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NUCLEONIC SYSTEMS AND IN-MEDIUM NUCLEONS

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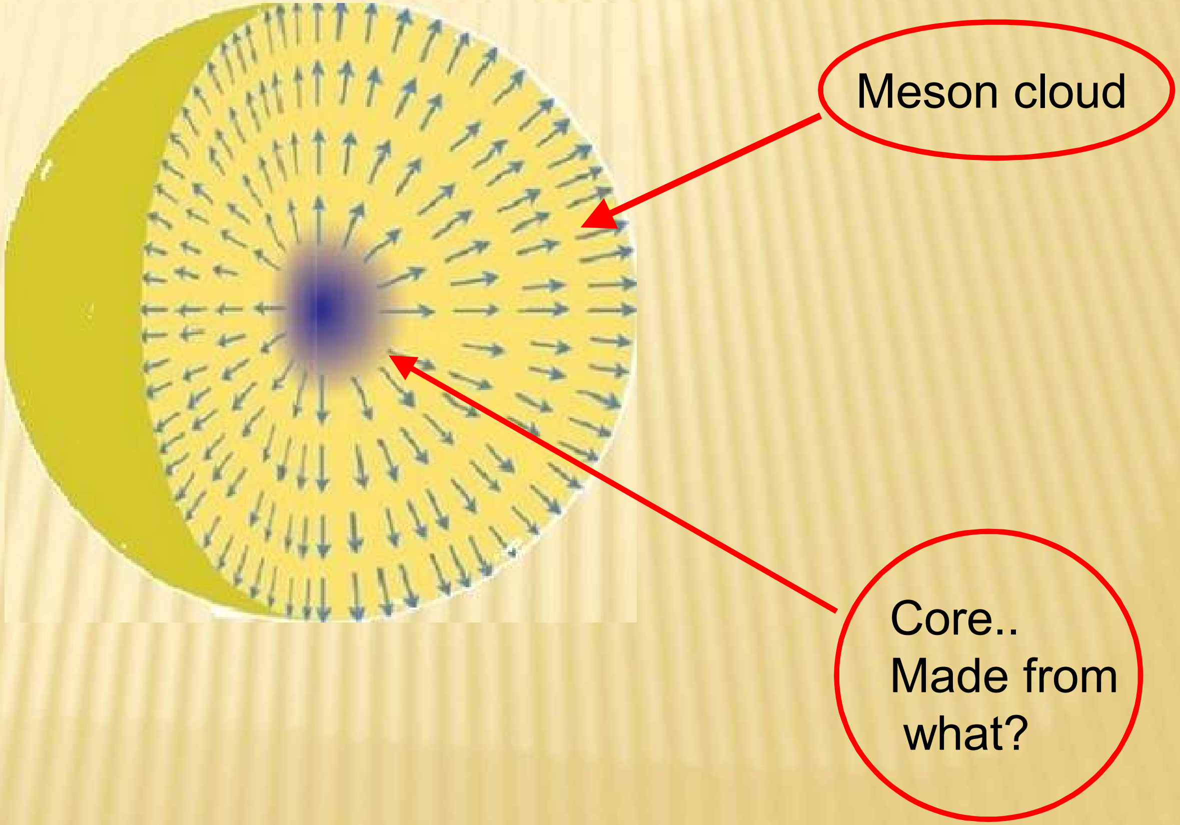
CONTENT

- Topological models and soliton
- Medium modifications
  - “Outer shell” modifications
  - “Inner core” modifications
- Nuclear matter
  - Symmetric matter
  - Asymmetric matter
- Hadron properties in nuclear matter
- Finite nuclei
- Summary
- Outlook

TOPOLOGICAL MODELS AND SOLITON

Structure

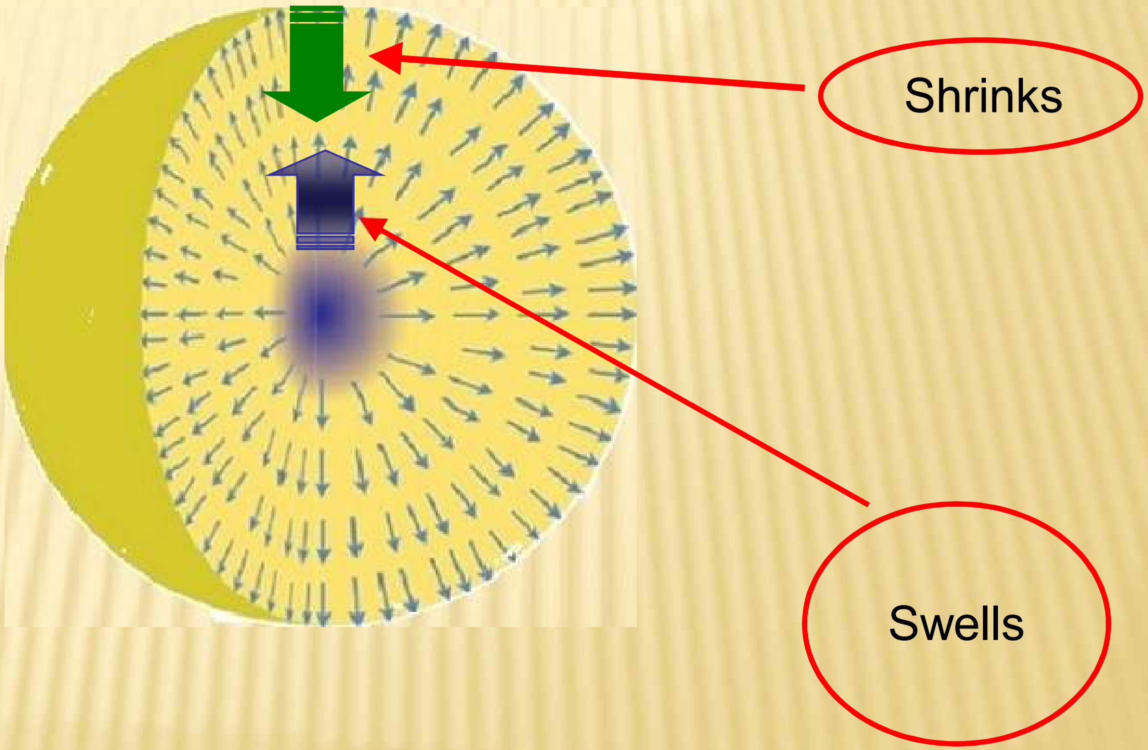
- What is a nucleon and, in particular, its core?
- Core treatment depends from the energy scale
- At large number of colors it still has the mesonic content



TOPOLOGICAL MODELS AND SOLITON

Stabilization

- Soliton has a finite size and finite energy
- One needs at least two contrterms in the effective Lagrangian



