance is usually blamed sitting at a wrong place or intruding the q^3 spectrum.
- It has been studied in any possible picture: normal q^3 first radial excitation, $q^4\bar{q}$ pentaquark, q^3g hybrid, $q^3(q\bar{q})$ resonance...
- Still an open question.

Helicity amplitudes for the $\gamma^* p \to N(1440)$ transition. The thick curves correspond to quark models assuming that $N(1440)$ is a q^3 first radial excitation: dashed (Capstick and Keister, 1995), solid (Aznauryan, 2007). The thin dashed curves are obtained assuming that $N(1440)$ is a $q^3 g$ hybrid state (Li et al., 1992). Figure courtesy to Rev. Mod. Phys. **82**, 1095.



- The sign change in the helicity amplitude as a function of Q^2 suggests a node in the wave function and thus a radially excited state.

$N_{1/2-}(1535)$

- This resonance is observed at a mass expected in quark models
- Large couplings to the $N\eta$, $N\eta'$, $N\phi$ and $K\Lambda$ but small couplings to the $N\pi$ and $K\Sigma$ are claimed.
- A large $N\eta$ coupling invites speculation that it might be created dynamically as $N\eta - \Sigma K$ coupled channel effect.
- A large $N\phi$ coupling leads to the proposal that the $N_{1/2-}(1535)$ may have a large component of $uuds\bar{s}$ pentaquark states.

Estimations of ground state pentaquark masses in OGE

Configurations	Spin (or J)	$\Delta m(q^4\overline{q})$	$M(q^4\overline{q})$ (MeV)
$\Psi^{sf}_{[31]_{FS}[211]_F[22]_S}$	$\frac{1}{2}$	-288	1760
$\Psi^{sf}_{[31]_{FS}[211]_F[31]_S}$	$\frac{1}{2}$, $\frac{3}{2}$	-480, -120	1570, 1930
$\Psi^{sf}_{[31]_{FS}[22]_F[31]_S}$	$\frac{1}{2}$, $\frac{3}{2}$	-336, 24	1710, 2070
$\Psi^{sf}_{[31]_{FS}[31]_F[22]_S}$	$\frac{1}{2}$	-48	2000
$\Psi^{sf}_{[31]_{FS}[31]_F[31]_S}$	$\frac{1}{2}$, $\frac{3}{2}$	64, 24	2110, 2070
$\Psi^{sf}_{[31]_{FS}[31]_F[4]_S}$	$\frac{3}{2}$, $\frac{5}{2}$	48, 240	2100, 2290
$\Psi^{sf}_{[31]_{FS}[4]_F[31]_S}$	$\frac{1}{2}$, $\frac{3}{2}$	528, 240	2580, 2290

Remarks

- The OGE model can give a $J = 1/2$, negative parity pentaquark with a mass ~ 1500 MeV.
- The GBE model can give a $J = 1/2$, positive parity pentaquark with a mass ~ 1440 MeV.
- How about OGE plus GBE interactions
- How to accommodate/ignore other pentaquark states of a large number.

Our Works

- OGE preliminary calculations indicate that $N(1440)$ might be mainly the q^3 first radial excitation, $N(1535)$ with a large $q^4\bar{q}$ component, and $N(1520)$ with a sizeable $q^4\bar{q}$ component.

- We aim at studying the whole baryon mass spectrum in steps, assuming that baryons consist of the q^3 as well as $q^4\bar{q}$ pentaquark components.

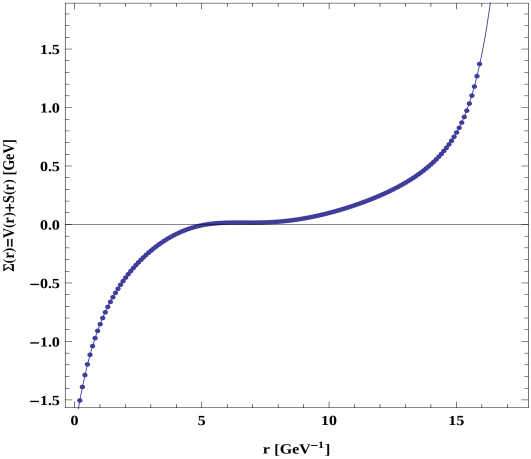
  1. Construct pentaquark wave functions to high-order excitations.