TOPOLOGICAL MODELS AND SOLITON

Skyrme model

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260(1961)]

- Nonlinear chiral effective meson (pionic) theory)

$$\mathcal{L} = \frac{F_\pi^2}{16}\mathrm{Tr}\left(\partial_\alpha U\right)\left(\partial^\alpha U^+\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2$$

Shrinks ← → Swells

- Hedgehog soliton (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\bar{n}F(r)\}$$

TOPOLOGICAL MODELS AND SOLITON

Original Lagrangian in use

[G.S. Adkins *et al.* Nucl. Phys. B228 (1983)]

$$\mathcal{L}_{\text{free}} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial^\alpha U\right)\left(\partial_\alpha U^+\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}\left(U + U^+ - 2\right)$$

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

Nontrivial mapping

- It has topologically nontrivial solitonic solutions (separated in the different topological sectors) with the corresponding conserved topological number *A*
- Nucleon is quantized state of the classical soliton-skyrmion

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|S = T, s, t\rangle = (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

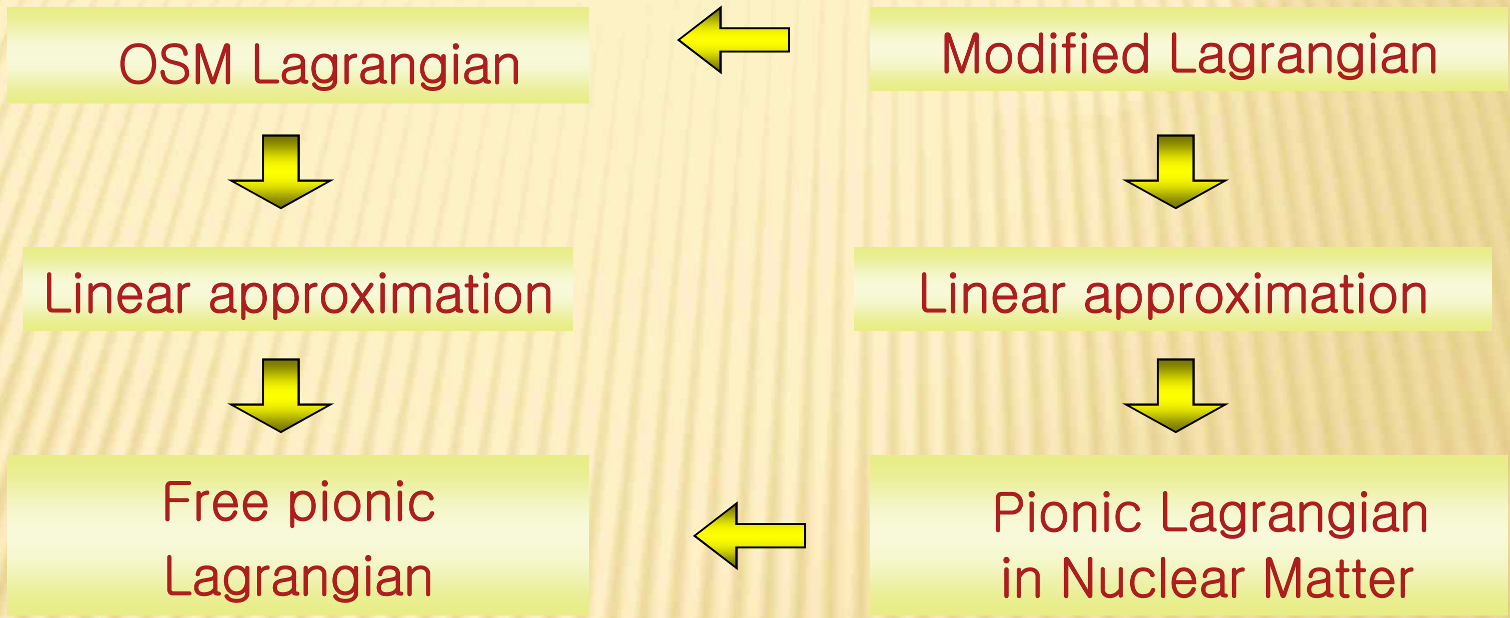
MEDIUM MODIFICATIONS

What happens in a nuclear medium?

- Medium effect
- One should be able to describe the possible phenomena
  - Deformations
  - Mass change
  - Swelling or shrinking
  - Change of NN interactions
  - Etc.