  2. Study baryon spectra with the Gaussian-form confinement potential plus OGE and/or GBE interactions

  3. Study baryon spectra with more realistic confinement potentials plus OGE and/or GBE interactions

A Potential

A qq interaction extracted by fitting the theoretical results in the chiral perturbative quark model to experimental data of electromagnetic and axial form factors of octet baryons



- That the pentaquark should be a color singlet demands that the color part of the pentaquark wave function must be a $[222]_1$ singlet.

$$\psi^c_{[222]}(q^4\bar{q}) = \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline & \\ \hline \end{array} \tag{5}$$

- The color part of the antiquark in pentaquark states is a $[11]_3$ antitriplet

$$\psi^c_{[11]}(\bar{q}) = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \tag{6}$$

- The color wave function of the four-quark configuration must be a $[211]_3$ triplet

$$\psi^c_{[211]_\lambda}(q^4) = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \psi^c_{[211]_\rho}(q^4) = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \psi^c_{[211]_\eta}(q^4) = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} \end{array} \tag{7}$$

q^4 Color Wave Functions

q^4 color wave functions can be derived by applying the λ-, ρ- and η-type projection operators of the S_4 $IR[211]$ in Yamanouchi basis,

$$P_{[211]_\lambda}(RRGB) \Longrightarrow \psi^c_{[211]_\lambda}(R)$$
$$P_{[211]_\rho}(RGRB) \Longrightarrow \psi^c_{[211]_\rho}(R)$$
$$P_{[211]_\eta}(BRGB) \Longrightarrow \psi^c_{[211]_\eta}(R)$$

$$\psi^c_{[211]_\eta}(R) = \frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle - |RBGR\rangle - |GRBR\rangle - |BGRR\rangle)$$

The singlet color wave function $\Psi^c_{[211]_j}$ $(j = \lambda, \rho, \eta)$ of pentaquarks is given by

$$\Psi^c_{[211]_j} = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\,\bar{R} + \psi^c_{[211]_j}(G)\,\bar{G} + \psi^c_{[211]_j}(B)\,\bar{B}\right]. \qquad (8)$$

- To form a fully antisymmetric q^4 wave function, the spatial-spin-flavor part must be $[31]$.

- The total wave function of q^4 systems may be written in the general form,

$$\psi = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\ \psi^{c}_{[211]_i}\psi^{\text{osf}}_{[31]_j} \tag{9}$$

- The coefficients of the antisymmetric ψ can be determined by applying the Yamanouchi-basis representations of the S_4 to the general form. One gets

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{10}$$

- In the same way we can write

$$\Psi^{osf}_{[31]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij}\Psi^{o}_{[X]_i}\Psi^{sf}_{[Y]_j} \quad (11)$$

$$\Psi^{sf}_{[Z]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij}\Phi^{f}_{[X]_i}\chi^{s}_{[Y]_j} \quad (12)$$

- The possible spatial-spin-flavor and spin-flavor configurations and explicit forms of the wave functions are determined by applying the S_4 representations in Yamanouchi basis.
- Spatial-spin-flavor configurations:

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}, [211]_{SF}$
$[211]_O$	$[31]_{SF}, [211]_{SF}, [22]_{SF}$
$[31]_O$	$[4]_{SF}, [31]_{SF}, [211]_{SF}, [22]_{SF}$

q^4 Spin-Flavor States

	$[4]_{FS}$		
$[4]_{FS}[22]_F[22]_S$	$[4]_{FS}[31]_F[31]_S$	$[4]_{FS}[4]_F[4]_S$	
	$[31]_{FS}$		
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	
	$[22]_{FS}$		
$[22]_{FS}[22]_F[22]_S$	$[22]_{FS}[22]_F[4]_S$	$[22]_{FS}[4]_F[22]_S$	$[22]_{FS}[211]_F[31]_S$
$[22]_{FS}[31]_F[31]_S$			
	$[211]_{FS}$		
$[211]_{FS}[211]_F[22]_S$	$[211]_{FS}[211]_F[31]_S$	$[211]_{FS}[211]_F[4]_S$	$[211]_{FS}[22]_F[31]_S$
$[211]_{FS}[31]_F[22]_S$	$[211]_{FS}[31]_F[31]_S$		

Spin-Flavor Wave Functions

- Taking $[31]_{FS}[22]_F[31]_S$ configuration as an example, the spin-flavor wave functions of various permutation symmetries take a general form,

$$\psi^{\rm sf} = \sum_{i=\lambda,\rho} \sum_{j=\lambda,\rho,\eta} a_{ij}\, \phi_{[22]_i} \chi_{[31]_j}$$

- a_{ij} can be determined by acting the permutations of S_4 on the general form. The spin-flavor wave functions for the q^4 cluster are derived as,

$$\begin{aligned}
\psi^{\rm sf}_{[31]_\rho} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\lambda} - \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\eta} \\
\psi^{\rm sf}_{[31]_\lambda} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\lambda} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\eta} \\
\psi^{\rm sf}_{[31]_\eta} &= \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\lambda}
\end{aligned}$$