MEDIUM MODIFICATIONS

- Modification of the mesonic sector modifies the baryonic sector

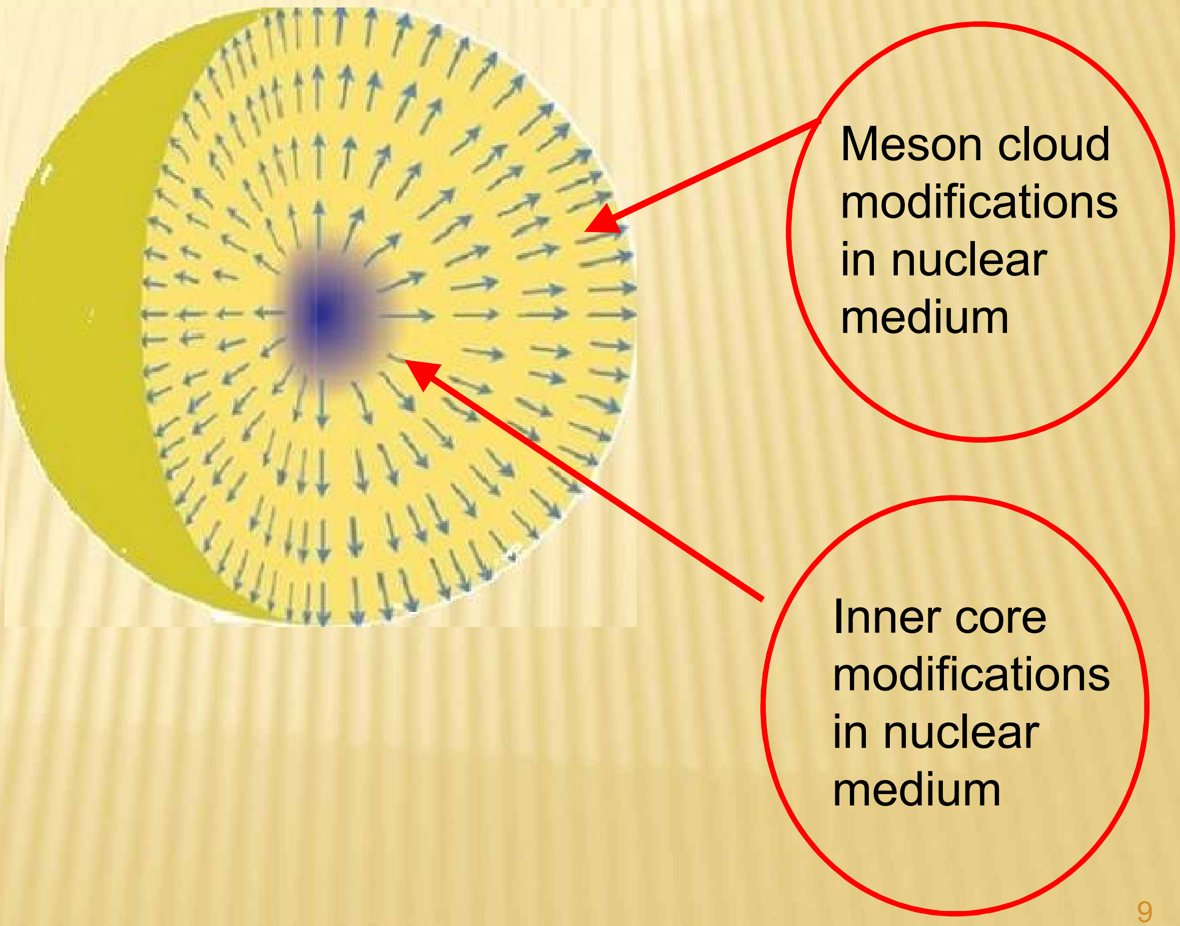


- Question arises: How to modify the mesonic sector?

MEDIUM MODIFICATIONS

Soliton in Nuclear Medium (structure changes)

- Outer shell modifications
- Inner core modifications (in particular, at higher densities)



MEDIUM MODIFICATIONS

"Outer shell" modifications

- Three types of pions can be treated separately
- In nuclear matter, one considers three types of polarization operators
- There will be some parameters which correspond to the isospin breaking effects in the surrounding environment

$$\left(\partial^{\mu}\partial_{\mu} + m^2_{\pi^{(\pm,0)}}\right)\vec{\pi}^{(\pm,0)} = 0$$

$$\left(\partial^{\mu}\partial_{\mu} + m^2_{\pi^{(\pm,0)}} + \hat{\Pi}^{(\pm,0)}\right)\vec{\pi}^{(\pm,0)} = 0$$

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

MEDIUM MODIFICATIONS

"Outer shell" modifications [U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = -\frac{F_\pi^2}{16}\left\{\alpha_s^{02}\mathrm{Tr}\left(\partial_0 U \partial_0 U^+\right) - \alpha_p^0 \mathrm{Tr}\left(\partial_i U \partial_i U^+\right)\right\}$$

$$\mathcal{L}_{\chi SB}^* = \frac{F_\pi^2 m_\pi^2}{8}\alpha_s^{00}\mathrm{Tr}\left(U-1\right)$$

$$\hat{\Pi} = 2\omega U_{opt} = \chi_s + \vec{\nabla}\cdot\chi_p\vec{\nabla}$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters following parts of the kinetic term is modified in different form:
  - Temporal part
  - Space part

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
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$c_0\ [m_\pi^{-3}]$	0.23	0.25
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MEDIUM MODIFICATIONS

"Inner core" modifications

[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

- May be related to
  - vector meson properties in nuclear matter
  - nuclear matter properties

$$\mathcal{L}_4^* = -\frac{1}{16e_\tau^{*2}}\mathrm{Tr}\left[L_0, L_i\right]^2 + \frac{1}{32e_s^{*2}}\mathrm{Tr}\left[L_i, L_j\right]^2$$

$$e \to e^* = e\,\gamma^{1/2}(\rho)$$

MEDIUM MODIFICATIONS

Final Lagrangian

[UY, JKPS62 (2013), UY, PRC88 (2013)]

- Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\text{sym}} + \mathcal{L}^*_{\text{asym}}$$

  - Isoscalar part

$$\mathcal{L}^*_{\text{sym}} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_{\chi\text{SB}}$$

  - Isovector part

$$\mathcal{L}^*_{\text{asym}} = \Delta\mathcal{L}^*_{\text{mes}} + \Delta\mathcal{L}^*_{\text{env}}$$

  - Nuclear matter stabilization
  - Asymmetric matter properties

$$\mathcal{L}^*_2 = -\frac{F_\pi^2}{16}\left\{\alpha_{\text{s}}^{02}\,\text{Tr}\left(\partial_0 U \partial_0 U^+\right) - \alpha_{\text{p}}^{0}\,\text{Tr}\left(\partial_{\text{i}} U \partial_{\text{i}} U^+\right)\right\}$$

$$\mathcal{L}^*_4 = -\frac{1}{16 e_\tau^{*2}}\text{Tr}\left[L_0, L_i\right]^2 + \frac{1}{32 e_s^{*2}}\text{Tr}\left[L_i, L_j\right]^2$$

$$\mathcal{L}^*_{\chi SB} = \frac{F_\pi^2 m_\pi^2}{8}\alpha_{\text{s}}^{00}\,\text{Tr}\left(U - 1\right)$$

$$\Delta\mathcal{L}^*_{\text{mes}} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}\left(m_{\pi_\pm}^2 - m_{\pi_0}^2\right)\text{Tr}\left(\tau_a U\right)\text{Tr}\left(\tau_a U^+\right)$$

$$\Delta\mathcal{L}^*_{\text{env}} = -\frac{F_\pi^2}{16} m_\pi \alpha_e \varepsilon_{\text{ab3}}\,\text{Tr}\left(\tau_a U\right)\text{Tr}\left(\tau_b \partial_0 U^+\right)$$

MEDIUM MODIFICATIONS

Reparametrization [UY, PRC88 (2013)]

- Five medium parameters

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad e_\tau \to e^*_\tau, \quad m_\pi \to m^*_\pi,$$
$$F_{\pi,s} \to F^*_{\pi,s}, \quad e_s \to e^*_s$$

Shell modifications   Core modifications

- Rearranging

$$1 + C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

MEDIUM MODIFICATIONS

Nucleon in nuclear matter

- Isoscalar mass

$$m^*_{N,s} = M^*_S + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2}\left(a^{*2} + \frac{\Lambda^{*2}_{\text{env}}}{\Lambda^{*2}}\right)$$

- Isovector mass

$$\Delta m^*_{np} = a^* + \frac{\Lambda^*_{\text{env}}}{\Lambda^*}$$

- Mass of the nucleon

$$m^*_{n,p} = m^*_{N,s} - \Delta m^*_{np} T_3$$

NUCLEAR MATTER

The binding-energy-formula terms in the present model

$$\varepsilon(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \dots$$

We are ready to reproduce

- Volume term
  - Infinite and asymmetric nuclear matter
- Asymmetry term
  - Isospin asymmetric environment
- Surface and Coulomb terms
  - Nucleons in a finite volume
- Finite nuclei properties
  - Local density approximation

NUCLEAR MATTER

Volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda,\delta)=\varepsilon_V(\lambda)+\varepsilon_S\delta^2+O(\delta^4)\equiv\varepsilon_V(\lambda)+\varepsilon_A(\lambda,\delta)$$

  - λ is normalized nuclear matter density
  - δ is asymmetry parameter
  - ε_S is symmetry energy

- In our model
  - Symmetric matter $\varepsilon_V(\lambda)=m^*_{N,s}(\lambda,0)-m_N^{\text{free}}$
  - Asymmetric matter

$$\varepsilon_A(\lambda,\delta)=\varepsilon(\lambda,\delta)-\varepsilon_V(\lambda)$$

$$=m^*_{N,s}(\lambda,\delta)-m^*_{N,s}(\lambda,0)+m^*_{N,V}(\lambda,\delta)\delta$$

NUCLEAR MATTER

Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left.\frac{\partial \varepsilon_V(\lambda)}{\partial \lambda}\right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left.\frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2}\right|_{\rho=\rho_0} \quad Q = 27\lambda^3 \left.\frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3}\right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \ldots$$

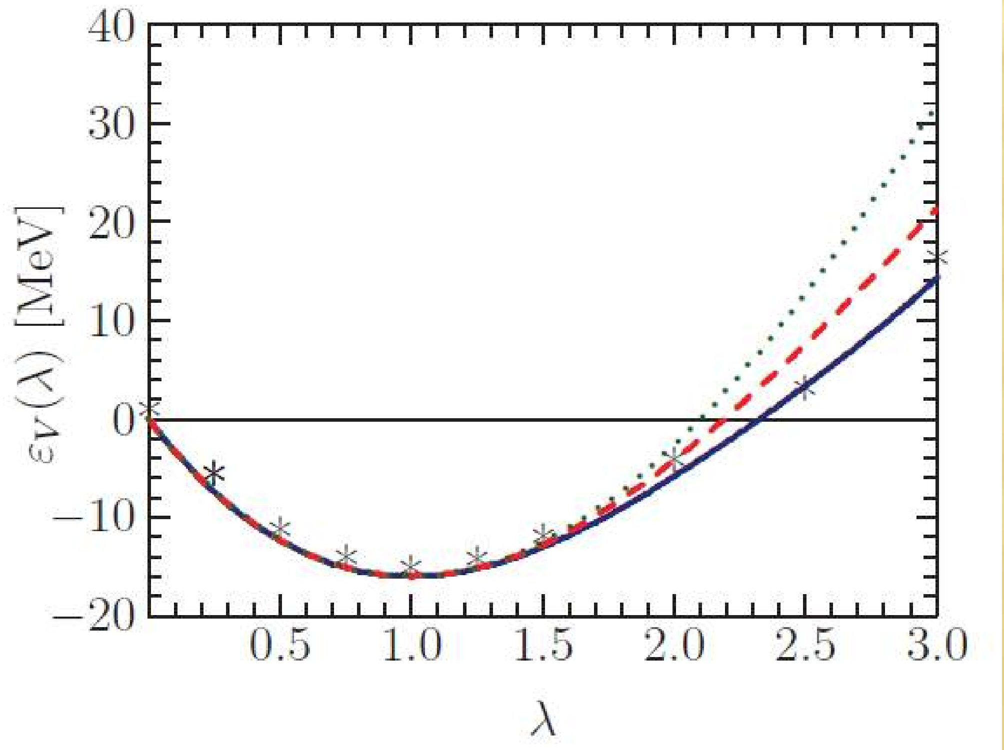
SYMMETRIC MATTER

Volume energy [UY, PRC88 (2013)]

- Set I – solid
- Set II – dashed
- Set III – dotted

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.

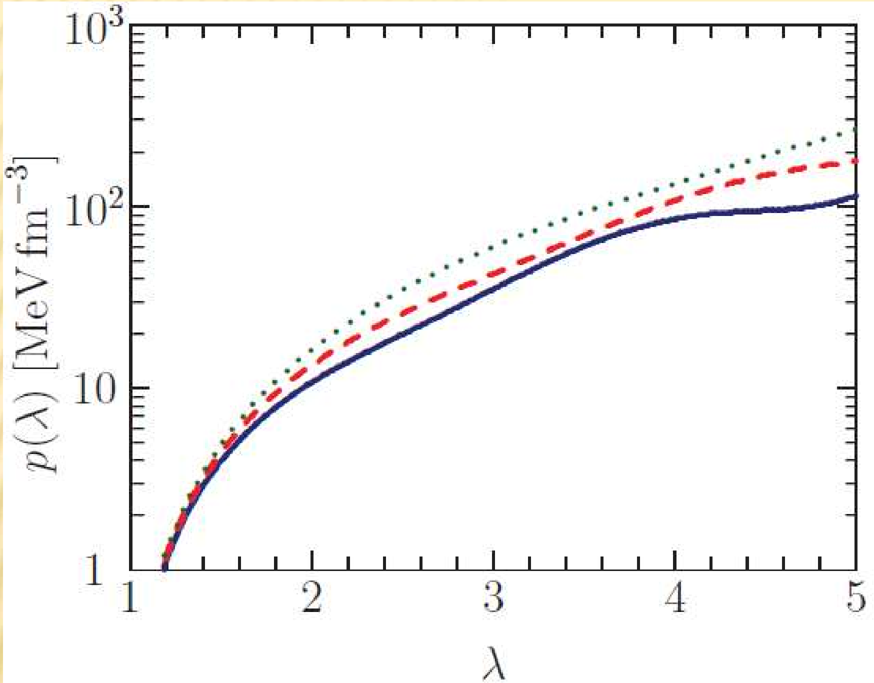
(From arigonna 2 body interactions + 3 body interactions)