Flavor Wave Functions

- Operate $P_{\lambda,\rho}$ on any q^4 flavor state, for example, $(uudd)$, the flavor wave functions are derived,

$$P_\rho(udud) \Longrightarrow \phi_{[22]_\rho} = \frac{1}{2}(dudu - duud + udud - uddu) \tag{13}$$

$$P_\lambda(uudd) \Longrightarrow \phi_{[22]_\lambda} = \frac{1}{2\sqrt{3}}(duud + udud - 2uudd + uddu + dudu - 2dduu) \tag{14}$$

- With $I = I_3 = 0$

$$\Phi_{[22]_\rho} = \phi_{[22]_\rho}\bar{s}, \quad \Phi_{[22]_\lambda} = \phi_{[22]_\lambda}\bar{s}. \tag{15}$$

- The flavor states with other values of isospin $(I,\, I_3)$ and strangeness S can be derived the same way.

Spin Wave Functions

- Operate $P_{\lambda,\rho,\eta}$ on any q^4 spin state, for example, the state spin=1

$$P_{[31]_\eta}(\uparrow\uparrow\uparrow\downarrow) \implies \chi_{[31]_\eta}(1,1) = \frac{1}{2\sqrt{3}} \mid 3 \uparrow\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\uparrow \rangle$$

$$P_{[31]_\rho}(\uparrow\downarrow\uparrow\uparrow) \implies \chi_{[31]_\rho}(1,1) = \frac{1}{\sqrt{2}} \mid \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow \rangle$$

$$P_{[31]_\lambda}(\uparrow\uparrow\downarrow\uparrow) \implies \chi_{[31]_\lambda}(1,1) = \frac{1}{\sqrt{6}} \mid 2 \uparrow\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow \rangle$$

- Combine the spin wave functions above with the one of the antiquark, the total spin wave function of the pentaquark states is

$$\chi(q^4\bar{s})_{[31]_\alpha} = \sqrt{\frac{2}{3}}\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=1)\chi_{\bar{s}}(-1/2) - \sqrt{\frac{1}{3}}\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=0)\chi_{\bar{s}}(1/2) \quad (16)$$

- with $\alpha = \rho,\ \lambda,\ \eta$. The states with other values of the projection m_s can be obtained the same way.

Spatial Wave Functions

A complete basis of certain symmetry may be constructed with pentaquark systems in the harmonic oscillator interaction,

$$H = \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m} + \frac{1}{2}C\left(\lambda^2 + \rho^2 + \eta^2 + \xi^2\right) \tag{17}$$

where

$$\begin{aligned} \vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \\ \vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \\ \vec{\eta} &= \frac{1}{\sqrt{12}}(\vec{r_1} + \vec{r_2} + \vec{r_3} - 3\vec{r_4}) \\ \vec{\xi} &= \frac{1}{\sqrt{20}}(\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} - 4\vec{r_5}) \end{aligned} \tag{18}$$

Spatial Wave Functions

Spatial wave functions take the general form,

$$\Psi^{\mathrm{o}}_{NLM} = \sum_{n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi} A(n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi)$$
$$\cdot\Psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda})\Psi_{n_\rho l_\rho m_\rho}(\vec{\rho})\Psi_{n_\eta l_\eta m_\eta}(\vec{\eta})\Psi_{n_\xi l_\xi m_\xi}(\vec{\xi})$$
$$\cdot C(l_\lambda,l_\rho,m_\lambda,m_\rho,l_{\lambda\rho},m_{\lambda\rho})$$
$$\cdot C(l_{\lambda\rho},l_\eta,m_{\lambda\rho},m_\eta,l_{\lambda\rho\eta},m_{\lambda\rho\eta})$$
$$\cdot C(l_{\lambda\rho\eta},l_\xi,m_{\lambda\rho\eta},m_\xi,LM) \qquad (19)$$

with $N = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi$

- The coefficients A are determined by applying the Yamanouchi basis representations of the S_4. Various types of spatial wave functions with the $[4], [31], [22], [211]$ and $[1111]$ symmetries are easily worked out.

Pentaquark Spatial Wave Function, $NLM = 322$

- Symmetric:

$$\Psi^S = \frac{1}{3}\left[-\Psi_{021}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{010}(\xi)\right.$$
$$-\Psi_{000}(\lambda)\Psi_{021}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{010}(\xi)$$
$$\left.-\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{021}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{010}(\xi)\right]$$

- Antisymmetric: Non
- λ and ρ types of [22]: Non
- λ, ρ and η types of [211]: Non