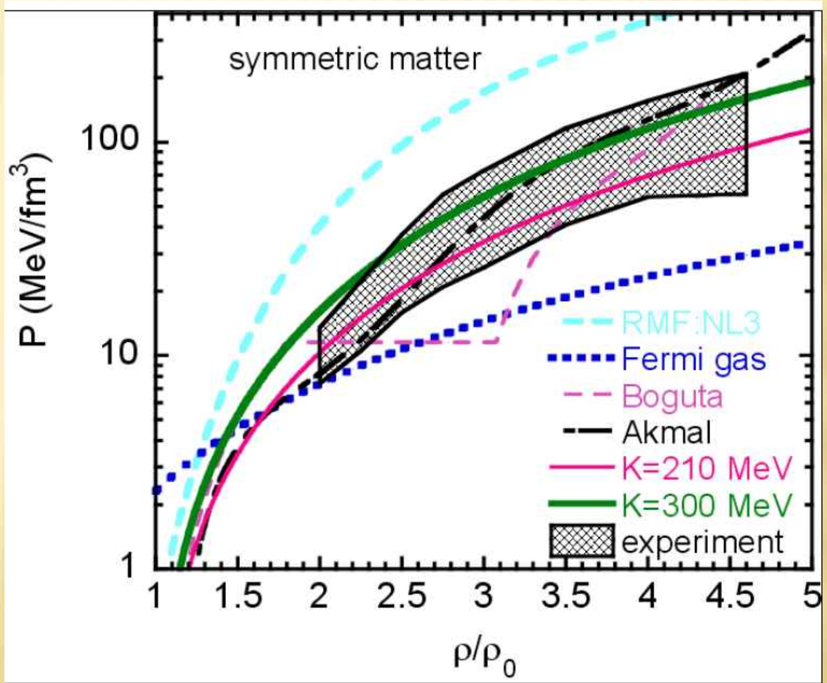


Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

SYMMETRIC MATTER

Pressure [UY, PRC88 (2013)]





For comparison: Right figure from

Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).

(Deduced from experimental flow data and simulations studies)

ASYMMETRIC MATTER

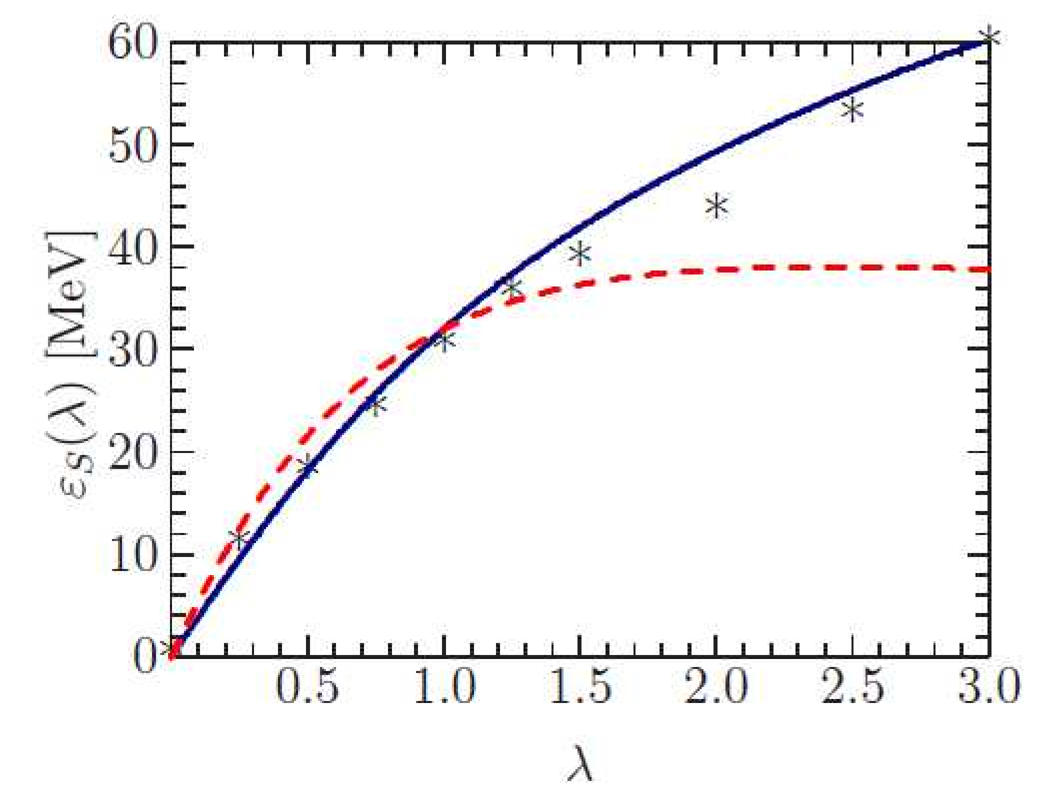
Symmetry energy

- Solid $L_s = 70\,\mathrm{MeV}$
- Dashed $L_s = 40\,\mathrm{MeV}$

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions

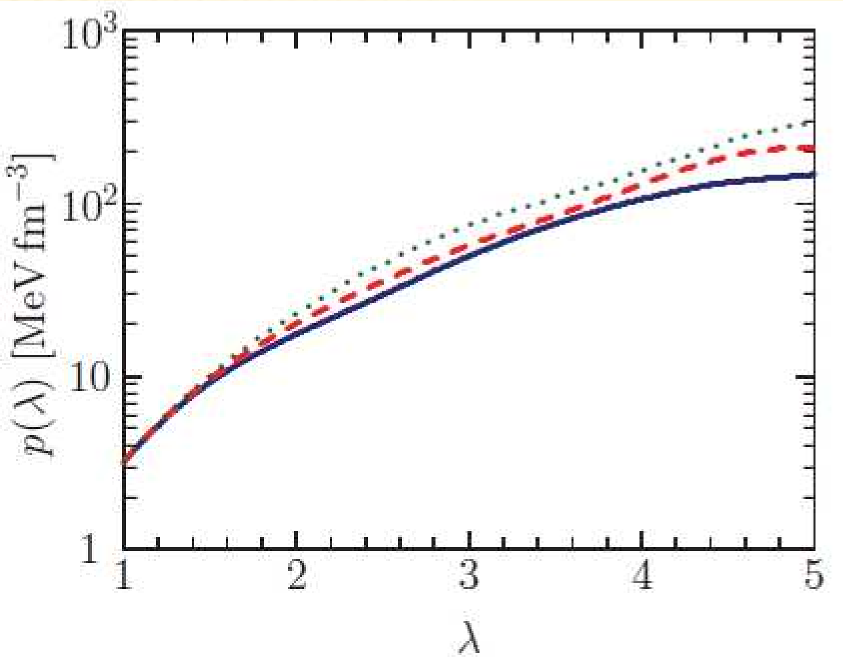
[PRC 58, 1804 (1998)] are given by stars.

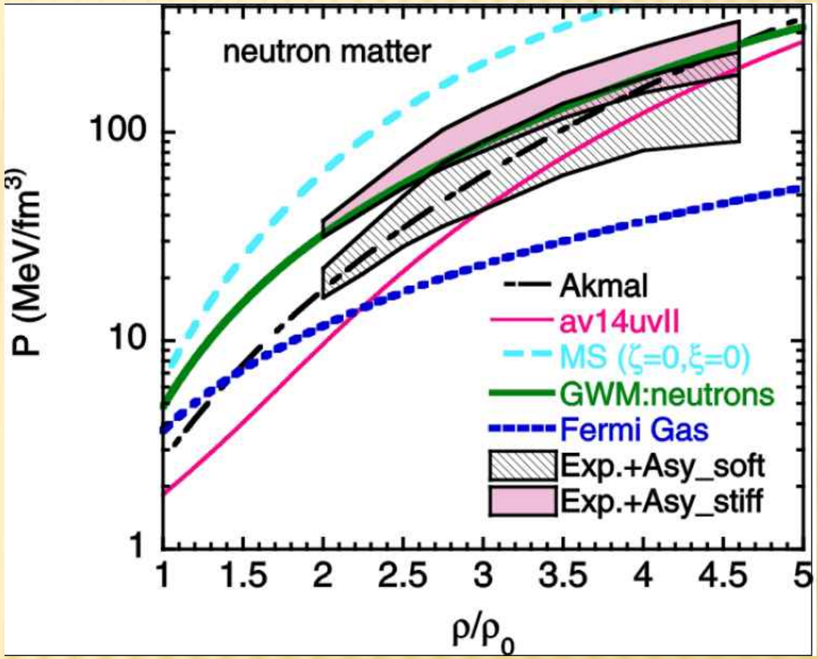
(From arigonna 2 body interactions + 3 body interactions)



ASYMMETRIC MATTER

Pressure in neutron matter [UY, PRC88 (2013)]





For comparison: Right figure from

Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).

(Deduced from experimental flow data and simulations studies)

ASYMMETRIC MATTER

Low density behavior of symmetry energy

For comparison:
Trippa-Colo-Vigezzi
[PRC 77, 061304 (2008)];
From analysis of GDR
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\text{fm}^{-3}) < 24.9 \text{ MeV}$$

$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\text{fm}^{-3})$ [MeV]
32	40	−181	−301	−257	25.15
32	50	−160	−310	−254	24.15
32	60	−126	−306	−239	23.22
32	70	−80	−290	−211	22.37
32	80	−21	−261	−172	21.57
32	90	50	−220	−119	20.82
32	100	134	−166	−55	20.13

Consequently one can predict in this model:

$$K_\tau = K_s - 6L_s$$

$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_S$$

NUCLEAR MATTER

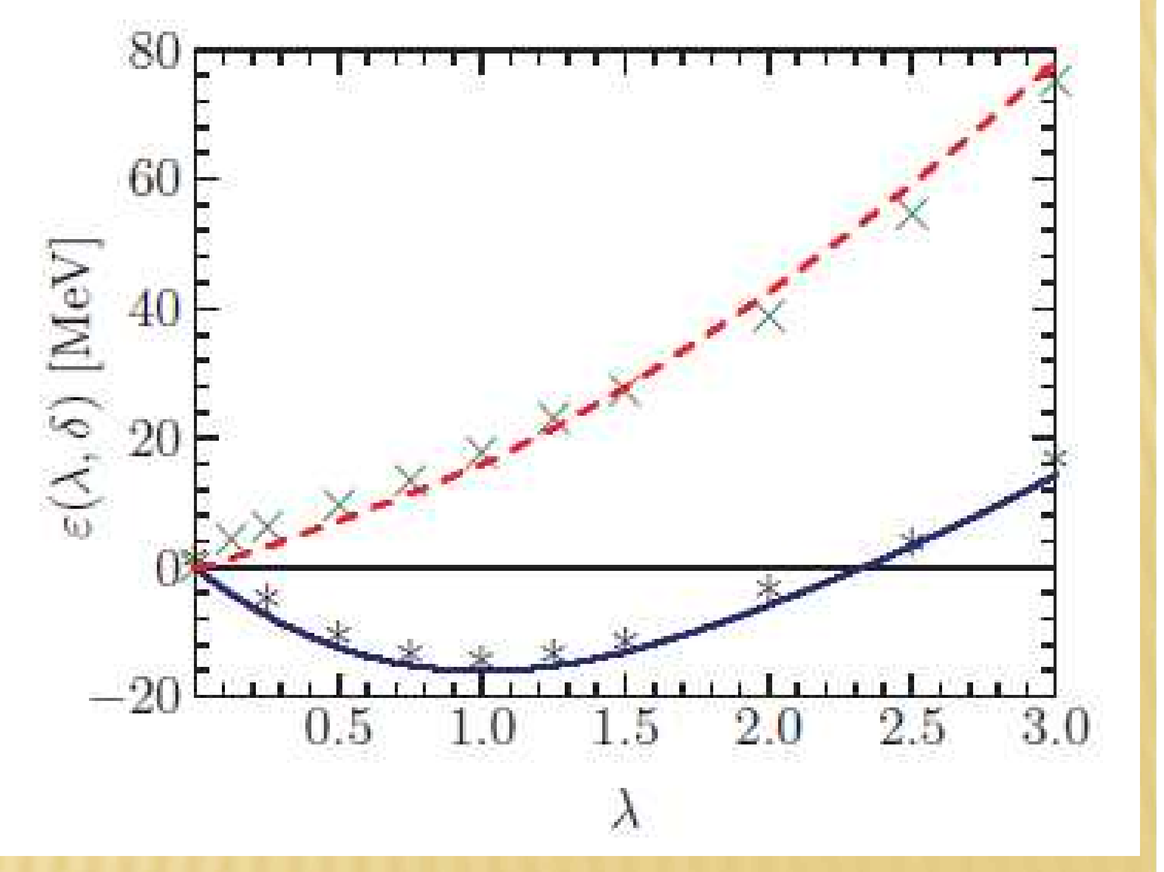
For completeness [UY, PRC88 (2013)]

- Symmetric matter – solid
- Neutron matter –dashed

For comparison:

APR predictions

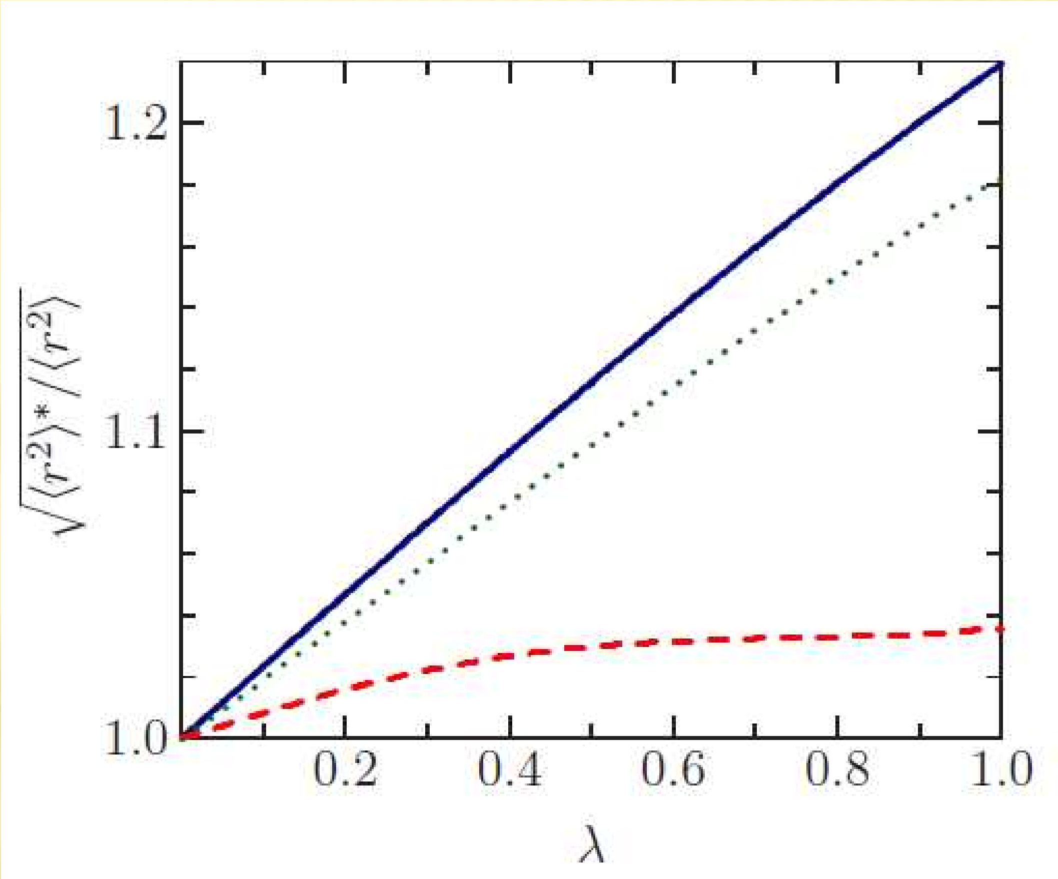
[PRC 58, 1804 (1998)] are given by stars.



HADRON PROPERTIES IN NUCLEAR MATTER

Mean square radii of nucleons

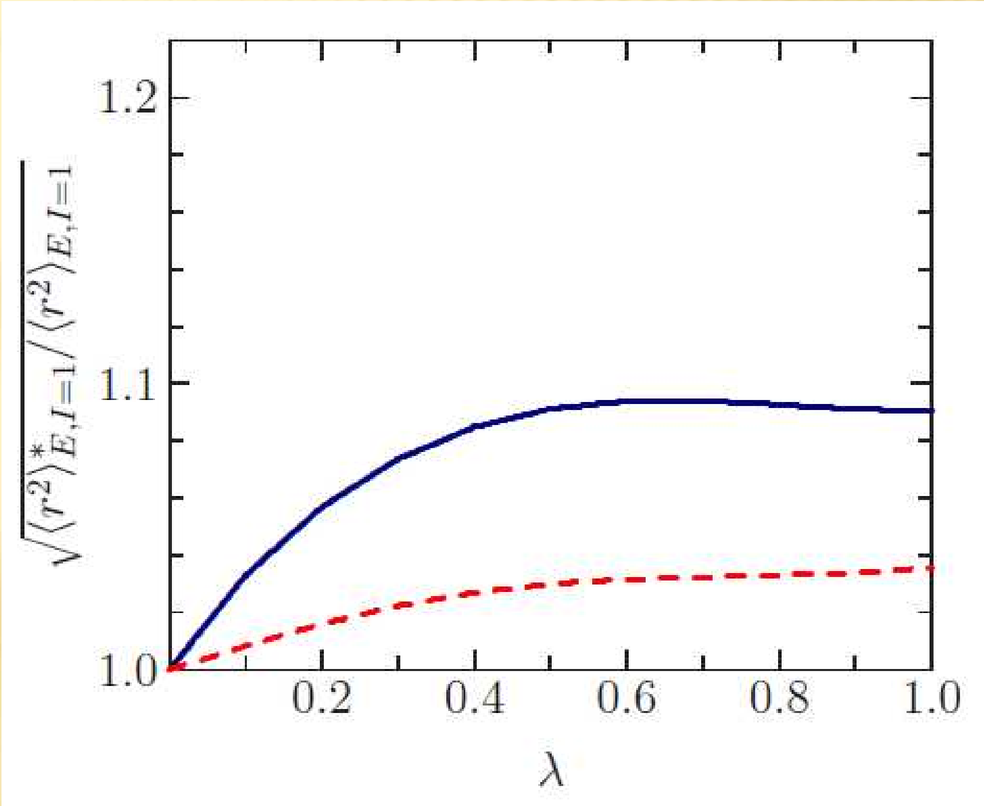
- Electric-isoscalar – solid curve
- Electric-isovector – dashed curve
- Magnetic-isoscalar – dashed curve
- Magnetic-isovector – dotted curve



HADRON PROPERTIES IN NUCLEAR MATTER

Mean square radii of nucleons

- Neutron matter – solid curve
- Symmetric matter – dashed curve



HADRON PROPERTIES IN NUCLEAR MATTER

Low energy constants in nuclear matter at ρ_0

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s}$$

	Present model	ChPT [1]	QCD sum rules [2]
$F^*_{\pi,t} / F_\pi$	0.37	0.74	0.79
$F^*_{\pi,s} / F_\pi$	0.72	< 0	0.78

[1] U. Meissner, J. Oller, A. Wirzba, Annals Phys. 297 (2002) 27.
[2] H. Kim, M. Oka, NPA720 (2003) 368.