Pentaquark Spatial Wave Function, $NLM = 322$

First Set of λ, ρ and η types of [31]:

$$\Psi^{\lambda[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{010}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{011}(\lambda)\Psi_{021}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \sqrt{2}\Psi_{010}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) - \Psi_{011}(\lambda)\Psi_{000}(\rho)\Psi_{021}(\eta)\Psi_{000}(\xi)]$$

$$\Psi^{\rho[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{022}(\lambda)\Psi_{010}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{021}(\lambda)\Psi_{011}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{010}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) - \Psi_{000}(\lambda)\Psi_{011}(\rho)\Psi_{021}(\eta)\Psi_{000}(\xi)]$$

$$\Psi^{\eta[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{010}(\eta)\Psi_{000}(\xi) - \Psi_{021}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{000}(\xi)]$$

Pentaquark Spatial Wave Function, $NLM = 322$

Second Set of λ, ρ and η types of [31]:

$$\Psi^{\lambda[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{010}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) - \Psi_{011}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{021}(\xi)]$$

$$\Psi^{\rho[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{000}(\lambda)\Psi_{010}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) - \Psi_{000}(\lambda)\Psi_{011}(\rho)\Psi_{000}(\eta)\Psi_{021}(\xi)]$$

$$\Psi^{\eta[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{010}(\eta)\Psi_{022}(\xi) - \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{021}(\xi)]$$

Pentaquark Spatial Wave Function, Symmetric $NLM = 444$

$$
\begin{aligned}
\Psi^{S_1} &= \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{044}(\xi) \\
\Psi^{S_2} &= \frac{1}{\sqrt{3}}[\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) + \Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) \\
&\quad +\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{022}(\xi)] \\
\Psi^{S_3} &= \sqrt{\frac{1}{17}}[\Psi_{033}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) - \sqrt{7}\Psi_{011}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) \\
&\quad +\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{033}(\eta)\Psi_{011}(\xi) - \sqrt{\frac{7}{2}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{011}(\xi) \\
&\quad -\sqrt{\frac{7}{2}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{011}(\eta)\Psi_{011}(\xi)] \\
\Psi^{S_4} &= \sqrt{\frac{5}{57}}[\Psi_{044}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{044}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&\quad +\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{044}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&\quad +\sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi)]
\end{aligned}
$$

Thank you for your patience!

Pentaquark Spatial Wave Function, Symmetric $NLM = 444$

$$\begin{aligned}
\Psi^{S_4} &= \sqrt{\frac{5}{57}}[\Psi_{044}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{044}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{044}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&+\sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi)]
\end{aligned}$$

$$\begin{aligned}
\Psi^{S_5} &= \sqrt{\frac{120}{1217}}[\Psi_{033}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{000}(\xi) - \sqrt{7}\Psi_{011}(\lambda)\Psi_{022}(\rho)\Psi_{011}(\eta)\Psi_{000}(\xi) \\
&-\frac{1}{2\sqrt{6}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{044}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{21}{20}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) \\
&+\sqrt{\frac{21}{20}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi)]
\end{aligned}$$

Applying the permutation (34) of S_4, we have

$$
\begin{aligned}
(34)\psi &= -a_{\lambda\rho}\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} \\
&+ a_{\rho\lambda}\left(-\frac{1}{3}\psi^{c}_{[211]_\rho}+\frac{2\sqrt{2}}{3}\psi^{c}_{[211]_\eta}\right)\left(\frac{1}{3}\psi^{osf}_{[31]_\lambda}+\frac{2\sqrt{2}}{3}\psi^{osf}_{[31]_\eta}\right) \\
&+ a_{\eta\eta}\left(\frac{2\sqrt{2}}{3}\psi^{c}_{[211]_\rho}+\frac{1}{3}\psi^{c}_{[211]_\eta}\right)\left(\frac{2\sqrt{2}}{3}\psi^{osf}_{[31]_\lambda}-\frac{1}{3}\psi^{osf}_{[31]_\eta}\right). \qquad (21)
\end{aligned}
$$

An antisymmetric ψ demands

$$a_{\rho\lambda} = -a_{\eta\eta}$$

Here we have used the $[31]$ and $[211]$ representation matrices for the permutation (34) of the S_4,

$$D^{[31]}(34) = \begin{pmatrix} 1/3 & 0 & 2\sqrt{2}/3 \\ 0 & 1 & 0 \\ 2\sqrt{2}/3 & 0 & -1/3 \end{pmatrix}, \; D^{[211]}(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/3 & 2\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{pmatrix}$$

q^4 Wave Function

- Applying the permutation (12) or (23) of the S_4 leads to

$$a_{\rho\lambda} = -a_{\lambda\rho} \tag{22}$$

- Finally, we derive a fully antisymmetric wave function for the q^4 configuration

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{23}$$

- In general we can write

$$\Psi^{osf}_{[31]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij}\Psi^{o}_{[X]_i}\Psi^{sf}_{[Y]_j} \tag{24}$$

- The possible configurations of spatial-spin-flavor wave functions are shown in the following table:

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}, [211]_{SF}$