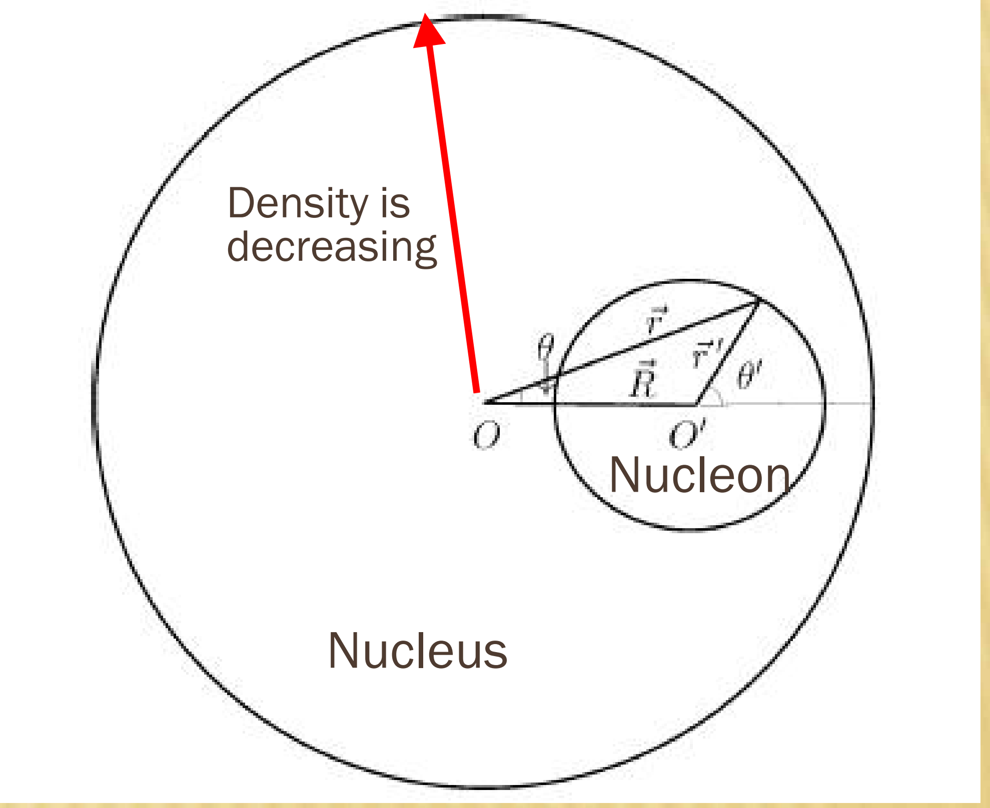
FINITE NUCLEI

Nucleon in a nucleus (local density approximation)

- The problem is coupled partial differential equations

$$f(F_{\bar{r}\bar{r}}, F_{\theta\theta}, F_{\bar{r}}, F_{\theta}, \Theta_{\theta}, F, \Theta) = 0,$$

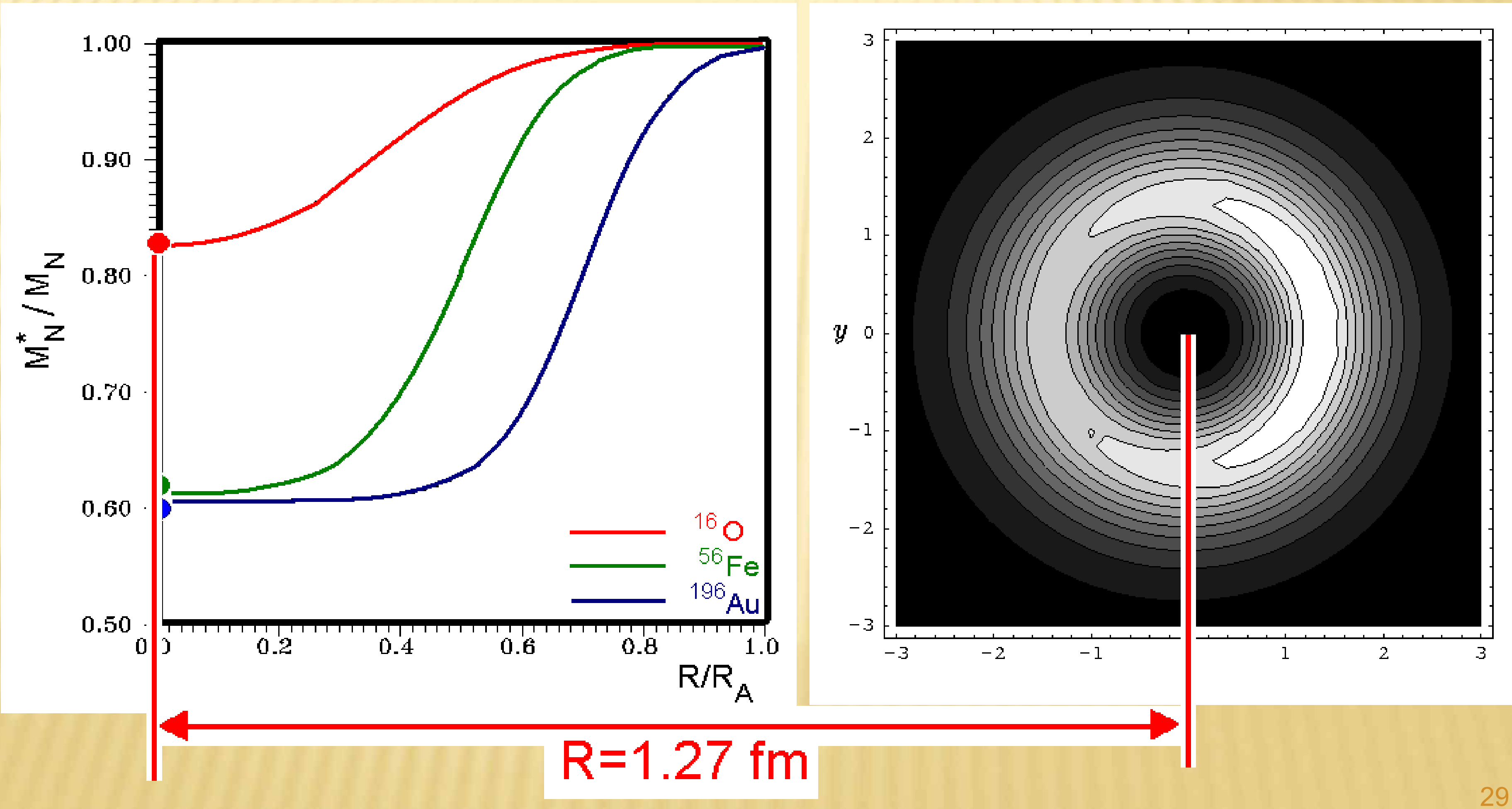
$$g(\Theta_{\theta\theta}, \Theta_{\theta}, F_{\bar{r}}, F_{\theta}, \Theta, F) = 0,$$



FINITE NUCLEI

Nucleon may deform [UY, *et al.*, NPA200 (2002) 403]

- The core modifications are not taken into account



SUMMARY

- Within the applicability range, the model describes

  - the single hadrons properties
    - in separate state
    - in the community of their partners
  - as well as the properties of that whole community at same footing

OUTLOOK

Extensions and applicability of the approach

- Nucleon tomography in nuclear matter
  [H.Ch. Kim, UY, PLB726 (2013), arXiv:1304.5926]
- NN interactions in nuclear matter
- Neutron stars
- Finite nuclei properties
  - Mirror nuclei
  - Exotic nuclei
  - Halo nuclei
- Nucleon-knock out reactions
- Vector mesons in nuclear matter
  [J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013), arXiv:1212.4616]

Thank you for your attention!